Banking regulation and collateral screening in a model of information asymmetry*

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Abstract

This paper explores the impact of banking regulation on a competitive credit market with ex-ante asymmetric information and aggregate uncertainty. I construct a model where the government imposes a regulatory constraint that limits the losses banks make in the event of their default. I show that the addition of banking regulation results in three deviations from the standard theory. First, collateral is demanded of both high and low risk firms, even in the absence of asymmetric information. Second, if banking regulation is sufficiently strict, there may not exist an adverse selection problem. Third, a pooling Nash equilibrium can exist.

Keywords: Banking; Adverse Selection; Collateral; Banking regulation.

JEL codes: D86; G21; G28

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1 Introduction

Following the Financial Crisis of 2007-2008, regulators and policy makers have increased their focus on ensuring stability in the banking sector. One key tool at the regulator's disposal is stress testing, which has become more widely used by regulators since the financial crisis. The empirical evidence suggests that the use of stress tests by the Federal Reserve and other banking regulators can have a negative impact on the lending conditions facing firms. For example, Acharya et al. (2018), focusing on lending to large firms in the US, find that stress tested banks tend to reduce the quantity of loans supplied to firms and tend to increase borrowing rates. Similarly, Cortés et al. (2018) complement this by documenting similar negative effects of stress testing on small business loans. Specifically, they provide evidence that stress tests conducted under the Comprehensive Capital Analysis and Review (CCAR) led to a decrease in affected banks' credit supply to small business. An overview of the recent history of stress testing in the financial sector can be found in Dent et al. (2016).

This paper seeks to contribute to the analysis of implementing more stringent banking regulation such as regulatory stress tests by offering a theoretical model that assesses the interaction between banking regulation and loan terms in a traditional model of loan contracts. I propose an adverse selection credit market model with aggregate uncertainty where firms have private information regarding the riskiness of their project. Firms operate a decreasing returns to scale production technology and fund their project by obtaining a bank loan. Banks have limited liability and are able to default on insured depositors. As banks do not internalize the social cost of default, they lend more than is socially optimal. This inefficiency can be corrected through banking regulation. I abstract from the implementation of banking regulation and assume that the government can impose a constraint on the level of systemic risk directly through a limit on the losses banks make conditional on their default. The regulatory constraint in my model can be interpreted as a condition that banks must be able to meet some minimum threshold following a regulatory stress-test.

The model allows banks flexibility in satisfying the regulatory requirements. Banks may reduce the size of the loans they offer, increase interest rates or reduce their loss given default through demanding more collateral from borrowers. Thus collateral now has two roles; as in traditional adverse selection models collateral can be used as a screening device but in addition it may also help the bank satisfy regulatory requirements by reducing the loss given default of a loan.

This paper highlights a novel channel through which banking regulation may distort equilibrium lending; the interaction of incomplete information and banking regulation. The paper has three main theoretical results which are contrary to standard adverse selection models.

First, I set out conditions under which collateral is demanded of both high and low risk firms, even in the absence of asymmetric information. Second, if banking regulation is sufficiently strict, there may not exist an adverse selection problem, as the difference in loan size at the full information contract is sufficient to separate the two firm types. Finally, I show that if there is insufficient pledgable collateral, the two firm types can receive the same contract in equilibrium, that is to say a pooling Nash equilibrium can exist.

The paper also sets out some empirical predictions of the model. First, the model predicts that increased regulation reduces loan size and that the fall in the size of loans is larger for higher-risk firms than lower-risk firms. This is consistent with the evidence in Acharya et al. (2018). Second, the model predicts that the collateral ratio of loans will increase following an increase in regulation. Finally, the model suggests that banking regulation may impact firms across different industries to varying decrees. Specifically, the model predicts that at high levels of regulation, collateral ratios will be larger for industries that feature higher returns to scale and less intangible capital.

This paper is directly related to the literature on adverse selection in credit markets. Papers that focus on the use of collateral as a screening device in credit markets featuring adverse selection include papers such as Stiglitz and Weiss (1981), Bester (1985a) and Lacker (2001). The addition of variable loan size to signalling models has also been studied previously by Bester (1985b) and Milde and Riley (1988). The existence of credit rationing equilibria, though not pooling equilibria, when there is insufficient collateral was raised by Besanko and Thakor (1987) and Clemenz (1993). The use of collateral in lending markets with information asymmetries has been extensively studied in the empirical literature. Both Lehmann and Neuberger (2001) and Jiménez et al. (2006) provide empirical evidence supporting the use of collateral as a screening device while Godlewski and Weill (2010) provides evidence that reconciles these papers with papers such as Berger and Udell (1990) that suggest high-risk borrowers pledge more collateral.

This paper also complements the empirical literature on the impact of regulatory stress testing on bank lending such as Acharya et al. (2018) and Cortés et al. (2018) by providing a theoretical mechanism through which more stringent regulation can impact lending outcomes. A related paper is Estrella (2004) who considers the impact of regulatory restrictions on a bank's value at risk on the probability of bank failure in a dynamic setting. His emphasis is on the portfolio choice of a bank choosing between safe and risky assets. In this paper, I emphasize the impact of banking regulation on the terms of loan contracts.

The paper is organized as follows. Section 2 presents the model. Section 3 derives the main results on the loan contracts in a competitive equilibrium. Section 4 discusses the optimal policy decision of the regulator, Section 5 discusses some possible extensions of the model and presents some empirical implications of the model and Section 6 concludes.

2 Model

2.1 Firms and Technology

Consider a credit market with a continuum of risk-neutral firms. Each firm has access to a project such that an investment of k will yield a cash-flow of φk^{α} if successful and zero if it fails. The curvature parameter $\alpha \in (0,1)$ is such that the cash-flow of a successful project features decreasing returns to scale and the productivity parameter $\varphi > 0$ is common to all firms. There exist two types of firms indexed by $i \in \{L, H\}$ that differ in the success probability of their projects. The probability a firm's project is successful is denoted by p_i with $0 < p_H < p_L < 1$ implying that H-type firms are high risk and feature a lower probability of success than low risk (L-type) firms. The fraction of firms of type i is denoted by $\mu_i \in (0,1)$ with $\sum_i \mu_i = 1$. The distribution of firms in the economy is public information.

Firms receive a known end-of-period endowment $W > 0$. The timing of the endowment means firms cannot use the endowment to invest in a project but instead must obtain a loan. Banks make loan offers to firms that consist of a loan size $k_i \geq 0$, an interest rate $R_i \geq 0$ and an amount of pledged collateral $C_i \in [0, W]$. The collateral is the amount of the firm's endowment sacrificed by the firm if it defaults on the loan payment $R_i k_i$.

In addition to the firm type, the probability of a project being successful also depends on the realization of an aggregate state $z \in \{z_B, z_G\}$. The aggregate state z_G occurs with probability $q \in (0,1)$ and z_B with probability $1 - q$. I denote the probability of firm i's project being successful conditional on z as $p_i(z)$. The probability of a project being successful is higher in the 'good' state (z_G) than in the 'bad' state (z_b) for both firm types such that

$$0 < p_i(z_B) < p_i(z_G) < 1 \quad \forall i \in \{L, H\}. \tag{1}$$

It follows from above that the expected probability of firm i's project being successful can be written as follows

$$p_i = q p_i(z_G) + (1-q) p_i(z_B). \tag{2}$$

To simplify the analysis, I assume that the ratio of success probabilities conditional on z_G

and z_B is the same across firm types such that

$$\frac{p_i(z_B)}{p_i(z_G)} = \xi \quad \forall i \in \{L, H\}, \tag{3}$$

where it follows from equation (1) that $\xi \in (0,1)$. It is assumed that the aggregate state is not known at the beginning of the period and thus loan contracts made between the bank and the firm cannot be made contingent on the realization of z.

The expected utility firm i receives from a loan contract (k_i, R_i, C_i) is

$$U_i(k_i, R_i, C_i) = p_i[\varphi k_i^{\alpha} - R_i k_i] - (1 - p_i) C_i + W. \tag{4}$$

To simplify the later analysis, I define the payoff the firm receives from a successful project as $\pi(k, R) = \varphi k_i^{\alpha} - R_i k_i$. The firm's marginal rate of substitution between the payoff from a successful project π and the collateral pledged is

$$\left.\frac{d\pi}{dC}\right|_{U_i} = \frac{1 - p_i}{p_i}. \tag{5}$$

As the marginal cost of collateral is lower for low-risk firms than high-risk firms, banks will be able to use collateral to screen between unobservable firm types.

2.2 Banking Sector and Regulation

There exist a large number of risk-neutral banks that fund loan contracts through deposits. Deposits are fully insured by the government and depositors earn a risk-free return which for simplicity is normalized to 1. Banks have limited liability and default on depositors if the proceeds from lending are less than what banks owe their depositors. If banks default, depositors are compensated by the government. The government does not charge banks an insurance premium but instead funds the deposit insurance by a lump-sum tax on households. The expected profit that a bank earns from a contract (k_i, R_i, C_i) that is accepted by type-i firms is given by the following equation

$$\begin{aligned} V_i(k_i, R_i, C_i) = {} & q\left(p_i(z_G) R_i k_i + \delta(1 - p_i(z_G)) C_i - k_i\right) \\ & + (1 - q)\max\left\{p_i(z_B) R_i k_i + \delta(1 - p_i(z_B)) C_i - k_i, 0\right\}, \end{aligned} \tag{6}$$

where $\delta \in (0,1)$ is a discount parameter on collateral implying that the use of collateral in a loan contract is costly. The parameter δ can be thought of as a reduced form way of capturing the agency and liquidation costs of transferring collateral to the banks. Competition in the banking sector will drive profits towards zero but I assume that banks do not default following the realization of the good aggregate state z_G. However, due to limited liability banks may default following the realization of z_B. I further assume that if a loan contract is accepted by at least one firm, it is accepted by a representative mass of firms, such that the law of large numbers holds. This assumption ensures that bank default would only occur due to aggregate risk and not due to the idiosyncratic firm risk.

If banks default following the realization of z_B, the government levies a lump-sum tax τ on households in order to make depositors whole again. Due to the presence of this deposit insurance banks do not fully endogenize the cost of default. In order to address the resulting externality I assume that the government can impose the following restriction on the riskiness of bank borrowing

$$p_i\left(z_B\right)R_ik_i+\delta\left(1-p_i\left(z_B\right)\right)C_i\geq\gamma k_i, \tag{7}$$

where $\gamma\in(0,1)$ is a parameter chosen by the government that determines how strict the regulatory regime is. The regulatory constraint set out by equation (7) is equivalent to stating that the bank only defaults on a fraction $(1-\gamma)$ of deposits if the bad aggregate state z_B is realized. One interpretation of this constraint is that it represents the requirement that banks pass a regulatory stress-test, with the parameter γ capturing how strict this stress-test is. Recent empirical studies such as Acharya et al. (2018) and Cortés et al. (2018) find that banks that fail stress tests adjust their lending in response. Furthermore, if the bank is publicly traded on a stock market, fully disclosing stress-test results as in the US is likely to create a strong incentive for the management of the bank to ensure the regulatory stress test is passed.

I focus on subgame perfect Nash equilibria of the following three-stage variant of the Rothschild and Stiglitz (1976) screening game. In the first stage, the government chooses the regulatory parameter γ. In the second stage, banks offer a single loan contract to firms. In the third stage, firms choose a single loan contract among those on offer. In this paper I discuss the existence of both separating and pooling Nash equilibria. A Nash equilibrium is a set of contracts $\{(k_i,R_i,C_i)\}_{i\in\{L,H\}}$ such that i) each contract earns non-negative profits for the bank, ii) the regulatory constraint defined by equation (7) is satisfied and iii) there exists no other set of contracts which, when offered in addition to the existing set of contracts, all earn non-negative profits with at least one offering strictly positive profits. I consider both separating and pooling equilibria. An equilibrium is separating if $(k_L,R_L,C_L)\neq(k_H,R_H,C_H)$ and is pooling otherwise.

3 Competitive Equilibrium

3.1 Equilibrium with identical customers

I first examine the case where there is a single type of firm and as a result banks have perfect information regarding the quality of the firms they are lending to. For an economy that consists of a single firm of type i, a competitive equilibrium must feature a contract that satisfies the regulatory constraint and ensures that banks make non-negative profits. A preliminary step in describing the equilibrium is to characterize the set of contracts that satisfy these two constraints in (π, C)-space. I will refer to this set as the *feasible set.*

First, I consider the case where the regulatory constraint is slack. In this case, the only constraint on the set of feasible contracts offered is that banks must make non-zero profits. Then the maximum payoff π that can be promised to firm i for a given pledge in collateral C that satisfies the following constraint binds

$$p_i\left(z_G\right) R_i k_i + \delta\left(1 - p_i\left(z_G\right)\right) C_i - k_i \geq 0, \tag{8}$$

which is simply the constraint that banks make non-negative profits if z_G is realized given that they default if z_B is realized. If banks did not default following z_B then that would imply that they would make non-negative profits following the realization of z_B, but then a competing bank could offer a lower interest rate R_i and thus a higher π such that equation (8) is satisfied while defaulting on any losses they make in z_B. By maximizing equation (4) subject to equation (8) the payoff maximizing loan size can be found as

$$\bar{k}_i = \left(\alpha\varphi p_i\left(z_G\right)\right)^{\frac{1}{1-\alpha}}. \tag{9}$$

Assuming equation (8) binds this can be substituted into equation (7) to yield the following inequality

$$C_i \geq \frac{1}{\delta}\left(\frac{\gamma - \xi}{1 - \xi}\right) k_i. \tag{10}$$

An immediate corollary of equation (10) is that the regulatory constraint will be satisfied for all $C_i \geq 0$ whenever $\gamma \leq \xi$. This establishes a lower-bound for γ below which the regulatory constraint will have no effect on the equilibrium. Furthermore, even if regulation is sufficiently high such that $\gamma > \xi$, if the collateral specified in the contract is sufficiently high, then the regulatory constraint will not bind. Specifically, there exists a cutoff level of collateral $\bar{C}_i$ such that for any $C_i > \bar{C}_i$ there is sufficient collateral to ensure that the regulatory constraint will be slack when banks offer a loan size of $\bar{k}_i$. This cutoff is defined by the following equation

$$\bar{C}_i = \frac{1}{\delta}\left(\frac{\gamma-\xi}{1-\xi}\right)\bar{k}_i. \tag{11}$$

Next, I consider the possibility that the regulatory constraint binds but where banks make positive profit and equation (8) is slack. By maximizing equation (4) subject to equation(7) the payoff maximizing loan size can be found as

$$\underline{k}_i = \left(\alpha\varphi p_i\left(z_B\right)\right)^{\frac{1}{1-\alpha}}. \tag{12}$$

It follows from the above discussion that if $\gamma > \xi$ and collateral is sufficiently low, banks offer a loan size equal to $\underline{k}_i$ and make strictly positive profits to ensure that the regulatory constraint binds. Specifically, there exists a cutoff level of collateral $\underline{C}_i$ such that for any $C_i < \underline{C}_i$ there is insufficient collateral available for competition to drive bank profits to zero while ensuring that the regulatory constraint will be satisfied. This cutoff is defined by the following equation

$$\underline{C}_i = \frac{1}{\delta}\left(\frac{\gamma-\xi}{1-\xi}\right)\underline{k}_i. \tag{13}$$

To understand this somewhat puzzling case, note that there are three dimensions along which contracts can be adjusted in order to meet the regulatory requirement; the loan size, the quantity of collateral and the interest rate. When the quantity of collateral is sufficiently low, firms would accept a higher interest rate in exchange for a larger loan size and thus banks are able to make positive profits.

By assumption $p_i\left(z_B\right) < p_i\left(z_G\right)$, thus it follows that $\underline{k}_i < \bar{k}_i$ and $\underline{C}_i < \bar{C}_i$. Thus when $\gamma > \xi$ there exists a range of collateral $C \in \left(\underline{C}_i, \bar{C}_i\right)$, where in order to maximize the payoff to firms, both equations (7) and (8) bind. The loan size can be found by rearranging the two binding constraints to get $k_i = \delta\left(\frac{1-\xi}{\gamma-\xi}\right)C_i$ such that this loan size is increasing in collateral and lies between the upper- and lower-bounds for the loan size $\underline{k}_i$ and $\bar{k}_i$.

The boundary of the feasible set of contracts on the interior of $\mathbb{R}^2_+$ can be summarized by the function $\Pi_i\left(C\right)$ that denotes the maximum payoff that can be offered to firms as a function of collateral . This function is characterized as follows

$$\Pi_i\left(C\right) = \begin{cases} \varphi\underline{k}_i^{\alpha} - \gamma\left(\frac{1}{p_i(z_B)}\right)\underline{k}_i + \delta\left(\frac{1-p_i(z_B)}{p_i(z_B)}\right)C & \text{when } C < \underline{C}_i \text{ and } \gamma > \xi \\ \varphi\left[\delta\left(\frac{1-\xi}{\gamma-\xi}\right)C\right]^{\alpha} - \delta\left[\frac{1}{p_i(z_G)}\left(\frac{1-\gamma}{\gamma-\xi}\right)+1\right]C & \text{when } \underline{C}_i \leq C \leq \bar{C}_i \text{ and } \gamma > \xi \\ \varphi\bar{k}_i^{\alpha} - \frac{1}{p_i(z_G)}\bar{k}_i + \delta\left(\frac{1-p_i(z_G)}{p_i(z_G)}\right)C & \text{otherwise.} \end{cases} \tag{14}$$

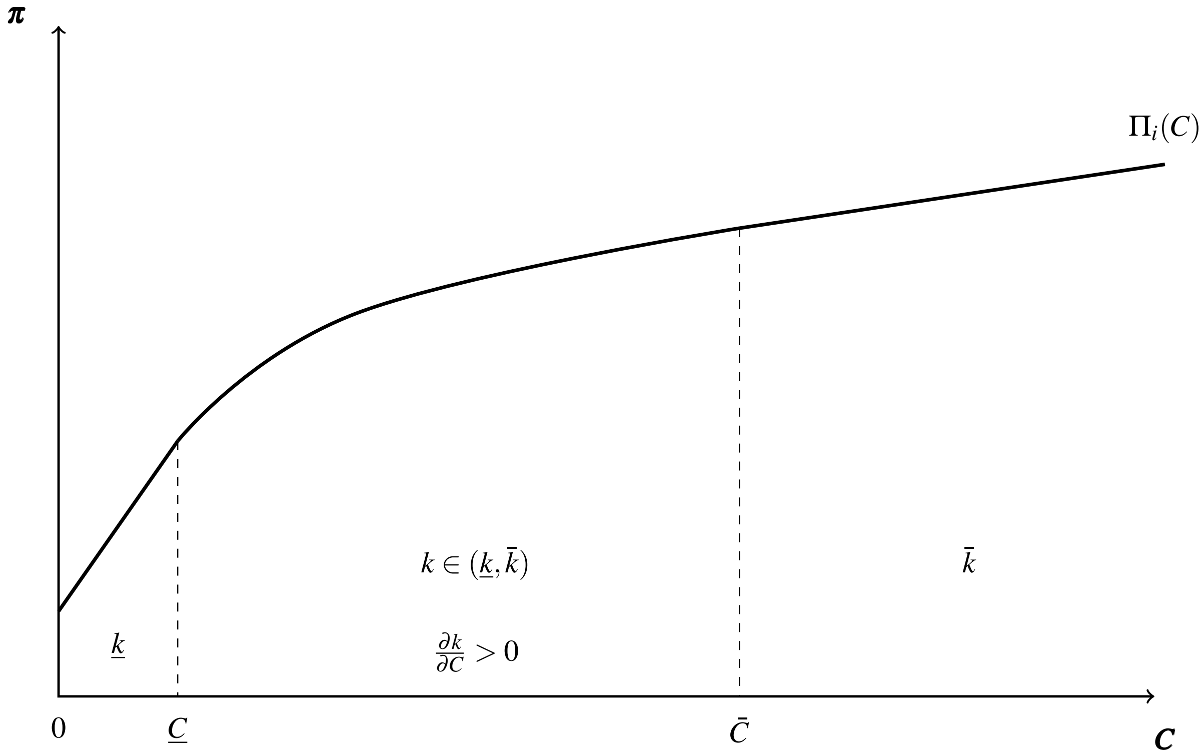

Figure 1: Set of feasible contracts for firm i

The set of feasible contracts is illustrated graphically for the case where $\gamma > \xi$ in figure 1.

Equation (14) denotes the largest possible payoff π that can be offered to firm i conditional on a collateral level C. As firm utility is strictly increasing in π, the competitive equilibrium is simply the point on equation (14) that maximizes firm utility.

In the case where $\gamma \leq \xi$, the regulatory constraint is always slack and the competitive equilibrium with identical customers is analogous to that in the Rothschild and Stiglitz (1976) case. The function $\Pi_i(C)$ is linear in C and with a gradient of $\delta\left(\frac{1-p_i(z_G)}{p_i(z_G)}\right)$, which is strictly lower than the gradient of firm i's indifference curves set out in equation (5) and thus the equilibrium contract will feature $C_i = 0$.

When $\gamma > \xi$, equation (14) is weakly concave in C and the gradient at $C_i = 0$ is $\delta\left(\frac{1-p_i(z_B)}{p_i(z_B)}\right)$. A sufficient condition for the competitive equilibrium to feature $C_i > 0$ is that $\delta\left(\frac{1-p_i(z_B)}{p_i(z_B)}\right)$ is strictly larger than the marginal rate of substitution of firm i, which holds whenever

$$\delta > \left(\frac{p_i(z_B)}{1-p_i(z_B)}\right)\left(\frac{1-p_i}{p_i}\right). \tag{15}$$

Thus if banks value collateral sufficiently highly, the equilibrium will feature positive collateral; the inequality set out in equation (15) will be satisfied as $\delta \to 1$. If equation 15 is

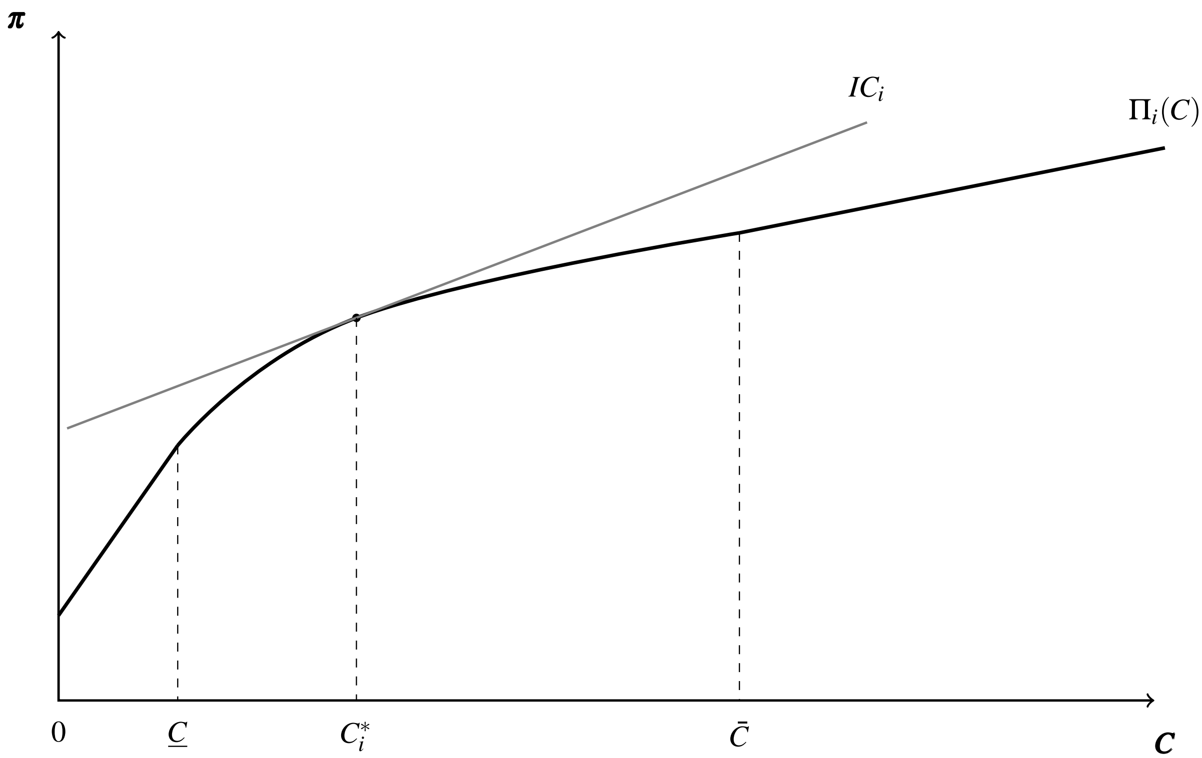

Figure 2: Competitive equilibrium with single firm type

satisfied, the competitive equilibrium will be the point of tangency between equation (14) and firm i's indifference curves. This is illustrated graphically in figure 2 where C_i^* denotes the collateral level at the competitive equilibrium.

This result is set out more formally in the following proposition.

Proposition 1. *If $\gamma > \xi$, $\delta > \left(\frac{p_i(z_B)}{1-p_i(z_B)}\right)\left(\frac{1-p_i}{p_i}\right)$ and W is sufficiently high, the competitive equilibrium contract for an economy featuring a single type of firm will feature strictly positive collateral $C_i^* > 0$ and a loan size $k_i^* < \bar{k}_i$ where C_i^* and k_i^* are given by the following equations*

$$k_i^* = \left(\frac{\alpha p_i \varphi}{(q + (1-q)\gamma) + \left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)^{\frac{1}{1-\alpha}}, \tag{16}$$

$$C_i^* = \frac{1}{\delta}\left(\frac{\gamma-\xi}{1-\xi}\right)k_i^*, \tag{17}$$

and the utility that the firm receives from the competitive equilibrium contract is

$$U_i^* = \left(\frac{1-\alpha}{\alpha}\right)\left[(q+(1-q)\gamma) + \left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)\right]k_i^*. \tag{18}$$

Proof. See Appendix. $\square$

Proposition 1 states that if the regulatory constraint is sufficiently strict and the cost of pledging collateral is sufficiently low, the competitive equilibrium will feature positive collateral even in the case of a single firm type. The reason for this is that pledging collateral, while costly, increases the loan size that firms are able to receive.

In order to focus on the novel aspects of this model, from now on I assume first that $\gamma > \xi$ such that the regulatory constraint is a relevant consideration for agents and second that equation (15) is satisfied so that firms are willing to use collateral to ensure that their credit contract satisfies any regulatory constraint.

An increase in γ tightens the regulatory constraint and as can be seen through differentiation of equation (16) decreases the loan size k_i^*. Thus the government is able to affect the size of firm loans through the regulatory constraint. An increase in γ increases the collateral ratio $c_i^* \equiv C_i^*/k_i^*$ as evidenced by equation (17) while firm utility U_i^* falls following an increase in γ. However, the change in the level of C_i^* is less clear cut as the increase in the collateral ratio may be offset by a fall in the loan size. Which of these effects dominates depends on the level of γ; at low levels of γ, the increase in the collateral ratio dominates and the level of collateral increases while this may not be the case at higher levels of γ. A more detailed analysis is provided in the Appendix.

3.2 Separating equilibrium with asymmetric information

I now return to the case of asymmetric information where there are two firm types $i \in \{H, L\}$ and a firm's type is known only to itself. Banks are unable to condition contracts on the firm type. Instead, in a separating equilibrium, there are two distinct contracts $\{(k_i, R_i, C_i)\}_{i\in\{L,H\}}$ chosen by firms in equilibrium, with each contract being chosen by a single firm type. In order for this to occur, firms must self-select into the contract intended for them and thus the following two incentive compatibility constraints must hold in equilibrium

$$p_H \left[\varphi k_H^\alpha - R_H k_H\right] - (1 - p_H)\, C_H \geq p_H \left[\varphi k_L^\alpha - R_L k_L\right] - (1 - p_H)\, C_L, \tag{19}$$

and

$$p_L \left[\varphi k_L^\alpha - R_L k_L\right] - (1 - p_L)\, C_L \geq p_L \left[\varphi k_H^\alpha - R_H k_H\right] - (1 - p_L)\, C_H. \tag{20}$$

To see which of these constraints is likely to bind, first note that if there is sufficient collateral the separating contract will lie on the boundary of the feasible set. Furthermore, for any loan that is not fully collateralized, $\Pi_L(C) > \Pi_H(C)$ and thus at a given C the separating payoff available to low-risk firms will be higher than that of high-risk firms. This can be shown by noting that if the amount of collateral is held fixed, low-risk firms can receive the same loan size as high-risk firms at a lower interest rate without violating either the zero profit condition or the regulatory constraint. Thus Π_H must lie on the interior of the feasible contract set for L-type firms. It follows immediately from this that the relevant incentive compatibility constraint to consider is equation (19). Low-risk firms will never prefer a contract intended for high-risk firms and equation (20) will always be satisfied in equilibrium.

In contrast to a more standard adverse selection model, equation (19) may not always bind. To see this, consider the following equation which can be found by substituting in the full information contracts $\{(k_i^*, R_i^*, C_i^*)\}_{i=\in\{L,H\}}$ into equation (19) and rearranging:

$$\Lambda_{IC}(\gamma) = \left(\frac{p_H}{p_L}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{(q+(1-q)\gamma) + \left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)}{(q+(1-q)\gamma) + \left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)^{\frac{\alpha}{1-\alpha}} + \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\frac{1}{\delta}\left(\frac{p_L}{p_H}-1\right)\left(\frac{\gamma-\xi}{1-\xi}\right)}{(q+(1-q)\gamma) + \left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right) - 1. \tag{21}$$

When $\Lambda_{IC}(\gamma) \geq 0$, the incentive compatibility constraint is slack and the set of contracts $\{(k_i^*, R_i^*, C_i^*)\}_{i=\in\{L,H\}}$ constitute the Nash equilibrium with imperfect information. On the other hand, if $\Lambda_{IC}(\gamma) < 0$, the incentive compatibility constraint will bind and low-risk firms must pledge a higher quantity of collateral in order to separate from high-risk firms.

Through inspection of equation (21) it is clear that the limit of $\Lambda_{IC}(\gamma)$ as $\gamma \to \xi$ is negative. Thus absent any regulation the incentive compatibility constraint would always bind. In this case, the full information contracts do not require any collateral to be pledged and as $\Pi_L(0) > \Pi_H(0)$ it follows that in the asymmetric information case equation (19) will bind.

Similarly, the limit of $\Lambda_{IC}(\gamma)$ as $\gamma \to 1$ is strictly positive, thus if the regulatory constraint is sufficiently strict, the incentive compatibility constraint no longer binds.

To understand why note that at the limit $\gamma \to 1$, loans are fully collateralized and hence both firm types are charged the risk free rate by the banks. As a result firms are able to fully distinguish their type solely through their choice of loan size and there is no incentive for high-risk firms to deviate as they will be charged the same interest rate as low-risk firms. While this means that sufficiently strict banking regulation may resolve the adverse selection problem, it may not be socially optimal to do this as the use of collateral is costly.

The Appendix proves that the function $\Lambda_{IC}(\gamma)$ is strictly increasing in γ and thus there exists a cutoff $\gamma^* \in (\xi, 1)$ such that the incentive compatibility constraint does not bind for regulation stricter than this value. The proposition below summarizes this result more formally.

Proposition 2. *If* $\gamma \geq \xi$, $\delta \geq \left(\frac{p_H(z_B)}{1-p_H(z_B)}\right)\left(\frac{1-p_H}{p_H}\right)$ *and* $W \geq C_L^*$, *there exists a unique cutoff* $\gamma^* \in (\xi, 1)$ *such that for any* $\gamma > \gamma^*$ *the incentive compatibility constraints will not bind and the contracts* $\{(k_i^*, R_i^*, C_i^*)\}_{i=\in\{L,H\}}$ *constitute the Nash equilibrium.*

Proof. See Appendix. □

Should equation (19) bind the separating contracts can then be illustrated graphically. In the separating equilibrium high-risk firms receive the same contract as they would in a single-type equilibrium. The separating contract for low-risk firms can then be found as the point on the frontier $\Pi_L(C)$ at which high-risk firms are indifferent between this contract and their separating contract (k_H^*, R_H^*, C_H^*). I denote the separating contract offered to low-risk firms by $\left(\hat{k}_L, \hat{R}_L, \hat{C}_L\right)$. As this contract must lie on the boundary of the feasible set, it follows from rearranging equation (19) that C_H^* and $\hat{C}_L$ have the following relationship

$$\Pi_L\left(\hat{C}_L\right) = \Pi_H\left(C_H^*\right) + \left(\frac{1-p_H}{p_H}\right)\left(\hat{C}_L - C_H^*\right). \tag{22}$$

An example of a separating contract is illustrated in figure 3.

The precise contract terms for low-risk firms in the case where equation (19) binds as their formulation depends on whether $\hat{C}_L$ is larger than $\bar{C}_L$ or not. However, from the properties of the boundary of the feasible sets Π_i described earlier it follows that the separating contract features a strictly larger loan size and larger quantity of collateral relative to the full information contract, that is $\hat{k}_L > k_L^*$ and $\hat{C}_L > C_L^*$. By rearranging equation (19) the quantity of collateral can be expressed as

$$\hat{C}_L = \frac{1}{\delta}\left(\frac{\frac{p_H}{p_L}\left(p_L\varphi\hat{k}_L^{\alpha} - (q + (1-q)\,\xi)\,\hat{k}_L\right) - U_H^*}{1 - \frac{p_H}{p_L}(q + (1-q)\,\xi) + \left(\frac{1-\delta}{\delta}\right)(1-p_H)}\right). \tag{23}$$

Through inspecting equation (23) it becomes clear that $\hat{C}_L$ is strictly increasing in γ so long as $\hat{C}_L \geq \bar{C}_L$. This occurs because at this point $\hat{k}_L = \bar{k}_L$ and the loan size offered to low risk firms is insensitive to changes in regulation. As discussed above the utility of the high-risk firms decreases as regulation becomes more strict and thus the collateral required for separation increases.

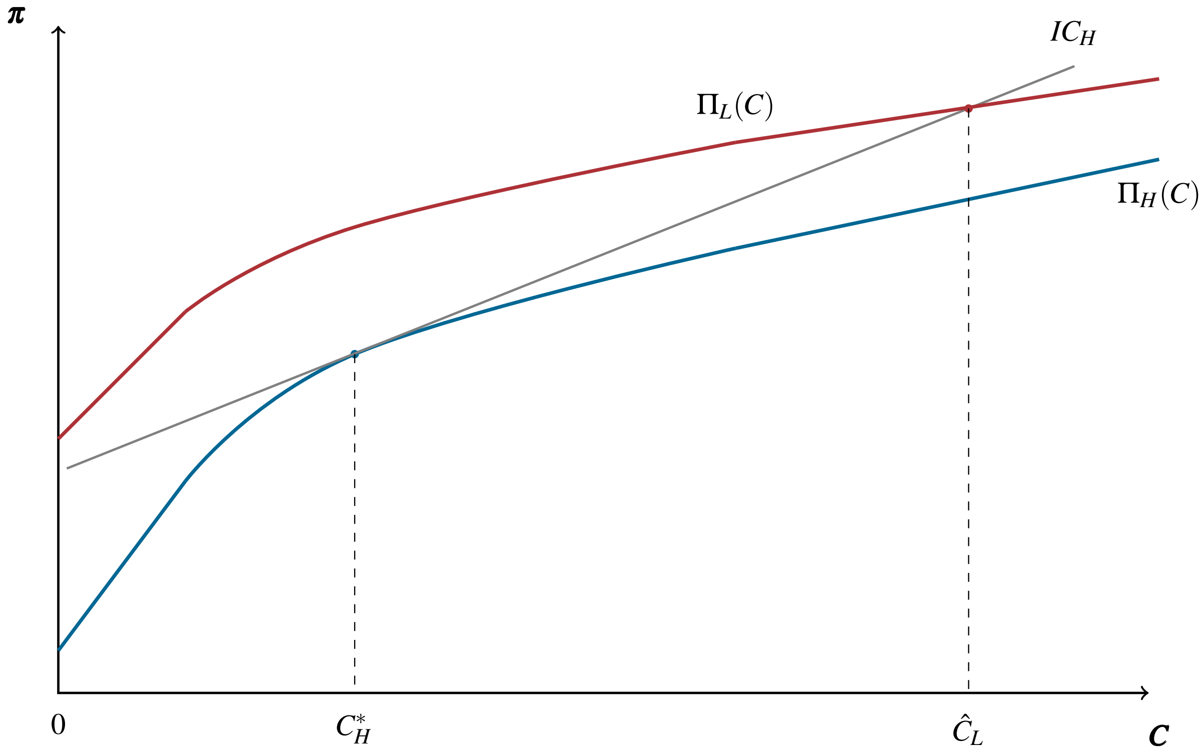

Figure 3: Asymmetric Information Contracts

In cases where regulation is sufficiently high $\gamma > \bar{\gamma}$ and $\hat{C}_L < \bar{C}_L$, the separating collateral increases at a lower rate and for sufficiently large values of γ may even start decreasing in γ. The reason for this is that when $\gamma > \bar{\gamma}$ the loan size $\hat{k}_L$ falls in response to an increase in regulation. Thus while the collateral-to-loan ratio remains increasing in γ, the absolute value of collateral may fall. More detailed discussion of the response of $\hat{C}_L$ to increases in regulation is provided in the section analyzing the optimal regulatory policy and in the Appendix.

To understand better the relationship between $\hat{C}_L$ and the point at which $\hat{k}_L$ starts to respond to tighter regulation, first consider the point where $\gamma = \xi$. At this point the regulatory constraint does not bind while positive collateral is still required in order to separate between high-risk and low risk-firms and thus $\hat{C}_L > \bar{C}_L = 0$. Next, note that when $\gamma = \gamma^*$, the regulatory constraint does bind while the incentive compatibility is slack and thus $\bar{C}_L > \hat{C}_L = C_L^*$. The following proposition establishes more formally that there exists a cutoff level of regulation $\bar{\gamma}$ above which $\hat{k}_L$ begins to respond to stricter regulation.

Proposition 3. *There exists a unique cutoff $\bar{\gamma} \in (\xi, \gamma^*)$ such that for any $\gamma > \bar{\gamma}$, the collateral required for separation $\hat{C}_L$ will be strictly lower than the cutoff $\bar{C}_L$ and thus $\hat{k}_L < \bar{k}_L$ implying that the loan size of the separating contract will be lower than in the case without regulation.*

Proof. See Appendix. □

An important implication of the above is that while the amount of collateral pledged by low-risk firms, $\hat{C}_L$, may not respond monotonically to changes in regulation there exists a unique separating contract for a given value of γ.

As pointed out by Rothschild and Stiglitz (1976) and Wilson (1977), a Nash equilibrium is not guaranteed to exist in an economy that features asymmetric information. This occurs when a pooling contract Pareto dominates separating contracts and thus would be preferred to the separating contract by both types of firms. Due to the curvature of the production function, the precise conditions required for the existence of a separating equilibrium cannot be found in closed form. However, even in the case where a Nash equilibrium does not exist, the separating equilibrium discussed in this section will exist as a Riley reactive equilibrium as set out in Riley (1979). Similarly, a pooling equilibrium would exist as a Wilson anticipatory equilibrium as in Wilson (1977).

3.3 Equilibrium when the wealth constraint binds

The analysis of the previous section assumed that available collateral W was sufficient so that the required collateral for separation could be supplied. I now discuss the equilibrium contracts under asymmetric information in the case where W is sufficiently low that the separating contract discussed earlier cannot be implemented.

First, consider the case where W is sufficiently large that $C_H^* < W$ but not so large that the low-risk firms can provide the level of collateral required for screening and thus $\hat{C}_L > W$. Then incentive compatibility requires that the k_L and R_L offered to the low-risk firm are such that the payoff low-risk firms receive is

$$\tilde{\pi}_L = \Pi_H \left(C_H^*\right) + \left(\frac{1-p_H}{p_H}\right)\left(C_H^* - W\right). \tag{24}$$

In a separating equilibrium when the wealth constraint binds, the low-risk firm's contract lies off the boundary of the feasible set of contracts and thus $\tilde{\pi}_L < \Pi_L(W)$. The firm is indifferent between any pair of contract terms (k_L, R_L) which yields the payoff $\tilde{\pi}_L$ specified above and banks will choose the combination of k_L and R_L that maximizes their profit subject to the regulatory constraint and supplying the firm a payoff of $\tilde{\pi}_L$. This possibility is illustrated in figure 4.

When the upper-bound on collateral binds there is the possibility that a pooling Nash Equilibrium exists. In a pooling equilibrium both high- and low-risk firms accept the same contract (k_P, R_P, C_P). The pooling contract can be found in an analogous way to the equilibrium

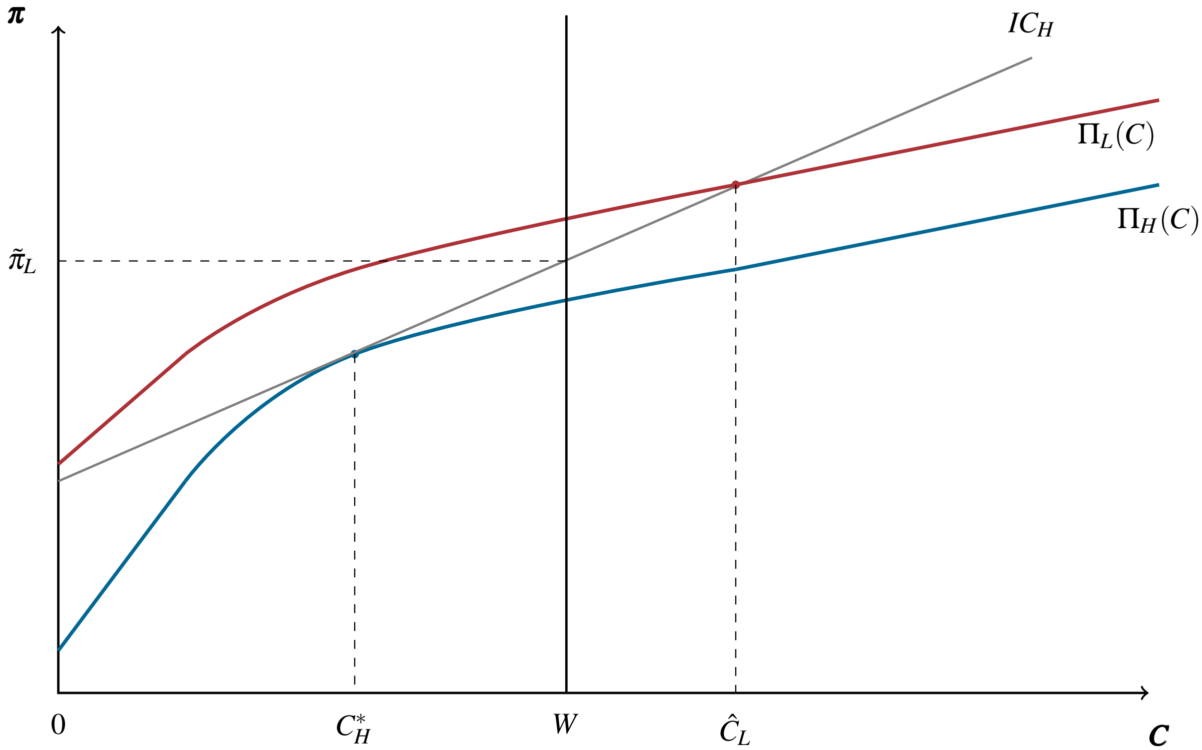

Figure 4: Separating Contract with binding wealth constraint

contract with a single firm type where the single firm type is composed of both high-risk and low-risk firms. The expected probability of success for this composition conditional on aggregate state z is simply a weighted average of the success probabilities for high- and low-risk firms, weighted by the proportion of that firm type in the economy

$$p_P(z_j) = \mu_H p_H(z_j) + \mu_L p_L(z_j) \quad \forall j \in \{G, B\}.$$

Similarly, the unconditional expected probability of success is also a weighted average of the success probabilities of the high- and low-risk firms

$$p_P = \mu_H p_H + \mu_L p_L.$$

It follows that the boundary of the set of feasible pooling contracts, $\Pi_P(C)$, is simply $\Pi_i(C)$ for the composite type P.

For a pooling contract (k_P, R_P, C_P) to exist as a Nash Equilibrium, there must exist no deviating contract that would satisfy equations (7) and (8) which would result in the pooling contract becoming either unprofitable or violate the regulatory constraint. That is, there cannot exist a cream-skimming contract that will attract only low-risk firms.

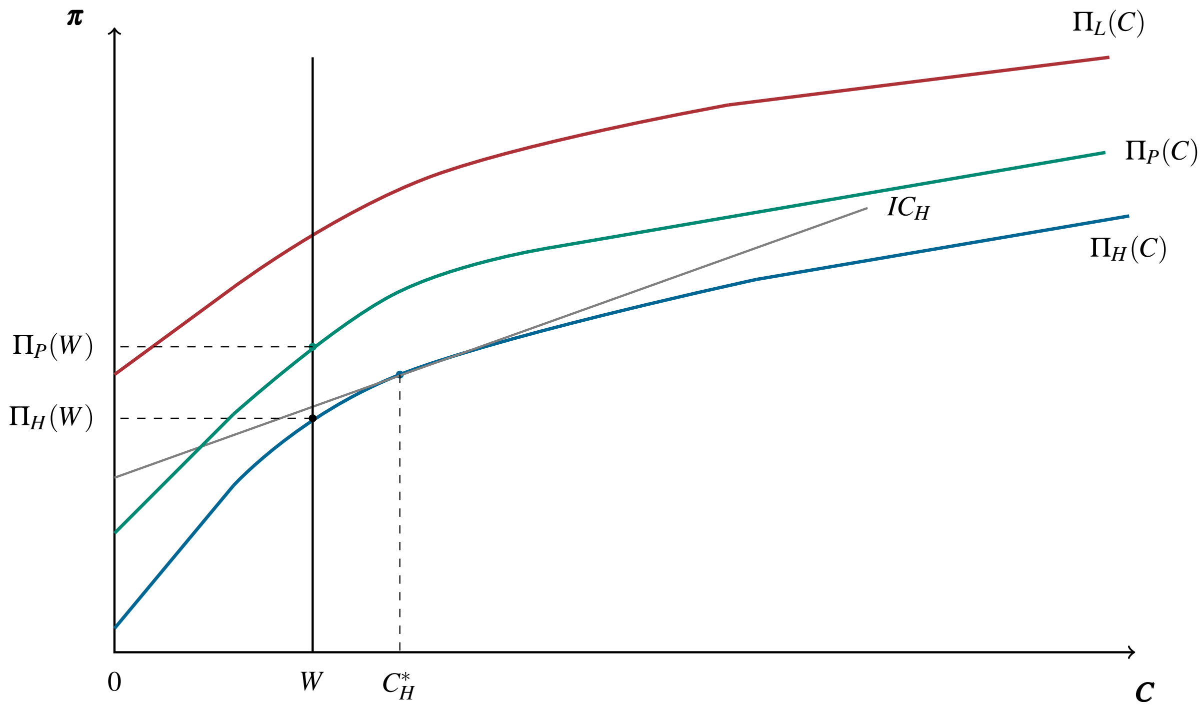

Figure 5: Pooling Contract with binding wealth constraint

A necessary condition for the existence of a pooling contract is for both high- and low-risk firms to prefer the pooling contract to the best separating contract available, otherwise firms would choose a separating contract over the pooling contract. Similarly, any pooling contract must lie on the boundary Π_P of pooling contracts, otherwise a better pooling contract could be found that would be preferred by at least one firm type.

A further necessary condition is that the upper-bound on collateral binds at the pooling contract such that $C_P = W$. In this case, no contract with higher collateral can be offered to low-risk firms and thus there exists no cream-skimming contract. A sufficient condition for the existence of a pooling equilibrium is that $C_H^* \leq W$ such that the separating contract for high-risk firms features a (weakly) binding wealth constraint. Then the pooling contract will lie strictly above the high-risk separating contract and no cream-skimming deviation exists. An example of a pooling contract existing as a Nash equilibrium is illustrated in figure 5. The precise terms of the pooling contract, as in the separating case, depend on where the boundary of feasible contracts intersects $C_P = W$. A sufficient condition for the existence of a pooling contract is $W \leq C_H^*$. This is summarized in the proposition below.

Proposition 4. *If $\gamma > \xi$, $\delta > \left(\frac{p_H(z_B)}{1-p_H(z_B)}\right)\left(\frac{1-p_H}{p_H}\right)$ and $W \leq C_H^*$ then a Nash equilibrium will consist of a single pooling contract (k_P, R_P, C_P) offered to both firms and where $C_P = W$ and $\pi = \Pi_P(W)$.*

Proof. It follows from Proposition 1, that if $\gamma > \xi$ and $\delta > \left(\frac{p_H(z_B)}{1-p_H(z_B)}\right)\left(\frac{1-p_H}{p_H}\right)$ that the high-risk firm's collateral in equilibrium will ideally be$C_H^* > 0$ and is defined by equation (17). Furthermore, it follows immediately from equation (14) that $\Pi_P(C) > \Pi_H(C)\ \forall C \leq C_H^*$. Thus if $W \leq C_H^*$ then $\Pi_P(W) > \Pi_H(W)$ and high-risk firms prefer a pooling contract that lies on the frontier $\Pi_P(W)$ over the best possible separating contract. It follows from the relative slope of the firm indifference curves that the only possible separating contract must offer low-risk firms both a higher payoff and higher collateral, but this would violate the restriction that $C_L \leq W$. □

The possibility of pooling equilibria existing as a Nash equilibrium arises in my model due to the interaction between the regulatory constraint and the adverse selection problem. In the classic Rothschild and Stiglitz (1976) screening game, a cream-skimming contract will always exist so long as $W > 0$. This is because no collateral is pledged by the pooling contract and thus there will always exist a deviating contract that features higher collateral and a higher payoff that would allow low-risk firms to separate from high-risk firms.[1] The introduction of the regulatory constraint means that if $\eta > \xi$ a pooling contract would pledge a positive amount of collateral and thus $W > 0$ is no longer a sufficient condition to ensure that a cream-skimming deviation exists.

4 Optimal Policy

4.1 Overview

Until this point, the regulatory parameter, γ, was taken as given. In this section, I consider the optimal policy decision of the government that can affect the economy only through the regulatory constraint described in equation (7).

For the sake of both brevity and simplicity, I assume that i) $\delta > \left(\frac{p_i(z_B)}{1-p_i(z_B)}\right)\left(\frac{1-p_i}{p_i}\right)\ \forall i$ so that both firms will pledge collateral; and ii) parameters are such that a Nash equilibrium always exists. I begin by focusing on the case where there is sufficient wealth that any contract can be implemented and thus a separating equilibrium exists. After this I analyze the case where the wealth constraint may bind and a pooling equilibrium may exist.

As discussed earlier if $\gamma \leq \xi$ the regulatory constraint will no longer bind and the model will

[1] A pooling contract with positive collateral would also not exist in a standard adverse selection model as collateral is costly and there would exist a profitable deviation from this contract to a pooling contract with zero collateral.

collapse to a standard adverse selection model. Thus, without loss of generality, I restrict the government's decision to choosing a parameter $\gamma \in [\xi, 1)$.

I now turn to the objective function of the government, which is assumed to be benevolent and maximizes the welfare of a risk-neutral household that owns both firms and banks. In order to provide deposit insurance, the bank levies a lump-sum tax τ on households. The objective function for the planner is thus

$$\mathcal{U} = \sum_i \mu_i \left(U_i \left(k_i, R_i, C_i\right) + V_i \left(k_i, R_i, C_i\right)\right) - (1-q)\,\tau, \tag{25}$$

where $U_i \left(k_i, R_i, C_i\right)$ is the firm expected profit set out in equation (4) and $V_i \left(k_i, R_i, C_i\right)$ is the expected profit of a bank contract set out in equation (6). The lump-sum τ is set to exactly cover the losses of the depositors in expectation and is set such that

$$\tau = \max \left\{ \sum_i \mu_i \left(k_i - p_i \left(z_B\right) R_i k_i - \delta \left(1 - p_i \left(z_B\right)\right) C_i\right), 0 \right\}. \tag{26}$$

Substituting the equation for τ into the government's objective function yields the following equation

$$\mathcal{U} = \sum_i \mu_i \left(p_i \varphi k_i^{\alpha} - (1-\delta)\left(1-p_i\right) C_i - k_i + W\right). \tag{27}$$

A useful benchmark to consider is the first best contract under full information, if the planner could choose directly the contracts provided to firms. Maximizing the above results in the following

$$k_i^{FB} = \left(\alpha p_i \varphi\right)^{\frac{1}{1-\alpha}} \text{ and } C_i^{FB} = 0.$$

Comparing this to the competitive equilibrium under full information as set out in Proposition 1, the collateral level will be higher than optimal whenever $\gamma > \xi$ while the loan size will be higher than optimal whenever $\gamma < \xi + \frac{(1-q)(1-\xi)}{(1-q)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1}{1-\xi}\right)}$. Thus when collateral is costly, even absent asymmetric information, the government is unable to implement the first best through adjusting the regulatory constraint.

The over-lending problem occurs because, due to the presence of deposit insurance, banks do not fully endogenize the cost of default. Instead, risk-shifting takes place and banks maximize profits only in states where they do not default. The government can reduce this over-lending problem by raising γ but does so at the cost of imposing higher collateral requirements on firms. As $\delta < 1$, collateral is assumed to be costly and thus the government faces a trade-off between reducing excessive lending and increasing the dead-weight loss from collateral usage.

4.2 Optimal Policy with a single firm type

To begin, I start by discussing the properties of the optimal policy decision in an economy with a single firm type. In this case, rewriting equation (27) yields the following optimization problem for the government

$$\mathcal{U}_i = \max_{\gamma} \left\{ \left(p_i \varphi \left(k_i^* \right)^{\alpha} - (1-\delta)(1-p_i) C_i^* - k_i^* + W \right) \right\}, \tag{28}$$

where k_i^* and C_i^* are defined in Proposition 2.

The partial derivative of equation (28) with respect to the policy parameter γ is

$$\frac{\partial \mathcal{U}_i}{\partial \gamma} = \left(p_i \varphi \alpha k_i^{\alpha-1} - 1 \right) \frac{dk_i^*}{d\gamma} - (1-\delta)(1-p_i) \frac{dC_i^*}{d\gamma}. \tag{29}$$

At the first best loan size, the marginal product of the project with respect to the loan size equals the interest rate and thus $p_i \varphi \alpha k_i^{\alpha-1} = 1$. As discussed earlier the derivative of k_i^* with respect to γ is negative and thus equation (29) states that at the optimal policy, the loan size will only be at the first best if at this point $\frac{dC_i^*}{d\gamma} = 0$. In the Appendix, I show that this will only occur if $(q + (1-q)\xi) = \alpha$. While it may seem surprising that it could be optimal for the government to set the loan size lower than first best when there is an over-borrowing problem, it follows from the fact that while the collateral to loan ratio is increasing in γ, the absolute quantity of collateral C_i^* may not be increasing in γ. Thus if, around the first best loan size an increase in γ will reduce the quantity of costly collateral used, then it will be optimal to further increase regulation at the expense of a lower loan size than in the first best.

By substituting out $\frac{dk_i^*}{d\gamma}$ and $\frac{dC_i^*}{d\gamma}$, the first order condition of the government's optimization problem can be rewritten as follows

$$\begin{aligned} \frac{\partial \mathcal{U}_i}{\partial \gamma} = {} & \left(\frac{1-q}{1-\alpha} \right) \left(\frac{(1-q)(1-\gamma) + \left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1-\gamma}{1-\xi}\right)}{(q + (1-q)\gamma) + \left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)} \right) k_i^* \\ & - \left(\frac{1-\delta}{\delta} \right)(1-p_i)\left(\frac{1}{1-\xi} \right) k_i^*. \end{aligned} \tag{30}$$

With a single firm type, the government maximizes welfare by choosing a value of γ that sets equation (30) equal to zero. Thus the optimal γ in the case with a single firm, denoted by γ_i^{OPT}, varies by the firm type.

To gain a greater understanding of the single firm optimum γ_i^{OPT}, first note that equation

(30) is strictly decreasing in γ. In addition, so long as $\delta < 1$ the point where $\gamma = 1$ is strictly negative. This implies that it is always welfare improving to set regulation lower than $\gamma = 1$ and thus loans will not be fully risk-free. If the optimal regulation γ_i^{OPT} is sufficiently low, the lower bound of $\gamma \geq \xi$ will bind and it is optimal for the government not to impose a binding regulatory constraint.

Next, through application of the implicit function theorem the Appendix shows that $\frac{d\gamma_i^{OPT}}{dp_i} > 0$ and thus $\gamma_H^{OPT} < \gamma_L^{OPT}$. The intuition for this is that the deadweight loss of collateral will be lower for low-risk firms as they are less likely to forfeit their collateral while the distortion from the optimal loan size will be relatively larger. Both of these factors contribute to it being optimal to impose higher regulation on low-risk firms.

It follows from the above that in the full information case with both firm types the optimal policy will lie somewhere between these two values of γ_i^{OPT} with the mass of high-risk firms, μ_H, determining how close the optimal will lie to γ_H^{OPT}.

Finally there is the possibility that in the two-firm case, if the optimal regulatory level denoted by γ^{OPT} is higher than γ^* as defined in Proposition 1, then the regulatory requirement will be sufficiently strict that the incentive compatibility constraint would not bind at this point. In this case the full information equilibrium coincides with the incomplete information equilibrium.

4.3 Optimal Policy with two firm types

I now return to the case of two firm types and where banks are unable to observe the firm type. The government then seeks to maximize equation (27) subject to the incentive compatibility constraint set out in equation (19) and the contract terms that firms receive in equilibrium. In a separating equilibrium the high-risk firm will receive (k_H^*, R_H^*, C_H^*) as described in Proposition 2. The low-risk firm receives a separating contract denoted by $\left(\hat{k}_L, \hat{R}_L, \hat{C}_L\right)$ where $\hat{R}_L$ and $\hat{C}_L$ can be found from equations (8) and (19) and $\hat{k}_L$ is given by the following equation

$$\hat{k}_L = \begin{cases} \delta\left(\frac{1-\xi}{\gamma-\xi}\right)\hat{C}_L & \text{if } \hat{C}_L < \bar{C}_L \\ \left(\alpha\varphi p_L\left(z_G\right)\right)^{\frac{1}{1-\alpha}} & \text{otherwise.} \end{cases} \tag{31}$$

The derivative of the government's optimization problem is

$$\frac{\partial \mathcal{U}}{\partial \gamma} = \mu_H \left(p_H \varphi \alpha \left(k_H^*\right)^{\alpha-1} - 1\right) \frac{dk_H^*}{d\gamma} - \mu_H \left(1-\delta\right)\left(1-p_H\right) \frac{dC_H^*}{d\gamma}$$
$$- \mu_L \left[\left(1 - p_L \varphi \alpha k_L^{\alpha-1}\right) \frac{d\hat{k}_L}{d\gamma} + \left(1-\delta\right)\left(1-p_L\right) \frac{d\hat{C}_L}{d\gamma}\right]. \qquad (32)$$

Substituting equation (30) into equation (32) yields the following equation

$$\frac{\partial \mathcal{U}}{\partial \gamma} = \mu_H \frac{\partial \mathcal{U}_H}{\partial \gamma} - \mu_L \left[\left(1 - p_L \varphi \alpha k_L^{\alpha-1}\right) \frac{d\hat{k}_L}{d\gamma} + \left(1-\delta\right)\left(1-p_L\right) \frac{d\hat{C}_L}{d\gamma}\right]. \qquad (33)$$

There are now two separate cases to consider, depending on whether $\hat{C}_L$ is less than $\bar{C}_L$ or not. This will yield two candidates for the optimal value of γ.

First consider the case where regulation is sufficiently low that $\gamma \leq \bar{\gamma}$ and thus $\hat{C}_L \geq \bar{C}_L$. In this case the size of the loan given to low-risk firms is not be affected by small changes in regulation and so $\frac{\partial \hat{k}_L}{\partial \gamma} = 0$. Furthermore, as shown in the Appendix any increase in γ results in a larger collateral requirement for low-risk firms as $\frac{d\hat{C}_L}{d\gamma} > 0$. From inspecting equation (33) it is clear that any solution to $\frac{\partial \mathcal{U}}{\partial \gamma} = 0$ would feature a value of γ that is strictly lower than γ_H^{OPT}, the optimal regulatory level if the economy consisted of only high-risk firms. Intuitively, this is because at low levels of regulation increasing γ results in a fall in the loan size only for high-risk firms while increasing the collateral that must be pledged by both firm types.

Next consider the case where regulation is sufficiently high that $\gamma > \bar{\gamma}$ and $\hat{C}_L < \bar{C}_L$. Now the size of the loan low-risk firms receive, $\hat{k}_L$ falls in response to an increase in regulation. In the Appendix it is shown that $\frac{d\hat{C}_L}{d\gamma}$ is strictly smaller than in the previous case where $\gamma \leq \bar{\gamma}$. As a result, the regulator has greater incentive to increase regulation. It is both less costly, with a smaller increase in collateral $\hat{C}_L$, and has greater benefits, with $\hat{k}_L$ falling and moving closer to the optimal level.

Which of the two candidates for the optimal value of γ described above is best cannot be determined analytically. However one key parameter determining if the optimum will lie above $\bar{\gamma}$ is μ_L. To understand why, note that as μ_L increases the weight that the regulator places on low-risk firms increases. As a result, it becomes more valuable for the regulator to increase regulation so that $\hat{k}_L$ falls closer to the optimum. Next, note that in cases where μ_L is relatively small, it will be optimal for the regulator to choose γ close to γ_H^{OPT}. Thus which of the two candidate solutions is optimal is likely to depend on the magnitude of

γ_H^{OPT}. If $\gamma_H^{OPT} < \bar{\gamma}$, then increasing the value of γ around this value will not affect $\hat{k}_L$ and the optimum will lie below $\bar{\gamma}$. On the other hand, if $\gamma_H^{OPT} > \bar{\gamma}$ then the regulator has an incentive to increase regulation in order to further reduce the loan size of the low-risk firms.

4.4 Optimal policy when wealth constraint binds

In the analysis of optimal policy I have up until this point assumed that there is sufficient collateral available to ensure that the optimal policy can be implemented. Now I consider the case where this may no longer be the case and the wealth constraint becomes a relevant consideration for the regulator. When the wealth constraint binds, it is possible for the government to implement a separating equilibrium by relaxing γ so that the wealth constraint no longer binds. In this case the binding wealth constraint forces the government to implement a strictly worse policy than would be the case if the wealth constraint did not bind. Another alternative for the government is to use the binding wealth constraint to implement a pooling equilibrium. In this subsection I discuss the properties of the welfare maximizing pooling equilibrium.

A pooling contract consists of one contract (k_P, R_P, C_P) that both firm types receive. As discussed earlier, for a pooling equilibrium to exist it must be the case that the wealth constraint binds and that the collateral offered by firms in the pooling contract is equal to the wealth available and that $C_P = W$. As a result, when focusing on the pooling contract the regulator's objective function set out in equation (27) can be rewritten as

$$\mathcal{U} = \sum_i \mu_i \left(p_i \varphi k_P^{\alpha} - (1-\delta)(1-p_i) C_P - k_P + W\right), \tag{34}$$

subject to $C_P = W$.

Equation (34) highlights that when the wealth constraint binds the regulator's choice of γ will serve to alter the loan size k_P as the collateral level is constrained to be equal to total wealth. Taking the first order condition of equation (34) with respect to k_P, the optimal pooling contract consists of setting γ such that

$$k_P^{FB} = (\alpha \varphi p_P)^{\frac{1}{1-\alpha}}. \tag{35}$$

As $k_P^{FB} < \bar{k}_P$ it follows that γ must be set sufficiently high that$C_P < \bar{C}_P$. Thus there are two candidates for an optimal pooling equilibrium; one where $C_P \in \left(\underline{C}_P, \bar{C}_P\right)$ and another where $C_P \leq \underline{C}_P$ in which the collateral cutoffs in the pooling case are defined as

$$\bar{C}_P = \frac{1}{\delta}\left(\frac{\gamma-\xi}{1-\xi}\right)\left(\frac{\alpha p_P \varphi}{q+(1-q)\,\xi}\right)^{\frac{1}{1-\alpha}}, \text{ and} \tag{36}$$

$$\underline{C}_P = \frac{1}{\delta}\left(\frac{\gamma-\xi}{1-\xi}\right)\left(\frac{\xi}{\gamma}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha p_P \varphi}{q+(1-q)\,\xi}\right)^{\frac{1}{1-\alpha}}. \tag{37}$$

I now consider the conditions necessary for these candidates to be an equilibrium.First consider the case where $C_P \leq \underline{C}_P$. In this case, the loan size is determined by the lower-bound $\underline{k}_P$ such that

$$k_P = \left(\frac{\xi}{\gamma\,(q+(1-q)\,\xi)}\right)^{\frac{1}{1-\alpha}}(\alpha p_P \varphi)^{\frac{1}{1-\alpha}}. \tag{38}$$

By combining equation (38) with equation (35) theγ such that these two equations both hold can be found

$$\gamma = \frac{\xi}{q+(1-q)\,\xi}. \tag{39}$$

For this to be an equilibrium it is require that $C_P \leq \underline{C}_P$

$$W \leq \frac{1}{\delta}\left(\frac{(1-q)\,\xi}{q+(1-q)\,\xi}\right)(\alpha p_P \varphi)^{\frac{1}{1-\alpha}}, \tag{40}$$

which imposes an upper-bound on wealth.

Next, consider the case, where $C_P \in \left(\underline{C}_P, \bar{C}_P\right)$. For this to hold, it must be the case that $C_P = W$ where,

$$C_P = \frac{1}{\delta}\left(\frac{\gamma-\xi}{1-\xi}\right)(\alpha\varphi p_P)^{\frac{1}{1-\alpha}}. \tag{41}$$

In order to implement k_P^{FB} a necessary condition is that the above equation is satisfied for some $\gamma < 1$ thus the following condition must hold

$$W < \frac{1}{\delta}(\alpha\varphi p_P)^{\frac{1}{1-\alpha}}, \tag{42}$$

which yields an upper-bound on wealth. Furthermore, for this to be a valid equilibrium it is also necessary that $C_P > \underline{C}_P$ and thus the following condition must hold

$$W > \frac{1}{\delta}\left(\frac{(1-q)\,\xi}{q+(1-q)\,\xi}\right)(\alpha p_P \varphi)^{\frac{1}{1-\alpha}}, \tag{43}$$

which is the converse of equation (40). Thus when W lies between the upper- and lower-bounds for wealth defined above, there exists some $\gamma \in \left(\frac{\xi}{q+(1-q)\xi}, 1\right)$ such that $C_P = W$ and the optimal loan size is satisfied.

Unlike the previous case, γ cannot be solved for analytically, however as wealth and thus collateral is larger when $C_P \in \left(\underline{C}_P, \bar{C}_P\right)$, it follows that a larger γ is required than specified in equation (39) in order to ensure that the wealth constraint binds. Intuitively, when there is less available collateral, banks must offer smaller loans for a given level of γ in order to meet the regulatory requirement. Thus a lower γ is required in order to implement the optimal loan size.

When available collateral is sufficiently low, the government is able to implement a pooling equilibrium. Whether or not this would lead to higher welfare than a separating equilibrium depends on several factors. First, the amount of available collateral W also determines the amount of collateral pledged by firms in the pooling equilibrium. Thus when W is large, it is unlikely for a pooling equilibrium to be optimal. Second, as both firm types receive the same size of loan in a pooling equilibrium, there is inefficient cross-subsidization between firm types as it is optimal for low-risk firms to receive larger loans than high-risk firms. The cost of this misallocation depends on the relative productivity of the two firm types; if the gap between p_H and p_L is large, a pooling equilibrium is likely to result in a loss of welfare.

5 Analysis

In this section I discuss some of the key assumptions made in the paper as well as possible extensions to the model. In addition I document some empirical implications of the model.

5.1 Returns to Scale

One of the key assumptions made in this paper that warrants further discussion is that the firm project features decreasing returns to scale. This is a departure from the classic adverse selection papers such as Stiglitz and Weiss (1981) and Bester (1985a) who assume a fixed project size and allow for separation to occur through differences in loan size. However in reality not all firms exhibit decreasing returns to scale. A recent paper by Gao and Kehrig (2017) finds that returns to scale are on average 0.96 and close to constant returns to scale. In addition they also document a large range of returns to scale across industries.

In the partial equilibrium framework presented in this paper, absent any frictions, the size of the firm under an assumption of constant returns to scale would be indeterminate. An alternative assumption that could be made would be to assume a convex adjustment cost of capital along with constant returns to scale of production as in the paper by Liu et al. (2009). This assumption would yield qualitatively similar results with firms able to separate by loan size.[2]

The above discussion raises an interesting question regarding how the results presented in this paper are affected by changes in the returns to scale parameter α. While the model cannot adopt constant returns to scale without additional modifications, I am able to consider the limiting case where $\alpha \to 1$ from below. In the discussion that follows I make the additional assumption that $\alpha p_H \varphi \geq 1$. This ensures that the project remains profitable as $\alpha \to 1$ and is equivalent to ensuring that k is strictly increasing in α. If this assumption did not hold, then the optimal scale of the project at the limit where $\alpha \to 1$ would be zero.

Given the above assumption, taking the derivatives of α show that k_H and C_H are increasing in α; high-risk firms receive larger loans as the returns to scale increase and require more collateral in order to meet an increase in regulation.

The impact on the contract terms for the low-risk firm are less obvious. The reason for this is that an increase in α has two effects that work in opposite directions. As α increases, the high-risk firms receive a larger expected payoff from their contract which in turn relaxes the incentive compatibility constraint. In cases where the regulatory parameter γ is low, this effect dominates and $\hat{C}_L$ falls while $\hat{k}_L$ becomes larger as α increases. When regulation is sufficiently high and $\gamma \geq \bar{\gamma}$, the amount of collateral required to meet the regulatory constraint is a fixed proportion of the loan size. In this case, the larger k_L brought about by increasing α leads to higher levels of collateral being needed in order to meet the regulatory requirement.

In the Appendix it is also shown that $\frac{d\bar{\gamma}}{d\alpha} < 0$. This implies that an increase in α makes it more likely that $\hat{C}_L$ will rise in response to increasing regulation. Thus as higher values of α tends to result in higher collateral requirements, especially at higher levels of regulation, it is more likely that the wealth constraint will bind. Thus pooling equilibria are more likely to occur in industries with higher returns to scale.

Now consider γ^*, the level of regulation at which point the incentive compatibility constraint no longer binds. The Appendix proves that $\frac{d\gamma^*}{d\alpha} < 0$ and thus as the α increases, the incentive compatibility constraint is less likely to bind at higher levels of regulation. The reason for this is that the model allows separation to occur through both the size of the loan and the

[2] There are some theoretical benefits to the approach taken by Liu et al. (2009); in particular the assumption of constant returns to scale allows them to generate a theoretical result linking the weighted average of the stock return and the after-tax corporate bond return which can be empirically tested. I am grateful to an anonymous referee for drawing this to my attention.

amount of collateral pledged. As α increases the optimal loan size for the two types of firms diverge and thus differences in loan size will have a greater separation effect.

Finally, consider the limiting case as $\alpha \to 1$. As discussed above, at the limit the loan size becomes unbounded and thus if $\gamma > \xi$, so do collateral requirements. With finite collateral available, a separating equilibrium will not be implementable and a pooling equilibrium becomes more likely.

5.2 Collateral

This paper focuses on the use of collateral in loan contracts. Specifically, I emphasize two theoretical roles for collateral; the first is the use of collateral ex ante in reducing adverse selection, while the second is the ex post role of collateral in reducing the loss given default of a particular loan and thus reducing the risk profile of a given loan. Both of these roles are explored empirically in Berger et al. (2011).

The ex post role of collateral in reducing risk allows the model to predict the impact of increasing bank regulation on collateral requirements. Specifically, the model predicts that the collateral to loan ratio is strictly increasing for all firm types following an increase in γ. In a separating equilibrium, when γ is sufficiently high that $\hat{C}_L < \bar{C}_L$, the collateral ratio for low-risk firms is

$$\hat{c}_L = \delta \left(\frac{\gamma - \xi}{1 - \xi} \right), \tag{44}$$

where $\hat{c}_L \equiv \hat{C}_L / \hat{k}_L$ is the collateral ratio and is strictly increasing in γ. Similarly, when γ is low such that $\hat{C}_L \geq \bar{C}_L$, the loan size k_L does not change in response to regulation while the collateral level is strictly increasing in γ and thus c_L is also strictly increasing in γ. This result also holds in a pooling equilibrium where collateral must be fixed at W while the loan size is strictly decreasing in γ.

A key assumption made in this paper is that lenders value collateral but at a rate discounted by δ. In cases where $\delta < 1$ this implies that the lender values collateral less than the borrower. In a typical screening model, $\delta < 1$ is needed to ensure that collateral is costly and can play a screening role. This is because if the borrower and lender valued collateral equally, then the competitive lenders would compensate borrowers for the pledging of collateral through lower interest rates and risk-neutral firms would fully collateralize their loans. It should be emphasized that in this model, with $\delta = 1$, the pledging of collateral will still be costly. This is because lenders have limited liability and so value profits conditional on the aggregate state where they do not default whereas firms value collateral in both aggregate states. This creates a wedge in the valuation of collateral that does not exist in models with no aggregate uncertainty.

In reality, δ, the factor at which lenders discount collateral is likely to vary across industries. One likely determinant of δ is the proportion of intangible capital used by a firm, with the valuation gap between borrower and lender likely to be larger for intangible than tangible capital. There is ample evidence of variations in the use of intangible capital For example, Demmou et al. (2019) provide evidence that the investment share of intangible assets varies across both countries and industries while Corrado and Hulten (2010) document a major shift in the composition of investment and capital formation from tangible to intangible assets over the 60 years prior to their study. Thus by analyzing the impact of the parameter δ the model may also be used in making predictions across industries with differing utilization of intangible capital.

By differentiating the collateral ratio defined by equation (44) it follows that the collateral ratio of high-risk firms is strictly decreasing. The reason for this is that as δ increases, loans require a lower collateral ratio in order to meet the regulatory requirement. The impact on the contract terms for the low-risk firms are less obvious. The reason for this is that in addition to the regulatory effect, there is an additional factor that works in the opposite direction. An increase in δ makes it relatively less costly for high-risk firms to pledge collateral and thus more collateral is required by low-risk firms in order to separate. The effect that dominates again depends on the level of regulation. As shown in the Appendix, at the point where $\gamma \leq \xi$ and the regulatory constraint does not bind, the second effect dominates and the collateral ratio is increasing in δ. At higher levels of regulation, where $\hat{C}_L < \bar{C}_L$ the first effect dominates and the collateral ratio is decreasing in δ.

The above analysis suggests that at high levels of regulation, the collateral requirements for all firm types increase and thus the wealth constraint is more likely to bind. Thus the model predicts that a pooling equilibrium is more likely to exist in industries that feature higher utilization of intangible collateral.

The model also provides a link between the cost of regulation and the amount of collateral pledged. This suggests that regulation may be more costly in more competitive banking markets. More competitive banking markets feature smaller and newer banks and the empirical evidence finds that the use of collateral in lending contracts is especially predominant in these smaller banks (Uchida, 2011) and newer banks (Gobbi and Lotti, 2004).

Another important assumption made in the paper is that the value of collateral does not vary over the business cycle and thus collateral features no systemic risk. Consider now the case where the collateral value is procyclical and that the value of collateral is expected to fall in a recession relative to a boom. If the value of collateral remains independent of unobservable firm type then the main results of the paper will be unaffected, the key impact would be that collateral would become less efficient in meeting the bank's regulatory requirements and would have a similar impact as a fall in δ. One difference from a fall in δ would be the interest rate banks charge to firms as, absent regulation, banks only value collateral in the good aggregate state and perfect competition would result in lower interest rates on bank loans.[3]

The results of this paper may be more sensitive to the case where collateral values differ across unobservable firm types. While a full analysis of this case is beyond the scope of this paper, a similar model has been considered by Niinimäki (2011). He finds that when the value of collateral increases with the firm's unobserved probability of success, firms with the highest expected probability of success may not be the most willing to pledge collateral. Instead he finds it is firms whose collateral value deviates the most and whose projects' probability of success features the highest variance that wish to pledge collateral. Thus, in this case collateral may become less effective at screening firm types and this would likely dampen the main channel of my model.

5.3 Banking Regulation

Banking regulation is modeled in this paper in a relatively simple way and takes the form of a binding constraint on banks' cash flow from lending which is not allowed to be below a certain proportion γ of their initial lending. This gives a role for collateral in meeting the regulatory requirement and thus generates an interaction between regulation and the screening problem.

As discussed earlier, the model predicts that loan size will be weakly lower following an increase in regulation. Additionally, the model predicts that, while an increase in regulation strictly reduces loan supply for high-risk firms, loan supply is only weakly falling for low-risk firms. The reason for this is that as low-risk firms are safer, the regulatory requirement needs to be higher for the constraint to become binding for these firms. This is a result that is consistent with the empirical evidence proposed in Acharya et al. (2018) who find evidence that lenders subject to a stress-test reduce credit and that riskier firms are affected to a greater extent.

[3] I do not wish to emphasize the implications of the model on interest rates as these are predominantly driven by the assumption of perfect competition in the banking sector. For the model to say something meaningful on interest rates it would be necessary to relax this assumption of perfect competition along the lines of Villas-Boas and Schmidt-Mohr (1999) and Hainz et al. (2012) who combine an adverse selection model with a spatial model of oligopolistic competition.

One possible criticism of the model is that the banks are assumed to always default in the bad aggregate state. This assumption greatly simplifies not only the bank default decision but also limits the model's ability to address regulatory issues such as bank closure policy. For example, the model could be extended along the lines of Mehran and Thakor (2011) where regulators receive a signal in an intermediate period and bank closure is a way for regulators to incentivize banks to put in costly monitoring effort. In Mehran and Thakor (2011) regulators choose to close down a bank only if they have insufficient liquidity. Seeing my model through this lens, one interpretation of the regulatory constraint is that of a regulator choosing to close down any bank that does not meet the constraint. As closed banks do not receive revenue, banks would then always choose to meet the regulatory requirement. One key difference is that there is no regulatory uncertainty in my model. Extending the model by introducing some uncertainty on the part of the regulator along the lines of Mehran and Thakor (2011) may be an interesting avenue to explore.

As the regulatory constraint presented in this model is very stylized it is worth discussing how the model may relate to other real world banking regulations such as the Basel II/III capital buffers. There is some relationship between the two; the Basel II/III capital buffers are determined as a percentage of the risk-weighted assets held by the bank. Banks that met certain certain criteria could calculate the credit risk of assets themselves using the so-called internal ratings-based (IRB) approach.[4] Thus insofar as higher collateral requirements result in lower risk-weights, increasing the collateral requirements of loans, and thus the quantity of risk-weighted assets held by the bank, would be one way to meet the regulatory capital requirements in the Basel II/III framework. In addition, Basel III introduced a countercyclical capital buffer (CCyB) which requires additional capital requirements in boom periods in order to avoid periods of excess credit growth. If correctly implemented, this policy would likely address the problem of overlending that is featured in this paper. However, it should be noted that the stated aim of the CCyB is to avoid the build-up of systemic risk and would better fit a model with a more complicated bank default decision.

In this paper I focus on banking regulation taking deposit insurance as given. Perhaps more importantly deposit insurance provides a reason for regulation to exist in the first place. The obvious implication of this is that it would be optimal for the regulator to simply remove deposit insurance and thus tackle the overlending problem at source. However, the main objective of the paper is to consider banking regulation in a world where deposit insurance is always provided. This can be motivated by the fact that deposit insurance is a reality in all advanced economies, as documented by Demirgüç-Kunt et al. (2008). Moreover, while

[4]For an overview of how collateral requirements contribute to the calculation of risk-weighted assets in the Basel II framework see De Lisa et al. (2012).

the need for deposit insurance in this paper is not micro-founded it would not be difficult to extend the model in order to do so. For example, as in Malherbe and McMahon (2020), I could add an intra-day period and household liquidity preferences to the model. This would create a rationale for Deposit Insurance in the vein of Diamond and Dybvig (1983).

6 Conclusion

This paper analyzed credit market equilibrium under private information when banks face a regulatory constraint that restricts the losses they can make in a recession. As in standard signalling models, borrowers are able to signal their type through both loan size and collateral in order to receive a lower loan interest rate. The addition of a regulatory constraint provides another role for collateral as higher collateral requirements also reduce banks' loss given default.

I highlight the interaction between the signalling problem and banking regulation. In particular several results differ significantly from more standard signalling models. First, collateral may be demanded of both high- and low-risk firms, even in the absence of asymmetric information. Secondly, if banking regulation is sufficiently strict, there may not exist an adverse selection problem. Additionally, if borrowers have sufficiently low pledgeable collateral, a pooling equilibrium may exist as a Nash equilibrium. This last result highlights how regulation may distort bank lending in ways that can have negative distributional effects by disrupting the ability of bank to screen borrowers.

Finally, the paper sets out some empirical predictions of the model. First, increased regulation reduces loan size and this fall affects higher-risk firms to a greater extent, which is consistent with the evidence in Acharya et al. (2018). Second, the model predicts that the collateral ratio of loans will increase following an increase in regulation. Finally, the model suggests that banking regulation may impact firms across different industries differently. The model predicts that at high levels of regulation, collateral ratios will be larger for industries that feature higher returns to scale and less intangible capital.

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Appendix

Proof of Proposition 1

In this proof it is assumed that W is sufficiently high that any quantity of collateral can be implemented in equilibrium. The Lagrangian that solves for the competitive equilibrium contract for an single firm type is

$$\begin{aligned}\mathcal{L}_i = {} & p_i\left(\varphi k_i^{\alpha} - R_i k_i\right) - \left(1-p_i\right) C_i + W \\ & + q\lambda_B\left[p_i\left(z_G\right) R_i k_i + \delta\left(1-p_i\left(z_G\right)\right) C_i - k_i\right] \\ & + \lambda_S\left[p_i\left(z_B\right) R_i k_i + \delta\left(1-p_i\left(z_B\right)\right) C_i - \gamma k_i\right] \\ & + \lambda_C^- C_i,\end{aligned} \tag{A.45}$$

where λ_S, λ_B and λ_C^- are the multipliers on equations (7), (8) and the non-negativity constraint on collateral. The first order conditions are

$$p_i = \lambda_B q p_i\left(z_G\right) + \lambda_S p_i\left(z_B\right), \tag{A.46}$$

$$\left(1-p_i\right) = \lambda_B q\delta\left(1-p_i\left(z_G\right)\right) + \lambda_S\delta\left(1-p_i\left(z_B\right)\right) + \lambda_C^-, \tag{A.47}$$

$$p_i\left(\alpha\varphi k_i^{\alpha-1} - R_i\right) + \lambda_B q\left[p_i\left(z_G\right) R_i - 1\right] + \lambda_S\left[p_i\left(z_B\right) R_i - \gamma\right] = 0. \tag{A.48}$$

First note that if the stress-test condition does not bind, $\lambda_S = 0$ and the contract terms that solve the first order conditions are given by $C_i = 0$, $R_i = 1$ and $k_i = \left(\alpha p_i\left(z_G\right)\right)^{\frac{1}{1-\alpha}}$. Plugging these equations into equation (7), the regulatory constraint will be satisfied only if $\gamma \leq \xi$.

Next, consider when λ_B will be strictly positive. To do this, suppose instead that $\lambda_B = 0$, then equation (A.46) implies that $\lambda_S = \frac{p_i}{p_i(z_B)}$. Substituting this into equation (A.47) and rearranging yields

$$\left(1-p_i\right) = \delta p_i\left(\frac{1-p_i\left(z_B\right)}{p_i\left(z_B\right)}\right) + \lambda_C^-. \tag{A.49}$$

This yields a contradiction whenever $\delta > \left(\frac{p_i(z_B)}{1-p_i(z_B)}\right)\left(\frac{1-p_i}{p_i}\right)$. Thus it follows that if $\gamma > \xi$ and $\delta > \left(\frac{p_i(z_B)}{1-p_i(z_B)}\right)\left(\frac{1-p_i}{p_i}\right)$ then $\lambda_S > 0$, $\lambda_B > 0$ and $\lambda_C^- = 0$. Thus equations (7) and (8) will bind in equilibrium and the equilibrium will feature a strictly positive amount of collateral. Solving the system of first order conditions then yields the equilibrium loan size and collateral size set out in equations (16) and (17). The interest rate charged to the firm follows from substituting equations (16) and (17) into equation (8) and the payoff U_i^* follows from substituting the contract terms into

$$U_i\left(k,R,C\right)=p_i\left(\varphi k_i^{\alpha}-R_ik_i\right)-\left(1-p_i\right)C_i.$$

Properties of the Equilibrium with identical customers

The derivative of $k_i^*$with respect to γ is

$$\frac{dk_i^*}{d\gamma}=-\left(\frac{1}{1-\alpha}\right)\left(\frac{(1-q)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1}{1-\xi}\right)}{(q+(1-q)\gamma)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)k_i^*, \tag{A.50}$$

which is strictly decreasing.

The derivative of U_i^* with respect to γ is

$$\frac{\partial U_i^*}{\partial\gamma}=-\left[(1-q)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1}{1-\xi}\right)\right]k_i^*, \tag{A.51}$$

which is strictly decreasing.

The derivative of C_i^* with respect to γ is

$$\frac{dC_i^*}{d\gamma}=\frac{1}{\delta}\left(\frac{1}{1-\xi}\right)\left[1-\left(\frac{1}{1-\alpha}\right)\left(\frac{(1-q)(\gamma-\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)}{(q+(1-q)\gamma)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)\right]k_i^*. \tag{A.52}$$

The derivative of this is strictly positive if

$$\left(\frac{1}{1-\alpha}\right)\left(\frac{(1-q)(\gamma-\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)}{(q+(1-q)\gamma)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)<1.$$

This holds at sufficiently low values of γ and the left hand side of the above is zero when $\gamma=\xi$. However, at high values of γ, this inequality may not hold, especially if α is high. In this case, the derivative will be negative and the level of collateral will fall in response to an increase in regulation. As discussed in the text, the reason for this is that k_i^* falls in response to γ and at sufficiently high levels of α and γ, this effect dominates the increased collateral ratio and the level of collateral required falls.

Proof of Proposition 2

First, if $\gamma \geq \xi$, $\delta \geq \left(\frac{p_H(z_B)}{1-p_H(z_B)}\right)\left(\frac{1-p_H}{p_H}\right)$ and $W \geq C_L^*$ then the equilibrium contracts if the incentive compatibility constraint does not bind are given by $\{(k_i^*, R_i^*, C_i^*)\}_{i=\in\{L,H\}}$. Then equation (21) can be derived by substituting these contract terms into equation (19) and rearranging. If for some value of γ equation (21) is weakly positive, then the incentive compatibility will not bind at γ, otherwise, it will. As discussed in the text, the limit of $\Lambda_{IC}(\gamma)$ as $\gamma \to \xi$ is negative and the incentive compatibility constraint will always bind for $\gamma \leq \xi$. On the other hand, as discussed in the text it is clear that the incentive compatibility constraint is slack at the upper-limit as $\gamma \to 1$.

The rest off the proposition follows so long as $\frac{\partial \Lambda_{IC}(\gamma)}{\partial \gamma} > 0$ and there exists a threshold $\gamma^* \in [\xi, 1)$ such that for any $\gamma > \gamma^*$ $\Lambda_{IC}(\gamma) > 0$.

To show that $\frac{\partial \Lambda_{IC}(\gamma)}{\partial \gamma} > 0$ note that the derivative of equation (21) with respect to γ can be written as

$$\begin{aligned}\frac{\partial \Lambda_{IC}(\gamma)}{\partial \gamma} &= \alpha\left(\frac{p_H}{p_L}\right)^{\frac{\alpha}{1-\alpha}}(p_L - p_H)\left(\frac{q+(1-q)\xi}{1-\xi}\right) \\ &\times \left(\frac{[q+(1-q)\gamma] + \left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)}{[q+(1-q)\gamma] + \left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)^{\frac{1}{1-\alpha}} \\ &+ \alpha\left(\frac{(q+(1-q)\xi)\frac{1}{\delta}\left(\frac{p_L}{p_H}-1\right)\left(\frac{1}{1-\xi}\right)}{\left[[q+(1-q)\gamma] + \left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)\right]^2}\right) \\ &\times \left[1 - p_H\left(\frac{p_H}{p_L}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{[q+(1-q)\gamma] + \left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)}{[q+(1-q)\gamma] + \left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)^{\frac{1}{1-\alpha}}\right]\end{aligned} \tag{A.53}$$

A sufficient condition for $\frac{\partial \Lambda_{IC}(\gamma)}{\partial \gamma} > 0$ is

$$p_H\left(\frac{p_H}{p_L}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{[q+(1-q)\gamma] + \left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)}{[q+(1-q)\gamma] + \left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)^{\frac{1}{1-\alpha}} \leq 1. \tag{A.54}$$

To show this, note that from the definition of k_i^* the following is true $\alpha\varphi p_i(k_i^*)^{\alpha-1} =$

$[q+(1-q)\gamma]+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)$. Thus equation (A.54) can be rewritten as

$$p_H\left(\frac{p_H}{p_L}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha\varphi p_L\left(k_L^*\right)^{\alpha-1}}{\alpha\varphi p_H\left(k_H^*\right)^{\alpha-1}}\right)^{\frac{1}{1-\alpha}}\leq 1, \tag{A.55}$$

which simplifies to

$$\left(\frac{p_L}{p_H}\right)^{\frac{1}{1-\alpha}}\frac{k_H^*}{k_L^*}\leq 1, \tag{A.56}$$

which will always be satisfied as $k_H^*<k_L^*$ and $p_L>p_H$.

Proof of Proposition 3

First, note that the cutoff $\bar{\gamma}$ as defined in Proposition 3 is simply the point where $\hat{C}_L=\bar{C}_L$ which is the point where the following equation must hold

$$\Lambda_{\bar{C}}(\gamma)\equiv\frac{1}{\delta}\left(\frac{\gamma-\xi}{1-\xi}\right)\bar{k}_L-\frac{1}{\delta}\left(\frac{\frac{p_H}{p_L}\left(p_L\varphi\bar{k}_L^{\alpha-1}-(q+(1-q)\xi)\right)\bar{k}_L-U_H^*}{1-\frac{p_H}{p_L}(q+(1-q)\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)}\right)=0. \tag{A.57}$$

First note that when $\gamma=\xi$, $\bar{C}_L=0$ and $\hat{C}_L>0$ implies that $\Lambda_{\bar{C}}(\xi)<0$. Next, note that when $\gamma=\gamma^*$, $C_L=C_L^*<\bar{C}_L$ and thus $\Lambda_{\bar{C}}(\gamma^*)>0$. It follows from this that for there to exists a unique $\bar{\gamma}\in(\xi,\gamma^*)$ such that $C_L=\bar{C}_L$ it is sufficient to show that $\frac{\partial\Lambda_{\bar{C}}}{\partial\gamma}>0$. To prove this note that differentiating equation (A.57) yields

$$\frac{\partial\Lambda_{\bar{C}}}{\partial\gamma}=\left(\frac{1}{1-\xi}\right)\bar{k}_L-\left(\frac{1}{1-\xi}\right)\left(\frac{(1-q)(1-\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)}{1-\frac{p_H}{p_L}(q+(1-q)\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)}\right)k_H^*. \tag{A.58}$$

Now note that as $\left(\frac{(1-q)(1-\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)}{1-\frac{p_H}{p_L}(q+(1-q)\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)}\right)<1$and $\bar{k}_L>k_H^*$ it follows that $\frac{\partial\Lambda_{\bar{C}}}{\partial\gamma}>0$.

Analysis of optimal policy with single firm type

Planner's First order condition Equation (28) yields a first order condition for the government's optimal policy problem when there is a single firm of type i. From equations(16) and (17) the derivatives of the equilibrium contracts with respect to the parameter γ are as follows

$$\frac{dk_i^*}{d\gamma}=-\left(\frac{1}{1-\alpha}\right)\left(\frac{(1-q)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1}{1-\xi}\right)}{p_i\varphi\alpha\left[k_i^*\right]^{\alpha-1}}\right)k_i^*, \tag{A.59}$$

$$\frac{dC_i^*}{d\gamma} = \frac{1}{\delta}\left(\frac{1}{1-\xi}\right)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\frac{1}{\alpha}\left(q+(1-q)\,\xi\right)-p_i\varphi\alpha\left[k_i^*\right]^{\alpha-1}}{p_i\varphi\alpha\left[k_i^*\right]^{\alpha-1}}\right)k_i^*. \tag{A.60}$$

Substituting these derivatives into equation (28) yields the following

$$\begin{aligned}\frac{\partial\mathcal{U}_i}{\partial\gamma} &= -\left(\left(\frac{1-q}{1-\alpha}\right)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1}{1-\xi}\right)\right)k_i^* \\ &\quad+\left(\frac{1-q}{1-\alpha}\right)\left(1+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\right)\frac{k_i^*}{p_i\varphi\alpha\left[k_i^*\right]^{\alpha-1}}.\end{aligned} \tag{A.61}$$

Thus the value of γ that sets the government's first order condition to zero is such that the marginal product of the project is given by the following equation

$$p_i\varphi\alpha\left[k_i^*\right]^{\alpha-1} = \left(\frac{1+\left(\frac{1-\delta}{\delta}\right)(1-p_i)}{1+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1-\alpha}{1-q}\right)\left(\frac{1}{1-\xi}\right)}\right). \tag{A.62}$$

As the marginal product at the first best loan size is 1, it follows that the first best loan size is achieved only if this will only occur if $(q+(1-q)\,\xi)=\alpha$. Substituting equation (17) into equation (A.62) and rearranging the optimal γ can be written as

$$\gamma = \xi + \left(\frac{(1-q)(1-\xi)+\left[1-(1-\alpha)\left(\frac{q+(1-q)\xi}{(1-q)(1-\xi)}\right)\right]\left(\frac{1-\delta}{\delta}\right)(1-p_i)}{\left[(1-q)+\left(\frac{1-\eta}{\eta}\right)(1-p_i)\left(\frac{1}{1-\xi}\right)\right]\left[1+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1-\alpha}{1-q}\right)\left(\frac{1}{1-\xi}\right)\right]}\right). \tag{A.63}$$

It follows from the above that a sufficient condition for the government to impose a binding regulatory constraint such that $\gamma > \xi$ is the following

$$\left(\frac{1-\delta}{\delta}\right)\left[1-(1-\alpha)\left(\frac{q+(1-q)\,\xi}{(1-q)(1-\xi)}\right)\right] > 0. \tag{A.64}$$

Proof of $\frac{d\gamma_i^{OPT}}{dp_i} > 0$ The implicit function theorem can be used to show that $\frac{d\gamma_i^{OPT}}{dp_i} > 0$. First, consider $\Lambda_{F,i}$, a simple transformation of equation (A.61) defined as

$$\Lambda_{F,i} \equiv \left(\frac{1-\alpha}{1-q}\right)\frac{\partial\mathcal{U}_i}{\partial\gamma}\frac{1}{k_i^*}. \tag{A.65}$$

Now note the partial derivatives of this function with respect to γ and p_i are

$$\frac{\partial \Lambda_{F,i}}{\partial \gamma} = -\left(\frac{\left(1 + \left(\frac{1-\delta}{\delta}\right)(1-p_i)\right)\left((1-q) + \left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1}{1-\xi}\right)\right)}{\left[(q + (1-q)\gamma) + \left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)\right]^2} \right) < 0, \tag{A.66}$$

and

$$\frac{\partial \Lambda_{F,i}}{\partial p_i} = \left[\left(\frac{1-\alpha}{1-q}\right) - \left(\frac{(q + (1-q)\xi)((1-\gamma))}{\left[(q + (1-q)\gamma) + \left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)\right]^2} \right) \right] \left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{1-\xi}\right). \tag{A.67}$$

The latter derivative is positive around the optimum, that is around the point where $\Lambda_{F,i} = 0$. To see this note that at $\Lambda_{F,i} = 0$ the following must hold

$$\left(\frac{1-\alpha}{1-q}\right)\left(\frac{1}{1-\xi}\right)\left(\frac{1-\delta}{\delta}\right) = \left(\frac{1}{1-p_i}\right)\left(\frac{(1-q)(1-\gamma) + \left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1-\gamma}{1-\xi}\right)}{(q + (1-q)\gamma) + (1-p_i)\left(\frac{1-\delta}{\delta}\right)\left(\frac{\gamma-\xi}{1-\xi}\right)} \right). \tag{A.68}$$

Substituting this into the above equation and rearranging shows that $\frac{\partial \Lambda_{F,i}}{\partial p_i} > 0$. It follows immediately from the implicit function theorem that $\frac{d\gamma_i^{OPT}}{dp_i} > 0$ and thus $\gamma_H^{OPT} < \gamma_L^{OPT}$.

Analysis of optimal policy with two firm types

Equation (32)is the first order condition for the government's optimal policy problem when there are two firm types. As the high-risk firm will receive (k_H^*, R_H^*, C_H^*), the derivatives $\frac{dk_H^*}{d\gamma}$ and $\frac{dC_H^*}{d\gamma}$ are given by equations (A.59) and (A.60) respectively.

The value of $\hat{C}_L$ can be found from combining equations (8) and (19) with the value of $\hat{k}_L$ defined in equation (31) such that $\hat{C}_L$ must satisfy the following equation

$$\hat{C}_L = \frac{1}{\delta}\left(\frac{\frac{p_H}{p_L}\left(p_L \varphi \left(\hat{k}_L\right)^{\alpha-1} - (q + (1-q)\xi)\right)\hat{k}_L - \pi_H^*}{1 - \frac{p_H}{p_L}(q + (1-q)\xi) + \left(\frac{1-\delta}{\delta}\right)(1-p_H)} \right). \tag{A.69}$$

The derivative $\frac{dC_H^*}{d\gamma}$ can then be found by totally differentiating the above equation with respect to γ.

In the first case where $\hat{C}_L \geq \bar{C}_L$, the derivative is simply

$$\frac{d\hat{C}_L}{d\gamma} = \frac{1}{\delta}\left(\frac{(1-q) + \left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{1}{1-\xi}\right)}{1-\left(\frac{p_H}{p_L}\right)(q+(1-q)\xi) + \left(\frac{1-\delta}{\delta}\right)(1-p_H)}\right) k_H^* > 0. \tag{A.70}$$

It follows that when $\hat{C}_L \geq \bar{C}_L$, $\frac{d\hat{C}_L}{d\gamma} > 0$ and an increase in regulation results in a larger collateral requirement.

Next consider the case where $\hat{C}_L < \bar{C}_L$. The derivative becomes

$$\frac{d\hat{C}_L}{d\gamma} = \frac{1}{\delta}\left(\frac{1}{1-\xi}\right)\left(\frac{(1-q)(1-\xi) + \left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{1}{1-\xi}\right) - \iota}{1-\left(\frac{p_H}{p_L}\right)(q+(1-q)\xi) + \left(\frac{1-\delta}{\delta}\right)(1-p_H) - \frac{\hat{k}_L}{k_H^*}\iota}\right) k_H^*, \tag{A.71}$$

where

$$\iota \equiv \left(\frac{1-\xi}{\gamma-\xi}\right)\frac{p_H}{p_L}\left(\alpha p_L \varphi (k_L)^{\alpha-1} - (q+(1-q)\xi)\right) \geq 0. \tag{A.72}$$

From equation (31) it follows that for any $\gamma \in [\xi, 1)$ that $\alpha p_L \varphi \left(\hat{k}_L\right)^{\alpha-1} \geq (q+(1-q)\xi)$ and thus $\iota > 0$ and is increasing in γ above $\bar{\gamma}$. As $\frac{d\hat{C}_L}{d\gamma}$ is decreasing in ι, it follows that the derivative will be decreasing in γ.

Analysis of Returns to scale

Impact of α on $\hat{C}_L$ and $\hat{k}_L$ First, note that the incentive compatibility constraint binds so long as $F = 0$ where the function F is defined below as

$$\begin{aligned} F \equiv\ & \frac{p_H}{p_L}\left(p_L \varphi \hat{k}_L^{\alpha} - (q+(1-q)\xi)\hat{k}_L\right) \\ & - \delta\left[1 - \frac{p_H}{p_L}(q+(1-q)\xi) + \left(\frac{1-\delta}{\delta}\right)(1-p_H)\right]\hat{C}_L - U_H^* = 0 \end{aligned} \tag{A.73}$$

When $C_L \geq \bar{C}_L$ the derivative for $\hat{k}_L$ with respect to α is

$$\frac{\partial \hat{k}_L}{\partial \alpha} = \frac{1}{(1-\alpha)^2 \alpha}\left(\alpha \ln\left(\frac{\alpha p_L \varphi}{q+(1-q)\xi}\right) + (1-\alpha)\right)\hat{k}_L, \tag{A.74}$$

thus given the assumption that $\alpha p_H \varphi \geq 1$ the derivative is strictly positive.

The derivative with respect to $\hat{C}_L$ can be found through application of the implicit function theorem to the function F. The partial derivative of F with respect to $\hat{C}_L$ is

$$\frac{\partial F}{\partial \hat{C}_L} = -\delta \left[1 - \frac{p_H}{p_L}\left(q + (1-q)\,\xi\right) + \left(\frac{1-\delta}{\delta}\right)(1-p_H)\right] < 0. \tag{A.75}$$

The partial derivative of F with respect to α is

$$\frac{\partial F}{\partial \alpha} = \frac{p_H}{p_L}\left(\alpha p_L \varphi \hat{k}_L^{\alpha-1} - (q + (1-q)\,\xi)\right)\frac{\partial \hat{k}_L}{\partial \alpha} - \frac{\partial U_H^*}{\partial \alpha}. \tag{A.76}$$

Note that when $C_L \geq \bar{C}_L$ then $\alpha p_L \varphi \hat{k}_L^{\alpha-1} = (q + (1-q)\,\xi)$ and thus,

$$\frac{\partial F}{\partial \alpha} = -\frac{\partial U_H^*}{\partial \alpha}, \tag{A.77}$$

where

$$\frac{\partial U_H^*}{\partial \alpha} = p_H \varphi \ln\left(k_H^*\right)\left(k_H^*\right)^{\alpha}, \tag{A.78}$$

is strictly positive due to the assumption that $\alpha p_H \varphi \geq 1$. Thus from the implicit function theorem it follows that $\frac{d\hat{C}_L}{d\alpha} < 0$.

When $C_L < \bar{C}_L$ the derivative for $\hat{k}_L$ with respect to α becomes

$$\begin{aligned}\frac{\partial F}{\partial \hat{C}_L} = {} & \frac{p_H}{p_L}\left(\alpha p_L \varphi \hat{k}_L^{\alpha-1} - (q + (1-q)\,\xi)\right)\delta\left(\frac{1-\xi}{\gamma-\xi}\right) \\ & - \delta\left[1 - \frac{p_H}{p_L}\left(q + (1-q)\,\xi\right) + \left(\frac{1-\delta}{\delta}\right)(1-p_H)\right].\end{aligned} \tag{A.79}$$

The sign of the above equation can be found by noting that $\hat{k}_L \geq k_L^*$ and thus

$$\alpha p_L \varphi \hat{k}_L^{\alpha-1} \leq \left[q + (1-q)\,\gamma\right] + \left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right). \tag{A.80}$$

Substituting this into inequality into the above derivative is enough to show that $\frac{\partial F}{\partial \hat{C}_L} < 0$ as before.

Next, note that the derivative of F with respect to α is

$$\frac{\partial F}{\partial \alpha} = \frac{p_H}{p_L}\left(p_L \varphi k_L^{\alpha} \ln\left(k_L\right)\right) - \frac{\partial U_H^*}{\partial \alpha}, \tag{A.81}$$

where

$$\frac{\partial U_H^*}{\partial \alpha} = p_H \varphi \ln\left(k_H^*\right)\left(k_H^*\right)^{\alpha}. \tag{A.82}$$

Thus the derivative can be written as

$$\frac{\partial F}{\partial \alpha} = p_H \varphi \left[k_L^{\alpha} \ln \left(k_L \right) - k_H^{\alpha} \ln \left(k_H \right) \right], \tag{A.83}$$

which is strictly positive. Hence through application of the implicit function theorem $\frac{d\hat{C}_L}{d\alpha} > 0$.

Proof that $\frac{d\bar{\gamma}}{d\alpha} < 0$ The derivative $\frac{d\bar{\gamma}}{d\alpha}$ can be found by applying the implicit function theorem to the function $\Lambda_{\bar{C}}$ as defined in equation (A.57). The partial derivative of $\Lambda_{\bar{C}}$ with respect to α is

$$\frac{\partial \Lambda_{\bar{C}}}{\partial \alpha} = \frac{1}{\delta} p_H \varphi \left(\frac{\ln \left(\bar{k}_L \right) \left(\bar{k}_L \right)^{\alpha} - \ln \left(k_H^{*} \right) \left(k_H^{*} \right)^{\alpha}}{1 - \frac{p_H}{p_L} \left(q + (1-q) \xi \right) + \left(\frac{1-\delta}{\delta} \right) \left(1 - p_H \right)} \right) > 0.$$

As this partial derivative is strictly positive, and it has already been found that $\frac{\partial \Lambda_{\bar{C}}}{\partial \gamma} > 0$, it follows from the implicit function theorem that $\frac{d\bar{\gamma}}{d\alpha} < 0$.

Proof that $\frac{d\gamma^{*}}{d\alpha} < 0$ The derivative $\frac{d\gamma^{*}}{d\alpha}$ can be found by applying the implicit function theorem to the function Λ_{IC} as defined in equation (21). The partial derivative of Λ_{IC} with respect to α is

$$\begin{aligned}
\frac{\partial \Lambda_{IC}}{\partial \alpha} = & \left(\frac{1}{1-\alpha} \right)^2 \left(\frac{p_H \left[\left(q + (1-q) \gamma \right) + \left(\frac{1-\delta}{\delta} \right) \left(1 - p_L \right) \left(\frac{\gamma - \xi}{1 - \xi} \right) \right]}{p_L \left[\left(q + (1-q) \gamma \right) + \left(\frac{1-\delta}{\delta} \right) \left(1 - p_H \right) \left(\frac{\gamma - \xi}{1 - \xi} \right) \right]} \right)^{\frac{\alpha}{1-\alpha}} \\
& \times \ln \left(\frac{p_H \left[\left(q + (1-q) \gamma \right) + \left(\frac{1-\delta}{\delta} \right) \left(1 - p_L \right) \left(\frac{\gamma - \xi}{1 - \xi} \right) \right]}{p_L \left[\left(q + (1-q) \gamma \right) + \left(\frac{1-\delta}{\delta} \right) \left(1 - p_H \right) \left(\frac{\gamma - \xi}{1 - \xi} \right) \right]} \right) \qquad \text{(A.84)} \\
& + \left(\frac{1}{1-\alpha} \right)^2 \left(\frac{\frac{1}{\delta} \left(\frac{p_L}{p_H} - 1 \right) \left(\frac{\gamma - \xi}{1 - \xi} \right)}{\left(q + (1-q) \gamma \right) + \left(\frac{1-\delta}{\delta} \right) \left(1 - p_L \right) \left(\frac{\gamma - \xi}{1 - \xi} \right)} \right). \qquad \text{(A.85)}
\end{aligned}$$

Now note that around $\Lambda_{IC} = 0$, that is around the point where $\gamma = \gamma^{*}$, the following must hold

$$\left(\frac{p_H\left[(q+(1-q)\gamma)+\left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)\right]}{p_L\left[(q+(1-q)\gamma)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)\right]}\right)^{\frac{\alpha}{1-\alpha}}$$
$$=\left[1-\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\frac{1}{\delta}\left(\frac{p_L}{p_H}-1\right)\left(\frac{\gamma-\xi}{1-\xi}\right)}{(q+(1-q)\gamma)+\left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)\right]. \tag{A.86}$$

Thus by substituting this equation into the partial derivative above, at the point $\gamma=\gamma^*$, the partial derivative can be written as

$$\frac{\partial\Lambda_{IC}}{\partial\alpha}=\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{1-\alpha}\right)^2\left[(1-x)\ln(1-x)+x\right], \tag{A.87}$$

where

$$x=\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\frac{1}{\delta}\left(\frac{p_L}{p_H}-1\right)\left(\frac{\gamma-\xi}{1-\xi}\right)}{(q+(1-q)\gamma)+\left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)>0. \tag{A.88}$$

Thus it follows that $\frac{\partial\Lambda_{IC}}{\partial\alpha}>0$ and as it has already been established that $\frac{\partial\Lambda_{IC}}{\partial\gamma}>0$ from application of the implicit function theorem it follows that $\frac{d\gamma^*}{d\alpha}<0$.

Analysis of Collateral

Impact of δ on $\hat{C}_L$ and $\hat{k}_L$ When $C_L\geq\bar{C}_L$ the derivative for $\hat{k}_L$ with respect to δ is equal to zero and thus the partial derivative of F, as defined by equation (A.73) can be written as

$$\frac{\partial F}{\partial\delta}=-\frac{\partial U_H^*}{\partial\delta}, \tag{A.89}$$

where

$$\frac{\partial U_H^*}{\partial\delta}=\left(\frac{1}{\delta}\right)^2(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)k_H^*\geq 0. \tag{A.90}$$

Thus it follows that $\frac{\partial F}{\partial\delta}<0$ and through equation (A.75) which states that $\frac{\partial F}{\partial\hat{C}_L}<0$ and application of the implicit function theorem it follows that $\frac{d\hat{C}_L}{d\delta}>0$. As $\frac{\partial\hat{k}_L}{\partial\delta}=0$ this implies that the collateral ratio is increasing in δ when $C_L\geq\bar{C}_L$.

When $C_L<\bar{C}_L$ the collateral ratio is determined by the following relationship

$$\frac{\hat{C}_L}{\hat{k}_L}=\frac{1}{\delta}\left(\frac{\gamma-\xi}{1-\xi}\right), \tag{A.91}$$

and thus the collateral ratio is decreasing in δ when $C_L < \bar{C}_L$.

Proof that $\frac{d\bar{\gamma}}{d\delta} > 0$ The derivative $\frac{d\bar{\gamma}}{d\delta}$ can be found by applying the implicit function theorem to the function $\Lambda_{\bar{C}}$ as defined in equation (A.57). The partial derivative of $\Lambda_{\bar{C}}$ with respect to δ can be written as

$$\begin{aligned}\frac{\partial \Lambda_{\bar{C}}}{\partial \delta} &= -\frac{1}{\delta}\Lambda_{\bar{C}} - \frac{1}{\delta^3}(1-p_H)\left(\frac{\frac{p_H}{p_L}\left(p_L\varphi\bar{k}_L^{\alpha-1} - (q+(1-q)\xi)\right)\bar{k}_L - U_H^*}{\left(1-\frac{p_H}{p_L}(q+(1-q)\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)\right)^2}\right) \\ &+ \frac{1}{\delta}\left(\frac{\frac{\partial U_H^*}{\partial \delta}}{1-\frac{p_H}{p_L}(q+(1-q)\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)}\right), \end{aligned} \tag{A.92}$$

where $\frac{\partial U_H^*}{\partial \delta} = \left(\frac{1}{\delta}\right)^2(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)k_H^* > 0$.

Note that at the point where $\gamma = \bar{\gamma}$ it must be the case that $\Lambda_{\bar{C}} = 0$ and thus the above equation can be written as

$$\frac{\partial \Lambda_{\bar{C}}}{\partial \delta} = -\frac{1}{\delta^3}(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)\left(\frac{\bar{k}_L - k_H^*}{1-\frac{p_H}{p_L}(q+(1-q)\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)}\right) < 0. \tag{A.93}$$

As this partial derivative is strictly negative, and it has already been found that $\frac{\partial \Lambda_{\bar{C}}}{\partial \gamma} > 0$, it follows from the implicit function theorem that $\frac{d\bar{\gamma}}{d\delta} > 0$.

Proof that $\frac{d\gamma^*}{d\delta} > 0$ The derivative $\frac{d\gamma^*}{d\delta}$ can be found by applying the implicit function theorem to the function Λ_{IC} as defined in equation (21). The partial derivative of Λ_{IC} with respect to δ is

$$\begin{aligned}\frac{\partial \Lambda_{IC}}{\partial \delta} &= B(q+(1-q)\gamma)\left(\frac{p_H\left[(q+(1-q)\gamma)+\left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)\right]}{p_L\left[(q+(1-q)\gamma)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)\right]}\right)^{\frac{1}{1-\alpha}} \\ &- B\frac{1}{p_L}\left[(q+(1-q)\gamma)-(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)\right], \end{aligned} \tag{A.94}$$

where

$$B = \frac{\frac{1}{\delta^2}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{p_L}{p_H}-1\right)\left(\frac{\gamma-\xi}{1-\xi}\right)p_L}{\left[(q+(1-q)\gamma)+\left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)\right]^2} > 0. \tag{A.95}$$

Now note that around $\Lambda_{IC} = 0$, that is around the point where $\gamma = \gamma^*$, equation (A.86) must hold and the partial derivative can be written as

$$\frac{\partial \Lambda_{IC}}{\partial \delta} = -B\left(\frac{1}{p_L} - 1\right)(q + (1-q)\xi)\left(\frac{1-\gamma}{1-\xi}\right)$$
$$- B(q + (1-q)\gamma)\left[1 - \left(\frac{p_H\left[(q+(1-q)\gamma) + \left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)\right]}{p_L\left[(q+(1-q)\gamma) + \left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)\right]}\right)^{\frac{1}{1-\alpha}}\right],$$

which is strictly negative. Thus it follows that $\frac{\partial \Lambda_{IC}}{\partial \delta} < 0$ and as it has already been established that $\frac{\partial \Lambda_{IC}}{\partial \gamma} > 0$ from application of the implicit function theorem it follows that $\frac{d\gamma^*}{d\delta} > 0$.