Lietuvos matematikos rinkinys
Proc. of the Lithuanian Mathematical Society, Ser. A
DOI: 10.15388/LMR.A.2015.17

ISSN 0132-2818
Vol. **56**, 2015
pages 95–100

Zeroes and poles of a characteristic function for Sturm–Liouville problem with nonlocal integral condition*

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Abstract. This paper presents some new results on a spectrum for the Sturm–Liouville problem with one integral type nonlocal boundary condition depending on tree parameters (γ, ξ_1, ξ_2). Some new results on distribution of the sets of special points (poles, zeros and constant eigenvalue points) are presented.

Keywords: integral boundary conditions, Sturm–Liouville problem, characteristic function.

Introduction

Boundary problems with nonlocal conditions are an area of the fast developing differential equations theory. Nonlocal boundary conditions (NBC's) arise when it is impossible to determine the boundary values of unknown function or/and its derivatives. There is an extensive interest of scientists in the eigenstructure of such type NBC's [1, 2].

In this paper we present some new results on a spectrum for the differential Sturm-Liouville problem with one integral type nonlocal boundary condition depending on tree unknowns. Many results of these investigations are presented as graphs of characteristic functions.

1 Problem formulation

Let us analyze the Sturm–Liouville problem with one classical boundary condition

$$-u'' = \lambda u, \qquad u(0) = 0, \quad t \in (0,1), \tag{1}$$

and another integral type NBC:

$$u(1) = \gamma \int_{\xi_1}^{\xi_2} u(t)\,dt \tag{2}$$

* The research was partially supported by the Research Council of Lithuania (grant No. MIP-047/2014).

with parameters $\gamma \in \mathbb{R}$, $\boldsymbol{\xi} = (\xi_1, \xi_2)$, $0 < \xi_1 < \xi_2 < 1$ and $\lambda \in \mathbb{C}_\lambda := \mathbb{C}$. If $\gamma = 0$, then we have the classical case and all eigenvalues have the following form $\lambda_k = (k\pi)^2$, $v_k(t) = \sin(k\pi t)$, $k \in \mathbb{N}$, where $\mathbb{N} := \{1, 2, 3, \ldots\}$. The same problem we get in the limit case $\xi_1 = \xi_2$. In this paper we use notation $\pi\mathbb{N} := \{\pi k \colon k \in \mathbb{N}\}$, $\mathbb{N}_o$ – for odd, $\mathbb{N}_e$ – even, and $\mathbb{Q}$ – rational numbers, notation $\gcd(n; m)$ for *greatest common divisor* of the integers n and m.

If $\lambda = 0$, then all the functions $u(t) = Ct$ satisfy the Eq. (1). Substituting this solution into NBC (2) we derive the existence of a nontrivial solution ($C \neq 0$). In consequence, eigenvalue $\lambda = 0$ exists if and only if $\gamma = 2/(\xi_2^2 - \xi_1^2)$.

In the general case, for $\lambda \neq 0$, eigenfunction $u(t) := C\sin(qt)$ satisfies Eq. (1) with $\lambda = q^2$, $q = x + \imath y \in \mathbb{C}_q \setminus \{0\}$, where $\mathbb{C}_q := \{q \in \mathbb{C} \colon x = 0, y \geqslant 0 \text{ or } x > 0\}$, then a map $\lambda = q^2$ is a bijection between $\mathbb{C}_q$ and $\mathbb{C}_\lambda$. If q is the root of the equation

$$u(1) = \gamma \int_{\xi_1}^{\xi_2} u(t)\,dt, \tag{3}$$

then there exists a nontrivial solution of the problem (1)–(2). We have

$$\frac{\sin q}{q} = \gamma \frac{\cos(q\xi_1) - \cos(q\xi_2)}{q^2}, \quad q \neq 0. \tag{4}$$

All *nonconstant eigenvalues* (which depend on the parameter γ) are γ-points of a meromorphic *complex characteristic function* (CF) defined on the set $\mathbb{C}_q$ [2]:

$$\gamma_c(q) = \frac{q\sin q}{\cos(q\xi_1) - \cos(q\xi_2)} = \frac{q\sin q}{2\sin\frac{q(\xi_2-\xi_1)}{2}\sin\frac{q(\xi_1+\xi_2)}{2}}. \tag{5}$$

We obtain this function expressing γ from Eq. (4). Zeroes, poles and critical points of the characteristic function are important for description of the spectrum. So, for this, from the function (5) we introduce two entire functions

$$Z(z) := \frac{\sin z}{z}, \qquad P_{\boldsymbol{\xi}}(z) := 2\frac{\sin(z(\xi_1+\xi_2)/2)}{z}\frac{\sin(z(\xi_2-\xi_1)/2)}{z}. \tag{6}$$

All zeroes $Z(z)$ are first order, real and nonnegative (except the point $z = 0$, that is the zero point of the second order). If $q \in \mathbb{C}_q$, then all zeroes of the function $Z(q)$ coincide with eigenvalue point of the classical case $\lambda = 0$.

We call the eigenvalue which do not depend on parameter γ as a *constant eigenvalue point* (CE point). We can define the CE point $q \in \mathbb{C}_q$ for any constant eigenvalue $\lambda = q^2$ [2]. For the problem (1)–(2) we can find CE point as a root of the system:

$$Z(q) = 0, \qquad P_{\boldsymbol{\xi}}(q) = 0. \tag{7}$$

If $q \in \pi\mathbb{N}$, i.e. $Z(q) \neq 0$, and q is a root of equation $P_{\boldsymbol{\xi}}(q) = 0$, then the point q is called a pole. Function $P_{\boldsymbol{\xi}}(z) = 2P^1_{\boldsymbol{\xi}}(z) \cdot P^2_{\boldsymbol{\xi}}(z)$ is a product of two functions

$$P^1_{\boldsymbol{\xi}}(z) := \frac{\sin(z(\xi_1+\xi_2)/2)}{z}, \qquad P^2_{\boldsymbol{\xi}}(z) := \frac{\sin(z(\xi_2-\xi_1)/2)}{z}. \tag{8}$$

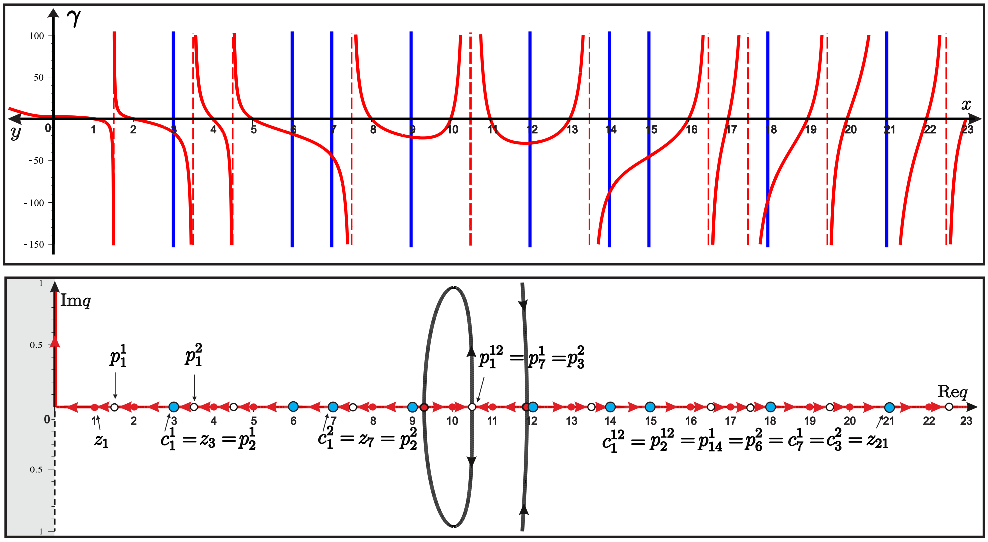


Fig. 1. Zeroes, poles, constant eigenvalues points and Real Characteristic Function for the problem (1)–(2), the *case* $\xi_1, \xi_2 \in \mathbb{Q}$, $\boldsymbol{\xi} = (8/21, 20/21)$.

In the domain $\mathbb{C}_q$ all zeros of the functions $P^1_{\boldsymbol{\xi}}$, $P^2_{\boldsymbol{\xi}}$ are simple and positive, so zeroes of function $P_{\boldsymbol{\xi}}$ can be simple or the second order. If $p \in \mathbb{C}_q$ satisfies conditions $P^1_{\boldsymbol{\xi}}(p) = 0$, $P^2_{\boldsymbol{\xi}}(p) \neq 0$ and $P^1_{\boldsymbol{\xi}}(p) \neq 0$, $P^2_{\boldsymbol{\xi}}(p) = 0$, then the point p is the first order pole. If both conditions $P^1_{\boldsymbol{\xi}}(p) = 0$, $P^2_{\boldsymbol{\xi}}(p) = 0$ are valid for fixed p, then the point p is called the second order pole point (see Fig.1, points p^1_1 and p^2_1 are first order poles, p^{12}_1 – the second order pole).

All *nonconstant eigenvalues* (which depend on the real parameter γ) are γ-points of *Complex-Real Characteristic Function* (Complex-Real CF). Complex-Real CF $\gamma(q)$ is a restriction of function (5) on the domain (net) $\mathcal{N} := \{q \in \mathbb{C}_q \colon \operatorname{Im}\gamma_c(q) = 0\}$. So, $\gamma(q)$ has real values on $\mathcal{N}$ and describes all eigenvalues for real γ.

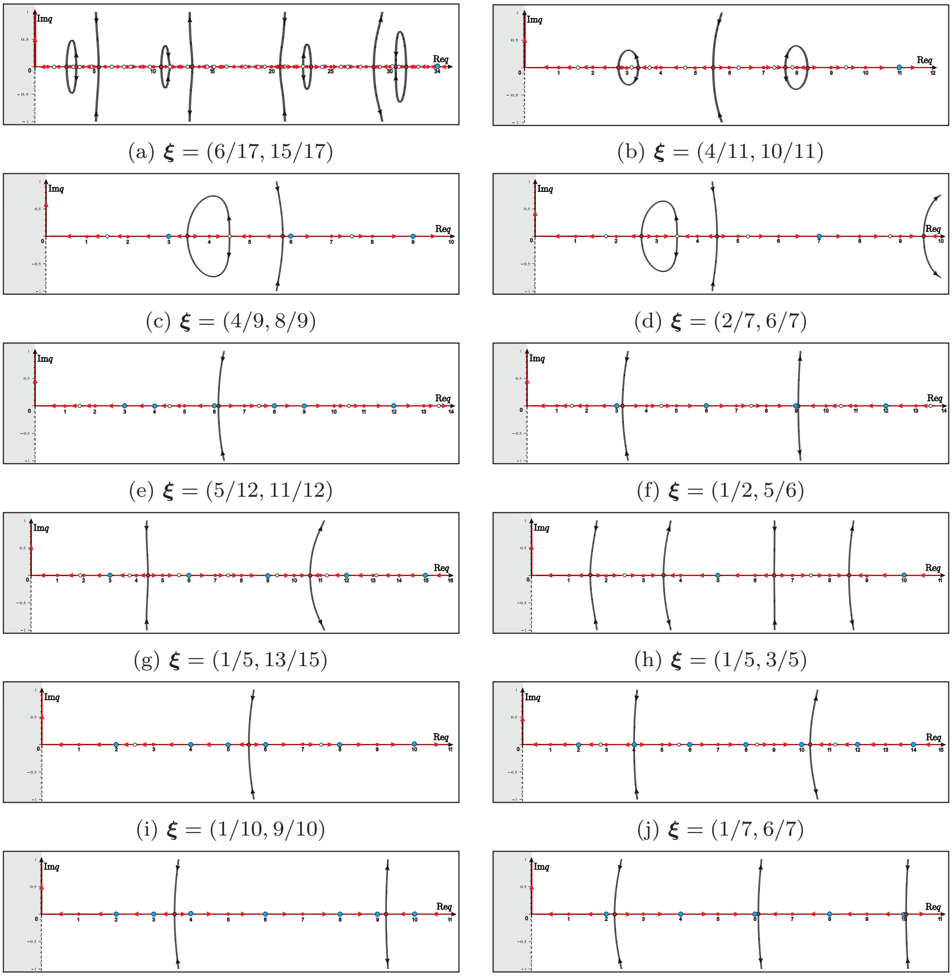
Real CF $\gamma(q)$ is defined on the domain $\{q \in \mathbb{C}_q \colon \lambda = q^2 \in \mathbb{R}\}$. *Real CF* $\gamma(q)$ describes only real eigenvalues ($\lambda = q^2$, where $q = x$, $x \geqslant 0$ and $q = y\imath$, $y \geqslant 0$).

2 Zeroes ant poles of Complex CF

For investigation of Complex CF (5) it is very important to find its zeroes and poles. Zeroes of the characteristic function are first order, real, nonnegative and fixed for all $\xi = (\xi_1, \xi_2)$: $z_k = k\pi$, $k \in \mathbb{N}$. The poles depend on ξ:

$$p^1_k = (2k\pi)/(\xi_1 + \xi_2), \quad k \in \mathbb{N}, \qquad p^2_k = (2k\pi)/(\xi_2 - \xi_1), \quad k \in \mathbb{N}. \tag{9}$$

For the corresponding points z_k, p^1_k, p^2_k we denominate sets $\overline{\mathcal{Z}}$, $\overline{\mathcal{Z}}^1_{\boldsymbol{\xi}}$, $\overline{\mathcal{Z}}^2_{\boldsymbol{\xi}}$. Then the set $\mathcal{Z}_{\boldsymbol{\xi}} = \mathcal{Z}^1_{\boldsymbol{\xi}} + \mathcal{Z}^2_{\boldsymbol{\xi}} + \mathcal{Z}^{12}_{\boldsymbol{\xi}}$ describes all zeroes of the function $P_{\boldsymbol{\xi}}$, where $\mathcal{Z}^1_{\boldsymbol{\xi}} := \overline{\mathcal{Z}}^1_{\boldsymbol{\xi}} \setminus \mathcal{Z}^{12}_{\boldsymbol{\xi}}$ and $\mathcal{Z}^2_{\boldsymbol{\xi}} := \overline{\mathcal{Z}}^2_{\boldsymbol{\xi}} \setminus \mathcal{Z}^{12}_{\boldsymbol{\xi}}$ are two families of the first order zeroes, $\mathcal{Z}^{12}_{\boldsymbol{\xi}} := \overline{\mathcal{Z}}^1_{\boldsymbol{\xi}} \cap \overline{\mathcal{Z}}^2_{\boldsymbol{\xi}}$ is a family of the second order zeroes. If ξ_1/ξ_2 is irrational number, then the second order zeroes



(a) $\boldsymbol{\xi} = (6/17, 15/17)$ (b) $\boldsymbol{\xi} = (4/11, 10/11)$

(c) $\boldsymbol{\xi} = (4/9, 8/9)$ (d) $\boldsymbol{\xi} = (2/7, 6/7)$

(e) $\boldsymbol{\xi} = (5/12, 11/12)$ (f) $\boldsymbol{\xi} = (1/2, 5/6)$

(g) $\boldsymbol{\xi} = (1/5, 13/15)$ (h) $\boldsymbol{\xi} = (1/5, 3/5)$

(i) $\boldsymbol{\xi} = (1/10, 9/10)$ (j) $\boldsymbol{\xi} = (1/7, 6/7)$

(k) $\boldsymbol{\xi} = (1/6, 5/6)$ (l) $\boldsymbol{\xi} = (1/4, 3/4)$

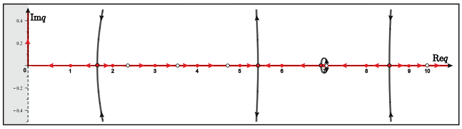
Fig. 2. Domain $\mathcal{N}$ for $\xi_1 = m_1/n_1, \xi_2 = m_2/n_2 \in \mathbb{Q}$.

do not exist. If $\xi_1/\xi_2 = m/n \in \mathbb{Q}$, where $m, n \in \mathbb{N}$, then the second order zeroes of the set $\mathcal{Z}_{\boldsymbol{\xi}}^{12}$ can be discovered by the following formula:

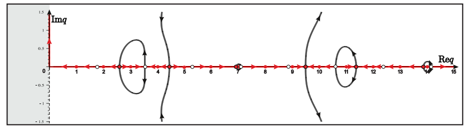
$$p_k^{12} = 2n/(\xi_2 d)k\pi = 2m/(\xi_1 d)k\pi, \quad k \in \mathbb{N},\ d = \gcd(n-m; n+m). \tag{10}$$

We consider the following sets: two families of the first order poles $\mathcal{P}_{\boldsymbol{\xi}}^1 := \mathcal{Z}_{\boldsymbol{\xi}}^1 \setminus \overline{\mathcal{Z}}$ and $\mathcal{P}_{\boldsymbol{\xi}}^2 := \mathcal{Z}_{\boldsymbol{\xi}}^2 \setminus \overline{\mathcal{Z}}$ and a set of the second order poles $\mathcal{P}_{\boldsymbol{\xi}}^{12} := \mathcal{Z}_{\boldsymbol{\xi}}^{12} \setminus \overline{\mathcal{Z}}$. Also we consider the set of the constant eigenvalues points $\mathcal{C}_{\boldsymbol{\xi}} := \mathcal{C}_{\boldsymbol{\xi}}^1 + \mathcal{C}_{\boldsymbol{\xi}}^2 + \mathcal{C}_{\boldsymbol{\xi}}^{12}$, where $\mathcal{C}_{\boldsymbol{\xi}}^1 := \mathcal{Z}_{\boldsymbol{\xi}}^1 \cap \overline{\mathcal{Z}}$ and $\mathcal{C}_{\boldsymbol{\xi}}^2 := \mathcal{Z}_{\boldsymbol{\xi}}^2 \cap \overline{\mathcal{Z}}$ are sets of the points with removable singularity, $\mathcal{C}_{\boldsymbol{\xi}}^{12} := \mathcal{Z}_{\boldsymbol{\xi}}^{12} \cap \overline{\mathcal{Z}}$ is the set of the points with the first order pole; a set of zeroes $\mathcal{Z}_{\boldsymbol{\xi}} := \overline{\mathcal{Z}} \setminus \mathcal{C}_{\boldsymbol{\xi}}$.

Case $\boldsymbol{\xi_1 = m_1/n_1, \xi_2 = m_2/n_2 \in \mathbb{Q}}$: In this case $\xi_1/\xi_2 \in \mathbb{Q}$ and for all $\boldsymbol{\xi}$, the set $\mathcal{C}_{\boldsymbol{\xi}}^{12} \neq \varnothing$ and there exist a few special cases for other sets of poles and constant

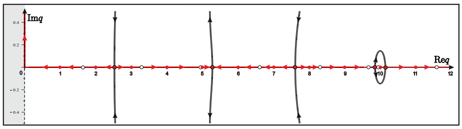


(a) $\boldsymbol{\xi} = (\sqrt{2}/10, \sqrt{2}/2)$

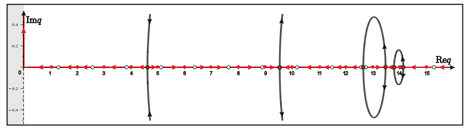


(b) $\boldsymbol{\xi} = (\sqrt{2}/5, (3\sqrt{2})/5)$

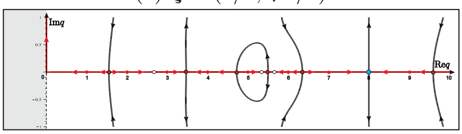
Fig. 3. Domain $\mathcal{N}$ for $\xi_1/\xi_2 = m/n \in \mathbb{Q}$, $\xi_1, \xi_2 \notin \mathbb{Q}$.



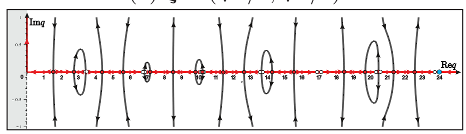
(a) $\boldsymbol{\xi} = (1/2, \sqrt{2}/2)$



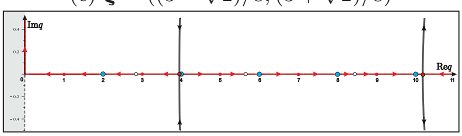
(b) $\boldsymbol{\xi} = (\sqrt{2}/2, \sqrt{3}/2)$



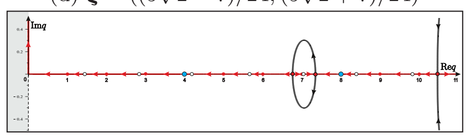
(c) $\boldsymbol{\xi} = ((3-\sqrt{2})/8, (3+\sqrt{2})/8)$



(d) $\boldsymbol{\xi} = ((5\sqrt{2}-7)/24, (5\sqrt{2}+7)/24)$



(e) $\boldsymbol{\xi} = ((2-\sqrt{2})/4, (2+\sqrt{2})/4)$



(f) $\boldsymbol{\xi} = ((2\sqrt{2}-1)/4, (2\sqrt{2}+1)/4)$

Fig. 4. Domain $\mathcal{N}$ for $\xi_1/\xi_2 \notin \mathbb{Q}$.

eigenvalues. For example if $\boldsymbol{\xi} = (8/21, 20/21)$, then all sets $\mathcal{P}^1_{\boldsymbol{\xi}}$, $\mathcal{P}^2_{\boldsymbol{\xi}}$, $\mathcal{P}^{12}_{\boldsymbol{\xi}}$, $\mathcal{C}^1_{\boldsymbol{\xi}}$, $\mathcal{C}^2_{\boldsymbol{\xi}}$, $\mathcal{C}^{12}_{\boldsymbol{\xi}}$ and $\mathcal{Z}_{\boldsymbol{\xi}}$ are not empty (see Fig. 1). The other example is if $\boldsymbol{\xi} = (6/17, 15/17)$, then all sets are not empty, except $\mathcal{C}^1_{\boldsymbol{\xi}}$, $\mathcal{C}^2_{\boldsymbol{\xi}}$ ($\mathcal{C}^1_1 = \mathcal{C}^2_1 = \mathcal{C}^{12}_1$, see Fig. 2(a)). Further, if $\boldsymbol{\xi} = (4/11, 10/11)$, then the sets $\mathcal{P}^{12}_{\boldsymbol{\xi}} = \mathcal{C}^1_{\boldsymbol{\xi}} = \mathcal{C}^2_{\boldsymbol{\xi}} = \varnothing$ and $\mathcal{P}^{12}_1 = \mathcal{C}^{12}_1$ (Fig. 2(b)). In the cases Figs. 2(c)–(h) the set $\mathcal{P}^1_{\boldsymbol{\xi}} \neq \varnothing$ however the set $\mathcal{P}^2_{\boldsymbol{\xi}}$ is empty. In the cases of Figs. 2(c)–(d) exist second order pole (there Figs. 2(e)–(h) $\mathcal{P}^{12}_{\boldsymbol{\xi}} = \varnothing$). For the $\boldsymbol{\xi} = (4/9, 8/9)$, $\boldsymbol{\xi} = (5/12, 11/12)$, $\boldsymbol{\xi} = (1/2, 5/6)$ the set $\mathcal{C}^1_{\boldsymbol{\xi}} \neq \varnothing$ and also for $\boldsymbol{\xi} = (5/12, 11/12)$ the set $\mathcal{C}^2_{\boldsymbol{\xi}} \neq \varnothing$. In the case Fig. 2(g) first family constant eigenvalue do not exist and for the case Figs. 2(d),(h) both sets $\mathcal{C}^1_{\boldsymbol{\xi}}$ and $\mathcal{C}^2_{\boldsymbol{\xi}}$ are empty. For the fixed $\boldsymbol{\xi}$ values, when $n_1 = n_2 = n$ and $m_1 + m_2 = n$ (see Figs. 2(i)–(l)): $\mathcal{P}^1_{\boldsymbol{\xi}} = \varnothing$ and $\mathcal{P}^{12}_{\boldsymbol{\xi}} = \varnothing$. In contrast to the case shown in Figs. 2(k) and (l), when $\boldsymbol{\xi} = (1/10, 9/10)$ and $\boldsymbol{\xi} = (1/7, 6/7)$, then the set $\mathcal{P}^2_{\boldsymbol{\xi}}$ is not empty (see Figs. 2(i)–(l)). For all examples (i)–(l) in Fig. 2, the set $\mathcal{C}^1_{\boldsymbol{\xi}} \neq \varnothing$, but the constant eigenvalue points depending on second order pole family are obtained only for examples (i),(k) in Fig. 2. For this instance, we can use expressions (9)–(10) and get poles

$$p^1_k = (2n_1n_2k\pi)/m_+, \quad p^2_k = (2n_1n_2k\pi)/m_-, \quad p^{12}_k = (2n_1n_2k\pi)/d, \quad k \in \mathbb{N}. \qquad (11)$$

From here, the constant eigenvalue points are

$$c^1_k = (2n_1n_2k\pi)/d_1, \quad c^2_k = (2n_1n_2k\pi)/d_2, \quad c^{12}_k = (2n_1n_2k\pi)/d_{12}, \quad k \in \mathbb{N}. \qquad (12)$$

where $m_+ = m_2n_1+m_1n_2$, $m_- = m_2n_1-m_1n_2$, $d = \gcd(m_-;m_+)$, $d_1 = \gcd(2n_\pm;m_+)$, $d_2 = \gcd(2n_\pm;m_-)$, $d_{12} = \gcd(2n_\pm;m_+;m_-)$.

Case $\boldsymbol{\xi_1/\xi_2 = m/n \in \mathbb{Q},\ \xi_1,\xi_2 \notin \mathbb{Q}}$: In this case $\xi_1+\xi_2,\ \xi_2-\xi_1 \notin \mathbb{Q}$. In consequence, there exist no constant eigenvalues ($\mathcal{C}_{\boldsymbol{\xi}} = \varnothing$, (see Fig. 3). So, in this case we have two families of the first order poles in $\mathcal{P}^1_{\boldsymbol{\xi}}$ and $\mathcal{P}^2_{\boldsymbol{\xi}}$, respectively, and the second order poles in $\mathcal{P}^{12}_{\boldsymbol{\xi}}$. Note, that for the special case $\xi_1/\xi_2 = (l-1)/(l+1)$, $1 < l \in \mathbb{N}$, $\mathcal{P}^2_{\boldsymbol{\xi}} = \varnothing$, in consequence $p^2_1 = p^{12}_1$ (see Fig. 3(b)).

Case $\boldsymbol{\xi_1/\xi_2 \notin \mathbb{Q}}$: In this case at least one number $\xi_1+\xi_2$ or $\xi_2-\xi_1$ is irrational. If $\xi_1+\xi_2 \notin \mathbb{Q}$ and $\xi_2-\xi_1 \notin \mathbb{Q}$ then CF have two families of the first order poles in $\mathcal{P}^1_{\boldsymbol{\xi}}$ and $\mathcal{P}^2_{\boldsymbol{\xi}}$, respectively. Second order poles and constant eigenvalue points do not exist (see Figs. 4(a), (b)). If $\xi_1+\xi_2 = m_+/n_+ \in \mathbb{Q}$ then $\mathcal{C}^1_{\boldsymbol{\xi}} \neq \varnothing$ and constant eigenvalue points exist $c^1_k = p^1_{m_+/d_1k} = z_{2n_+/d_1k} = 2n_+/d_1k\pi$, $k \in \mathbb{N}$, $d_1 = \gcd(2n_+;m_+)$ where $n_\pm = n_1n_2$, $m_+ = m_2n_1+m_1n_2$ (see Figs. 4(c) and 4(e)). In this case if $\xi_1+\xi_2 \neq 2/l$ then $\mathcal{P}^{12}_{\boldsymbol{\xi}} = \varnothing$, $\mathcal{C}^2_{\boldsymbol{\xi}} = \varnothing$ and $\mathcal{C}^{12}_{\boldsymbol{\xi}} = \varnothing$. In addition, if $\xi_1+\xi_2 \neq 2/l$ is not satisfied, then the set $\mathcal{P}^1_{\boldsymbol{\xi}}$ is also empty. If $\xi_2-\xi_1 = m_-/n_- \in \mathbb{Q}$ then $\mathcal{C}^2_{\boldsymbol{\xi}} \neq \varnothing$ and $c^2_k = p^2_{m_-/d_2k} = z_{2n_-/d_2k} = 2n_-/d_2k\pi$, $k \in \mathbb{N}$, $d_2 = \gcd(2n_-;m_-)$, where $n_\pm = n_1n_2$, $m_- = m_2n_1-m_1n_2$. The other sets of constant eigenvalue points ($\mathcal{C}^1_{\boldsymbol{\xi}}$, $\mathcal{C}^{12}_{\boldsymbol{\xi}}$) and poles ($\mathcal{P}^{12}_{\boldsymbol{\xi}}$) are empty if $\xi_1+\xi_2 \neq 2/l$ (Fig. 4(d)). If this condition is not satisfied then, additionally, $\mathcal{P}^2_{\boldsymbol{\xi}} = \varnothing$ (see Fig. 4(f)).

3 Conclusion

The main result of the article is the classification of poles and zero points. The dependence of zeros and poles on the integral BC parameters ξ_1 and ξ_2 is investigated. The constant eigenvalues non-existence condition (sets $\mathcal{C}^1_{\boldsymbol{\xi}}$, $\mathcal{C}^2_{\boldsymbol{\xi}}$ and $\mathcal{C}^{12}_{\boldsymbol{\xi}}$ are empty) is $\xi_1/\xi_2 = m/n \in \mathbb{Q}$, $\xi_1,\xi_2 \notin \mathbb{Q}$. If the following condition $\xi_1/\xi_2 \notin \mathbb{Q}$ is satisfied, then $\mathcal{P}^{12}_{\boldsymbol{\xi}} = \varnothing$ and $\mathcal{C}^{12}_{\boldsymbol{\xi}} = \varnothing$. For all ξ_1 and ξ_2 satisfying condition $\xi_1,\xi_2 \in \mathbb{Q}$, the set $\mathcal{C}^{12}_{\boldsymbol{\xi}}$ is not empty.

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REZIUMĖ

Šturmo ir Liuvilio uždavinio su nelokaliąja integraline kraštine sąlyga charakteristinės funkcijos nuliai ir poliai

A. Skučaitė ir A. Štikonas

Straipsnyje pateikiami nauji rezultatai, gauti tiriant diferencialinio Šturmo ir Liuvilio uždavinio su nelokaliąja integraline kraštine sąlyga spektro struktūrą. Pateikti nauji rezultatai, aprašantys nulių, polių ir pastoviųjų tikrinių reikšmių pasiskirstymą ir priklausomybę nuo parametrų ξ_1 ir ξ_2.

Raktiniai žodžiai: integralinės kraštinės sąlygos, Šturmo ir Liuvilio uždavinys, charakteristinė funkcija.