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# FRACTAL ANALYSIS OF TIME SERIES OF THE CRYPTOCURRENCIES PRICE

**Master's thesis**

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## Abstract

The examination of financial markets behaviour crucial part of the financial investments theory. The methods for analyzing the financial, markets established in the 1960's and 1970's, were valid only during periods of stable market conditions. They are based on the assumption that the financial market's behaviour is subject to the normal distribution law. In the nineties began to look at this problem from the point of view of fractal analysis. It was observed that financial time series have the property of self-similarity. In this paper, we have tested long memory property for the five biggest cryptocurrencies: Bitcoin, Ethereum, Binance Coin, XRP, and Cardano. This thesis studies persistence and volatility using R/S analysis was carried out. Our findings show that four out of five cryptocurrencies have a significant long memory, supporting the use of fractional Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH) extensions as a suitable modeling technique. In this paper, the Fractionally Integrated GARCH (FIGARCH) models, with skewed student distribution, were produced and compared with GARCH models. Models were compared using Akaike information criteria, which indicated the improvement of the model's fitness. The paper ends with some concluding remarks and future directions of research.

**Keywords:** Fractal analysis, R/S analysis, GARCH, Fractionally integrated GARCH, volatility of financial series.

## Santrauka

Finansų rinkų elgesio tyrimas yra svarbi finansinių investicijų teorijos dalis. Finansinės analizės metodai, šeštajame ir aštuntajame dešimtmečiuose įkurtose rinkose, galiojo tik stabilių rinkos sąlygų laikotarpiais. Jie buvo pagrįsti prielaida, kad finansų rinkos elgsena yra taikomas normalaus paskirstymas. Praėjusio amžiaus devintajame dešimtmetyje į šią problemą buvo pradėta žiūrėti iš fraktalinės analizės pusės ir pastebėta, kad finansinės laiko eilutės yra atsikartojančios į save. Šiame darbe išbandėme penkių didžiausių kriptovaliutų ilgosios atminties savybę: Bitcoin, Ethereum, Binance Coin, XRP ir Cardano. Buvo atliktas patvarumo ir nepastovumo eilučių tyrimas naudojant R/S analizę. Mūsų išvados rodo, kad keturios iš penkių kriptovaliutų turi didelę atmintį, o tai leidžia naudoti fraktalinius generalizuoto autoregresyvaus sąlyginio heteroskedastiškumo (GARCH) plėtinius kaip tinkamą modeliavimo metodą. Šiame darbe buvo sukurti fraktališkai integruoti GARCH (FIGARCH) modeliai su iškreiptu studentų pasiskirstymu ir palyginti su GARCH modeliais. Modeliai buvo lyginami naudojant Akaike informacinius kriterijus, kurie parodė modelio tinkamumo pagerėjimą. Rašto darbas yra baigiamas pastebėjimais ir būsimomis tyrimų kryptimis.



**Raktažodžiai:** Fraktalinė analizė, R/S analizė, GARCH, Fraktališkai integruotas GARCH, laiko eilutės nepastovumas.

# 1 Introduction

By December of 2021, all cryptocurrencies, measured in market capitalization, had a combined market value of 2.2 trillion dollars. Bitcoin alone made up nearly 895 billion dollars of this value, while Ethereum – the second-biggest cryptocurrency, was catching up with Bitcoin, and made up over 461 billion dollars. Given the significant value of currencies, some people see their value as actual gold-based currencies. In contrast, others view them as investment opportunities. The result has been large swings in the value for both currencies over short periods of time.

In 2017, the value of a single Bitcoin increased 2000%, going from $863 on January 9, 2017, to the highs of $17,550 in December 2017. Eight weeks later, in February 2018, the price of a single Bitcoin had plunged to a value of $7,9643. Blockchain, the promising technology that underpins cryptocurrencies, makes it probable that they will continue to be utilized in some capacity and that their use will expand. The variability in the price of cryptocurrencies creates uncertainty for people and investors who use them as a currency rather than an investment. Cryptocurrencies are a comparably new store of value (Bitcoin was launched in 2009) relative to the well-established currencies such as the U.S. dollar. What causes price changes in this new store of value is an area of debate. Researcher Ladislav Kiroufek found that Bitcoin is a unique asset. Its price behaves similarly to both standard financial assets and speculative ones. Given that cryptocurrency prices do not behave like traditional currencies, the prices are tough to model and predict.

This paper focuses on analyzing time series from the point of view of fractal analysis and creating a forecasting model to predict the returns of cryptocurrencies. For this paper, Bitcoin, Ethereum, Binance Coin, XRP, and Cardano were chosen as analysis objects, which have the biggest capitalization market and have more than four years of historical data. Such cryptocurrencies are more predictable and more interesting from the point of view of investment.

Fractal analysis, for a long time, was used to analyze and recognize when features of complex ecological systems are altered, since fractals can characterize the natural complexity in such systems. Thus, fractal analysis helps quantify patterns in nature and identify deviations from these natural sequences. At the beginning of the 90s, the study of financial markets began in fractal analysis. Financial time series, with the property of self-similarity, began to be regarded as fractals [13]. In this paper, Rescaled range analysis (R/S analysis), one of the fractal analysis methods, was used to observe the presence of long-term memory in time series.

The Hurst index ($H$ index) captures a strong trend in data, if it exists. The $H$ index, also known as the index of long-range dependence, can extrapolate a future value or average for the data. The Hurst index ranges between zero and one, measuring persistence, randomness, or mean reversion. Time series that displays a random stochastic process has the $H$ index close to 0.5. When $H$ is greater than 0.5, the data is exhibiting a strong long-term trend, and when $H$ is less than 0.5, it is likely to reverse the trend over the time frame considered.[13]

Chapter 1 provides a brief overview of the working principles and history of cryptocurrency. It quickly introduces the reader to all the cryptocurrencies and data, which will be mentioned or used in this paper. Chapter 2 introduces the reader to the methodology used and the steps to execute it. Chapter 3 consists of the descriptive statistics of the data and the results after R/S analysis was done.

Chapter 4 compares and shows the best GARCH and FIGARCH models, created with the hourly data and checks whether including long memory into the model improves the model's accuracy. Finally, Section 5 summarizes our conclusions and the opportunities for future work.

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## 2 Cryptocurrencies

The research, outlined further in this article, requires understanding where and why the data was acquired and how cryptocurrencies differ from typical fiat currency or company stocks traded on regular stock exchanges. This section will go into the history of these data sources and why they were picked so that the reader can understand the final analysis.

### 2.1 Working principle of the Cryptocurrencies and history

This research examines market capitalization statistics for the world's most valuable cryptocurrencies. Bitcoin is the most popular, and Ethereum is the second most popular. The first cryptocurrency, Bitcoin, was founded in 2009 by a person or group known only as "Satoshi Nakamoto." With the launch of Bitcoin, "Satoshi Nakamoto" has published a study, "Bitcoin: A Peer-to-Peer Electronic Cash System," which described a peer-to-peer payment system, based on electronic cash (cryptocurrencies) that could be sent without a middleman from one party to another without the need for a third party to verify the transaction. This breakthrough is made possible by using "blockchain," which functions as a shared ledger on a peer-to-peer network, with all transactions validated by the network and hence impossible to counterfeit. Blockchain technology has applications that go beyond peer-to-peer payment systems. The technology offers security, anonymity, and a distributed ledger, making it ideal for internet-of-things applications, distributed storage systems, healthcare, and other applications. Many more blockchains and cryptocurrencies have been formed due to the blockchain's wide range of applications. Cryptocurrencies are linked to the blockchain because they incentivize machines to run and validate the blockchain and the electricity they require. Cryptocurrencies will become more popular as the use of blockchains grows. This provides them intrinsic value, but a variety of variables can influence the scale of said value. Because this is a new sort of money and a new type of storing value, gaining a better knowledge of what can drive price shifts is valuable.

### 2.2 Bitcoin

Bitcoin (BTC) is a decentralized cryptocurrency launched in 2009. It is a peer-to-peer online currency, meaning that all transactions happen directly between equal, independent network participants, without the need for any intermediary to permit or facilitate them [4]. According to Nakamoto's own words, Bitcoin was created to allow "online payments to be sent directly from one party to another without going through a financial institution." Some concepts for a similar type of decentralized electronic currency precede BTC [6]. Still, Bitcoin holds the distinction of being the first-ever cryptocurrency to come into actual use.

The Bitcoin network uses the Proof-of-Work (PoW) consensus algorithm, which is used to validate transactions, secure the network, and mine new tokens. The critical drawback of the PoW mechanism is extra computational work, which leads to a large amount of excess electricity consumption.

The market cap of Bitcoin stood over 900 billion USD when this paper was written. It has a circulating supply of over 18.9 million BTC coins and a maximum supply of 21 million BTC coins. The cryptocurrency price has reached an all-time high of $68,789.63$ USD. Historical BTC prices are shown in Figure 1.

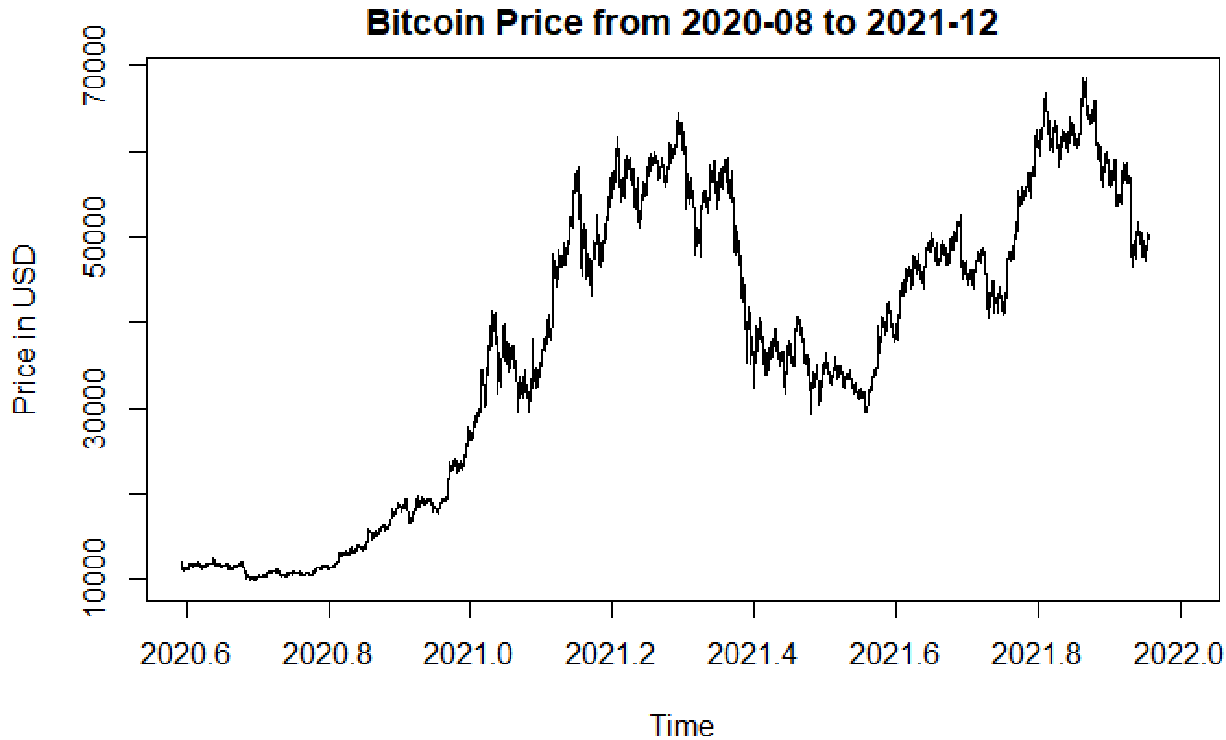


Figure 1: Bitcoin historical price

### 2.3 Ethereum

Ethereum (ETH) is a decentralized open-source blockchain system that features its cryptocurrency, Ether [9]. Ethereum works as a platform for numerous other cryptocurrencies and executing decentralized smart contracts. Ethereum was first described in 2013 in the white paper by Vitalik Buterin. Buterin, along with other co-founders, secured funding for the project in an online public crowd sale in the summer of 2014. The project team managed to raise $18.3 million in Bitcoin. Ethereum's price in the Initial Coin Offering (ICO) was $0.311, with over 60 million Ether sold. Taking Ethereum's price now puts the return on investment (ROI) at an annualized rate of around 270%, essentially almost quadrupling investment every year since the summer of 2014.

Same as Bitcoin, Ethereum currently uses a Proof-of-Work (PoW) consensus mechanism. Ethereum network will switch to a different, more efficient consensus algorithm – Proof-of-Stake (PoS). The selected algorithm will fix issues arising from the original PoW protocol, such as scalability limitations and high energy consumption.

The market cap of Ethereum stands over 450 billion USD, making it the second-biggest cryptocurrency when this paper was written. It has a circulating supply of 118 million ETH coins and does not have the maximum limit of the available coins. Historical ETH prices are shown in Figure 2.

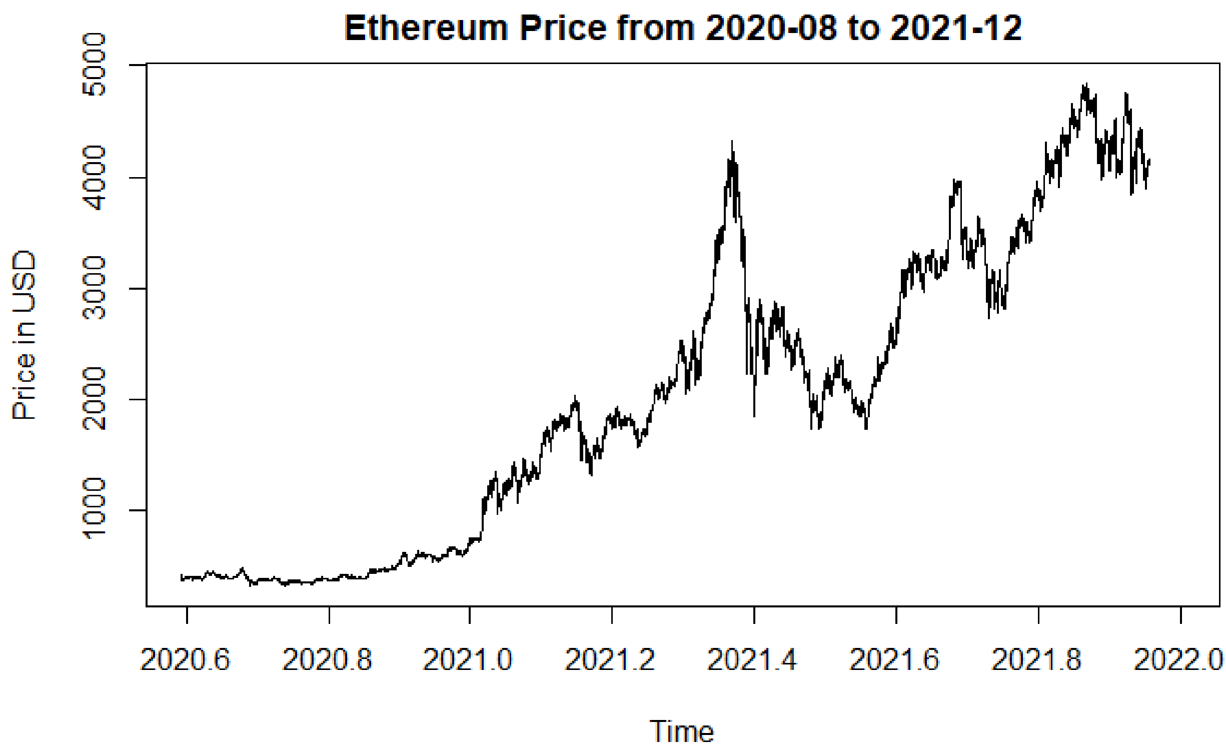


Figure 2: Ethereum historical price

### 2.4 Binance Coin

Binance Coin (BNB) has the most significant cryptocurrency daily exchange volume globally. Binance brings cryptocurrency exchanges to the forefront of financial activity globally. Binance's idea is to show this new paradigm in global finance.

Binance Chain relies on a system of 21 validators with proof-of-Staked-Authority (PoSA) consensus that can support short block times and lower fees. As Ethereum price rises and network usage increases, Binance Chain achieves significantly lower fees and a faster transaction rate relative to the Ethereum network.

Aside from being the largest cryptocurrency exchange globally, Binance has launched a whole ecosystem of functionalities for its users. The Binance network includes the Binance Chain, Binance Smart Chain, Binance Academy, Trust Wallet, and Research projects, which all employ the powers of blockchain technology to bring new-age finance to the world. Binance Coin is an integral part of the successful functioning of many of the Binance sub-projects. [17]

The market cap of Binance Coin stands over 88 billion USD, which makes it the third biggest cryptocurrency at the time this paper was written. It has a circulating supply of 166 million BNB coins, which is the maximum limit of the available coins. Historical BNB prices are shown in Figure 3.

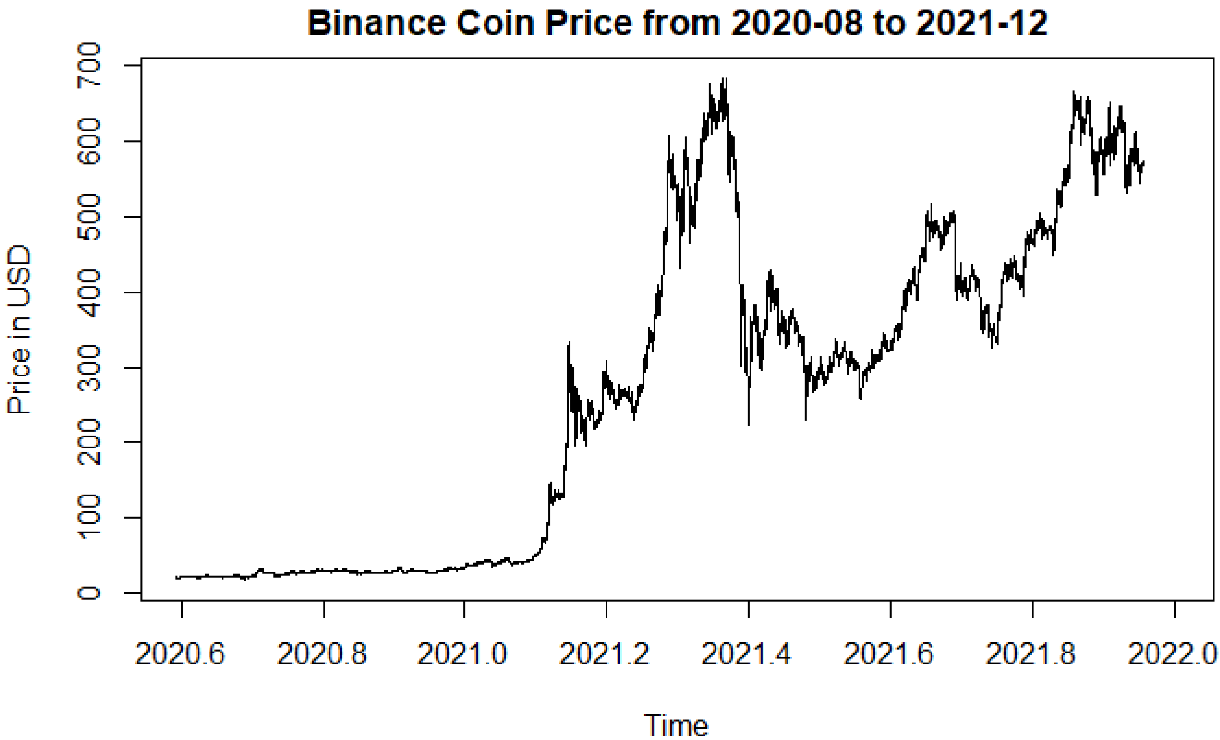


Figure 3: Binance Coin historical price

### 2.5 XRP

XRP is the currency that runs on a digital payment platform called RippleNet, which is on top of a distributed ledger database called XRP Ledger. The XRP Ledger is open-source and is not based on blockchain but rather the previously mentioned distributed ledger database. The idea behind the Ripple payment platform was first voiced in 2004 by Ryan Fugger [14]. It was not until Jed McCaleb, and Chris Larson took over the project in 2012, that Ripple began to be built.

The market cap of XRP stands over 38 billion USD, making it the eighth biggest cryptocurrency when this paper was written. It has a circulating supply of 47 billion XRP coins, and the maximum limit of available coins is 100 billion XRP. Historical XRP prices are shown in Figure 4.

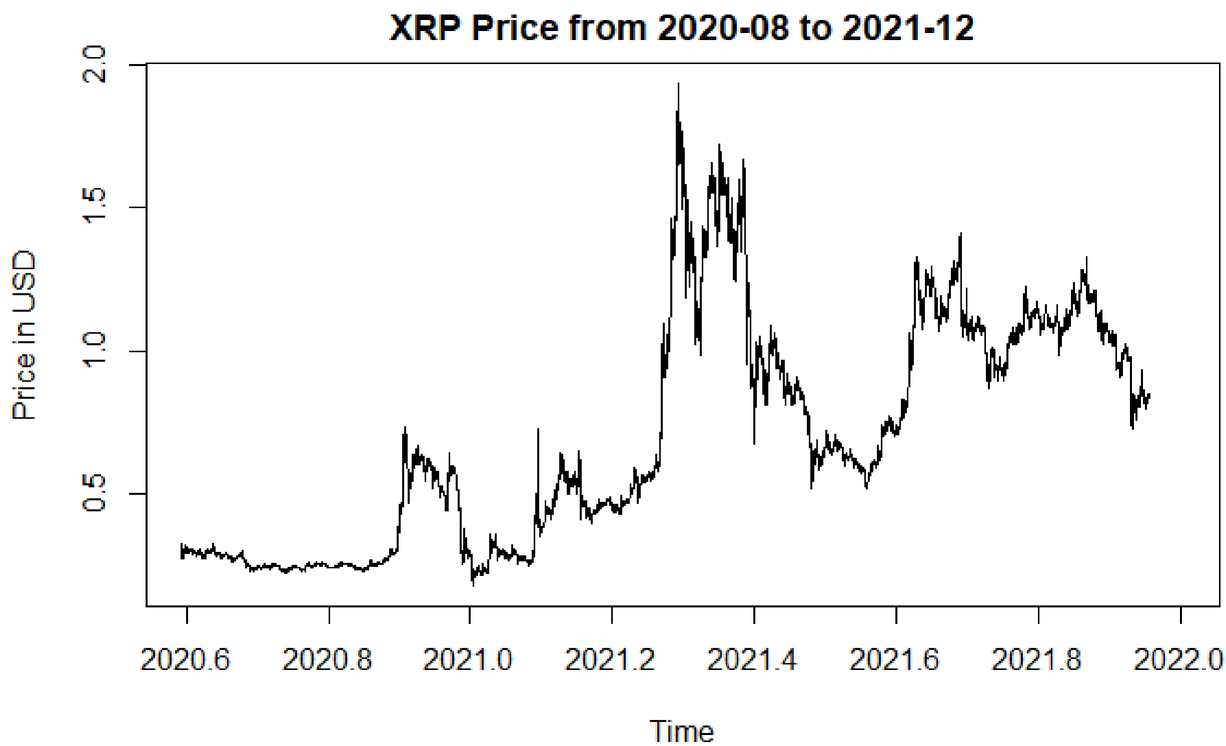


Figure 4: XRP historical price

### 2.6 Cardano

Cardano (ADA) is a proof-of-stake blockchain platform. The open-source project aims to "redistribute power from unaccountable structures to the margins to individuals".

Cardano was founded in 2017 and named after the 16th-century Italian polymath Gerolamo Cardano. The ADA token is designed to ensure that owners can participate in the operation of the network. Because of this, those who hold the cryptocurrency have the right to vote on any proposed changes to the software [8].

Cardano uses the proof-of-Stake (PoS) consensus mechanism, which is less energy-intensive than the proof-of-Work algorithm relied upon by Bitcoin. Although the much larger Ethereum is going to be upgrading to PoS, this transition is going to take place gradually [7].

The market cap of Cardano stands over 42 billion USD, which makes it the 6th biggest cryptocurrency at the time this paper was written. It has a circulating supply of 33.5 billion ADA coins, and the maximum limit of available coins is 45 billion ADA. The historical price of Cardano is shown in Figure 5.

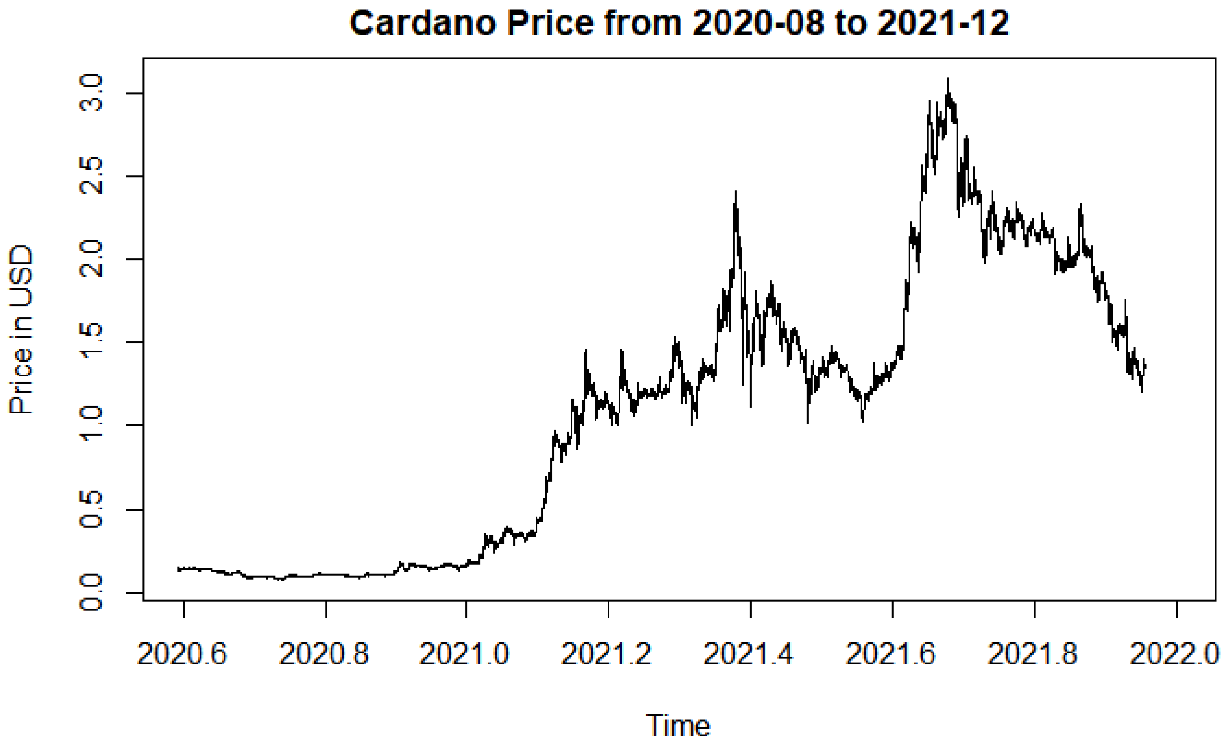


Figure 5: Cardano historical price

## 2.7 Data

https://www.cryptodatadownload.com/data/binance/ website has already collected cryptocurrency data gathered from the exchange platform ”Binance”. Datasets used for this paper consist of hourly prices of Bitcoin, Ethereum, Binance Coin, XRP, and Cardano during the pandemic from August 2020 up until December 2021, which consists of more than 11 000 values per cryptocurrency. The data was used for R/S analysis to check whether the time series has a long memory and to make GARCH models for forecasting. Daily data was used to make forecasting models. In this paper, we compare the results of two models: GARCH and Fractional Integrated GARCH models.

# 3 Methodology

To better detect various characteristics of cryptocurrency returns, we employ long memory tests before modelling the volatility of these series. According to the results of these long memory tests, we later estimate the appropriate GARCH and Fractionally Integrated GARCH (FIGARCH) class models, which accurately take into account the asymmetry in the relevant series. The long memory process could be described as a slowly decaying autocovariance function. At this point, it is necessary to remind that volatility is highly persistent in high-frequency financial time series.

## 3.1 $R/S$ Analysis procedure

Re-scaled range statistic (R/S) was introduced by Hurst (1951) and later revised by Mandelbrot and Wallis (1969) to detect the presence of long-term memory in time series. According to Mandelbrot (1971), R/S statistics could be used in economic and financial analysis. Essentially, the re-scaled range statistic R/S is the range of partial sums of deviations of a time series from its mean, re-scaled by its standard deviation (Zivot and Wang 2003). Consequently, the Hurst exponent H symbolizes the scaling behaviour of the range of cumulative departures of a time series from its mean.

To determine the type of memory of financial time series, $R/S$ analysis was used, which consists of performing the following steps:

- The Source series with length $M$ is converted using logarithmic ratios. The result is a time series of length $N = M - 1$ with the following values:

$$N_i = \log\left(\frac{M_{i+1}}{M_i}\right),$$

  where $i = 1, 2, ..., M - 1$. The requirements for the sample are that its volume $N$ must be large enough and be a multiple of 2.

- Further, this series is divided into $A$ adjacent sub-periods of length $n$, such that $An = N$. Each of them is denoted by $I_a$, where $a = 1, 2, ..., A$. We denote every item in $I_a$ through $N_{k,a}$ at $k = 1, 2, ..., n$. The average value $N_{k,a}$ is determined in each sub-period according to the formula:

$$e_a = \frac{1}{n}\sum_{k=1}^{n} N_{k,a},$$

  where $a = 1, 2, ..., A$.

- Next, a series of accumulated deviations ($X_{k,a}$) are compiled for each sub-period $I_a$. It is defined as follows:

$$X_{k,a} = \sum_{i=1}^{k}(N_{i,a} - e_a),$$

  where $k = 1, 2, ..., n$.

- In the next step, the range of the accumulated frequency of each sub-period $I_a$ is determined:

$$R_a = \max_{1 \leq k \leq n} X_{k,a} - \min_{1 \leq k \leq n} X_{k,a}.$$

- Then we calculate the sample standard deviation for each sub-period $I_a$ according to the formula:

$$S_a = \sqrt{\frac{1}{n} \sum_{i=1}^{k} (N_{i,a} - e_a)^2}.$$

- The average value $R/S$ is determined for length n according to the following formula:

$$(R/S)_n = \frac{1}{A} \sum_{a=1}^{A} \frac{R_a}{S_a}.$$

- The last step is to increase the length $n$ to the next higher value. Steps 1-6 are repeated until $n = N/2$. Finally, linear regression is constructed, where the variable $\log(n)$ is taken as an argument, and the dependent value is $\log\left(\frac{R}{S}\right)$. The slope of the equation is an estimate of the Hurst index, $H$.

The values of the Hurst index can take the following values:

- Whether the process is characterized by long-term memory, that is persistence. It means that subsequent indicators are highly dependent on previous ones. This is close to the sensitivity to the initial conditions, which is characteristic of chaos.

$$0,5 < H \leq 1.$$

- If the sample is random.

$$H = 0,5.$$

- If the system is changing faster than random. The Hurst indicator means not a persistent process.

$$0 \leq H < 0,5.$$

Then, the financial time series was investigated concerning chaotic cycles. For this, the V-statistic was calculated, which gives a more accurate measurement of the cycle length. It can be said that the higher the Hurst index is, the smaller the number of “notches” in the time series. This indicator can be used to get a good performance in the presence of noise. It is defined as follows: [13]

$$V_n = \frac{(R/S)_n}{\sqrt{n}}.$$

This ratio will lead to a constant line if the $R/S$ statistics changes the scale in proportion to the square root of time [18]. For example, function graph $V$ will be flat if the process is independent probabilistic.

If the process is persistent and $R/S$ changes its scale faster than the square root of time ($H > 0,5$), the graph has a slope directed upwards. If the process is not persistent ($H < 0,5$), the graph has a slope directed downwards. Plotting $V$-statistics is as follows: values $V_n$ are put on the $Oy$ axis, and $\log(n)$ are arranged along axis $Ox$. At the points in which the graph becomes straightforward, the process with long-term memory dissipates.

To test the null hypothesis that the system is an independent process, these values must be compared with the theoretical values $E((R/S)_n)$. These values are calculated by the formula:

$$E((R/S)_n) = \frac{(n-0,5)}{n} \cdot \sqrt{\frac{2}{\pi n}} \cdot \sum_{r=1}^{n-1} \sqrt{\frac{n-r}{r}}$$

Next, the same series of financial indicators are checked for volatility. For this, the values of the original series were transformed into a series of logarithmic differences:

$$S_i = \ln\left(\frac{M_i}{M_{i-1}}\right), \text{ where } i = 2, 3, ..., M.$$

Volatility is the deviation of adjacent increments $S_i$. These increments are disjoint and independent:

$$V_n = \frac{\sum_{i=1}^{n}(S_i - \overline{S})^2}{n-1},$$

where $V_n$ is dispersion for $n$ days, $\overline{S}$ is an average value for $S_i (i = 1, 2, ..., n)$. Change in volatility over time $n$ is calculated:

$$L_n = \ln\left(\frac{V_n}{V_{n-1}}\right).$$

A study was conducted in the presence of cycles. The entire period was divided into sub-periods according to the schedule of V-statistics of the financial series. For each sub-period, the Hurst index was calculated, and the significance of the regression equations was determined. After interpreting the result obtained, the investigated persistent financial series cycles were determined.

### 3.2 GARCH Model

Data used for GARCH models must be stationary. The data is stationary if the mean, variance, and autocorrelation structure are constant over the time interval. A stationary series does not contain trends, and it has no seasonality. The stationarity of the data is essential to describe the future behaviour of the process. If the data is not stationary, it is necessary to transform it using the first difference. The first differences are the data changes from one period to the next. Plotting the data of the first difference can reveal whether the data has been transformed to a stationary series or not. If it is still not stationary, the second difference is taken [11]. The model fitting can be carried out once the stationarity of the series has been achieved. In the current study, two-time series models will be considered.

The central assumption underlying Bollerslev's (1986) GARCH modelling is that the market variance depends not only on historical conditional market variance but also on market shocks. Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH, p, q) equation is written in the following form:

$$R_t = \alpha_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{j=1}^{h} \phi_j R_{t-j} + \varepsilon_t, \tag{1}$$

where $\varepsilon_t|\Omega_{t-1} \sim N(0, h_t)$.

$$h_t = \omega + \sum_{i=1}^{p} \beta_i h_{t-i} + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2, \tag{2}$$

where $h_t = \sigma_t^2|\Omega_{t-1}$.

The satisfying conditions for the equations are $\omega > 0, \alpha > 0, \beta \geq 0, (\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i < 1)$. $\varepsilon_t$ stands for disturbance term for mean equation, $R_t$ describes the return of the asset at time $t$, and $X$'s denote explanatory variables. Equations (1) and (2) are the mean and the conditional variance equations, respectively. In this model, estimated parameters should satisfy non-negativity constraints to assure a positive conditional variance. Hence, GARCH models consider only the magnitude of the shock - not its sign.

### 3.3 The Fractionally Integrated GARCH (FIGARCH) Model

The primary purpose of introducing the FIGARCH model was to develop a more flexible class of processes for the conditional variance, capable of explaining and representing the observed temporal dependencies in financial market volatility. In particular, the FIGARCH model allows only a slow hyperbolic rate of decay for the lagged, squared or absolute innovations in the conditional variance function. This model can accommodate the time dependence of the variance and a leptokurtic unconditional distribution for the returns with a long memory behaviour for the conditional variances. [2]

Most of the time, high-frequency financial data follows a pattern that yields a sum of $\alpha_1$ and $\beta_1$ close to one, with $\alpha_1$ small and $\beta_1$ large. Therefore, the effect of shocks on the conditional variance diminishes very slowly. In these situations, Baillie et al. (1996) suggest the class of Fractionally Integrated GARCH (FIGARCH) models. This model captures slowly decaying volatility and recognizes conditional variance's long memory and short memory characteristics (Chkili et al., 2014). Fractionally integrated processes are significantly different from stationary and unit-root processes with persistence and mean-reverting features. Formally, the FIGARCH $(1, d, 1)$ can be defined with lag operator "$L$" as follows:

$$h_t = \omega + \beta h_{t-1} + [1 - (1 - \beta L^{-1})(1 - \lambda L)(1 - L)^d]\varepsilon_t^2,$$

where $\omega > 0, \beta > 1$ and $\lambda < 1$.

The fractional integration parameter $d$ reflects the degree of long memory, or the persistence of shocks to conditional variance, and satisfies the condition $0 \leq d \leq 1$. If $0 < d < 1$, the model implies an intermediate range of persistence and indicates the volatility shocks only at a hyperbolic rate. If the integration parameter $d = 0$, the model has a short memory and reduces to a GARCH $(1, 1)$ model. On the other hand, if $d = 1$, the model transforms to Integrated GARCH (IGARCH) $(1, 1)$.

# 4 Results

## 4.1 R/S Analysis

The entire cryptocurrency period was split into subperiods according to the schedule of V-statistics of the financial series. A study was conducted in the presence of cycles to check for persistency. The Criterion was the slope of the V-statistics curve. The Hurst index was calculated for each subperiod, and the best fit was chosen for the FIGARCH model. After interpreting the obtained result, the cycles for the investigated persistent financial series were determined:

An R/S analysis was performed from hourly data, and the Hurst index was calculated. The number of elements in the subgroups was chosen to be 24. This division gave the best or better values of the Hurst index for each cryptocurrency. In addition, this number of elements can indicate that cryptocurrencies have some sort of dependency on the local hours. As a method for the study, a program was written in the programming language R, see Appendix C. The result of the Hurst index for Bitcoin is shown in Table 1. The value of the Corrected R over S Hurst exponent index is 0.55,

Table 1: Hurst index for cryptocurrencies

| Cryptocurrency | Bitcoin | Ethereum | Binance Coin | XRP | Cardano |
|---|---|---|---|---|---|
| Simple R/S Hurst estimation: | 0.5366088 | 0.555031 | 0.552313 | 0.5207716 | 0.5476281 |
| Corrected R over S Hurst exponent: | 0.5575797 | 0.5659764 | 0.5722721 | 0.5407198 | 0.5599986 |
| Theoretical Hurst exponent: | 0.5271419 | 0.5282946 | 0.5331905 | 0.5331905 | 0.5331905 |

which is greater, but not by much, than the Corrected empirical Hurst exponent value 0.52. It allows us to conclude that the hourly data of the Bitcoin price is persistent and possesses long-term memory. However, the values of the indexes do not differ by much.

Next, we built a graph of V-statistics and a $E((R/S)_n)$ graph of theoretically calculated indicator $E((R/S)_n)$ to the null hypothesis and shows the behaviour of a system that is a completely independent process. For comparison, Appendix A, Figure 36 show the V- statistics graphs for the bitcoin series under consideration. These graphs also confirm the presence of persistence for the financial series of Brent crude oil price quotes and the dollar/ruble exchange rates.

The same R/S analysis was performed, and the Hurst index was calculated for the rest of the cryptocurrencies. The number of elements in the subgroups was 24 as with bitcoin because the same pattern was noticed in the rest. The Hurst index was the highest for Ethereum and Binance at approximately 0.55, with the lowest and close to $H = 0.5$ for XRP.

In addition, this number of elements can possibly indicate that cryptocurrencies depend on the local hours. It allows us to conclude that the hourly data of the other coin prices are persistent and possesses long-term memory, see Appendix Figures 37-40, where $(R/S)$ always surpasses $E((R/S)_n)$. Nevertheless, the values of the indexes do not differ significantly.

## 5 Models

The hourly return measures the price changes of the cryptocurrencies in a U.S. dollar price as a percentage change of the previous hour closing price. A positive return means the cryptocurrency has grown in value, while a negative return has lost value. Suppose there is a lot of movement in daily returns. In that case, the cryptocurrency is more volatile and is riskier as an investment. Bitcoin prices started to gradually increase at the beginning of 2020, as displayed in Figure 1. This large and sudden increase corresponds to the beginning of the lockdown during the pandemic.

Hourly return series is calculated as log differences of price levels as follows:

$$r_t = 100 \cdot (\ln P_t - \ln P_{t-1}),$$

where $P_t$ represents the price of the cryptocurrency at the moment $t$, $P_{t-1}$ represents the price of the cryptocurrency at the previous hour and $r_t$ represents return at time $t$.

### 5.1 Bitcoin

#### 5.1.1 Descriptive statistics

We discuss the modelling of time-series data obtained from the "Binance" exchange rate for August 1, 2020, through December 12, 2021: a total of 11936 observations. Figure 6 represents the percentage returns of the exchange rate series. It clearly indicates the high volatility in the series during the period under consideration. Analysing short lag, up to 50 hours (see Figure 7), it can be observed that a lag of 24 hours is significant. Hence it confirms a choice of parameter $d = 24$ for Hurst estimation. The long autocorrelation function of the absolute returns, displayed in Figure 8, has got hyperbolic decay, indicating the possibility of modelling the series with a long memory model.

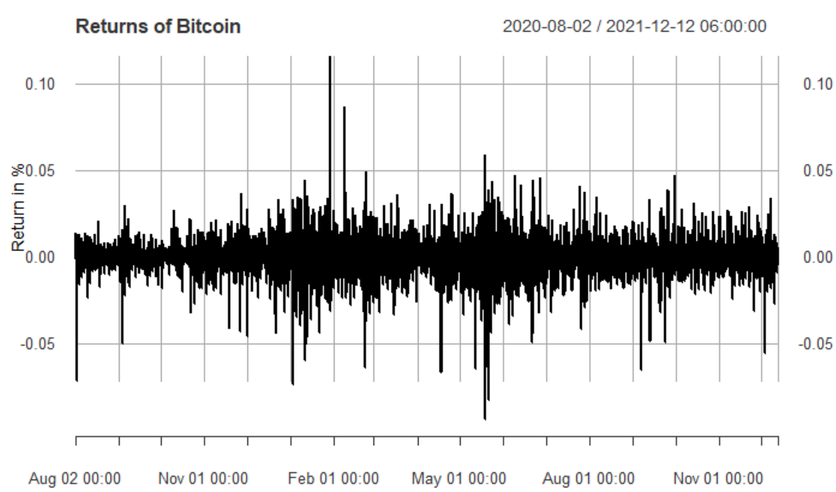


Figure 6: Returns of Bitcoin

As it is hard to have a deterministic trend in the long run for financial data, we will not look for trend models here and going to directly jump into GARCH models, more specifically FIGARCH and GARCH, in order to capture the auto-correlation structure of historical prices.

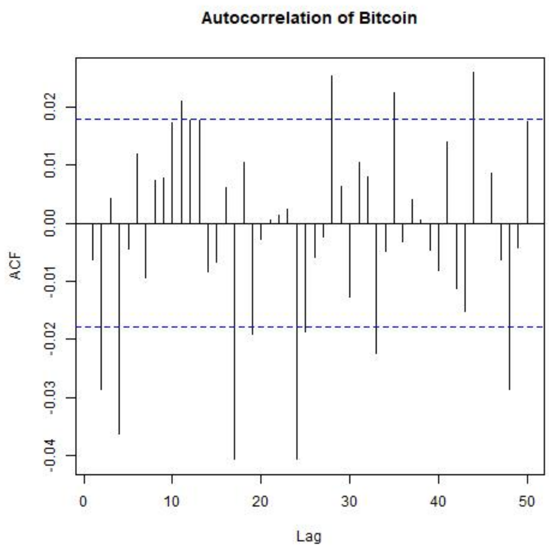


Figure 7: Autocorrelation of Bitcoin (lag 50)

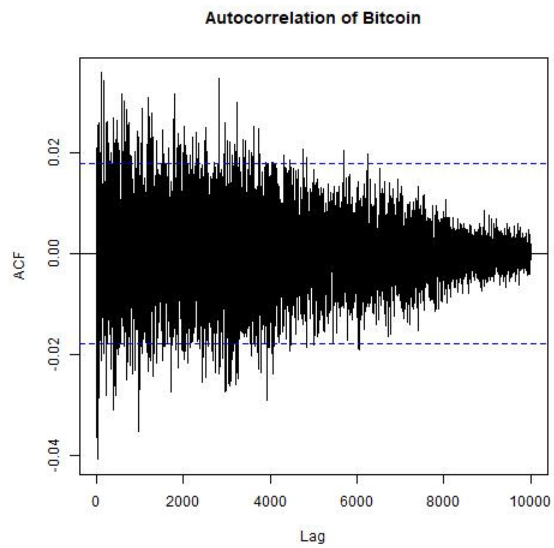


Figure 8: Autocorrelation of Bitcoin (lag 10000)

Figure 9 shows the Q-Q normal plot for return in Figure 6 with fat skewed tails and a significant Shapiro-Wilk normality test, confirming the series is not normally distributed. Student-t and skewed student-t distributions were used since the returns have heavier tails than the normal distribution.

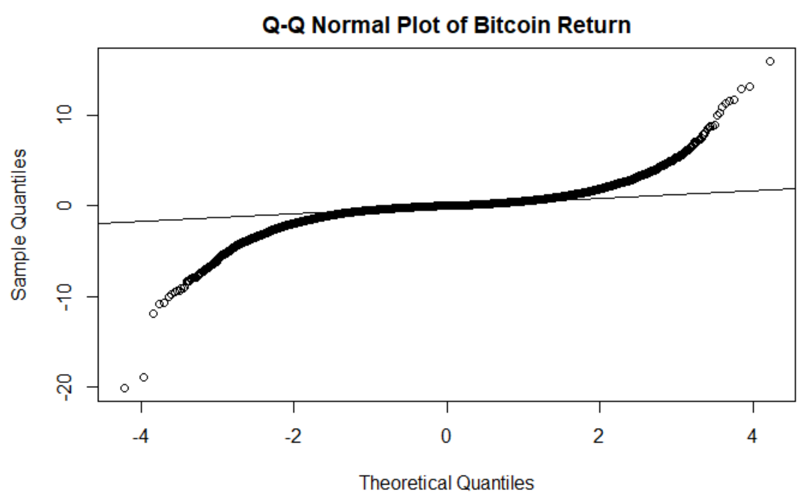


Figure 9: Q-Q plot of Bitcoin

### 5.1.2 GARCH and FIGARCH models

Fitted GARCH model is written in form of:

$$\begin{cases} r_t &= 0.000125 + \varepsilon_t \\ \varepsilon_t &= 0.000001\sigma_t \\ \sigma_t^2 &= 0.000001 + 0.077393\varepsilon_{t-1}^2 + 0.921267\sigma_{t-1}^2 \end{cases}$$

Model's results for volatility forecasting are observed in Figure 10. There is no significant difference between the fitted values and the forecasted values. The model decently responds to volatility jumps. However, post 2021 August, observation accuracy decreases.

We have fit the FIGARCH model to this data set:

$$y_t = 100log(s_t/s_{t-1}) = 0.000202 + \varepsilon_t, \quad \varepsilon_t h_t^{-1/2} \sim \mathbb{N}(0,1)$$
$$h_t = 0.000000 + 0.957744h_{t-1} + [1 - 0.957744L - (1-\psi_1 L)(1-L)^d)]\varepsilon_t^2$$

It is seen in Figure 11 that the FIGARCH model approaches the same issues – further forecasting decreases in accuracy and can not follow higher volatility periods in late periods. Compared to the GARCH model described previously, the FIGARCH model is better for forecasting by Akaike estimate (see Appendix B, Table 3) -7.3114 than the GARCH model and its Akaike estimate being equal to -7.1065.

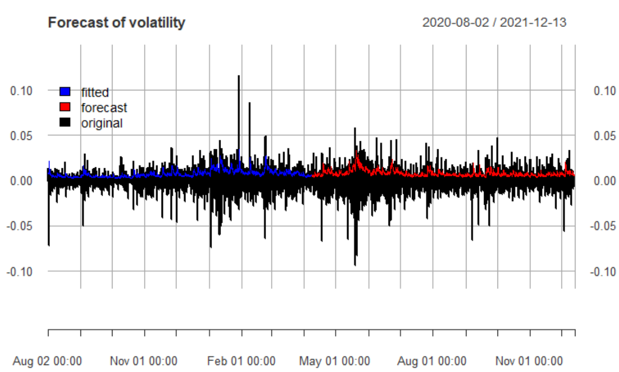


Figure 10: GARCH forecast of volatility

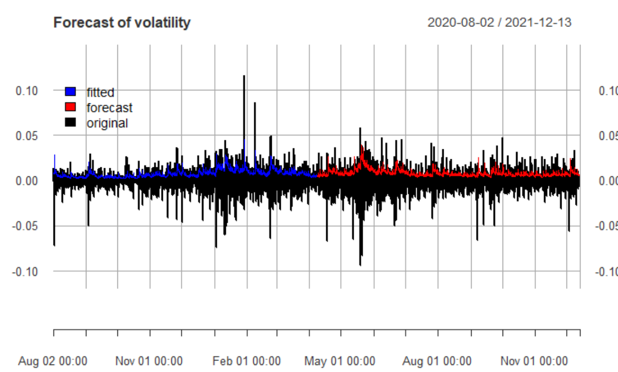


Figure 11: FIGARCH forecast of volatility

Using the information criteria computations given in Table 3, we conclude that the FIGARCH (p,d,q) process fits well for the cryptocurrency data. Thus, we propose FIGARCH (0,d,1) as a suitable model for this data set. Model residual analysis shows degradation in model forecasting after only a few months.

## 5.2 Ethereum

### 5.2.1 Descriptive statistics

The time-series data of Ethereum were obtained from the "Binance" exchange rate for the period of August 1, 2020, through December 12, 2021: a total of 11936 observations. Figure 12 represents the percentage returns of the exchange rate series. The hourly returns of the Ethereum had stayed within the 20 percent range throughout the period, with only one case, where the returns of Ethereum were down 14 percent just in one hour. During the pandemic, the volatility of the time series has increased significantly, and it must be considered.

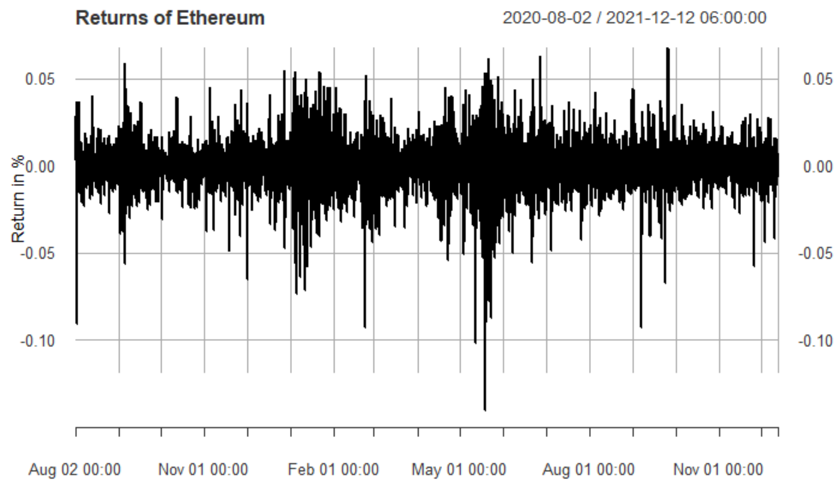


Figure 12: Returns of Ethereum

Analysing short lag, up to 50 hours (see Figure 13), it can be observed that a lag of 24 hours is significant. Hence it confirms a choice of parameter $d = 24$ for Hurst estimation. The long autocorrelation function of the absolute returns, displayed in Figure 14, has got hyperbolic decay, indicating the possibility of modelling the series with a long memory model.

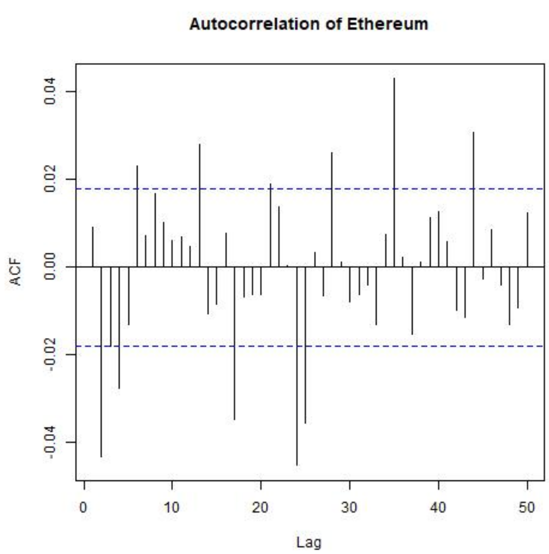


Figure 13: Autocorrelation of Ethereum (lag 50)

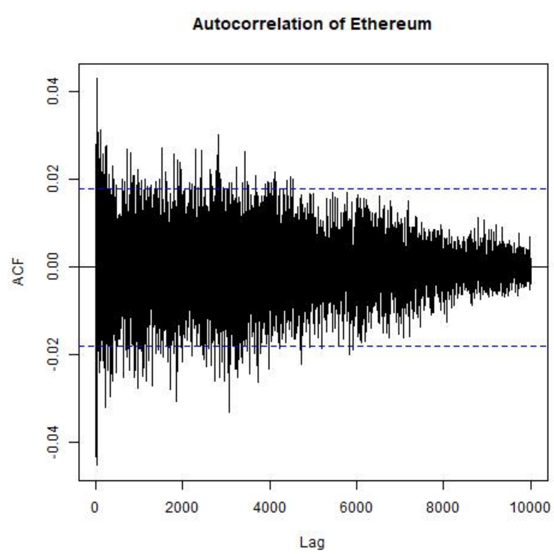


Figure 14: Autocorrelation of Ethereum (lag 10000)

As it is hard to have a deterministic trend in the long-run for financial data, we are not going to look for trend models here and, as in the case of Bitcoin, we are directly jumping into GARCH models, more specifically FIGARCH and GARCH, to capture the auto-correlation structure of historical prices.

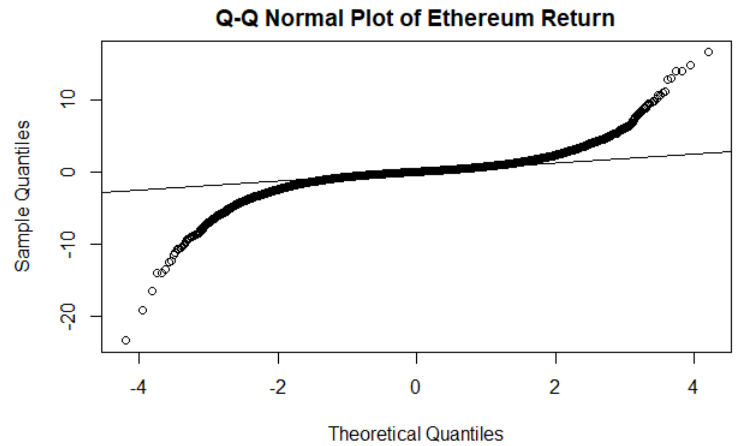


Figure 15: Q-Q plot of Ethereum

The Q-Q normal plot in Figure 15 for return of Ethereum with fat tails and significant Shapiro-Wilk normality test confirms, that the series is not normally distributed. Student-t and skewed student-t distributions were used since the returns have heavier tails than the normal distribution.

### 5.2.2 GARCH and FIGARCH models

Fitted GARCH model is written in form of:

$$\begin{cases} r_t &= 0.000226 + \varepsilon_t \\ \varepsilon_t &= 0.000002\sigma_t \\ \sigma_t^2 &= 0.000002 + 0.070181\varepsilon_{t-1}^2 + 9.147769\sigma_{t-1}^2 \end{cases}$$

Results of the GARCH model for volatility forecasting are observed in Figure 16. There is no significant difference between the fitted values and the forecasted values. The model decently responds to volatility jumps. However, post August 2021, observation accuracy decreases.

We have fit the FIGARCH model to this data set:

$$y_t = 100log(s_t/s_{t-1}) = 0.000289 + \varepsilon_t, \quad \varepsilon_t h_t^{-1/2} \sim \mathbb{N}(0,1)$$
$$h_t = 0.000000 + 0.954726h_{t-1} + [1 - 0.954726L - (1-\psi_1 L)(1-L)^d)]\varepsilon_t^2$$

It is seen in Figure 17 that the FIGARCH model approaches the same issues – further forecasting decreases in accuracy and can not follow higher volatility periods in late periods. Compared to the GARCH model described previously, the FIGARCH model is better for predicting by Akaike estimate (see Appendix B, Table 3) -6.7019 compared to the GARCH model and its Akaike estimate being equal to -6.5549.

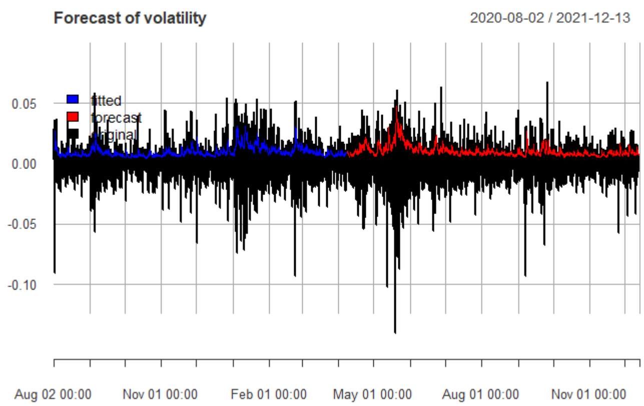


Figure 16: GARCH forecast of volatility

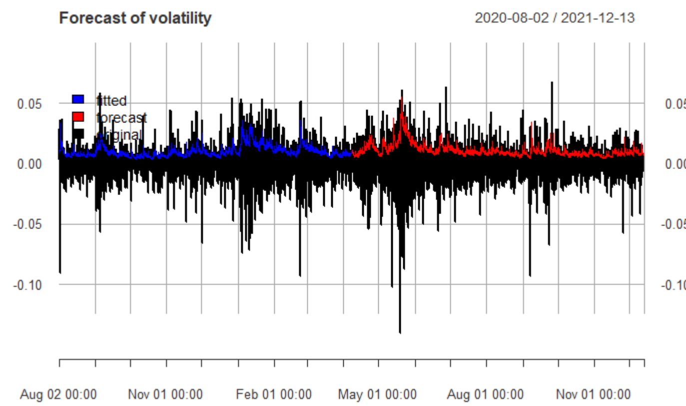


Figure 17: FIGARCH forecast of volatility

Using the information criteria computations given in Table 3, we conclude that the FIGARCH (p,d,q) process fits well for the cryptocurrency data. Thus, we propose FIGARCH (0,d,1) as a suitable model for this data set. Model residual analysis shows degradation in model forecasting only after a few months.

## 5.3 Binance Coin

### 5.3.1 Descriptive statistics

The time-series data of Binance Coin were obtained from the"Binance" exchange rate for the period of August 1, 2020 through December 12, 2021: a total of 11936 observations. Figure 18 represents the percentage returns of the exchange rate series. The hourly returns of the Binance Coin have been more volatile than Bitcoin and Ethereum.

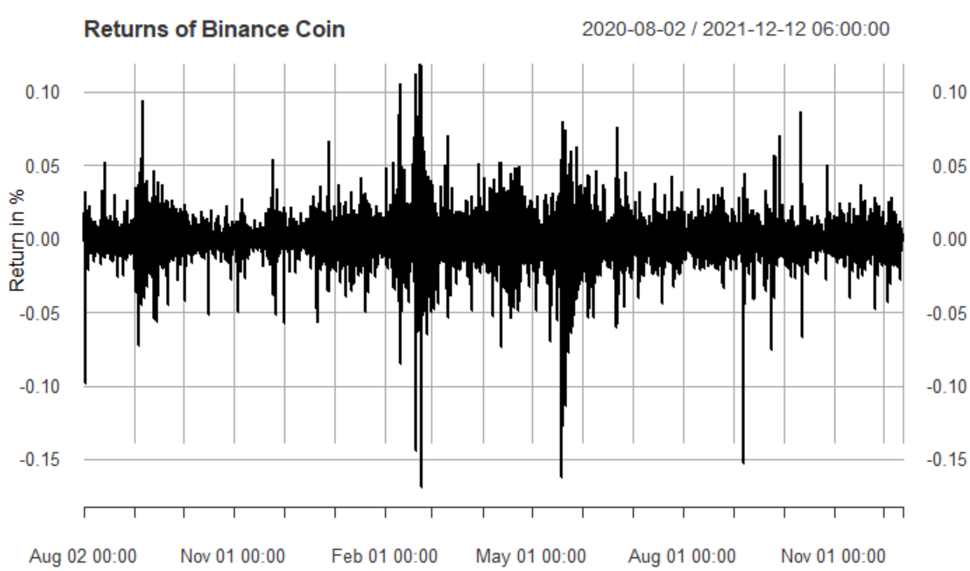


Figure 18: Returns of Binance Coin

During the pandemic, the volatility of the time series has increased significantly, and it must be considered. On several occasions the returns have reached over a 15% difference in the price in only one hour. Analyzing short lag, up to 50 hours (see Figure 19), it can be observed that a lag of 24 hours is significant. Hence it confirms a choice of parameter d = 24 for Hurst estimation. The long autocorrelation function of the absolute returns, displayed in Figure 20, has got hyperbolic decay, indicating the possibility of modelling the series with a long memory model.

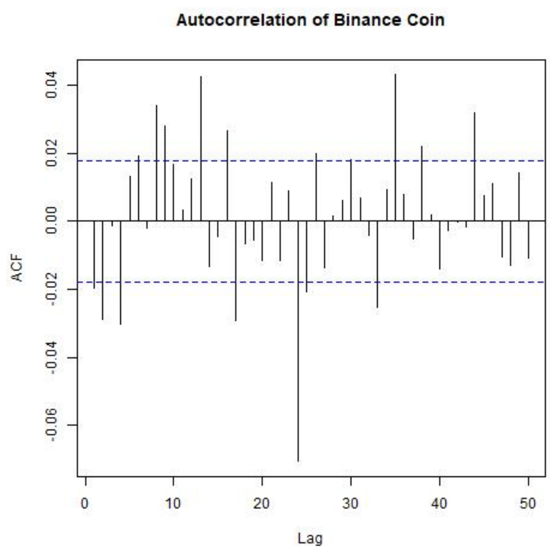


Figure 19: Autocorrelation of BNB (lag 50)

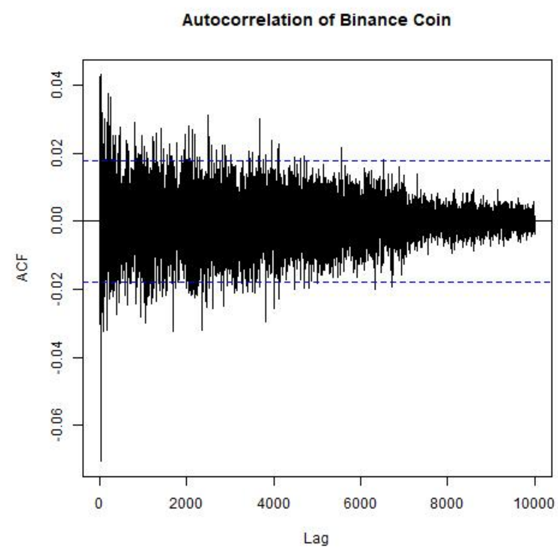


Figure 20: Autocorrelation of BNB (lag 10000)

The autocorrelation function of the absolute returns has got hyperbolic decay (see Figure 20), when more data values are taken. As a result, it indicates the possibility of modelling the series with a long memory model. It is hard to have a deterministic trend in the long run for financial data. We will not look for trend models here and going to directly jump into GARCH models to capture the auto-correlation structure of historical prices.

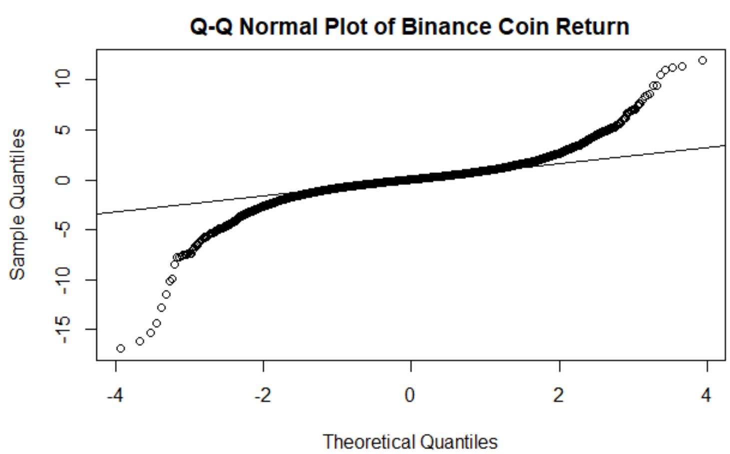


Figure 21: Q-Q Plot of Binance Coin

The Q-Q normal plot for returns in Figure 21 with fat tails and significant Shapiro-Wilk normality test confirms, that the series is not normally distributed. Student-t and skewed student-t distributions were used since the returns have heavier tails than the normal distribution.

### 5.3.2 GARCH and FIGARCH models

Fitted GARCH model is written in form of:

$$\begin{cases} r_t &= 0.000125 + \varepsilon_t \\ \varepsilon_t &= 0.000002\sigma_t \\ \sigma_t^2 &= 0.000002 + 0.107997\varepsilon_{t-1}^2 + 0.891003\sigma_{t-1}^2 \end{cases}$$

Results of the GARCH model for volatility forecasting are observed in Figure 22. There is no significant difference between the fitted values and the forecasted values. The model decently responds to volatility jumps. However, post August 2021, observation accuracy decreases.

We have fit the FIGARCH model to this data set:

$$y_t = 100log(s_t/s_{t-1}) = 0.000206 + \varepsilon_t, \quad \varepsilon_t h_t^{-1/2} \sim \mathbb{N}(0,1)$$
$$h_t = 0.000001 + 0.926249h_{t-1} + [1 - 0.926249L - (1-\psi_1 L)(1-L)^d)]\varepsilon_t^2$$

It is seen in Figure 23 that the FIGARCH model approaches the same issues – further forecasting decreases in accuracy and can not follow higher volatility periods in late periods. Compared to the GARCH model described previously, the FIGARCH model is better for forecasting by Akaike estimate (see Appendix B, Table 3) -6.4959 compared to the GARCH model and its Akaike estimate being equal to -6.0615.

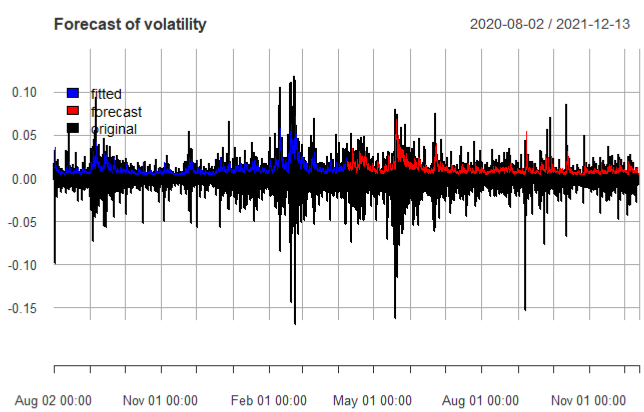


Figure 22: GARCH forecast of volatility

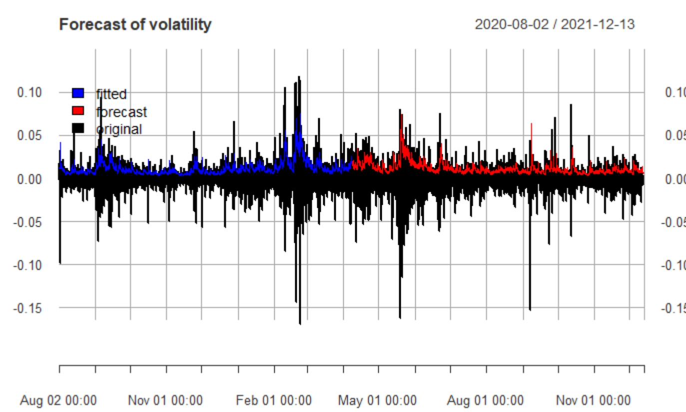


Figure 23: FIGARCH forecast of volatility

Model residual analysis shows degradation in model forecasting only after a few months. Using the information criteria computations given in Table 3, we conclude that the FIGARCH (p,d,q) process fits well for the cryptocurrency data. Thus, we propose FIGARCH (0,d,1) as a suitable model for this data set.

### 5.4 XRP

#### 5.4.1 Descriptive statistics

The time-series data of XRP were obtained from the "Binance" exchange rate for the period August 1, 2020 through December 12, 2021: a total of 11936 observations.

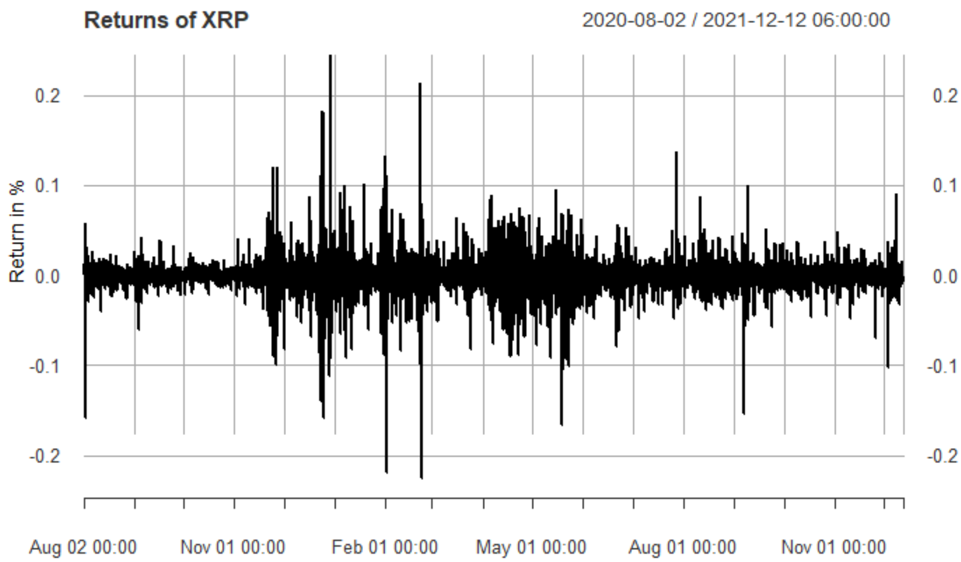


Figure 24: Returns of XRP

Figure 24 represents the percentage returns of the exchange rate series. The hourly returns of the XRP had stayed within a 40% change throughout the period, with only a few cases where the returns of the XRP were more than 20% up and more than 20% down in one hour. During the pandemic, the volatility of the time series has increased significantly, and it must be considered.

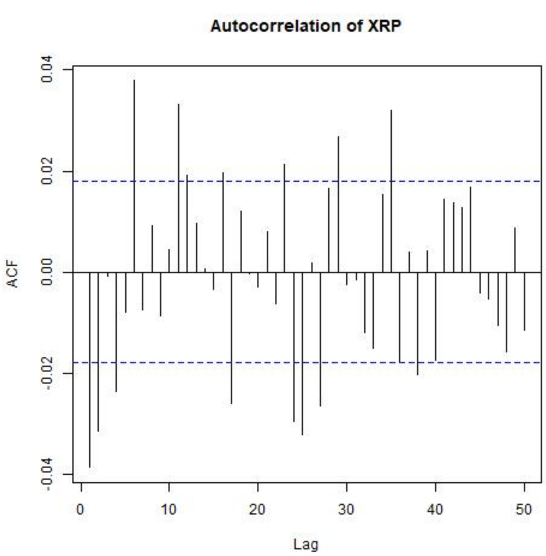


Figure 25: Autocorrelation of XRP (lag 50)

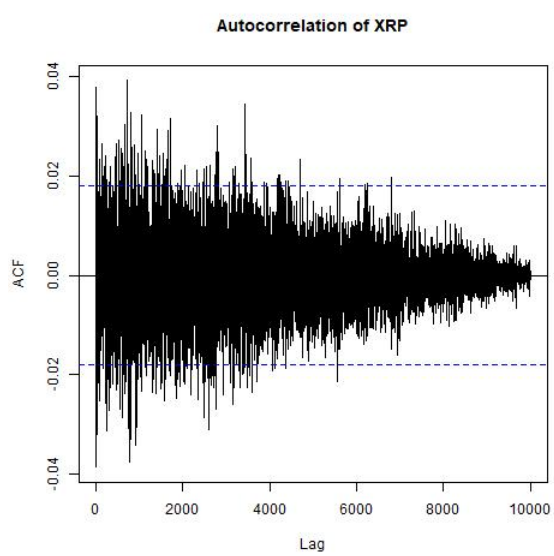


Figure 26: Autocorrelation of XRP (lag 10000)

Analyzing short lag, up to 50 hours (see Figure 25), it can be observed that a lag of 24 hours is significant. However, it is not the biggest spike in the autocorrelation of XRP. Nevertheless, it confirms a choice of parameter d = 24 for Hurst estimation. The long autocorrelation function of the absolute returns, displayed in Figure 26, has got hyperbolic decay, indicating the possibility of modelling the series with a long memory model.

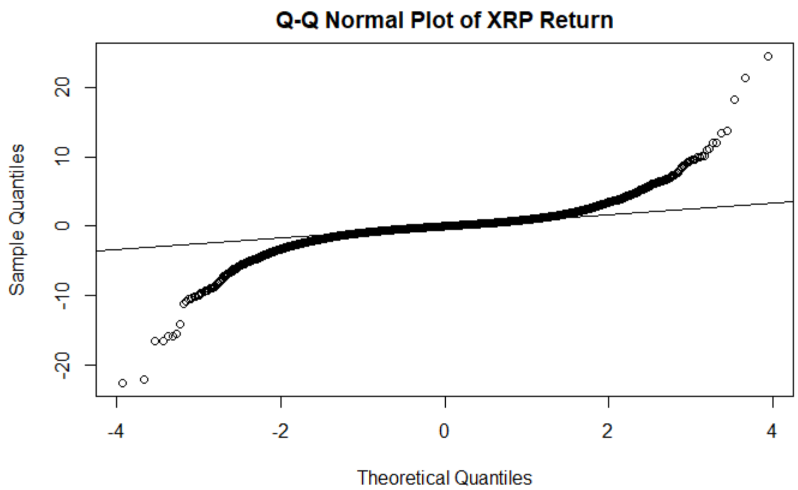


Figure 27: Q-Q Plot of XRP

The Q-Q normal plot for return in Figure 27 with fat tails and significant Shapiro-Wilk normality test confirms, that the series is not normally distributed. Student-t and skewed student-t distributions were used since the returns have heavier tails than the normal distribution.

### 5.4.2 GARCH and FIGARCH models

Fitted GARCH model is written in form of:

$$\begin{cases} r_t &= 0.000279 + \varepsilon_t \\ \varepsilon_t &= 0.000002 \cdot \sigma_t \\ \sigma_t^2 &= 0.000002 + 0.120534\varepsilon_{t-1}^2 + 0.875746\sigma_{t-1}^2 \end{cases}$$

Results of the GARCH model for volatility forecasting are observed in Figure 28. There is no significant difference between the fitted values and the forecasted values. The model decently responds to volatility jumps. However, post 2021, observation accuracy decreases.

We have fit the FIGARCH model to this data set:

$$y_t = 100log(s_t/s_{t-1}) = 0.000064 + \varepsilon_t, \quad \varepsilon_t h_t^{-1/2} \sim \mathbb{N}(0,1)$$
$$h_t = 0.000002 + 0.712234h_{t-1} + [1 - 0.712234L - (1 - \psi_1 L)(1 - L)^d)]\varepsilon_t^2$$

It is seen in Figure 29 that the FIGARCH model approaches the same issues – further forecasting decreases in accuracy and can not follow higher volatility periods in late periods. Compared to the GARCH model described previously, the FIGARCH model is not better for forecasting by Akaike estimate (see Appendix B, Table 3) -6.3381 compared to the GARCH model. Its Akaike estimate is equal to -6.3717.

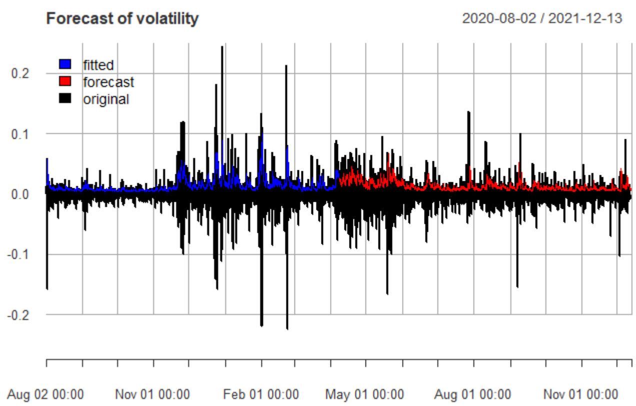


Figure 28: GARCH forecast of volatility

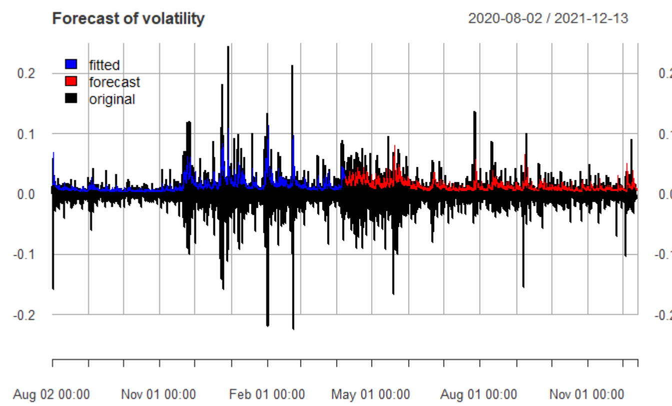


Figure 29: FIGARCH forecast of volatility

Using the information criteria computations given in Table 3, we conclude that the FIGARCH (p,d,q) process fits almost equally as good as the GARCH model. Thus, we propose FIGARCH (0,d,1) as a still suitable model for this data set.

## 5.5 Cardano

### 5.5.1 Descriptive statistics

The time series data of Cardano were obtained from the "Binance" exchange rate for the period of August 1, 2020 through December 12, 2021: a total 11936 observations. Figure 30 represents the percentage returns of the exchange rate series.

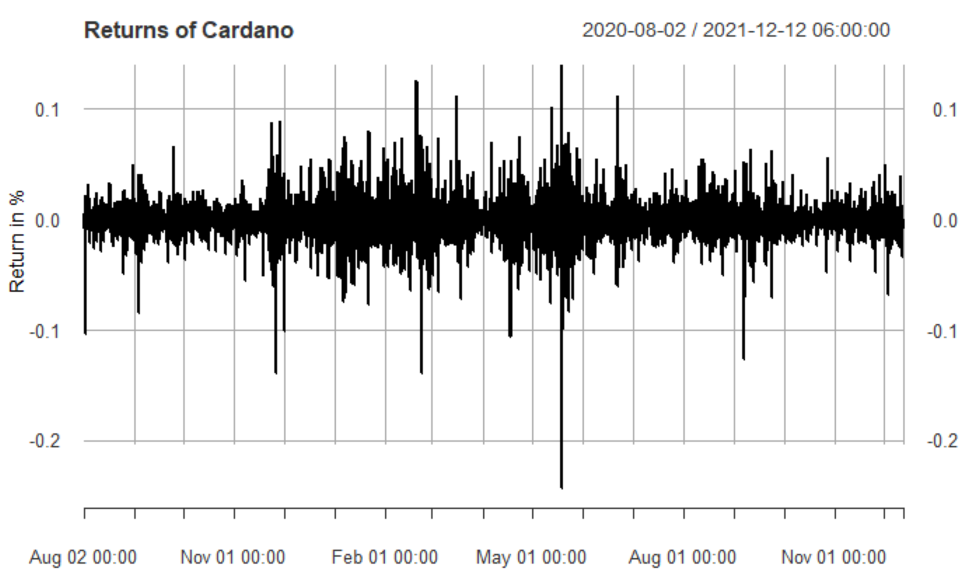


Figure 30: Returns of Cardano

During the pandemic, the volatility of the time series has increased significantly, and it must be considered. The hourly returns of Cardano have been more volatile compared to Bitcoin and Ethereum. On one occasion, the value of the coin dropped over 20% in only one hour.

Analyzing short lag, up to 50 hours (see Figure 31), it can be observed that a lag of 24 hours is significant. Hence it confirms a choice of parameter d = 24 for Hurst estimation. The long autocorrelation function of the absolute returns, displayed in Figure 32, has got hyperbolic decay, indicating the possibility of modelling the series with a long memory model.

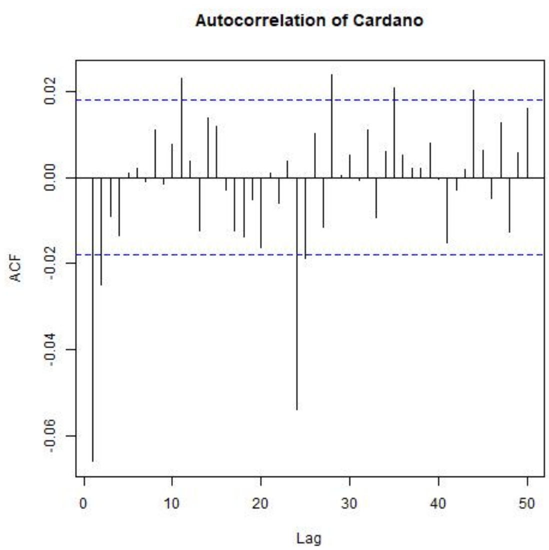


Figure 31: Autocorrelation of Cardano (lag 50)

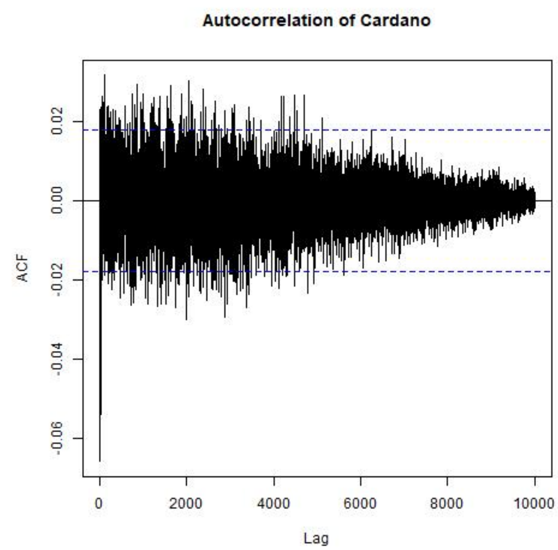


Figure 32: Autocorrelation of Cardano (lag 10000)

As it is hard to have a deterministic trend in the long run for financial data, we will not look for trend models here and going to directly jump into GARCH models to capture the auto-correlation structure of historical prices.

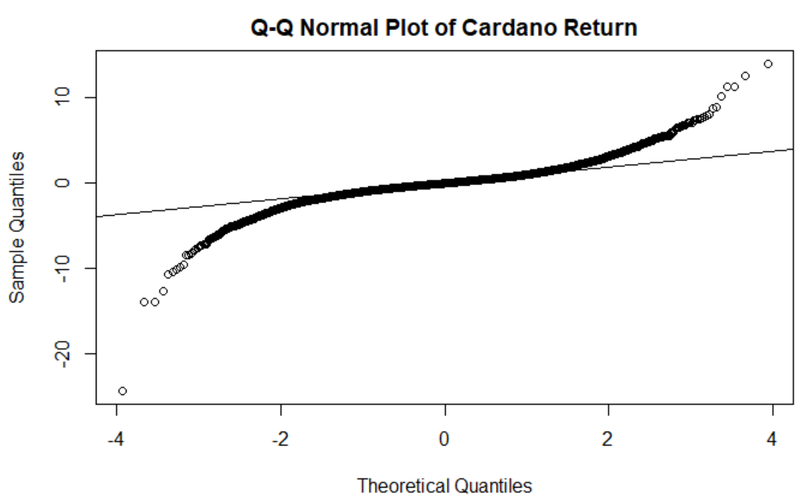


Figure 33: Q-Q Plot of Cardano

The Q-Q normal plot for return in Figure 33 with fat tails and significant Shapiro-Wilk normality test confirms the series is not normally distributed. Student-t and skewed student-t distributions were used since the returns have heavier tails than the normal distribution.

### 5.5.2 GARCH and FIGARCH models

Fitted GARCH model is written in form of:

$$\begin{cases} r_t &= 0.000123 + \varepsilon_t \\ \varepsilon_t &= 0.000004\sigma_t \\ \sigma_t^2 &= 0.000004 + 0.128723\varepsilon_{t-1}^2 + 0.863925\sigma_{t-1}^2 \end{cases}$$

Results of the GARCH model for volatility forecasting are observed in Figure 34. There is no significant difference between the fitted values and the forecasted values. The model decently responds to volatility jumps. However, post August 2021, observation accuracy decreases.

We have fit the FIGARCH model to this data set:

$$y_t = 100log(s_t/s_{t-1}) = -0.000029 + \varepsilon_t, \quad \varepsilon_t h_t^{-1/2} \sim \mathbb{N}(0,1)$$
$$h_t = 0.000002 + 0.766887h_{t-1} + [1 - 0.766887L - (1-\psi(L))(1-L)^d)]\varepsilon_t^2$$

It is seen in Figure 35 that the FIGARCH model approaches the same issues – further forecasting decreases in accuracy and can not follow higher volatility periods in late periods. Compared to the GARCH model described previously, the FIGARCH model is better for forecasting by Akaike estimate (see Appendix B, Table 3) -6.1549 compared to the GARCH model and its Akaike estimate being equal to -5.9728.

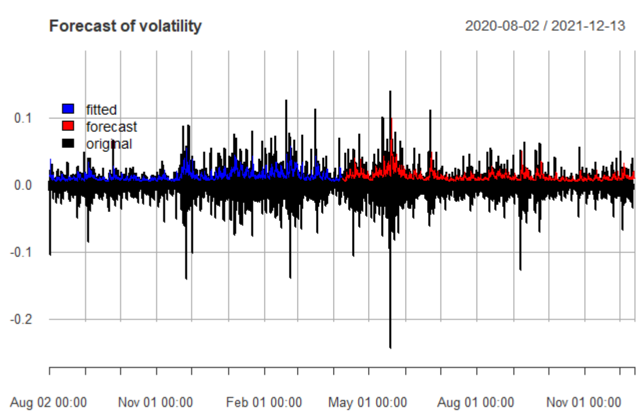


Figure 34: GARCH forecast of volatility

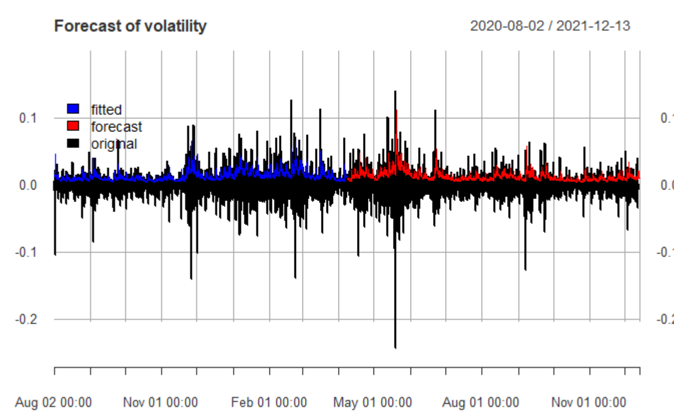


Figure 35: FIGARCH forecast of volatility

Model residual analysis shows degradation in model forecasting only after a few months. Using the information criteria computations given in Table 3, we conclude that the FIGARCH (p,d,q) process fits well for the cryptocurrency data. Thus, we propose FIGARCH (0,d,1) as a suitable model for this data set.

## 6 Conclusions

To conclude, the main goal of this paper was to investigate whether the time series of the cryptocurrencies are persistent and predictable by using fractal analysis. The persistence hypothesis of the studied series was tested using R/S analysis, and the Hurst index of fractal analysis and cycles were also found there. The results showed that most of the cryptocurrencies' returns depend on the local hours. The most significant Hurst index was acquired with subgroups of 24 elements, with the hourly returns. The Hurst index was higher than 0.5 for 4 out of 5 cryptocurrencies, indicating that processes are not random. However, the value of the Hurst index never reaches more than 0.6, which concludes that cryptocurrencies are not strongly persistent with the given data.

Moreover, the GARCH and the FIGARCH models were fitted on the hourly cryptocurrencies' returns. The Fractionally Integrated GARCH models slightly improved the accuracy of forecasting for Bitcoin, Ethereum, Binance Coin and Cardano, according to information criteria.

Apart from regular equities, commodities, and precious metals, cryptocurrency may become a mainstream financial tool in the future of global financial markets. The blockchain as a technology is still in the development process, and all cryptocurrencies have not yet reached their stabilized values. Therefore, future studies could analyze at what level of stability cryptocurrencies could be predicted with former methods. As a result, this study will give investors a better understanding of the cryptocurrency markets and stimulate more research.

For the future works the similar R/S analysis could be applied for the other cryptocurrencies and different periods of time. In addition, analysis of the returns of cryptocurrency during the specific hour could be performed, since, the evidence of the a lag of 24 hours were observed.

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# Appendices

## Appendix A

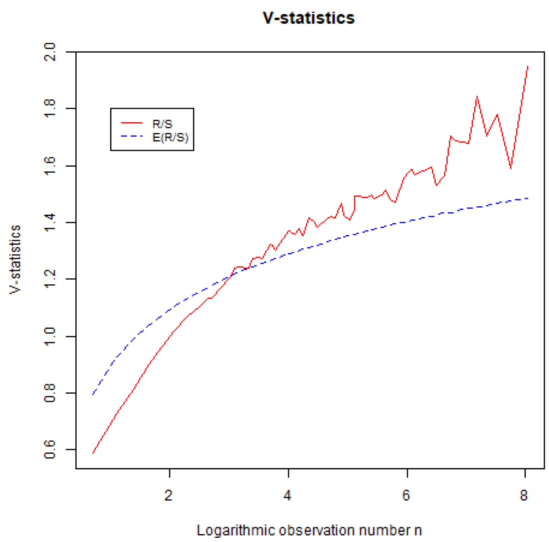


Figure 36: V-Statistics of Bitcoin

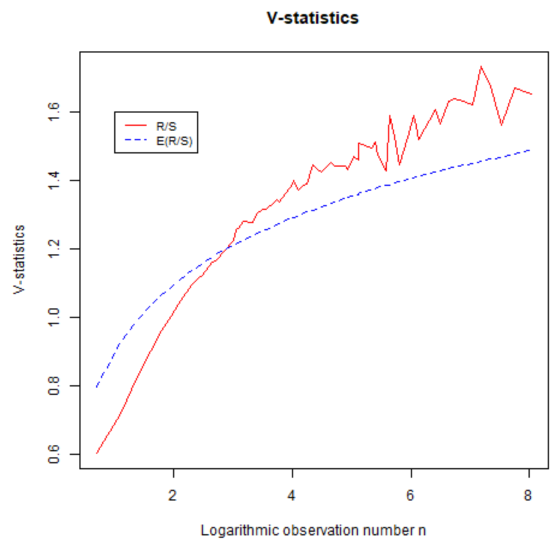


Figure 37: V-Statistics of Ethereum

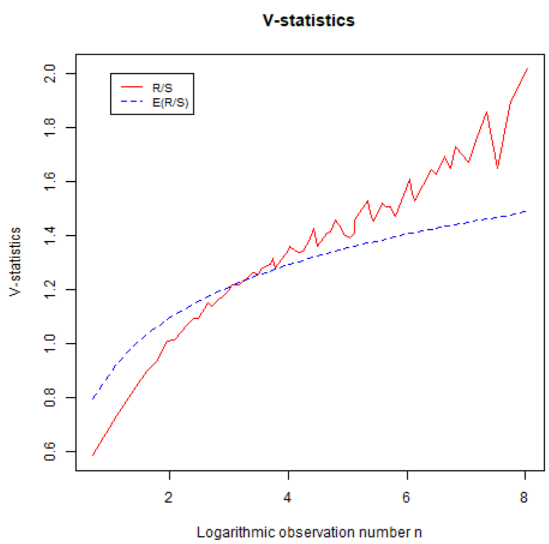


Figure 38: V-Statistics of Binance Coin

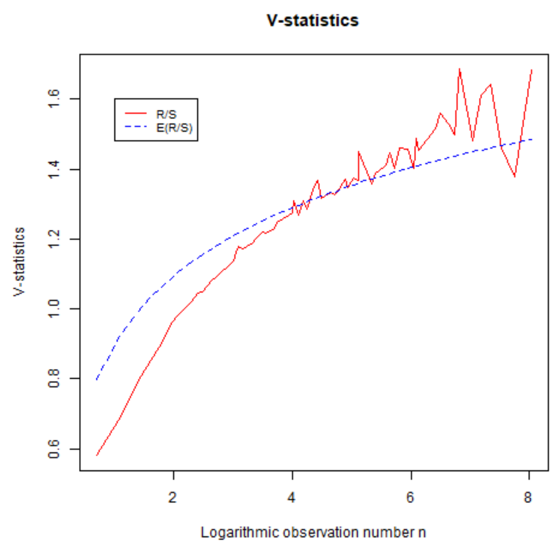


Figure 39: V-Statistics of XRP

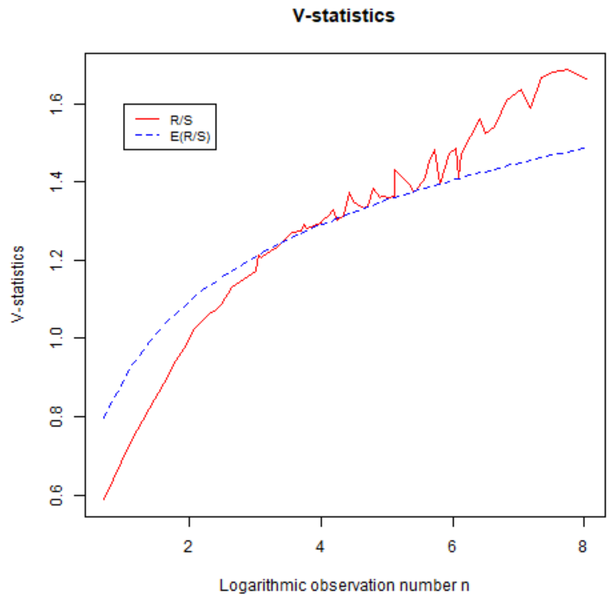


Figure 40: V-Statistics of Cardano Coin

## Appendix B

Table 2: Information Criteria of GARCH models

| Information Criteria | Bitcoin | Ethereum | Binance Coin | XRP | Cardano |
|---|---|---|---|---|---|
| Akaike | -7.1065 | -6.5549 | -6.0615 | -6.3717 | -5.9728 |
| Bayes | -7.1009 | -6.5493 | -6.0559 | -6.3661 | -5.9671 |
| Shibata | -7.1065 | -6.5549 | -6.0615 | -6.3717 | -5.9728 |
| Hannan-Quinn | -7.1046 | -6.5530 | -6.0595 | -6.3698 | -5.9708 |

Table 3: Information Criteria of GARCH models

| Information Criteria | Bitcoin | Ethereum | Binance Coin | XRP | Cardano |
|---|---|---|---|---|---|
| Akaike | -7.3114 | -6.7019 | -6.4959 | -6.3381 | -6.1549 |
| Bayes | -7.3036 | -6.6940 | -6.4881 | -6.3302 | -6.1470 |
| Shibata | -7.3114 | -6.7019 | -6.4959 | -6.3381 | -6.1549 |
| Hannan-Quinn | -7.3087 | -6.6992 | -6.4932 | -6.3353 | -6.1521 |

## Appendix C

```
r.s.function = function(n){
  #  Find A
  A = length(r.df)/n

  N.ka = matrix(r.df, ncol = A, nrow = n)

  #  Calculate e.a which is means
  e.a = colMeans(N.ka)


  #  Step 3, series of accumulated deviations are compiled for each sub-period

  X.ka = matrix(rep(0), ncol=A, nrow=n)
  for (k in 1:n){

    for (a in 1:A){
      X.ka[k,a] = sum(N.ka[1:k,a]) - e.a[a]
    }
  }

  #  Step 4, the range of the accumulated frequency of each sub-period is determined
  R.a = matrix(0, nrow=1,ncol=A)

  for (a in 1:A){
    R.a[1,a] = max(X.ka[,a]) - min(X.ka[,a])
  }


  #  Step 5, we calculate the sample standard deviation for each sub-period according to the
  S.a  = matrix(0, nrow=1,ncol=A)

  S.a_initial = colSums((N.ka - e.a)^2)
  S.a = sqrt(S.a_initial / n)

  #  Step 6, The average value is determined for length n according to the following
  RS.n = (1/A) * sum(R.a / S.a)

  return(RS.n)
}
```

## Appendix D

```
library(xts)
library(rugarch)
#hourly data
df <- read.csv("C:/...", header=TRUE, sep= ";", dec=",")

plot(df, xlab="Time", ylab="Price in US dollar", main="Price of Bitcoin")
df$date = paste0(df$date, ":00")
df$date <- as.POSIXct(df$date,
                      format = "%d/%m/%Y %T", tz = "UTC")
# Removing non-numeric characters in the series
df$close <- (as.numeric(df$close))

inds = seq(
  from=as.POSIXct("01/08/2020 23:00:00","%d/%m/%Y %T", tz="UTC"),
  to=as.POSIXct("13/12/2021 00:00:00", "%d/%m/%Y %T", tz="UTC"),
  by="hour"
)

df$close[2:11936] = diff(log(df$close), lag=1)
plot(r.df, xlab="Time", ylab="Return in %", main="Returns of Bitcoin")

df.ts = xts(df$close, inds[1:11936])

plot(df.ts,as.POSIXct("01/08/2020 23:00:00","%d/%m/%Y %T", tz="UTC"), xlab="Time",

ylab="Price in USD",
     main="Bitcoin Price from 2020 to 2022")


r.df =df.ts # Continuous compound return
plot(r.df, xlab="Time", ylab="Return in %", main="Returns of Bitcoin")

r.df = r.df[-1]
out_of_sample <- round(length(r.df)/2)
dates_out_of_sample <- tail(inds, out_of_sample)


df = df$close[-1]
setwd("C:/...")
AutoCorrelation = acf(df, lag = 2, lag.max = 10000)
plot(AutoCorrelation , main = "Autocorrelation of Bitcoin")
jpeg('BTC1autocorrelation.jpg')
plot(AutoCorrelation, main = "Autocorrelation of Bitcoin")
dev.off()
```

```
setwd("C:/...")
PartialAutoCorrelation = pacf(df, lag = 1, lag.max = 50)
plot(PartialAutoCorrelation , main = "Partial Autocorrelation of Bitcoin")
jpeg('pavadinimas.jpg')
plot(AutoCorrelation, main = "Partial Autocorrelation of Bitcoin")
dev.off()


garch_spec2 <- ugarchspec(mean.model = list(armaOrder = c(1,0), include.mean = TRUE),
                          variance.model = list(model = "sGARCH", garchOrder = c(1,1)))

garch_spec3 = ugarchspec(variance.model=list(model="fiGARCH", garchOrder=c(1,1), submodel="GARCH"),
                mean.model=list(armaOrder=c(1,0), include.mean=TRUE), distribution.model="std")

garch_fit <- ugarchfit(spec = garch_spec3, data = r.df, out.sample = out_of_sample)
garch_fit
coef(garch_fit)

# forecast log-returns along the whole out-of-sample
garch_fore <- ugarchforecast(garch_fit, n.ahead = 1, n.roll = out_of_sample-1 )
#?ugarchfit
forecast_log_returns <- xts(garch_fore@forecast$seriesFor[1, ], dates_out_of_sample)
forecast_volatility <- xts(garch_fore@forecast$sigmaFor[1, ], dates_out_of_sample)

fitted_garch = xts(fitted(garch_fit), inds[1:out_of_sample-1])
colnames(fitted_garch) = "fitted"

# plot of log-returns
plot(cbind("fitted"   = sigma(garch_fit),
           "forecast" = forecast_volatility,
           "original" = r.df),
     col = c("blue", "red", "black"), lwd = c(0.5, 0.5, 2),
     main = "Forecast of volatility", legend.loc = "topleft", ylim = c(-0.15,0.15))

# plot of log-returns
plot(cbind("fitted"   = fitted(garch_fit)*exp(1),
           "forecast" = forecast_log_returns*exp(1),
           "original" = r.df),
     col = c("blue", "red", "black"), lwd = c(0.5, 0.5, 2),
     main = "Forecast of log-returns", legend.loc = "topleft", ylim = c(-0.15,0.15))
```