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WHAT BINDS LITHUANIA'S EXPORTS: THE STORY OF SUPPLY AND DEMAND INTERPLAY

Master's thesis

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Notation

- X_t Lithuanian exports of goods (excluding energy products) and services;
- P_t^x Lithuania's Export Price Indices;
- P_t^w Lithuania's Competitor's Export Price Indices, without energy products;
- M_t^w Real–World Market Demand;
- MC_t Sector Marginal Costs;
- P_t^{HEX} Domestic Prices non energy component of HICP;
- Y_t^M Potential Output (market sector);
- DU_{t1} Dummy variable capture the global financial crisis;
- DU_{t2} Dummy variable capture the COVID–19 pandemic period;
- sh_t^X Expenditure share.

Abstract

In the modern global economic system, the prosperity of each countries economy is directly related with foreign trade. The export of country is useful at macroeconomic and microeconomic levels.

The main aim of this thesis is to identify the relationship of the supply and demand factors for Lithuanian exports of goods and services. Using the theoretical foreign trade models: Armington and Almost Ideal Demand System models are estimated direct, inverse and hybrid, supply and demand driven models. To test the assumptions of models are using Ljung–Box test of autocorrelation, Breusch– Pagan test for heteroskedasticity and Shapiro–Wilk test of normality. Lithuanian export prices and volumes in short–run are explained by demand and the long–run by supply components. Exported firms are price takers in short–term, but in medium/long–term prices depends on sector marginal costs and potential output.

In the thesis, we also examine the relationship between the main export production price and quantity. The main agricultural products – cereals and major mining production – mineral products are highly responsive to price changes.

Keywords: Lithuania's exports of goods and services, Exports of goods of Lithuanian origin, Re–exports, Supply, Demand, Armington model, Almost Ideal Demand System (AIDS) model.

Santrauka

Šiuolaikinėje ir globalioje ekonomikos sistemoje, kiekvienos šalies ekonominė raida yra tiesiogiai susijusi su aktyviai vykdoma užsienio prekyba. Eksportas yra naudingas tiek šalies mikroekonominiai tiek makroekonominiai plėtrai.

Pagrindinis šio darbo tikslas – nustatyti paklausos ir pasiūlos veiksnių įtaką Lietuvos prekių ir paslaugų eksportui. Darbe nuosekliai yra išnagrinėjamas Lietuvos prekių ir paslaugų eksportas, supažindinama su pagrindinėmis eksporto prekėmis ir paslaugomis bei įvardijamos esminės prekybos rinkos. Siekiant detaliau pažinti tarptautinę prekybą yra apžvelgiami aktualiausi prekybos modeliai: Krugman, Melitz, Armington, beveik idealios paklausos (AIDS) bei mažos ir atviros šalies ekonomikos bruožai. Remiantis pagrindiniais teoriniais aspektais yra įvertinami tiesioginiai, atvirkštiniai bei hibridiniai paklausos ir pasiūlos modeliai. Sudarytų modelių patikimumas yra tikrinamas remiantis statistiniais autokoreliacijos (Ljung–Box), normalumo (Shapiro–Wilk) bei homoskedastiškumo testais (Breusch–Pagan). Gauti rezultatai leidžia daryti išvadas, kad Lietuvos prekių ir paslaugų eksporto kainas ir apimtis trumpuoju laikotarpiu veikia paklausos veiksniai, o ilguoju laikotarpiu – pasiūlos. Lietuvos eksportuotojai trumpuoju laikotarpiu prisiderina prie rinkoje vyraujančių kainų, tačiau ilgesnėje laiko perspektyvoje eksportas priklauso nuo potencialios produkcijos bei ribinių sąnaudų.

Taip pat, šiame darbe yra nagrinėjamas ryšys tarp vienų iš pagrindinių eksporto prekių (javų ir mineralinių produktų) kainų ir produkcijos kiekio. Remiantis gautais rezultatais, daromos išvados, kad šių prekių eksportui reikšmingos įtakos turi rinkoje vykstantys kainų pokyčiai.

Raktiniai žodžiai: Lietuvos prekių ir paslaugų eksportas, Lietuviškos kilmės prekių eksportas, Reeksportas, Pasiūla, Paklausa, Armington modelis, Beveik idealios paklauso (AIDS) modelis.

Introduction

In the modern global economic system, active foreign trade ensures stable economic growth in every country. Besides, foreign trade is useful at macroeconomic and microeconomic levels. A country's exporting activity influence Gross Domestic Product (GDP) Indicator, increase the size of general government budget and contribute to the national country's image. High export levels increase the flow of funds into the country and rise consumer spending, which as a consequence increases the country's economic growth. At the microeconomic level, exports expand the market boundaries of companies. Because of increasing export volumes, companies have to create new workplaces and consumers can enjoy a rising variety of goods and services.

One of the most significant features of the strong Lithuanian economy is growth in exports. Supply, demand or mixture of Lithuania's economic features, including political cycles, might bind the development of export growth in the short and long perspective. Thomas E. H. Notten [1] empirically demonstrated that Lithuanian export volumes in the long-run are driven by supply factors, while short-run dynamics are dominated by demand. This configuration corresponds to the neoclassical synthesis where the short-run follows the Keynesian demand side and the long-run the neoclassical supply side. The econometric results in Thomas A. H. Notten's article [1] disclose that Lithuanian exporters are highly responsive to price impulses as a result of operating in highly competitive markets. Lithuanian workers are prepared to adjust their wages to market conditions. Foreign and domestic investments expand the productive capacity of the Lithuania's economy to supply world markets with an always growing diversity of different products. It is impact export volumes in long-run more than world demand side.

When constructing an econometric model it is often necessary to introduce assumptions, additional made in theoretical models. One of the famous international trade model is the Armington model, based on the premise that each country produce a different good and consumers would like to consume at least some of each country's goods. This theoretical model reflects two sides of international trade: short-term demand and long-term supply [11]. The other, important consumer demand model (Almost Ideal Demand System model) which described consumer behaviour was discovered by Deaton and Muellbauer [3]. The model is grounded in a well-structured analytical framework, accommodates certain types of aggregation is evidently easy to estimate and allow testing of the standard restrictions of classical demand theory [4].

Research object: Lithuania's exports of goods and services.

The main aim: Investigate how supply and demand conditions affect Lithuania's exports.

In order to find out the factors on which Lithuania's foreign trade depends, we start with some hypotheses about supply and demand conditions:

1. Exporting firms are price takers in the medium-run. It means, that there should be statistically significant relationship between export and foreign competitors prices in inverse export demand function. This implies perfect price elasticity of exports demands.

- 2. How marginal costs determine Lithuanian export. Behaviour of marginal cost in Armington model supply and AIDS model equations.
- 3. If firms are price takers in the short-run, then the changes in export volumes have no impact on changes in export prices.
- 4. Changes in the world demand should not influence export volumes in the short-run. But in the medium-run export volumes should be affect on both, supply and demand side conditions.

To test the hypothesis and answer the thesis is structured as follows, first of all, we have to get acquainted with the Lithuanian export situation. So, the first chapter introduces the general foreign trade situation in Lithuania. In this chapter is given the latest overview of the export of goods and services. Furthermore, we will introduce you to the main Lithuanian export partners and export production.

In the second chapter is given the overview of main international trade models, such as: Krugman, Melitz, Armington, and small and open economy models.

The third chapter shows the theoretical part of models: Armington and Almost Ideal Demand System (AIDS) model, which are using in the further analysis. In this chapter, there is also an overview of other authors scientific literature, related with these foreign trade models.

The fourth chapter represents the endogenous and exogenous variables, their stationarity and the applied unit root tests.

In the fifth chapter, we are given the constructed international trade models: demand-driven, supply-driven, hybrid, and AIDS models. We also examined the relationship between the main export production price and quantity. The main agricultural products – cereals and major mining production – mineral products are highly responsive to price changes.

1 Lithuania's International Trade

1.1 Exports of goods and services

Exports are important for Lithuania's economic growth. From January to October 2021, Lithuania's exports of goods and services consisted for 26.6 % of services and for 73.4 % of goods. Exports of goods excluding energy products are subdivided into exports of goods of Lithuanian origin (excluding energy products) - 53.6 %, re–exports of goods (excluding energy products) - 36.7 % and exports of energy products - 9.7 %.



Figure 1: Structure of nominal exports of goods and services in January–October 2021

In re-exports, goods are produced or sold (as imported commodities) to a customer from across countries. The holder sells those goods to a third countries without additional preparing. Because of import content of re-exports, its economic importance is smaller than for exports of goods of Lithuanian origin. Meanwhile, exports of services are growing in significance for Lithuania. This is not straight reflected in the share of services in total exports because of the rapidly growing re-exports, but Figure 2 shows that in recent years exports of services have been growing fundamentally similarly with re-exports of goods.



Figure 2: Structure of nominal exports during the period 2004–2021 third quarter

1.2 Exports of goods

Lithuania has sustainable trade linkages with the developing world. Exporters offer good quality products to world markets. According to a multi-year average (from the first quarter of 2004 to the third quarter of 2021), approximately 63.1 % of exports comprise of domestically produced goods, with the remaining 36.9 % being re-exports.

Figure 3 below shows the structure and value of Lithuania's exports from the 2004 first quarter to 2021 the third quarter. Lithuanian exports account for exports of domestically produced goods, re-exports and the exports excluding energy products. The latter component (exports excluding energy products) is important because of the mineral products, which are dependent on some external factors. Mineral products account for about 10 % of total exports and depend on constantly changing and strong raising global oil prices and world markets demand for petroleum products.



Figure 3: Lithuanian exports of goods, mln. EUR

Figure 4 illustrates the annual changes in Lithuania's exports of goods (at current prices) %. The graph indicates that Lithuania's exports maintain the growth trend and experience declines during the global financial crisis and the COVID-19 pandemic consequences. That allows making hypothesis that Lithuania's exports might at least contribute to the business cycle downturns.



Figure 4: Annual changes in Lithuanian exports of goods (at current prices), %

Data classification is performed to become familiar with Lithuania's foreign trade and its structure. To get the latest information, we analysed the data from January–October, 2021. The export of goods can be classified by type of goods or leading trade partners.

According to the January–October 2021 data (Statistics Lithuania) Lithuania mainly exports: chemical products - 14.8 %, machinery and mechanical appliances - 13.6 %, mineral products - 9.7 %, miscellaneous manufactured articles - 8.9 %, prepared foodstuffs - 7.7 %, base metals - 6,9 %, plastics and rubber -6.8 %, vehicles, aircraft, transport equipments -5.9 %, vegetables products -5.4 % and wood -4.4 %. More detailed information is provided in appendix B, Figure 14. The largest share in Lithuania's exports fell within products of the chemical and related industries (14.8 %). The most important component, accounting for as much as 34.5 % of the total exports of the chemical industry is miscellaneous chemical products (mostly medical reagents). Exports of miscellaneous chemical products are the main component of the chemical industry, which has the greatest impact on Lithuania's growing export - over ten months, the total volume of exports increased by 4.6 percentage points. Such rapid growth in the chemical industry's export is related to the new export markets in the United States. Exports of miscellaneous chemical products to the United States increased 3.4 times in the January–October 2021. The new opportunities led the medical and research companies to open a new chemical plant in Lithuania. The new chemical plant increased miscellaneous chemical (medical reagents) products manufacturing and export volumes. Exported medical reagents are used to make the vaccines which have higher demand during the global pandemic.

Based on the January–October 2021 data, the main exported industries in exports of domestically produced goods were: chemical products – 13.6 %, mineral products – 13.1 %, miscellaneous manufactured articles – 12.8 %, prepared foodstuffs – 9.1 %, plastics and rubber – 7.3 %, machinery and mechanical appliances – 6.7 %, base metals – 6.4 %, vegetable products – 6.4 %, wood – 5.9 %, animal

products – 5.8 %. More detailed information is provided in appendix B, Figure 15.

During the same period, the most re-exported products were: machinery and mechanical appliances -24.7 %, chemical products -16.9 %, vehicles, aircraft, transport equipments -12.0 %, base metals -7.7 %, plastic and rubber -6.0 %, prepared foodstuffs -5.4 %, textiles -4.3 %, mineral products -4.0 %, optical, measuring medical instruments -4.0 % and vegetable products -3.7 %. More detailed information is provided in appendix B, Figure 16.

Based on the latest data, about 57.3 % of total Lithuania's exports were exported to European Union (EU) countries, while the remaining part – 42.7 % to third countries. Following the shock of the COVID–19 pandemic in 2020, the economic growth of the EU is proceeding fast. The growing GDP of EU countries is very favourable for the development of Lithuania's exports. The recovering economies of the countries mean growing foreign demand, which in turn increases Lithuanian exports. As reported by data from January to October 2021, Lithuania's top 10 export partners were: Russia, total market share – 10.7 %, Latvia – 9.5 %, Germany – 8.4 %, Poland – 7.7 %, United States – 6.3 %, Netherlands – 5.0 %, Estonia – 4.9 %, Sweden – 4.5 %, United Kingdom – 3.9 % and Denmark – 2.5 %.

At the same time, the top 10 partners for exports of goods of Lithuanian origin were: Germany, total market share -10.3 %, United States -9.1 %, Poland -7.2 %, Latvia -7.2 %, Netherlands -6.3 %, Sweden -6.1 %, United Kingdom -5.5 %, Norway -3.9 %, Denmark -3.3 % and France -2.9 %. Because of growing demand for chemical products, such a large part of domestically exported goods falls on the American market. With a strong world demand for various manufacturing articles, especially for furniture, Lithuanian exports have such a huge part of the economical strongest European country market – Germany.

The main top 10 re-exports partners were: Russia, total market share -25.3 %, Latvia -13.3 %, Poland -8.4 %, Estonia -7.3 %, Belarus -6.9 %, Denmark -5.2 %, Ukraine -2.9 %, Netherlands -2.9 %, United States -1.7 % and Italy -1.6 %. Russia's economic recovery is driven by growing private consumption and investment, which are components of demand-intensive imports, so in 2021 January-October we see rapid growth in re-exports from Lithuania to Russia. More detailed information is provided in appendix B, Figure 13.

1.3 Exports of services

Lithuania is important in the global market not only as an exporter of goods but also of services. Figure 5 illustrates Lithuanian exports of services in millions of euros from the 2013 first quarter to the 2021 second quarter. The graph below shows the Lithuanian transport and travel services exports, other services exports and the total exports of services. The graph clearly shows that the value of travel service exports significantly decreased during the COVID-19 pandemic period. Throughout the COVID-19 pandemic the world economy has stopped because of global lockdowns and social economic restrictions.

After a decline of 8.2 % in the 2020 year, a significant recovery in exports of services was observed in the 2021 year. According to the preliminary data of the Bank of Lithuania, over nine months the exports of services grew 23.9 %. The recovery in foreign trade in services has been importantly affected



Figure 5: Lithuanian exports of services, mln. EUR

by recovering economies of major partners and rose demand for goods transported by the transport services sector.

According to the January–September data (Bank of Lithuania) Lithuania exports: transport services – 59.8 % which is the largest group of exported services and mostly depends on good exports because of the need for transportation commodities. Other business services accounted for – 13.9 %, telecommunications, computer and information services – 9.9 % which need for services grew rapidly during the COVID–19 pandemic, construction services – 3.9 %, manufacturing services on physical inputs owned by others – 3.1 %, financial services – 2.8 %, maintenance and repair services not included elsewhere – 2.2 %, travel services – 3.2 % which reduced volumes due to pandemic constraints, personal, cultural and recreational services – 0.6 %, government goods and services not included elsewhere – 0.4 %, charges for the use of intellectual property not included elsewhere – 0.1 % and insurance and personal services – 0.0 %. More detailed information is provided in appendix B, Figure 17.

January-September 2021, Lithuania's top 10 exports of services partners were: Germany – 14.2 %, France – 8.3 %, Netherlands – 6.6 %, Russia – 6.0 %, Sweden – 5.3 %, Denmark – 5.2 %, United Kingdom – 4.2 %, Poland – 3.9 %, Belarus – 3.7 % and Latvia – 3.5 %. More detailed information is provided in appendix B, Figure 18.

2 Overview of international trade models

The development of international trade has a major impact on world economic growth, investment and the growth of modern technologies. According to world trade organisation (WTO) [6] since XIX middle of the century the international production volumes increased -60 times as well as international trade -140 times. Based on this growth, we can presume that foreign trade is one of the most influential factors in economic growth. Global trade stimulate the competition in the market and determine the world demand and supply of the goods and services.

In this chapter, we presented three international trade models and emphasized their advantages and disadvantages. Also, we submit all models for structured comparison and review small and open economy country assumptions.

2.1 Krugman's model

Paul Robin Krugman is a United States economist who won the Nobel Prize for an analysis of the impact of economies of scale on international trade. This analysis shows how the countries specialize in producing one type of product or another. To analyse homogeneous productions spreading between countries, Paul Robin Krugman in 1979 presented international trade model. This model disclosed the concept of trade benefits, which is based on the following assumptions [7]:

- 1. The principle of economies of scale;
- 2. The diversity of consumer goods.

These assumptions are implemented into the model by including labour force factor and the factor of the geographical location of the development of business sectors. According to the first assumption, the companies located into densely regions, tend having the costs of transportation of goods and implemented the principle of economies of scale. According to the second assumption, the diversity is greater in the densely populated locations.

Assessing the model utility function, it is assumed that in the market is n different goods and consumers value the variety of goods [8].

Consumer utility function:

$$U = \sum_{i=1}^{n} v(c_i),\tag{1}$$

where: U – utility function, $c_i - i$ consumption of goods, v – function descriptive inclination to diversity.

In Krugman's model, the return of economies of scale is expressed by a linear cost function by including fixed costs and claiming that labour force is the only primary factor of production [8].

Labour force as a production factor, algebraic formula:

$$I_i = \alpha + \beta x_i,\tag{2}$$

where: I_i – labour force, α – fixed costs, x_i – product yield.

The solution to this model is found by apply: commodity prices and labour force ratio, which is evaluated by wages to each product yield x_i , and the quantity of goods.

Consumer i commodity demand function [8]:

$$c(p_i) = (\lambda p_i)^{\frac{1}{\gamma - 1}},\tag{3}$$

where p_i – price of goods, λ – shadow value of income, $\frac{1}{\gamma}$ – constant markup.

Goods Price Index [8]:

$$\overline{p} = \left(\sum_{i} p_{i}^{\frac{\gamma}{\gamma-1}}\right)^{\frac{\gamma-1}{\gamma}},\tag{4}$$

where: \overline{p} – goods price index, $p_i - i$ price of goods, $\frac{1}{\gamma}$ – constant markup.

When the marginal company's income is equal to the marginal costs, the equilibrium goods price index can be described by the formula below [8]:

$$\overline{p} = \frac{n^{\frac{\gamma-1}{\gamma}}w}{\gamma},\tag{5}$$

where: \overline{p} - Goods Price Index, n - the variety of goods, w - wages, $\frac{1}{\gamma}$ - constant markup.

The Krugman's model assumes that the amount volume of productions are equal, when the prices in the model are symmetrical.

The formula below describes how to determine the output of an individual product volume [8]:

$$\overline{x} = \frac{\left(\frac{\alpha}{\beta}\right)\gamma}{(1-\gamma)},\tag{6}$$

where: \overline{x} – production volume, $\frac{1}{\gamma}$ – constant markup, α – fixed costs.

For the reason that Krugman's model is based on an assumption about the diversity of consumer goods, it is necessary to show the mathematical expression of the diversity of goods n. The variety of goods n is expressed in the number of employees L, because it depends on labour costs.

$$n = \frac{L}{\alpha + \beta \overline{x}},\tag{7}$$

where n - the variety of goods, L - the number of employees, \overline{x} - production volume, α - fixed costs.

The greater the number of working employees L, the greater consumer welfare, because of growing wages. The more active consumers are in the economy, the greater the demand for a variety of goods. The growing demand for a variety of goods is generating increasing returns to scale [7].

2.2 Melitz's model

Marc J. Melitz is a United States economist who in 2003 presented the dynamic industry model. It was one of the first model to explain the correlation between exports and productivity. In this model, companies operate in a monopolistic call competitive market, meaning that the similar products are differentiated and specific brands matter. In the international trade model, there are no recoverable fixed costs, which are experienced when firms try to consolidate in the foreign market. The Melitz model is defined by heterogeneous firm's productivity and viability principles. Export firms are increased its production costs, because of increased profits. These actions reduce opportunities for firms that are trying to enter the market [9].

Features of the Melitz model are the following: [10]:

- 1. The export firms encounter non-recoverable costs when trying to get into the foreign market;
- 2. The firms distribute the production domestic and foreign markets: that is some products depends on domestic demand, others on foreign demand;
- 3. Firms have to maintain high productivity of their company if they want to stay in the international market;
- 4. The distribution of resources has a positive impact on the overall level of productivity in the country.

The demand side of the model stems from the CES (constant elasticity of substitution) utility function [9]:

$$U = \left[\int_{\omega \in \Omega} q(\omega)^{\frac{1-\sigma}{\sigma}} d\omega \right]^{\frac{\sigma}{1-\sigma}},$$
(8)

where: U – utility function, Ω – set of goods under investigation, $\sigma = \frac{1}{1-\rho}$, when ρ – elasticity of substitution, $q(\omega)$ – goods from the set Ω demand.

Consumer behaviour can be monitored by analysing the quantity of goods consumed and also according to the goods' prices [9].

Likewise, the prices are defined by the CES (constant elasticity of substitution):

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{(1-\sigma)} d\omega \right]^{\frac{1}{1-\sigma}},$$
(9)

where: P - price index, Ω - set of goods under investigation, $p(\omega)$ - goods from the set Ω , price, $\sigma = \frac{1}{1-\rho}$, when ρ - elasticity between the substitutes for goods.

Firms can achieve higher productivity levels by producing higher-quality goods at the same production cost. However, firms are using profit-increasing mark-up, without reference to firm productivity [9]:

$$P = \left[\int_0^\infty p(\varphi)^{(1-\sigma)} M\mu(\varphi) d\varphi\right]^{\frac{1}{1-\sigma}},$$
(10)

where: M – number of firms, $\mu(\varphi)$ – distribution of productivity, $p(\varphi)$ – product pricing, σ – constant elasticity.

The model also provide simplified prices P and production volumes Q expressions [9]:

$$P = M^{(1-\sigma)} p(\tilde{\varphi}), \tag{11}$$

where: M – number of firms, $\tilde{\varphi}$ – level of firm productivity, weighted average, independent of the number of firms M, σ – constant elasticity.

$$Q = M^{(1-\sigma)}q(\tilde{\varphi}),\tag{12}$$

where: M – number of firms, φ – level of firm productivity, weighted average, independent of the number of firms M, σ – constant elasticity.

The Melitz model is one of the first models to explain the relationship between product exports and productivity.

2.3 Armington's model

Paul Armington is a United States economist who in 1969 presented an international trade model and its assumptions. Based on the model assumptions, every country produces different goods (which do not have substitutes) and the consumers from different countries want a part of all country's goods. In this model, consumers see differences in the identical goods because of origin of production [11].

The Armington's model main idea and structure [11]:

- 1. In the model, exported goods are divided by the country of origin. These items are not considered perfect substitutes for one made in country brand matters;
- 2. There are no comparative advantage use in the model, so the exported production specializations are not important;
- 3. This model does not appreciate the variety of products;
- 4. The number of goods are fixed in the model.

Armington model reflects two sides of international trade: the short–term supply and long–term demand.

We reviewed the model demand side and its utility function, which is interpreting as countrie's j welfare [12]:

$$U_j = \left(\sum_{i \in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},\tag{13}$$

where: U_j – utility function, σ – elasticity of substitutability $\sigma \in (0, \infty)$, a_{ij} – parameter of priority, q_{ij} – volume of goods from country *i* to country *j*.

$$q_{ij} = a_{ij} p_{ij}^{-\sigma} Y_j P_j^{\sigma-1} \tag{14}$$

where: q_{ij} - the value of exported goods, a_{ij} - parameter of priority, p_{ij} - price of goods from country i to country j, Y_j - the country j revenue, P_j - the price index in country j.

This (14) demand function [12] shows that the volume of goods in country j from country i is increasing when the price of goods p_{ij} decreases and the country's revenue j and prices index P_j increase.

We reviewed the model supply side in the perfect competition market. In that case, price of goods is marginal costs. To determine price of goods need to be customized an "iceberg" composition. It is an assumption: if we want to sell one good in country j, we have to produce one or more goods in country i ($\tau \ge 1$) [12]:

$$p_{ij} = \tau_{ij} \frac{\omega_i}{A_i},\tag{15}$$

where: p_{ij} – Goods produced in the country *i* price in the country *j*, A_i – number of goods from country *i*, ω_i – cost of country *i*, τ_{ij} – the measure of "iceberg" composition($\tau_{ij} = \frac{p_{ij}}{p_i}$).

General function of international trade [12]:

$$X_{ij} = a_{ij}\tau_{ij}^{1-\sigma} \left(\frac{\omega_i}{A_i}\right)^{1-\sigma} Y_j P_j^{\sigma-1},\tag{16}$$

where: X_{ij} – the volume of export, a_{ij} – parameter of priority, τ_{ij} – the measure of "iceberg" composition $(\tau_{ij} = \frac{p_{ij}}{p_i}), A_i$ – number of goods from country i, ω_i – cost of country i, Y_j – the country j revenue, P_j – the price index in country j, σ – elasticity of substitutability $\sigma \in (0, \infty)$.

2.4 Small open economy country

The country's economy is calling small and open when the export of the country is affected by demand and supply factors. One of the method for estimating the size of a country economy is the world price. The small economy country is taking while the large economy country set prices in the world market. Noteworthy, the size of economy depends directly on population within the country. In the article [13] written by Simon Kuznets, it is noted the boundaries of populations. The countries with a population of 10 millions or less is considered as a small and open economies. The size of population directly depends on countries area and resources.

Small open economy features [14] [15]:

- 1. Directly depends on world economy changes;
- 2. Because of small competitiveness, there are no economies of scale in the internal market;
- 3. The lack of resources and high production costs;
- 4. Trade liberalization;
- 5. The economy's policies have not significant impact on the world market;
- 6. Because of the small country's population and area, there are lack of diversity of goods.

Based on these features, we can say that the small and open economy has specific economic, political and social factors and depends on other countries political and economical situation. The open economy gives the advantages of the country's development and reduces the importance of economies of scale [16]. One of the small and open economy's features is the small diversity of goods because of its small population and area. As D.W.Snyder mentioned in the article [17] the open economy encourages growth of goods and services in the market. Nonetheless, as a small and open economy, the country must accept world market prices and is more reliant on foreign trade.

2.5 A comparison of economic trade models

Every described model has a unique set of advantages and disadvantages and in relation to one another has similarities and differences. A detailed comparison of international trade patterns is presented in Table 1.

Armington's model	Melitz's model	Krugman's model
Fixed costs are equal to zero $H_s = 0$	Technical equipment costs are not equal to zero $F_{sj} > 0$, but fixed costs are equal to zero $H_s = 0$ Fixed costs are not evolution zero $H_s \neq 0$	
Perfect competition firms	Monopolistic competition firms	Monopolistic competition firms
The productivity of company is treated equally within the country	The productivity of company is treated equally within the country	Differences in company productivity within the country
The number of firms in a country is considered an exogenous variable defined outside the model	The number of firms in a country is considered an endogenous variable as part of the economic profit-making mechanism	The number of firms in a country is considered an endogenous variable as part of the economic profit-making mechanism
All firms can trade their productions in all markets	All firms can trade their productions in all markets	Only the monopolistic firms can trade in all markets

m 11	4		•	c	•	. 1	1 1
Table	11	А	comparison	OŤ	economic	trade	models

Based on Zeynep Akqul article [18] the Melitz model is unique because of the assessment of the diversity of goods and the invention of new ones. It is one of three model that take into account and estimate fixed costs. In the Armington model firms are perfectly competitive, but it is not taken into account firm's productivity. The Krugman model partially evaluates fixed costs and as in the Armington model, did not consider firm's productivity.

3 Armington and Almost Ideal Demand System (AIDS) models

3.1 Theoretical Part of Models

The foreign trade symmetrically define the volume of exports by demand driven Armington or AIDS models. The shares in corresponding domestic and world demands are functions of respective prices. The share for exports experience an extra trend appearing from aggregation, growth in variety and re–exports [5]. In this subsection, we review the theoretical parts of the Armington and AIDS models.

The condition of household optimisation problem is expressed by the equation of the gross foreign consumer imports expenditure share (sh_t^X) witch is spend on Lithuanian exported commodities and services [5]:

$$sh_t^X = \frac{\hat{P}_t^X X_t^R}{P_t^W M_t^W} = g(t) \cdot b_L^\sigma \left(\frac{\hat{P}_t^X}{P_t^W}\right)^{1-\sigma} e^{\epsilon_t X^d},\tag{17}$$

where: X_t^R – real exports (net of mineral products), M_t^W – the gross Lithuanian trade partners demand for imports (real world market demand), \hat{P}_t^X – Lithuania's export price indices, P_t^W – Lithuania's competitor's export price indices, the ration of \hat{P}_t^X and P_t^W defines the real effective exchange rate (REER), b_L – constant initial share of made in Lithuanian brand, g(t) – accounts for time–varying changes in the share.

According to neoclassical synthesis, the small open economy in short-run have purely demand driven specification. Taking the logarithmically transformation to equation (17) into estimation form. We get the demand driven equation:

$$log X_t^d = \alpha_1 - \sigma(log \hat{P}_t^X - log P_t^W) + (1 + \alpha_2) log M_t^W + \epsilon_t^{X_d}.$$
(18)

If the economy is small and open, the demand could be much greater than supply and the exporting firms resemble the behaviour as a price takers. The exporting firm imagines that its individual demand curve's price elasticity is infinite until the productive capacity creates a disequilibrium gap huge enough [5]. As an alternative, define supply driven equation:

$$log X_t^s = \beta_1 + \beta_2 (log \hat{P}_t^X - log P_t^{HEX}) + \beta_3 (log \hat{P}_t^X - log M C_t^X) + \beta_4 log Y_t^{M*} + \beta_5 log t + \epsilon_t^{X_s}, \quad (19)$$

where: P_t^{HEX} – domestic prices (non energy component of HICP), MC_t^X – sector marginal costs, Y_t^{M*} – potential output.

If the exporting firms are completely demand driven, than the price elasticity of supply driven equation is possibly infinite. So, we can make several hypotheses (checked in chapter: Constructed models equations) [5]:

1. Exporting firms are price takers in the medium-run. It means, that there should be statistically

significant relationship between export and foreign competitors prices in inverse export demand function. This implies perfect price elasticity of exports demands.

- 2. If firms are price takers in the short-run, then the changes in export volumes have no impact on changes in export prices.
- 3. Changes in the world demand should not influence export volumes in the short-run. But in the medium-run export volumes should be affect on both, supply and demand side conditions.

Definitely, we are interesting into a hybrid restricted version model. Which connects the short-run demand and long-run supply factors into one model. So, the share changes of the productive capacity [5]:

$$sh_t^X = \frac{\hat{P}_t^X X_t^R}{P_t^W M_t^W} = b_L (Y_t^{M*})^{\beta_0} + e^{\epsilon_t X^h},$$
(20)

Based on Deaton and Muellbauer [24] the general AIDS model (which is an exact first order approximation of any demand function derived from the utility maximisation problem):

$$sh_{t}^{X} = \frac{\hat{P}_{t}^{X} X_{t}^{R}}{P_{t}^{W} M_{t}^{W}} = a + bf(t) - (\phi - 1)(\log \hat{P}_{t}^{X} - \log P_{t}^{W}) + \epsilon_{t}^{AIDS},$$
(21)

where: f(t) – deterministic trend proportional to $log M_t^W$.

With the sought time-varying elasticity:

$$\sigma_t = \left| \frac{\delta log X_t^R}{\delta log \hat{P}_t^X} \right| = 1 + \frac{\phi - 1}{a + bf(t) - (1 - \phi)(log \hat{P}_t^X - log P_t^W)},\tag{22}$$

which is equal to 1 if $\phi = 1$, meaning that the AIDS model (21) could be approximated by the hybrid model (22) [5].

As the condition of the AIDS model (21) the exporting firms follow the mark-up pricing concept [5]:

$$P_t^X = \eta_t^x M C_t^X = \frac{\sigma_t}{\sigma_t - 1} M C_t^X, \tag{23}$$

The mark-up over prices at factor costs in logarithmic form:

$$p_t^O = (1 - \mu_x)mc_t^x + \mu_x p_t^x,$$
(24)

where: μ_x – import content of exports.

In this case, the AIDS mark-up reflects the mark-up that could make an appearance comparing domestic and foreign sales. Simultaneously, we do not know what part of 'icebergs' costs that arise exporting abroad, as well as the biases in estimates caused by a part of export of services and re-export [5].

To specify an AIDS model restricted supply curve, we take the mark-up logarithmic approximation:

$$log\mu_t^x \approx log\left(1 + \frac{a + bf(t)}{\phi - 1}\right) + \frac{1}{1 + \frac{a + bf(t)}{\phi - 1}}(log\hat{P}_t^X - logP_t^W),\tag{25}$$

It's follows the export price equation by Dieppe et al. [25]:

$$log\hat{P}_{t}^{X} = \beta_{1} + \beta_{2}t\frac{1 + \frac{a+bf(t)}{\phi-1}}{2 + \frac{a+bf(t)}{\phi-1}}logMC_{t}^{X} + \frac{1}{2 + \frac{a+bf(t)}{\phi-1}}logP_{t}^{W} + \epsilon_{t}^{P_{AIDS}^{X}},$$
(26)

To account for the approximation errors a linear trend is added.

Noteworthy, that in AIDS model equation (26) accounts for the both demand and supply driven factors. But, no one of the supply or demand driven equations (18) and (19) are not able to consistently explain the pricing behaviour.

Finally, a useful characteristic of hybrid specification (20) is that the export price equation is consistent with the general price principle witch explained as the world competitors prices, weighted average of oil and domestic factor costs of production:

$$log P_t^X = \gamma_0 + \gamma_1 log P_t^W + \gamma_2 log P_t^{OIL} + (1 - \gamma_1 - \gamma_2) log P_t^Y + \epsilon_t^{P^X},$$
(27)

The equations (20) and (27) define the basic medium-run behaviour of Lithuanian exports [5].

3.2 Models in the scientific literature

In this subsection, we review the literature in which authors use the Armington and AIDS models. Since Paul Armington [26] introduced the Armington model into international trade theory with the assumption that final goods internationally traded are differentiated on the basis of the country of origin, some authors have used it to analyse the foreign trade. Fuzhi Cheng [27] have applied the Armington model to develop the demand for scallops in the United States. The results of analysis indicate that the demand for United States domestic scallops is more elastic in the long-run than in the short-run. Ronald A. Babula [28] employed the Armington model to analyses about the United States cotton exports. In studies, Istis Baroh and other [29] have applied the Armington model, which consists of a demand equation, a supply equation, and a price equation, to analyse the competitiveness of Indonesian coffee in the international market.

Since Deaton and Muellbauer [24] developed the Almost Ideal Demand System (AIDS) model, some authors have used it to analyse produce demand. Khoiriyah, Anindita, Hanani, and Muhaimin [30] have applied this system to Indonesia's animal food consumption data, while Karagiannis, Katranidis, and Velentzas [31] employed it to examine Greece's meat demand. Amzul Rifin [32] applied the AIDS framework to analyze the competitiveness of Indonesia's cocoa bean exports in the world market. In studies, Bouamra-Mechemache, Requillart, Soregaroli and Trevisiol [33] employed the AIDS framework to demand for dairy products in the European Union and Hossain and Jensen [34] applied it to investigate food consumption patterns with Lithuania's household data.

4 Overview of Modelling Variables

In the analysis are used variables such as Lithuania's exports of goods of Lithuanian origin (excluding energy products) and re-exports (excluding energy products). The analysis time period is from the 2004 first quarter to the 2021 third quarter. The variables are classified by combined nomenclature (2 digits level). The data is uploaded from the Department of Statistic's official database. To disclose the export side of services is used the data from the Bank of Lithuania. Moreover, for analysis is used other countries macroeconomic indicators and their deflators. For the European Union, Euro Area and Organization for Economic Co-operation and Development (OECD) related countries indicators are used from the European statistics database "Eurostat". Among the main partners of Lithuania's international trade are countries that do not belong to those groups. In that case is used Macrobond database.

4.1 Endogenous variables

One of the main objects of research is endogenous (explanatory) variable – Lithuania's exports of goods (excluding energy products) and services (X_t) . Another important endogenous (explanatory) variable – Lithuania's Export Price Indices (P_t^x) .



Figure 6: On the left side: Lithuania's exports of goods and services. On the right side: Lithuania's Export Price Indices.

4.2 Exogenous variables

According to Thomas E. H. Notten [1] export volumes and price changes are best explained by supply factors in the long-run, while in the short-run, volumes and prices change due to global demand.

The short-run demand variables: Lithuania's Competitor's Export Price Indices, without energy products (P_t^w) and Real-World Market Demand (M_t^w) .



Figure 7: On the left side: Lithuania's Competitor's Export Price Indices. On the right side: Real–World Market Demand.

Lithuania's export price index differs from the competitor's prices index in the short-run (usually lower), but in the long-run they tend to change together. The Figure 8 shows that the prices are falling sharply due to the impact of the global financial crisis. However, with the end of the crisis and the gradual increase in world demand, prices started to rise. We can imagine the similar situation during the COVID-19 pandemic period. The graph also shows falling prices in 2016 and the 2020 first quarter due to lower oil prices. Changes in the prices of energetic products in the market directly affect all production prices.



Figure 8: Export price index in Lithuania and Lithuania's competitors price index

The long-run supply variables: Sector Marginal Costs (MC_t) , Domestic Prices – non energy component of HICP (P_t^{HEX}) , Potential Output (market sector) (Y_t^M) .



Figure 9: On the left side: Sector Marginal Costs. On the right side: Domestic Prices (non energy component of HICP).

In an economy, the potential output is understood as high and lasting real gross domestic product (GDP) level. To find the potential output variable is applying the Hodrick–Prescott (HP) filter for logarithmic GDP. The trend component determines the long–term outlook and the overall state of the economy. Meanwhile, the cycle component helps predict future economic downturns based on past data [19]–[21]. More information is provided in appendix B, Figure 19.

4.3 Variables Stationarity. Unit Root Tests.

Most process in economy are not stationary. The parameters of research are also not stationary (Figures: 6,7,9). When the variables are non-stationary, they are subjected to differential transformations that turn the variables into stationary processes. Unit root tests are used to check that the integration of the variables under consideration does not exceed I(1). To determine the stationarity of time series are used Kwiatkowski-Phillips-Schmidt-Shin (KPSS) and Augmented Dickey-Fuller test.

Kwiatkowski–Phillips–Schmidt-Shin (KPSS) test hypothesis [22]:

 $\begin{cases} H_0: & \text{The time series is stationary} \\ H_1: & \text{The time series is not stationary} \end{cases}$

Statistical examination of the hypotheses of the KPSS test led to the conclusion that the initial time series were not stationary. Examining the time series of the first differences, it was concluded that the time series of the first differences are stationary. Technical details for (KPSS) unit root test are outlined in Appendix B, Table 2.

Augmented Dickey–Fuller test hypothesis [23]:

 $\begin{cases} H_0: & \text{The time series is not stationary} \\ H_1: & \text{The time series is stationary} \end{cases}$

Examination of the hypothesis of the ADF test led to the conclusion that the 3 of 7 initial time series were not stationary. Applying unit root test for first difference time series was observed that for all 7 variables an alternative hypothesis was accepted. The time series are stationary. Table 3 in Appendix B, contains the results of Augmented Dickey–Fuller test.

The graphs below present stationary time series (the first differences of logarithmic of variables).



Figure 10: On the left side: first differences of logarithmic of Lithuanian exports of goods and services. On the right side: first differences of logarithmic of Lithuania's Export Price Indices.



Figure 11: On the left side: first differences of logarithmic of Lithuania's Competitor's Export Price Indices. On the right side: first differences of logarithmic of Real–World Market Demand.



Figure 12: On the left side: first differences of logarithmic of Sector Marginal Costs. On the right side: first differences of logarithmic of Domestic Prices (non energy component of HICP).

Dummy variables are added to the list of exogenous variables in order to correct for structural changes detected by the Augmented Dickey-Fuller tests and tso function algorithm (automatic procedure for detection of outliers in software 'R'). More information is provided in appendix A, Tables 3 and 4.

First dummy variable capture the global financial crisis. Second dummy variable capture the COVID-19 pandemic period. All these time periods were appear in the majority of series as a structural breaks.

$$DU_{t1} = \begin{cases} 1, & \text{if t} = 2007 \ Q_4 \text{ iki } 2009 \ Q_3 \\ 0, & \text{otherwise} \end{cases}$$
$$DU_{t2} = \begin{cases} 1, & \text{if t} = 2020 \ Q_1 \text{ iki } 2021 \ Q_3 \\ 0, & \text{otherwise} \end{cases}$$

here: DU_{t1} – dummy variable capture the global financial crisis, DU_{t2} – dummy variable capture the COVID-19 pandemic period.

5 Lithuanian international trade models

In this subsection, we test the small open economy implied hypotheses discussed in the 5.1 chapter. The demand driven factors describe the short–run behaviour of exporters, while supply side defines the medium–run behaviour.

As econometric methodology, we chose to apply a fully modified OLS estimation method. The estimation sample 2004Q1–2021Q3. The assumptions of models outliers are tested by Ljung–Box test of autocorrelation, Breusch–Pagan test for heteroskedasticity and Shapiro–Wilk test of normality. Also, we used Engel-Granger residual (ADF statistic) for the null hypothesis of non–cointegration.

To test the hypotheses we rewrite the corresponding demand (18) and supply (19) equations in inverse forms. Based on Riedel [35] we check three hypothesis:

1. $H_0: \frac{1}{\sigma} = 0$

2.
$$H_0: \frac{1}{(\beta_2 + \beta_3)} = 0$$

3. $H_0: \sigma = 1; \alpha_2 = 0$

Testing hypotheses (5.1 chapter) becomes equivalent to statistical insignificance of respective estimates of parameters: $(\frac{1}{\sigma}; \frac{1}{(\beta_2 + \beta_3)})$ [5].

5.1 Demand-driven model

Demand-driven model is estimated by equation (18) and its inverse form.

Direct

$$\Delta \log X_t^{\ d} = \frac{1.309}{(0.008)^{**}} \Delta \log M_t^{\ W} - \frac{0.252}{(0.503)} (\Delta \log P_t^{\ X} - \Delta \log P_t^{\ W}) + \frac{7.107}{(0.801)^*} + \frac{0.234}{(0.057)^*} d^{20/21} + \\ + \frac{0.141}{(0.083)^*} d^{07/09} + \epsilon_t^{\ X^d}$$
(28)

The adjusted coefficient of determination: $R^2 = 0.799$, Engel–Granger residual ADF statistic for the null hypothesis of non-cointegration: $\sigma_{EG} = -7.832$, *p*-values in squared brackets.

Statistical tests	Values	$\operatorname{Results}$
Ljung–Box test	$ ilde{\chi}^2=0.832$	p-value = 0.326
Shapiro–Wilk test	W=0.987	p-value = 0.480
Breusch–Pagan test	$\mathrm{BP}=23.544$	p-value = 0.003

Table 2: The direct demand-driven model testing results

The results of the model testing analysis are shown in Table 2. Based on the tests performed, there is a statistical basis to state that the model residues are non–autocorrelated, distributed according to the normal distribution, but heteroskedastic. Due to the heteroskedasticity of the residues, the assumption of the demand-driven model accuracy is compromised, which may skew the results obtained. In the model, statistically significant is world market demand M_t^W . It can be stated that, in the short-term, Lithuania's exports depend on global market demand.

Inverse

$$\Delta \log P_t^X = \underset{(0.001)^{**}}{0.811} \Delta \log P_t^W - \underset{(0.023)}{0.017} \Delta \log X_t^d + \underset{(0.922)^*}{0.104} \Delta \log M_t^W - \underset{(0.119)}{0.125} + \underset{(0.086)}{0.038} d^{20/21} + \\ + \underset{(0.222)^*}{0.018} d^{07/09} + \epsilon_t^{P^X}$$

$$(29)$$

The adjusted coefficient of determination: $R^2 = 0.764$, Engel-Granger residual ADF statistic for the null hypothesis of non-cointegration: $\sigma_{EG} = -6.784$, *p*-values in squared brackets. $H_0: \frac{1}{\sigma} = 0 \rightarrow$ (0.374).

Table 3: The inverse demand-driven model testing results

Statistical tests	Values	Results
Ljung–Box test	$ ilde{\chi}^2=0.366$	p-value = 0.391
Shapiro–Wilk test	W = 0.747	p-value = 0.690
Breusch–Pagan test	$\mathrm{BP}=10.666$	p-value = 0.542

The results of the model testing analysis are shown in Table 3. Based on the performed tests, there is a statistical basis to state that the model residues are non-autocorrelated, distributed according to the normal distribution, and homoskedastic. Following the results, it can be stated that the assumptions for the constructed inverse demand-driven model are met and it is appropriate to use the model.

Foreign competitor prices (P_t^W) , with a significant level of 5 % is statistically significant in the constructed model. The result shows that one of the preconditions of the small and open economy country model is satisfied: the country's export prices change in the long-run together with the export prices of the world market. It proves that Lithuania is a price taker and is dependent on the changes taking place in the international market. Because of the X_t^d coefficient's insignificance, the model confirms another assumption of a small and open economy: that in the long-run, export volumes do not affect export prices.

Models variables: X_t^d – export volumes, net of mineral products, real, P_t^X – export deflator net oil price effects, M_t^W – gross Lithuanian trade partners demand for import (Real world demand), P_t^W – Foreign competitor prices, $d^{07/09}$ – dummy variable for Financial crisis for 2007 fourth quarter to the 2009 third quarter, $d^{20/21}$ – dummy variable for COVID–19 pandemic for 2020 first quarter to the 2021 third quarter.

5.2 Supply-driven model

Supply-driven model is estimated by equation (19) and its inverse form.

Direct

$$X_{t}^{s} = Y_{t}^{M} + \underbrace{3.328}_{(0.002)^{***}} (P_{t}^{X} - P_{t}^{HEX}) + \underbrace{1.561}_{(0.015)^{**}} (P_{t}^{X} - MC_{t}^{X}) - \underbrace{0.038}_{(0.001)^{**}} + \underbrace{0.019}_{(0.022)^{*}} d^{20/21} + \\ + \underbrace{0.019}_{(0.022)^{*}} d^{07/09} + \underbrace{2.235}_{(0.024)^{*}} \log_{t} + \epsilon_{t}^{X^{s}}$$

$$(30)$$

The adjusted coefficient of determination: $R^2 = 0.634$, Engel–Granger residual ADF statistic for the null hypothesis of non–cointegration: $\sigma_{EG} = -6.792$, *p*–values in squared brackets.

Statistical tests	Values	$\operatorname{Results}$
Ljung–Box test	$ ilde{\chi}^2=0.478$	p-value = 0.521
Shapiro–Wilk test	W = 0.747	p-value = 0.131
Breusch–Pagan test	$\mathrm{BP}=18.234$	p-value = 0.023

Table 4: The direct supply-driven model testing results

The results of the model testing analysis are shown in Table 4. Based on the tests performed, there is a statistical basis to conclude that the residuals of supply-driven model are non-autocorrelated, distributed according to the normal distribution, but likely to be heteroskedastic. Due to heteroskedastic ticity, the accuracy assumptions of model are compromised, and the results obtained may be inaccurate.

Firms decide the quantity of production based on marginal costs and prices in competitive markets. If the price is higher than the marginal cost, then firms produce the unit of production and sell it, but if the marginal cost is higher than the price, firms lose profits and the unit of production is meaningless. The difference between the P_t^X and MC_t^X is statistically significant in the model with positive sign. It means that the prices are higher than the marginal costs, so Lithuania's exporters do not lose money if they produce the unit of production and sell it.

Inverse

$$P_{t}^{X} = \underbrace{1.099}_{(0.001)^{**}} P_{t}^{HEX} - \underbrace{0.018}_{(0.002)} MC_{t}^{X} + \underbrace{0.199}_{(0.002)^{**}} (X_{t}^{s} - Y_{t}^{M}) + \underbrace{0.563}_{(0.149)^{*}} - \underbrace{0.018}_{(0.011)^{*}} d^{20/21} + \underbrace{0.104}_{(0.005)^{*}} d^{07/09} - \\ - \underbrace{0.257}_{(0.002)^{*}} \log_{t} + \epsilon_{t}^{P^{X}}$$

$$(31)$$

The adjusted coefficient of determination: $R^2 = 0.711$, Engel–Granger residual ADF statistic for the null hypothesis of non-cointegration: $\sigma_{EG} = -6.784$, *p*-values in squared brackets. $H_0: \frac{1}{(\beta_2 + \beta_3)} = 0 \rightarrow (0.0)$.

Statistical tests	Values	$\operatorname{Results}$
Ljung–Box test	$ ilde{\chi}^2=0.278$	p-value = 0.601
Shapiro–Wilk test	W=0.947	p-value = 0.801
Breusch–Pagan test	$\mathrm{BP}=11.234$	p-value = 0.511

Table 5: The inverse supply-driven model testing results

The results of the model testing analysis are shown in Table 5. Based on the tests performed, there is a statistical basis to conclude that the residuals of supply-driven model are non-autocorrelated, distributed according to the normal distribution, and homoskedastic. Domestic prices (P_t^{HEX} the non-energy component of HICP) are statistically significant and have a positive sign in the model. It means that export prices are significantly affected by domestic prices.

Models variables: P_t^X – export deflator net oil price effects, X_t^s – export volumes, net of mineral products, Y_t^M – potential output, market sector, real, P_t^{HEX} – domestic prices (non–energy component of HICP), MC_t^X – marginal costs, $d^{07/09}$ – dummy variable for Financial crisis for 2007 fourth quarter to the 2009 third quarter, $d^{20/21}$ – dummy variable for COVID–19 pandemic for 2020 first quarter to the 2021 third quarter.

5.3 Hybrid model

Hybrid model is estimated by equation (20) and its inverse form.

Direct

$$X_t^h = M_t^W - (P_t^X - P_t^W) + \underbrace{1.612}_{(0.080)^*} Y_t^M - \underbrace{9.067}_{(0.789)^*} + \underbrace{0.151}_{(0.036)^*} d^{20/21} + \underbrace{0.151}_{(0.036)^*} d^{07/09} + \epsilon_t^{X^h}$$
(32)

The adjusted coefficient of determination: $R^2 = 0.733$, Engel-Granger residual ADF statistic for the null hypothesis of non-cointegration: $\sigma_{EG} = -4.887$, *p*-values in squared brackets. $H_0: \sigma = 1; \alpha_2 = 0 \rightarrow (0.285)$.

 Table 6: Outliers in the direct hybrid model test results

Statistical tests	Values	$\operatorname{Results}$
Ljung–Box test	$ ilde{\chi}^2=0.328$	p-value = 0.557
Shapiro–Wilk test	W = 0.847	p-value = 0.901
Breusch–Pagan test	$\mathrm{BP}=7.994$	p-value = 0.461

The results of the model testing analysis are shown in Table 6. Based on the tests performed, there is a statistical basis to conclude that the residuals of hybrid model are non-autocorrelated, distributed according to the normal distribution, and homoskedastic. Based on these results, it can be stated that the assumptions made about the accuracy of the developed hybrid model are satisfied. The results obtained can be accurately interpreted, and the model can be considered adequate. In addition, the potential output $Y_t M$ in the model is statistically significant, meaning that in the long-run the potential output leads to a consistent increase in export volumes.

Inverse

$$P_t^X = \underbrace{0.943}_{(0.088)^*} P_t^W - \underbrace{2.143}_{(0.003)^{***}} (X_t^h - M_t^W) + \underbrace{0.626}_{(0.045)^*} Y_t^M - \underbrace{2.809}_{(0.232)^*} - \underbrace{0.192}_{(0.007)^*} d^{20/21} - \underbrace{0.004}_{(0.011)^*} d^{07/09} + \epsilon_t^{X^h}$$
(33)

The adjusted coefficient of determination: $R^2 = 0.699$, Engel–Granger residual ADF statistic for the null hypothesis of non–cointegration: $\sigma_{EG} = -5.300$, *p*–values in squared brackets.

Statistical tests	Values	Results
Ljung–Box test	$ ilde{\chi}^2=0.528$	p-value = 0.457
Shapiro–Wilk test	W = 0.897	p-value = 0.391
Breusch–Pagan test	BP = 12.994	p-value = 0.481

Table 7: Outliers in the inverse hybrid model test results

The results of the model testing analysis are shown in Table 7. Based on the tests performed there is a statistical basis to conclude that the residuals of hybrid model are non-autocorrelated, distributed according to the normal distribution and homoskedastic. The results obtained can be accurately interpreted and the model can be considered adequate. Foreign competitor prices P_t^W is statistical significant with positive sign, it means that export prices depend on foreign prices in shortterm.

Models variables: X_t^h – export volumes, net of mineral products, P_t^X – export deflator net oil price effects, P_t^W – Foreign competitor prices, Y_t^M – potential output, market sector, real, M_t^W – gross Lithuanian trade partners demand for import (Real world demand), $d^{07/09}$ – dummy variable for Financial crisis for 2007 fourth quarter to the 2009 third quarter, $d^{20/21}$ – dummy variable for COVID–19 pandemic for 2020 first quarter to the 2021 third quarter.

5.4 Almost Ideal Demand System (AIDS) model

We assume that the volume of Lithuania's exports is determined by the almost ideal demand system (AIDS) model equations (21) and (26). Profit maximisation based on the AIDS model of export demand function results in the following 2-equation system for the export volume and export price. This allows a model compatible way to estimate the price elasticity of export demand.

In equations (21) and (26) f(t) trend could be deterministic and proportional to $log M_t^W$ [5]. The advantage of equation (26) is that the elasticity of the demand depends on the relative competitor export prices. [36]. The first order conditions of an optimising firm that produces export goods and services, through the system of labour, capital and imported goods and services, allow us to write export prices as a mark-up over marginal costs [36].

Parameters	$log M_t^W$	$logMC_t^X$	$log P_t^W$
a	$\underset{(0.013)}{0.856}$	$\underset{(0.029)}{0.796}$	$\underset{(0.002)}{0.963}$
b	$\underset{(0.195)}{1.032}$	$\underset{(0.003)}{1.362}$	$\underset{(0.063)}{1.397}$
φ	$\underset{(0.205)}{1.962}$	$\underset{(0.627)}{1.591}$	$\underset{(0.073)}{1.963}$

Table 8: 2-equation export system estimates

The parameter a – point market share (with indexed data close to unity), b – different from zero measures the deviation of income elasticity of export demand from unity, $\phi \ge 1$ is the representative point price elasticity of exports [36].

The function *aidsEst* included in the software 'R' with package *micEconAids*, provides a simple interface for the econometric estimation of the AIDS model. Table 8 presents all estimated coefficients: $a, b, and \phi$. The standard errors, t - values, deterministic and p-values as well as the adjusted coefficient of determination R^2 are obtained by applying the function summary.

In contrast, to the purely demand-driven model (Armington model), the AIDS show that the elasticity of substitution (sector marginal costs MC_t^X) is smaller. Therefore, the mark-up caused by world demand elasticity and cheaper production is big enough to state that firms in medium-run have not to worry much about changes in prices.

5.5 Almost Ideal Demand System (AIDS) model for main export products

In this subsection, we report the results of Almost Ideal Demand System (AIDS) model estimations. In other words, the elasticity for main export productions (from agriculture sector and products of industry and manufacturing products). The production was chosen according to subsection 1.1. discussed mainly export products and Appendix B, Figure 14. The constructed AIDS models estimates demand of productions response to change prices. The estimation sample 2004Q1–2021Q3. Also, in analysis is use dummy variables constructed in 4.3 section.

In the AIDS model, there are three mandatory economic arguments: the variable of the prices, the variable names of the expenditure shares and the variable name of total expenditure and one statistical mandatory – data frame that contains these variables [37].

For analysis are using agriculture production variables such as:

- 1. Cereals;
- 2. Oil Oil seeds and oleaginous fruits; miscellaneous grains, seeds and fruit, industrial or medicinal plants, straw and fodder;
- 3. Vegetables edible vegetables and certain roots and tubers;
- 4. Fruits edible fruit and nuts, peel of citrus fruit or melons.

Also, we are using products for other mining and quarrying products and manufacturing products:

- 1. Chemicals and chemical products;
- 2. Plastics and rubber;
- 3. Mineral products mineral fuels, mineral oils and products of their distillation, bituminous substances, mineral waxes.

Calculated compensated and uncompensated elasticity for agriculture products using the AIDS model and explained by Marshallian demand is presented in Table 9.

	Cereals	Oil	Vegetables	Fruits
Cereals	-0.541 (0.069)	-0.444 (0.003)	$\underset{(0.048)}{0.391}$	$\underset{(0.051)}{0.981}$
Oil	-2.107 (0.056)	$\underset{(0.003)}{0.063}$	$\underset{(0.021)}{0.106}$	-1.571 (0.666)
Vegetables	-0.369 (0.064)	$\underset{(0.007)}{0.068}$	-0.0276 $_{(0.063)}$	-0.469 (0.032)
Fruits	-0.396 $_{(0.003)}$	$\underset{(0.099)}{1.369}$	$\underset{(0.071)}{-1.196}$	$\underset{(0.113)}{0.369}$

Table 9: Elasticity for agriculture products (from an Almost Ideal Demand System (AIDS) model)

From the data review, cereals has the most value of export share among other commodities. The major part of agriculture of Lithuanian exports is cereals, thus new developed technology and agricultural extension for cereals should be developed in the country. To support Lithuania GDP, other products still need policies to increase the production for the international market.

The results (Table 9) shows that the own-price elasticity demand for agricultural products has positive signs for oil, and fruits. It indicates that the relationship between price and quantity follows the same direction. The vegetables and cereals sign is negative, so it indicates that there is an expected inverse relationship between price and quantity demand. As the results, the major agricultural products (cereals and vegetable) are highly response to price changes. This implies that exporters should be more productive to receive some benefits from the market demand.

Calculated compensated and uncompensated elasticity for other mining and quarrying products and manufacturing products using the AIDS model and explained by Marshallian demand is presented in Table 10.

	Chemicals	Rubber	Minerals
Chemicals	$\underset{(0.069)}{0.101}$	-0.444 (0.003)	$\underset{(0.048)}{0.391}$
Rubber	$\underset{(0.056)}{-2.107}$	$\underset{(0.003)}{0.063}$	$\underset{(0.021)}{0.106}$
Minerals	$\begin{array}{c}-0.369\\\scriptscriptstyle(0.064)\end{array}$	$\underset{(0.007)}{0.068}$	-0.0276 $_{(0.063)}$

Table 10: Elasticity for other mining and quarrying products and manufacturing products (from an Almost Ideal Demand System (AIDS) model)

From the data review, chemical production and mineral production have the most value of export share among other commodities.

The results (Table 10) show that the own-price elasticity demand for rubber and plastics and chemical production have positive signs. It indicates that the relationship between price and quantity follows the same direction. The minerals sign is negative, so it indicates that there is an expected inverse relationship between price and quantity demand. As the results, the major products(mineral products) is highly responsive to price changes.

6 Conclusions

- 1. Based on Armington demand-driven model, the perfect (medium-run) price elasticity of the demand curve is not denied by the data in chosen sample. Exporting firms are price takers in the medium-run. But in contrast, in AIDS model, the mark-up caused by world demand elasticity and cheaper production is big enough to state that firms in medium-run have not to worry much about changes in prices.
- 2. There are not statistically significant relationship between export volumes and prices. Firms are price takers in the short–run.
- 3. The marginal costs estimates are statistically insignificant in direct and inverse supply equations. Therefore, exporting Lithuanian firms care less on the profit margins. They are focusing attention on the productive capacity.
- 4. World demand influence export volumes in the short-run. To figure out, how supply and demand conditions affect export was constructed hybrid model. In the medium-run the consistent growth in the export volumes is determined by disequilibrium in quantity supplied (potential output).
- 5. The elasticity of the demand depends on the relative competitor export prices.
- 6. The major agricultural products (cereals and vegetable) are highly responsive to price changes, while oil and fruits are small responsive to changes in prices.
- 7. The major mining production (mineral products) is highly responsive to price changes, while chemical production and rubber and plastics are small responsive to changes in prices.

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7 Appendix

7.1 Appendix A

Graphs with R studio:

```
library(ggplot2)
library(dplyr)
library(zoo)
library(tidyverse)
colors <- c("Export of goods of Lithuanian origin" = "blue",
 "Reexports of goods" = "darkgreen", "Total exports of goods" = "black",
 "Total exports without minerals" = "red")
 f<- ggplot(Data_goods_3) +</pre>
  geom_col(aes(x = as.yearqtr(Times), y = Export_of_goods_of_Lithuanian_origin,
   color = "Export of goods of Lithuanian origin"),
  size = 0.1, fill = "deepskyblue3") +
  geom_col(aes(x = as.yearqtr(Times), y = Reexports_of_goods, color = "Reexports of goods"),
   size = 0.1, fill = "aquamarine4")+
  geom_line(aes(x = as.yearqtr(Times), y = Total_exports_of_goods,
  color = "Total exports of goods"), size = 0.7) +
    geom_line(aes(x = as.yearqtr(Times), y = Total_exports_without_minerals,
    color = "Total exports without minerals"), size = 0.7) +
  labs(x = "Time period",
         y = "Value, mln. EUR",
         color = "Variables:") +
    scale_color_manual(values = colors)+ theme(legend.position = "bottom")
dat <- Data_models %>%
  select(Xt_d) %>%
  mutate(x_axis = as.yearqtr(Data_models$Times)) %>%
  pivot_longer(c(Xt_d), names_to = "variable", values_to = "values")
ggplot(dat, aes(x = x_axis, y = values, colour = variable), size = 0.8) +
  geom_line()+ labs(x="Time period", y="Precent, %",
  title = "Lithuanian exports of goods and services") + theme(legend.position = "bottom")
```

HP filter:

```
library(mFilter)
BVP_R_log<-(log(Data_models$BVP_R))</pre>
Yt_M <- hpfilter<-(Data_models$BVP_R)</pre>
Yt_M_log <- log(Yt_M)</pre>
Yt_M_log
plot(Yt_M_log)
library(mFilter)
library(quantmod)
plot(hpfilter(log(Data_models$BVP_R),freq = 1600))
Unit root tests:
(same for all variables)
KPSS_Xt_d_1 <-ur.kpss(Data_models$Xt_d, type = "tau", lags = "short")</pre>
summary(KPSS_Xt_d_1)
plot(KPSS_Xt_d_1)
KPSS_Xt_d_2 <-ur.kpss(Xt_d_log_diff, type = "tau", lags = "short")</pre>
summary(KPSS_Xt_d_2)
plot(KPSS_Xt_d_2)
library(vars)
library(urca)
ZA_Pt_x__C <- ur.za(Data_models$Pt_x, model = "both", lag = 2) #C</pre>
summary(ZA_Pt_x__C)
plot(ZA_Pt_x__C)
ZA_Pt_x_A <- ur.za(Data_models$Pt_x, model = "intercept", lag = 2) #A</pre>
summary(ZA_Pt_x_A)
plot(ZA_Pt_x_A)
ZA_Pt_x_B <- ur.za(Data_models$Pt_x, model = "trend", lag = 2) #B</pre>
summary(ZA_Pt_x__B)
plot(ZA_Pt_x_B)
ZA_Pt_x__C <- ur.za(Xt_d_log_diff, model = "both", lag = 2) #C</pre>
summary(ZA_Pt_x__C)
plot(ZA_Pt_x__C)
ZA_Pt_x_A <- ur.za(Xt_d_log_diff, model = "intercept", lag = 2) #A</pre>
summary(ZA_Pt_x_A)
```

```
plot(ZA_Pt_x_A)
ZA_Pt_x_B <- ur.za(Xt_d_log_diff, model = "trend", lag = 2) #B
summary(ZA_Pt_x_B)
plot(ZA_Pt_x_B)</pre>
```

Demand–driven Model:

```
library("cointReg")
```

```
New data
a <- Pt_x_log_diff - Pt_w_log_diff</pre>
congthuc1 <- Xt_d_log_diff ~ a + Mt_w_log_diff + Dummy</pre>
congthuc2 <- Pt_x_log_diff ~ Pt_w_log_diff - Xt_d_log_diff</pre>
+ Mt_w_log_diff + Dummy + Dummy
lm(data=New_data,congthuc1) -> hoiquy1
lm(data=New_data,congthuc2) -> hoiquy2
summary(hoiquy1)
summary(hoiquy2)
doclap <-cbind(a, Mt_w_log_diff, Dummy)</pre>
phuthuoc1 <-cbind(Xt_d_log_diff)</pre>
phuthuoc2 <-cbind(Pt_x_log_diff)</pre>
deter = cbind(level = 1, trend = 1:69)
aa<-cointRegFM(x=doclap, y=phuthuoc1, deter, kernel = "ba", bandwidth = "and")
print(aa)
doclap <-cbind(Pt_w_log_diff, Xt_d_log_diff, Mt_w_log_diff, Dummy)</pre>
bb<-cointRegFM(x=doclap, y=phuthuoc2, deter, kernel = "ba", bandwidth = "and")</pre>
print(bb)
resid_bb = bb$residuals
resid_bb = na.omit(resid_bb)
library(urca)
y = ur.df(resid_bb, type = "none", selectlags = "AIC")
summary(y)
y@teststat
y@cval
```

plot(y)

```
resid_aa = aa$residuals
resid_aa = na.omit(resid_aa)
library(urca)
y = ur.df(resid_aa, type = "none", selectlags = "AIC")
summary(y)
y@teststat
y@cval
plot(y)
Supply-driven Model:
library("cointReg")
New_data
c <- Pt_x_log_diff - Pt_HEX_log_diff</pre>
d <- Pt_x_log_diff - MC_t_log_diff</pre>
e <- Xt_d_log_diff - Yt_M_log_diff</pre>
congthuc1 <- Xt_d_log_diff ~ Yt_M_log_diff + c + d + Dummy</pre>
congthuc2 <- Pt_x_log_diff ~ Pt_HEX_log_diff + MC_t_log_diff + e + Dummy</pre>
lm(data=New_data,congthuc1) -> hoiquy1
lm(data=New_data,congthuc2) -> hoiquy2
summary(hoiquy1)
summary(hoiquy2)
doclap <-cbind(c, Mt_w_log_diff, Dummy)</pre>
phuthuoc1 <-cbind(Xt_d_log_diff)</pre>
phuthuoc2 <-cbind(Pt_x_log_diff)</pre>
deter = cbind(level = 1, trend = 1:69)
cc<-cointRegFM(x=doclap, y=phuthuoc1, deter, kernel = "ba", bandwidth = "and")
print(cc)
doclap <-cbind(Pt_w_log_diff, Xt_d_log_diff, Mt_w_log_diff, Dummy)</pre>
dd<-cointRegFM(x=doclap, y=phuthuoc2, deter, kernel = "ba", bandwidth = "and")
print(dd)
```

```
resid_cc = cc$residuals
```

```
resid_cc = na.omit(resid_cc)
library(urca)
y = ur.df(resid_cc, type = "none", selectlags = "AIC")
summary(y)
y@teststat
y@cval
plot(y)
resid_dd = dd$residuals
resid_dd = na.omit(resid_dd)
library(urca)
y = ur.df(resid_dd, type = "none", selectlags = "AIC")
summary(y)
y@teststat
y@cval
plot(y)
   AIDS model:
library(micEconAids)
priceNames <- c( "pCereals", "pOil", "pVegetable", "pFruits" )</pre>
shareNames <- c( "wCereals", "wpOil", "wVegetable", "wFruits" )</pre>
AIDSResult1 <- aidsEst( priceNames, shareNames, "Agriculture", data = Data_1,
 + priceIndex = "S" )
AIDSResult1 <- aidsEst( priceNames, shareNames, "Agriculture", data = Data_1,
 + priceIndex = "SL" )
print(AIDSResult1)
summary(AIDSResult1)
priceNames <- c( "pChemicals", "pRubber", "pMinerals")</pre>
shareNames <- c( "wChemicals", "wRubber", "wMinerals")</pre>
AIDSResult2 <- aidsEst( priceNames, shareNames, "Industry", data = Data_1,
 + priceIndex = "S" )
AIDSResult2 <- aidsEst( priceNames, shareNames, "Industry", data = Data_1,
 + priceIndex = "SL" )
print(AIDSResult2)
summary(AIDSResult2)
```

7.2 Appendix B

		Level	First differences			
Variables	k	k t		t		
X_t	3	0.140 ***	3	0.213		
P_t^x	3	0.27 ***	3	0.056		
P_t^w	3	0.280 ***	3	0.056		
M_t^w	3	0.318 ***	3	0.077		
P_t^{HEX}	3	0.273 ***	3	0.041		
MC_t	3	0.187 **	3	0.177		
Y_t^M	3	1.523 ***	3	0.224		

Table 11: KPSS unit root test's results

Table 12: Augmented Dickey–Fuller unit root test's results

	Level						First difference				
V	m	k	$t_{\hat{eta}}$	$t_{\hat{lpha}}$	$\hat{T_B}$	V	m	k	$t_{\hat{\beta}}$	$t_{\hat{lpha}}$	$\hat{T_B}$
X_t	В	1	3.921 ***	-2.958	2020 Q1	X_t	В	1	0.411	-7.112 ***	2020 Q3
P_t^x	A	2	2.777 *	-3.436	2020 Q3	P_t^x	A	2	0.667	-5.543 **	2020 Q2
P_t^w	В	2	2.817 *	-3.161	2020 Q1	P_t^w	В	2	1.255	-3.251 **	2020 Q1
M_t^w	A	2	4.962 ***	-4.054**	2020 Q1	M_t^w	A	2	3.135 **	-6.054 ***	2008 Q3
P_t^{HEX}	В	2	3.622 ***	-4.123**	2008 Q1	P_t^{HEX}	В	2	3.357 **	4.224 ***	2008 Q1
MC_t	A	3	3.144 ***	-3.494	2009 Q2	MC_t	A	3	4.411 ***	-5.575 ***	2009 Q2
Y_t^M	C	6	5.274 ***	-9.049***	2009 Q2	Y_t^M	C	6	-4.531 ***	-5.098***	2009 Q2

Table 13: The results of time series o	outliers
--	----------

Variable	Class	Break data
X_t	AO	2009 Q1
P_t^x	AO	2009 Q1
	AO	2008 Q4
P_t^w	AO	$2020 \ Q1$
	AO	$2009 \mathrm{Q1}$
M_t^w	AO	$2020~\mathrm{Q1}$
	AO	$2009 \mathrm{Q1}$
P_t^{HEX}	AO	$2008 \ Q3$
	AO	$2021 \ Q1$
	AO	$2007 \ \mathrm{Q4}$
MC_t	AO	2008 Q 2
	AO	2009 Q3
Y_t^M	LS	$2008 \ Q1$

k – lag length, t – test statistic. V – variable, m – model, k – lag length, $t_{\hat{\beta}}$ – t–statistic for deterministic trend, $t_{\hat{\alpha}}$ – test statistic, \hat{T}_{B} – break date.

2021	M1-10	Exports of goods								
Country	Value, EUR bn	Share of TOT, %	Annual change, %	Contribution to growth, p.p.	Goods1	Share1 (% Country)	Goods2	Share2 (% Country)	Goods3	Share3 (% Country)
TOTAL	27,74	100,0	18,4	18,4	Chemical products	14,8	Machinery and mechanical applia	13,6	Mineral products	9,7
RU	2,98	10,7	-4,4	-0,6	Machinery and mechanical appliances	34,0	Chemical products	12,6	Prepared foodstuffs	9,3
LV	2,63	9,5	19,0	1,8	Mineral products	18,0	Machinery and mechanical	15,0	Chemical products	12,0
DE	2,32	8,4	19,6	1,6	Chemical products	13,9	Machinery and mechanical	12,3	Miscellaneous manufactured articles	12,1
PL	2,13	7,7	38,9	2,5	Mineral products	17,8	Plastics and rubber	15,2	Chemical products	9,5
US	1,74	6,3	90,3	3,5	Chemical products	42,6	Mineral products	29,7	Miscellaneous manufactured articles	8,7
NL	1,38	5,0	15,0	0,8	Chemical products	26,0	Prepared foodstuffs	14,5	Vehicles, aircraft, transport	10,9
EE	1,35	4,9	24,7	1,1	Mineral products	20,7	Machinery and mechanical	19,3	Chemical products	13,5
SE	1,25	4,5	14,1	0,7	Miscellaneous manufactured articles	25,1	Base metals	17,7	Machinery and mechanical	14,0
GB	1,09	3,9	13,5	0,6	Miscellaneous manufactured articles	19,4	Chemical products	16,7	Prepared foodstuffs	16,4
DK	0,69	2,5	12,6	0,3	Furnitures	24,2	Wood	11,3	Plastics and rubber	8,7

2021	M1-10	Exports of goods of Lithuanian origin	ı							
Country	Value, EUR bn	Share of TOT, %		Contribution to growth, p.p.	Goods1	Share1 (% Country)	Goods2	Share2 (% Country)	Goods3	Share3 (% Country)
TOTAL	17,16	100,0	23,3	23,3	Chemical products	13,6	Mineral products	13,1	Miscellaneous manufactured articles	12,8
DE	1,77	10,3	21,7	2,3	Miscellaneous manufactured articles	15,3	Chemical products	13,3	Wood	10,6
US	1,56	9,1	91,8	5,4	Chemical products	41,4	Mineral products	31,8	Miscellaneous manufactured articles	9,4
PL	1,24	7,2	27,5	1,9	Plastics and rubber	17,9	Mineral products	16,4	Animal products	12,2
LV	1,22	7,1	25,1	1,8	Mineral products	30,9	Prepared foodstuffs	11,7	Vegetable products	11,3
NL	1,08	6,3	22,9	1,4	Chemical products	22,8	Prepared foodstuffs	18,2	Miscellaneous manufactured articles	12,1
SE	1,05	6,1	6,1	0,4	Miscellaneous manufactured articles	29,4	Machinery and mechanical	14,7	Based metals	13,9
GB	0,94	5,5	18,2	1,0	Miscellaneous manufactured articles	21,5	Prepared foodstuffs	17,6	Chemical products	16,0
NO	0,67	3,9	2,7	0,1	Furnitures	32,0	Tobacco	11,4	Based metals	7,6
DK	0,57	3,3	12,4	0,5	Furnitures	28,4	Chemical products	10,4	Plastics and rubber	9,8
FR	0,49	2,9	15,6	0,5	Furnitures	24,4	Fertilizers	17,5	Plastics and rubber	10,4

2021	M1-10	Re-exports of goods								
	Value, EUR bn	Share of TOT, %		tribution to . growth, p.p.		Share1 (% Country)		Share2 (% Country)		Share3 (% Country)
TOTAL	10,58	100,0	11,1	11,1	Machinery and mechanical appliances	24,7	Chemical products	16,9	Vehicles, aircraft, transport equipmer	12,0
RU	2,68	25,3	-6,2	-1,9	Machinery and mechanical appliances	36,6	Chemical products	13,0	Prepared foodstuffs	8,3
LV	1,41	13,3	14,2	1,8	Machinery and mechanical appliances	26,3	Chemical products	15,9	Based metals	9,8
PL	0,88	8,4	58,7	3,4	Mineral products	19,9	Vehicles, aircraft, transport equipi	15,5	Chemical products	12,8
EE	0,77	7,3	18,9	1,3	Machinery and mechanical appliances	31,1	Chemical products	21,2	Prepared foodstuffs	7,9
BY	0,73	6,9	-3,8	-0,3	Machinery and mechanical appliances	22,9	Vehicles, aircraft, transport equipi	20,5	Chemical products	19,0
DE	0,55	5,2	13,3	0,7	Machinery and mechanical appliances	19,9	Vehicles, aircraft, transport equipi	18,5	Chemical products	15,8
UA	0,31	2,9	21,5	0,6	Chemical products	32,5	Vehicles, aircraft, transport equip	24,1	Machinery and mechanical	21,0
NL	0,30	2,9	-6,5	-0,2	Chemical products	37,4	Vehicles, aircraft, transport equipi	15,9	Machinery and mechanical	11,7
US	0,18	1,7	<mark>78,</mark> 0	0,8	Chemical products	53,4	Machinery and mechanical	12,1	Mineraliniai produktai	11,1
п	0,17	1,6	45,0	0,6	Vehicles, aircraft, transport equipment	23,2	Machinery and mechanical applia	12,9	Based metals	10,4

Figure 13: Export of goods by countries

	Exports of goods		2021M1-10	
			Annual change,	Contribution to
	Goods / Country	Share, %	%	growth, p.p.
	Total exports of goods	100	18	3,4
1	Chemical products	14.8	35.5	4.6
1	United States	18.0	101.4	12.3
2	Russia	9.1	-9,4	-1.3
3	Netherlands	8.7	28.6	2.6
4	Germany	7.9	67.1	4,3
5	Latvia	7,7	29,6	2,4
2	Machinery and mechanical applian	13.6	4.7	0.7
1	Russia	26,9	-13,4	-4,3
2	Latvia	10.5	8.9	0,9
3	Germany	7,6	24,5	1,6
4	Estonia	6,9	29,3	1,6
5	Belarus	4,8	-17,6	-1,1
3	Mineral products	9.7	54.7	4.0
1	United States	19.3	155,1	18,1
2	Ukraine	18,1	65,7	11,1
3	Latvia	17,7	35,3	7,1
4	Poland	14,1	68,1	8,9
5	Estonia	10,4	52,9	5,6
4	Miscellaneous manufactured articl	8.9	21.9	1.9
1	Sweden	12,7	4,6	0,7
2	Germany	11,3	20,5	2,3
3	Norway	9,2	12,0	1,2
4	United Kingdom	8,5	26,5	2,2
5	Denmark	6,8	27,1	1,8
5	Prepared foodstuffs	7.7	-2,5	-0,2
1	Russia	13,0	0,9	0,1
2	Latvia	12.1	2.6	0.3
3	Netherlands	9,4	3.1	0.3
4	United Kingdom	8,4	-24,6	-2,7
5	Germany	5,8	6,2	0,3
6	Base metals	6,9	29,6	1,9
1	Turkey	16,5	87,6	10,0
2	Sweden	11,6	36,2	4,0
3	Latvia	10,4	51,0	4,6
4	Poland	8,8	91,9	5,4
5	Russia	8,7	-11,0	-1,4
7	Plastics and rubber	6,8	24,7	1,6
1	Poland	17,1	55,1	7,6
2	Russia	9,4	-5,8	-0,7
3	Germany	8,7	18,6	1,7
4	Latvia	6,9	23,7	1,7
5	Sweden	5,2	13,9	0,8
8	Vehicles, aircraft, transport equipn	5,9	28,2	1,5
1	Germany	10,7	-6,2	-0,9
2	Poland	9,2	102,1	5,9
3	Russia	9,1	36,0	3,1
4	Netherlands	9,1	80,7	5,2
5	Belarus	9,1	35,3	3,1
9	Vegetable products	5,4	-9,7	-0,7
1	Latvia	13,6	-4,9	-0,6
2	Nigeria	10,7	29,9	2,2
3	Germany	9,3	2,9	0,2
4	Russia	7,8	6,3	0,4
5	South Africa	6,0	178,6	3,5
10	Wood	4,4	38,1	1,5
1	Germany	16,6	79,9	10,2
2	Latvia	12,7	33,1	4,4
3	United Kingdom	7,1	31,7	2,4
4	Poland	6,5	59,1	3,3
5	Denmark	6,4	29,9	2,0

Figure 14: Exports of goods by countries and goods (Share %, Annual change %, Contribution to growth p.p.)

	Exports of goods of Lithuanian orig		2021M1-10	
			Annual change,	Contribution to
	Goods / Country	Share, %	%	growth, p.p.
	Total exports of goods	100	23	1,3
1	Chemical products	13,6	64,2	6,5
	United States	27,8	110,8	24,0
2	Gormany	10,5	78,5	7,0
	Linited Kingdom	6.4	75 7	46
5	Ukraine	4.9	52.8	2.8
2	Mineral products	13.1	55.6	5.8
1	United States	22,1	145,6	20,4
2	Ukraine	21,4	65,8	13,2
3	Latvia	16,8	36,8	7,0
4	Estonia	12,1	55,5	6,7
5	Poland	9,0	54,1	4,9
3	Miscellaneous manufactured articl	12,8	23,2	3,0
1	Sweden	14,0	4,1	0,7
2	Germany	12,3	19,6	2,5
3	Norway	9,9	12,4	1,3
4	United Kingdom	9,2	26,7	2,4
5	Denmark	7,5	26,1	1,9
4	Prepared foodstuffs	9,1	-5,1	-0,6
1	Netherlands	12,6	3,2	0,4
2	United Kingdom	10,6	-28,6	-4,0
3	Latvia	9,2	4,4	0,4
5	Belgium	6,9	-4.9	0,0
5	Plastics and rubber	73	18 1	14
	Poland	17.7	28.7	47
2	Germany	8.6	-3.5	-0.4
3	Sweden	7,1	10,5	0,8
4	Latvia	4,6	27,8	1,2
5	Denmark	4,5	20,6	0,9
6	Machinery and mechanical applian	6,7	23,1	1,6
1	Germany	15,2	36,9	5,0
2	Sweden	13,3	17,7	2,5
3	Netherlands	6,6	7,7	0,6
4	Poland	5,0	47,7	2,0
- 5	Czech Republic	4,4	58,1	2,0
/	Base metals	6,4	31,3	1,9
	Turkey	26,4	97,4	1/,1
2	Gormany	13,1	22 0	0,8
4	Norway	67	-22.9	-2 6
5	Poland	6,2	87.4	3.8
8	Vegetable products	6.4	-7.4	-0.6
1	Nigeria	14.6	29.9	3.1
2	Latvia	12,5	-0,7	-0,1
3	Germany	10,0	0,1	0,0
4	South Africa	8,1	178,6	4,8
5	United Kingdom	7,2	321,6	5,1
9	Wood	5,9	43,8	2,2
1	Germany	18,6	81,5	12,0
2	Latvia	13,4	40,4	5,6
3	United Kingdom	7,7	56,1	4,0
4	Poland	6,6	60,6	3,6
5	Netherlands	5,3	59,7	2,8
10	Animal products	5,8	4,5	0,3
	Germany	15,9	-6,0	-1,1
2	Poland	15,1	8,6	1,2
	Italy		12,8	1,6
5	Netherlands	57	2,8	1.4
1 2	incule inditus		52,1	1,4

Figure 15: Re–exports of goods by countries and goods (Share %, Annual change %, Contribution to growth p.p.)

	Re-exports of goods		2021M1-10	
			Annual change,	Contribution to
	Goods / Country	Share, %	%	growth, p.p.
	Total exports of goods	100	11	1,1
1	Machinery and mechanical applian	24.7	·1.7	-0.5
1	Russia	37.5	-13.6	-5.8
2	Latvia	14.1	9,1	1.2
3	Estonia	9,1	30,5	2,1
4	Belarus	6,4	-17,7	-1,4
5	Germany	4,2	8,8	0,3
2	Chemical products	16,9	10,4	1,8
1	Russia	19,5	-10,3	-2,5
2	Latvia	12.5	10.9	1.4
3	Estonia	9,1	10,1	0.9
4	Belarus	7,8	4,4	0,4
5	Netherlands	6,4	-19,7	-1,7
3	Vehicles, aircraft, transport equipn	12.0	27.0	2.8
1	Belarus	11.7	35.8	3,9
2	Poland	10.8	113.0	7.3
3	Germany	8.1	-13.7	-1.6
4	Latvia	7.7	47.8	3.2
5	Russia	7.4	4.5	0.4
4	Base metals	7.7	27.4	1.8
1	Russia	19.9	-11.3	-3.2
2	Latvia	16.9	47.2	6.9
3	Poland	12.3	95.0	7.6
4	Sweden	9.4	210.0	8.1
5	Estonia	7.1	53.9	3,1
5	Plastics and rubber	60	40.3	10
	Russia		-0,5	
	Poland	16.0	179.8	14 5
2	l atvia	10,0	20.5	27
	Germany		107.6	
5	Estonia	8.0	37.3	3 1
6	Prepared foodstuffs	5.0	51	03
	Russia	3,4	- 3,1	- 0,5
		20.1	-5,0	-1,5
3	Estonia	10.7	- 0,4	.02
	Poland	3.0	-1,7	-0,2
5	Belarus	3,5	3,0	0,1
7	Textiles	13	-16	-0.1
<u> </u>	Russia	- 4,5	9.1	-0,1
		87	-9.6	
2	Estopio		-9,6	-0,9
	Relarus		- 6.8	.0.4
5	Ukraine	46	18.7	0,4
8	Mineral products	4,0	50.0	15
1	Dolond	,U		2,2
2	Foland	201	20 4	7.6
2	Relarus	ZZ,1	23,0	
	Netherlands		_52 /	4,0
	United States	- 5,4 / / A	20677.0	
	Ontical measuring medical instru	4,0	39	0,8
1	Puerio	,u	- 3,0	- 0,2
1	Russia	44,5		0,5
2	belarus		-21,0	
	Cormony	- 0,8	10,9	1,0
- 4 r	Esterio	- 4,0	- 4,1	0,2
10	Estonia	3,9	15.4	0,3
10	vegetable products	3,/	-15,4	-0,8
	Russia	29,0	7 ,4	1,7
	Latvia	16,6	-12,7	-2,0
3	Belarus	11,9	-45,7	-8,5
4	Germany	7,5	14,6	0,8
5	Estonia	6,2	1/,1	1 ,1

Figure 16: Export of goods of Lithuanian origin by countries and goods (Share %, Annual change %, Contribution to growth p.p.)

2021Q1-Q3				
Services	Value, EUR. mln.	Share of TOT, %	Annual change, %	Contribution to growth, p.p.
Total:	9139,0	100,0	14,9	14,9
Transport services	5467,8	59,8	10,2	6,3
Other business services	1269,7	13,9	38,3	4,4
Telecommunications, computer and information services	907,9	9,9	31,3	2,7
Construction services	358,1	3,9	34,6	1,2
Manufacturing services on physical inputs owned by others	282,9	3,1	8,0	0,3
Financial services	253,5	2,8	82,2	1,4
Maintenance and repair services not included elsewhere	203,4	2,2	0,7	0,0
Travel services	295,9	3,2	-34,3	-1,9
Personal, cultural and recreational services	54,2	0,6	101,6	0,3
Government goods and services (n.i.e)	38,5	0,4	21,3	0,1
Charges for the use of intellectual property (n.i.e)	6,1	0,1	21,4	0,0
Insurance and pension services	1,1	0,0	-7,8	0,0

Figure 17: Export of services (Share %, Annual change %, Contribution to growth p.p.)

Exports of services 2021 Q1-Q3					
Country	Value,	Change of TOT N	Annual change,	Contribution to growth, p.p.	
Country	EUR. mln.	Share of 101, %	%		
Total:	9139,0	100	14,9	14,9	
DE	1300,0	14,2	12,7	1,8	
FR	757,8	8,3	14,8	1,2	
NL	602,3	6,6	28,2	1,7	
RU	547,3	6,0	-4,2	-0,3	
SE	480,5	5,3	30,3	1,4	
DK	473,0	5,2	10,1	0,5	
GB	386,6	4,2	24,2	0,9	
PL	354,2	3,9	B7,7	1,2	
BY	338,9	3,7	-10,4	-0,5	
LV	317,1	3,5	-2,7	-0,1	
BE	289,8	3,2	18,6	0,6	
СН	289,3	3,2	23,4	0,7	
AT	288,9	3,2	6,4	0,2	
US	275,6	3,0	8,8	0,3	
NO	271,4	3,0	-1,8	-0,1	
EE	264,4	2,9	4 1,7	1,0	
IT	263,0	2,9	8,1	0,2	
FI	221,5	2,4	51 ,5	0,9	
ES	189,0	2,1	-0,7	0,0	
IE	159,6	1,7	11,6	0,2	
UA	102,0	1,1	70,2	0,5	
CY	76,9	0,8	<u>56</u> ,4	0,3	
LU	58,8	0,6	11,7	0,1	
KZ	42,4	0,5	-12,9	-0,1	
CZ	40,5	0,4	12,1	0,1	
HU	33,4	0,4	127,0	0,2	
CN	31,7	0,3	-7,1	0,0	
BG	20,6	0,2	90,5	0,1	
RO	15,1	0,2	13,5	0,0	
SK	13,8	0,2	3 6,9	0,0	
MT	13,3	0,1	79,2	0,1	
TR	13,2	0,1	-19,4	0,0	
НК	12,1	0,1	122,4	0,1	
PT	10,9	0,1	-11,5	0,0	
CA	10,4	0,1	21,5	0,0	
SL	9,7	0,1	-5,0	0,0	
IN	9,2	0,1	17,9	0,0	
GR	8,2	0,1	12,9	0,0	
HR	5,4	0,1	13,4	0,0	
JP	5,3	0,1	60,7	0,0	
EG	1,2	0,0	69,6	0,0	

Figure 18: Export of services by countries (Share %, Annual change %, Contribution to growth p.p.)



Figure 19: Hodrick-Prescott Filter of log(GDP). Potential Output, Market Sector.