VILNIUS UNIVERSITY
FACULTY OF MATHEMATICS AND INFORMATICS
MODELLING AND DATA ANALYSIS
MASTER'S STUDY PROGRAMME

Hierarchical Modelling of Mathematical Achievements

Matematinių Pasiekimų Hierarchinis Modeliavimas

Master's thesis

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Vilnius

2022

Abstract

In education, students tend to be more homogenous and share characteristics within their respected hierarchies such as schools, than the entire student population. Hierarchical modeling becomes the best solution in dealing with hierarchical data structures to account for the group effects that ordinary regressions tend to over/underestimate. This paper aimed to construct and analyze hierarchical linear and logistic regression models for mathematics achievement scores from Lithuanian 4th-grade data provided by IEA's TIMSS 2019 using HLM 8 software. The analysis included socio-economic status and other significant independent variables (including school location, early skills and education, school climate and safety, and student attitude and instructional clarity). TIMSS 2019 models were compared with TIMSS 2015 data tested with the same variables. In the TIMSS 2019 models, we found that the mean socio-economic status of schools, school location, and school's emphasis on success variables were significant and explained a large amount of variance for mathematics achievement at the school level.

Keywords: Hierarchical data Structures, Trends in Mathematics and Science Study, hierarchical linear models, HLM, hierarchical logistic models, HGLM, TIMSS 2015, TIMSS 2019, HLM 8, socioeconomic status, urban vs rural schools, mathematics achievement.

Contents

List of Figures

1 Introduction

Hierarchical structures have been a big part of human society. From family structures to business organizations, a hierarchical social structure has existed within. People tend to share qualities (such as values, morals, socioeconomic status) within hierarchical structures compared to being sampled from an entire population [27]. Students are part of a hierarchical structure that consists of classrooms, grades, schools, and countries in education. Analyzing these structures would account for homogeneity and a less basis result. Analyzing the whole student population would disregard this homogeneity, leading to over/underestimation. Utilizing hierarchical modeling will aid in observing within and between hierarchical educational structures such as the school level.

Mathematical data for Lithuanian fourth-graders was obtained from Trends in Mathematics and Science Study (TIMSS). TIMSS is an assessment of student achievement in mathematics and science for fourth and eighth grades internationally directed by TIMSS & PIRLS International Student Center at Boston College. Lithuania has been a participant for TIMSS since 1995 which Lithuanian students had scored above the international mean in TIMSS since 2003 when Lithuanian 4th-grade data was first introduced.

This thesis will analyze the socioeconomic status and various other student and school-related variables that could explain the variance between and within schools for mathematics achievement with 2015 and 2019 TIMSS data. The other variables include school location, early skills and education, school climate and safety, and student attitude and instructional clarity. The use of hierarchical models and hierarchical logistic models will aid in explaining the variance and the significance of the introduced variables using HLM 8 (a statistical tool for hierarchical data modeling). The purpose of this study is to find significant variables that could benefit policymakers and educators in identifying possible educational inequities within and between schools in Lithuania.

Since TIMSS 2011, TIMSS has introduced a Home Resources for Learning variable. This socioeconomic status variable (SES) will be included in the hierarchical models accompanied by the additional student and school-level variables. Even though Lithuanian students perform well in mathematics by TIMSS standards, monitoring the changes of students' and schools' socioeconomic status still creates value in spotting any potential discrepancies that could potentially cause problems and educational inequalities in the future.

1.1 Objectives

1.1.1 Aim

This thesis aims to construct and analyze hierarchical linear and logistic regression models for mathematics achievement scores from Lithuanian 4th-grade data provided by IEA's TIMSS 2019 using HLM 8 software. The uniqueness of this thesis is to analyze the effects of socioeconomic status and other significant independent variables (including school location, early skills and education, school climate and safety, and student attitude and instructional clarity) on Lithuanian mathematics achievement scores within and between schools.

1.1.2 Goals

- Prepare and clean TIMSS data using IEA IDBAnalyzer and IBM SPSS Statistic 25.
- Construct a two-level hierarchical linear model on Lithuanian Mathematical Achievement including the socioeconomic status and other significant independent variables on a student and school level with TIMSS 2019 Data.
- Summarize the results obtained from the TIMSS 2019 hierarchical linear models and compare results with the same variables tested with IEA's TIMSS 2015 data.
- Construct a two-level hierarchical logistic model on student achieving mathematics achievement scores above TIMSS international mean, including the socioeconomic status and other significant independent variables on a student and school level.
- Compare the results of the two-level hierarchical linear and logistic models explaining the variance between and within school levels.

2 Background

In this section, concepts related to hierarchical modelling that will be used in the analysis and educational concepts related to the variables used in the method will be discussed.

2.1 Hierarchical Data Structures

Educational data is a common area to encounter hierarchical data structures. For example, in figure 1, students are nested within classrooms, schools, grades, and districts. The presence of these structures can introduce problems in analysis for ordinary regressions. Students tend to be more homogeneous in their respected group than students in the entire population. Students are not randomly assigned to their corresponding hierarchies. For example, students will go to the closest school based on their region or location. In these hierarchies, individuals may share qualities (such as values, morals, socio-economic status).

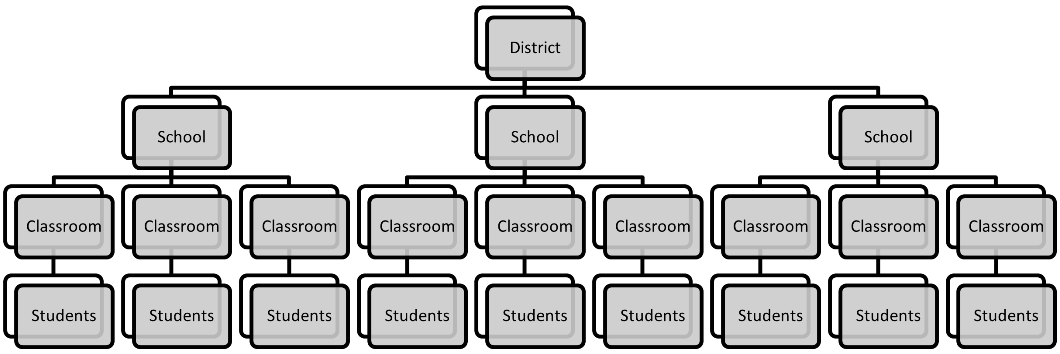


Figure 1: Example of an educational hierarchical data structure

Ordinary regressions assume that each observation is independent and does not incorporate group effects [23]. Since hierarchical structures have group effects, this effect needs to be incorporated into the analysis to observe the differences between hierarchical levels. Ordinary regressions that deal with cross-level data would bring down the higher-level variables down to the lower level or aggregate lower level up to the higher level. The problem with both of these approaches is that they do not accurately depict the individual and group effects on the dependent variable [27]. Hierarchical modeling becomes the best solution in dealing with hierarchical data structures.

2.2 Hierarchical Models

In this section, concepts related to the hierarchical Linear model and logistic models are discussed using educational examples. This section introduces the concepts around hierarchical modeling used in the analysis of IEA's TIMSS data shown in the method section.

2.2.1 Unconditional Model

The hierarchical modeling is similar to multiple regression, at least for the first level. There is a dependent variable Y_{ij} and one or more independent variables β_{ij}. The hierarchical model assumes that the data set has a hierarchical structure with the dependent variable measured at the lowest level [21].

When starting with the hierarchical linear model, the analysis must begin with an unconditional model to show how much variation in the dependent variable lies within and between level 2 (for example, schools) [2].

Within-school level 1 model (for example, student level) is indicated by the following equation:

$$Y_{ij} = \beta_{0j} + r_{ij} \tag{1}$$

In equation (1), Y_{ij} indicates the dependent variable (mathematical achievement) for student i in school j. β_{0j} represents the intercept for the average mathematical achievement for school j. r_{ij} is the residual error for the mathematical achievement of student i in school j. r_{ij} is normally distributed with a mean of zero and a constant variance σ^2 [28].

Between-School level-2 model (for example, school level) is indicated by the following equation:

$$\beta_{0j} = y_{00} + u_{0j} \tag{2}$$

In equation (2), y_{00} indicates the grand mean of mathematical achievement between all schools. u_{0j} is the random effect of school j on the average mathematical achievement. u_{0j} has a mean of zero and variance τ_{00}.

Substituting equation (2) into equation (1) will result in a mixed model

$$Y_{ij} = y_{00} + u_{0j} + r_{ij} \tag{3}$$

which is an unconditional model with a grand mean y_{00}, a level 2 effect u_{0j}, and a level 1 effect r_{ij} [28]. In equation (3), the model is considered random since u_{0j} is a random effect.

An unconditional model can show whether our dependent variable (mathematical achievement) is significant at the school level. This model can show how much variance is within-school σ^2 and between-school τ_{00} [2]. Within-school and between-school variance can be used to find the Intraclass Correlation Coefficient (ICC). The ICC is the proportion of variance in the dependent variable that is explained by levels in the model [1]. The ICC is calculated as a ratio of between-school variance τ_{00} over the total error variance:

$$ICC = \tau_{00}/\tau_{00} + \sigma^2 \tag{4}$$

The ICC shows the variation in the dependent variable accounted for between schools.

An unconditional model will be the baseline for tested models with variables like the models explained in the following two sections. This baseline model will test how much level-1 and level-2 independent variables explain the variation that we obtained from the ICC describing the unexplained variance between levels.

2.2.2 Random-coefficient Regression Model

After creating the baseline model with the unconditional model, we test for variables that would explain the variance of level-1 and level-2. The random-coefficient regression model is one of the models that test the level-1 independent variables on the dependent variable (mathematical achievement). The results are compared with the unconditional model to observe the variance explained by introducing a level-1 independent variable. This model will use students' socioeconomic status (SES) as an example level-1 independent variable.

The level-1 within-school model:

$$Y_{ij} = \beta_{0j} + \beta_{1j}(X_{ij} - \overline{X}_{ij}) + r_{ij} \tag{5}$$

In equation (5), the independent variable slope is introduced with β_{1j} which shows the association between the independent variable and the dependent variable (mathematical achievement) in school j. Group mean centering is applied in the following parenthesis $(X_{ij} - \overline{X}_{ij})$, where X_{ij} is SES for student i in school j, and $\overline{X}_j$ is school j average in SES [2]. Grand mean centering and grand mean will be explained further in section (2.2.6).

Between-School model is indicated by the following equation:

$$\beta_{0j} = y_{00} + u_{0j}$$
$$\beta_{1j} = y_{10} + u_{1j} \tag{6}$$

In equation (6), y_{10} indicates the slope of average SES within all schools. u_{0j} is the random effect of school j on the average mathematical achievement.

We can use the "variance explained" index introduced by Raudenbush and Bryk to compare the level-1 σ^2 estimates between the unconditional model as a baseline and random-coefficient regression Model with the introduced independent variable (SES).

The "variance explained" index:

$$"VE" = \frac{\sigma^2(Baseline) - \sigma^2(SES)}{\sigma^2(Baseline)} \tag{7}$$

In our example, Random-coefficient Regression Model tests whether SES significantly impacts the dependent variable (mathematical achievement) within schools.

2.2.3 Mean-as-Outcomes Regression Model

The Mean-as-Outcomes Regression Model tests the level-2 independent variables on the dependent variable (mathematical achievement). Results are compared with the unconditional model to observe the variance explained by introducing a level-2 independent variable. In this model, School's mean socioeconomic status (MSES) will be used as an example level-2 independent variable. Equation (1) will be the level-1 model of this example.

Between-School model is indicated by the following equation:

$$\beta_{0j} = y_{00} + y_{01}W_j + u_{0j} \tag{8}$$

In equation (8), W_j represents level-2 independent variable MSES [28]. By substituting equation (8) into Equation (1), we obtain the following mixed model:

$$Y_{ij} = y_{00} + y_{01}W_j + u_{0j} + r_{ij} \tag{9}$$

In equation (9), the random effect from earlier models u_{0j} becomes the residual variance of the grand mean of schools [28].

Just like the variance explained index introduced in the random-coefficient regression model section, we can apply this index to the mean-as-outcomes regression model to see how much of the variance was explained with the level-2 variable:

If we replace the level-1 variance σ^2 with level-2 variance τ_{00} in equation (10), we obtain the proportion of variance between schools explained with the introduction of the level-2 variable (MSES).

$$"VE" = \frac{\tau_{00}(Baseline) - \tau_{00}(MSES)}{\tau_{00}(Baseline)} \tag{10}$$

In our example, the mean-as-outcomes regression model tests whether MSES significantly impacts the outcome variable (mathematical achievement) between schools [2].

2.2.4 Full Hierarchical Linear Model

The models Unconditional Model, Random-coefficient Regression Model, and Mean-as-Outcomes Regression Model introduced before are considered sub-models used in Hierarchical modeling. We can introduce level-1 and level-2 independent variables that we observed in random-coefficient regression and mean-as-outcomes regression models to create a full hierarchical linear model and observe both levels simultaneously.

Within-school with level 1 variable (SES) is indicated by the following equation:

$$Y_{ij} = \beta_{0j} + \beta_{1j}(X_{ij} - \overline{X}_{ij}) + r_{ij} \tag{11}$$

Between-School model with level 2 variable (MSES) is indicated by the following equation:

$$\begin{aligned} \beta_{0j} &= y_{00} + y_{01}W_j + u_{0j} \\ \beta_{1j} &= y_{11} + u_{1j} \end{aligned} \tag{12}$$

By substituting equation (12) into Equation (11), we obtain the following mixed model:

$$Y_{ij} = y_{00} + y_{01}W_j + u_{0j} + y_{11}(X_{ij} - \overline{X}_{ij}) + u_{1j}(X_{ij} - \overline{X}_{ij}) + r_{ij} \tag{13}$$

The Full Hierarchical Linear model can indicate whether a 1-level variable SES is significant after controlling for MSES and vice versa. Using equations (7) and (10), we can observe whether the level 1 and level 2 variables accounted for any variance in the dependent variable (mathematical achievement).

2.2.5 Two-Level Models for Binary Outcomes

The hierarchical linear models introduced before are represented as a linear function, and the random effects are normally distributed when the outcome variable is continuous. The hierarchical generalized linear model (HGLM) is used when the outcome variable is binary since the residuals are not normally distributed. The hierarchical generalized linear model can be considered a form of HLM. The difference between HGLM and HLM is that HGLM utilizes a logistic regression. Similar to linear regression, logistic regression uses one or more independent variables to estimate the effect on the dependent variable [18]. HGLM introduces a hierarchical structure to the logistics regression similarly to hierarchical linear models.

According to Raudenbush and Bryk [28], HGLM is made up of three parts:

- Level-1 Sampling Model (Binary Outcome)

$$
\begin{aligned}
E(Y_{00}|\varphi_{ij}) &= m_{ij}\varphi_{ij} \\
Var(Y_{00}|\varphi_{ij}) &= m_{ij}\varphi_{ij}(1-\varphi_i ij)
\end{aligned}
\tag{14}
$$

In equation (14), i represents the student and j represents the school. m_{ij} represents the trials and φ_{ij} is the probability of "Success" for the ith student of jth school. Since linear models are normally distributed, the sampling model refers to the distribution of the Bernoulli since the outcome is binary.

- Level-1 Link Function (Logit Link)

A link function transforms a Bernoulli dependent variable to be used in a linear model and follow linear model assumptions since a Bernoulli variable has a non-normal distribution.

$$
\eta_{ij} = log(\frac{\varphi_{ij}}{1-\varphi_i ij}) \tag{15}
$$

In equation (15), η_{ij} represents the log of odds of success.

- Level-1 Structural Model

$$\eta_{ij} = \beta_{0j} + \beta_{1j}X_{ij} + \beta_{2j}X_{2ij} + ... + \beta_{pj}X_{pij} \tag{16}$$

HGLM Level-1 model uses the standard level-1 model HLM with a link function (equation 15) which is substituted into equation 16 for the dependent variable. The level-2 model is the same as the HLM model that was discussed in the previous section.

Linear regression and hierarchical linear models generate beta coefficients β that show the relationship between the independent and dependent variables. In logistics regression and HGLM, the beta coefficients β need to be transformed through exponentiation to be interpreted. Exponentiation is achieved by using the constant e and raising it to the power of the coefficient. This exponentiated coefficient is then interpreted using odd ratios, which estimate the odds of success (Outcome variable = 1) and the effect of the variables [18].

2.2.6 Centering

Variables in hierarchical modeling have the option to be centered. Variables can be left uncentered or centered by either group mean or grand mean. Centering by group means transforms the student level variable to a measure of the difference between the student variable and the school's mean. Grand mean centers student variable with the average for all students. Centering provides a more straightforward interpretation of the models and can give different coefficients and meanings depending on the research question [11]. The school level is usually centered around the grand mean since the other option is to leave it uncentered. Level 1 variables require a centering decision whether to leave them uncensored or center them with group mean or grand mean. Group mean is appropriate when observing level 1 association to the dependent variable. Grand mean is appropriate when observing level 2 predictors while controlling with level 1 covariates. Both group and grand mean can observe the influence of a variable at both level 1 and level 2 [12].

2.2.7 Model Fit Indices

Model fit indices are used to compare two or more models accounting for the best fit and complexity. A deviance measure defines fit, and complexity is estimated by the number of parameters included in the model. When comparing two models, the index that is closer to zero is the best-fit model [5].

$$\begin{aligned} AIC &= d + 2p \\ BIC &= d + p * ln(n) \end{aligned} \tag{17}$$

Here in equation 17, deviance is represented with d. The number of predictors is represented with p, and n is for the sample size. The smaller the AIC, BIC, and deviance represent a better fit model and a lower predictive error when comparing models.

2.3 Factors Influencing Student Achievement

This section discusses the factors that influence student achievement from the literature review, which helped choose the TIMSS variables used in the hierarchical models in the analysis discussed in the method.

2.3.1 Socioeconomic status

Socioeconomic status (SES) is usually indicated through various factors such as parents' education, income, occupational class, and the number of books at home. There is a lot of research backing family SES as a strong predictor in student achievement [33] [35]. In education, researchers have documented the association with student achievement with regards to socioeconomic status extensively ever since the Coleman report was published in 1966 [9]. Student socioeconomic status has contributed to education achievement gaps over the recent decades, which was affected by differences in socioeconomic status among students' families rather than schooling systems [8]. The explanation of this gap can be potentially explained by income inequality, school choice, and increased parental investment to their children [6] [30] [32]. The Lithuanian school system has shown a trend in maintaining low within-school socioeconomic status inequality and keeping a high-average overall performance before and after being part of the Soviet Union [7]. Gustafsson research showed that Lithuanian schools that have a more significant emphasis on academic success helped mitigate within-school socioeconomic status inequality [19]. Inequities can be seen between schools with factors such as school location and school socioeconomic composition [7].

2.3.2 School Location

Since the end of the socialist era, inequalities between rural and urban areas in eastern Europe have been increasing [24]. Urban areas experience growth and development in post-socialist countries, and the rural regions experience fewer changes. Academic achievement in rural areas of eastern Europe is lower than their urban counterparts. An OECD educational performance report shows that the education gap between rural and urban students still exists in Lithuania. According to OECD 2018 survey [26], income inequality is relatively high among those who live in rural areas and older Lithuanians. Comparing other OECD countries household income inequality, Lithuania is one of the highest due to unequal earnings, low social benefits, and a low re-distributive tax system.

2.3.3 Early Skills & Education

Early childhood education is an essential factor in a child's early development and long-term potential in their student life. Children that have had quality early education have had better cognitive performance compared to those who had none [13]. Early mathematical skills were strongly correlated with mathematical and reading achievement. Preschoolers' knowledge of number systems predicted their mathematical achievement in later stages of school when controlling for their background and working memory [4]. Lastly, parental involvement provides children with early mathematical and literacy input and other learning environments [20].

2.3.4 School Climate & Safety

A positive school climate is seen to benefit students' engagement and academic achievement [34]. A positive school climate consists of students, families, and teachers working together to nurture a positive learning environment. A positive school climate also promotes school safety to feel physically and emotionally safe. Students in a violent and non-supportive environment can cause them to be less engaged, isolated, and have lower academic achievement [3].

2.3.5 Student Attitude & Instructional Clarity

Mathematical achievement depends not only on cognitive abilities (IQ, working memory... etc.) but also attitude plays a part [10]. According to Hwang [22], students' attitude is significant and helps achieve high performance in the related subject. Students with a positive attitude toward mathematics perform higher due to enjoying the subject, understanding the importance, and having confidence. According to Fung [17], students with high levels of attitude and cognitive engagement were seen to have higher levels of achievement.

3 Method

In this section, we describe TIMSS 2019 method and procedures, TIMSS fourth grade Lithuanian data, and the analysis conducted that was mentioned in the aim using the concepts discussed in section 2.

3.1 TIMSS Research design

TIMSS uses a cross-sectional research design. TIMSS applies a stratified two-stage cluster sample design based on international standards. In the first sampling stage, schools are selected randomly throughout the population, including important demographic variables. The second sampling stage selects a fourth-grade class from each participating school. Lithuanian fourth-grade sample stratification variables consisted of school type, languages, urbanization, and school type [25].TIMSS 2019 uses a matrix sampling approach that packages the entire assessment pool of mathematics and science items at each grade level into a set of 14 student achievement booklets. This approach helps reduce the load on the students during the examination and decreases the burden for schools administering the test[25]. In this thesis, the relationship between students' mathematics achievement and student and school variables was observed using two-level hierarchical models and hierarchical logistic models.

3.2 Sample and Weights

TIMSS 2015 for Lithuanian data contains a sample of 225 schools and 4529 students from a population of 843 schools and 26,375 students with an average age of testing of 10.7 years. TIMSS 2019 for Lithuanian data contains a sample of 207 schools and 3741 students from a population of 827 schools and 28,035 students with an average age of testing 10.7 years. [25].

In order to use the Stratified two-stage cluster sample in an analysis, sampling weights are needed to represent the population accurately depending on the levels being analyzed [16][31]. The HLM models' weights will be total student weight (TOTWGT) and school weight (SCHWGT). TOTWGT is the product of weighting components for schools classes and students. SCHWGT is designed for use in school-level analysis, a product of school weighting factor (WGTFAC1) and school weighting adjustment (WGTADJ1). SCHWHT will be used for level-2 between-school weights. Level-1 within-school weight will be acquired by dividing TOTWGT by SCHWGT, which we will call TOTSCH. The division is necessary since total student weight is inverse of the joint probability of selecting a student within a given school[14][31].

3.3 TIMSS Variables

In this section, TIMSS data variables that were chosen with the help of the literature review in section 2.3 on the factors affecting student achievement are discussed in this section [25].

3.3.1 Socioeconomic status

In TIMSS, the scale Home Resources for Learning (SES) combines data consisting of students and their parents. The students filled out questions about the number of books at home and study supports, while parents filled out questions about the number of children's books at home, their education, and their occupation. To represent school level SES, Home Resources for Learning was aggregated to the school level to create the mean socioeconomic status variable (MSES).

3.3.2 School Location

In TIMSS, School location was created from a variable in the school background questionnaire. This ordinal variable consisted of 5 value labels: urban, suburban, medium-size city, small town, and remote rural. The variable was re-coded to a binary variable called School in Urban Location (URBAN), with 1 being urban and suburban and 0 being the other three value labels.

3.3.3 Early Skills & Education

The models included student-level variables to address Early Skills & education: Early Literacy and Numeracy Activities (EARLY) and Student Attended Preschool (PRESCHOOL). Early Literacy and Numeracy Activities variable consisted of 18 questions on literacy and numeracy activities on students before they began schools with the help of their parents engaging their children. The Student Attended Preschool variable was a nominal variable that measured how many years of pre-primary education the student received. School-level Literacy and Numeracy Tasks When Beginning Primary School (PREPARED) variable measured children's performance when they began primary school. Both variables were taken from the home questionnaire in TIMSS, which parents filled out.

3.3.4 School Climate & Safety

In TIMSS, some scales were created to address school climate & safety. School emphasis on academic success (EMPHASIS) variable was obtained from principals of schools to observe information about the school's expectations on academic achievement. Safe and orderly schools (SOS) variable was obtained from the teachers who were asked about their perception of their school's safety and order. Finally, Mean of Instructional Clarity in Mathematics Lessons (MCLARITY) variable was aggregated to the school level. Students were asked about their teacher's instruction during mathematics class. All three variables were tested for the school level for the hierarchical models.

3.3.5 Student Attitude & Instructional Clarity

Students Like Learning Mathematics (LIKE) variable was used to account for student attitude towards mathematics achievement. This scale was obtained from the student questionnaire in TIMSS which students were asked 9 items that covered student's attitudes towards mathematics.

Student engagement was included for the student level with the variable called Instructional Clarity in Mathematics Lessons (CLARITY). Students were asked 7 questions about whether their teacher's lessons were engaging and clear.

3.3.6 Mathematics Achievement

Mathematics achievement scale was the dependent variable used in this thesis for the hierarchical linear models. The scale had a range of 0 to 1000 with the center point being the international mean of mathematics achievement of 500.For the binary outcome model, the dependent variable was a re-coded binary mathematics achievement variable. This binary variable was coded 1 for students achieving mathematics achievement score above 500 and 0 for below. The mathematics assessment given to fourth grades consisted of 175 questions on pre-algebra, geometry and data.

3.4 Analysis

IEA International Database Analyzer (IDB Analyzer) is a free software tool used to combine student, home, and school data from TIMSS questioners into SPSS Statistics (*.sav) files. Two SPSS files were created for the student and school levels. Using IDB Analyzer, variables and weights corresponding to each level were selected. SPSS was then used for data preparation and check for model assumptions. After preparing student and school-level SPSS files, HLM 8 was used for descriptive analysis, visualization of SES and Mathematics achievement scores, and Hierarchical linear modeling. HLM 8 is one of the products created by Scientific Software International Inc. (SSI), which is a statistical tool for hierarchical data modeling [29].

TIMSS 2015 and TIMSS 2019 data at level-2 had some missing values, which accounted for 2% and 4.3% in missing cases. Missing data at the student level for TIMSS 2015 and TIMSS 2019 accounted for 18% and 25% of the cases. Cases with missing data were deleted in preparation for

HLM 8. In HLM 8, Student-level variables were group-mean centered, and school-level variables were grand-mean centered, and weights were applied. After accounting for missing data, model assumptions (linearity, normality, and independence) were held, which can be shown in appendix A. Figure (5) shows the tests for linearity with residuals vs. predicted value plot. There is no obvious trend shown. Figure (6) shows a test for normality which is checked with q-q plots. The residuals were plotted linearly, which means normality was held [11].

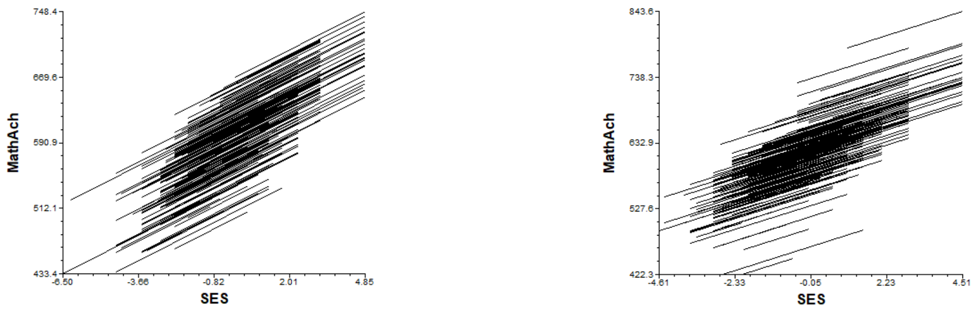
The main purpose of the analyses was to analyze the socioeconomic status and various other student and school-related variables that could explain the variance between and within schools for mathematics achievement with 2015 and 2019 TIMSS data using HLM8. The final HLM and HGLM models were constructed with significant variables which are given in appendices B, C, and D. The dependent variable for TIMSS 2015 and 2019 models in appendices B, and C was mathematical achievement. For the binary outcome model in appendix D, the dependent variable was whether mathematics achievement scores were above 500. This dependent variable was a dummy variable which was coded 1 for mathematics achievement score were above 500 and 0 for below. Model fit indices were calculated with the deviance generated from HLM 8. Variance explained which was introduced in the background was calculated for each model with variables to compare with the unconditional model for how much variance was explained with the introduced variables.

4 Results

In this section, we discuss the descriptive analyses results and model results obtained by the final HLM and HGLM models which are given in appendices G, H, and I.

4.1 Descriptive analyses

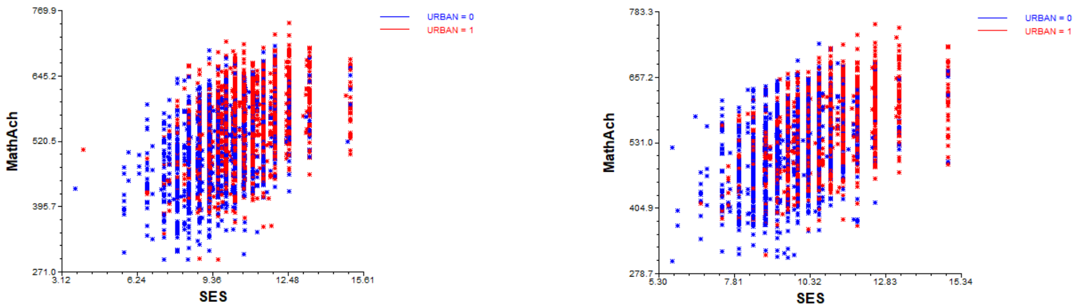
TIMSS 2015 Lithuanian fourth-grade data had 4529 students and 225 schools in the original sample. The final sample had 3704 students and 221 schools after excluding missing values. TIMSS 2019 Lithuanian fourth-grade data had 3741 students and 207 schools in the original sample. The final sample had 2814 students and 221 schools after excluding missing values. TIMSS 2015 and TIMSS 2019 descriptive statistics of the student and school-level variables are provided in appendices E , and F. Mathematics achievement scores were scaled with an international average of 500. Lithuanian fourth graders' mathematics achievement scores for TIMSS 2015 and 2019 were above the international average. All other variables excluding urban and preschool were scaled with an international average mean of 10. Lithuanian fourth-graders were above the international mean for all variables, including student-level SES and School level MSES.



(a) 2015 TIMSS (b) 2019 TIMSS

Figure 2: Associations between SES and mathematics achievement

Figure 2 shows TIMSS 2015 and 2019 estimated slopes for SES and mathematics achievement for each school. When SES increases, schools' slopes show an upward trend in schools' average mathematics achievement scores. TIMSS 2015 slopes are more concentrated together while TIMSS 2019 slopes are more spread out, with a few outliers schools performing very well at the top and poorly at the bottom of the graph.

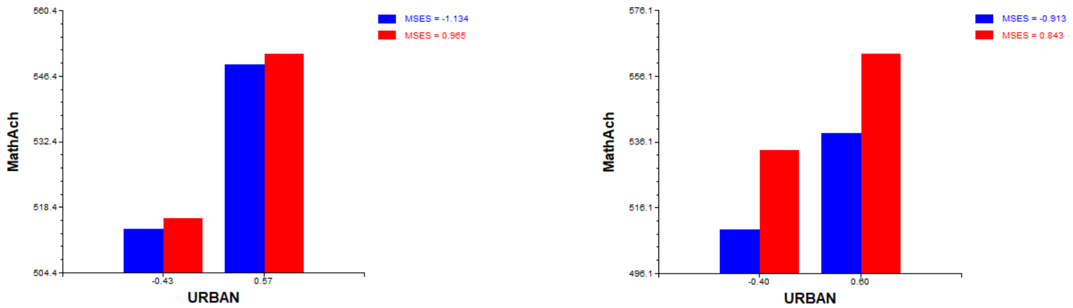


(a) 2015 TIMSS (b) 2019 TIMSS

Figure 3: Urban and Rural differences in SES and mathematics achievement

Figure 3 shows TIMSS 2015 and 2019 Rural (Blue) vs Urban (Red) within school SES differences. Each point on the graph represents a student's mathematics achievement score in relation to their SES. When comparing both graphs, there is a gap between rural and urban students. TIMSS 2015 both Urban and rural areas were concentrated in the middle, while in TIMSS 2019 urban students are no longer overlapping. There is a clear distinction between higher mathematical achievement and higher SES in urban students. Rural vs Urban discrepancies still exists and increasing between Lithuanian 4th graders in relation to Mathematics achievement and SES.

Figure 4 shows TIMSS 2015 and 2019 rural vs urban when comparing schools upper 25% (Red) and the lower 75% (Blue) in mean socioeconomic status. In TIMSS 2015, there is a gap in mathematics achievement performance when comparing rural vs. urban. Schools in the upper 25% and



(a) 2015 TIMSS (b) 2019 TIMSS

Figure 4: Urban and Rural differences in MSES and mathematics achievement

the lower 75% of mean socioeconomic status within rural and urban performed equally. In TIMSS 2019 graph, a big gap is emerging between higher and lower mean socioeconomic status schools in mathematics achievement within rural and urban areas. The upper 25% in rural mean socioeconomic status schools perform on par with the lower 75% in urban areas. Overall, 2019 TIMSS within urban and rural schools are experiencing mean socioeconomic gaps for schools not present in TIMSS 2015 data.

4.2 TIMSS 2019 Models

In this section, results of TIMSS 2019 models are discussed, which can be referred to appendix G for the full table with model results.

4.2.1 Unconditional Model

The unconditional model (Model 1) is the first model to analyze which creates a baseline when comparing models with variables. This model examines how much variation was within and between schools for the dependent variable mathematical achievement. In the TIMSS 2019 unconditional model, variation between schools was found statistically significant on mathematical achievement. The estimated ICC was .3912, which indicates that 39.12% of variation in mathematical achievement scores was between schools.

4.2.2 SES only model

The SES-only model (Model 2) introduces student variable SES and school variable MSES. This model will test the relationship between and within schools of SES and mathematical achievement. The SES-only model shows that SES was statistically significant in relation to students' mathematical achievement scores within schools with a coefficient of 17.553 with a p-value $< .001$. This means for each 1 point increase in SES, a student's mathematical achievement score increases by 17.553 points. MSES was statistically significant in relation to the school's mathematical achievement score with a coefficient of 20.072 with a p-value $< .001$. For each 1 point increase in School's MSES, School's mathematical achievement score increases by 20.072 points. Variance explained index was calculated to see how much variance was explained by the introduced variables. Level 1 SES explained 10.50% of the variance within schools compared to the base model. Level 2 MSES explained 43.07% of the unexplained variance between schools compared to the unconditional model.

4.2.3 Full Hierarchical Linear Model

The full hierarchical linear model (Model 3) shows that schools variables MSES, EMPHASIS, and URBAN were statistically significant. Student variables that were statistically significant were SES and LIKE. All other variables were not statistically significant and were dropped from the final model. The BIC and AIC were lower than the other three models, indicating the best fit model.

MSES had a positive coefficient of 13.704, which for every point of increase in MSES, school's mathematical achievement scores increased by 13.704 between schools. EMPHASIS has a positive coefficient of 7.586, which for every point of increase, EMPHASIS raised the school's mathematical achievement scores by 7.586 points. If a school was in an urban area, the coefficient for URBAN was 29.290. This means that a school's mathematical achievement score would be increased by 29.290 if the school was located in an urban area. These school-level variables accounted for 52.81% of the unexplained variance compared to the unconditional model. The addition of the other school variables with MSES increased the variance explained by 9.74% compared to model 2.

The student-level variable SES has a positive coefficient of 16.895, which for every point of increase in SES, increased student's mathematical achievement score by 16.895. LIKE also had a positive coefficient of 8.354, which for every point of increase in LIKE, increased students' mathematical achievement scores by 8.354. These two student variables accounted for 15.72% of the unexplained variance from the unconditional model and accounted for 5.22% when comparing model 2.

4.3 TIMSS 2019 Binary Outcome Models

In this section, results of TIMSS 2019 binary outcome models are discussed, which can be referred to appendix H for the full table with model results and odds ratios.

4.3.1 Unconditional Model

In the TIMSS 2019 binary outcome unconditional model, variation between schools was found statistically significant on students achieving mathematics achievement scores above the international mean. The estimated ICC was .1973, which indicates that 19.73% of variation for achieving mathematics achievement score above international mean was between schools.

4.3.2 SES only model

The SES-only model (Model 2) introduces student variable SES and School variable MSES. This model will test between and within schools' relationship of student and school-level variables for achieving mathematics achievement scores above the international mean. The SES-only model (Model 2) shows that SES was statistically significant in achieving mathematics achievement scores above the international mean within schools with an odds ratio of 1.644. The Odds Ratio tells us that SES within schools increases the odds of achieving above the international mean for mathematical achievement scores by 1.644:1. The higher the SES of the student, the more likely students are to achieve mathematics achievement scores above the international mean. The Odds Ratio is above 1, so there is an increase in the odds of achieving a mathematics achievement score above the international mean of 500. Also, we see that MSES coefficient between schools is positive, and the odds ratio is above 1 with a ratio of 1.545:1. The higher the MSES of the school, the more likely schools are to achieve mathematics achievement scores above the international mean. Since level 1 variance has a constant of 3.29 for hierarchical logistic regression, Variance explained index cannot be calculated except for Level 2. Level 2 MSES explained 16.50% of the unexplained variance between schools compared to the unconditional model for achieving mathematics achievement scores above the international mean of 500. Model 2 was the best fitting model comparing model 1 and model 2.

4.3.3 Full hierarchical logistics model

The full hierarchical logistics model (Model 3) introduces other student and school variables to the SES-only model. This model will test between and within the school's relationship with SES and other variables for students achieving a mathematics achievement score above the international mean of 500. The full hierarchical logistics model shows that school variables MSES, URBAN and ORDERLY were statistically significant. Student variables that were statistically significant were SES and LIKE. All other variables were not statistically significant and were dropped from the final model. The Odds Ratio tells us that MSES between schools increases the odds of achieving above the international mean for mathematical achievement scores by 1.401:1. The higher the MSES of the school, the more likely the school is achieving mathematics achievement scores above the international mean. If the school was in an urban area, the Odds Ratio for URBAN was 2.195:1. If the school is in an urban area, the more likely the school achieves a mathematics achievement score above the international mean. The Odds Ratio of ORDERLY between schools increases the odds of achieving above the international mean for mathematical achievement scores by 1.107701:1. The higher the school's ORDERLY variable, the more likely schools will achieve mathematics achievement scores above the international mean. These school-level variables accounted for 38.24% of the unexplained variance compared to the unconditional model. The student-level variable SES has an odds ratio of 1.65:1 which means the higher SES of the students, the more likely the students are to achieve mathematics achievement scores above the international mean. The student-level variable LIKE has an odds ratio of 1.192:1 which means the higher LIKE variable of the students, the more likely the students are to achieve mathematics achievement scores above the international mean. Since level 1 variance has a constant of 3.29 for hierarchical logistic regression, variance explained index cannot be calculated except for Level 2.

4.4 TIMSS 2015 Models

In this section, results of TIMSS 2015 models are discussed, which can be referred to appendix I for the full table with model results.

4.4.1 Unconditional Model

In the TIMSS 2015 unconditional model, variation between schools was found statistically significant on mathematical achievement scores. The estimated ICC was .2738, which indicates that 27.38% of variation in mathematical achievement scores was between schools.

4.4.2 SES only model

The SES-only model (Model 2) introduces student variable SES and School variable MSES. This model will test the relationship between and within schools between student and school-level variables to mathematical achievement scores.

The SES-only model shows that SES was statistically significant in relation to mathematical achievement within schools with a coefficient of 16.33 with a p-value $< .001$. This means for each 1 point of increase in SES, "mathematical achievement" increases by 16.33. MSES was statistically significant in relation to "mathematical achievement" between schools with a coefficient of 5.87

with a p-value $< .001$. This means for each 1 point increase in the school's MSES, mathematical achievement increases by 5.87. Variance explained index was calculated to see how much variance was explained by the introduced variables. Level 1 SES explained 12.34% of the variance within schools compared to the base model. Level 2 MSES explained 1.89% of the unexplained variance between schools compared to the unconditional model.

4.4.3 Full hierarchical linear model

The full hierarchical linear model (Model 3) introduces other student and school variables to the SES-only model. This model will test between and within school's relationship with SES and other variables to mathematical achievement scores.

The full hierarchical linear model shows that school variables EMPHASIS and URBAN were statistically significant. Student variables that were statistically significant were SES, LIKE, and PRESCHOOL. All other variables were not statistically significant and were dropped from the final model. EMPHASIS has a positive coefficient of 8.755, which for every point of increase, EMPHASIS raised the school's mathematical achievement scores by 8.755. If the school was in an urban area, the coefficient for URBAN was 36.10. This means that a school's mathematical achievement score would increase by 36.10. These school-level variables accounted for 38.24% of the unexplained variance compared to the unconditional model. The student-level variable SES has a positive coefficient of 16.04, which for every point of increase in SES, increased student's mathematical achievement score by 16.04. LIKE also had a positive coefficient of 6.487, which for every point of increase in LIKE, increased students' mathematical achievement scores by 6.487. These two student variables accounted for 15.91% of the unexplained variance from the unconditional model.

5 Conclusion

Lithuania had scored above the international mean in TIMSS since 2003 when Lithuanian 4th-grade data was introduced. This thesis analyzed socioeconomic status and various other student and school-related variables that could explain the variance between and within schools for mathematics achievement with 2015 and 2019 TIMSS data. 2015 TIMSS data provided a baseline model to compare if any variables' significance changed after the 4 years compared to the 2019 TIMSS results. The use of hierarchical and logistic models was performed to explain this variance and the significance of the introduced variables. Two-level hierarchical models and logistic models were tested for the school and student levels.

Comparing TIMSS 2015 and 2019 models for within-school results, the attitude and the family-SES were significant in our research. Within 4 years of the two surveys being conducted, the variance within Lithuanian 4 the grade schools remained the same with minor increases to the coefficients for students liking mathematics and family-SES. This study can further support the previous study mentioned [7] that the Lithuanian school system in 2019 continues the trend in maintaining low within school socioeconomic status inequality and keeping a high-average overall performance. Also, the school's emphasis on the success variable was a significant variable on the school level in models tested. Gustafsson mentioned in a study that in more equity school systems like Lithuania, school's emphasis on success minimizes the slope of within-school family-SES which could explain the low portion of variance within schools and minimal changes after 4 years [19]. Since the significant variables within schools only accounted for a small portion of the variance of mathematical achievement. It would be interesting to explore other possible school variables that are significant and could increase the variance explaining mathematics achievement that could improve the models tested in this thesis.

Between Schools' variation explained for mathematics achievement increased over the 4 years between TIMSS 2015 and TIMSS 2019. One interesting outcome is that school-SES was significant in TIMSS 2019 model, which was not the case in TIMSS 2015 model. Schools with a higher mean school-SES performed better and explained a big portion of variance between schools for mathematics achievement. Higher School-SES schools tend to have better quality instructional clarity and school climate since students are better prepared and parents are more involved [19]. There could also be a peer effect present in which students who are in a similar SES environment could experience a performance boost in mathematics achievement [15].

Another outcome is that the rural vs. urban effect in Lithuanian education still exists in Lithuania, as shown in TIMSS 2015 and TIMSS 2019 models. This means that urban school locations will experience a higher achievement just by the school's location, which has been a problem in Lithuania for years. There is a gap between rural and urban schools forming between schools with higher mean socioeconomic status performing better than lower mean socioeconomic status in the TIMSS 2019 model. The explanation of this gap can be potentially explained by income inequality, school choice, and increased parental investment to their children since average wages has doubled since 2015 in Lithuania [6] [30] [32]. Also, this rural vs. urban effect could be due to several factors such as the number of teachers decreasing, class size, and school funding.

5.1 Future Works

Lithuania has had several different funding models over the decades, such as the pupil's basket and the class basket. Since 2018, class basket has been used, incorporating class size into its allocation. This could benefit larger class size schools compared to rural areas with smaller class sizes. Some rural areas' schools have been seen consolidated over the years to increase the class size in some regions. In future works, it would be interesting if these assumptions could be investigated further with TIMSS data for the class level since this study aimed to build a hierarchical linear model and observe the effects of socioeconomic status on mathematical achievement while utilizing the most recent TIMSS 2019 data.

Overall, TIMSS has a lot of different scales and variables that can help monitor the performance of Lithuanian students' mathematical achievement. The data is structured conveniently to create hierarchical models on various levels such as teacher, classroom, school, and country. TIMSS 2023 data will be the 8th assessment cycle of TIMSS, which would be interesting whether new types of research questions can be constructed and tested using TIMSS data. In future works, it would be interesting to see how socioeconomic status changes for Lithuanian 4th graders in the next 4 years after TIMSS 2019.

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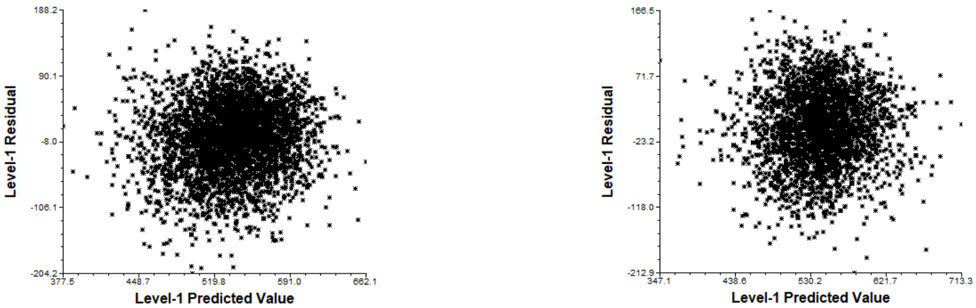
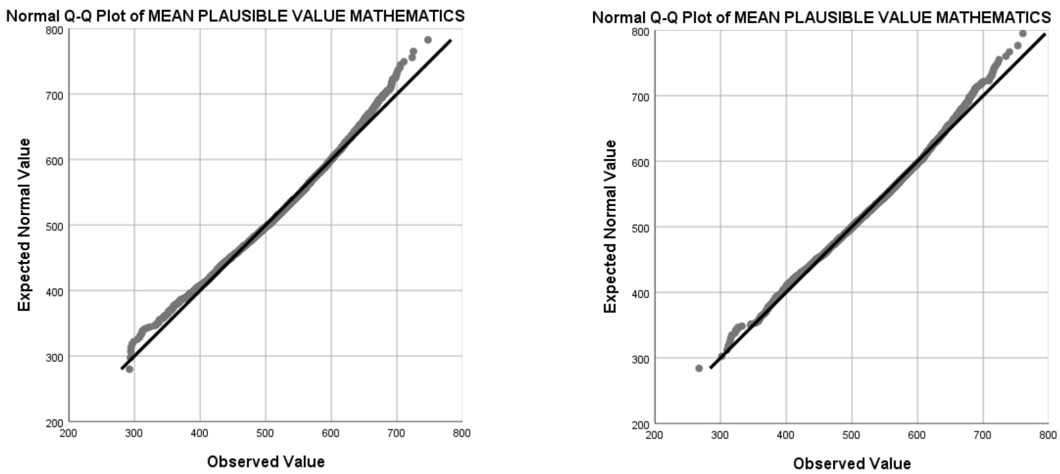
Appendix A Model Assumptions



(a) 2015 TIMSS (b) 2019 TIMSS

Figure 5: Testing for Linearity



(a) 2015 TIMSS (b) 2019 TIMSS

Figure 6: Testing for Normality

Appendix B TIMSS 2019 Model Equations

TIMSS 2019 Model
Model 1: Unconditional Model
$$MathAch_{ij} = \beta_{0j} + r_{ij} \quad (18)$$ $$\beta_{ij} = y_{00} + u_{0j} \quad (19)$$
Model 2: SES Only
$$MathAch_{ij} = \beta_{0j} + \beta_{1j} * (SES_{ij}) + r_{ij} \quad (20)$$ $$\beta_{1j} = y_{00} + y_{10} * (MSES_j) + u_{0j} \quad (21)$$ $$\beta_{0j} = y_{10} \quad (22)$$
Model 3: Full Hierarchical Linear Model
$$MathAch_{ij} = \beta_{0j} + \beta_{1j} * (SES_{ij}) + \beta_{2j} * (LIKE_{ij}) + r_{ij} \quad (23)$$ $$\beta_{0j} = y_{00} + y_{01} * (URBAN_j) + y_{02} * (EMPHASIS_j) + y_{03} * (MSES_j) + u_{0j} \quad (24)$$ $$\beta_{1j} = y_{10} \quad (25)$$ $$\beta_{2j} = y_{20} \quad (26)$$

Appendix C TIMSS 2015 Model Equations

TIMSS 2015 Model
Model 1: Unconditional Model
$$MathAch_{ij} = \beta_{0j} + r_{ij} \quad (27)$$ $$\beta_{ij} = y_{00} + u_{0j} \quad (28)$$
Model 2: SES Only
$$MathAch_{ij} = \beta_{0j} + \beta_{1j} * (SES_{ij}) + r_{ij} \quad (29)$$ $$\beta_{0j} = y_{00} + y_{01} * (MSES_j) + u_{0j} \quad (30)$$ $$\beta_{1j} = y_{10} \quad (31)$$
Model 3: Full Hierarchical Linear Model
$$MathAch_{ij} = \beta_{0j} + \beta_{1j} * (SES_{ij}) + \beta_{2j} * (LIKE_{ij}) + \beta_{3j} * (PRESCHOOL_{ij}) + r_{ij} \quad (32)$$ $$\beta_{0j} = y_{00} + y_{01} * (URBAN_j) + y_{02} * (EMPHASIS_j) + u_{0j} \quad (33)$$ $$\beta_{1j} = y_{10} \quad (34)$$ $$\beta_{2j} = y_{20} \quad (35)$$

Appendix D TIMSS 2019 Binary Outcome Model Equations

TIMSS 2019 Binary Outcome Model
Model 1: Unconditional Model
$Prob(MathAch_{ij} = 1 \vert \beta_j) = \varphi_{ij}$ (36) $Log[\varphi_{ij}/(1-\varphi_{ij})] = \eta_{ij}$ (37) $\eta_{ij} = \beta_{0j} + r_{ij}$ (38) $\beta_{ij} = y_{00} + u_{0j}$ (39)
Model 2: SES Only
$\eta_{ij} = \beta_{0j} + \beta_{1j} * (SES_{ij}) + r_{ij}$ (40) $\beta_{0j} = y_{00} + y_{01} * (MSES_j) + u_{0j}$ (41) $\beta_{1j} = y_{10}$ (42)
Model 3: Full Hierarchical Linear Model
$\eta_{ij} = \beta_{0j} + \beta_{1j} * (SES_{ij}) + \beta_{2j} * (LIKE_{ij}) + r_{ij}$ (43) $\beta_{0j} = y_{00} + y_{01} * (URBAN_j) + y_{02} * (ORDERLYj) + y_{03} * (MSES_{ij}) + u_{0j}$ (44) $\beta_{1j} = y_{10}$ (45) $\beta_{2j} = y_{20}$ (46)   (47)

Appendix E TIMSS 2015 Descriptive Statistics

Table 1: TMISS 2015 Data

LEVEL-1 DESCRIPTIVE STATISTICS

Variable	N	Mean	SD	MIN	MAX
MathAch	3704	535.34	69.13	293.69	747.24
SES	3704	10.19	1.41	3.69	15.04
LIKE	3704	10.18	1.79	4.41	12.66
EARLY	3704	10.42	1.73	3.21	15.30
CLARITY	3704	10.23	1.79	2.29	12.75
PRESCHOOL	3704	2.39	0.96	0	3
LEVEL-2 DESCRIPTIVE STATISTICS					
MSES	221	10.05	1.46	5.70	15.04
EMPHASIS	221	10.29	1.48	6.75	16.74
URBAN	221	0.43	0.50	0.00	1.00
PREPARED	221	10.14	1.55	5.92	14.39
MCLARITY	221	10.23	1.84	2.29	12.75
ORDERLY	221	10.42	1.63	6.66	13.41

Appendix F TIMSS 2019 Descriptive Statistics

Table 2: TMISS 2019 Data

LEVEL-1 DESCRIPTIVE STATISTICS

Variable	N	Mean	SD	MIN	MAX
MathAch	2814	541.15	71.73	301.69	760.40
MathAchD	2814	0.72	0.45	0.00	1.00
SES	2814	10.37	1.37	5.76	14.88
LIKE	2814	10.18	1.72	3.85	13.14
EARLY	2814	10.55	1.80	1.07	16.21
CLARITY	2814	10.24	1.69	4.59	12.26
PRESCHOOL	2814	2.55	0.83	0	3
LEVEL-2 DESCRIPTIVE STATISTICS					
MSES	198	10.15	1.45	5.92	14.88
EMPHASIS	198	10.34	1.43	5.66	16.40
URBAN	198	0.40	0.49	0.00	1.00
PREPARED	198	10.28	1.90	3.07	14.63
MCLARITY	198	10.43	1.67	6.70	12.26
ORDERLY	198	10.36	1.66	6.70	13.35

Appendix G TIMSS 2019 Results

Table 3: TMISS 2019 Dependent Variable: Mathematics Achievement

Fixed effects	**Model 1 Coefficient**	**Model 2 Coefficient**	**Model 3 Coefficient**
School mean achievement, β_{0j}			
Intercept,y_{00}	528.262***	531.285***	533.739***
	(6.72)	(4.485)	(3.926)
MSES, y_{01}		20.072***	13.704***
		(3.266)	(3.166)
EMPHASIS, y_{02}			7.586**
			(3.347)
URBAN, y_{03}			29.290***
			(6.926)
PREPARED, y_{04}			−−
MCLARITY, y_{05}			−−
ORDERLY, y_{05}			−−
SES slope		17.553***	16.895***
Intercept		(1.260)	(1.290)
LIKE slope			8.354***
Intercept			(.0864)
EARLY slope			−−
Intercept			
CLARITY slope			−−
Intercept			
PRESCHOOL slope			−−
Intercept			
Random Effects	**Variance component**	**Variance component**	**Variance component**
Student level residual, r_{ij}	3762.09	3367.19	3170.54
Mean achievement, u_{0j}	2417.72	1376.37	1140.99
ICC	0.3912	0.29015	0.2646
Model Fit Indices	**Model 1**	**Model 2**	**Model 3**
AIC	31680.68	31297.46	31113.90
BIC	31670.74	31267.63	31054.25
Explained variance	**Model 1**	**Model 2**	**Model 3**
From Model 1			
Level-1		0.1050	0.1572
Level-2		0.4307	0.5281

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Appendix H TIMSS 2019 Binary Outcome Results

Table 4: TMISS 2019 Dependent Variable: Mathematics Achievement Above International Mean

Fixed effects	**Model 1 Coefficient**	**Model 2 Coefficient**	**Model 3 Coefficient**
School mean achievement, β_{0j}			
Intercept,y_{00}	0.699070***	0.847315***	0.913422***
	(0.114117)	(0.098622)	(0.093125)
MSES, y_{01}		0.435350***	0.337509***
		(0.074947)	(0.074283)
EMPHASIS, y_{02}			−−
URBAN, y_{03}			0.786324***
			(0.175187)
PREPARED, y_{04}			−−
MCLARITY, y_{05}			−−
ORDERLY, y_{05}			0.102287**
			(0.047269)
SES slope		0.497537***	0.502344***
Intercept		(0.043929)	(0.045111)
LIKE slope			0.175701***
Intercept			(0.033134)
EARLY slope			−−
Intercept			
CLARITY slope			−−
Intercept			
PRESCHOOL slope			−−
Intercept			
Random Effects	**Variance component**	**Variance component**	**Variance component**
Student level residual, r_{ij}	3.29	3.29	3.29
Mean achievement, u_{0j}	0.8086	0.67519	0.59408
ICC	0.1973	0.1703	0.1530
Model Fit Indices	**Model 1**	**Model 2**	**Model 3**
AIC	7863.99	7847.77	7878.74
BIC	7854.05	7817.96	7842.91
Explained variance	**Model 1**	**Model 2**	**Model 3**
From Model 1			
Level-1			
Level-2		0.1650	0.2653

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 5: TMISS 2019 Dependent Variable: Mathematics Achievement Above International Mean

Fixed effects	**Model 1 Odds Ratio**	**Model Odds Ratio**	**Model 3 Odds Ratio**
School mean achievement, β_{0j}			
Intercept,y_{00}	2.011880	2.333373	2.492840
	(1.606,2.520)	(1.921,2.835)	(2.074,2.996)
MSES, y_{01}		1.545504	1.401452
		(1.333,1.792)	(1.210,1.623)
EMPHASIS, y_{02}			−−
URBAN, y_{03}			2.195311
			(1.554,3.102)
PREPARED, y_{04}			−−
MCLARITY, y_{05}			−−
ORDERLY, y_{05}			1.107701
			(1.009,1.216)
SES slope		1.644666	1.652590
Intercept		(1.509,1.793)	(1.513,1.805)
LIKE slope			1.192082
Intercept			(1.117,1.272)
EARLY slope			−−
Intercept			
CLARITY slope			−−
Intercept			
PRESCHOOL slope			−−
Intercept			

Appendix I TIMSS 2015 Results

Table 6: TMISS 2015 Dependent Variable: Mathematics Achievement

Fixed effects	**Model 1 Coefficient**	**Model 2 Coefficient**	**Model 3 Coefficient**
School mean achievement, β_{0j}			
Intercept,y_{00}	525.547870***	525.649771***	530.251240***
	(3.753288)	(3.854078)	(2.651871)
MSES, y_{01}		5.878305**	−−
		(2.542805)	
EMPHASIS, y_{02}			8.755499**
			(2.155520)
URBAN, y_{03}			36.102950***
			(5.563776)
PREPARED, y_{04}			−−
MCLARITY, y_{05}			−−
ORDERLY, y_{05}			−−
SES slope		16.330478***	16.045039***
Intercept		(1.003542)	(0.966087)
LIKE slope			6.487429***
Intercept			(0.709955)
EARLY slope			−−
Intercept			
CLARITY slope			−−
Intercept			
PRESCHOOL slope			3.339842**
Intercept			(1.307333)
Random Effects	**Variance component**	**Variance component**	**Variance component**
Student level residual, r_{ij}	3362.69320	2947.62356	2827.70984
Mean achievement, u_{0j}	1268.04356	1243.97913	783.01818
ICC	0.2738	0.2968	0.2169
Model Fit Indices	**Model 1**	**Model 2**	**Model 3**
AIC	41115.36	40652.24	40428.18
BIC	41105.14	40621.58	40366.88
Explained variance	**Model 1**	**Model 2**	**Model 3**
From Model 1			
Level-1		0.1234	0.1591
Level-2		0.0189	0.3824

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$