ON-SHELL RENORMALIZATION OF FERMION MASSES, FIELDS, AND MIXING MATRICES AT 1-LOOP*

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In the case of fermion mixing, we propose to use only one of the usual on-shell renormalization conditions at 1-loop and to use off-diagonal mass counterterms. These new counterterms allow for a natural separation of gauge-dependent parts as well as UV divergent parts, for example, the off-diagonal mass counterterms can be chosen to be gauge-independent and to contain all the UV divergences that would otherwise be included in field renormalization. Containment of UV divergences in off-diagonal mass counterterms prevents the migration of said divergences from the mass term to other terms in the Lagrangian. This naturally allows to not associate counterterms with mixing matrices and take them to be always renormalized. In addition, we argue that it is more consistent to not have counterterms for mixing matrices and instead have off-diagonal mass counterterms. Finally, the renormalization scheme is truly universal as it is based on mass structures and also includes absorptive parts where possible.

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1. Introduction

Renormalization in models without particle mixing is already a textbook material (at least at 1-loop), while models that do contain particle mixing pose a more challenging case. The usual on-shell (OS) approach is to require a diagonal propagator on the mass shell

$$\frac{1}{\not{p} - m_j} \Sigma_{ji}\left(p^2\right) u_i \;=\; 0\,, \tag{1a}$$

$$\bar{u}_j \Sigma_{ji}\left(p^2\right) \frac{1}{\not{p} - m_i} \;=\; 0 \tag{1b}$$

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for $(i \neq j)$ as proposed in [1] in combination with the usual renormalization

$$\begin{aligned} \psi^0_{\mathrm{L,R}j} &\to Z_{\mathrm{L,R}ji}\psi_{\mathrm{L,R}i}\,, \\ m^0_i &\to m_i + \delta m_i\,. \end{aligned} \tag{2}$$

Here, it is understood that i and j are flavour indices and that the mass counterterm is diagonal. By applying the above no-mixing conditions, one gets a solution for field renormalization Z as well as its Hermitian conjugate $Z^\dagger$ in terms of self-energy scalar functions. However, it has been noted that the solutions suffer from over-specification due to absorptive parts above particle production thresholds [2, 3]. The absorptive parts break the pseudo-hermiticity of the self-energy, namely $\Sigma \neq \gamma^0\Sigma^\dagger\gamma^0$, therefore, naive Hermitian conjugation of the field counterterms does not produce the Hermitian conjugate solution, *i.e.* $(Z_{\mathrm{L,R}})^\dagger \neq Z^\dagger_{\mathrm{L,R}}$. In other words, absorptive parts break the hermiticity of the Lagrangian. A simple way out of this is to drop the absorptive parts as suggested in [4]. However, even with the absorptive parts dropped, the field renormalization constants enter the renormalization of mixing matrices quite generally as one can relate mixing angle and field renormalizations [5], hence, care must be taken in fixing field counterterms. In addition, there are criteria for mixing matrix counterterms that are not trivially satisfied if one uses field counterterms for defining mixing counterterms. For example, it has been formulated that the CKM matrix [6–8] in the Standard Model (SM) should:

1. correctly cancel UV divergences in the Wud vertex,
2. be gauge-independent, and
3. preserve unitarity of the bare CKM matrix [9].

Points 1. and 3. seem to be easily achievable already in the very first attempt to renormalize the CKM matrix [10], where

$$\delta V \sim -\delta Z^{\mathrm{A},u}_{\mathrm{L}} V + V\delta Z^{\mathrm{A},d}_{\mathrm{L}} \tag{3}$$

and Z^{A} denote the anti-Hermitian parts of field renormalization. However, the above counterterm definition is gauge-dependent, $\partial_\xi \delta V \neq 0$, due to the gauge dependence of field counterterms. There have been quite a few attempts [9, 11–16] to get rid of this gauge dependence, most of which are fairly unnatural, non-universal, and hard to implement.

There is one approach that stands out as it takes care of gauge dependence by separating the gauge-independent part into off-diagonal mass counterterms and using those to define the CKM counterterm via a "1-loop rotation" instead of using field counterterms [17, 18]. While the approach naturally gets rid of gauge dependence, there are other problems: there is no explicit field renormalization, the definition of the mixing matrix counterterm actually leaves the Lagrangian unrenormalized, the approach is non-universal, and the counterterms are expressed in terms of self-energies only for the SM. Similarly, there the authors chose to drop the absorptive parts.

On the other hand, one may include the absorptive parts by introducing an additional set of field renormalization constants, although, at the price of hermiticity. This additional set of counterterms ensures the OS no-mixing conditions and gauge independence of the Wud vertex as long as the CKM matrix counterterm is gauge-independent. In turn, a third set of field renormalization constants has to be used for the CKM counterterm to ensure its gauge independence. All of this has been noted in [3].

Finally, there is a comparable multitude of mixing renormalization approaches for multi-Higgs as well as SUSY models ([19–27] to name a few), although, it is worth mentioning that more often than not the mixing renormalization in SUSY models is done by using off-diagonal mass counterterms and not introducing any counterterms for mixing matrices. We will discuss the consistency of both scenarios before the conclusions.

2. Setup

In the light of the introduction, we set up our renormalization in the following way. We include the absorptive parts and also keep the Lagrangian Hermitian by dropping the no-mixing condition for outgoing particles such that only

$$\frac{1}{\not{p} - m_j} \Sigma_{ji}\left(p^2\right) u_i = 0\,, \qquad (i \neq j) \tag{4}$$

remains. In order to separate the off-diagonal gauge-dependent contributions, we use off-diagonal mass counterterms

$$\delta m_i \to \delta m_{ji} = P_{\mathrm{L}} \delta m_{ji}^{\mathrm{L}} + P_{\mathrm{R}} \delta m_{ji}^{\mathrm{R}}\,. \tag{5}$$

Here, $P_{\mathrm{L,R}}$ are left- and right-handed projectors, in addition, hermiticity of the Lagrangian requires

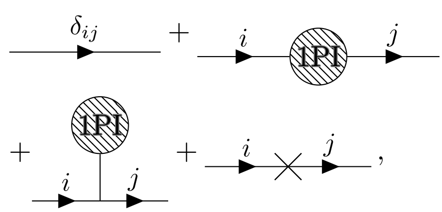
$$\left(\delta m^{\mathrm{L}}\right)^{\dagger} = \delta m^{\mathrm{R}} \tag{6}$$

and it should be understood that the renormalized mass m is diagonal (this means that the bare mass is not diagonal due to the counterterm). Otherwise, the renormalization is standard and we can write down the self-energy decomposition as follows:

$$
\begin{aligned}
\Sigma_{ji}\left(p^2\right) \; = \; & \Sigma^{\mathrm{L}}_{ji}\left(p^2\right)\not{p}P_{\mathrm{L}} + \Sigma^{\mathrm{R}}_{ji}\left(p^2\right)\not{p}P_{\mathrm{R}} + \Sigma^{s\mathrm{L}}_{ji}\left(p^2\right)P_{\mathrm{L}} + \Sigma^{s\mathrm{R}}_{ji}\left(p^2\right)P_{\mathrm{R}} \\
& + \frac{1}{2}\left(\delta Z^{\dagger}_{\mathrm{L}ji} + \delta Z_{\mathrm{L}ji}\right)\not{p}P_{\mathrm{L}} + \frac{1}{2}\left(\delta Z^{\dagger}_{\mathrm{R}ji} + \delta Z_{\mathrm{R}ji}\right)\not{p}P_{\mathrm{R}} \\
& - \left(\delta m^{\mathrm{L}}_{ji} + \frac{1}{2}\delta Z^{\dagger}_{\mathrm{R}ji}m_i + \frac{1}{2}m_j\delta Z_{\mathrm{L}ji}\right)P_{\mathrm{L}} \\
& - \left(\delta m^{\mathrm{R}}_{ji} + \frac{1}{2}\delta Z^{\dagger}_{\mathrm{L}ji}m_i + \frac{1}{2}m_j\delta Z_{\mathrm{R}ji}\right)P_{\mathrm{R}} \,. \qquad (7)
\end{aligned}
$$

One can also write down the self-energy in terms of Feynman diagrams

$\Sigma_{ji}\left(p^2\right) \; =$



(8)

where we also include unrenormalized tadpoles, this is equivalent to using the FJ scheme [19, 28].

Having the self-energy decomposition it is fairly simple to apply the no-mixing condition on incoming particles and arrive at a relation between the off-diagonal mass and field counterterms in terms of self-energy scalar functions

$$
\begin{aligned}
& \delta Z_{\mathrm{L}ji} \\
& = -\frac{2}{m_i^2 - m_j^2}\left(m_i^2\Sigma^{\mathrm{L}}_{ji}\left(m_i^2\right) + m_i m_j \Sigma^{\mathrm{R}}_{ji}\left(m_i^2\right) + m_j\Sigma^{s\mathrm{L}}_{ji}\left(m_i^2\right) + m_i\Sigma^{s\mathrm{R}}_{ji}\left(m_i^2\right)\right) \\
& + \frac{2}{m_i^2 - m_j^2}\left(m_j\delta m^{\mathrm{L}}_{ji} + m_i\delta m^{\mathrm{R}}_{ji}\right)\,, \qquad (i \neq j) \qquad (9)
\end{aligned}
$$

with an analogous equation for the right-handed part. Further, using $(\delta m^{\mathrm{L}})^{\dagger} = \delta m^{\mathrm{R}}$, one can find that the Hermitian part of the field renormalization remains unchanged w.r.t. the usual approach (except for the absorptive parts), while the mass counterterms only contribute to the anti-Hermitian part of field renormalization

$$
\begin{aligned}
& \left(m_i^2 - m_j^2\right)\delta Z^{\mathrm{A}}_{\mathrm{L}ji} - 2m_j\delta m^{\mathrm{L}}_{ji} - 2m_i\delta m^{\mathrm{R}}_{ji} \\
& = -\left(m_i^2\Sigma^{\mathrm{L}}_{ji}\left(m_i^2\right) + m_i m_j\Sigma^{\mathrm{R}}_{ji}\left(m_i^2\right) + m_j\Sigma^{s\mathrm{L}}_{ji}\left(m_i^2\right) + m_i\Sigma^{s\mathrm{R}}_{ji}\left(m_i^2\right)\right) + \mathrm{H.C.} \qquad (10)
\end{aligned}
$$

Here, H.C. stands for Hermitian conjugation and there is an analogous expression for $\delta Z^{\mathrm{A}}_{\mathrm{R}}$.

Note that $\delta Z^{\rm A}_{\rm L}$ in the above equation is multiplied by a very distinct mass structure, namely $m_i^2 - m_j^2$, which is seen neither in front of mass counterterms nor on the r.h.s. of the equation. In addition, this mass structure causes numerical problems as well as singularities in the degenerate mass limit, therefore, it makes sense to explore gauge and UV properties of the above equation and see how the terms multiplied by $m_i^2 - m_j^2$ behave.

3. Exploration

3.1. Gauge dependence

Investigation of gauge dependence may be easily done by using the Nielsen identities [29]. For fermion self-energy scalar functions at 1-loop, one may arrive at the following gauge parameter derivatives [13, 30]:

$$\begin{aligned}
\partial_\xi \Sigma^{\rm L}_{ji}\left(p^2\right) &= -m_i \Lambda^{\rm L}_{ji} - m_j \bar{\Lambda}^{\rm L}_{ji} + \Lambda^{s{\rm R}}_{ji} + \bar{\Lambda}^{s{\rm L}}_{ji}\,, \\
\partial_\xi \Sigma^{\rm R}_{ji}\left(p^2\right) &= -m_i \Lambda^{\rm R}_{ji} - m_j \bar{\Lambda}^{\rm R}_{ji} + \Lambda^{s{\rm L}}_{ji} + \bar{\Lambda}^{s{\rm R}}_{ji}\,, \\
\partial_\xi \Sigma^{s{\rm L}}_{ji}\left(p^2\right) &= p^2 \Lambda^{\rm R}_{ji} + p^2 \bar{\Lambda}^{\rm L}_{ji} - m_i \Lambda^{s{\rm L}}_{ji} - m_j \bar{\Lambda}^{s{\rm L}}_{ji}\,, \\
\partial_\xi \Sigma^{s{\rm R}}_{ji}\left(p^2\right) &= p^2 \Lambda^{\rm L}_{ji} + p^2 \bar{\Lambda}^{\rm R}_{ji} - m_i \Lambda^{s{\rm R}}_{ji} - m_j \bar{\Lambda}^{s{\rm R}}_{ji}\,.
\end{aligned} \tag{11}$$

Here, Λs and $\bar{\Lambda}$s are vertex functions containing BRST sources and are decomposed analogously to the self-energies as should be evident from the superscripts. This allows to take a derivative w.r.t. the gauge parameter in Eq. (10), and we arrive at

$$\begin{aligned}
&\left(m_i^2 - m_j^2\right) \partial_\xi \left(\delta Z^{\rm A}_{{\rm L}ji}\right) - 2m_j \partial_\xi \delta m^{\rm L}_{ji} - 2m_i \partial_\xi \delta m^{\rm R}_{ji} \\
&= \left(m_i^2 - m_j^2\right)\left(-m_i \bar{\Lambda}^{\rm R}_{ji}\left(m_i^2\right) - \bar{\Lambda}^{s{\rm L}}_{ji}\left(m_i^2\right)\right) + {\rm H.C.}
\end{aligned} \tag{12}$$

Since momenta and renormalized masses do not depend on the gauge, all of the gauge dependence is encoded by the Λs, hence, it is evident from the above equation that all of the gauge dependence comes multiplied by the $m_i^2 - m_j^2$ mass structure. Although, one should be careful as gauge-independent terms may also carry the same mass structure. If one does not include the off-diagonal mass counterterms, the above equation shows the cancellation of $m_i^2 - m_j^2$ in the gauge-dependent part of field renormalization which has been noted in [13] for fermions and in [22] for scalars. In our case, the counterterms are not yet fixed so Eq. (12) only shows that the gauge-dependent parts of counterterms should provide appropriate gauge dependence. Of course, the present $m_i^2 - m_j^2$ mass structure on both hand sides already implies a natural way to achieve this.

3.2. UV divergences

Another question that we may tackle is that of UV properties. At 1-loop, this is made fairly simple because of two things: (1) the Passarino–Veltman functions and their UV parts are well known [31], (2) contributions to self-energies at 1-loop are also known [32]. The contributions from a boson (scalar or vector) and a fermion running in the loop are as follows:

$$\begin{aligned}\Sigma^{\mathrm{L,R}}\left(p^2\right) &= f_{\mathrm{L,R}} B_1\left(p^2, m^2_{\psi\mathrm{loop}}, m^2_{\mathrm{bos}}\right) \overset{\mathrm{UV}}{\Longrightarrow} -f_{\mathrm{L,R}}\frac{1}{\epsilon_{\mathrm{UV}}}\,,\\ \Sigma^{s\mathrm{L},(s\mathrm{R})}\left(p^2\right) &= m_{\psi\mathrm{loop}} f_s^{(\dagger)} B_0\left(p^2, m^2_{\psi\mathrm{loop}}, m^2_{\mathrm{bos}}\right) \overset{\mathrm{UV}}{\Longrightarrow} m_{\psi\mathrm{loop}} f_s^{(\dagger)}\frac{2}{\epsilon_{\mathrm{UV}}}\,.\end{aligned} \tag{13}$$

Here, $m_{\psi\mathrm{loop}}$ denotes the mass of the fermion running in the loop and m_{bos} that of a boson, fs are appropriate couplings ($f_{\mathrm{L,R}}$ are Hermitian), and ϵ_{UV} is from dimensional regularization with $D = 4 - \epsilon_{\mathrm{UV}}$. It is clear that the UV divergent parts do not depend on the arguments of the Passarino–Veltman functions, which makes the application simpler. In addition, since we also include tadpoles, they also contribute to divergent parts

$$\Sigma^{s\mathrm{L},(s\mathrm{R})}\left(p^2\right) = f_{\mathrm{T}}^{(\dagger)} m_{\psi\mathrm{loop}} A_0\left(m^2_{\psi\mathrm{loop}}\right) \overset{\mathrm{UV}}{\Longrightarrow} f_{\mathrm{T}}^{(\dagger)} m_{\psi\mathrm{loop}} \frac{2m^2_{\psi\mathrm{loop}}}{\epsilon_{\mathrm{UV}}}\,. \tag{14}$$

Here, we listed the contribution from fermion tadpoles as they are the only ones that could possibly contribute to the fermion mass structures. On the other hand, particles in the loop receive an index distinct from those of external particles and cannot be used for the mass structures. In addition, the coupling f_{T} includes a Yukawa coupling with indices corresponding to external particles, however, this coupling is either non-diagonal and not proportional to masses, or diagonal and proportional to masses. Since we take the non-diagonal case, we see that even fermion tadpoles have no effect on mass structures.

Putting all of this into Eq. (10), we arrive at the following divergent parts:

$$\begin{aligned}&\left[\left(m_i^2 - m_j^2\right)\delta Z^{\mathrm{A}}_{\mathrm{L}ji} - 2m_j \delta m^{\mathrm{L}}_{ji} - 2m_i \delta m^{\mathrm{R}}_{ji}\right]_{\mathrm{div}}\\ &= -\frac{1}{\epsilon_{\mathrm{UV}}}\left(-f_{\mathrm{L}}\left(m_i^2 + m_j^2\right) - f_{\mathrm{R}} 2m_i m_j + 4f_s^\dagger m_i m_{\psi\mathrm{loop}} + 4f_s m_j m_{\psi\mathrm{loop}}\right)\\ &-\frac{1}{\epsilon_{\mathrm{UV}}}\left(4m^3_{\psi\mathrm{loop}} f_{\mathrm{T}} m_j + 4m^3_{\psi\mathrm{loop}} f_{\mathrm{T}}^\dagger m_i\right)\,.\end{aligned} \tag{15}$$

Looking at the r.h.s., it is immediately seen that there are no divergences with the $m_i^2 - m_j^2$ mass structure, in turn, this means that there are no gauge-dependent UV divergences in Eq. (10). In contrast, if we were to look at UV divergences of the Hermitian part of the field renormalization, we would see UV divergences multiplied by $m_i^2 - m_j^2$ only. On the other hand, there seem to be new mass structures on the r.h.s. of the above equation, namely $m_i^2 + m_j^2$ and $2m_i m_j$. On the l.h.s., these may be produced by rewriting the mass counterterms such that the mass is renormalized by shifting as well as multiplicatively, *i.e.*

$$m^0 \to m + P_{\mathrm{L,R}}\delta m^{\mathrm{L,R}} + \left(1 + P_{\mathrm{L}}\delta m^+ + P_{\mathrm{R}}\delta m^-\right) m \left(1 + P_{\mathrm{L}}\delta m^- + P_{\mathrm{R}}\delta m^+\right) , \tag{16}$$

then the newly appearing mass structures multiply $\delta m^{\pm}$ on the l.h.s. of Eq. (15). However, it does not seem practical to use $\delta m^{\pm}$ and so we only keep $\delta m^{\mathrm{L,R}}$.

Again, the counterterms are not yet fully fixed but UV divergences also imply a natural way of fixing them.

4. Definitions

Having the previous sections in mind, we define the anti-Hermitian part of the field renormalization as the coefficient of $m_i^2 - m_j^2$ in Eq. (10) and in an analogous equation for the right-handed part

$$\begin{aligned}
\delta Z^{\mathrm{A}}_{\mathrm{L}ji} \equiv\ & \Big[-\left(m_i^2 \Sigma^{\mathrm{L}}_{ji}\left(m_i^2\right) + m_i m_j \Sigma^{\mathrm{R}}_{ji}\left(m_i^2\right)\right. \\
& \left.+\, m_j \Sigma^{s\mathrm{L}}_{ji}\left(m_i^2\right) + m_i \Sigma^{s\mathrm{R}}_{ji}\left(m_i^2\right)\right) + \mathrm{H.C.}\Big]\Big|_{\left(m_i^2 - m_j^2\right)} , \\
\delta Z^{\mathrm{A}}_{\mathrm{R}ji} \equiv\ & \Big[-\left(m_i^2 \Sigma^{\mathrm{R}}_{ji}\left(m_i^2\right) + m_i m_j \Sigma^{\mathrm{L}}_{ji}\left(m_i^2\right)\right. \\
& \left.+\, m_j \Sigma^{s\mathrm{R}}_{ji}\left(m_i^2\right) + m_i \Sigma^{s\mathrm{L}}_{ji}\left(m_i^2\right)\right) + \mathrm{H.C.}\Big]\Big|_{\left(m_i^2 - m_j^2\right)} .
\end{aligned} \tag{17}$$

For the mass counterterms, the situation is made more simple by the fact that the field renormalization is now defined and we can simply solve for the mass counterterms

$$\begin{aligned}
\delta m^{\mathrm{L}}_{ji} =\ & \tfrac{1}{2}\left(m_i \Sigma^{\mathrm{R}}_{ji}\left(m_i^2\right) + \Sigma^{s\mathrm{L}}_{ji}\left(m_i^2\right) + m_j \Sigma^{\mathrm{L}\dagger}_{ji}\left(m_j^2\right) + \Sigma^{s\mathrm{R}\dagger}_{ji}\left(m_j^2\right)\right) \\
& + \tfrac{1}{2}\left(m_i \delta Z^{\mathrm{A}}_{\mathrm{R}ji} - m_j \delta Z^{\mathrm{A}}_{\mathrm{L}ji}\right) , \\
\delta m^{\mathrm{R}}_{ji} =\ & \tfrac{1}{2}\left(m_i \Sigma^{\mathrm{L}}_{ji}\left(m_i^2\right) + \Sigma^{s\mathrm{R}}_{ji}\left(m_i^2\right) + m_j \Sigma^{\mathrm{R}\dagger}_{ji}\left(m_j^2\right) + \Sigma^{s\mathrm{L}\dagger}_{ji}\left(m_j^2\right)\right) \\
& + \tfrac{1}{2}\left(m_i \delta Z^{\mathrm{A}}_{\mathrm{L}ji} - m_j \delta Z^{\mathrm{A}}_{\mathrm{R}ji}\right) .
\end{aligned} \tag{18}$$

There is a number of features that should be noted about these definitions. First, the anti-Hermitian part of the field renormalization contains all the possible gauge dependence and is also UV finite, while the mass counterterms are UV divergent and gauge-independent. Second, the expressions are truly universal and written in terms of self-energy scalar functions, although, for field renormalization, one has to take the coefficient of $m_i^2 - m_j^2$. In addition, there is a fairly similar scheme in the MSSM for squarks [24], which only shows that problems and their solutions are very similar for fermions and bosons. Third, since the definitions rely on the $m_i^2 - m_j^2$ mass structure, there are no singularities for the anti-Hermitian part of field renormalization or mass counterterms in the degenerate mass limit — a feature wanted for mixing renormalization [22].

Finally, the absorptive parts are taken into account such that there is no OS mixing for incoming particles (outgoing antiparticles). Even though the scheme seems to *maximally* include the absorptive parts given the hermiticity constraints such an inclusion is not without its problems. For example, since only one Lehmann–Symanzik–Zimmermann (LSZ) [33] factor has been incorporated into field renormalization, the gauge dependence in the Wud vertex amplitude is still not fully canceled even with a gauge-independent CKM counterterm; for the full gauge cancellation one needs to incorporate both LSZ factors but, again, the cost is hermiticity [3]. Another problem is that the singularity in the degenerate mass limit appears for the Hermitian part of field renormalization only in the absorptive parts (a similar problem also appeared in [15]). On top of that, the absorptive parts also spoil the bare Majorana condition, which requires $\delta Z_{\mathrm{L}} = \delta Z_{\mathrm{R}}^{\star}$ at 1-loop. Luckily, the main goal of our scheme is that of proposing a universal set of counterterms (and their definitions) such that they have an appropriate gauge and UV properties in the case of particle mixing, hence, the mentioned problems may be avoided by dropping the absorptive parts by the reader. In addition, this set of counterterms is trivially adaptable to MS schemes and even to schemes where two sets of field renormalization constants are used. For the latter, instead of the anti-Hermitian part of field renormalization, one has to use $\frac{1}{2}(\delta Z_{\mathrm{L,R}} - \delta\bar{Z}_{\mathrm{L,R}})$ and similarly the Hermitian part becomes $\frac{1}{2}(\delta Z_{\mathrm{L,R}} + \delta\bar{Z}_{\mathrm{L,R}})$, where $\delta\bar{Z}$ comes from imposing the OS no-mixing condition also for outgoing particles.

5. Renormalization of mixing matrices

Finally, having the explicit definitions of mass and field counterterms, we may discuss the renormalization of mixing matrices. For concreteness, let us first consider the CKM matrix in the SM. In the initial approach [10], the form of the CKM counterterm in Eq. (3) is dictated by UV divergences appearing in the Wud vertex and unitarity of the CKM matrix. However, this is because the anti-Hermitian parts of field renormalization are divergent in the usual approach and there is nothing else to cancel these UV divergences, in contrast, in our approach, the anti-Hermitian part of field renormalization is finite and there is nothing to cancel. Therefore, we can choose to set the CKM counterterm to 0 and this is perfectly in line with the 3 criteria listed in the introduction.

We again note that having off-diagonal mass counterterms and no mixing matrix counterterms seems to be a fairly common approach in SUSY theories, but rather rare in non-SUSY ones. On the other hand, having or not having mixing matrix counterterms are often presented as viable *alternative* methods (*e.g.* [5]), however, this begs to answer the question of consistency (or compatibility). We consider two scenarios: (1) renormalization after diagonalization and (2) diagonalization after renormalization. Further, as a consistency condition, we take that the two scenarios should be related by a basis rotation at all times — renormalization in that sense commutes with basis rotations.

Let us take the first scenario. Consider diagonalizing the bare mass matrix, this makes mixing matrices appear in the Lagrangian (of course, only if particle mixing is present in a given model). Upon renormalization, the mixing matrix receives a counterterm and the mass counterterms are diagonal. Schematically, we may write

$$\begin{array}{ccccc} m^0_{ji} & \xrightarrow{\text{diagonalization}} & m^0_i & \xrightarrow{\text{renormalization}} & m_i + \delta m_i\,, \\ \cancel{V^0} & \xrightarrow{\text{diagonalization}} & V^0 & \xrightarrow{\text{renormalization}} & V + \delta V\,. \end{array} \tag{19}$$

On the other hand, we may take the second scenario and renormalize the theory in a basis where the bare-mass matrix is not diagonal and there is no mixing matrix in the Lagrangian, naturally, there are no mixing matrix counterterms. Afterwards, we diagonalize the *renormalized* mass, but the counterterm in general remains non-diagonal. Schematically, this is

$$\begin{array}{ccccc} m^0_{ji} & \xrightarrow{\text{renormalization}} & m_{ji} + \delta m_{ji} & \xrightarrow{\text{diagonalization}} & m_i + (V^\dagger \delta m V)_{ji}\,, \\ \cancel{V^0} & \xrightarrow{\text{renormalization}} & \cancel{V + \delta V} & \xrightarrow{\text{diagonalization}} & V \cancel{+\delta V}\,. \end{array} \tag{20}$$

Now we look for compatibility between the two scenarios. It is fairly evident that one cannot switch between scenarios via a basis rotation. For example, rotating the final step of the first scenario such that the renormalized mixing matrix disappears (undiagonalization) and comparing with the second step of the second scenario, it can be seen that the first scenario contains a leftover mixing matrix counterterm (*and no associated parameter!*), while there is no such counterterm in the second scenario. Similarly, the "final states" between scenarios also differ in their counterterms although the bases are the same (renormalized masses are diagonal). Hence, both scenarios are only compatible if one sets δV to 0 and always uses non-diagonal mass counterterms even if the renormalized mass is diagonal.

The usual approach where diagonal mass counterterms are used is only made possible by field renormalization which takes care of non-diagonal contributions. The usual field counterterms are defined by the 2-point function but appear in other terms (*e.g.* the Wud vertex in the SM), hence, carrying contributions to other terms in the Lagrangian, which causes migration of UV divergences and this is why renormalization of mixing matrices becomes necessary. On the other hand, if one uses non-diagonal mass counterterms, the UV divergences stay in the mass term such that there is no need to renormalize mixing matrices and, as we argue, this is a step towards consistency. This consistency implies that only the renormalized part of the bare-mass matrix should be diagonalized, while the counterterms are, in general, non-diagonal. In addition, it seems to us that the more consistent approach is also easier to implement and, at least up to absorptive parts, straightforwardly gives the required properties: gauge dependence, UV cancellations, unitarity, and non-singularity in the degenerate mass limit.

6. Conclusions

We have presented a universal OS renormalization scheme for fermions which relies on the OS no-mixing condition for incoming particles, non-diagonal mass counterterms, and mass structures. The scheme naturally gives gauge-independent mass counterterms, UV finite and gauge-dependent field counterterms, and no counterterms for mixing matrices. In addition, the scheme includes absorptive parts as much as possible, although, these parts still cause some problems: gauge-dependence is not fully cancelled in the Wud amplitude, there are singularities in the Hermitian part of field renormalization (but not in the anti-Hermitian part or mass counterterms) when the degenerate mass limit is taken. Fortunately, the main takeaway of the scheme is the set of used counterterms, which means that the counterterms are easily adaptable to MS schemes, schemes where the absorptive parts are dropped, or schemes where two sets of field renormalization constants are used. Moreover, we have argued that using non-diagonal mass counterterms instead of mixing matrix counterterms is more consistent and should be the preferred way.

We hope to publish a more detailed discussion including analytical examples of the proposed scheme in the Grimus–Neufeld model [34] in a future publication.

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