LOW SEESAW SCALE IN THE GRIMUS–NEUFELD MODEL*

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Received 23 December 2021, accepted 28 December 2021, published online 28 February 2022

We present the possibility of the low seesaw scenario in the Grimus–Neufeld model. We argue that it can be natural and phenomenologically interesting, while consistent with neutrino data. We present the approximated expressions for neutrino masses and estimate the magnitude of the Yukawa couplings. We show that they can be sizable and can lead to possible restrictions on the scalar sector.

DOI:10.5506/APhysPolBSupp.15.2-A17

1. Introduction

Direct evidence of physics beyond the Standard model (SM) are the non-vanishing neutrino masses [1–4]. Addressing their nature by various models has already quite a long history. Nevertheless, these realizations are usually not very predictive and hence, large parameter regions in these models are not probed by the experiments. As a result, even the very minimal extensions of the SM, which include neutrino masses, are neither excluded nor confirmed. As the neutrino sector must inevitably be an integral part of any model that has an ambition to explain the full picture of particle physics, it makes sense to think of these minimal realizations of neutrino mass models as part of some bigger model. Yet this leaves the neutrino sector a somewhat unessential part in these models, which is simply just there. In this context, it is worthwhile to rethink the minimal realizations for neutrino masses with bigger scrutiny and try to look for parameter regions which can actually be sensitive to the current experimental limits.

By the term *minimal,* we usually refer to the extension which adds something to the model that is just enough to explain the phenomena under consideration — in this case, the neutrino masses and mixings. In this sense,

* Presented at the XLIV International Conference of Theoretical Physics “Matter to the Deepest”, 15–17 September, 2021.

the seesaw mechanism [5] is minimal, as it introduces only one additional particle to the SM per needed mass parameter. A slightly more sophisticated mechanism is the radiative mass generation, which includes a larger scalar sector. In the context of the widespread belief that the scalar sector is probably larger than the one of the SM anyways, this also can be thought of being minimal. This approach is known as the scotogenic model [6], which postulates the existence of sterile neutrinos, but prevents them from acquiring tree-level masses by a Z_2 symmetry.

In both of these scenarios, seesaw or scotogenic, it is customary to study only the parameter region which has the non-SM neutrino (or sterile neutrino) very heavy, a way heavier than the electroweak scale[1]. Moreover, both of the models are rather idealistic: the seesaw model alone is idealistic in the sense that it does not consider the possibility of a larger scalar sector, which immediately gives the possibility of additional radiative mass generation; while the scotogenic model relies on the exact Z_2 symmetry, which, if even slightly broken, can allow for the seesaw mechanism. We find that the latter option might be particularly interesting in a phenomenological sense, as it naturally gives the low seesaw scale and suppressed Yukawa couplings to the SM Higgs, while allowing sizable ones to the 2^{nd} Higgs doublet. In addition, since it has two mechanisms for generating neutrino masses interplaying with each other, it is enough to have only a single sterile neutrino to explain both of the experimentally observed mass squared differences. This scenario is just a specific parameter region of a more general model, called the Grimus–Neufeld [8] (GN) model, which is simply the general 2HDM, extended with a single seesaw neutrino. In contrast to the more restrictive approaches (scotogenic or pure seesaw), this is quite general, but it is "truly minimal" as it can be realized with only a single sterile neutrino instead of two or three.

The low seesaw scale was never studied in the GN model before, yet it is an interesting scenario due to the following reasons:

— It has a similarity to the "idealistic" scenario of the scotogenic model [6] by having an approximate Z_2 symmetry in the neutrino Yukawa sector (instead of an exact one), but has a lower number of postulated particles, thus does not make the study more complicated. There is also a limiting case of it to the scoto-seesaw model [9, 10], in which the seesaw mechanism is strictly separated by an opposite Z_2 charge of the two sterile neutrinos. There are no restrictions in the GN model, and thus studying its parameter space around the mentioned limiting scenarios can give us an insight into the situation how phenomenologically distinguishable these exact limits really are.

[1] There exist exceptions to this, for example, [7].

— A high seesaw scale induces a naturalness problem in the scalar sector [11]. It is rather surprising that this issue is rarely even mentioned and often just ignored in the studies of various models with the seesaw mechanism. Ironically, the seesaw mechanism is introduced to be a "natural" explanation of the smallness of neutrino masses and by "natural" it is meant that the Yukawa couplings are kept to be of $O\,(1)$, or simply not extremely small. The neutrino mass in Type I seesaw is then

$$m_\nu = \frac{y^2 v^2}{2M}\,, \qquad y = O\,(1) \Rightarrow M \approx M_{\mathrm{GUT}}\,, \tag{1}$$

where M is the sterile neutrino mass, y is the Yukawa coupling for the neutrino, and v is the v.e.v. of the Higgs boson. One can immediately see that this can be a problem in the scalar sector: for $M > 10^7$ GeV, we get that the correction to the Higgs mass is higher than the mass itself [11], since

$$\delta m_h \sim \frac{y^2}{16\pi^2} M^2\,. \tag{2}$$

Hence, there is no really a natural way to address the neutrino masses with Type I seesaw. However, from the 't Hooft naturalness point of view, the limit of $y \to 0$ actually increases the symmetry of the Lagrangian, which again is a strong motivation for Type I seesaw with a small M and gives a new meaning to the natural seesaw mechanism.

— Values for M in the keV, MeV, or GeV range are simply not excluded.

In this report, we show that the low seesaw scale is a perfectly viable parameter region in the GN model, which can reproduce the observed neutrino data with almost any scalar potential[2].

2. Quick introduction to the model

A detailed presentation of the model can be found in [8, 12–14]. For completeness and setting the notation, we present only the essential parts of the model.

The scalar sector consists of the 2HDM. We pick the Higgs basis, in which

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} v\,, \qquad \langle H_2 \rangle = 0\,. \tag{3}$$

Hence, when we say "first Higgs doublet", it always means the SM-like one, in which the pseudoscalar and the charged components of the doublet are the Goldstone bosons.

[2] There is only one fine-tuned exception, which is described in the paper below.

We pick the flavour basis for the Yukawa couplings for charged fermions and, for simplicity, we set the Yukawa couplings of charged fermions to the second Higgs doublet to zero, which reads

$$Y^{\mathrm{E}1}_{\ell_i} = \frac{\sqrt{2}m_i}{v}\,, \qquad Y^{\mathrm{E}2}_{\ell_i} = 0\,, \qquad i = e, \mu, \tau\,. \tag{4}$$

The only fermion, N, that is added is a singlet under all gauge groups with a mass M. This allows the Yukawa couplings for neutrinos, which are then 2 general complex 3-vectors, $Y^{\nu 1}$ and $Y^{\nu 2}$. The Lagrangian part of the main interest is then

$$\begin{aligned} \mathcal{L}_{\mathrm{Yuk}+M} &= -Y^1_{\nu_i}\ell_i H_1 N - Y^2_{\nu_i}\ell_i H_2 N - \frac{1}{2}MN^2 + \mathrm{H.c.}\,, \\ \ell_i &= \begin{pmatrix} \nu_i \\ e^-_i \end{pmatrix}, \qquad i = e, \mu, \tau\,, \end{aligned} \tag{5}$$

where all fermions are written in terms of left-handed Weyl spinors. The product ℓH has to be understood as the invariant SU(2) product. By applying a unitary transformation V on neutrinos, we pick a new basis

$$\nu^F = V\nu\,, \quad Y^1_\nu = (0, 0, iy)\,V^\dagger\,, \quad Y^2_\nu = \big(0, d, id'\big)\,V^\dagger\,, \quad y, d \in \mathbb{R}^+\,, \quad d' \in \mathbb{C}\,. \tag{6}$$

In this basis, there is a clear zero-mass state, which does not interact with the scalar sector at all and a clear seesaw-mass state, which is the only one which interacts with the first Higgs doublet. Thus at tree level, two neutrinos are massless

$$m_1 = m_2 = 0\,, \qquad m_4 > m_3 > 0\,, \tag{7}$$

where the non-zero masses are given by the seesaw relations

$$M = m_4 - m_3\,, \qquad \frac{y^2 v^2}{2} = m_3 m_4\,. \tag{8}$$

At loop level, only the neutrino that does not interact with Higgses at all stays massless. For simplicity, we always order the neutrinos by their mass, hence at one loop, we will have

$$\begin{aligned} m_1^{\mathrm{pole}} &= 0\,, \\ m_4^{\mathrm{pole}} > m_3^{\mathrm{pole}} > m_2^{\mathrm{pole}} &> 0\,. \end{aligned} \tag{9}$$

To be consistent with neutrino data [15], we must have

$$\begin{aligned} \text{Normal hierarchy:} \quad m_2^{\mathrm{pole}} &= \sqrt{\Delta m^2_{21}}\,, \quad m_3^{\mathrm{pole}} = \sqrt{\Delta m^2_{21} + \left|\Delta m^2_{32}\right|}\,, \\ \text{Inverted hierarchy:} \quad m_2^{\mathrm{pole}} &= \sqrt{\left|\Delta m^2_{32}\right| - \Delta m^2_{21}}\,, \quad m_3^{\mathrm{pole}} = \sqrt{\left|\Delta m^2_{32}\right|}\,. \end{aligned} \tag{10}$$

The idea is to make these masses as input parameters and define 2 free parameters in terms of them, by solving the 1-loop equations for masses. V is then also related to the PMNS matrix at one-loop level. For simplicity, we will concentrate on masses only, as not all the parameter values can be consistent with Eq. (10), while the PMNS matrix can always be reproduced. From the solutions that satisfy Eq. (10), we can then see which parameter regions are allowed in the GN model.

3. One-loop mass calculations

To calculate the one-loop neutrino masses in general, we must first calculate the 4×4 matrix of self-energies. However, since we are interested in light-neutrino masses only, this can be radically simplified by using the Grimus–Lavoura approximation [16]. It can be summarized as follows:

— Evaluate all self-energies at $p^2 = 0$.

— Assume $m_4 \gg m_3$, make a large mass expansion, and pick the leading order. This also gives $m_4^{\rm pole} \approx m_4$. In GN at one loop, it is equivalent to expanding the loop functions in terms of Yukawa couplings [14].

We note that since m_3 is really small, m_4 can be relatively small, certainly smaller than the electron mass for the GL approximation to be applicable. Then the 4×4 matrix becomes block diagonal and the 2×2 block for light neutrinos can be studied separately. All the diagrams that are contributing to this mass matrix are shown in Fig. 1.

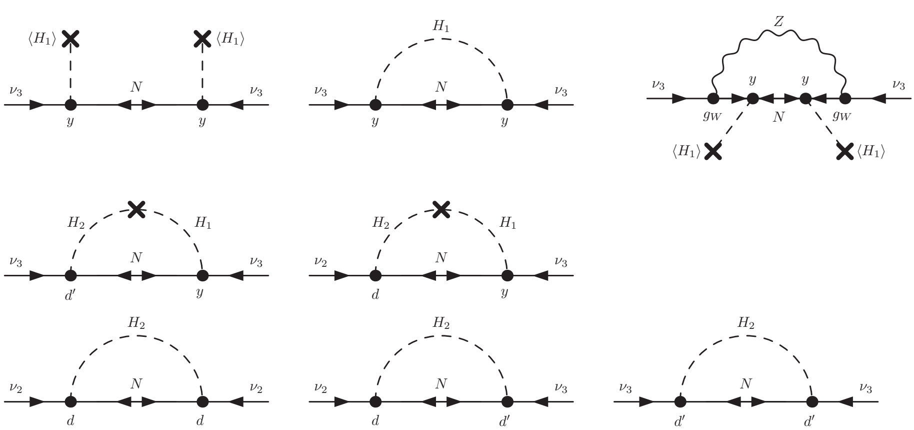

Fig. 1. Feynman diagrams that contribute to light neutrino masses in the basis defined by Eq. (6).

The resulting 2×2 mass matrix for light neutrinos can be then written as [13]

$$M_\nu^{\mathrm{GL}} = \begin{pmatrix} 0 & 0 \\ 0 & m_3 \end{pmatrix} + \frac{m_4}{32\pi^2} \begin{pmatrix} -d^2 F_{22} & -i\left(dd' F_{22} + dy F_{12}\right) \\ -i\left(dd' F_{22} + dy F_{12}\right) & y^2 F_{11} + \left(d'\right)^2 F_{22} + 2yd' F_{12} \end{pmatrix}, \quad (11)$$

where F_{ij}s are the dimensionless loop functions to be read out from Fig. 1 with i, j being Higgs family indices[3]. This mass matrix is finite and gauge invariant and thus, in principle, we can directly relate any of the two free parameters that this matrix depends on to the measured neutrino mass squared differences. The loop functions are

$$\begin{aligned} F_{11} &= -3f_Z - f_H \cos^2(\beta - \alpha) - f_h \sin^2(\beta - \alpha)\,, \\ F_{22} &= f_A - f_H \sin^2(\beta - \alpha) - f_h \cos^2(\beta - \alpha)\,, \\ F_{12} &= (f_H - f_h) \sin(\beta - \alpha) \cos(\beta - \alpha)\,, \end{aligned} \quad (12)$$

where, assuming the low seesaw scale, we have

$$f_i = \frac{m_4^2}{m_4^2 - m_i^2} \ln\left(\frac{m_4^2}{m_i^2}\right) \approx \ln\left(\frac{m_4^2}{m_i^2}\right), \quad \text{for } m_3 \ll m_4 < v \sim m_i\,, \qquad i = h, H, A\,. \quad (13)$$

β–α is the mixing angle between H_1 and H_2. From $\left|\det M_\nu^{\mathrm{GL}}\right| = m_3^{\mathrm{pole}} m_2^{\mathrm{pole}}$, one can express the coupling d^2 in terms of one loop, GL-approximated pole masses [13]. Assuming $\frac{m_4}{4\pi v} < 1$, we can make an expansion and take only the leading term

$$\begin{aligned} d^2 &= \left(\frac{4\pi v}{m_4}\right)^4 \frac{2m_3^{\mathrm{pole}} m_2^{\mathrm{pole}}}{v^2} \frac{m_4}{m_3} \frac{1}{\left|F_{12}^2 - F_{22}\left[F_{11} + \left(\frac{4\pi v}{m_4}\right)^2\right]\right|} \\ &= \frac{32\pi^2 m_3^{\mathrm{pole}} m_2^{\mathrm{pole}}}{m_4 m_3} \left(\frac{1}{|F_{22}|} + O\left(\left(\frac{m_4}{4\pi v}\right)^2\right)\right). \end{aligned} \quad (14)$$

Then, using Eq. (8), we get

$$\frac{y^2}{d^2} = \frac{2m_3^2 m_4^2}{(4\pi)^2\, 2m_3^{\mathrm{pole}} m_2^{\mathrm{pole}} v^2} |F_{22}| = \left(\frac{m_4}{4\pi v}\right)^2 \frac{m_3^2}{m_3^{\mathrm{pole}} m_2^{\mathrm{pole}}} |F_{22}|\,, \quad (15)$$

[3] We define the Z-boson contribution absorbed into F_{11}.

which shows that for small m_4, the y coupling is significantly smaller than d. We thus can neglect the diagrams that are $\sim y$ in the loop, which means that F_{11} and F_{12} are neglected in the 2×2 mass matrix for light neutrinos, which corresponds to neglecting the diagrams having H_1, Z and H_1–H_2 mixing in the loop in Fig. 1. This leads to the following simplified matrix:

$$M_\nu = \frac{1}{32\pi^2} m_4 F_{22} d^2 \times \begin{pmatrix} -1 & -i\,\frac{d'}{d} \\ -i\,\frac{d'}{d} & \left(\frac{1}{F_{22}}\left(\frac{4\pi v}{m_4}\right)^2\right)\frac{y^2}{d^2} + \left(\frac{d'}{d}\right)^2 \end{pmatrix}, \tag{16}$$

$$F_{22} = \ln\left(\frac{m_H^2}{m_A^2}\right) - \ln\left(\frac{m_H^2}{m_h^2}\right)\cos^2(\beta-\alpha)\,. \tag{17}$$

Note that the limit of $y \to 0$ makes this matrix singular, which automatically means that one of the pole masses for light neutrino vanishes. This limit corresponds to the exact Z_2 symmetry in the Yukawa Lagrangian, thus it is consistent with the statement that one cannot reproduce the two non-vanishing light neutrino masses with a single sterile neutrino in the scotogenic (exactly Z_2-symmetric) model. Hence, we have a lower bound on y, even though it can be really small (*e.g.* $y = O\left(10^{-9}\right)$ for $m_4 = O$ (MeV)), thus we can talk of an "approximate" Z_2 symmetry, while still reproducing neutrino masses in the GN model.

Putting Eq. (14) and Eq. (15) into Eq. (16), we get

$$M_\nu = \frac{m_3^{\rm pole} m_2^{\rm pole}}{m_3} \mathrm{sign}(F_{22}) \times \begin{pmatrix} -1 & -i\,\frac{d'}{d} \\ -i\,\frac{d'}{d} & \frac{m_3^2}{m_3^{\rm pole} m_2^{\rm pole}}\mathrm{Sign}(F_{22}) + \left(\frac{d'}{d}\right)^2 \end{pmatrix}. \tag{18}$$

In this form, no dependence on the size of F_{22} is present in the mass matrix, but it depends on the sign of F_{22}. The size of F_{22} only controls the size of d via Eq. (14) and thus the overall size of the Yukawa couplings. Equation (14) has to give perturbative d for the description to make sense, which we will talk about in the next section. If this is the case, for checking whether the model can consistently reproduce the observed neutrino data, it is enough to show for which parameter regions the singular values of Eq. (18) are consistent with the observed mass squared differences. For this, we first abbreviate

$$Z_{m3} = \frac{m_3}{m_3^{\rm pole}}\,, \qquad \kappa = \arg\left(d'\right)\,, \qquad x = \left|\frac{d'}{d}\right|^2\,, \qquad m_{32} = \frac{m_3^{\rm pole}}{m_2^{\rm pole}}\,, \tag{19}$$

and then from $\mathrm{Tr}\left(M^\dagger M\right) = \left(m_2^{\mathrm{pole}}\right)^2 + \left(m_3^{\mathrm{pole}}\right)^2$, we get the equation for the real parameter x

$$(1+x)^2 \frac{1}{Z_{m3}^2} + \mathrm{sign}(F_{22}) \left(2\cos^2(\kappa) - 1\right) m_{32} x + m_{32}^2 \left(Z_{m3}^2 - 1\right) - 1 = 0\,. \tag{20}$$

The parameters κ and Z_{m3} are then the free input parameters. However, they are restricted, as $x \geq 0$. From this, we get the lower bound on Z_{m3} (or y)

$$Z_{m3} \geq \frac{m_2^{\mathrm{pole}}}{m_3^{\mathrm{pole}}} \Rightarrow m_3 \geq m_2^{\mathrm{pole}} \Rightarrow y^2 \geq \frac{2m_2^{\mathrm{pole}}}{v^2} m_4\,, \tag{21}$$

which tells us how close we can approach the scotogenic model. The parameter Z_{m3} has to be restricted from above to avoid a large cancellation between the loop level and the tree-level diagrams. For instance, the restriction that no more than 50% of the tree-level mass is canceled by loop corrections translates into $Z_{m3} < 2$. The solutions of Eq. (20) as a function of κ at different values of Z_{m3} are shown in Fig. 2. As can be seen from Fig. 2, the parameter κ is only restricted for $Z_{m3} > 1$. Moreover, if we now put in the values of Eq. (10), from Eq. (21) we see that the allowed values for Z_{m3} and κ in the inverted hierarchy have a smaller parameter space than for the normal hierarchy. Nevertheless, given these values and provided that $F_{22} \neq 0$, Eq. (20) always has real and positive solutions, which means that the GN model can always reproduce the correct neutrino data, both in the normal and inverted hierarchy. The question is, whether the Yukawa couplings can be large enough to give signatures elsewhere, and: can we actually give a restriction on the scalar sector from that?

4. The magnitude of d^2 and restrictions on the scalar sector

Up until now, we assumed that $F_{22} \neq 0$, but we did not look how small it can be. Since $\left|\frac{d'}{d}\right|$ does not depend on the magnitude of F_{22} and the value of $\left|\frac{d'}{d}\right|$ is limited, as can be seen from Fig. 2, the size of d can be used as an estimate of a possible size of the Yukawa couplings of neutrinos to the second Higgs doublet, see Eq. (6). Since $d^2 \sim \frac{1}{F_{22}}$, the description breaks down if d^2 is no longer perturbative. Using Eq. (14) and limiting $d^2 < 4\pi$, gives the theoretical lower bound on F_{22}

$$\frac{8\pi m_3^{\mathrm{pole}} m_2^{\mathrm{pole}}}{m_4 m_3} < |F_{22}|\,. \tag{22}$$

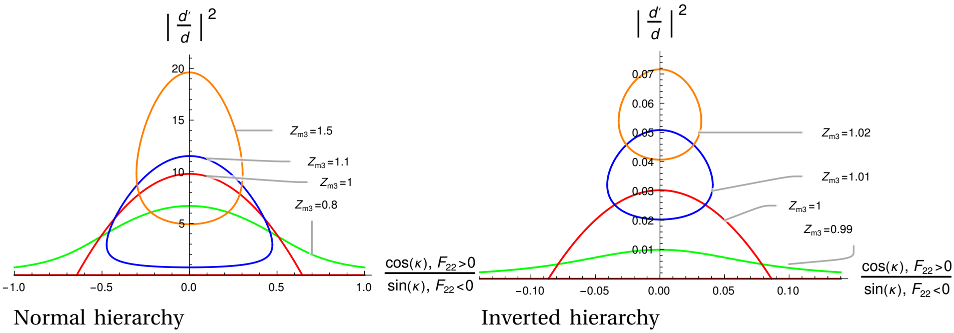

Fig. 2. Solutions of Eq. (20), as a function of $\sin(\kappa)$, for $F_{22} < 0$ (or $\cos(\kappa)$, for $F_{22} > 0$) at different values of Z_{m3} for NH (left) and IH (right). For $Z_{m3} > 1$, κ is no longer a completely free parameter and becomes restricted by the consistency with the neutrino data. Note the scale change on the x-axis for IH, as κ is way more restricted for IH than for NH for $Z_{m3} > 1$.

Since $m_2^{\rm pole} \approx 10^{-2}$eV and $Z_{m3} = \frac{m_3^{\rm pole}}{m_3} = O(1)$, this limit is rather weak. For instance, if we consider the case of no mixing between the Higgs doublets, we have

$$O\left(\frac{\rm eV}{m_4}\right) < |F_{22}| = \left|\ln\left(\frac{m_H}{m_A}\right)\right| = \left|\ln\left(1 + \frac{m_H - m_A}{m_A}\right)\right| \approx \left|\frac{m_H - m_A}{m_A}\right| . \tag{23}$$

We see that it is not satisfied only if the degeneracy between m_A and m_H holds to at least in the $\sim \log_{10}\frac{m_4}{\rm eV}$ digit accuracy, which, for $m_4 \sim O$ (MeV–GeV), is rather fine-tuned. It is unrealistic to expect to see such accuracy in any near-future experiment, even if this would turn out to be a realistic scenario. On the other hand, d^2 gives an overall magnitude of the Yukawa couplings, thus if, instead of restricting its size from perturbativity, it is restricted from somewhere else, we can make this limit on the scalar sector stronger by orders of magnitude. To illustrate what size for d^2 we can expect, for normal and inverted hierarchy, it is

$$\text{NH: } d^2 = \frac{2.7\text{ eV}}{Z_{m3} m_4}\frac{1}{|F_{22}|} , \qquad \text{IH: } d^2 = \frac{16\text{ eV}}{Z_{m3} m_4}\frac{1}{|F_{22}|} . \tag{24}$$

Assuming $Z_{m3} = O(1)$, we get

$$m_4 = O\ (\text{MeV}) \text{ and } d^2 = O\left(10^{-2}\right) : \begin{cases} \text{NH:} & |F_{22}| = O\left(10^{-4}\right) , \\ \text{IH:} & |F_{22}| = O\left(10^{-3}\right) . \end{cases} \tag{25}$$

We see that couplings are relatively large, while the mass of the heavy neutrino is light enough so that it does not give a kinematical suppression in contrast to the usual, large-scale seesaw mechanism. This indeed might give some effect already, and we expect to exclude a larger parameter space of the scalar sector in this regime and put a stronger bound than the one in Eq. (22).

5. Conclusions

In the GN model, the Yukawa sector is almost completely determined by neutrino oscillations, with only 2 free degrees of freedom, which we parametrized by Z_{m3} and $\arg(d')$. The scalar potential must satisfy the limit defined in Eq. (22), but this limit is quite weak, and we can say that, to a good approximation, the neutrino sector can be realized with any scalar potential in the GN model. However, in the low seesaw regime, the Yukawa couplings to the second Higgs doublet can become significant and we might expect tensions with other experiments. This could potentially lead to stronger constraints on the scalar potential than the bound given in Eq. (22). We expect the strongest constraints to come from flavour violating decays, such as $\mu \to e\gamma$, which we plan to investigate in a near future.

This project has received funding from the European Social Fund (project No. 09.3.3-LMT-K-712-19-0013) under the grant agreement with the Research Council of Lithuania (LMTLT).

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