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Analysis of Government Bond Spreads in the Euro Area

Master's thesis

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Abstract

This thesis investigates the spreads of 10-year bond yields of sovereign euro area countries versus Germany as the way to highlight the differences or similarities among its member states. For this purpose, the thesis, first, examines the convergence and divergence patterns of 13 euro area countries over the period from 2003 to 2022, implementing the Phillips and Sul method. Second, since the property of dependence on distant events unfolds in the long memory parameter, the ARFIMA model finds the fractional part of the differencing parameter. The latter approach is tested against the periodogram-based methods, which serve as a robustness check of the assessed potentially long memory feature having parameters. The empirical findings suggest the acceptance of the overall convergence of euro area government bond spreads in the long-run. Although there are divergence features in a certain period in which the significant shocks occur, the common treatment of the euro area as the unity is confirmed. The investigation of long memory in the euro area discloses the absence of dependent distant events in the spreads residuals after removing ARIMA impacts or this dependence is weak. According to the results, the proposed two-step procedure that involves fitting both ARIMA and ARFIMA models to estimate the long memory parameter of the government bond spreads in the euro area is more reliable in narrower confidence intervals sense than periodogram-based alternatives.

Key words: government bonds, spreads, convergence, divergence, long memory, ARFIMA model

Euro zonos vyriausybių obligacijų pajamingumų skirtumų analizė

Santrauka

Skirtumai tarp pinigų sąjungos narių gali būti vertinami įvairiais metodais. Vienas iš jų – nagrinėti euro zonos vyriausybių obligacijų pajamingumų skirtumus nuo Vokietijos. Naudojantis Phillips ir Sul pasiūlytu metodu, šiame darbe nagrinėjami 13 euro zonos šalių konvergencijos ir divergencijos ypatumai periodu nuo 2003 m. iki 2022 m. Be to, kadangi sąsaja tarp tolimų įvykių atskleidžiama per ilgos atminties parametrą, darbe ARFIMA modeliu nustatoma trupmeninė skirtumo parametro dalis. ARFIMA modeliu rasti galimai ilgos atminties savybę vaizduojantys parametrai yra lyginami su koeficientais, rastais naudojant periodograma grįstus metodus. Empiriniai rezultatai rodo, kad ilgame periode euro zonos suverenių šalių obligacijų pelningumų skirtumai pasižymi konvergencijos savybe. Nors divergencijos požymiai ir yra pastebimi atsiradus pasauliniams ar didelio masto lokaliems sukrėtimams, tačiau rezultatai statistiškai neprieštarauja tam, kad euro zona gali būti laikoma vieningu dariniu. Ilgos atminties tyrime atskleidžiama, kad euro zonos vyriausybės obligacijų pelningumų skirtumų liekanos po ARIMA poveikio šalinimo neturi sąsajos tarp tolimų įvykių arba ta priklausomybė yra silpna. Be to, šiame darbe siūloma euro zonos vyriausybės obligacijų pelningumų skirtumų vertinimui naudoti ne periodograma pagrįstus būdus, o dviejų žingsnių metodą, kuriame, visų pirma, yra pritaikomas ARIMA modelis, o tada ARFIMA modeliu yra pervertinamas ilgos atminties parametras, kuris statatistiškai duoda siauresnius pasikliautinuosius intervalus nei periodograma grįsti metodai.

Raktiniai žodžiai: vyriausybės obligacijos, pajamingumų skirtumai, konvergencija, divergencija, ilga atmintis, ARFIMA modelis

Contents

Abbreviations

ADF – Augmented Dickey-Fuller test
AIC – Akaike Information Criteria
ARFIMA – Autoregressive Fractionally Integrated Moving Average
ARIMA – Autoregressive Integrated Moving Average
CI – Confidence Interval
EA – the Euro Area
EU – the European Union
GPH – Geweke and Porter-Hudak method
KPSS – Kwiatkowski-Phillips-Schmidt-Shin Test
PP – Phillips-Perron Test
ZA – Zivot and Andrews Test

Notations

d – fractional differencing parameter
$\tilde{d}$ – integer part of differencing parameter
p, q – short memory parameters
r – time-trimming fraction

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1 Introduction

In general, the state of being in full agreement and harmony defines the underlying idea of unity concept, such as the euro area (EA). Furthermore, the amalgamation process gives the reason to believe that members of the community behave the same or, at least, have similar characteristics. Since the beginning of the EA formation, each country in the block is deemed to implicate the same behaviour or features in the respective fields. Consequently, no exceptions could be made to the expectations of the European Union (EU) when forming the rules to join the EA club or scrutinizing the features of the EA member states. The question arises: do the euro area countries possess this phenomenon?

The introduction of the euro area was followed by the common belief in the existence of a solid club of single-minded EU member states. As a consequence, the countries in the euro area were treated equally by the market participants for a respectively long period. However, in the past two decades, several events completely changed investors' and markets' expectations of the euro area unity patterns. First of all, the financial crisis divulged the first signs of disparity between the club. As it is widely known, the latter historical event was accompanied by a sovereign debt crisis, in which the market's expectations of governments in the euro area countries became perceptibly diverse.

The importance of the market's treatment of each country's ability to repay the debts is another key point to recognize. Since Greece was considered unreliable in the context of paying off debts, the market had no desire to lend money to such a country. Therefore, after the end of the sovereign debt crisis, the Greek debt predicament occurred, emphasising the variant expectations from the market compared to other EA countries. The occurrence of such significant shocks did not recur till now and after the above-mentioned phenomena. Hence, the market started to revive the unity concept in the euro area. Even though the equal reliability of the countries slowly returns, notable global events such as the coronavirus pandemic and the Russian invasion of Ukraine indicate the fluctuations in the market, emphasizing the market participants' concerns about the credibility of certain countries.

From another perspective, unity can be analysed through the dissimilarity between the members of the alliance. There are many ways to determine the differences among the countries. However, in this particular research, the spreads between 10-year bond yields of sovereign countries in the euro area are investigated. The yield is the key indicator that investors use to gauge the level of expense for a bond or group of bonds. Conventionally, the German government bond is considered a benchmark in the euro area. The factors that determine government bond spreads are connected to the risks that an investor faces when buying a government bond compared to the hazards that an investor takes on when purchasing a German government bond.

Bearing in mind the previously mentioned historical events, another question appears: Are there any methods that can inspect such characteristics of the unity concept? Essentially, in this thesis, the relevant issue is analysed through convergence and divergence patterns. The research contains a comprehensive analysis of the converging and diverging behaviour in both: different periods and the whole sample from 2003 to 2022. For the purpose of convergence analysis, the Phillips and Sul method [16] is considered appropriate to capture the unity property in the euro area government bond spreads. To be more precise, the method is based on filtering the trend from the seasonal components by using the Hodrick-Prescott filter and then the $\log t$ regression is performed to obtain the results for the hypothesis testing. The method is able to catch the long-run behaviour and consequently discloses the subgroups with similar patterns, even if, for instance, the convergence overall is rejected.

Second, to obtain a thorough analysis, the thesis investigates the long memory feature of the spreads. The long memory feature is characterized by a strong dependence between distant events. In this regard, the examination of the long memory feature in the government bond spreads is tested through the autoregressive fractional integrated moving average (ARFIMA) model denoted as ARFIMA(p,d,q). To be more precise, the main objective of the ARFIMA model is to estimate the long memory parameter d which is widely known as a differencing parameter that, in general, has integer and fractional parts. Before the analysis of ARFIMA, the standard tests of time series are performed and the important characteristics are reviewed. In particular, the ARFIMA model requires the examination of unit root, autocorrelation and partial autocorrelation functions, selection of the appropriate p, $\tilde{d}$ and q parameters by using autoregressive integrated moving average ARIMA(p, $\tilde{d}$, q) models, where $\tilde{d}$ is the integer part of d, and ultimately the estimation of the fractional differencing parameter applying maximum likelihood approach.

The thesis possesses the following two tasks:

1. Detection of the convergence or divergence behaviour in the euro area sovereign bond spreads using the Phillips and Sul method;

2. Examination whether the property of dependence between distant events can be attributed to the spreads between government bond yields.

The empirical findings suggest the acceptance of the overall convergence of euro area government bond spreads. Although there are divergence features in a certain period in which the significant shocks occur, the common treatment of the EA as the unity is confirmed. The investigation of long memory in the euro area discloses the absence of dependent distant events in the spreads residuals after removing ARIMA impacts or this dependence is weak. Thus, the shocks that affected certain members of the EA more in the past do not imply that they will be more exclusively affected than other members in the future.

The thesis is organized as follows. Section 2 provides background on the convergence and long memory analysis of sovereign bonds or other relevant financial instruments. Section 3 develops the main theoretical knowledge of the used methods. Section 4 describes the data and performs the investigation of the spreads. Section 5 discusses the results of the econometric analysis, and Section 6 concludes the research. Furthermore, the results are computed with the help of R (version 4.1.3) and the relevant code is presented in the Appendix E.

2 Literature review

The yield term is associated with the return that investors expect to gain on a bond. Government bond spreads can indicate various signals in financial markets or a country's economic situation, and there are numerous methods for analysing and interpreting these spreads. In some papers, the most often considered benchmark is the German government bond in the euro area. The debt securities with higher yields determine the higher risks associated with them. Subsequently, the wide spreads between bonds underline the significantly higher level of risk for one of the considered instruments.

Mauro Costantinia and Ricardo M. Sousa [4] demonstrate that increased uncertainty causes a flight to safety and a flight to quality in the sovereign bond markets in the 10 euro area countries. Forecasting of sovereign bond risk premia is more accurate when assuming global, macroeconomic, and common euro area uncertainty, as opposed to country-level, financial, and euro area idiosyncratic uncertainty, respectively. Sovereign bondholders ask for a larger premium to hold risky long-term government bonds compared to the safe-haven bond when faced with greater uncertainty. Therefore, a flight to quality is sparked by uncertainty. According to Mauro Costantinia and Ricardo M. Sousa, models using uncertainty measures, particularly at long horizons, predict around one-third of the volatility in sovereign bond risk premia. Similar to other authors, Carlo A. Favero [6] analyses three main factors: local factor, driven by fiscal fundamentals and growth, international factor, driven by the market's appetite for risk and the expectations of exchange rate devaluations.

The spreads of government bonds in the euro area can be classified into groups or subgroups based on their characteristics using different techniques. For example, Nikolaos Antonakakis, Christina Christou, Juncal Cunado, and Rangan Gupta [1] examine the convergence patterns of 17 euro area countries using Phillips and Sul method. The authors analyse the period between 2002 and 2015 and find no full convergence over selected euro area countries. Despite the rejection of full convergence, Phillips and Sul's method indicates the existence of convergence in three subgroups over euro area countries. Even though transitional curves might imply divergence of bond yield spreads in the short-run, the same spreads tend to converge in the long-run except for Greece and Cyprus.

Robinson Kruse and Christoph Wegener [13] investigate a simple autoregressive model, which contains a unit root or even explosive behaviour, and the innovations of the model are strongly correlated in the sense of the long memory model. Robinson Kruse and Christoph Wegener conduct Monte Carlo simulations and study the finite-sample properties of the Phillips unit root test against explosive alternatives. In the case of the strongly autocorrelated residuals, the authors demonstrate the benefit of adjusting critical values in the unit root test, which leads to a size-controlled test with increased power. Moreover, the authors perform the Lagrange Multiplier test to examine long memory in time series. Robinson Kruse and Christoph Wegener analyse France and Greece government bond spreads versus Germany, and their findings show the existence of long memory and explosive behaviour in Greek-German spreads, while both the unit root behaviour and no evidence of the long memory are demonstrated in the example of French-German spreads.

The investigation of Lucio Della Ratta and Giovanni Urga [17] takes into account parametric and semiparametric long memory estimators. The main objectives of their research are US Treasury and corporate yields and the spreads between them. Their research concludes that the fractional difference parameter discloses the nonstationary long memory ARFIMA process, while the first-order of difference has the short or long memory stationary features in some cases. Due to the fractional cointegration in the bivariate systems of yields, the authors suggest the existence of a long-term equilibrium relationship between them. Furthermore, Philipp Sibbertsen, Christoph Wegener, and Tobias Basse [19] highlight the alteration of the integration order of the government bond yield spreads during the crisis period. The analysed spreads of the sovereign bonds issued by Germany, Spain, Italy, and France are closer to the process with the first-order of difference rather than $I(0)$.

The presented literature review hints at the appropriate methods and approaches for the analysis of government bond spreads in the euro area. The following sections of the thesis are dedicated to further investigation of the Autoregressive Fractionally Integrated Moving Average model and to testing the convergence or divergence behaviour using Phillips and Sul's approach in the context of EA sovereign bond spreads and the most recent shocks, including coronavirus outbreak and Russia's invasion of Ukraine.

3 Methodology

3.1 Methodology of Phillips and Sul method

This section reveals the approach suggested by Phillips and Sul [16] to test the long-run convergence among the variables related to, for instance, economies or countries that are referred to as individuals. The main idea of this method is that it discovers groups of individuals who possess the resembling behaviour in the long-run. The technique is able to disclose the subgroups with similar patterns and at the same time allows some individuals to diverge. As Kris Ivanoviski, Sefa Awaworyi Churchill, and Russell Smyth state [11], the method has the following advantages. First, it does not require prior assumptions and allows for heterogeneity among the time series. Second, the approach is robust regardless of whether the time series is trend-stationary or not. Third, the Phillips and Sul [16] method renders the framework for modelling both transitional dynamics and long-run behaviour through a nonlinear time-varying factor model.

3.1.1 $\log t$ regression

Suppose s_{it} is panel data. The starting point is the rearrangement of the data. Accordingly, suppose s_{it} consists of two components:

$$s_{it} = a_{it} + v_{it}, \tag{1}$$

where the component a_{it} holds the systematic part of the data and variable v_{it} denotes the temporary behaviour of s_{it}. Since the preceding equation can contain both common and temporal components in both variables, Phillips and Sul [16] suggest rearranging (1) as:

$$s_{it} = \left(\frac{a_{it} + v_{it}}{\mu_t}\right)\mu_t = \beta_{it}\mu_t, \tag{2}$$

where μ_t is both a steady-state trend function of the group and mutual for everyone in a panel, β_{it} is a unit-specific component that varies over time. In other words, the element μ_t catches the trending behaviour of s_{it} while β_{it} gauges the range between the common component μ_t and s_{it}. The β_{it} represents the relative share of individual i in common trend component μ_t at time t. The (2) equation is a dynamic factor model.

The modification of the (1) equation to the form of (2) allows capturing the long-run equilibrium patterns. The key idea of the Phillips and Sul [16] is that relative temporal behaviour is sufficient to estimate convergence over a long period into the future. Many authors analyse the distant future behaviour by using cointegration methods, even though cointegration is not an obligatory condition to account for long-run behaviour. In the case of cointegration deficiency, the common stochastic trends accommodate long-run co-movement in aggregate behaviour in the time-varying model. Consequently, it allows the modelling of the transitional effects. To find the convergence patterns of the relevant variable, the main task is to estimate the parameter β_{it}.

The following representation accounts for the existence of relatively long-run equilibrium or convergence in the context of individual heterogeneity:

$$\lim_{k\to\infty} \frac{s_{it+k}}{s_{jt+k}} = 1, \forall i, j,$$

where s_{jt} and s_{it} are two different time series. The condition of relative convergence of s_{it} is equivalent to:

$$\lim_{t\to\infty} \beta_{it} = \beta, \forall i. \tag{3}$$

Likewise to other econometric models, some restrictions are necessary for the construction of this method. Phillips and Sul [16] propose to eliminate the common trend component while defining the relative transition parameter as:

$$\psi_{it} = \frac{s_{it}}{\frac{1}{N}\sum_{i=1}^{N} s_{it}}. \tag{4}$$

Given (2) equation, the relative transition parameter is rearranged to:

$$\psi_{it} = \frac{s_{it}}{\frac{1}{N}\sum_{i=1}^{N} s_{it}} = \frac{\beta_{it}\mu_t}{\frac{1}{N}\sum_{i=1}^{N} \beta_{it}\mu_t} = \frac{\beta_{it}}{\frac{1}{N}\sum_{i=1}^{N} \beta_{it}}. \tag{5}$$

The condition of relative convergence (3) indicates the convergence of the relative transition parameter (5) to unity:

$$\lim_{t\to\infty} \psi_{it} = \lim_{t\to\infty} \frac{\beta_{it}}{\frac{1}{N}\sum_{i=1}^{N} \beta_{it}} = \frac{\beta}{\beta} = 1.$$

From a different viewpoint, the convergence term implies that relative transition parameters should become of the same level over a long period of time. To put it in another way, in the case of convergence, the limit of cross-sectional variance between relative transition parameters approaches zero when $t \to \infty$. Therefore, premise Ψ_t is cross-sectional variance defined as:

$$\Psi_t = \frac{1}{N} \sum_{i=1}^{N} (\psi_{it} - 1)^2, \tag{6}$$

then:

$$\lim_{t \to \infty} \Psi_t = \lim_{t \to \infty} \frac{1}{N} \sum_{i=1}^{N} (\psi_{it} - 1)^2 = 0, \forall i.$$

Despite the decrease of the cross-sectional variance when $t \to \infty$, there are cases when the convergence overall cannot be obtained for the whole sample. More specifically, the method should be able to identify local convergence within subgroups, as local convergence cannot be treated in the same way as overall convergence. To solve this issue, Phillips and Sul [16] propose to define loading coefficient β_{it} in the semi-parametric form as:

$$\beta_{it} = \beta_i + \frac{\sigma_i \xi_{it}}{L(t) t^{\alpha}}, \tag{7}$$

where β_i is the part of the factor loading parameter which does not vary over time, $L(t)$ is a slowly varying function that satisfies the subsequent condition:

$$\frac{L(\lambda t)}{L(t)} \to 1, t \to \infty \tag{8}$$

and increases over time ($L(t) \to \infty$), ξ_{it} are independent identically distributed variables (i.i.d. (0,1)), although weakly dependent over time and α is the speed of convergence. The elements of the equation (7) satisfy several conditions, which are thoroughly introduced in the initial Phillips and Sul paper [16]. Although, it is worth mentioning that $L(t)$ is usually considered to be an increasing but slowly varying function, for instance, $\log(t+1)$. Even if the speed of convergence is equal to zero, the expression of $L(t)$ through slowly varying and yet still increasing function guarantees that $\beta_{it} \underset{p}{\to} \beta_i$ when $t \to \infty$.

In the case of convergence or divergence, the following conditions must correspondingly hold:

$$\lim_{k \to \infty} \mathbb{P}(|\beta_{it+k} - \beta| > \epsilon) = 0 \iff \beta_i = \beta \text{ and } \alpha \geq 0,$$

$$\lim_{k \to \infty} \mathbb{P}(|\beta_{it+k} - \beta| > \epsilon) \neq 0 \iff \beta_i \neq \beta \text{ and } \alpha < 0.$$

The behaviour over a long period of time is analyzed through the test, in which the null hypothesis H_0 supports the convergence patterns:

$$H_0 : \beta_i = \beta, \forall i \text{ and } \alpha \geq 0, \tag{9}$$

versus the alternative H_1 that discloses divergent behaviour:

$$H_1 : \beta_i \neq \beta, \forall i \text{ and } \alpha < 0. \tag{10}$$

Given (9) and (10), the rejection of the null hypothesis does not lead to the fact that the individual units cannot converge to the subgroups. In simple words, H_0 accounts for the temporary divergence and there is a possibility of different transitional paths. Phillips and Sul [16] propose to test the above-mentioned phenomenon of convergence through the model with least ordinary squares, which is known as OLS, so that:

$$\log\left(\frac{\Psi_1}{\Psi_t}\right) - 2\log L(t) = c + b\log t + u_t, \text{ for } t = [rT], [rT]+1, [rT]+2, ..., T, \tag{11}$$

where Ψ_t as in (6), $L(t) = \log(t+1)$, $b = 2c$, the estimate of α in H_0 is c, b is the speed of convergence parameter of β_{it}, r is the fraction between the interval $(0, 1]$ and subsequently $[rT]$ denotes the integer number of the multiplication of the sample size T and chosen r. Referring to the views of Kerui Du [5], the results depend on the fraction r and it is proven using Monte Carlo simulations that the algorithm performs satisfactorily when r is between the interval 0.2 and 0.3, the value of r is chosen accordingly for the large sample ($T \geq 100$) and small sample ($T \leq 50$). In the convergence estimation, the share represented by the r variable is eliminated from the whole sample.

The hypothesis of the convergence is tested by applying heteroskedasticity and autocorrelation consistent (HAC) robust one-sided t-test of the weak inequality $\alpha \geq 0$ (using the estimated b). As Kerui Du emphasizes [5], the limit distribution of the regression t statistic is:

$$t_b = \frac{\hat{b} - b}{e_b} \to N(0, 1),$$

where e_b is a conventional HAC estimate established from the regression residuals. Such residuals are expressed as:

$$e_b^2 = \widehat{lvar}(\hat{u}_t)\left[\sum_{t=[Tr]}^{T}\left(\log t - \frac{1}{T - [Tr] + 1}\sum_{t=[Tr]}^{T}\log t\right)^2\right]^{-1},$$

where $\widehat{lvar}$ is a conventional autocorrelated and heteroskedastic estimate formed from the regression residuals and

$$\hat{u}_t = \log\left(\frac{\Psi_1}{\Psi_t}\right) - 2\log L(t) - \hat{c} - \hat{b}\log t, \text{ for } t = [Tr], ..., T.$$

Therefore, the null hypothesis H_0 is rejected at the 5% level when the t-value satisfies $t_b < -1.65$.

3.1.2 Clustering and merging algorithms

In the previous section, the focal point is the single equilibrium in the whole sample. But is the condition of only one equilibrium decent in the analysis of long-run behaviour? Is there a possibility of multiple equilibria? Indeed, the existence of multiple equilibria is a widespread phenomenon. Persuasive, the non-acceptance of the general convergence (the rejection of the null hypothesis) does not imply that all research subjects diverge to different levels. Conversely, there is a big chance of club convergence in the sample. The multiple convergences exhibit homogeneous behaviour among the groups. Therefore, the existence of those convergence clubs enlarges the understanding of long-run behaviour. To deal with this issue, Phillips and Sul [16] propose the following clustering algorithm to find the existing convergence clubs (if any exist). As described in Roberto Sichera, Pietro Pizzuto [20], the process of grouping and merging the units into clubs consists of six steps:

1) Sort all N countries in descending order regarding the last observation in the time series. As a consequence, the country with the highest value of the latest observation becomes the first while the country with the lowest value turns into the last in the row.

2) Find the first subgroup consisting of k individuals with the highest values of the latest observations such that $t_k > -1.65$ for $\{k, k+1\}$. The existence of such a subgroup is not always defined. In the case of the absence of such subgroup k for which $t_k > -1.65$, the conclusion of no club convergence is drawn.

3) Run the regression for the remaining $N-k$ subgroups with individuals $\{k, k+1, k+2, ..., k+m\}$, where $m \in \{1, 2, .., N-k\}$. The next step is selecting the value of m that subgroups with individuals $\{k, k+1, .., k+m\}$ grant the biggest value of test statistic of log t regression and defining that group as the core. In other words, the core group formation is expressed as:

$$m = arg \max_k \{t_k\} \text{ subject to } \min\{t_k\} > -1.65.$$

4) Gradually add individuals one by one that are not included in the core group and run the log t regression to the new groups. The new unit is added to the group if the test statistic t_k is greater than a critical value. This suggests that the units satisfying the latter condition create the convergence club.

5) Collect all the units that do not hold the latter condition and run the log t regression to find out if $t_k > -1.65$ is implemented. In case $t_k > -1.65$ is fulfilled, it is concluded that there are two convergence clubs. If not, repeat the procedure defined above on this subgroup. If the conditions are satisfied, a new convergence club is formed. In the absence of other convergence clubs, the conclusion of the divergence of remaining individuals is drawn.

6) Since a high critical value $c*$ sometimes leads to the over-determination of the groups, perform

the log t regression test for all pairs of the previously found convergence clubs. If the condition $t_k > -1.65$ is satisfied jointly, merge those clubs into one.

3.2 Methodology of finding the long memory in time series

This section describes how the long memory feature could be tackled by using Autoregressive Fractionally Integrated Moving Average (ARFIMA) models. The generic approach is, first, to consider the standard Autoregressive Integrated Moving Average (ARIMA) model and then assess the fractional difference parameter.

As it is stated in [12], the ARIMA model combines the autoregressive model with the moving average model and includes a differencing component to account for non-stationarity in the data. To be more precise, the model may consist of three types of parts:

1. Autoregressive (AR) component that reflects the dependence of the time series on its own past values;

2. Moving average (MA) component that represents the error of the forecast as a linear combination of past errors;

3. Integrated (I) component that specifies the integer order of differencing d applied to the initial time series in order to attain weak stationarity in the covariance sense.

The main drawback of the ARIMA model is that it can obtain only integer values for the differencing parameter d. Hence, the ARFIMA model is introduced to account for long-term dependencies and overcome this limitation. The main difference between ARIMA and ARFIMA is that the parameter d can obtain fractional values. Subsequently, ARFIMA may tackle the class of long-memory models and these models extend beyond the presence of random walks and unit roots in univariate time series processes.

3.2.1 Stationary ARFIMA model

ARFIMA $\{X_t\}$ process is defined through the ARFIMA(p,d,q) in the following form:

$$\Phi(L)(1-L)^d X_t = \Psi(L) Z_t, \tag{12}$$

where Z_t is a white noise process with (finite or infinite) variance σ_Z and zero mean ($\mathbb{E}Z_t = 0$). Moreover, the autoregressive polynomial $\Phi(L)$ of the order p is expressed as:

$$\Phi(L) = (1 - \phi_1 L - \phi_2 L^2 - ... - \phi_p L^p), \tag{13}$$

and the moving average polynomial $\Psi(L)$ of the order q takes the shape of:

$$\Psi(L) = (1 - \psi_1 L - \psi_2 L^2 - ... - \psi_q L^q). \tag{14}$$

L denotes backward shift or lag operator that holds $LX_t = X_{t-1}$. (12) can be transformed as:

$$\Phi(L)X_t = \Psi(L)(1-L)^{-d}Z_t,$$

where the fractional differencing parameter $(1-L)^{-d}$ is expressed as:

$$(1-L)^{-d} = \sum_{j=0}^{\infty} b_j(d)L^j$$

and $b_j(d)$ denotes the coefficients in the expansion function $f(z) = (1-z)^{-d}, |z| < 1$. Therefore, $b_j(d)$ can be defined through gamma functions:

$$b_j(d) = \frac{\Gamma(j+d)}{\Gamma(d)\Gamma(j+1)}, \quad j = 0, 1, 2, \ldots,$$

so that

$$(1-L)^{-d} = \sum_{j=0}^{\infty} \frac{\Gamma(j+d)}{\Gamma(d)\Gamma(j+1)} L^j.$$

When j is large, an asymptotic approximation of $\frac{\Gamma(j+d)}{\Gamma(d)\Gamma(j+1)}$ is derived in the following way:

$$\frac{\Gamma(j+d)}{\Gamma(d)\Gamma(j+1)} \sim \frac{j^{d-1}}{\Gamma(d)}.$$

Furthermore, by defining $(1-L)^{-d}X_t$ as U_t, the ARMA(p, q) process is derived. Subsequently, if both polynomials $\Phi(B)$ and $\Psi(B)$ have roots that lie outside of the unit circle and if the absolute value of d is less than 0.5, then the process $\{X_t\}$ is stationary and invertible. The spectral density of the stationary and invertible $\{X_t\}$ process is given by the following formula:

$$f_X(\lambda) = f_U(\lambda)\left[2\sin\left(\frac{\lambda}{2}\right)\right]^{-2d},$$

where $\lambda \in [-\pi, \pi]$.

There are three interpretation cases of the d parameter:

- $d \in (0, 0.5)$ – the long memory exists in the $\{X_t\}$ process,
- $d \in (-0.5, 0)$ – the process is characterized by intermediate memory,
- $d = 0$ – $\{X_t\}$ process is a short memory process.

Furthermore, Niels Handrup and J. Eduardo Vera Valdes [8] state that the hyperbolic decay of the autocorrelation function at a sufficient pace indicates the presence of long memory in the process. To be more explicit, in the existence of the long memory, the autocorrelation function slowly approaches zero. Figure 1 depicts the example of the autocorrelation function in the presence of long memory.

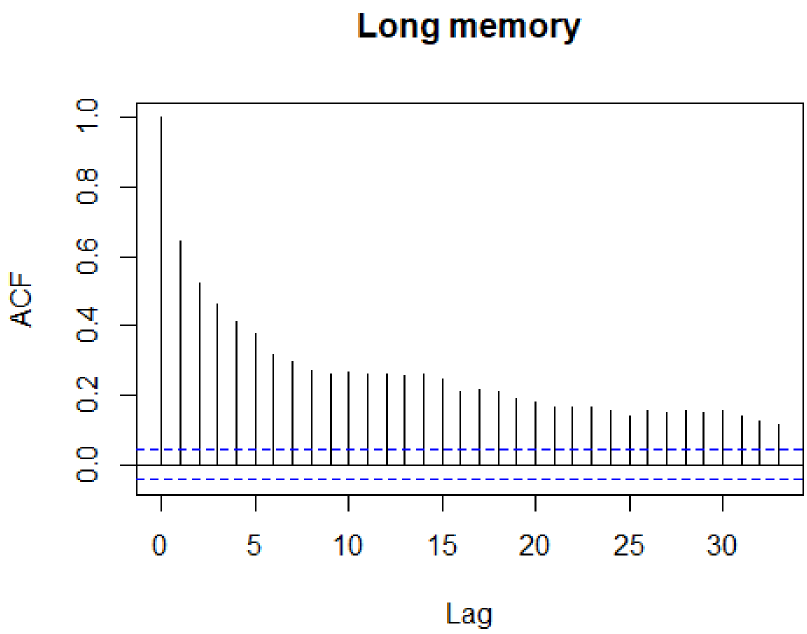

Figure 1: The example of the autocorrelation function in the presence of the long memory

3.2.2 Non-stationary ARFIMA model

Controversially to stationary ARFIMA, the non-stationary ARFIMA model is characterized by time-varying statistical properties, such as a changing mean, variance, or autocovariance. In the most common case of non-stationary ARFIMA, the source of non-stationarity is the integer integration part, similar to the standard ARIMA model, when (12) is rewritten as:

$$\Phi(L)(1-L)^{v}X_t = \Psi(L)Z_t, \tag{15}$$

where $v = d + \tilde{d}$, $\tilde{d} \in \mathbb{Z}$, $d \in (0, 0.5)$, and $\tilde{d} > 0$, so that (15) is rearranged as:

$$\Phi(L)(1-L)^{(d+\tilde{d})}X_t = \Phi(L)(1-L)^{d}(1-L)^{\tilde{d}}X_t = \Phi(L)(1-L)^{d}Y_t,$$

where $Y_t = (1-L)^{\tilde{d}}X_t$ is a stationary process. Therefore, if $v \geq 0.5$ – the process is the mean-reverting yet with time-varying variance and, hence, is considered non-stationary. Subsequently, the conclusion of no long memory is drawn in this case. On the other hand, one can study if the long memory phenomenon is present in the residuals after the appropriate number of integer differences $\tilde{d}$ were applied.

3.2.3 ARIMA model

If the order of the fractional differencing parameter d is equal to zero in an ARFIMA model, then the model becomes equivalent to an ARIMA model, which is sufficient to model short-term dependencies but also, due to the integrated part, may admit a description of non-stationary processes. To be more precise, the long-dependencies are not present if the fractional differencing parameter is set to zero and, therefore, the ARIMA model is appropriate to fit. Taking into account the absence of the long memory, (15) can be transformed to the form (16), in which only the short-dependencies are considered:

$$\Phi(L)(1-L)^{\tilde{d}}X_t = \Psi(L)Z_t, \tag{16}$$

where the notations are the same as previously defined.

3.2.4 Unit root tests

Before fitting both ARFIMA and ARIMA models, it is crucial to test for the integer level of integration of the data. There are many ways to test if the data have unit roots, this thesis takes into account the following tests:

1. Augmented Dicky Fuller test (ADF),
2. Phillips-Perron test (PP),
3. Kwiatkowski-Phillips-Schmidt-Shin test (KPSS),
4. Zivot and Andrews test (ZA).

The appropriate unit root test selection is another key point to master. Unit root tests help to find the proper integer order of differencing. Since the number of the latter tests is quite large, the question arises: which methods are suitable for use? Of course, the eligibility of unit root tests depends on the time series types. The number of studies compares ADF, PP, and KPSS tests. For example, Markéta Arltová and Darina Fedorová [2] analyse time series with AR(1) process without a constant but with positive ϕ_1 values varying from 0.01 to 0.99. Their findings conclude that the ADF test is the most appropriate method to test the unit root existence in the case of long time series. The KPSS and PP tests are recommended to apply for the shorter time series.

The latter tests are expressed in the following forms:

1. ADF:
$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{j=1}^{p} \sigma_j \Delta y_{t-j} + \epsilon_t,$$
where p the AR process lag order, α is an intercept $,\beta, \gamma$ – coefficients of time trend and process root accordingly, t – time index, y_t – data, and ϵ_t denotes i.i.d. residuals;

2. PP:
$$\Delta y_t = \alpha + \gamma y_{t-1} + \epsilon_t,$$
where ϵ_t is serially correlated;

3. KPSS:
$$\Delta y_t = \beta t + y_{t-1} + e_t + \epsilon_t,$$
where the expression $y_{t-1} + e_t$ may be considered random walk;

4. ZA:
$$\Delta y_t = \alpha + \gamma y_{t-1} + \beta t + aDU_t + bDT_t + \sum_{j=1}^{p} \sigma_j \Delta y_{t-j} + \epsilon_t,$$
where DU_t – indicator of the occurring mean shift at break-date, DT_t – indicator of the occurring trend shift at break-date.

The details of ADF, PP, KPSS, and ZA tests are presented in the papers [15], [14], [22], and [23] accordingly.

3.2.5 Periodogram-based methods

The default method to estimate the fractional difference parameter after a sufficient level of integration is reached is the maximum likelihood approach. However, the pioneering fractional difference estimation approaches stem from the use of periodogram-based methods.

For instance, the Geweke and Porter-Hudak (GPH) method and the Sperio technique are applied in this research in order to derive the alternative estimate of the long memory-related fractional difference parameter. As Tetiana Stadnytska, Simone Braun and Joachim Werner [21] state, the Geweke and Porter-Hudak algorithm estimates long memory by performing a linear regression of the log periodogram on a deterministic predictor, while the Sperio method assesses the long memory parameter through a regression equation that uses the smoothed periodogram function to model the spectral density. Furthermore, these methods are not restricted to the limitation of the stationary time series. Therefore, the latter techniques are employed regardless of whether the time series is stationary or not. On the other hand, the periodogram-based methods might suffer from the presence of unit roots in the data that significantly disturb periodograms by dragging the mass closer to zero frequency. Hence, the pre-differencing step is deemed crucial in investigating the long memory feature.

The explanations of the latter approaches are found in [7] and [18].

4 Empirical results

4.1 Data

The main objectives of this research are spreads between government bond yields in the euro area. Even though the euro area consists of 19 European Union Member States, only 14 of them are included in this analysis. The euro area is formed from the European Union countries, whose national currency is the euro. The reason for eliminating Cyprus, Estonia, Luxembourg, Malta, and Slovenia from the sample is the deficiency of the available data. The daily data of government bond yields are sourced from Bloomberg and cover the period from January 2003 to November 2022. In addition, the sourced data are yields to maturity, frequently known as YTMs. YTM shows the rate of return that a bond earns after making all interest payments and repaying the original principal. YTM is calculated as follows:

$$YTM = \sqrt[n]{\frac{FV}{CP}} - 1,$$

where FV is the face value, CP is the current price of the bond and n is the number of years to maturity. To be more precise, face value represents the nominal value of the bond that is defined during the issuance of the debt security. The current price of the bond is computed in the following way:

$$CP = \left(COUPON \cdot \frac{1 - \frac{1}{(1+YTM)^n}}{YTM} \right) + \left(FV \cdot \frac{1}{(1+YTM)^n} \right),$$

where $COUPON$ is the rate paid on the bond annually. The latter rate is exhibited as the percentage of the nominal value.

Moreover, the sourced data are 10-year government bond yields. It is common knowledge that a 10-year yield is often deemed as being reflective of investor confidence in the market, roughly encompassing two political cycles. Since the availability of such data is limited for some countries, the information about the government bond yields is obtained in two ways. To exemplify, there are countries (especially the smaller ones) that have a very restricted number of government bonds and the remaining maturity of those bonds is not exactly 10 years, therefore there are a lot of missing values in the extracted time series. To solve this issue, the interpolation and extrapolation methods are implemented. The interpolation method is able to produce the values between the known observations, meanwhile, the extrapolation method is capable of constructing the unknown values beyond the observations range. As a result, the extrapolation method comes with a higher risk and unreliability than the interpolation. Consequently, the method producing values between the known observations is preferred to the method producing values beyond the observed range. The interpolation formula is written as:

$$y = \frac{(y_2 - y_1)}{(x_2 - x_1)} (x - x_1) + y_1, \tag{17}$$

where y_1 is the yield of the bond with a shorter maturity than 10-year, y_2 is the yield of the bond with a longer maturity than 10-year, x_1 is the remaining maturity of the bond that matures earlier than in 10-year, x_2 is the remaining maturity of the bond that matures later than in 10-year and x is exactly ten years in this case.

Date	y_1	y_2	x_1	x_2
01/01/2015	1.820	1.541	11.833	9.063

Table 1: Lithuanian government bond yield on 01/01/2015

In light of Table 1, Lithuanian 10-year government bond yield is interpolated in the following way.

$$\frac{1.820 - 1.541}{11.833 - 9.063}(10 - 9.063) + 1.541 = 1.635$$

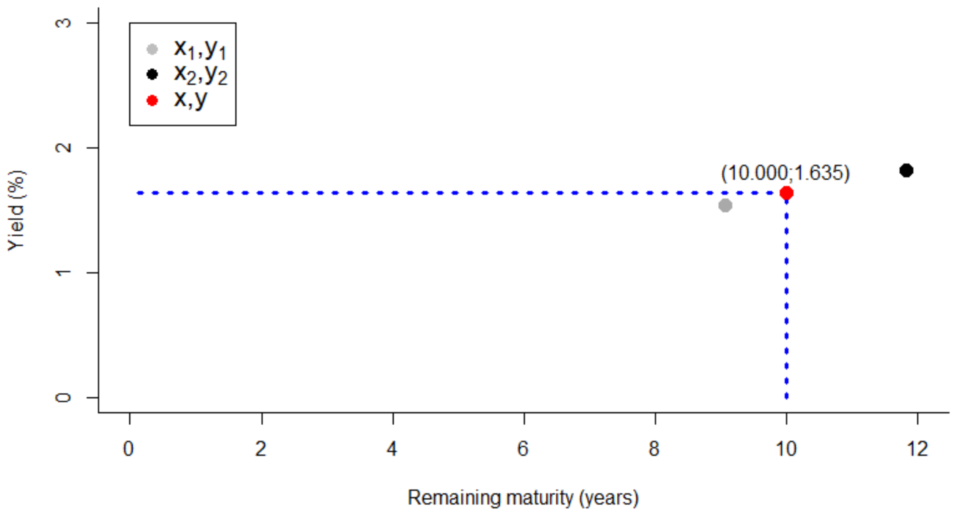

Figure 2: Interpolation of Lithuania's yield

The extrapolation is implemented as follows:

$$y = \frac{(x - x_1)}{(x_2 - x_1)}(y_2 - y_1) + y_1,$$

where y_1, y_2, x_1, x_2, x as in (17). For illustration purposes, the Lithuanian government bond yield on 01/01/2003 is extrapolated using the data provided in Table 2.

Date	y_1	y_2	x_1	x_2
01/01/2003	4.969	4.214	9.362	5.140

Table 2: Lithuanian government bond yield on 01/01/2003

Referring to the values provided in the Table 2, 10-year Lithuanian government bond yield is calculated as:

$$y = \frac{10 - 5.140}{9.362 - 5.140}(4.969 - 4.214) + 4.214 = 5.083.$$

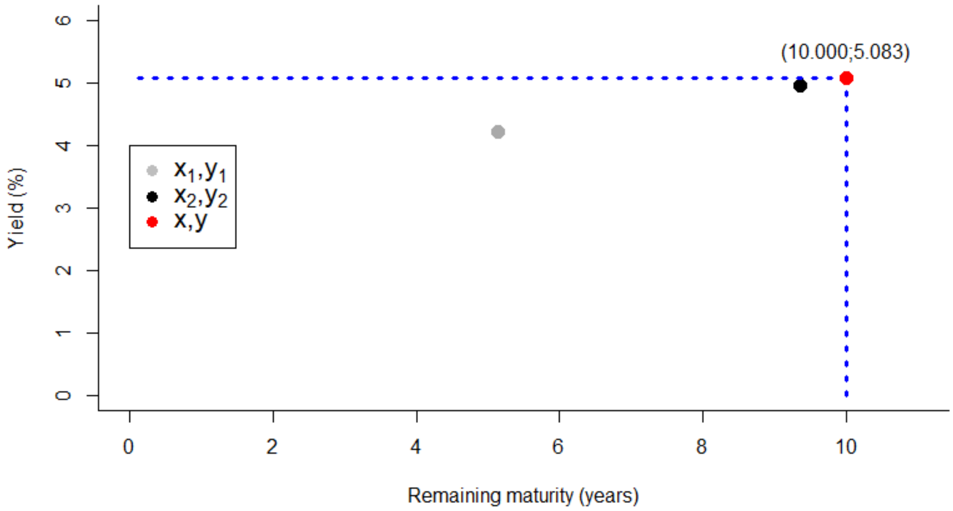

Figure 3: Extrapolation of Lithuania's yield

It is widely known that German government bonds, also known as Bunds, are commonly considered a benchmark of the euro area – the analysis of this thesis is no exception. The 10-year Bund yield is chosen to be a reference against the other euro area countries. The spreads between countries are computed by subtracting the Bunds yield from each country's government bond yield. First of all, let us study the following Table 3 sample of euro area government bond yields.

The euro area government bond spreads versus Germany are computed according to (18).

$$s_{EA} = (y_{EA} - y_{DE}) \cdot 100, \tag{18}$$

where y_{DE} is the German government bond yield and y_{EA} is the euro area country's yield (excluding Germany). For the sake of brevity, the countries will be denoted as their country codes later in this research (the table of the country codes is presented in the Appendix B). Therefore, the spreads are

Country	Yield 05/01/2003	Yield 06/01/2003	Yield 07/01/2003
Austria	4.394	4.373	4.353
Belgium	4.450	4.430	4.396
Finland	4.422	4.399	4.378
France	4.932	4.399	4.345
Germany	**4.333**	**4.312**	**4.286**
Greece	4.582	4.562	4.535
Ireland	4.441	4.419	4.392
Italy	4.552	4.531	4.492
Latvia	5.410	5.427	5.299
Lithuania	5.166	5.138	5.110
Netherlands	4.386	4.431	4.304
Portugal	4.456	4.431	4.397
Slovakia	5.117	5.117	5.117
Spain	4.410	4.389	4.358

Table 3: Euro area government bond yields on 05/01/2003 - 07/01/2003

calculated as:

$$s_{AT} = (4.394 - 4.333) \cdot 100 = 6.1,$$

$$s_{BE} = (4.450 - 4.333) \cdot 100 = 11.7.$$

By using the same approach, the spreads for sampled euro area countries are found. The measurement units of spreads are selected to be the basis points (bps). To clarify, the basis points are often used in finance to sketch the percentage change in the value of financial instruments, where one basis point is equal to 0.01%.

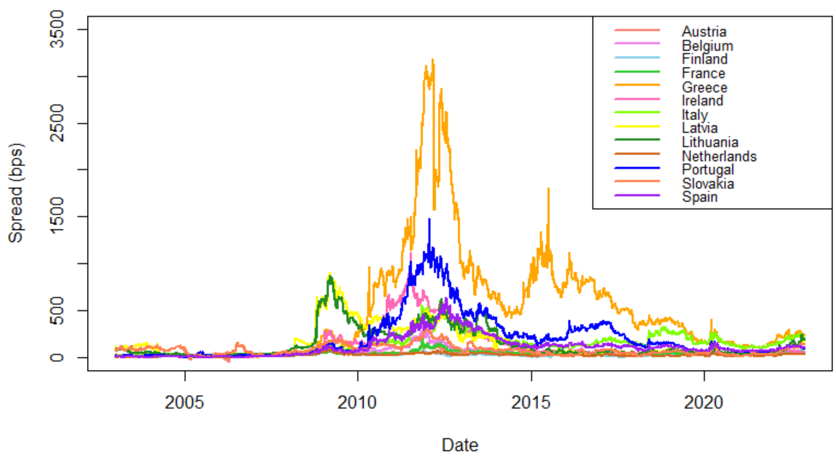

Figure 4: Euro area government bond yield spreads versus Germany

To sum up, spreads of sovereign bond yields (versus Germany) in the whole analysed period are presented in Figure 4. Individual graphs of euro area government bond yield spreads are provided in Figure 5.

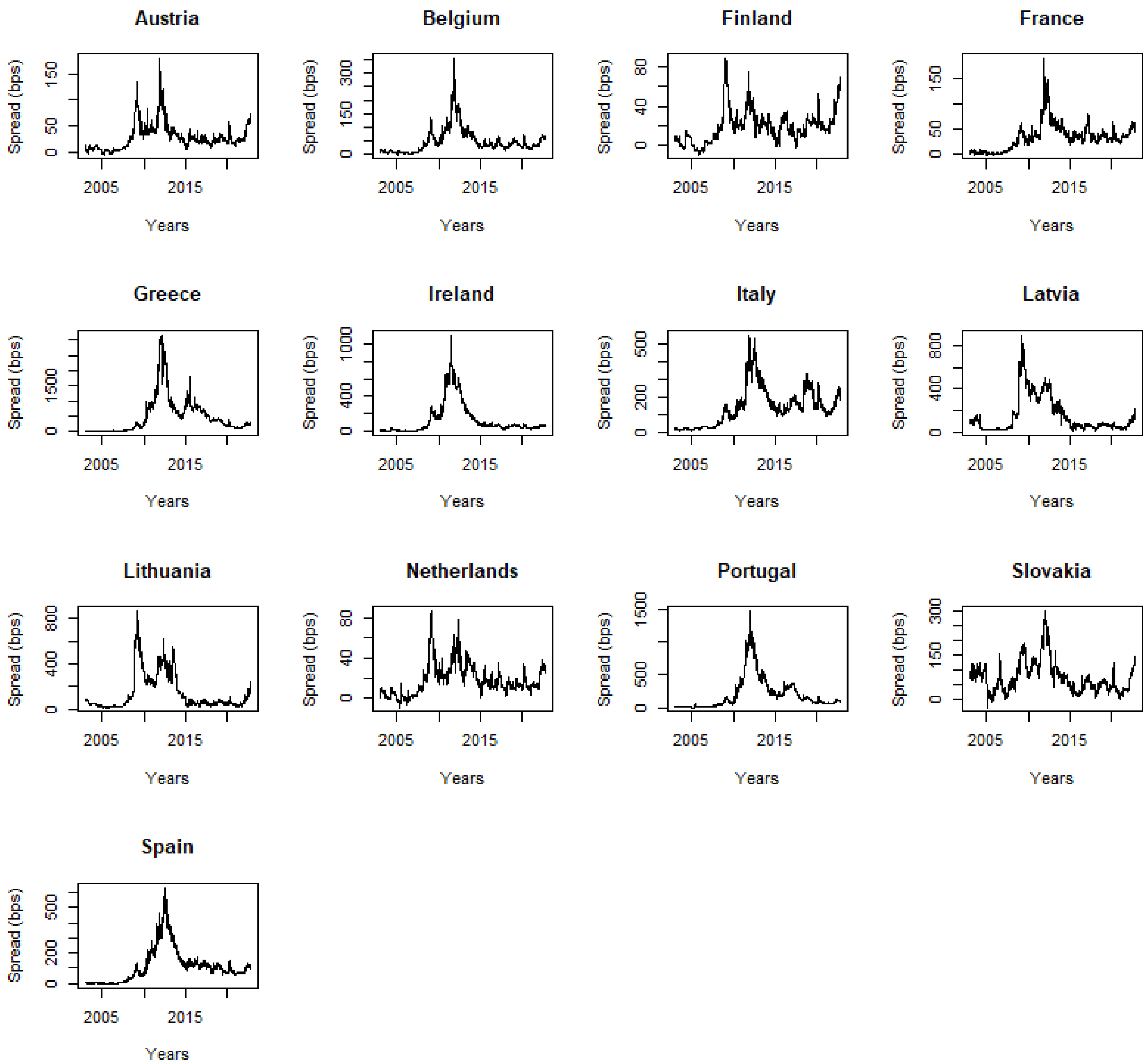

Figure 5: Individual graphs of euro area government bond yield spreads versus Germany

4.2 Application of Phillips and Sul method

This section provides the convergence analysis of the government bond spreads in the euro area. To begin with, the data is divided into four periods for this particular convergence analysis. The first period covers the entire timeline from January 2003 to November 2022. The second period embraces the interval from the start of 2003 to the end of 2008. The third period begins in 2009 and ends in the middle of 2015, encompassing notable fluctuations in government bond yields. The fourth period covers the rest of the time span, from the middle of 2015 to November 2022, and includes events such as the coronavirus pandemic and the Russian invasion of Ukraine, during which there were smaller fluctuations in the spreads. The motivation for dividing the data into smaller periods is to analyse the different behaviours of the spreads in each stage. As there were clear periods of convergence and divergence throughout the entire term (see Figure 4), these contradictory phenomena may balance each other out in the long-run. Therefore, the composition of clubs can vary and change over time depending on which of the above time periods are analysed.

Besides, the time-trimming coefficient presented in equation (11) is denoted as the variable r.

4.2.1 Hodrick and Prescott filter

The initial data is transformed using the method proposed by Hodrick and Prescott (1997) to separate it into trend and cyclical components. According to Hodrick and Prescott [9], the cyclical components represent deviations from the trend and have an average of approximately zero in the long-run. The trend is smoothed using a smoothing parameter, which is defined as the sum of squares of the second differences of the trend. This transformation is used in the proposed method of Phillips and Sul to analyse the spreads, which are composed of two components. The Hodrick and Prescott filter is suitable for this purpose due to its ability to penalize the variability of the trend.

Several smoothing parameters are considered in this research. The selection of an appropriate level of smoothness is done by considering the variation of the cut-off frequency. The cut-off frequency can be defined as the point in a system's frequency response at which energy flowing through the system starts to diminish rather than pass through. The smoothing parameter and the cut-off frequency are related according to the following equation: $\lambda = \left(2 \cdot \sin\left(\frac{\pi}{freq}\right)\right)^{-4}$, where λ is the smoothing parameter and $freq$ is the cut-off frequency. Three cut-off frequencies are analysed, and the appropriate level of smoothness is chosen based on the graphs of the Hodrick-Prescott filter with different relevant parameters (see Appendix C). To summarise, a cut-off frequency of $freq = 30$ is determined as the proper level of smoothness for this research.

4.2.2 Analysis of convergence in the period from 2003 to 2022

With reference to (6), the cross-sectional variances are computed for the period from the start of 2003 to the end of 2008. Subsequently, the $\log t$ regression (11) is performed and the following results are derived.

r	b coefficient	standard deviation	t-statistic	p-value
$\frac{1}{3}$	0.45	0.04	10.63	1.00
$\frac{1}{5}$	-0.34	0.06	-5.68	0.00

Table 4: The $\log t$ regression results for the period 2003-2022

The results of the statistical analysis suggest that the null hypothesis, which proposes the existence of convergence in the spreads among euro area countries, cannot be rejected at the 5% significance level when the time-trimming parameter is $\frac{1}{3}$. This indicates that the spreads exhibit converging behaviour, and the unity of the euro area is not challenged by the data. The t-statistic, which is a measure of the strength of the evidence against the null hypothesis, is found to be greater than -1.65.

In this analysis, a relatively long time series of daily data is considered comprising 5192 observations. If the time-trimming fraction $\frac{1}{5}$ is used, the results are contradictory, with the t-statistic for the convergence overall being less than the 5% significance value. This leads to both the rejection of the hypothesis of convergence overall and the construction of two convergence clubs. Lithuania and Italy are assigned to the first club, while the remaining countries are designated to the second club. However, upon closer examination of the data in Figure 4, the similar behaviour of Lithuania and Italy is observed towards the end of the series and influenced by the small time-trimming fraction, which is insufficient to minimise the impact of the final trend on the convergence estimate. As a result, the overall hypothesis of convergence cannot be rejected and the relative transition paths are presented in Figure 6.

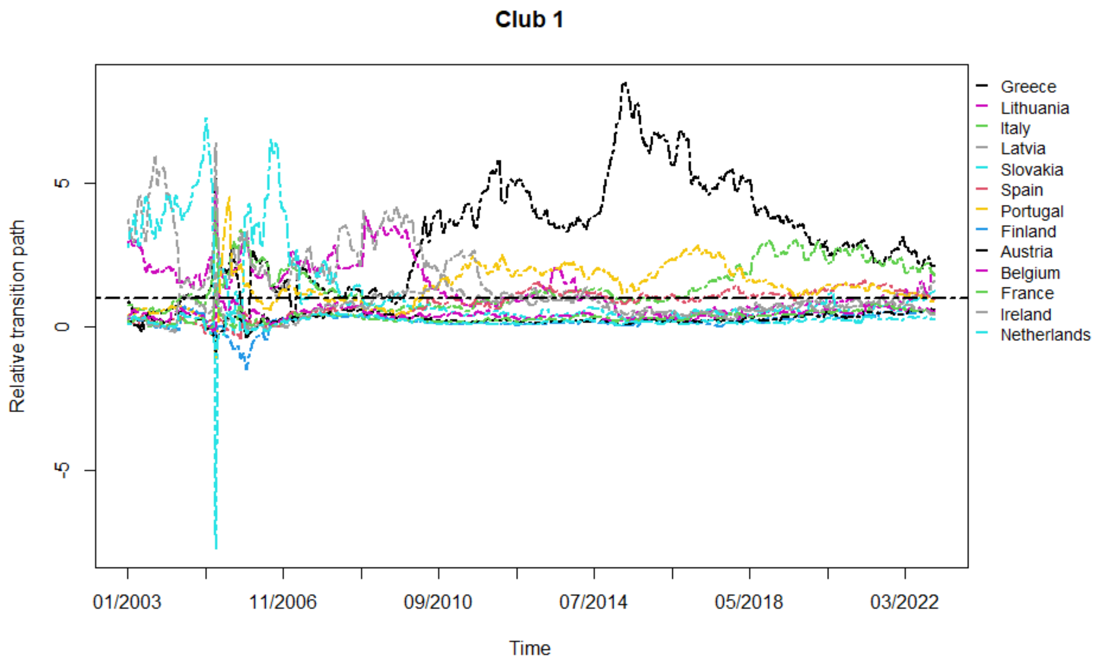

Figure 6: Relative transition paths 2003-2022

4.2.3 Analysis of convergence in the period from 2003 to 2008

The convergence analysis of separate periods is conducted in the same manner as for the whole sample. Therefore, the selected smoothing parameter of the Hodrick-Prescott filter is the same as previously defined and the results of implemented $\log t$ regression are provided in Table 5.

r	b coefficient	standard deviation	t-statistic	p-value
$\frac{1}{3}$	1.21	0.02	78.83	1.00
$\frac{1}{5}$	0.56	0.08	7.22	1.00

Table 5: The $\log t$ regression results for the period 2003-2008

According to Table 5, the results of the analysis support the existence of a convergence process, as the t-statistics for both time-trimming fractions of $\frac{1}{3}$ and $\frac{1}{5}$ are greater than -1.65. Thus, the concept of unity is not rejected for the period from 2003 to 2008. The relative transition paths are derived from the relative transition parameter (5). The relative transition paths for both cases are shown in Figure 7.

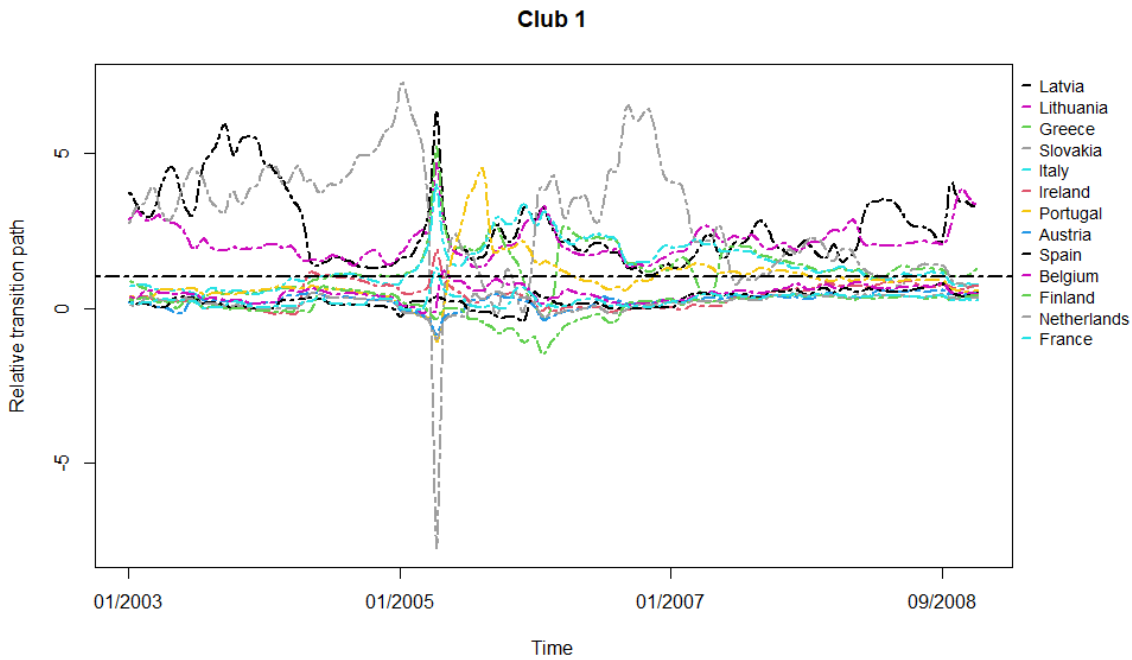

Figure 7: Relative transition paths 2003-2008

4.2.4 Analysis of convergence in the period from 2009 to 2015

Referring to the views of presented historical events, the second subperiod includes two significant occurrences: the financial crisis and the sovereign debt crisis. It is worth mentioning that the largest variation of government bond yields is observed from 2009 to 2015. As a consequence, in the beginning, the spreads of government bonds in the euro area increase by a very large amount and then begin to fall rapidly in the second half of the period.

The $\log t$ regression gives the following results.

r	b coefficient	standard deviation	t-statistics	p-value
$\frac{1}{3}$	-0.52	0.11	-4.832	0.00
$\frac{1}{5}$	-0.62	0.04	-14.28	0.00

Table 6: The $\log t$ regression results for the period 2009-2015

In this case, the t-statistics for both r variables are less than -1.65, indicating the rejection of H_0. Since the overall convergence is rejected, the club convergence is tested for the period from 2009 to 2015 with the time-trimming parameter that is equal to $\frac{1}{3}$. The convergence clubs are given in Table 7.

Club1	Portugal, Italy, Spain, Ireland, Lithuania, Latvia, Belgium, France, Slovakia, Netherlands, Austria, Finland
Divergent	Greece

Table 7: Convergence clubs for the period 2009-2015 ($r = \frac{1}{3}$)

The relative transition paths, excluding the divergent country, are presented in Figure 8.

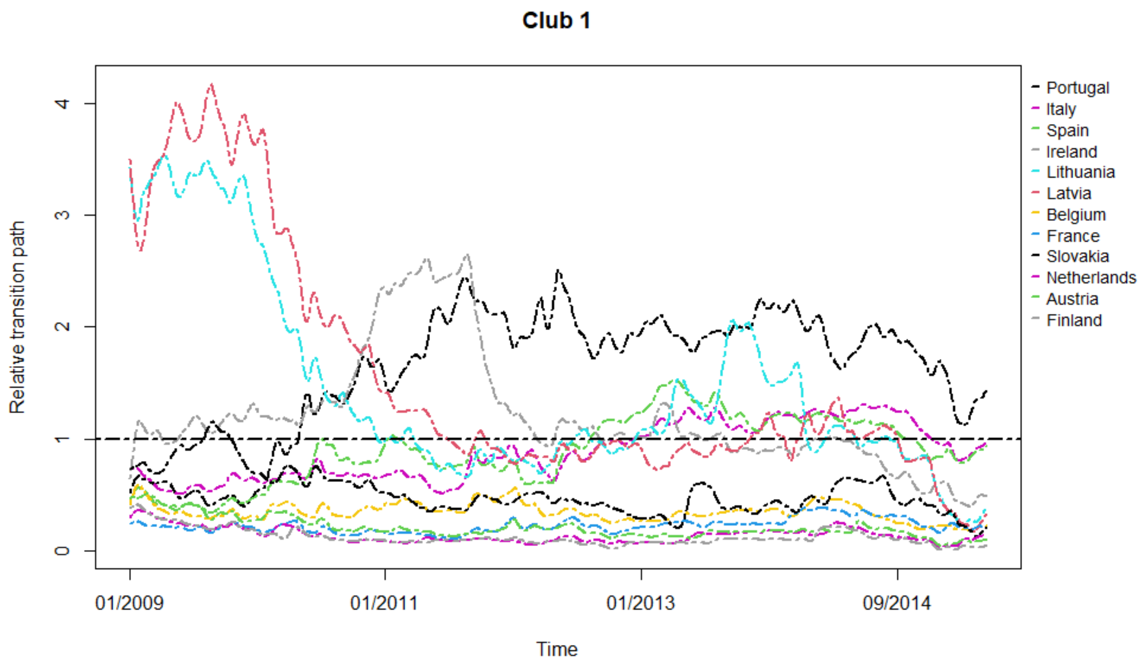

Figure 8: Relative transition paths 2009-2015 $\left(r = \frac{1}{3}\right)$

Since the t-statistics computed for both time-trimming parameters differ from each other, the detection of convergence clubs is repeated with the time-trimming parameter $r = \frac{1}{5}$. The results are provided in Table 8.

	Number of units	b coefficient	std.err	t-statistics
club1	7	0.04	0.01	8.14
club2	5	-0.03	0.04	-0.67

Table 8: Convergence clubs for the period 2009-2015 ($r = \frac{1}{5}$)

The table implies the following results:

1. the first convergence club includes Portugal, Italy, Spain, Ireland, Lithuania, Latvia, and France;
2. the second convergence club consists of Belgium, Slovakia, Netherlands, Austria, and Finland,
3. the divergent behaviour of Greece is confirmed this time as well.

The relative transition paths of these convergence clubs are depicted in Figure 9.

Figure 9: Relative transition paths 2009-2015 $\left(r = \frac{1}{5}\right)$

Since two convergence clubs were identified, the method of clustering and merging algorithm (described in Section 3.1.2) is applied. The algorithm determines if it is possible to merge these clubs into one. Based on the results, it appears that it is not possible to merge the convergence clubs. Therefore, the conclusion of the existence of two convergence clubs is derived.

To conclude, the convergence overall is rejected for the period from 2009 to 2015. Despite the overall divergence of the spreads, the existence of club convergence is confirmed. Even though both $r = \frac{1}{3}$ and $r = \frac{1}{5}$ cases show a different number of existing clubs, the divergent behaviour of Greece is confirmed for both of them. Bearing in mind that Phillips and Sul suggest choosing the larger time-trimming parameter of the considered ones when the sample of time series is relatively large, the conclusion of one convergence club and one divergent unit is drawn.

4.2.5 Analysis of convergence in the period from 2015 to 2022

During the fourth period, the spreads of most sovereign bonds decreased moderately, tending towards a similar level. However, with the emergence of the coronavirus pandemic and the Russian invasion of Ukraine, the yields on these bonds began to react differently. For this reason, the spreads have increased unevenly, especially for Lithuania and Italy (see Figure 4).

To examine whether the variance of the difference between successive values of the spreads for the period from 2015 to 2022 is decreasing, $\log t$ regression is applied. The results of this regression are provided in Table 9.

	b coefficient	standard deviation	t-statistics	p-value
$r = \frac{1}{3}$	1.22	0.00	438.86	1.00
$r = \frac{1}{5}$	0.93	0.07	14.13	1.00

Table 9: The $\log t$ regression results for the period 2015-2022

The results for both time-trimming cases indicate the non-rejection of H_0 of an overall convergence. However, it is worth mentioning that the t-statistics for the two time-trimming fractions differ significantly. The reason for such a significant difference can be attributed to the fluctuation of the yields for Lithuania and Italy at the end of the period, as discussed in the analysis of the entire time span. Despite the differences in the t-statistics for the two time-trimming fractions, the overall convergence is not rejected in either case.

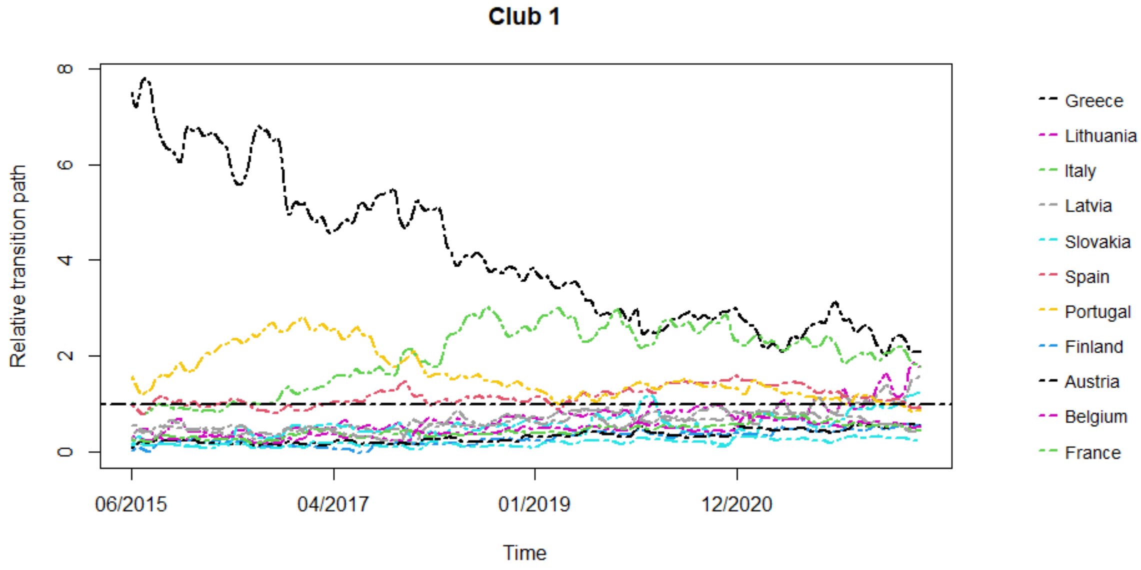

Figure 10: Relative transition paths 2015-2022

4.3 Long memory analysis

In this section, the procedure of the long memory testing is described and practically implemented on the government bond spreads in the euro area. In this thesis, the algorithm for testing long memory consists of three main stages:

1. Testing the unit roots in the time series;
2. Fitting the ARIMA model;
3. Assessing the fractional difference parameter using the ARFIMA model with integer model parameters determined in step 2.

Later, in this research, the procedure of fitting the ARIMA model and assessing the fractional difference parameter using the ARFIMA model is denoted as a two-step procedure. The obtained results from the two-step procedure are compared to results derived from the periodogram-based methods.

4.3.1 Unit roots

First, unit root tests described in Section 3.2.4 for the spreads in the euro area are executed. The null hypothesis of the ADF and PP test specifies the existence of the unit root, whereas H_0 of the KPSS test indicates the absence of a unit root in time series. ZA is an alternative test with the unit root null hypothesis that also admits a single structural break in the intercept. The results of the subsequent tests are presented in Table 10 with p-values returned for ADF, PP and KPSS tests and t-statistic reported for the ZA test.

	ADF	PP	KPSS	ZA
Austria	0.33	0.06	0.01	-4.34
Belgium	0.49	0.36	0.01	-3.79
Finland	0.21	0.04	0.01	-4.05
France	0.29	0.04	0.01	-4.77**
Greece	0.63	0.66	0.01	-3.35
Ireland	0.76	0.81	0.01	-2.69
Italy	0.45	0.47	0.01	-3.63
Latvia	0.63	0.67	0.01	-4.12
Lithuania	0.34	0.67	0.01	-3.88
Netherlands	0.16	0.02	0.01	-5.51*
Portugal	0.90	0.79	0.01	-3.17
Slovakia	0.08	0.27	0.01	-4.11
Spain	0.78	0.75	0.01	-3.54

The rejection of the ZA test is indicated by one and two asterisks, respectively, for the 1% and 10% levels of significance.

Table 10: Unit root tests

Referring to the views of Table 10, the p-values of the ADF test are higher than 0.05, indicating the non-rejection of the null hypothesis for the whole sample at the 5% level of significance. Likewise, the PP test shows the unit root existence in the time series, even though there are several exceptions. To be more precise, the obtained p-values from the PP test are 0.04, 0.04 and 0.02 respectively for Finland, France and Netherlands. Since the p-values point to statistically significant deviation from the null hypothesis, the H_0 is rejected, meaning that FI, GR, and NL time series can be handled later as having no unit roots. Furthermore, the KPSS test indicates the rejection of the null hypothesis that the time series may be trend-stationary.

Since the results of unit root existence are deceptive in the PP test compared to the ADF and KPSS tests, the execution of an additional test is performed. The unit root test of ZA is adopted as the supplemental one. Since the latter test accounts for the single structural break, the testing procedure contains the following hypotheses: the null hypothesis states that the time series is unit root series with drift versus the alternative hypothesis of the process with a one-time break in the level and without unit roots. The conclusion of rejection or non-rejection of the null hypothesis in the Zivot and Andrews test can only be drawn by determining the critical values of the test statistic at the appropriate level of significance. Due to the importance of selecting the significance levels, the critical values are elected at 1%, 5% and 10% significance levels. Table 11 exhibits the chosen critical values.

Critical value (1%)	Critical value (5%)	Critical value (10%)
-5.34	-4.80	-4.58

Table 11: Critical values of the Zivot and Andrews test statistic

The test statistics of Zivot and Andrews are less than 10% critical value for France and the Netherlands. Since the examined level of confidence is chosen to be 5% in ADF, PP and KPSS tests, the consequence of the rejection of the null hypothesis for the Netherlands is obtained.

Government bond spreads actively demonstrate the existence of unit root for the whole sample with the inconsistency for the Netherlands. To summarize, two out of four tests designate the rejection of the hypothesis of the unit root for the case of the Netherlands. Since the integrated process in data is obtained from the unit root tests in most cases, the cogitation of both integer difference and fractional difference parameters should be taken into account. Even though PP and ZA tests disclose the contradictory behaviour of the Netherlands compared to the rest of the EA countries, for homogeneity treatment of the time series, disregard the results of the rejection of the null hypothesis based on the PP and ZA tests. The unit root feature is reflected in the autocorrelation graphs (see Appendix D.1).

Since the existence of unit root is non-rejected for the initial data, the first-order of differencing parameter ($d = 1$) is analysed. The results of the unit root tests for the first-order of difference are presented in Table 12.

	ADF	PP	KPSS	ZA
Austria	0.01	0.01	0.10	-65.95
Belgium	0.01	0.01	0.10	-62.62
Finland	0.01	0.01	0.10	-66.65
France	0.01	0.01	0.10	-72.59
Greece	0.01	0.01	0.10	-63.63
Ireland	0.01	0.01	0.10	-55.64
Italy	0.01	0.01	0.10	-68.98
Latvia	0.01	0.01	0.10	-85.30
Lithuania	0.01	0.01	0.10	-94.33
Netherlands	0.01	0.01	0.10	-69.61
Portugal	0.01	0.01	0.10	-61.30
Slovakia	0.01	0.01	0.10	-78.92
Spain	0.01	0.01	0.10	-65.15

Table 12: Unit root tests for the first-order of differencing

Using the same reasoning about the rejection and the non-rejection of the null hypothesis in respective tests as previously described, the absence of unit root is considered in the spreads when the order of difference is equal to one. Subsequently, the autocorrelation graphs reflect the latter phenomenon (see Appendix D.2).

The question comes up: Is it appropriate order of differencing? For instance, over-differencing removes the trend patterns and can result in the loss of important information. Rob J. Hyndman and Yeasming Khandakar point out [10] that over-differencing has adverse effects on forecasting and predicting confidence intervals. In the following sections, it is examined whether the first-order of difference is an appropriate parameter for the spreads in the EA countries.

4.3.2 ARFIMA

Before fitting the ARFIMA model, it is necessary to determine the parameters of the short memory to more accurately estimate the parameters of the long memory. The short-term parameters can be fully and properly estimated using the ARIMA model. The most appropriate values for the p, $\tilde{d}$ and q parameters are selected using the Akaike Information Criteria (AIC), which is described in detail in [3]. To find ARIMA p, $\tilde{d}$ and q parameters, the KPSS unit root test is used. The appropriate coefficients are found using R and the relevant code is presented in Appendix E. To discover more accurate estimates, the exact method is used to fit the model. Moreover, all the possible combinations of models are considered and the optimal ARIMA model is selected based on AIC.

Once the ARIMA parameters are determined, the fractional difference parameter d is assessed using the ARFIMA model. Since the ARIMA parameter $\tilde{d}$ is equal to one for all government bond spreads in the euro area, the initial value of the difference parameter in the ARFIMA model is set to one as well. The results are given in Table 13.

	ARIMA p	ARIMA d	ARIMA q	ARFIMA d	95% CI
Austria	1	1	4	-0.14	(-0.20;-0.09)
Belgium	2	1	1	0.08	(-0.08;0.25)
Finland	2	1	1	-0.08	(-0.13;-0.03)
France	3	1	2	-0.19	(-0.25;-0.12)
Greece	2	1	3	-0.03	(-0.08;0.02)
Ireland	2	1	3	0.02	(-0.01;0.05)
Italy	1	1	2	-0.05	(-0.13;0.02)
Latvia	5	1	0	-0.01	(-0.08;0.06)
Lithuania	4	1	1	0.15	(0.09;0.21)
Netherlands	4	1	1	-0.16	(-0.22;-0.10)
Portugal	0	1	5	-0.05	(-0.11;0.00)
Slovakia	1	1	3	0.08	(0.02;0.14)
Spain	1	1	4	-0.01	(-0.11;0.09)

Table 13: Estimated $p, \tilde{d}, q$, d parameters and 95% CI of d

In view of Table 13, the estimated d parameters in the ARFIMA model in most of the cases are less than 0, indicating that there is no sign of long memory in the residuals of analysed EA government bond spreads obtained after fitting ARIMA models. As described in Section 3.2.1, the range between -0.5 and 0 refers to the intermediate memory, meaning that the spreads fall between short and long memory. The assessed fractional difference parameters for BE, IE, LT, and SK are higher than 0 but less than 0.5. Therefore, these countries implicate the persisting memory in the residuals. Although the d parameter is relatively small for those countries, weak signs of long memory are considered.

4.3.3 Periodogram-based methods

To justify the assessed fractional differencing parameters, two additional tests are considered. The tests for the verification of the results are selected to be Geweke and Porter-Hudak and Sperio. The latter tests are based on the regression equation that uses the smoothed periodogram function as an estimate of the spectral density. The results of those tests are presented in Table 14.

	GPH d	GPH 95% CI	Sperio d	Sperio 95% CI
Austria	1.01	(0.83;1.18)	0.98	(0.91;1.05)
Belgium	1.06	(0.94;1.19)	1.03	(0.98;1.08)
Finland	0.85	(0.71;1.00)	0.90	(0.83;0.96)
France	0.96	(0.83;1.09)	0.97	(0.92;1.02)
Greece	1.05	(0.91;1.20)	1.04	(0.96;1.13)
Ireland	1.08	(0.94;1.23)	1.07	(0.99;1.15)
Italy	1.01	(0.87;1.15)	1.00	(0.95;1.05)
Latvia	1.02	(0.85;1.19)	1.01	(0.93;1.08)
Lithuania	0.94	(0.79;1.09)	0.93	(0.84;1.02)
Netherlands	0.96	(0.78;1.15)	0.92	(0.84;1.00)
Portugal	1.25	(1.10;1.40)	1.19	(1.12;1.26)
Slovakia	0.89	(0.76;1.03)	0.83	(0.78;0.89)
Spain	1.04	(0.88;1.20)	1.07	(0.99;1.14)

Table 14: Long memory estimates by GPH and Sperio methods

In the context of the ARFIMA model, the integer differencing parameter $\tilde{d}$ obtained from the ARIMA model is equal to one, indicating that the first-order of differences is chosen for the initial data. However, the GPH and Sperio tests do not take into account that the data should be differenced, and, therefore, return results for undifferenced data. To compare the results between Table 13 and Table 14, the proper solution is to either add a unit to the d parameter obtained from the ARFIMA model or subtract a unit from the d estimates derived from GPH and Sperio tests. The selected assertion is the subtraction of unity from the GPH and Sperio tests. With the consideration of the property of results being biased for GDP and Sperio, the long memory is considered in more countries than in ARFIMA for both GPH and Sperio methods. Moreover, the methods of GPH and Sperio do not indicate the long memory in Lithuania's and Slovakia's spreads, even though such a feature is obtained by the ARFIMA model. The GPH and Sperio parameters have wider confidence intervals indicating that the error in the estimation of the d parameter could lead to completely different interpretations. In the evaluation of differencing parameter methods based on the periodogram, the most striking difference is observed for the spread of Portugal. The fractional difference estimate of the ARFIMA model does not indicate long memory for Portugal, however, the d parameter found by the GPH and Sperio method falls into the long memory range. In this case, 95% confidence intervals confirm the long memory for the case of Portugal. When evaluating the results in general, all three methods (ARFIMA, GPH, Sperio) show that the d parameter does not indicate long memory in most of the cases, and if it does, the memory is weak and observed only for several countries.

Looking from another perspective, since pre-differencing is the crucial step to assess the long memory parameter, the estimates of long memory are calculated for the first-order differenced spreads. The results are presented in Table 15.

	GPH d	GPH 95% CI	Sperio d	Sperio 95% CI
Austria	0.07	(-0.09;0.22)	0.01	(-0.07;0.08)
Belgium	0.10	(-0.06;0.26)	0.03	(-0.02;0.08)
Finland	-0.02	(-0.17;0.13)	-0.03	(-0.09;0.03)
France	0.01	(-0.11;0.13)	-0.04	(-0.09;0.01)
Greece	0.05	(-0.10;0.20)	0.04	(-0.05;0.13)
Ireland	0.09	(-0.05;0.23)	0.06	(-0.02;0.15)
Italy	0.03	(-0.09;0.16)	-0.01	(-0.06;0.04)
Latvia	0.04	(-0.11;0.20)	0.00	(-0.08;0.08)
Lithuania	-0.06	(-0.20;0.09)	-0.08	(-0.16;0.01)
Netherlands	0.00	(-0.19;0.19)	-0.09	(-0.16;-0.01)
Portugal	0.23	(0.10;0.37)	0.21	(0.13;0.29)
Slovakia	-0.08	(-0.22;0.07)	-0.13	(-0.19;-0.07)
Spain	0.11	(-0.06;0.28)	0.05	(-0.03;0.14)

Table 15: Long memory estimates by GPH and Sperio methods for the first-order differenced spreads

Little differences are observed comparing Table 14 with Table 15. To be more precise, the disparities are found in the interpretation of: FR and NL for the GPH method, AT and IT for the Sperio method. Again, the confidence intervals computed for the first-order differenced data include the existence and absence of long memory. Despite tiny discrepancies between these two methods, the remaining results are very similar. Therefore, the results of the two-step procedure are compared to the estimates of GPH and Sperio methods for undifferenced data.

5 Discussion of the results

The first part of the work presents the methodologies of convergence patterns and long memory testing. The convergence of the government bond spreads in the euro area is examined via Phillips and Sul method. Furthermore, the analysis of long memory is implemented through both ARIMA, ARFIMA models and periodogram-based approaches. The government bond yield spreads versus Germany are computed manually and some modifications are adapted to the initial data.

Examining the unity concept in the context of the government bond spreads in the euro area considers the importance of selecting the appropriate parameters. For example, in the case of Hodrick and Prescott's filter, several smoothing parameters are considered, even though the chosen cut-off frequency is $freq = 30$. Furthermore, two cases are analyzed, considering that Phillips and Sul [16] recommend trimming some portion of the period when running the log t regression model. In the first case, the selected fraction to trim is considered $\frac{1}{3}$ and later results are recomputed with the trimming parameter $\frac{1}{5}$. The importance of selecting the appropriate time-trimming parameter is most evident when analyzing the entire period from 2003 to 2022 where the overall convergence is challenged by the notable fluctuations of the spreads at the end. As it is known, the key role of the time-trimming parameter is to eliminate the impact of the initial and final trends in the data. The results indicate rejection of convergence overall when considering a smaller fraction of trimming, although the outcome of log t regression is entirely different with a larger time-trimming particle. Therefore, the conclusion of the non-rejection of H_0 is drawn based on the larger of considered time-trimming parameters. The findings remain identical in the analyzed subperiods except for the interval from 2009 to 2015. Since the values of t-statistics are -4.83 and -14.28 respectively for time-trimming coefficients $\frac{1}{3}$ and $\frac{1}{5}$, the overall convergence is rejected. In the latter episode, the $\log t$ regression with the time-trimming $\frac{1}{3}$ indicates the convergence of all countries, excluding Greece. The divergent patterns of Greece are confirmed by running the $\log t$ regression with the time-trimming coefficient $\frac{1}{5}$. Hence, two converging clubs are found in this case. For the shorter time-trimming, the Phillips and Sul method combines Portugal, Italy, Spain, Ireland, Lithuania, Latvia, and France into one club and the rest of the countries are assigned to the second club, leaving Greece as a divergent country. The values of t-statistics in the final period vary remarkably while assuming different fractions to trim. Even though the results do not indicate the rejection of convergence, in the long-run, the fluctuations in yields at the end of the period significantly affect the regression results.

Continuing the analysis from a long memory perspective, the unit root existence in the spreads of euro area countries is confirmed in most cases. Particularly, four unit root tests (ADF, PP, KPSS, ZA) are performed. For the case of the Netherlands, two of four tests (PP and ZA) show that the null hypothesis of the unit root is statistically significantly rejected. Therefore, the time series of the Netherlands government bonds can be considered as having no unit roots, although, for the sake of homogeneity treatment, the results of PP and ZA tests are disregarded. Since the fundamental fractional parameter assessment methods are restricted to the stationary process in the R, the research scrutinizes the parameter of long-range dependence in two steps. The first step consists of the investigation of the short memory components (p and q parameters) and integer part of differencing parameter $\tilde{d}$ using ARIMA. The second step is to re-evaluate the d parameter by providing the p, $\tilde{d}$ and q parameters found in the previous step. The fraction parts of the long memory parameter d for most EA countries are between the interval -0.5 and 0, indicating a mean-reverting process and the absence of long memory. However, the fractional parts of differencing coefficients for Belgium, Ireland, Lithuania and Slovakia are respectively 0.08, 0.02, 0.15, and 0.08, disclosing weak signs of long memory since d coefficients obtained from the ARFIMA model fall between 0 and 0.5. Moreover, the results obtained through periodogram-based methods, such as the Geweke and Porter-Hudak (GPH) and Sperio approaches, discover the long memory feature in more countries than in the two-step procedure. However, the outcomes of the latter methods are likely to be shifted due to periodogram smoothing and the unit root residual impact. As a consequence, the associated confidence intervals for periodogram-based approaches are, in general, wider than those produced by the two-step approach.

6 Conclusions

This thesis confirms the unity concept in the euro area. The unity concept is examined through the dissimilarities between the members of the EA while measurements of disparity are spreads between government bond yields versus Germany in the euro area. This research provides statistically significant evidence of united behaviour by equating the process of convergence to the same level among the group to the unity patterns. Even though there are some signs of divergence for Greece in the period from 2009 to 2015, the research, in general, concludes the convergence of the euro area countries in the long-run. Although all series are first-order integrated, hence, possess no long memory feature in levels of spreads, some weak evidence of long memory in the residuals is obtained after taking the first difference. Moreover, this research proposes to use the two-step procedure that involves fitting both ARIMA and ARFIMA models to estimate the long memory parameter of the government bond spreads in the euro area rather than relying on periodogram-based methods. This proposal is reasoned by the fact that the order of difference is not taken into account and that the confidence intervals of the methods based on the periodogram are significantly wider and include both the existence and absence of long memory cases.

The investigation shows that even if the market treats particular countries differently in certain periods, in general, the EA is considered a unitary club. The occurrence of shocks, for example, significant global events, such as financial or sovereign debt crises, may show different short-term expectations for the euro area members' abilities to remain stable and repay their debts. Despite the temporary variant treatment, the common belief of monetary union long-run stability in the EA is observed. The arising shocks do not contain the feature of the strong dependence between themselves. Thus, even if certain major events disproportionately impact one country or a group of countries, it does not necessarily mean that the upcoming shocks will be equally severe as prior ones for particular countries.

References

[1] Nikolaos Antonakakis, Christina Christou, Juncal Cunado, and Rangan Gupta. Convergence patterns in sovereign bond yield spreads: Evidence from the euro area. *Journal of International Financial Markets, Institutions and Money*, 49:129–139, 2017.

[2] Markéta Arltová and Darina Fedorová. Selection of unit root test on the basis of length of the time series and value of ar(1) parameter. *Statistika - Statistics and Economics Journal*, 96:47–64, 09 2016.

[3] Joseph E. Cavanaugh and Andrew A. Neath. The akaike information criterion: Background, derivation, properties, application, interpretation, and refinements. *WIREs Computational Statistics*, 11(3):e1460, 2019.

[4] Mauro Costantini and Ricardo M Sousa. What uncertainty does to euro area sovereign bond markets: Flight to safety and flight to quality. *Journal of International Money and Finance*, 122:102574, 2022.

[5] Kerui Du. Econometric convergence test and club clustering using stata. *The Stata Journal: Promoting communications on statistics and Stata*, 17:882–900, 01 2018.

[6] Carlo A Favero. Modelling and forecasting government bond spreads in the euro area: a gvar model. *Journal of Econometrics*, 177(2):343–356, 2013.

[7] John Geweke and Susan Porter-Hudak. The estimation and application of long memory time series models. *Journal of Time Series Analysis*, 4(4):221–238, 1983.

[8] Niels Haldrup and J. Eduardo Vera Valdés. Long memory, fractional integration, and cross-sectional aggregation. *Journal of Econometrics*, 199(1):1–11, 2017.

[9] Robert Hodrick and E.C. Prescott. Post-war us business cycles: An empirical investigation. *Journal of Money, Credit and Banking*, 8:231–247, 01 1981.

[10] Rob J. Hyndman and Yeasmin Khandakar. Automatic time series forecasting: The forecast package for r. *Journal of Statistical Software*, 27(3):1–22, 2008.

[11] Kris Ivanovski, Sefa Awaworyi Churchill, and Russell Smyth. A club convergence analysis of per capita energy consumption across australian regions and sectors. *Energy Economics*, 76:519–531, 2018.

[12] Vijay Kotu and Bala Deshpande. Chapter 12 - time series forecasting. In Vijay Kotu and Bala Deshpande, editors, *Data Science (Second Edition)*, pages 395–445. Morgan Kaufmann, second edition edition, 2019.

[13] Robinson Kruse and Christoph Wegener. Explosive behaviour and long memory with an application to european bond yield spreads. *Scottish Journal of Political Economy*, 66(1):139–153, 2019.

[14] Chingnun Lee and Fu-Shuen Shie. Fractional integration and the phillips-perron test. *Academia Economic Papers*, 32(2):273–312, 2004.

[15] Rizwan Mushtaq. Augmented dickey fuller test. *Econometrics: Mathematical Methods & Programming eJournal*, 2011.

[16] Peter C. B. Phillips and Donggyu Sul. Transition modeling and econometric convergence tests. *Econometrica*, 75(6):1771–1855, 2007.

[17] Lucio Ratta and Giovanni Urga. Modelling credit spread: A fractional integration approach. *SSRN Electronic Journal*, 08 2005.

[18] Valderio A. Reisen. Estimation of the fractional difference parameter in the arima(p, d, q) model using the smoothed periodogram. *Journal of Time Series Analysis*, 15(3):335–350, 1994.

[19] Philipp Sibbertsen, Christoph Wegener, and Tobias Basse. Testing for a break in the persistence in yield spreads of emu government bonds. *Journal of Banking & Finance*, 41:109–118, 2014.

[20] Roberto Sichera and Pietro Pizzuto. Convergenceclubs: A package for performing the phillips and sul's club convergence clustering procedure. *The R Journal*, 12 2019.

[21] Tatjana Stadnitski, Simone Braun, and Joachim Werner. Analyzing fractal dynamics employing r. *Nonlinear dynamics, psychology, and life sciences*, 14:117–44, 04 2010.

[22] Ewa M. Syczewska. Empirical power of the Kwiatkowski-Phillips-Schmidt-Shin test. Working Papers 45, Department of Applied Econometrics, Warsaw School of Economics, September 2010.

[23] Eric Zivot and Donald W. K Andrews. Further evidence on the great crash, the oil-price shock, and the unit-root hypothesis. *Journal of Business & Economic Statistics*, 20(1):25–44, 2002.

Appendices

A Some explanations of Phillips and Sul method

Notoriously, μ_t can have both stochastic and deterministic constituents. To account for both components, μ_t must fit both the convergence and divergence behaviour of each i. To achieve the desired result, Phillips and Sul [16] propose employing the standardization process. The essence is to standardize s_{it} and, respectively, the relevant variable μ_t acquires the standardized form.

Suppose v_{it} is the function that varies regularly at infinity and $f_i(t) = v_{it}$. Then $f_i(t)$ satisfies

$$f_i(t) = t^{\gamma_i} W_i(t), \tag{19}$$

where t^{γ_i} is power exponent and $W_i(t)$ is slowly varying function. In addition, $W_i(t)$ satisfies (8).

Similarly to $f_i(t)$, μ_t is denoted as

$$\mu_t = t^{\gamma} Z(t), \tag{20}$$

where $Z(t)$ is another slowly varying function.

In light of introduced denotations (19) and (20), the method holds for a more extensive diversity of asymptotic behaviour. Considering (19) and (20) equations, assume $t = [Tr]$ where T denotes the whole sample and r is the share of the sample T, then:

$$\frac{v_{it}}{T^{\gamma_i}} = \frac{f_i(t)}{T^{\gamma_i}} = \frac{t^{\gamma_i} W_i(t)}{T^{\gamma_i}} = \frac{[Tr]^{\gamma_i}}{T^{\gamma_i}} \frac{W_i(Tr)}{W_i(T)} W_i(T) \sim r^{\gamma_i} W_i(T) \tag{21}$$

and

$$\frac{\mu_t}{T^{\gamma}} = \frac{t^{\gamma} Z(t)}{T^{\gamma}} = \frac{[Tr]^{\gamma}}{T^{\gamma}} \frac{Z(Tr)}{Z(T)} Z(T) \sim r^{\gamma} Z(T) \tag{22}$$

In view of (21) and (22), the following equations are implemented:

$$\frac{s_{it}}{T^{\gamma_i}} = \frac{a_{it} + v_{it}}{T^{\gamma_i}} = \frac{a_{it}}{T^{\gamma_i}} + \frac{v_{it}}{T^{\gamma_i}} \sim r^{\gamma_i} W_i(T), \tag{23}$$

$$\frac{\mu_t}{T^{\gamma_i}} \sim r^{\gamma} Z(T). \tag{24}$$

From this point premise $r^{\gamma} = \mu(r)$. Considering (2) and (21)-(24) equations, the following one is derived:

$$\begin{aligned} \frac{s_{it}}{T^{\gamma_i}} &= \frac{(a_{it} + v_{it})\mu_t}{T^{\gamma_i}\mu_t} = \frac{a_{it}}{T^{\gamma_i}} + \frac{v_{it}}{T^{\gamma_i}} \frac{\mu_t}{T^{\gamma}} \frac{T^{\gamma}}{\mu_t} = o(1) + \frac{v_{it}}{T^{\gamma_i}} \frac{\mu_t}{T^{\gamma}} \frac{T^{\gamma}}{\mu_t} \\ &\sim r^{\gamma_i} W_i(T) r^{\gamma} Z(T) \frac{1}{r^{\gamma} Z(T)} \sim r^{\gamma_i - \gamma} \frac{W_i(T)}{Z(T)} r^{\gamma} Z(T) = r^{\gamma_i - \gamma} J_i(T) r^{\gamma} Z(T), \end{aligned}$$

where $J_i(t) = \frac{W_i(T)}{Z(T)}$. Presume the constituent $r^{\gamma_i - \gamma} J_i(T)$ as $\beta_{iT}^J(r)$ and $r^{\gamma} Z(T)$ as $\mu_T^Z(r)$. Since $J_i(t)$ is a slowly varying function, then both $\beta_{iT}^J(r)$ and $\mu_T^Z(r)$ are regularly varying functions and act asymptotically like power exponents $r^{\gamma_i - \gamma}$ and r^{γ} respectively.

Taking into consideration the standardization factor $d_{iT} = T^{\gamma_i} W_i(t)$, where $\gamma_i > 0$, the standardized form of s_{it} is obtained in the following way:

$$
\begin{aligned}
\frac{1}{d_{iT}} s_{it} &= \frac{1}{T^{\gamma_i} W_i(T)} \left(\frac{a_{it} + v_{it}}{\mu_t} \right) \mu_t = \frac{a_{it}}{T^{\gamma_i} W_i(T) \mu_t} \mu_t + \frac{v_{it}}{T^{\gamma_i} W_i(T) \mu_t} \mu_t \\
&= \frac{a_{it}}{T^{\gamma_i} W_i(T)} + \frac{v_{it}}{T^{\gamma_i} W_i(T)} \frac{\mu_t}{T^{\gamma} Z(T)} \frac{T^{\gamma} Z(T)}{\mu_t} \\
&= o(1) + \frac{t^{\gamma_i} W_i(t)}{T^{\gamma_i} W_i(T)} \frac{T^{\gamma} Z(T)}{\mu_t} \frac{\mu_t}{Z(T) T^{\gamma}} = \\
&= o(1) + \left(\frac{t}{T} \right)^{\gamma_i - \gamma} \frac{W_i\left(\frac{tT}{T}\right)}{W_i(T)} \frac{Z(T)}{Z\left(\frac{tT}{T}\right)} \left(\frac{t}{T} \right)^{\gamma} \frac{Z\left(\frac{tT}{T}\right)}{Z(T)} \\
&\sim \beta_{iT}\left(\frac{t}{T}\right) \mu_T \left(\frac{t}{T}\right),
\end{aligned}
\tag{25}
$$

where

$$
\beta_{iT}\left(\frac{t}{T}\right) = \left(\frac{t}{T}\right)^{\gamma_i - \gamma} \frac{W_i\left(\frac{tT}{T}\right)}{W_i(T)} \frac{Z(T)}{Z\left(\frac{tT}{T}\right)}
$$

and

$$
\mu_T\left(\frac{t}{T}\right) = \left(\frac{t}{T}\right)^{\gamma} \frac{Z\left(\frac{tT}{T}\right)}{Z(T)}.
$$

As formerly suppose $t = [Tr]$, then:

$$
\beta_{iT}\left(\frac{t}{T}\right) = \beta_{iT}\left(\frac{Tr}{T}\right) = \left(\frac{Tr}{T}\right)^{\gamma_i - \gamma} \frac{W_i(Tr)}{W_i(T)} \frac{Z(T)}{Z(Tr)} \sim r^{\gamma_i - \gamma} = \beta_i(r)
$$

as well as

$$
\mu_T\left(\frac{t}{T}\right) = \mu_T\left(\frac{Tr}{T}\right) = \left(\frac{Tr}{T}\right)^{\gamma} \frac{Z(Tr)}{Z(T)} \sim r^{\gamma} = \mu(r).
$$

The above-stated expressions of the relative transition parameter and common trend component apply only to the non-stochastic version. To consider the stochastic features, a standardized form of s_{it} arises using $o_p(1)$ error uniformly in $t \leqslant T$ so that the following conditions hold:

$$
\begin{aligned}
\beta_i T(r) &\underset{p}{\rightarrow} r^{\gamma_i - \gamma} = \beta_i(r), \\
\mu_T(r) &\underset{p}{\rightarrow} r^{\gamma} = \mu(r).
\end{aligned}
\tag{26}
$$

Referring to the views of (25) and (26), the following conclusion is drawn:

$$
\frac{1}{d_{iT}} s_{iT} \rightarrow \beta_i(r)\mu(r) = s_i(r).
$$

To account for asymptotic relative transition paths, the next step is to deduce the asymptotic behaviour of the relative transition parameter ψ_{it}. In light of (4) and (25), the following adjustments of ψ_{it} are fitted:

$$\psi_{iT}\left(\frac{t}{T}\right) = \frac{\frac{s_{it}}{d_{iT}}}{\frac{1}{n}\sum_{j=1}^{n}\left(\frac{s_{jt}}{d_{jT}}\right)} = \frac{\beta_{iT}\left(\frac{t}{T}\right)}{\frac{1}{n}\sum_{j=1}^{n}\beta_{iT}\left(\frac{t}{T}\right)}. \tag{27}$$

Recurrently, presume $t = [Tr]$, then:

$$\psi_{iT}\left(\frac{[Tr]}{T}\right) \underset{p}{\rightarrow} \frac{\beta_i(r)}{\frac{1}{n}\sum_{j=1}^{n}\beta_i(r)} = \psi_i(r), \text{ when } T \rightarrow \infty. \tag{28}$$

The elimination of the business cycle component in the initial data is another key point in the investigation of the long-run equilibrium. To recall, the s_t has two components: μ_t and β_{it}. (2) can be extended by adding the business cycle component ω_{it} so that:

$$s_{it} = \beta_{it}\mu_t + \omega_{it}. \tag{29}$$

To remove the effect of the business cycle, the most popular and appropriate way is to use smoothing methods. One of the most often used methods of smoothing is the Whittaker-Hodrick-Prescott smoothing filter, in which the nature of μ_t and s_{it} does not have to be specified previously in order to use this tool. To simplify, the (29) equation consists of two parts: the first constituent $\beta_{it}\mu_t$ embodies the trend information while the second one, ω_{it}, depicts the business effects. By using Whittaker-Hodrick-Prescott smoothing filter, the trend estimate is calculated as $\hat{\theta}_{it} = \widehat{v_{it}\mu_t}$. The following expression of $\hat{\theta}_{it}$ is derived:

$$\hat{\theta}_{it} = \theta_{it} + \epsilon_{it} = \left(\beta_{it} + \frac{\epsilon_{it}}{\mu_t}\right)\mu_t, \tag{30}$$

where the error in the filter estimate is denoted as ϵ_{it} and θ_{it} is the trend. The relative transition path ψ_{it} is estimated as $\hat{h}_i(r) = \hat{h}_{i[Tr]}$ when both a common standardization and $t = [Tr]$ conditions are implemented. As Phillips and Sul [16] indicates, the condition $\frac{\epsilon_{it}}{\mu_t} \underset{p}{\rightarrow} 0$ uniformly in i, is sufficiently reasoned to use in this case, so that when $T \rightarrow \infty$:

$$\hat{\psi}_i(r) = \frac{\left(\beta_{i[Tr]} + \frac{\epsilon_{i[Tr]}}{\mu_{[Tr]}}\right)}{\frac{1}{n}\sum_{i=1}^{n}\left(\beta_{j[Tr]} + \frac{\epsilon_{j[Tr]}}{\mu_{[Tr]}}\right)} = \frac{\beta_{iT}\left(\frac{t}{T}\right)}{\frac{1}{n}\sum_{j=1}^{n}\beta_{iT}\left(\frac{t}{T}\right)} + o_p(1) \underset{p}{\rightarrow} \frac{\beta_i(r)}{\frac{1}{n}\sum_{j=1}^{n}\beta_i(r)}. \tag{31}$$

To conclude, the approximately calculated value of the transition path is sufficient to estimate the relative transition parameter.

B Country codes

Code	Country
AT	Austria
BE	Belgium
DE	Germany
FI	Finland
FR	France
GR	Greece
IE	Ireland
IT	Italy
LV	Latvia
LT	Lithuania
NL	Netherlands
PT	Portugal
SK	Slovakia
ES	Spain

C The selection of Hodrick Prescott's smoothing parameter


Hodrick-Prescott Filter of BE
2003-01-01 / 2022-11-24
300
200
100
0
Jan 01 2003
Jul 01 2004
Jan 02 2006
Jul 02 2007
Jan 01 2009
Jul 01 2010
Jan 02 2012
Jul 01 2013
Jan 01 2015
Jul 01 2016
Jan 01 2018
Jul 01 2019
Jan 01 2021
Jun 30 2022


Hodrick-Prescott Filter of FI
2003-01-01 / 2022-11-24
80
60
40
20
0
Jan 01 2003
Jul 01 2004
Jan 02 2006
Jul 02 2007
Jan 01 2009
Jul 01 2010
Jan 02 2012
Jul 01 2013
Jan 01 2015
Jul 01 2016
Jan 01 2018
Jul 01 2019
Jan 01 2021
Jun 30 2022


Hodrick-Prescott Filter of FR
2003-01-01 / 2022-11-24
150
100
50
0
Jan 01 2003
Jul 01 2004
Jan 02 2006
Jul 02 2007
Jan 01 2009
Jul 01 2010
Jan 02 2012
Jul 01 2013
Jan 01 2015
Jul 01 2016
Jan 01 2018
Jul 01 2019
Jan 01 2021
Jun 30 2022


Hodrick-Prescott Filter of GR
2003-01-01 / 2022-11-24
3000
2500
2000
1500
1000
500
0
Jan 01 2003
Jul 01 2004
Jan 02 2006
Jul 02 2007
Jan 01 2009
Jul 01 2010
Jan 02 2012
Jul 01 2013
Jan 01 2015
Jul 01 2016
Jan 01 2018
Jul 01 2019
Jan 01 2021
Jun 30 2022


Hodrick-Prescott Filter of IE
2003-01-01 / 2022-11-24
1000
800
600
400
200
0
Jan 01 2003
Jul 01 2004
Jan 02 2006
Jul 02 2007
Jan 01 2009
Jul 01 2010
Jan 02 2012
Jul 01 2013
Jan 01 2015
Jul 01 2016
Jan 01 2018
Jul 01 2019
Jan 01 2021
Jun 30 2022


Hodrick-Prescott Filter of IT
2003-01-01 / 2022-11-24
500
400
300
200
100
Jan 01 2003
Jul 01 2004
Jan 02 2006
Jul 02 2007
Jan 01 2009
Jul 01 2010
Jan 02 2012
Jul 01 2013
Jan 01 2015
Jul 01 2016
Jan 01 2018
Jul 01 2019
Jan 01 2021
Jun 30 2022


Hodrick-Prescott Filter of LV
2003-01-01 / 2022-11-24
800
600
400
200
Jan 01 2003
Jul 01 2004
Jan 02 2006
Jul 02 2007
Jan 01 2009
Jul 01 2010
Jan 02 2012
Jul 01 2013
Jan 01 2015
Jul 01 2016
Jan 01 2018
Jul 01 2019
Jan 01 2021
Jun 30 2022


Hodrick-Prescott Filter of LT
2003-01-01 / 2022-11-24
800
600
400
200
Jan 01 2003
Jul 01 2004
Jan 02 2006
Jul 02 2007
Jan 01 2009
Jul 01 2010
Jan 02 2012
Jul 01 2013
Jan 01 2015
Jul 01 2016
Jan 01 2018
Jul 01 2019
Jan 01 2021
Jun 30 2022


Hodrick-Prescott Filter of NL
2003-01-01 / 2022-11-24
80
60
40
20
0
Jan 01 2003
Jul 01 2004
Jan 02 2006
Jul 02 2007
Jan 01 2009
Jul 01 2010
Jan 02 2012
Jul 01 2013
Jan 01 2015
Jul 01 2016
Jan 01 2018
Jul 01 2019
Jan 01 2021
Jun 30 2022


Hodrick-Prescott Filter of PT
2003-01-01 / 2022-11-24
1000
500
0
Jan 01 2003
Jul 01 2004
Jan 02 2006
Jul 02 2007
Jan 01 2009
Jul 01 2010
Jan 02 2012
Jul 01 2013
Jan 01 2015
Jul 01 2016
Jan 01 2018
Jul 01 2019
Jan 01 2021
Jun 30 2022


Hodrick-Prescott Filter of SK
2003-01-01 / 2022-11-24
300
250
200
150
100
50
0
Jan 01 2003
Jul 01 2004
Jan 02 2006
Jul 02 2007
Jan 01 2009
Jul 01 2010
Jan 02 2012
Jul 01 2013
Jan 01 2015
Jul 01 2016
Jan 01 2018
Jul 01 2019
Jan 01 2021
Jun 30 2022


Hodrick-Prescott Filter of AT
2003-01-01 / 2022-11-24
150
100
50
0
Jan 01 2003
Jul 01 2004
Jan 02 2006
Jul 02 2007
Jan 01 2009
Jul 01 2010
Jan 02 2012
Jul 01 2013
Jan 01 2015
Jul 01 2016
Jan 01 2018
Jul 01 2019
Jan 01 2021
Jun 30 2022

D Autocorrelation functions

D.1 Initial spreads

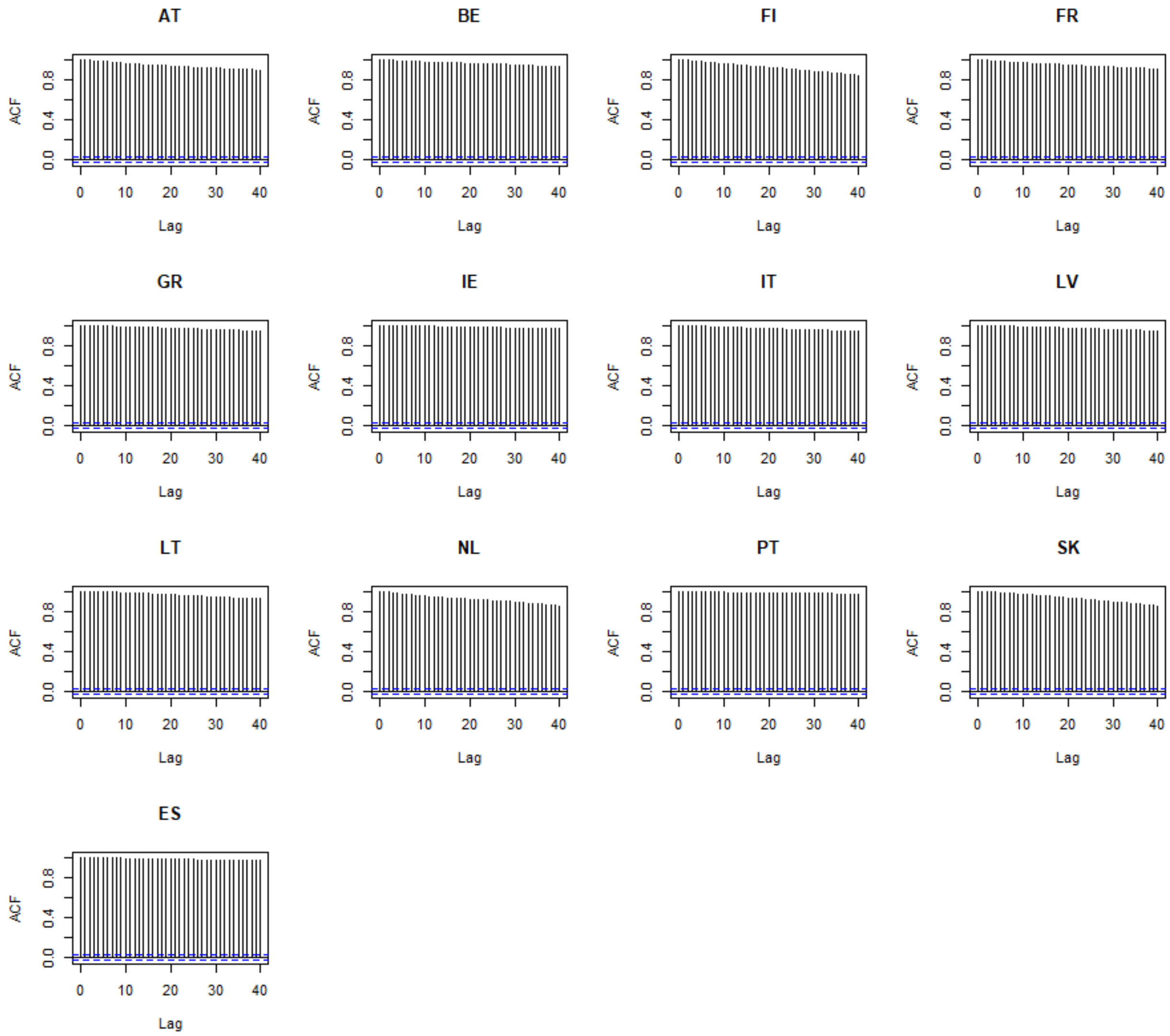

Figure 11: Autocorrelation functions for spreads

D.2 First-order differenced spreads

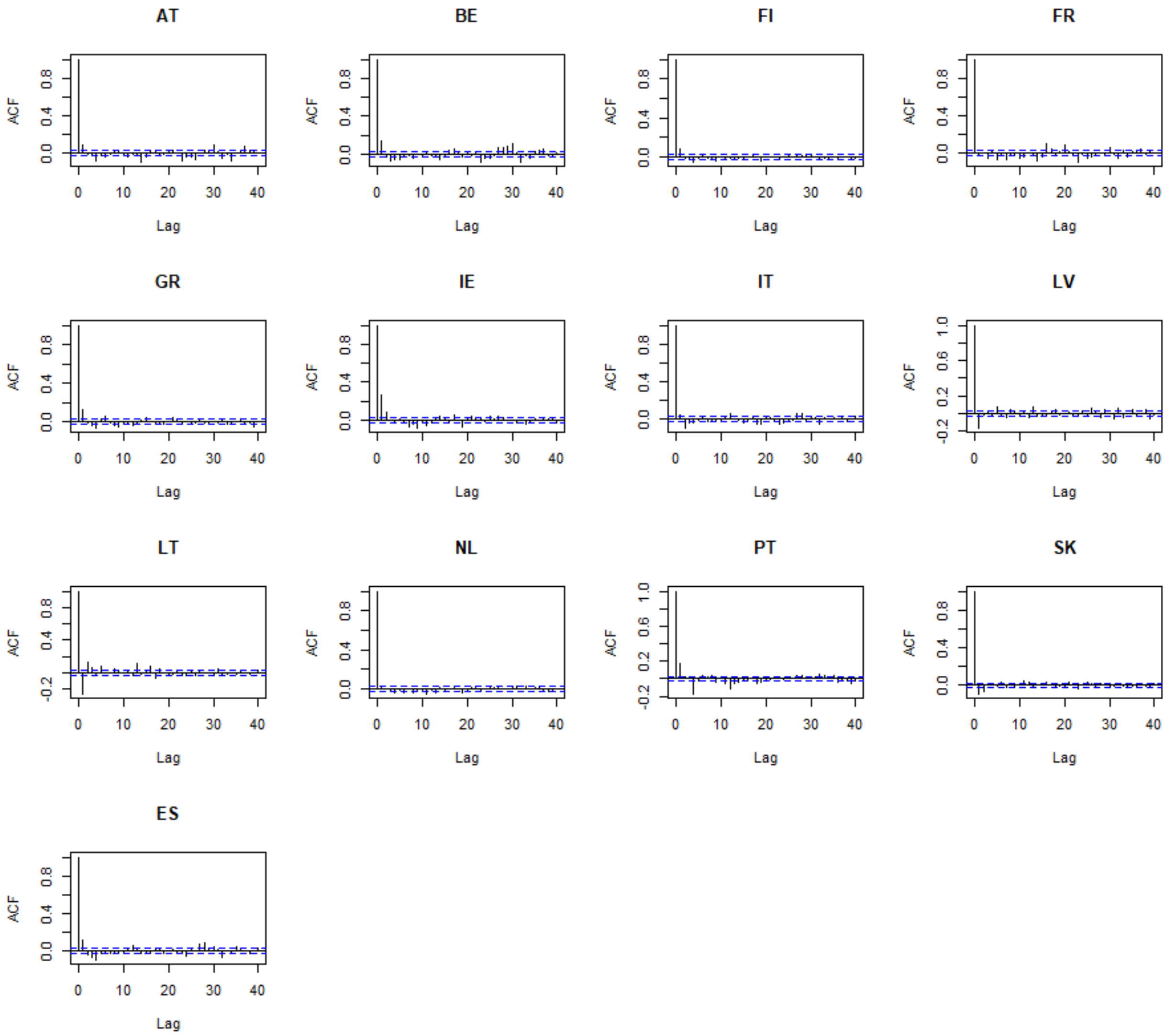

Figure 12: Autocorrelation functions for the first-order differenced spreads

E R code

R code

Ineta Beriozovaitė

2023-01-07

```
library(ConvergenceClubs)
library(mFilter)
library(readxl)
library(arfima)
library(fracdiff)
library(forecast)
library(stats)
library(lmtest)
library(tseries)
library(data.table)
library(urca)
library(psych)
library(FCVAR)
library(urca)
library(smoots)
library(forecast)
library(xtable)
yields <- read_excel("bloomberg_data_nuo_2003_be _cyprus.xlsx")
yields <- data.frame(yields)
yields <- yields[(weekdays(yields$Date) != c('Saturday') &
                    weekdays(yields$Date) != c('Sunday')),]
spreads <- data.frame(yields$Date,
                      (yields$Austria-yields$Germany)*100,
                      (yields$Belgium-yields$Germany)*100,
                      (yields$Finland - yields$Germany)*100,
                      (yields$France - yields$Germany)*100,
                      (yields$Greece - yields$Germany)*100,
                      (yields$Ireland - yields$Germany)*100,
                      (yields$Italy - yields$Germany)*100,
                      (yields$Latvia - yields$Germany)*100,
                      (yields$Lithuania-yields$Germany)*100,
                      (yields$Netherlands - yields$Germany)*100,
                      (yields$Portugal-yields$Germany)*100,
                      (yields$Slovakia-yields$Germany)*100,
                      (yields$Spain-yields$Germany)*100)
colnames(spreads) <- c("Date",  "Austria", "Belgium", "Finland", "France",
                       "Greece", "Ireland", "Italy", "Latvia", "Lithuania",
                       "Netherlands", "Portugal", "Slovakia", "Spain")

spreads1 <- spreads[,2:11]
spreads2 <- spreads[,12:14]
par(mfrow =c(2,1))
```

```
plot.ts(spreads1,plot.type = "multiple", col = "black")
plot.ts(spreads2,plot.type = "multiple", col = "black")

#spreads_2003-2022
spreads_euro_area <- as.data.frame(t(sapply(spreads[,-1], as.numeric)))
countries<-rownames(spreads_euro_area)
rownames(spreads_euro_area) <- NULL
colnames(spreads_euro_area) <- spreads[,1]
filteredspreads_euro_area <- apply(spreads_euro_area, 1,
                                   function(x){mFilter::hpfilter(x,freq=30,
                                                                 type="frequency")$trend})
filteredspreads_euro_area <- data.frame(Countries = countries,
                                        t(filteredspreads_euro_area),
                                        stringsAsFactors=FALSE )
colnames(filteredspreads_euro_area) <- colnames(spreads_euro_area)
H_euro_area <- computeH(filteredspreads_euro_area[,-1], quantity = "H")
round(estimateMod(H_euro_area, time_trim=1/3, HACmethod = "FQSB"), 3)
clubs_euro_area <- findClubs(filteredspreads_euro_area, dataCols=2:5193,
                             unit_names = 1, refCol=5193, time_trim=1/3, cstar=0,
                             HACmethod = 'FQSB')
summary(clubs_euro_area)
clubs_euro_area
plot(clubs_euro_area, avgTP = FALSE, legend = TRUE,
     plot_args=list(xlabs=c("01/2003","12/2004","11/2006","10/2008","09/2010",
                            "08/2012","07/2014","06/2016","05/2018","04/2020",
                            "03/2022"), lty = 6, lwd = 2),
     legend_args = list(lwd = 2))
mclubs_euro_area <- mergeClubs(clubs_euro_area, mergeMethod='PS')
summary(mclubs_euro_area)

#spreads_2003-2008
spreads_2003_2008 <- spreads[match(as.POSIXct("2003-01-01 00:00",
                                              tz = "UTC"), spreads$Date):
                               match(as.POSIXct("2008-12-31 00:00",
                                                tz = "UTC"), spreads$Date), ]
spreads_2003_2008 <- as.data.frame(t(sapply(spreads_2003_2008[,-1],
                                            as.numeric)))
countries<-rownames(spreads_2003_2008)
rownames(spreads_2003_2008) <- NULL
colnames(spreads_2003_2008) <- spreads[match(as.POSIXct("2003-01-01 00:00",
                                                        tz = "UTC"),
                                             spreads$Date):
                                         match(as.POSIXct("2008-12-31 00:00",
                                                          tz = "UTC"),
                                               spreads$Date),1]
filteredspreads_2003_2008 <- apply(spreads_2003_2008, 1,
                                   function(x){mFilter::hpfilter(x, freq=30,
                                                                 type="frequency")$trend})
filteredspreads_2003_2008 <- data.frame(Countries = countries,
                                        t(filteredspreads_2003_2008),
                                        stringsAsFactors=FALSE )
colnames(filteredspreads_2003_2008) <- c("Countries",
                                         colnames(spreads_2003_2008))
```

```
H_2003_2008 <- computeH(filteredspreads_2003_2008[,-1], quantity = "H")
describe(head(H_2003_2008))
round(estimateMod(H_2003_2008, time_trim=1/3, HACmethod = "FQSB"), 3)

clubs_2003_2008 <- findClubs(filteredspreads_2003_2008, dataCols=2:1567,
                             unit_names = 1, refCol=1567, time_trim=1/3, cstar=0,
                             HACmethod = 'FQSB')
summary(clubs_2003_2008)
clubs_2003_2008
plot(clubs_2003_2008, avgTP = FALSE, legend = TRUE,
     plot_args=list(xlabs=c("01/2003","01/2005","01/2007","09/2008"), lty = 6, lwd = 2),
     legend_args = list(lwd = 2))

#spreads_2009-2015
spreads_2009_2015 <- spreads[match(as.POSIXct("2009-01-01 00:00",
                                              tz = "UTC"), spreads$Date):
                               match(as.POSIXct("2015-06-01 00:00",
                                                tz = "UTC"), spreads$Date), ]

spreads_2009_2015 <- as.data.frame(t(sapply(spreads_2009_2015[,-1],
                                            as.numeric)))
countries<-rownames(spreads_2009_2015)
rownames(spreads_2009_2015) <- NULL
colnames(spreads_2009_2015) <- spreads[match(as.POSIXct("2009-01-01 00:00",
                                                        tz = "UTC"),
                                             spreads$Date):
                                         match(as.POSIXct("2015-06-01 00:00",
                                                          tz = "UTC"),
                                               spreads$Date),1]

filteredspreads_2009_2015 <- apply(spreads_2009_2015,1,
                                   function(x){mFilter::hpfilter(x,freq=30,
                                                                 type="frequency")$trend})
filteredspreads_2009_2015 <- data.frame(Countries = countries,
                                        t(filteredspreads_2009_2015),
                                        stringsAsFactors=FALSE )
colnames(filteredspreads_2009_2015) <- c("Countries",
                                         colnames(spreads_2009_2015))
H_2009_2015 <- computeH(filteredspreads_2009_2015[,-1], quantity = "H")
round(estimateMod(H_2009_2015, time_trim=1/3, HACmethod = "FQSB"), 3)
clubs_2009_2015 <- findClubs(filteredspreads_2009_2015, dataCols=2:1674,
                             unit_names = 1, refCol=1674, time_trim=1/3, cstar=0,
                             HACmethod = 'FQSB')
summary(clubs_2009_2015)
clubs_2009_2015
t(data.frame(clubs_2009_2015$club1$unit_names, clubs_2009_2015$divergent$unit_names))[1,]
plot(clubs_2009_2015, avgTP = FALSE, legend = TRUE,
     plot_args=list(xlabs=c("01/2009","01/2011","01/2013","09/2014"), lty = 6, lwd = 2),
     legend_args = list(lwd = 2))
mclubs_2009_2015 <- mergeClubs(clubs_2009_2015, mergeMethod='PS')
summary(clubs_2009_2015)

#spreads_2015-2022
```

```r
spreads_2015_2022 <- spreads[match(as.POSIXct("2015-06-02 00:00",
                                              tz = "UTC"), spreads$Date):
                               match(as.POSIXct("2022-11-24 00:00",
                                                tz = "UTC"), spreads$Date), ]
spreads_2015_2022 <- as.data.frame(t(sapply(spreads_2015_2022[,-1],
                                            as.numeric)))
countries<-rownames(spreads_2015_2022)
rownames(spreads_2015_2022) <- NULL
colnames(spreads_2015_2022) <- spreads[match(as.POSIXct("2015-06-02 00:00",
                                                        tz = "UTC"),
                                             spreads$Date):
                                         match(as.POSIXct("2022-11-24 00:00",
                                                          tz = "UTC"),
                                               spreads$Date),1]
filteredspreads_2015_2022 <- apply(spreads_2015_2022, 1,
                                   function(x){mFilter::hpfilter(x, freq=30,
                                                                 type="frequency")$trend})
filteredspreads_2015_2022 <- data.frame(Countries = countries,
                                        t(filteredspreads_2015_2022),
                                        stringsAsFactors=FALSE )
colnames(filteredspreads_2015_2022) <- c("Countries",
                                         colnames(spreads_2015_2022))
H_2015_2022 <- computeH(filteredspreads_2015_2022[,-1], quantity = "H")
round(estimateMod(H_2015_2022, time_trim=1/3, HACmethod = "FQSB"), 3)
clubs_2015_2022 <- findClubs(filteredspreads_2015_2022, dataCols=2:1954,
                             unit_names = 1, refCol=1954, time_trim=1/3, cstar=0,
                             HACmethod = 'FQSB')
summary(clubs_2015_2022)
clubs_2015_2022
plot(clubs_2015_2022, avgTP = FALSE, legend = TRUE,
     plot_args=list(xlabs=c("06/2015","04/2017","01/2019","12/2020"), lty = 6, lwd = 2),
     legend_args = list(lwd = 2))
#ARFIMA
AT <- spreads[,2]
BE <- spreads[,3]
FI <- spreads[,4]
FR <- spreads[,5]
GR <- spreads[,6]
IE <- spreads[,7]
IT <- spreads[,8]
LV <- spreads[,9]
LT <- spreads[,10]
NL <- spreads[,11]
PT <- spreads[,12]
SK <- spreads[,13]
ES <- spreads[,14]

#Unit root tests
#The Augmented Dickey and Fuller's (1981) test
adftest <- function(y){
  adf_p_value <- adf.test(y)$p.value
  return(adf_p_value)
}
```

```r
adftest_pvalues<-apply(spreads[,2:14], 2, adftest)

#Phillips and Perron's (1988) test.
pptest <- function(y){
  pp_p_value <- pp.test(y)$p.value
  return(pp_p_value)
}
pptest_pvalues<-apply(spreads[,2:14], 2, pptest)

#Kwiatkowski et al.'s (1992) test
kpsstest <- function(y){
  kpss_p_value <- kpss.test(y)$p.value
  return(kpss_p_value)
}
kpsstest_pvalues<-apply(spreads[,2:14], 2, kpsstest)

unitroots <- data.frame(adftest_pvalues, pptest_pvalues, kpsstest_pvalues)
colnames(unitroots) <- c("ADF", "PP", "KPSS")
#unitroots

#Zivot and Andrews Unit Root Test
zivotandrewstest <- function(y){
  zivotandrews <- ur.za(y)
  zivotandrews_teststatistic <- zivotandrews@teststat
  zivotandrews_criticalvalues <- zivotandrews@cval
  return(c(zivotandrews_teststatistic, zivotandrews_criticalvalues))
}
zivotandrewstest_results<-apply(spreads[,2:14], 2, zivotandrewstest)
zivotandrewstest_results
zivotandrewstest_results

Tstatistic <- data.frame(zivotandrewstest_results[1,])
colnames(Tstatistic) <- c("Zivot & Andrews test statistic")
critical_values <- data.frame(zivotandrewstest_results[2,1],
                              zivotandrewstest_results[3,1],
                              zivotandrewstest_results[4,1])
rownames(critical_values) <- NULL
colnames(critical_values) <- c("Critical value (1%)","Critical value(5%)",
                               "Critical value(10%)")

#The Augmented Dickey and Fuller's (1981) test
adftest_diff <- function(y){
  y <- diff(y)
  adf_p_value_diff <- adf.test(y)$p.value
  return(adf_p_value_diff)
}
adftestdiff_pvalues<-apply(spreads[,2:14], 2, adftest_diff)

#Phillips and Perron's (1988) test.
pptest_diff <- function(y){
  y <- diff(y)
  pp_p_value_diff <- pp.test(y)$p.value
  return(pp_p_value_diff)
```

```
}
pptest_pvalues_diff<-apply(spreads[,2:14], 2, pptest_diff)

#Kwiatkowski et al.'s (1992) test
kpsstest_diff <- function(y){
  y <- diff(y)
  kpss_p_value_diff <- kpss.test(y)$p.value
  return(kpss_p_value_diff)
}
kpsstest_pvalues_diff<-apply(spreads[,2:14], 2, kpsstest_diff)

#Zivot and Andrews Unit Root Test
zivotandrewstest_diff <- function(y){
  y <- diff(y)
  zivotandrews_diff <- ur.za(y)
  zivotandrews_teststatistic_diff <- zivotandrews_diff@teststat
  zivotandrews_criticalvalues_diff <- zivotandrews_diff@cval
  return(c(zivotandrews_teststatistic_diff, zivotandrews_criticalvalues_diff))
}
zivotandrewstest_results_diff<-apply(spreads[,2:14], 2, zivotandrewstest_diff)
Tstatistic_diff <- data.frame(zivotandrewstest_results_diff[1,])
colnames(Tstatistic_diff) <- c("Zivot & Andrews test statistic")

unitroots_diff <- data.frame(adftestdiff_pvalues, pptest_pvalues_diff,
                             kpsstest_pvalues_diff, Tstatistic_diff)
colnames(unitroots_diff) <- c("ADF", "PP", "KPSS", "ZA test statistic")
unitroots_diff

autoarima_arfima <- function(y){
  coeffic <- c(forecast::auto.arima(y, approximation = FALSE,
                                    stepwise=FALSE, ic = c("aic"),
                                    stationary = FALSE, test = c("kpss"))$arma)
  pdq <- c(coeffic[1], coeffic[6], coeffic[2])
  fit <- arfima::arfima(y, order = c(coeffic[1],coeffic[6],coeffic[2]), useC=3)
  cov<-vcov(fit, type = "o")
  cov<- cov$`Mode 1`$observed
  se <- sqrt(diag(cov))
  t <- 1.96
  dfrac <- fit$modes[[1]][9][[1]]
  CI_upper <- fit$modes[[1]][9][[1]] + t*se
  CI_lower <- fit$modes[[1]][9][[1]] - t*se
  CI_upper <- tail(CI_upper, 2)[-2]
  CI_lower <- tail(CI_lower, 2)[-2]
  return(c(pdq, dfrac, CI_lower, CI_upper))
}
arima_arfima <- apply(spreads[,2:14], 2, autoarima_arfima)
results <- t(data.frame(arima_arfima))
results
colnames(results) <- c("ARIMA p", "ARIMA d", "ARIMA q", "ARFIMA d",
                       "Lower bound of 95% CI", "Upper bound of 95% CI")
results[,4] <- results[,4] + 1
results[,5] <- results[,5] + 1
results[,6] <- results[,6] + 1
```

```
results
#GPH
GPH <- function(y){
  GPHfit <-  fdGPH(y,bandw.exp=0.5)
  se<-GPHfit$sd.reg
  t<- 1.96
  upper<- GPHfit$d + t*se
  lower<- GPHfit$d - t*se
  return(c(GPHfit$d, lower, upper))
}
GPH_res <- apply(spreads[,2:14], 2, GPH)
GPH_results <- t(data.frame(GPH_res))
colnames(GPH_results) <- c("GPH d", " 95% CI",  "95% CI")

SPERIO <- function(y){
  Speriofit <-  fdSperio(y,bandw.exp=0.5)
  se<-Speriofit$sd.reg
  t<- 1.96
  upper<- Speriofit$d + t*se
  lower<- Speriofit$d - t*se
  return(c(Speriofit$d, lower, upper))
}
Sperio_res <- apply(spreads[,2:14], 2, SPERIO)
Sperio_results <- t(data.frame(Sperio_res))
colnames(Sperio_results) <- c("Sperio d", "Sperio 95% CI",  "Sperio 95% CI")

GPH_diff <- function(y){
  GPHfit <-  fdGPH(diff(y),bandw.exp=0.5)
  se<-GPHfit$sd.reg
  t<- 1.96
  upper<- GPHfit$d + t*se
  lower<- GPHfit$d - t*se
  return(c(GPHfit$d, lower, upper))
}
GPH_res_diff <- apply(spreads[,2:14], 2, GPH_diff)
GPH_results_diff <- t(data.frame(GPH_res_diff))
colnames(GPH_results_diff) <- c("GPH d", " 95% CI",  "95% CI")

dtfr<- data.frame(GPH_results, Sperio_results)
colnames(dtfr) <- c("GPH d","GPH 95% CI","GPH 95% CI","Sperio d",
                    "Sperio 95% CI","Sperio 95% CI")

SPERIO_diff <- function(y){
  Speriofit <-  fdSperio(diff(y),bandw.exp=0.5)
  se<-Speriofit$sd.reg
  t<- 1.96
  upper<- Speriofit$d + t*se
  lower<- Speriofit$d - t*se
  return(c(Speriofit$d, lower, upper))
}
Sperio_res_diff <- apply(spreads[,2:14], 2, SPERIO_diff)
Sperio_results_diff <- t(data.frame(Sperio_res_diff))
colnames(Sperio_results_diff) <- c("Sperio d", "Sperio 95% CI",  "Sperio 95% CI")
```

```
Sperio_results_diff
dtfr_diff<- data.frame(GPH_results_diff, Sperio_results_diff)
colnames(dtfr_diff) <- c("GPH d","GPH 95% CI","GPH 95% CI","Sperio d",
                    "Sperio 95% CI","Sperio 95% CI")
dtfr_diff
```