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Nonlocal optical potential with core excitation in $^{10}\mathrm{Be}(d,p)^{11}\mathrm{Be}$ and $^{11}\mathrm{Be}(p,d)^{10}\mathrm{Be}$ reactions

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ABSTRACT

We propose a new nonlocal form of the nucleon-nucleus optical potential and demonstrate its reliability. We extend the nonlocal potential to include the excitation of the nuclear core and develop energy-independent proton-^{10}Be potential reasonably reproducing the experimental data at low energies. We apply the new potential to the study of deuteron stripping and pickup reactions $^{10}\mathrm{Be}(d,p)^{11}\mathrm{Be}$ and $^{11}\mathrm{Be}(p,d)^{10}\mathrm{Be}$ using rigorous three-body Faddeev-type equations for transition operators that are solved in the momentum-space partial-wave framework. The achieved description of the experimental data is considerably more successful as compared to previous studies with local potentials. The values of spectroscopic factors consistent with the data are determined, exhibiting only weak energy dependence. The results possibly indicate an increased predicting power of the proposed calculational scheme.

1. Introduction

Deuteron pickup and stripping reactions $^{A+1}\mathrm{Z}(p,d)^{A}\mathrm{Z}$ and $^{A}\mathrm{Z}(d,p)^{A+1}\mathrm{Z}$ are extensively used to extract the information on structure of the involved nucleus $^{A+1}$Z, that can be either stable such as ^{17}O or an exotic short-living nucleus such as ^{11}Be. Since these reactions are dominated by three-body degrees of freedom, their theoretical calculations rely on three-body approaches for the system consisting of a proton (p), a neutron (n), and a nuclear core ^{A}Z. The simplest ones and therefore most widely used are the distorted-wave Born approximation (DWBA) and adiabatic distorted-wave approximation (ADWA) reviewed in Ref. [1]. Typical dynamics, mainly due to the calculational simplicity, is assumed to be determined by local potentials between nucleons and the nuclear core, either real (for bound states) or complex (for scattering states). The uncertainties due to an approximate solution of the three-body problem have been overcome by rigorous three-body Faddeev scattering theory [2] in the version of integral equations for the transition operators proposed by Alt, Grassberger and Sandhas [3]. Since it was practically implemented in the momentum-space framework [4], it was possible to include the potentials that are nonlocal in the coordinate space. Those pioneering calculations [4] revealed important effects of the optical potential nonlocality in $^{A+1}\mathrm{Z}(p,d)^{A}\mathrm{Z}$ and $^{A}\mathrm{Z}(d,p)^{A+1}\mathrm{Z}$ reactions and thereby triggered the developments of the DWBA and ADWA approaches to include nonlocal optical potentials [5,6]. The ADWA extension [5,7] claimed even more spectacular nonlocality effects like an enhancement of the differential cross section by a factor of two in particular cases and induced strong sensitivity to the neutron-proton potential model [7]. However, more refined Faddeev-type [8] and continuum-discretized coupled-channel (CDCC) calculations [9] have not confirmed those findings, indicating unreliability of the ADWA in the context of nonlocal optical potentials.

Another important dynamic ingredient in few-body nuclear reactions is the excitation of the nuclear core ^{A}Z. In standard reaction approaches the core excitation (CeX) as well as other inelastic processes are accounted for implicitly via optical potentials. Explicit inclusion of the CeX generates coupling potentials and states with several components. When the deuteron stripping and pickup reactions are analyzed using standard DWBA and ADWA approaches, the different components are assumed to participate in the reaction independently, and the differential cross section for a given transfer channel simply scales with the respective spectroscopic factor (SF), the weight of the respective component in the bound core plus neutron system. However, the Faddeev-type calculations extended for the dynamical excitation of the nuclear core [10] proved for a number of cases like $^{10}\mathrm{Be}(d,p)^{11}\mathrm{Be}$ and $^{20}\mathrm{O}(d,p)^{21}\mathrm{O}$ that the SF factorization in the differential cross section is a good approximation at low energies only, roughly up to 10 MeV per nucleon, while the deviation from the SF factorization as-

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sumption increases with increasing reaction energy. Furthermore, the direction of the quadrupole CeX effect depends on the angular momentum transfer ℓ, suppressing $\ell = 0$ and enhancing $\ell = 2$ reactions [11]. The CeX effect is a result of multi-step transitions leading to a complicated interplay between its contributions of the two- and three-body nature. Consistent conclusions have been drawn also in the CDCC-type study [12].

Those Faddeev-type calculations with CeX were limited so far to local potentials between nucleons and the core. A further shortcoming in the case of ^{10}Be$(d,p)^{11}$Be and ^{11}Be$(p,d)^{10}$Be reactions was the use of standard optical potential parametrizations such as Chapel Hill 89 (CH89) [13] or Koning and Delaroche (KD) [14], fitted to heavier nuclei. Those parametrizations do not provide accurate description of $p + ^{10}$Be scattering data, examples for elastic scattering at 6 to 11 MeV are shown in Ref. [15]. Furthermore, as the parameters of local optical potentials are energy-dependent, there is an ambiguity in fixing them, especially in transfer reactions where the effective nucleon-core energy differs in the initial and final states. Thus, the predictions of previous calculations for three-body reactions have the associated uncertainties. Therefore, the goals of the present work are (i) developing an energy-independent nonlocal optical potential for the nucleon-^{10}Be system including explicitly the dynamical excitation of the ^{10}Be core, consistent with the experimental data in a possibly broader energy regime, and (ii) applying this new potential to the description of ^{10}Be$(d,p)^{11}$Be and ^{11}Be$(p,d)^{10}$Be reactions using the rigorous three-body Faddeev formalism. Given their sophistication and advantages over widely employed standard potentials and approaches, the combination of the two above goals would constitute a new state-of-the-art description of deuteron stripping and pickup reactions.

Section 2 introduces the Faddeev equations with the excitation of the core while Section 3 describes and validates the proposed nonlocal optical potential. Section 4 presents the results for ^{10}Be$(d,p)^{11}$Be and ^{11}Be$(p,d)^{10}$Be reactions at several energy values, whereas the conclusions are drawn in Sec. 5. Note that we use natural units $\hbar = c = 1$.

2. Three-body AGS equations with core excitation

We consider the $p + n + ^A$Z three-particle system with masses m_p, m_n, and m_A, respectively, in its center-off-mass (c.m.) frame. The nuclear core ^{A}Z can be either in its ground (g) or excited (x) state, with the respective components of the operators being indicated by Latin superscripts in the following. The transitions between the internal states of the core are induced by nucleon-core potentials V_α^{ba}, where the Greek subscript α labels the spectator particle in the odd-man-out notation, e.g., the spectator n means the interacting ^{A}Z$+p$ pair, and so on. For each interacting pair the two-particle transition operator is obtained from the Lippmann-Schwinger equation

$$T_\alpha^{ba} = V_\alpha^{ba} + \sum_{j=g,x} V_\alpha^{bj} G_0^j T_\alpha^{ja} \tag{1}$$

where the free resolvent $G_0^j = (E + i0 - \delta_{jx}\Delta m_A - K)^{-1}$ has contributions not only from the kinetic energy operator K but also from the core excitation energy Δm_A in the respective sector of the Hilbert space, E being the available energy.

Dynamic equations to be solved in the present work are the Faddeev equations [2] in the AGS version [3] for three-body transition operators, extended in Ref. [10] to include CeX via coupling of different sectors in the Hilbert space, i.e.,

$$U_{\beta\alpha}^{ba} = \bar\delta_{\beta\alpha}\,\delta_{ba} G_0^{a\,-1} + \sum_{\sigma=p,n,A}\sum_{j=g,x} \bar\delta_{\beta\sigma}\, T_\sigma^{bj}\, G_0^j U_{\sigma\alpha}^{ja}, \tag{2}$$

with $\bar\delta_{\beta\alpha} = 1 - \delta_{\beta\alpha}$.

On-shell matrix elements of $U_{\beta\alpha}^{ba}$ taken between the two-cluster channel states determine the physical transition amplitudes for the respective reactions. E.g., for the deuteron induced reactions the initial channel state $|\nu_A, \mathbf{q}_A\rangle$ is a product of the deuteron wave function and a free wave for the deuteron-nucleus motion with the relative momentum q_A, the set of discrete quantum numbers being abbreviated by ν_A. In the $p + ^A$Z channel with the relative proton-nucleus momentum $\mathbf{q}_p$ the channel state $|\nu_p, \mathbf{q}_p\rangle = |\nu_p^g, \mathbf{q}_p\rangle + |\nu_p^x, \mathbf{q}_p\rangle$ has two components whose norms are given by the respective spectroscopic factors, their summ being equal to unity. Thus, the amplitudes for the deuteron stripping reaction are

$$\mathcal{T}(\nu_p, \mathbf{q}_p; \nu_A, \mathbf{q}_A) = \sum_{b=g,x} \langle \nu_p^b, \mathbf{q}_p | U_{pA}^{bg} | \nu_A^g, \mathbf{q}_A \rangle. \tag{3}$$

and the corresponding differential cross sections are

$$\frac{d\sigma(\nu_A \to \nu_p)}{d\Omega_p} = (2\pi)^4 M_A M_p \frac{q_p}{q_A} |\mathcal{T}(\nu_p, \mathbf{q}_p; \nu_A, \mathbf{q}_A)|^2, \tag{4}$$

where M_α is the spectator-pair reduced mass for the partition α. For unpolarized observables one has to perform a corresponding spin averaging (summation) over the initial (final) states.

The solution of the scattering equations (2) is performed in the momentum-space partial-wave framework, with technical details explained in Refs. [10,11].

3. Nonlocal potential with core excitation

Since our Faddeev-type calculations are performed in the momentum-space basis, the treatment of nonlocal potentials, once they are transformed to momentum space, is the same as of local ones. The most often used nonlocal form in the coordinate space is the one proposed by Perey and Buck [16], i.e.,

$$V_N(\mathbf{r}', \mathbf{r}) = H(|\mathbf{r}' - \mathbf{r}|) V(y), \tag{5}$$

where $y = |\mathbf{r}' + \mathbf{r}|/2$, and

$$H(x) = \pi^{-3/2} \rho^{-3} e^{-(x/\rho)^2} \tag{6}$$

is the nonlocality function with the nonlocality range ρ. The local part

$$\begin{aligned} V(y) = &- V_V\, f_V(y) - iW_V\, f_W(y) - i4W_S\, f_S(y)[1 - f_S(y)] \\ &+ V_s \frac{2}{y} \frac{df_s(y)}{dy}\, \boldsymbol{\sigma} \cdot \mathbf{L} \end{aligned} \tag{7}$$

has real volume, imaginary volume and surface, and real spin-orbit terms with the respective strength parameters V_V, W_V, W_S, and V_s, while the radial dependence is given by the corresponding Woods-Saxon functions

$$f_k(y) = [1 + e^{(y - R_k)/a_k}]^{-1} \tag{8}$$

with radius $R_k = r_k A^{1/3}$ and diffuseness a_k. This type of potential, with parameters of Giannini et al. [17,18], has been used in a number of studies [4,8,19], demonstrating important nonlocality effects in nucleon transfer reactions.

Perey and Buck [16] proposed also an alternative version of the nonlocal potential, given by Eq. (5) but with $y = (r' + r)/2$. We make one step further and introduce the nonlocal potential

$$V_N(\mathbf{r}', \mathbf{r}) = \frac{1}{2}\big[H(|\mathbf{r}' - \mathbf{r}|) V(r) + V(r') H(|\mathbf{r}' - \mathbf{r}|)\big]. \tag{9}$$

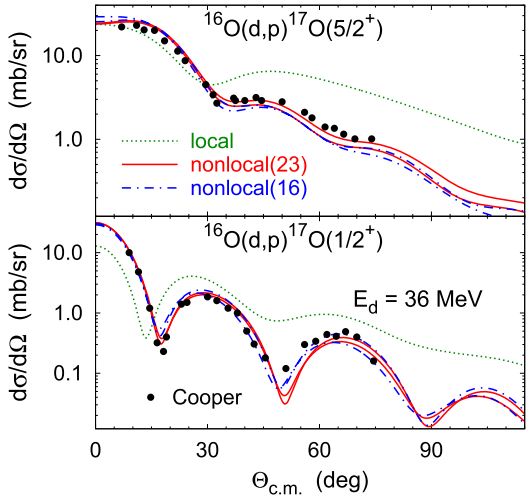

Fig. 1. Differential cross section for ^{16}O$(d,p)^{17}$O transfer reactions at $E_d = 36$ MeV leading to ^{17}O ground $\frac{5}{2}^+$ (top) and excited $\frac{1}{2}^+$ (bottom) states. Predictions obtained using several versions of nonlocal optical potentials of type (5) and (9), displayed by solid and dashed-dotted curves, respectively, and those using the local optical potential from Ref. [20] (dotted curves), are compared with the experimental data from Ref. [21].

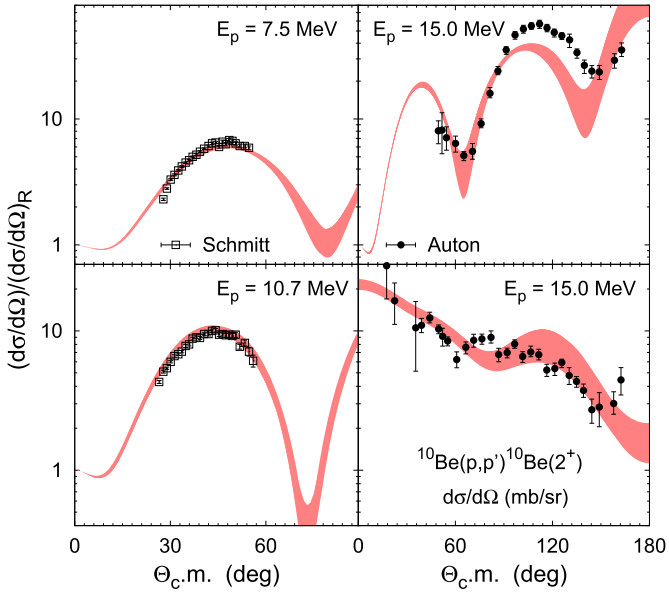

Fig. 2. Differential cross sections $d\sigma/d\Omega$ divided by the Rutherford cross section for elastic $p + {}^{10}$Be scattering at 7.5, 10.7 and 15 MeV/nucleon beam energies as functions of the c.m. scattering angle $\Theta_{\rm c.m.}$. The bottom-right panel shows the differential cross section for the inelastic $p + {}^{10}$Be scattering at 15 MeV/nucleon energy. Predictions with different parameter sets of the nonlocal optical potential are combined into bands and compared with the experimental data from Refs. [15] and [25].

By expanding in Taylor series it is an easy exercise to show that the leading term in the difference between the two latter versions is $\frac{1}{4}H(|\mathbf{r}' - \mathbf{r}|)V^{(2)}(r)(r' - r)^2$, i.e., a small quantity.

In summary, all three discussed nonlocal potential versions are phenomenological, become local in the limit $\rho \to 0$, and their parameters have to be determined by fitting the two-body data. Here we compare the potentials of type (5) and (9) in the $p + n + {}^{16}$O system that has been extensively studied in earlier works without CeX. First, for the two-body scattering $p + {}^{16}$O and $n + {}^{16}$O the reproduction of the experimental data is of comparable quality as achieved in Ref. [19] with the nonlocality type (5) (excluding backward angles that need special terms). Second, we also calculated ^{16}O$(d,p)^{17}$O reactions using several versions of type (5) and (9) nonlocal potentials, labeled in Fig. 1 by nonlocal(16) and nonlocal(23), respectively. We show examples at the deuteron beam energy $E_d = 36$ MeV for transfer reactions leading either to the ^{17}O ground state $\frac{5}{2}^+$ or excited state $\frac{1}{2}^+$. In addition, we include the results from Ref. [19] obtained with a local potential [20]. It is obvious that both types of the nonlocal potential predict very similar nonlocality effects, and are clearly superior in accounting for the experimental data [21] as compared to the local one.

Thus, the proposed nonlocal potential (9) captures well the essential features of a more traditional nonlocal potential (5). The next step is its extension to include the excitation of the core. The inclusion of the CeX for local potentials with the rotational model [22,23], as appropriate for ^{10}Be, assumes a permanent quadrupole deformation of the core nucleus characterized by the deformation parameter β_2, resulting in the Woods-Saxon radius $R_k = R_{k0}[1 + \beta_2 Y_{20}(\hat{\xi})]$ depending on the internal core degrees of freedom $\hat{\xi}$ in the body-fixed frame. This renders the potential noncentral; it is expanded into multipoles retaining the $\lambda = 2$ multipole as a dominant one for the CeX. The new nonlocal form (9) has an advantage in performing partial-wave and multipole expansion

$$\langle r'L'S'J|V_N^{ba}|rLSJ\rangle = \frac{1}{2}\Big[H_{L'}(r',r)V^{ba}_{L'S',LS,J}(r) + V^{ba}_{L'S',LS,J}(r')H_L(r',r)\Big] \tag{10}$$

since the respective integration angular variables are not coupled, and consequently the terms $H(|\mathbf{r}' - \mathbf{r}|)$ and $V(r)$ can be expanded separately, resulting in $H_L(r',r)$ and $V^{ba}_{L'S',LS,J}(r)$, with orbital momentum L, total spin S and conserved total angular momentum J. $V^{ba}_{L'S',LS,J}(r)$ is exactly the standard local potential with the CeX [22,23], employed also in previous Faddeev-type calculations. The transformation of the potential (10) to the momentum space is straightforward. Note that for the nonlocal potential of type (5) the angular variables of partial-wave and multipole expansion are coupled, since deformed $V(y)$ depends both on $\hat{\xi}$ and the angle between $\mathbf{r}'$ and $\mathbf{r}$. This precludes taking over directly the result derived for the local potential.

The parameters of the new nonlocal potential for the $p + {}^{10}$Be system are partly assumed and partly determined by fitting the experimental data [15]. The Coulomb force and its deformation [22] is included as well. We introduced some constraints before the fit such that our potential has no more free parameters than standard optical potentials. First, we fix the nonlocality range $\rho = 1$ fm, close to a typical value for nonlocal potentials [17,18]. Second, as for several standard local potentials [13,24] we assume the same geometric parameters for both imaginary terms, i.e., $f_W(y) = f_S(y)$. Third, since no polarization data are available to constrain the spin-orbit force, we assume $f_s(y) = f_V(y)$ and $V_s = 7.5\,\text{MeV}\,\text{fm}^2$, close to the value for other nonlocal potentials [19], and do not deform this term. Thus, the parameters to be determined from the fit are the strengths V_V, W_V, and W_S, reduced radii r_V and r_W, diffuseness a_V and a_W, and the quadrupole deformation parameter β_2. We fitted them to the elastic data at 6, 7.5, 9, 10.7 MeV per nucleon [15] and to both elastic and inelastic data at 12, 13, 14, 15 and 16 MeV per nucleon [25]. In order to estimate the uncertainties we developed about 10 parameter sets with a comparable quality of the fit, and show the corresponding predictions as bands in Fig. 2. An example set is $V_V = 91.17$ MeV, $W_V = 1.35$ MeV, $W_S = 7.61$ MeV, $r_V = 1.14$ fm, $r_W = 1.26$ fm, $a_V = 0.50$ fm, $a_W = 0.65$ fm, and $\beta_2 = 0.72$; the parameter tables can be obtained from the authors upon request. The value of the quadrupole deformation parameter β_2 ranges between 0.71 and 0.78, quite consistent with 0.70 to 0.74 from the DWBA analysis [25] using local potentials. We stress that we achieve quite

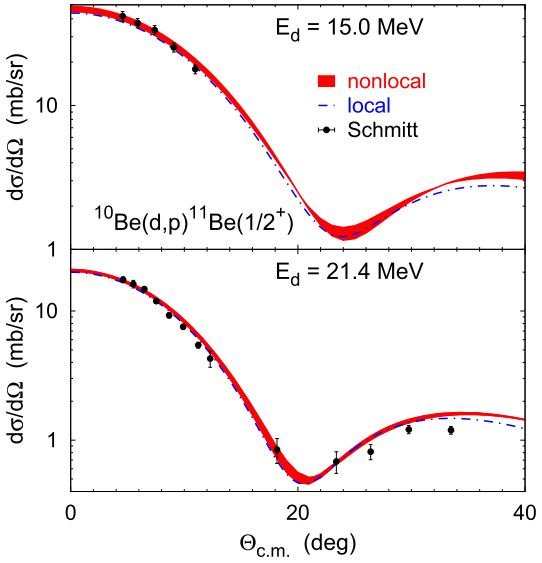

Fig. 3. Differential cross section for the transfer reaction ^{10}Be$(d,p)^{11}$Be at 15 and 21.4 MeV deuteron beam energy, the final ^{11}Be nucleus being in its ground state $\frac{1}{2}^+$. Predictions with the spectroscopic factor $S(0^+,\frac{1}{2}^+)=0.754$ and different parameter sets of nonlocal optical potentials are combined into a band and compared with the experimental data from Ref. [15].

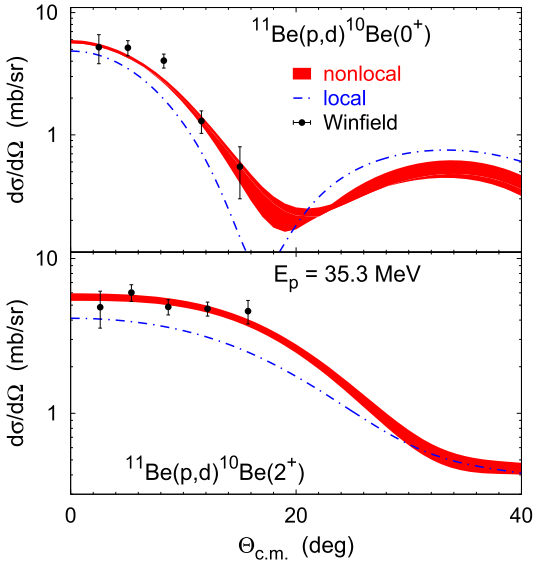

Fig. 4. Differential cross section for ^{11}Be$(p,d)^{10}$Be transfer reactions at $E_p=35.3$ MeV leading to the ground (0^+) and excited (2^+) states of ^{10}Be. Predictions with spectroscopic factors $S(0^+,\frac{1}{2}^+)=0.807$, $S(2^+,\frac{1}{2}^+)=0.193$ and different parameter sets of nonlocal optical potentials are combined into a band and compared with the experimental data are from Ref. [33].

satisfactory description of the data by using energy-independent potentials.

Regarding the $n+^{10}$Be potential we are not aware of the available experimental data. We therefore fix all the geometry parameters to those of the $p+^{10}$Be potential, as done for most of the local potentials, and refit only the strengths V_V, W_V, and W_S, aiming to make the predictions consistent with those of standard local optical potentials. The achieved consistency is comparable to the one between the $p+^{10}$Be predictions and data, and is not shown separately.

4. Results

In this section we present results for ^{10}Be$(d,p)^{11}$Be and ^{11}Be$(p,d)^{10}$Be reactions obtained solving the Faddeev-type three-body equations and including the quadrupole excitation of the ^{10}Be core via the potentials developed in the previous section. In the $n+^{10}$Be partial waves with total angular momentum/parity $\frac{1}{2}^+$ $(\frac{1}{2}^-)$ where the ground (excited) state of ^{11}Be resides we use real nonlocal potential with geometric parameters taken over from Refs. [10,23] and fit the strengths to reproduce the binding energy of 0.504 MeV (0.184 MeV) for the ground (excited) state. In the $\frac{1}{2}^+$ case we add a weak (few percent of the central volume part) $\mathbf{L}^2$ term as in Refs. [11,26] adjusted to reproduce the desired binding energy and the SF. Finally, the realistic CD Bonn potential [27] is used for the neutron-proton interaction, and the $p+^{10}$Be Coulomb force is included via the screening and renormalization method [28–31] as in Refs. [10,11].

We start with the study of the ^{10}Be$(d,p)^{11}$Be reaction at deuteron beam energies $E_d=15$ and 21.4 MeV where the final ^{11}Be nucleus is in its ground state $\frac{1}{2}^+$. We performed calculations using different parameter sets of $p+^{10}$Be and $n+^{10}$Be nonlocal optical potentials (10) and combined them into a band. The transfer cross section depends also on the spectroscopic factor $S(S_A^{\pi_A},J^\Pi)$ where $S_A^{\pi_A}$ and J^Π are spin/parity of ^{10}Be and ^{11}Be, respectively. Since for small variations of the SF that dependence is nearly linear, in Fig. 3 we show the predictions for a single fixed $S(0^+,\frac{1}{2}^+)=0.754$ value that is found to be consistent with the experimental data [15]. The width of the band reflects the theoretical uncertainty due to the potential model, and therefore also the theoretical error bar for the SF, which is around 5% (3%) at 15 MeV (21 MeV). Remarkably, the predictions from Ref. [32] based on the local CH89 potential with $S(0^+,\frac{1}{2}^+)=0.855$ and displayed by dashed-dotted curves turn out to be very close. For comparison, the SFs extracted in Ref. [15] using ADWA with CH89 and KD potentials [13,14], range from 0.77 to 0.81 and from 0.67 to 0.74 at $E_d=15$ and 21.4 MeV, respectively. Thus, our results show considerably weaker energy-dependence.

The above agreement between local and nonlocal potentials is likely accidental as it disappears at higher energy. In Fig. 4 we study the deuteron pickup reaction ^{11}Be$(p,d)^{10}$Be at $E_p=35.3$ MeV/nucleon beam energy with the final ^{10}Be nucleus being in its ground (excited) state 0^+ (2^+). This energy is equivalent to $E_d=40.9$ MeV in the time-reversed ^{10}Be$(d,p)^{11}$Be reaction. Again, predictions with different parameter sets of nucleon-nucleus nonlocal optical potentials (10) are combined into a band, but this time a better agreement with the experimental data [33] is obtained with 5% change in the SF, i.e., $S(0^+,\frac{1}{2}^+)=0.807$ and, consequently, $S(2^+,\frac{1}{2}^+)=0.193$, which quite reasonably compare with 0.90 and 0.16, respectively, as predicted by the ab initio no-core shell model with continuum (NCSMC) with an extra adjustment to neutron separation energies [34]. The uncertainties associated with the band width are around 3% and 5%, respectively. With this model and nonlocal potentials we are able to provide rather good description of the experimental data simultaneously for both transfer reactions leading to either ground or excited state of ^{10}Be. In contrast, the local potential predictions from Ref. [32] underestimate the differential cross section in both cases, and the description can not be repaired by an adjustment of SFs (assuming their sum is one), since increasing the cross section for one channel would reduce it for another. As shown in Ref. [32] none of the three different local optical potentials was able to reproduce the data [33] well. DWBA analysis [33] reports $S(0^+,\frac{1}{2}^+)$ values from 0.65 to 0.80.

Finally, Fig. 5 presents our results for the differential cross section in the ^{10}Be$(d,p)^{11}$Be reaction at $E_d=15$ and 21.4 MeV

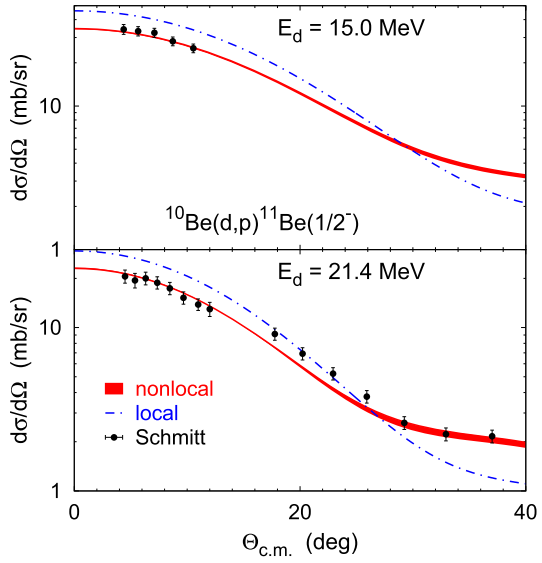

Fig. 5. Differential cross section for transfer reactions $^{10}\mathrm{Be}(d,p)^{11}\mathrm{Be}$ at $E_d = 15$ and 21.4 MeV leading to the excited $\frac{1}{2}^-$ state of ^{11}Be. Predictions with the spectroscopic factor $S(0^+, \frac{1}{2}^-) = 0.654$ and different parameter sets of nonlocal optical potentials are combined into a band and compared with the experimental data from Ref. [15].

but with the final ^{11}Be nucleus being in its excited state $\frac{1}{2}^-$. In this partial wave we used a real nonlocal potential that yields $S(0^+, \frac{1}{2}^-) = 0.654$, while predictions with different parameter sets of nucleon-nucleus nonlocal optical potentials (10) form a band. At a first glance it seems to be narrower than the $p + ^{10}$Be band in Fig. 2, however, the width of bands for angles up to 40 deg is quite comparable. With $S(0^+, \frac{1}{2}^-) = 0.654$ the nonlocal potential predictions describe the experimental data [15] quite well at both energies, where the error bar associated with the band width is below 1%. The local potential results taken from Ref. [32] correspond to $S(0^+, \frac{1}{2}^-) = 0.786$ and fail to reproduce the data. The agreement for forward angles could be improved by rescaling the predictions with smaller $S(0^+, \frac{1}{2}^-) \approx 0.60$ instead, but the disagreement at $\Theta_{\mathrm{c.m.}} > 20$ deg would increase. There is an important difference in the shape of the angular distribution obtained with local and nonlocal potentials, consistent with findings of earlier works without the CeX. Our $S(0^+, \frac{1}{2}^-) = 0.654$ value is in agreement with ADWA results [15] ranging from 0.63 to 0.71.

We finally note that the present calculations cannot explain systematically lower cross sections at $E_d = 18$ MeV [15], a problem present also in all earlier ADWA and Faddeev-type calculations [10,15].

Since the nucleon transfer reactions at low energies and forward angles are peripheral to a good approximation, the differential cross sections should scale with the square of the asymptotic normalization coefficient (ANC) [35]. We therefore collect in Table 1 the ANC values corresponding to the ^{11}Be wave functions used in this work, in an earlier Faddeev-type study with local potentials [32], the ones extracted in the ADWA study [35] with the CH89 optical potential, and those resulting from the NCSMC calculations [34]. Where available, we include in Table 1 also SF values. Despite differences in dynamics and SF values, we observe a good agreement for the ground state ANC. The excited state ANC obtained with the nonlocal model deviates from others by 10%, but despite this fits the experimental data at forward angles as good as other models, an provides even a better description at larger angles. This indicates that beside the ANC also the continuum dynamics plays important role in transfer reactions.

Table 1
Spectroscopic factors and asymptotic normalization coefficients (in units of $\mathrm{fm}^{-1/2}$) for ground and excited states of ^{11}Be obtained by several reaction and structure models. The ANC values corresponding to Figs. 3 and 4 are 0.779 and 0.809 $\mathrm{fm}^{-1/2}$, respectively. The results of Ref. [32] for the excited state were rescaled to fit the data at forward angles.

	SF	ANC
$S_A^{\pi_A} = 0^+, \quad J^\Pi = \frac{1}{2}^+$		
Faddeev, nonlocal	0.78 ± 0.04	0.79 ± 0.02
Faddeev, local [32]	0.854	0.785
ADWA [35]		0.785 ± 0.03
NCSMC [34]	0.90	0.786
$S_A^{\pi_A} = 0^+, \quad J^\Pi = \frac{1}{2}^-$		
Faddeev, nonlocal	0.65 ± 0.01	0.149 ± 0.005
Faddeev, local [32]	0.60	0.13
ADWA [35]		0.135 ± 0.005
NCSMC [34]		0.129

5. Conclusions

We proposed a new nonlocal form of the nucleon-nucleus optical potential, and using the $^{16}\mathrm{O}(d,p)^{17}\mathrm{O}$ reaction as example demonstrated that it reproduces well the essential features of traditional nonlocal potential. We extended the new nonlocal potential to account for collective degrees of freedom of the nuclear core via the rotational quadrupole excitation. For the $p + ^{10}$Be system the potential parameters were determined by fitting the experimental data for elastic and inelastic scattering. In contrast to standard local potentials, developed nonlocal parametrizations are energy-independent but nevertheless are able to provide quite a good description of the experimental data over a broader energy range. For the $n + ^{10}$Be system, owing to the lack of the experimental data, the potential parameters were determined demanding consistency with the predictions by several global parametrizations of standard optical potentials.

Nonlocal potentials with explicit excitation of the core for the first time were applied to the study of deuteron stripping and pickup reactions using rigorous three-body scattering theory. Faddeev-type equations extended for the core excitation were solved in the momentum-space partial-wave representation leading to well-converged results. We studied deuteron stripping and pickup reactions $^{10}\mathrm{Be}(d,p)^{11}\mathrm{Be}$ and $^{11}\mathrm{Be}(p,d)^{10}\mathrm{Be}$ at energies corresponding to 15, 21.4 and 40.9 MeV deuteron beam energy. A good description of the experimental data [15,33] was achieved, though for the reactions involving the ^{11}Be ground state a slight readjustment of the SF by $\sim 5\%$ was required at the highest energy. On the other hand, the predictions using local optical potentials have been less successful. The SF values found to be consistent with the experimental data are $S(0^+, \frac{1}{2}^+) = 0.78 \pm 0.04$ and $S(0^+, \frac{1}{2}^-) = 0.65 \pm 0.01$, in reasonable agreement with some of earlier determinations [15,33] based on DWBA or ADWA. However, an important feature of our results is smaller spread of values and weaker energy dependence, likely due to more sophisticated potentials and treatment of the three-body dynamics. The results also suggest that with the nonlocal potential parameters adjusted to the experimental data in a limited energy regime it may be possible to make reliable predictions outside that regime, indicating an increased predicting power of our calculational scheme. This is expected to hold also for other reaction channels such as inelastic and breakup, and the future studies should clarify this question.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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