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Closure Properties for Heavy-Tailed and Related Distributions

An Overview

Springer

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Preface

... then, prudence cannot be science. Nor can it be skill. It is not science because the matters of conduct may vary; and it is not skill because the genus of action is different from that of production. It remains, therefore, that prudence is a disposition with truth, involving reason and concerned with action about things that are good or bad for a human being.

Aristotle, Nicomachean Ethics, VI.5, 1140b

The classification of scientific subjects has its origin in early antiquity and up to now continues to participate in active research. In fact, it provides the correct framework in which the research activity becomes exact science. Recently, even the social sciences take the luxury of rigorous methods of testing assumptions and employ accurate quantitative expressions to go deeper into the nature of the human beings.

The closure properties in probability theory have a long history, back from the middle of the previous century. They appear as substantial supports in reliability theory, queueing theory, branching processes, risk theory, stochastic control, asset pricing, and others. Especially in the context of heavy-tailed distributions, they play a crucial role in the classification of the random variables and eventually of various risks. Therefore, a systematic presentation of the subject has already been mature.

The lack of closure properties is a strong motivation for enlarging the class of distributions. In this way, some new classes of theoretical or practical interest may appear. On the other hand, the presence of closure properties can bring up other possibilities for the investigation of the origin of the class. In both cases, the study of the closure properties can be instrumental for a deeper understanding of the class.

We find it necessary to restrict our overview in the space of *heavy-tailed* and related distributions, as they present special interest in applications, like in actuarial and financial issues, where economical stability is under threat. For this purpose, we take advantage of the recent accumulation of a vast amount of results on this topic. Eventually, the modelling of extremal events can serve as a tool for investigation on other subjects, for example, physics, chemistry, biology, or geology.

This book is written as a consequence of rather intensive research work of the authors in ruin theory and other problems of insurance mathematics, which is a rapidly developing area. The recent project started about 5 years ago and was intended as a survey paper. However, later it was decided to write a book devoted to the overview of closure properties, which although mostly known for several decades and are reported in the papers and books but were not presented in a unified manner and systematically classified. In a sense, this book can serve as a handbook.

The reader is supposed to have advanced knowledge of calculus and probability theory, as also some familiarity with stochastic processes. In the references are found most of the classical works related to the topic, which can be used for deeper study. Most statements are without proofs but there are clear indications of where these proofs can be found. From numerous examples and counterexamples, we kept only the most representative, which provide substantial insight into the theory. This short book is written primarily to help graduate students and young researchers to enter quickly into the subject. Furthermore, it can be used by applied scientists and industrial scholars, or people in the market, to optimize their preferences and decisions. Any comments or criticism are encouraged and welcomed, as they are helpful for the further development of the state-of-the-art. Practical implementation of this theory is of paramount importance and much anticipated.

The first two authors are grateful for financial support from the Research Council of Lithuania (grants No. S-MIP-17-72, No. S-MIP-20-16). Finally, the text would not be possible without the direct and indirect contribution of our colleagues working in similar areas. We are grateful to Qihe Tang, Yuebao Wang, Yang Yang, Vytautas Kazakevičius, Vygantas Paulauskas, and, of course, our graduate and doctoral students for their incentive role, new problems, and ideas. Finally, we would like to thank all reviewers and editorial team (especially Veronika Rosteck) for their remarks and suggestions, which helped us substantively to improve an earlier version of this book.

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Contents

Acronyms

Distribution classes:

- Heavy-tailed distributions: $\mathscr{H}$.
- Strongly heavy-tailed distributions: $\mathscr{H}^*$.
- Regularly varying distributions: $\mathscr{R}(\alpha)$.
- Extended regularly varying distributions: $\mathscr{E}\mathscr{R}\mathscr{V}(\alpha,\ \beta)$.
- Rapidly varying distributions: $\mathscr{R}(\infty)$.
- Consistently varying distributions: $\mathscr{C}$.
- Dominatedly varying distributions: $\mathscr{D}$.
- Positively decreasing-tailed distributions: $\mathscr{P}\mathscr{D}$.
- Long-tailed distributions: $\mathscr{L}$.
- Exponential-like-tailed distributions: $\mathscr{L}(\gamma)$.
- Generalized long-tailed distributions: $\mathscr{O}\mathscr{L}$.
- Subexponential distributions: $\mathscr{S}$.
- Strong subexponential distributions: $\mathscr{S}^*$.
- Strongly subexponential distributions: $\mathscr{S}_*$.
- Convolution equivalent distributions: $\mathscr{S}(\gamma)$.
- Generalized subexponential distributions: $\mathscr{O}\mathscr{S}$.
- Generalized strong subexponential distributions: $\mathscr{O}\mathscr{S}^*$.
- Subexponential positively decreasing-tailed distributions: $\mathscr{A}$.
- Generalized subexponential positively decreasing-tailed distributions: $\mathscr{O}\mathscr{A}$.

Methods:

- Strong tail-equivalence principle: STEP.
- Weak tail-equivalence principle: WTEP.