

Synthetic Landau levels and robust chiral edge states for dark-state polaritons in a static and scalable continuum media

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We demonstrate the generation and dynamical control of synthetic Landau levels and robust chiral edge states for neutral dark-state polaritons using electromagnetically induced transparency in our theoretical studies. We adopt an optical approach to produce synthetic magnetic fields for dark-state polaritons in the static laboratory frame. In our scalable system, an Aharonov-Bohm phase is obtained along a closed loop in a continuous material rather than a sophisticated lattice structure. Our scheme paves the way toward versatile quantum simulators, dynamically controllable photonic circuits, and generators for exotic states of light carrying topological winding numbers.

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The discovery of the quantum Hall effects [1–6] inspired the remarkable progress in emulating the synthetic gauge fields for electrically neutral particles which do not experience the Lorentz force. Successful implementations include ultracold atoms [7–13], photons [14–27], and electronic circuits [28]. Most pioneering works in photonics use sophisticated periodic structures of resonators [14–19, 21–24, 27, 29] or an elaborated optical trajectory [20, 26] to produce synthetic gauge fields and robust chiral edge states for photons [25]. Among photonic systems, electromagnetically induced transparency (EIT) [30–35] has attracted extensive studies on slow [36–40] and stationary [41–44] light in continuum media. EIT leads to electrically neutral quasiparticles known as dark-state polaritons (DSPs) [38, 39, 42, 43]. DSPs can interact strongly via atom-atom interaction [26, 45–52] and should facilitate creation of strongly correlated DSP quantum states under synthetic gauge fields, e.g., the fractional quantum Hall states [26, 53]. Yet, up to now, only mechanical rotation [54] is considered for creating the synthetic gauge fields for DSPs in the rotating frame. Here, we demonstrate an optical method to

generate Landau levels (LLs) and robust chiral edge states for the atomic coherence of the DSPs in a lattice-free medium. Our scheme generates synthetic gauge fields in the static laboratory frame and thus avoids problems associated with mechanical rotations [54], such as excitations [55] and destabilization [56]. Moreover, our lattice-free system is scalable; namely, one can scale up the size of the synthetic quantum states and study the gauge fields in the expanded system. This makes a building block for more complex systems, such as bosonic quantum Hall states. The lattice-free platform also allows for easier access to interactions with Rydberg EIT. Yet, unlike static photonic structures, our system is dynamically controllable and can lead to fast control over photonic flow like a circuit. Furthermore, the present EIT-based system can potentially simulate a charged particle in a magnetic field [26, 53, 54], nonlinear optics [57], negative-mass dynamics [58], and non-Hermitian quantum mechanics [59, 60].

Figure 1 depicts our system. A two-dimensional atomic medium is coupled to four control (probe) fields propagating along four directions indicated by thick red (blue) arrows. Figure 1(a) illustrates the typical stationary-light setup where counterpropagating control fields are aligned [41–44]. We consider the three-level Λ -type system in Fig. 1(b), and each control (probe) field drives a $|2\rangle \rightarrow |3\rangle$ ($|1\rangle \rightarrow |3\rangle$) transition with Rabi frequency $\Omega_{c(p)}^j$, where index $j = F, B, R$, and L indicates the forward, backward, rightward, and leftward quantity, respectively. The imaging beam (green thick arrow) in Figs. 1(a) and 1(b) drives the $|2\rangle \rightarrow |4\rangle$ transition and yields the absorption imaging of DSPs [61]. $\Delta_{c(p)}$ is the one-photon

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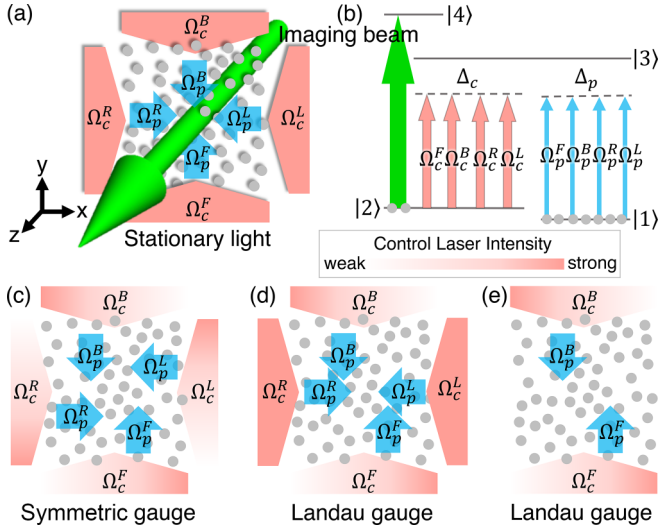


FIG. 1. (a) Two-dimensional EIT-stationary-light system. Gray dots illustrate atoms, and red (blue) thick arrows depict control (probe) fields. (b) Red (blue) upward-pointing arrows represent the control (probe) fields driving transition $|2\rangle \rightarrow |3\rangle$ ($|1\rangle \rightarrow |3\rangle$) with detuning $\Delta_{c(p)}$. The color gradient of each red arrow illustrates the transverse intensity of a control laser. An aligned $\Omega_p^F - \Omega_p^B$ or $\Omega_p^R - \Omega_p^L$ blue arrow pair represents stationary light, and a misaligned blue arrow indicates the slow light. Green thick arrows [along z in (a), and upward in (b)] depict the imaging beam which drives transition $|2\rangle \rightarrow |4\rangle$ for the visualization of DSPs. (c)–(e) Generation of the synthetic vector potential of the symmetric gauge (c) and Landau gauge [(d) and (e)].

detuning of the control (probe) fields. Our system is described by optical-Bloch equations [38,41,44,62,63]

$$\frac{\partial \rho_{21}}{\partial t} = i(\Delta_c - \Delta_p)\rho_{21} + \frac{i}{2} \sum_j \Omega_c^{j*} \rho_{31}^j, \quad (1)$$

$$\frac{\partial \rho_{31}^j}{\partial t} = \frac{i}{2} \Omega_p^j + \frac{i}{2} \Omega_c^j \rho_{21} - \left(\frac{\Gamma}{2} + i\Delta_p \right) \rho_{31}^j, \quad (2)$$

$$\frac{1}{c} \frac{\partial \Omega_p^j}{\partial t} + \epsilon_j \frac{\partial \Omega_p^j}{\partial X_j} = i\eta \rho_{31}^j, \quad (3)$$

where ρ_{21} (ρ_{31}^j) is the coherence between state $|2\rangle$ ($|3\rangle$) and $|1\rangle$. ρ_{31}^j quantifies the polarization and the dispersion relation to Ω_p^j . Γ is the spontaneous decay rate of state $|3\rangle$. $\eta = \frac{\Gamma \xi_x}{2L_x} = \frac{\Gamma \xi_y}{2L_y}$ is the light-matter coupling constant. ξ_x (ξ_y) and L_x (L_y) are the optical depth and the medium length in the x (y) direction, respectively. $\epsilon_{F(R)} = 1$, $\epsilon_{B(L)} = -1$, $X_{R(L)} = x$, and $X_{F(B)} = y$ indicate the Ω_p^j propagating direction. The ground-state coherence ρ_{21} dominates the DSP, and we derive its governing equation in the following steps. Neglecting $\frac{1}{c} \frac{\partial \Omega_p^j}{\partial t}$ in Eq. (3) as typically adopted in EIT [38], multiplying both sides of Eq. (3) by Ω_c^{j*} , and then summing Eq. (3) over j lead to $\sum_j \epsilon_j \Omega_c^{j*} \frac{\partial \Omega_p^j}{\partial X_j} = i\eta \sum_j \Omega_c^{j*} \rho_{31}^j$. By replacing the right-hand side of the above relation with Eq. (1), and substituting the adiabatic condition [38] $\Omega_p^j = -[1 + \epsilon_j (\frac{2\Delta_p - i\Gamma}{i\eta}) \frac{\partial}{\partial X_j}] \Omega_c^j \rho_{21}$ on

its left-hand side, we obtain a Schrödinger-like equation

$$i\hbar \frac{\partial \rho_{21}}{\partial t} = \frac{(\hbar \nabla + \vec{A})^2}{2m} \rho_{21} + U \rho_{21} + i \left(\frac{\Gamma}{2\Delta_p} \right) \frac{\hbar^2}{2m} \nabla^2 \rho_{21} \quad (4)$$

when $\frac{\partial \Omega_c^j}{\partial X_j}$ and $\frac{\partial^2 \Omega_c^j}{\partial X_j^2}$ are negligible. Here, ρ_{21} plays the role of wave function, the synthetic vector potential $\vec{A} = m\vec{V}_g$ for charge $q = -1$, the scalar potential $U = \hbar(\Delta_p - \Delta_c) - \frac{m}{2} |\vec{V}_g|^2$, and \hbar is the reduced Planck constant. $\vec{V}_g = (V_R - V_L, V_F - V_B) = \frac{1}{2\eta} (|\Omega_c^R|^2 - |\Omega_c^L|^2, |\Omega_c^F|^2 - |\Omega_c^B|^2)$ is the EIT propagation velocity [44]. We use constant Δ_p , η , Ω_c , and $V_R + V_L = V_F + V_B = \frac{\Omega_c^2}{2\eta}$ to get the uniform effective mass $m = \frac{\hbar\eta^2}{2\Delta_p\Omega_c^2}$. Equation (4) suggests that our scheme can create versatile quantum simulators by optically engineering the Hamiltonian. Furthermore, the \vec{A}^2 and Δ_p -dependent m can potentially simulate nonlinear optics [57] and negative-mass dynamics [58]. The last diffusion term allows for the study of non-Hermitian quantum mechanics [59,60]. We emphasize that the design of our system is guided by Eq. (4), and all of the following results are given by full numerical calculations of Eqs. (1)–(3).

We first demonstrate three possible schemes to produce LLs [4–6,20,25,64,65] for DSPs. One can engineer different gauges for \vec{A} , e.g., by choosing our first scheme with $\Omega_c^R = \frac{\Omega_c}{2} \sqrt{1 - \frac{y}{L_y}}$, $\Omega_c^L = \frac{\Omega_c}{2} \sqrt{1 + \frac{y}{L_y}}$, $\Omega_c^F = \frac{\Omega_c}{2} \sqrt{1 + \frac{x}{L_x}}$, $\Omega_c^B = \frac{\Omega_c}{2} \sqrt{1 - \frac{x}{L_x}}$, and $L_x = L_y = L$, which leads to the symmetric-gauge $\vec{A} = \frac{\hbar\eta}{8L\Delta_p} (-y, x)$ as depicted in Fig. 1(c).

Given that the control laser intensity is proportional to $|\Omega_c^j|^2$, our \vec{A} can be realized by using four control lasers with gradients of transverse intensity; e.g., the forward one is proportional to $1 + \frac{x}{L_x}$ as illustrated in Fig. 1(c). The transverse intensity profile remains unchanged along the beam propagating direction within Rayleigh range > 1 m, given by beam size $\approx L_x, L_y > 1$ mm in our system and optical wavelength $\lesssim 1$ μm . Thus the control beams remain collimated within the atomic cloud. For simplicity, we utilize the Landau-gauge $\vec{A} = \frac{\hbar\eta}{4L_x\Delta_p} (0, x)$ using the second design with $\Omega_c^R = \Omega_c^L = \frac{\Omega_c}{\sqrt{2}}$, $\Omega_c^F = \frac{\Omega_c}{\sqrt{2}} \sqrt{1 + \frac{x}{L_x}}$, and $\Omega_c^B = \frac{\Omega_c}{\sqrt{2}} \sqrt{1 - \frac{x}{L_x}}$ as illustrated in Fig. 1(d). We also use $\Delta_c = \Delta_p - \frac{\Omega_c^2}{16\Delta_p L_x^2} x^2$ to make $U = 0$, which can be implemented by position-dependent Zeeman or Stark shifts of atomic levels [54,66,67]. These lead to LLs in the Landau gauge, $\rho_{21}^{(n)}(x, y, t) = H_n(\frac{x+kl_B^2}{l_B}) \exp\{-\frac{(x+kl_B^2)^2}{2l_B^2} + i[ky - (n + \frac{1}{2})\omega_B t]\}$, where H_n is the Hermite polynomial. The cyclotron frequency $\omega_B = \Omega_c^2/(\xi_x\Gamma)$, and the magnetic length $l_B = \sqrt{\hbar/m\omega_B} = L_x \sqrt{8\Delta_p/(\xi_x\Gamma)}$. Moreover, the third scheme, depicted in Fig. 1(e), utilizes only two control beams $\Omega_c^F = \frac{\Omega_c}{\sqrt{2}} \sqrt{1 + \frac{x}{L_x}}$ and $\Omega_c^B = \frac{\Omega_c}{\sqrt{2}} \sqrt{1 - \frac{x}{L_x}}$. Here one can introduce the transverse effective mass $m_{\perp} = \frac{2\eta\hbar k_p}{\Omega_c^2}$ differing from the longitudinal m , where k_p is the angular wave number of the probe field [54]. The above three schemes result in a uniform synthetic magnetic field $\vec{B} = \frac{\hbar\eta}{4L_x\Delta_p} \hat{e}_z$.

Noting that the optical depth is a key parameter in EIT physics [68–70], we envisage using $\xi_{x(y)} \leq 2000$ for atoms trapped in a pancake geometry with size $L_{x(y)} \sim$ millimeters.

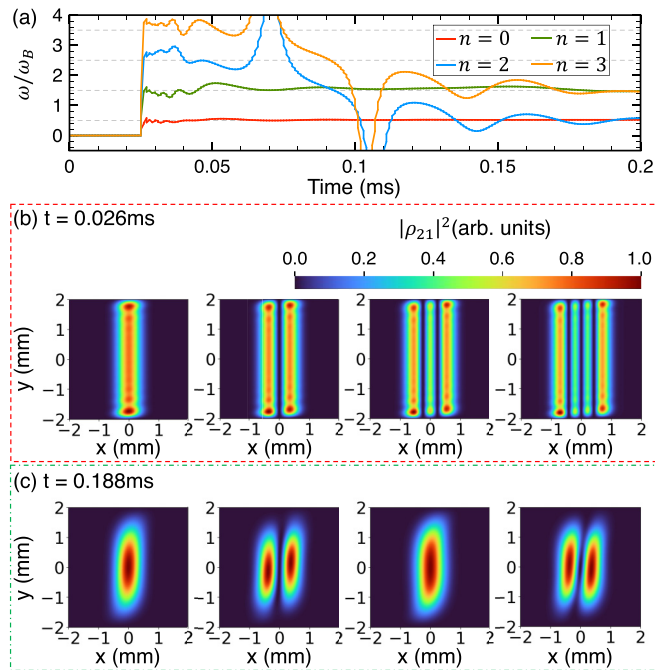


FIG. 2. (a) Time evolution of the angular frequency of the DSP in the n th LL. Solid red, green, blue, and orange curves depict $n = 0, 1, 2,$ and $3,$ respectively. (b) and (c) The spatial profile of the n th LL at $t = 0.026$ ms, when the Landau-gauge environment is switched on (b), and at $t = 0.188$ ms (c). The maximum $|\rho_{21}|^2$ is normalized to 1.

The size can be made larger under suitable design of the optical setup. To have a spatially uniform $\eta \propto n_{3D}$, one can implement a uniform three-dimensional (3D) particle density n_{3D} using a box potential [71,72] for the x and y confinement by the optical relay. The millimeter-size trap is achievable by using large aperture optics along with sufficient optical power. Indeed, optical depth over 1300 has been realized in nonuniform clouds [68–70]. One can start with a nonuniform 3D cloud with $n_{3D} = 2.7 \times 10^{11} \text{ cm}^{-3}$ and 1.3×10^{10} atoms [68] and perform forced-evaporative cooling to increase n_{3D} to $2.7 \times 10^{12} \text{ cm}^{-3}$. We estimate that with such n_{3D} one can reach $\xi_{x(y)} = 2000$ with $L_{x(y)} = 2$ mm and $L_z \approx 0.2$ mm [73]. Moreover, DSPs' dynamics can be probed by directly measuring $\rho_{22}(x, y, t) \approx |\rho_{21}(x, y, t)|^2$ via the imaging beam in Fig. 1 [61], or by detecting $\rho_{21}(x, y, t)$ via density matrix reconstruction [74] or the EIT retrieval [34,70,75–77] at different directions as is done in tomography. We prepare the n th LL $\rho_{21}^{(n)}(x, y, t = 0)$ with $k = 0$ and observe its evolution. The dynamics of $\rho_{21}^{(n)}$ in the presence of \vec{A} shows two features: (i) Its eigen angular frequency $\omega = (n + 1/2)\omega_B$, and (ii) the last diffusion term in Eq. (4) causes the decay from $\rho_{21}^{(n+2)}$ to $\rho_{21}^{(n)}$, as illustrated in Fig. 2. $\rho_{21}^{(n)}$ is initially prepared by the EIT light storage for $t < 0.025$ ms [34]. Subsequently, we switch on the Landau-gauge EIT environment at $t = 0.025$ ms and analyze its evolution. In Fig. 2(a), gray dashed lines depict the theoretical $\omega_n/\omega_B = (n + 1/2)$. We calculate the angular frequency of ρ_{21} via $\omega = i(\partial_t \rho_{21})/\rho_{21}$ and illustrate them with solid curves for $n = 0, 1, 2, 3,$ with the following parameters: $\Gamma = 2\pi \times 5.2$ MHz, $\Delta_p = 1.26\Gamma$, $\Omega_c = 1.84\Gamma$, $\xi_x = \xi_y = 1400$, and $L_x = L_y = 4$ mm, which

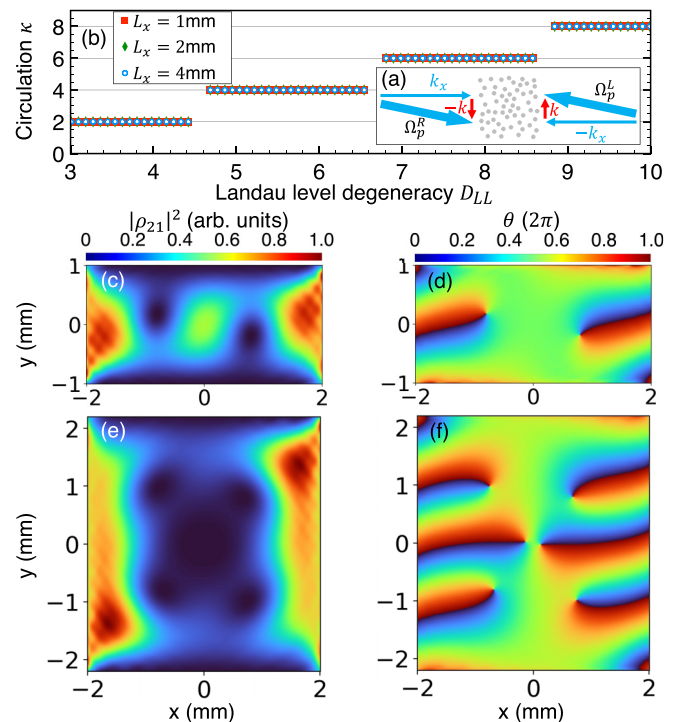


FIG. 3. Steady state ρ_{21} under suitable boundary conditions in (a) with slightly tilted probe beams. (b) D_{LL} -dependent circulation quanta κ for $L_x = 1$ mm (red squares), 2 mm (green diamonds), and 4 mm (blue circles) for $(l_B/L_x, \xi_x) = (0.15, 2000)$. (c) $|\rho_{21}|^2$ and (d) phase θ for $L_y = 2$ mm and $D_{LL} = 3.5$. (e) $|\rho_{21}|^2$ and (f) θ for $L_y = 4.4$ mm and $D_{LL} = 7.8$.

result in $\omega_B = 2.4 \times 10^{-3}\Gamma$, and $l_B = 0.34$ mm. During $0.025 \text{ ms} < t < 0.08 \text{ ms}$ there are four branches of the numerically solved ω matching the theoretical values very well up to 0.06 ms. Later on, the four branches converge in only two bands, which manifests prediction (ii). Figure 2(b) demonstrates the snapshot of $|\rho_{21}|^2$ for different input n at $t = 0.026$ ms, and Fig. 2(c) shows the results at $t = 0.188$ ms. One can observe that only $\Delta n = 2$ transitions occur. We then investigate the DSP characteristics with suitable injection of probe fields. We initially turn on all control fields to generate an in-plane synthetic \vec{B} and then pump the system by probe fields with Landau level boundary conditions $\Omega_p^R(-L_x/2, y) = \exp(-i\omega_B t -iky)$ and $\Omega_p^L(L_x/2, y) = \exp(-i\omega_B t +iky)$ with $k = L_x/(2l_B^2)$. This can be implemented by using two slightly tilted probe fields with wave vectors $(\mp k_x, \pm k)$ in Fig. 3(a), where $k_x \approx k_p$. The $\exp(-i\omega_B t)$ term can be realized by a phase modulation. The pumping frequency ω_B is chosen in the gap between $n = 0$ and $n = 1$ LL such that no eigenstate is excited in the bulk region. We analyze the steady-state $\rho_{21}(x, y) = |\rho_{21}(x, y)|e^{i\theta(x, y)}$ [78] by calculating the circulation $\kappa = \frac{1}{2\pi} \oint \vec{V}_p \cdot d\vec{l}$ along the system edge as a function of LL degeneracy $D_{LL} = L_x L_y / (2\pi l_B^2)$, where the current $\vec{V}_p = \nabla\theta$. With $(\xi_x, l_B/L_x, \omega_B, \Omega_c, \Delta_p) = (2000, 0.15, 1.7 \times 10^{-3}\Gamma, 1.84\Gamma, 5.62\Gamma)$, we sweep D_{LL} via elongating L_y and fixing $L_x = 1, 2,$ and 4 mm, respectively. Figure 3(b) depicts the quantization of κ for a fixed $l_B/L_x = 0.15$, i.e., the relative size of $n = 1$ LL wave function to the

medium length. Remarkably, our scalable system reveals a general feature that the κ plateaus are identical for different L_x . Each plateau indicates an equal number of vortices forming under \bar{A} . Using $(L_x, L_y) = (4 \text{ mm}, 2 \text{ mm})$, where $D_{LL} = 3.5$, we demonstrate $|\rho_{21}|^2$ and the phase θ in Figs. 3(c) and 3(d), respectively. Figure 3(c) illustrates an array of two vortices whose cores are located in each hollow of $|\rho_{21}|^2$. Figure 3(d) indicates that the topological winding number is 1 for each loop enclosing a vortex core. We numerically calculate the circulation $\kappa = \frac{1}{2\pi} \iint (\nabla \times \vec{V}_p) \cdot \hat{z} dx dy = 2$, which coincides with the number of vortices. Next, we increase D_{LL} to 7.8 by increasing L_y to 4.4 mm in Figs. 3(e) and 3(f). The phase in Fig. 3(f) manifests that the total vortex number and κ become 6. Notably, Fig. 3(e) shows the emergence of a DSP edge state and suggests a transition from a vortex array to an edge flow upon increasing L_y . We highlight the scalability of our system and its advantage over other schemes [26,54] in two aspects: Fig. 3(b) shows identical physics with scaled-up sizes, and Figs. 3(c)–3(f) manifest our studies with increasing L_y .

We now demonstrate a dynamical control over DSP edge states with $(\xi_x, l_B, \omega_B, \Omega_c, \Delta_p) = (2000, 0.4 \text{ mm}, 1.7 \times 10^{-3} \Gamma, 1.84\Gamma, 10\Gamma)$. Initially, only Ω_c^R is turned on, and then we pump the system by injecting Ω_p^R of transverse width $w = 0.5 \text{ mm}$, with the boundary condition $\Omega_p^R(-L_x/2, y) = \exp[-(y/w)^2 - i(\omega_B t - ky)]$ and $k = L_x/(2l_B^2)$ at the left edge. As depicted in Fig. 4(a), ρ_{21} occurs at 0.06 ms in both the edge ($|x| > 0.5 \text{ mm}$) and the bulk region ($|x| \leq 0.5 \text{ mm}$). Subsequently, Ω_c^L , Ω_c^F , and Ω_c^B are switched on at 0.07 ms to generate a synthetic out-of-plane \bar{B}_1 . $|\rho_{21}|^2$ subsequently vanishes in the bulk and clockwise propagates along the edge; see Fig. 4(b). We invert \bar{B}_1 at 0.27 ms and observe that the DSP edge flow becomes counterclockwise under the in-plane \bar{B}_1 in Fig. 4(c). Moreover, the DSP edge state even bypasses the vacuum area (white dashed square) in Fig. 4(d); namely, it is robust and immune to the defect [14–19,21–24,27,29]. We emphasize that in Figs. 4(e)–4(h) we use $(\xi_x, l_B, \omega_B, \Omega_c, \Delta_p) = (400, 0.5 \text{ mm}, 8.4 \times 10^{-3} \Gamma, 1.84\Gamma, 3.13\Gamma)$, confirming that the above robust chiral edge states can be clearly observed for $\xi_x \geq 400$ in our simulations. Figures 4(i)–4(l) illustrate the results of random ξ fluctuating around the average value 400 with maximum 10% variation [Fig. 4(i)]. The density [Figs. 4(j) and 4(l)] and θ [Fig. 4(k)] show that the winding topology is immune to the nonuniformity. Figure 4(m) shows the time sequence of the synthetic field \bar{B}_1 , which is used for the above results. Figure 4(m) also illustrates the time sequence of another in-plane field, \bar{B}_2 , which leads to the same results as in Figs. 4(c), 4(d), 4(g), 4(h), and 4(l). Given that the EIT-retrieved light inherits the phase of ρ_{21} [79–81], one can shine a control field along z to retrieve a probe beam carrying topological winding numbers [82–85].

We put forward an EIT-based optical approach to generate synthetic LLs and robust chiral edge states for neutral EIT DSPs in lattice-free media with experimentally realizable parameters. Our system is scalable and provides the capability of dynamical control. Moreover, our scheme can be a versatile quantum simulator [86] for different Hamiltonians and non-Hermitian systems [59,60,87], and it also can generate exotic

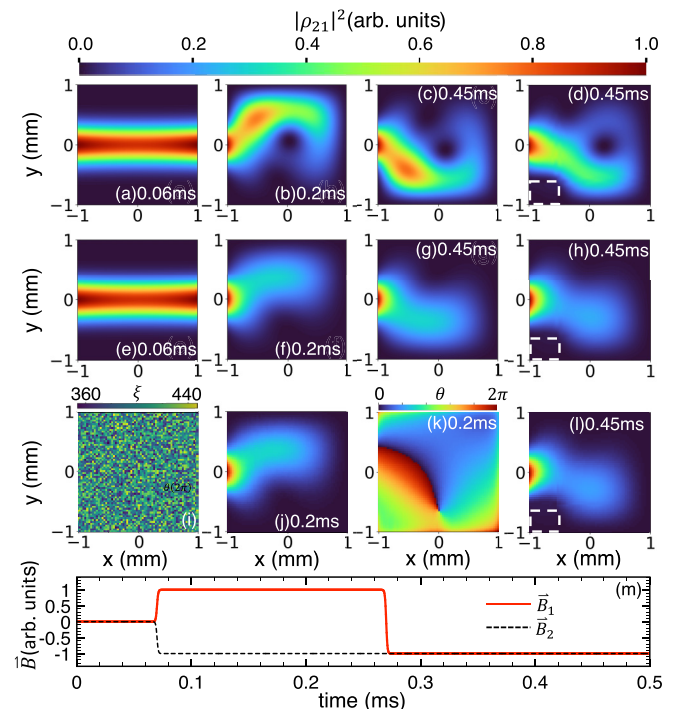


FIG. 4. Dynamical manipulation of initially rightward slow-light DSP entering the medium at $(x, y) = (-1 \text{ mm}, 0)$ [(a) and (e)]. Snapshots of the DSP edge state $|\rho_{21}|^2$ with out-of-plane \bar{B}_1 at 0.2 ms [(b), (f), and (j)] and in-plane synthetic \bar{B}_1 at 0.45 ms [(c), (d), (g), (h), and (l)]. In (d), (h), and (l) the DSP bypasses the vacuum area illustrated by the white dashed square. (j)–(l) Results with fluctuating optical depth $\xi(x, y)$ [shown in (i)] and $l_B = 0.5 \text{ mm}$, where (k) depicts the θ of (j). (m) The time sequence of the synthetic magnetic field \bar{B}_1 (solid red curve) for (a)–(l) where \bar{B}_1 is turned on at 0.07 ms and inverted at 0.27 ms. Another in-plane synthetic magnetic field, \bar{B}_2 (dashed black curve), also gives identical results to those in (c), (d), (g), and (h). We have uniform $(\xi_x, l_B) = (2000, 0.4 \text{ mm})$ for (a)–(d) and uniform $(\xi_x, l_B) = (400, 0.5 \text{ mm})$ for (e)–(h).

lights carrying topological phase winding numbers [82–85]. In our system, the scalability and forbidden decay between $n = 1$ and $n = 0$ LL also suggest the application of having many qubits by generating remote DSPs with Rydberg interactions [26]. Analogous to the realization of the Laughlin state of light in Ref. [26], Rydberg EIT [26,52] and a single photon source are viable combinations to introduce strong interaction and engineer a desired filling factor of bosonic quantum Hall state for our scheme.

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- $n_{3D} = 2.7 \times 10^{11} \text{ cm}^{-3}$ and then increase n_{3D} to $2.7 \times 10^{12} \text{ cm}^{-3}$ with a reduced number $N = 2.2 \times 10^9$. The volume is about 0.8 mm^3 , where we make $(L_x, L_y, L_z) = (2, 2, 0.2) \text{ mm}$. To make $L_x = 4 \text{ mm}$ with a fixed $\xi_x = 2000$, we use a smaller density $n_{3D} = 1.35 \times 10^{12} \text{ cm}^{-3}$ and a larger number $N = 3.7 \times 10^9$ with $(L_x, L_y, L_z) = (4, 2, 0.3) \text{ mm}$.
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