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Tomas Plankis

COMPUTER CALCULATIONS FOR SOME SEQUENCES AND POLYNOMIALS

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VILNIAUS UNIVERSITETAS

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KOMPIUTERINIAI SKAIČIAVIMAI KAI KURIOMS SEKOMS IR POLINOMAMS

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Su disertacija galima susipažinti Vilniaus universiteto bibliotekoje.

Summary of doctoral thesis

Scientific problem

In this thesis we will consider divisibility properties of some recurrent sequences, Newman polynomials and computer calculations in those and related questions of number theory.

Aims and problems

First we will investigate divisibility properties of recurrent sequences first. For this we consider the following questions:

- When is the sequence periodic?
- Has an infinite bounded sequence infinitely many zeroes?
- How the amount of zeros in the sequence depends on the choice of the first element?

Second, we will search for effective algorithms. Those algorithms are related to:

- recurrent sequences
- calculation of $\inf_{P \in \mathbb{P}_n} Q_2(P)$, where P is Newman polynomial and

$$Q_2(P) = (\deg(P) + 1)H(P^2)/P(1)^2.$$

Actuality

Mathematic and calculations are related. First mechanisms for calculations were used in antic times by mathematicians, engineers and merchants. Developing technologies mechanisms evolved. Informatics - the new branch of science which research information processing was born after computers development. Computers have big potential in calculations, especially for complex mathematical calculations or modelling. Such tasks have computational complexity problem. Computational complexity theory is a branch of the theory of computation in computer science that investigates the problems related to the resources required to run algorithms, and the inherent difficulty in providing algorithms that are efficient for both general and specific computational problems. Often it is important to know how much time an algorithm will run. One of the oldest questions in number theory is to find the integer sequence with infinitely many prime numbers. Thus, we decided to research the calculations of recurrent sequences. The other part of this thesis is related to

Newman polynomials. In the last few years they got much attention, because they appear in various questions of number theory.

Methods

In the first part of this work we are using analytical and elementary methods of number theory: modulo properties, induction, and some classical results of number theory like Ferma and Euler theorems. We used mathematical package Maple for algorithms and computational complexity theory to estimate the complexity of acquired algorithms.

In the second part we used C++ programming language, some combinatorial and statistical methods like regression analysis.

Novelty

All results presented in this thesis are new. They were presented at conferences (see "Approbation") and published in the refereed journals and others publications (see "Principal publications"). The results have both theoretical and practical aspects.

Structure of thesis. The thesis is written in lithuanian language. It consists of introduction, two sections, conclusions, bibliography and the list of publications. The size of the work - 67 pages.

Review and main results. There are several unsolved problems concerning the sequence $[\alpha^n]$, $n = 1, 2, 3, \ldots$, where $\alpha > 1$ is a fixed real number and where $[x]$ stands for the integral part of a real number x. It is indeed surprising that some of them are unsolved. For instance, if α is not an integer, even a simple question like whether or not the sequence $[\alpha^n]$, $n = 1, 2, 3, \ldots$, contains infinitely many composite numbers remains unsolved except for very few special cases. It is expected that there are infinitely many composite numbers of the form $[\alpha^n]$, $n \in \mathbb{N}$, for every $\alpha > 1$. Forman and Shapiro proved that for $\alpha = 3/2$ and $\alpha = 4/3$ and Dubickas and Novikas proved that for $\alpha = 5/4$. No other rational noninteger number α for which the sequence $[\alpha^n]$, $n = 1, 2, 3, \ldots$, is proved to contain infinitely many composite numbers is known. Guy has raised the question whether the sequence $[\alpha^n]$, $n = 1, 2, 3, \ldots$, where α is a rational noninteger number, contains infinitely many prime numbers. Naturally, this problem is expected to be even more difficult. It remains unsolved: not a single α with this property is known. However, Mills, Wright and later Alkauskas and Dubickas gave some existence results. One should remark that metrical results are well known. They are not difficult to obtain and follow from an old work of Koksma: for every $\xi \neq 0$, the sequence of fractional parts $\{\xi\alpha^n\}$, $n = 1, 2, 3, \dots$, is uniformly distributed in the interval $[0, 1)$ for almost all $\alpha > 1$. Applying this result to $\xi = 1/2$, we easily see that, for such $\alpha, \{\alpha^n/2\} < 1/2$ for infinitely many n. So, the sequence $[\alpha^n]$, $n = 1, 2, 3, \dots$, has infinitely many even numbers for almost all $\alpha > 1$. Hence the results for integral parts are closely related to relevant results for fractional parts.

The beginning of this work is related to the theorem of Alkauskas and Dubickas:

THEOREM. *Let*

$$x_n \equiv (x_{n-1}^n + n - 1) \pmod{g}, \tag{1}$$

where $n \in \mathbb{N}$, $x_0 \in \mathbb{N}$, *be the recurrent sequence. If* g *is prime number* > 2 *then the sequence have infinitely many zeroes .*

We are interested in the following questions:

- is this sequence periodic?
- has this sequence infinitely many zeroes?
- is there some dependence between the zeroes and the first element x_0?

First we considered the recurrent sequence of the theorem. Some results were proved and some computer calculations were made.

The next proposition is interesting because it is possible to prove it in a simple analytic way.

PROPOSITION 1.1. *Let* x_n *be the sequence (1) and* x_0 *is even positive integer. If* $g = 2^m$, $m > 1$ *and* $m \in \mathbb{N}$, *then the sequence has infinitely many zeroes.*

Separate group is made from Carmichael numbers.

DEFINITION 1.2. *A Carmichael number is a composite positive integer* n *which satisfies*

$$a^{n-1} \equiv 1 \pmod{n}$$

for every integer $a \geqslant 2$ *coprime to* n.

KORSELT'S THEOREM. *A composite integer* $n \in \mathbb{N}$ *is a Carmichael number if and only if* n *is square-free, and for all prime divisors* p *of* n, *it is true that* $(p-1)|(n-1)$.

Carmichael numbers are sometimes called "absolute pseudoprime" and satisfying Korselt's criterion. R. D. Carmichael was the first who found the first and smallest such number in 1910. The first few Carmichael numbers are 561, 1105, 1729, 2465, 2821, 6601, 8911, 10585, 15841, 29341, ...

PROPOSITION 1.3. *If g is a Carmichael number then the sequence (1) has infinitely many zeroes.*

DEFINITION 1.4. *We call the sequence $x_1, x_2, x_3, ...$ periodic if there exists a $t \in \mathbb{N}$ such that $x_n = x_{n+t}$, for every $n > N$, $N \in \mathbb{N}$.*

PROPOSITION 1.5. *Let x_n be the sequence (1). If $g \geqslant 2$ is a prime number then the sequence is periodic with period $g(g-1)$. If $g = 2$ then the sequence is periodic with period 4.*

PROPOSITION 1.7. *If $g = 2^m$, $m > 1$, then the sequence (1) is periodic with period g.*

PROPOSITION 1.8. *The sequence (1) is periodic for Carmichael numbers with period $g(g-1)$*

According to the statistical calculations we can guess for sure that the questions above are true for the sequence. Data was acquired for different groups of numbers: primes, the power of 2 and so on. Sequence is periodic but the zeroes in the sequence depends from the first element. We will formulate few conjectures.

CONJECTURE 1.9. *Let x_n be the sequence (1). If $x_0 = 2k$, $k = 0, 1, 2, ...$ then the sequence x_n has infinitely many zeroes.*

CONJECTURE 1.13. *Let x_n be the sequence (1). If $g = p^a$, where $a = 2, 3, ...$, then the sequence x_n has infinitely many zeroes.*

CONJECTURE 1.14. *Let x_n be the sequence (1). If $g = p^a$, where $a = 2, 3, ...$, then the sequence x_n is periodic with period $p^a(p-1)$.*

We must note that the last two Conjectures are related. Although we cannot prove them, but one can explain why they are expected to be correct. Both of them depends on Euler's theorem.

EULER'S THEOREM. *Let $\varphi(n)$ be Euler's totient function and $a, n \in \mathbb{N}$. If a is coprime to n then*

$$a^{\varphi(n)} \equiv 1 \pmod{n}$$

Some groups of sequences have interesting properties.

DEFINITION 1.10. *Let $g > 1$ is fixed. We call recurrent sequence g-stable if there exists such $N \in \mathbb{N}$ that all elements in the sequence are independent from x_0.*

EXAMPLE 1.11. *Let $a_0 = 2$, $g = 5$. Then*

Dubickas proved periodicity for more general case.

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
0	1	3	4	3	4	0	2	0	4	4	2	4	4	3
1	2	0	3	2	4	0	2	0	4	4	2	4	4	3
2	0	2	4	3	4	0	2	0	4	4	2	4	4	3
3	0	2	4	3	4	0	2	0	4	4	2	4	4	3
4	2	0	3	2	4	0	2	0	4	4	2	4	4	3

THEOREM. *Let x_0 and $m > 1$ be two integers, and let $F(z)$ be an arbitrary polynomialwith integer coefficients. Then the sequence defined by the recurrent relation $x_n = x_{n-1}^n + F(n), n = 1, 2, 3, \ldots,$ is ultimately periodic modulo g.*

Next we will study some more complicated sequences and will search for the effective algorithm to calculate them. We studied next recurrent sequences.

(1) $x_n \equiv x_{n-1}^{n^r} + 1 \pmod g$
(2) $x_n \equiv x_{n-1}^{n!} + 1 \pmod g$
(3) $x_n \equiv x_{n-1}^{r^n} + 1 \pmod g$

The following theorems were formulated.

THEOREM 1.15. *The sequence $x_n \equiv x_{n-1}^{n^r} + 1 \pmod g$ is periodic with the period $T \leqslant g\varphi(g)$ and pre-periodic part $t \leqslant [\sqrt[r]{\log_2(g)}] + 1 + g$.*

THEOREM 1.16. *The sequence $x_n \equiv x_{n-1}^{n!} + 1 \pmod g$ is periodic with the period $T \leqslant 2$ and pre-periodic part $t \leqslant \varphi(g)$.*

THEOREM 1.17. *The sequence $x_n \equiv x_{n-1}^{r^n} + 1 \pmod g$ is periodic with the period $T \leqslant g\varphi(\varphi(g))$ and pre-periodic part $t \leqslant [\log_2 \log_2(g)] + 1 + g\varphi(\varphi(g))$.*

According to the theorems above we developed and improved the algorithm to calculate sequences length and pre-periodic part's length. The improved algorithm has polynomial complexity instead of exponential, which is characteristic in the similar tasks of exhaustive search. One of the main theorems of the thesis is the following:

THEOREM 1.18. *Let d be a positive integer, $F(z_0, \ldots, z_{d-1}) \in \mathbb{Z}[z_0, \ldots, z_{d-1}]$, $f : \mathbb{N} \mapsto \mathbb{N}$ and $g : \mathbb{Z} \mapsto \mathbb{Z}$. Suppose that f and g are ultimately periodic modulo q for every integer $q \geqslant 2$, and $\lim_{n\to\infty} f(n) = \infty$. Let $x_1, \ldots, x_d \in \mathbb{Z}$ and*

$$x_{n+1} = F(x_n, \ldots, x_{n-d+1})^{f(n)} + g(n)$$

for $n = 1, 2, 3, \ldots$. Then, for each $m \geqslant 2$, the sequence $x_n \pmod m$, $n = 1, 2, 3, \ldots,$ is ultimately periodic.

The following corollary generalizes the main result in one of the previous papers of A. Dubickas.

Corollary 1.19. *Let $f : \mathbb{N} \mapsto \mathbb{N}$ and $g : \mathbb{Z} \mapsto \mathbb{Z}$ be two functions which are ultimately periodic modulo q for every integer $q \geqslant 2$, and $\lim_{n\to\infty} f(n) = \infty$. Suppose that $x_1 \in \mathbb{N}$ and*

$$x_{n+1} = x_n^{f(n)} + g(n)$$

for $n = 1, 2, 3, \ldots$. Then, for each $m \geqslant 2$, the sequence $x_n \pmod{m}$, $n = 1, 2, 3, \ldots,$ is ultimately periodic.

We also give a statement in the opposite direction:

Proposition 1.20. *Let $m \geqslant 3$ be an integer, which is not a power of 2, and let $f : \mathbb{N} \mapsto \mathbb{N}$. Suppose that $x_1 \in \mathbb{N}$ and*

$$x_{n+1} = x_n^{f(n)} + 1$$

for $n = 1, 2, 3, \ldots$. If the sequence $x_n \pmod{m}$, $n = 1, 2, 3, \ldots,$ is ultimately periodic, then there are positive integers q, n_0, t, where $2 \leqslant q \leqslant m - 1$, such that the sequence $f(n_0 + ut) \pmod{q}$, $u = 0, 1, 2, \ldots,$ is purely periodic.

The condition that m is not a power of 2 is essential. Evidently, any sequence given by $x_{n+1} = x_n^{f(n)} + 1$, where $f : \mathbb{N} \mapsto \mathbb{N}$, is purely periodic modulo 2. If $m = 2^s$, where $s \geqslant 2$, we can take any function $f : \mathbb{N} \mapsto \mathbb{N}$ satisfying $f(n) \geqslant s$ for each sufficiently large n. It is easy to see that, starting from some n_0, the sequence $x_n \pmod{2^s}$ is $1, 2, 1, 2, 1, 2, \ldots,$ so $x_n \pmod{2^s}, n = 1, 2, 3, \ldots,$ is ultimately periodic.

Lemma 1.22. *Let $f : \mathbb{N} \mapsto \mathbb{N}$ be a non-decreasing function satisfying*

$$\lim_{n\to\infty} f(n) = \infty$$

with the property that, for every $m \in \mathbb{N}$, there is an integer n_m such that $f(n+m) - f(n) \leqslant 1$ for each $n \geqslant n_m$. Then there is no arithmetic progression $au+b$, $u = 0, 1, 2, \ldots,$ with $a, b \in \mathbb{N}$ such that, for some $q \geqslant 2$, the sequence $f(au + b) \pmod{q}$, $u = 0, 1, 2, \ldots,$ is ultimately periodic.

Now we can construct more examples of non-periodic sequences. It is easy to see that the functions $f(n) = 1 + [\log n]$, $f(n) = [\alpha n^\sigma]$, where $\alpha \geqslant 1$ and $0 < \sigma < 1$, satisfy the conditions of the lemma. Hence, by proposition, the sequences given by $x_1 \in \mathbb{N}$ and

$$x_{n+1} = x_n^{1+[\log n]} + 1,$$

or

$$x_{n+1} = x_n^{[\alpha n^\sigma]} + 1,$$

are ultimately periodic modulo $m \in \mathbb{N}$, if and only if, $m = 2^s$ with some integer $s \geqslant 0$.

For a subset $\mathcal{A}$ of an abelian group (usually $\mathbb{Z}$), define

$$\mathcal{A}^*(k) := |\{(a_1, a_2) \in \mathcal{A} \times \mathcal{A} : a_1 + a_2 = k\}|$$

and

$$\mathcal{A}^\circ(k) := |\{(a_1, a_2) \in \mathcal{A} \times \mathcal{A} : a_1 - a_2 = k\}|.$$

In 1932, Simon Sidon considered sets of integers with both $\mathcal{A}^*$ and $\mathcal{A}^\circ$ bounded. It is easily shown that $\mathcal{A}^*(k) \leqslant 2$ for all k if and only if $\mathcal{A}^\circ(k) \leqslant 1$ for all $k \neq 0$. This led Sidon to ask how large a subset of $\{1; 2; \ldots; n\}$ can be with the property that $\mathcal{A}^*(k) \leqslant 2$? Since that time such sets, e.g., $\{1, 2, 5, 7\}$, have been known as Sidon sets.

Now we will give the definition of the Sidon set.

Definition. *Sidon sequence (or Sidon set) is a sequence $A = a_0, a_1, a_2, \ldots$ of natural numbers in which all pairwise sums $a_i + a_j, i \leqslant j$ are different.*

Some time ago Yu considered the quantity

$$\liminf_{k\to\infty} \deg(P_k) H(P_k^2)/P_k^2(1),$$

where $P_k, k = 1, 2, \ldots,$ is a sequence of Newman polynomials with $\deg(P_1) < \deg(P_2) < \ldots.$ He conjectured that this limit is always at least 1 if

$$P_k(1)/\deg(P_k) \to 0,$$

as $k \to \infty$. Martin and O'Bryant in their paper gave a lower bound for Sidon sets which was obtained using probabilistic methods. This result was also obtained by Cilleruelo for Newman polynomials. It was shown that a corresponding limit can be as small as $\pi/4$ and so the conjecture was disproved.

We will remind that Newman polynomials are those that have coefficients in the set $\{0, 1\}$.

Dubickas proved that it is sufficient to consider the quantity

$$Q_2(P) = (\deg(P) + 1) H(P^2)/P(1)^2,$$

because there is always a sequence of Newman polynomials $P_k, k = 1, 2, ...,$ with increasing degrees such that

$$\liminf_{k\to\infty} Q_2(P_k) = Q_2(P).$$

The purpose of the last work was to create an effective algorithm to calculate this quantity. The main result is given below.

Theorem 2.3. *For each Newman polynomial P of degree at most 36 we have*

$$Q_2(P) \geqslant Q_2(P_0) = 432/529 = 0.816635\ldots,$$

where the coefficients of the polynomial P_0 *of degree* 35 *in ascending order are given by*

$$110111010111111110101000000110101111.$$

Also we answered one of the few questions asked by Berenhaut and Saidak. They asked if is it true that $Q = 8/9$ occurs only for families of polynomials for which

$$\frac{P(1)}{\deg(P)} = \frac{3}{4}?$$

As we see in the following table, the answer is no.

Table 1: $Q_2(P)$ values for some polynomials P

Q_2	$\deg P$	$\frac{P(1)}{\deg P}$
$\frac{8}{9}$	3, 7, 11, 15	$1, \frac{6}{7}, \frac{9}{11}, \frac{4}{5}$
$\frac{15}{16}$	4, 9	$1, \frac{8}{9}$
$\frac{21}{25}$	13, 27, 34	$\frac{10}{13}, \frac{20}{27}, \frac{25}{34}$
$\frac{5}{6}$	19, 26	$\frac{12}{19}, \frac{9}{13}$
$\frac{240}{289}$	23, 29	$\frac{17}{23}, \frac{17}{29}$

We did not managed to create an effective algorithm but we managed to found some properties which can be useful to create one. For example, we found out that values $P(1)$ form the linear regression

$$y = \frac{9003}{6545} + \frac{4336}{6545}x.$$

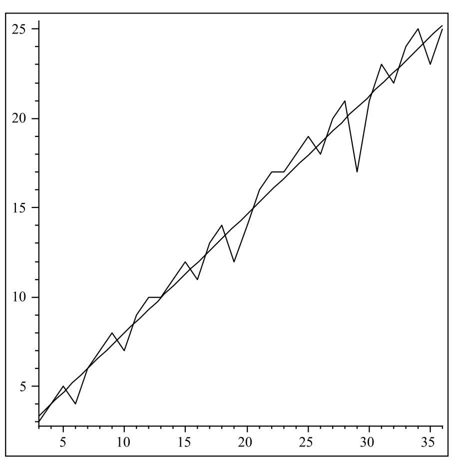

Figure 1: Graph for $P(1)$

Also, if our conjecture is correct then it is sufficient to check only those Newman polynomials whose degrees satisfy

$$\deg P = 2 \pmod 3.$$

Conclusions

In the thesis, the following results are established (see the subsection "Aims and problems"):

- We investigated the conditions for periodicity of recurrent sequences modulo m.
- We proved that there are infinitely many zeros in some sequences.
- We proved that the existence of zeros depends from the first element of sequence for some sequences.
- We found good enough algorithm for finding periodicity of recurrent sequences.
- We answered one of the few questions asked by Berenhaut and Saidak.
- We created algoritm to calculate $\inf_{P \in \mathbb{P}_n} Q_2(P)$ and found some dependences to create better algorithm.

Principal publications

The main results of the thesis are published in the following papers:

- T. Plankis, *Divisibility properties of a recurrent sequence,* in A. Laurinčikas and E. Manstavičius (eds.): Analytic and Probabilistic Methods in Number Theory (Proceedings of the fourth international conference in honour of J. Kubilius, Palanga, Lithuania, 25-29 September 2006), 150–155.
- T. Plankis, *Divisibility properties of recurrent sequences,* Nonlinear analysis : modelling and control, **13 (4)** (2008), 503–511.
- T. Plankis, A. Dubickas, *Periodicity of some recurrence sequences modulo m,* Integers, **8 (1)** (2008), #A42, 6 p.
- T. Plankis, *Calculations for Newman polynomials,* Šiauliai mathematical seminar **4 (12)** (2009), 157–165.

Approbation

Main results of the thesis were presented at the seminar on Number Theory of the Department of Probability theory and Number Theory of the Faculty of Mathematics and Informatics of Vilnius University, at the fourth international conference (Palanga, Lithuania, 2006) and at the PhD summer school in Number Theory and Probability (Druskininkai, Lithuania, 2007).

Reziumė

Šiame darbe aptariamos problemos, su kuriomis susidūriau studijuodamas doktorantūroje Vilniaus universitete. Aš tyrinėjau rekurenčiąsias sekas, Niumano polinomus ir kompiuterių galimybes skaičiuojant sekų periodus ir tam tikrą dydį Niumano polinomams, susijusį su Sidono sekomis. Darbą sudaro įvadas, 2 skyriai, išvados ir bibliografija. Pirmame skyriuje įrodomos kai kurių rekurenčiųjų sekų savybės ir pateikiamas pakankamai efektyvus algoritmas apskaičiuojantis sekos periodo ir priešperiodinės dalies ilgius. Antrame skyriuje pateikiami rezultatai apskaičiuojant dydį

$$Q_2(P) = (\deg(P) + 1)H(P^2)/P(1)^2.$$

Taip pat buvo atsakyta į vieną iš anksčiau Berenhauto ir Saidako iškeltų klausimų.

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