VILNIUS UNIVERSITY
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FINANCIAL AND ACTUARIAL MATHEMATICS MASTER'S STUDY PROGRAM

Master's thesis

On $n \times n$ recurrent determinants originating from survival probabilities in homogeneous discrete time risk model

Rekurentieji $n \times n$ determinantai kylantys iš išgyvenimo tikimybių homogeniniam diskretaus laiko rizikos modeliui

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1 Introduction

This is an analysis of certain $n \times n$ recurrent determinants arising ultimately from the Sparre Andersen risk model presented in [1], as follows:

$$W(t) = u + ct - \sum_{i=1}^{\Theta(t)} Z_i$$

where:

- $t \geq 0$;
- $u \geq 0$ denotes the initial surplus;
- $c > 0$ denotes the premium rate per unit of time;
- The cost of claims $\{Z_1, Z_2, ...\}$ are independent copies of a nonnegative random variable Z;
- The interoccurence times of claims $\{\theta_1, \theta_2, ...\}$ are independent copies of a nonnegative random variable θ, which further not degenerate at zero;
- The sequences $\{Z_1, Z_2, ...\}$ and $\{\theta_1, \theta_2, ...\}$ are independent;
- $\Theta(t) = \#\{n \geq 1 : T_n \leq t\}$ is the renewal process generated by the random variable θ, where $T_n = \theta_1 + \theta_2 + ... + \theta_n$.

In [2], particular focus was given to a special case of this model (referred to as the generalized premium discrete risk time model):

$$W(T) = u + \kappa t - \sum_{i=1}^{t} X_i \tag{1}$$

with the following additional conditions:

- $c = \kappa \in \mathbb{N}$;
- $\theta = 1$;
- $Z_i \stackrel{d}{=} X_i, i \in \mathbb{N}$, where X_i are independent copies of an integer-valued nonnegative random variable X;
- $u \in \mathbb{N}_0$;
- $t \in \mathbb{N}$;
- $W(0) = u$.

In this model, the ultimate time survival probability is defined as follows:

$$\varphi(t) := \mathbb{P}\left(\bigcap_{t=1}^{\infty}\{W(t) > 0\}\right)$$

It is also helpful to denote the local probabilities of the random variable X as $h_i = \mathbb{P}(X = i)$. With this denotation, and via a proof in [2], we can rewrite this equation recursively as:

$$\varphi(t) = \sum_{i=1}^{u+\kappa} h_{u+\kappa-i}\varphi(i). \tag{2}$$

Finally, we establish a net profit condition, $\mathbb{E}W(t) > 0$, such that we can always expect $W(t) > 0$ to have a nonzero probability for any $t \in \mathbb{N}$ — that is, that there is at least *some* chance that ruin will be avoided. By reformulating, we arrive at $\mathbb{E}X < \kappa$ as a way of satisfying this condition.

With this established, we thus proceed from the formulation in (2).

2 Conjectures

2.1 First conjecture

Again in [2], this recursive definition of φ in (2) is used in conjunction with certain recurrent equalities as part of a series of theorems involved in the calculation of the ultimate time survival probability. In particular, for the case where $\kappa = 2$, we are interested in the following:

$$\bar{\alpha}_0^{(0)} = 1, \bar{\alpha}_1^{(0)} = 0, \bar{\alpha}_n^{(0)} = \frac{1}{h_0}\left(\bar{\alpha}_{n-2}^{(0)} - \sum_{i=1}^{n-1} h_{n-i}\bar{\alpha}_i^{(0)}\right), n \geq 2,$$

and

$$\bar{\alpha}_0^{(1)} = 0, \bar{\alpha}_1^{(1)} = 1, \bar{\alpha}_n^{(1)} = \frac{1}{h_0}\left(\bar{\alpha}_{n-2}^{(1)} - \sum_{i=1}^{n-1} h_{n-i}\bar{\alpha}_i^{(1)}\right), n \geq 2.$$

These are used in the following relationship, which holds true if $h_0 > 0$ and $\mathbb{E}X < \kappa = 2$:

$$\begin{pmatrix} \bar{\alpha}_n^{(0)} & \bar{\alpha}_n^{(1)} \\ \bar{\alpha}_{n+1}^{(0)} & \bar{\alpha}_{n+1}^{(1)} \end{pmatrix} \times \begin{pmatrix} \varphi(0) \\ \varphi(1) \end{pmatrix} = \begin{pmatrix} \varphi(n) \\ \varphi(n+1) \end{pmatrix}. \tag{3}$$

The 2×2 matrix that appears in this equation is of particular interest in this analysis. The determinant of this matrix, $\bar{D}_n$, is defined as follows:

$$\bar{D}_n = \bar{\alpha}_n^{(0)}\bar{\alpha}_{n+1}^{(1)} - \bar{\alpha}_n^{(1)}\bar{\alpha}_{n+1}^{(0)}.$$

Conjecture 1. It is conjectured in [2] that the following two properties of $\bar{D}_n$ hold:

$$1 \leq \bar{D}_{2n} \leq \bar{D}_{2n+2} \tag{4}$$

and

$$-\frac{1}{h_0} \geq \bar{D}_{2n+1} \geq \bar{D}_{2n+3}. \tag{5}$$

While this conjecture is stated as unproven in [2], some progress has since been made, for example in [3].

2.2 Second conjecture

We can expand beyond the 2×2 case here to a general $n \times n$ (with correspondingly larger values of κ) by adjusting the sequences as follows:

n	$x_n^{(0)}$	$x_n^{(1)}$	$\cdots$	$x_n^{(k-1)}$
0	1	0	$\cdots$	0
1	0	1	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$k-1$	0	0	$\cdots$	1
$n \geq k$	$\frac{x_{n-k}^{(0)} - \sum_{i=1}^{n-1} h_{n-i} x_i^{(0)}}{h_0}$	$\frac{x_{n-k}^{(1)} - \sum_{i=1}^{n-1} h_{n-i} x_i^{(1)}}{h_0}$	$\cdots$	$\frac{x_{n-k}^{(k-1)} - \sum_{i=1}^{n-1} h_{n-i} x_i^{(k-1)}}{h_0}$

where $n \in \mathbb{N}_0$ and $k \geq 2$.

This then allows us to construct a new matrix (which is equivalent to the matrix in (3) in the case when $k = 2$):

$$\begin{pmatrix} x_n^{(0)} & x_n^{(1)} & \cdots & x_n^{(k-1)} \\ x_{n+1}^{(0)} & x_{n+1}^{(1)} & \cdots & x_{n+1}^{(k-1)} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n+k-1}^{(0)} & x_{n+k-1}^{(1)} & \cdots & x_{n+k-1}^{(k-1)} \end{pmatrix}$$

and consider its determinant D_n.

Conjecture 2. The second conjecture investigated in the following section is that (i) this matrix is nonsingular, and (ii) the following three properties of D_n hold, for all $n = 0, 1, 2...$:

$$0 < D_n \leq D_{n+1}, k \text{ is odd}, \tag{6}$$

$$0 < D_{2n} \leq D_{2n+2}, k \text{ is even} \tag{7}$$

and

$$0 > D_{2n+1} \geq D_{2n+3}, k \text{ is even}. \tag{8}$$

3 Calculations and examples

3.1 Conjecture 1

We start with a very simple choice for X:

$$\mathbb{P}(X=0)=\mathbb{P}(X=1)=\tfrac{1}{2}. \tag{9}$$

Applying this to the matrix from (3) gives us the following matrices for n:

$$\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}0&1\\2&-1\end{pmatrix},\begin{pmatrix}2&-1\\-2&3\end{pmatrix},\begin{pmatrix}-2&3\\6&-5\end{pmatrix},\begin{pmatrix}6&-5\\-10&11\end{pmatrix},\dots$$

By calculating the determinants of these matrices, we can then investigate the first part of the conjecture, (4):

$$1\le\bar{D}_0=1\le\bar{D}_2=4\le\bar{D}_4=16\le\bar{D}_6=64\le\cdots$$

and the second part, (5):

$$-\frac{1}{h_0}=-2\ge\bar{D}_1=-2\ge\bar{D}_3=-8\ge\bar{D}_5=-32\ge\bar{D}_7=-128\ge\cdots$$

We can then go on to examine a less straightforward distribution:

$$\begin{aligned}\mathbb{P}(X=0)=\mathbb{P}(X=10)&=\tfrac{1}{36},\\ \mathbb{P}(X=1)=\mathbb{P}(X=9)&=\tfrac{1}{18},\\ \mathbb{P}(X=2)=\mathbb{P}(X=8)&=\tfrac{1}{12},\\ \mathbb{P}(X=3)=\mathbb{P}(X=7)&=\tfrac{1}{9},\\ \mathbb{P}(X=4)=\mathbb{P}(X=6)&=\tfrac{5}{36},\\ \mathbb{P}(X=5)&=\tfrac{1}{6}.\end{aligned} \tag{10}$$

We get the following matrices:

$$\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}0&1\\36&-2\end{pmatrix},\begin{pmatrix}36&-2\\-72&37\end{pmatrix},\begin{pmatrix}-72&37\\1332&-144\end{pmatrix},\dots$$

and the following evaluations of the determinants:

$$1\le\bar{D}_0=1\le\bar{D}_2=1188\le\bar{D}_4=1267488\le\cdots$$

$$-\frac{1}{h_0}=-36\ge\bar{D}_1=-36\ge\bar{D}_3=-38916\ge\bar{D}_5=-41243904\ge\cdots$$

showing that the conjecture still holds.
We can go further by examining a distribution with infinite support (in this case, a geometric distribution with $p = \frac{1}{2}$):

$$\mathbb{P}(X = n) = (\tfrac{1}{2})^{n+1}, n = 0, 1, 2, 3, \ldots \tag{11}$$

with the matrices:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 2 & -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} 2 & -\frac{1}{2} \\ -1 & 2 \end{pmatrix}, \begin{pmatrix} -1 & 2 \\ 4 & -2 \end{pmatrix}, \cdots$$

and the determinants:

$$1 \le \bar{D}_0 = 1 \le \bar{D}_2 = \tfrac{7}{2} \le \bar{D}_4 = 10 \le \bar{D}_6 = 27 \le \cdots$$

$$-\frac{1}{h_0} = -2 \ge \bar{D}_1 = -2 \ge \bar{D}_3 = -6 \ge \bar{D}_5 = -\tfrac{33}{2} \ge \bar{D}_7 = -44 \ge \cdots$$

Finally, we consider a Poisson distribution, with $\lambda = 1$:

$$\mathbb{P}(X = n) = \frac{1}{e \cdot (n!)}, n = 0, 1, 2, 3, \ldots \tag{12}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ e & -1 \end{pmatrix}, \begin{pmatrix} e & -1 \\ -e & e + \frac{1}{2} \end{pmatrix}, \begin{pmatrix} -e & e + \frac{1}{2} \\ e^2 + \frac{e}{2} & 2e + \frac{1}{6} \end{pmatrix}, \cdots$$

$$1 \le \bar{D}_0 = 1 \le \bar{D}_2 = e^2 - \tfrac{e}{2} \le \bar{D}_4 \approx 27.221 \le \cdots$$

$$-\frac{1}{h_0} = -e \ge \bar{D}_1 = -e \ge \bar{D}_3 \approx -12.922 \ge \bar{D}_5 \approx -57.089 \ge \cdots$$

In all cases, the conjecture still holds.

3.2 Conjecture 2

We can go further and examine the general case for matrices beyond 2×2. We will use the same distributions in the previous subsection, to reflect a small sampling of distributions with different properties.

First, we'll use the simple distribution in (9) and the case $k = 3$. In this case, we get the matrices:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 2 & 0 & -1 \\ -2 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 & -1 \\ -2 & 2 & 1 \\ 2 & -2 & 1 \end{pmatrix}, \cdots$$

and can evaluate the determinants as they relate to (6), as k is here odd:

$$0 < D_0 = 1 \le D_1 = 2 \le D_2 = 4 \le D_3 = 8 \le D_4 = 16 \le \cdots$$

If we instead look at the case where $k = 4$, we get the following matrices:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & -1 \\ -2 & 2 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & -1 \\ -2 & 2 & 0 & 1 \\ 2 & -2 & 2 & -1 \end{pmatrix}, \cdots$$

and can similarly evaluate their determinants as compared to (7) and (8):

$$0 < D_0 = 1 \leq D_2 = 4 \leq D_4 = 16 \leq D_6 = 64 \leq \cdots$$

$$0 > D_1 = -2 \geq D_3 = -8 \geq D_5 = -32 \geq D_7 = -128 \geq \cdots$$

While the matrices are here omitted for space, we can also check the determinants for $k = 5$ (which for this distribution are the same as those for $k = 3$):

$$0 < D_0 = 1 \leq D_1 = 2 \leq D_2 = 4 \leq D_3 = 8 \leq D_4 = 16 \leq \cdots$$

and for $k = 6$ (which are likewise the same as for $k = 4$:

$$0 < D_0 = 1 \leq D_2 = 4 \leq D_4 = 16 \leq D_6 = 64 \leq \cdots$$

$$0 > D_1 = -2 \geq D_3 = -8 \geq D_5 = -32 \geq D_7 = -128 \geq \cdots$$

and so on.

Proceeding in the same way for the distribution in (10), for $k = 3$:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 36 & -3 & -2 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 36 & -3 & -2 \\ -72 & 38 & 1 \end{pmatrix}, \begin{pmatrix} 36 & -3 & -2 \\ -72 & 38 & 1 \\ 36 & -72 & 36 \end{pmatrix}, \cdots$$

$$0 < D_0 = 1 \leq D_1 = 36 \leq D_2 = 1152 \leq D_3 = 36324 \leq \cdots$$

and for $k = 4$:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 36 & -4 & -3 & -2 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 36 & -4 & -3 & -2 \\ -72 & 39 & 2 & 1 \end{pmatrix}, \cdots$$

$$0 < D_0 = 1 \leq D_2 = 1116 \leq D_4 = 995400 \leq D_6 = 816599520 \leq \cdots$$

$$0 > D_1 = -36 \geq D_3 = -33732 \geq D_5 = -28750608 \geq \cdots$$

and so on for $k = 5$ (which unlike the previous distribution has distinct determinants from $k = 3$:

$$0 < D_0 = 1 \le D_1 = 36 \le D_2 = 1080 \le D_3 = 31500 \le \cdots$$

and $k = 6$:

$$0 < D_0 = 1 \le D_2 = 1116 \le D_4 = 995400 \le D_6 = 816599520 \le \cdots$$

$$0 > D_1 = -36 \ge D_3 = -33732 \ge D_5 = -28750608 \ge \cdots$$

As before, we move on to a distribution with infinite support, (11), and $k = 3$:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -\frac{1}{4} & -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 2 & -\frac{1}{4} & -\frac{1}{2} \\ -1 & 2 & 0 \end{pmatrix}, \begin{pmatrix} 2 & -\frac{1}{4} & -\frac{1}{2} \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}, \cdots$$

$$0 < D_0 = 1 \le D_1 = 2 \le D_2 = \tfrac{15}{4} \le D_3 = 7 \le D_4 = 13 \le \cdots$$

$k = 4$:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & -\frac{1}{8} & -\frac{1}{4} & -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & -\frac{1}{8} & -\frac{1}{4} & -\frac{1}{2} \\ -1 & 2 & 0 & 0 \end{pmatrix}, \cdots$$

$$0 < D_0 = 1 \le D_2 = \tfrac{31}{8} \le D_4 = \tfrac{29}{2} \le D_6 = 54 \le \cdots$$

$$0 > D_1 = -2 \ge D_3 = -\tfrac{15}{4} \ge D_5 = -28 \ge D_7 = -\tfrac{833}{8} \ge \cdots$$

$k = 5$:

$$0 < D_0 = 1 \le D_1 = 2 \le D_2 = \tfrac{63}{16} \le D_3 \approx 7.751 \le D_4 \approx 15.252 \le \cdots$$

and $k = 6$:

$$0 < D_0 = 1 \le D_2 \approx 3.969 \le\approx 15.627 \le D_6 \approx 61.508 \le \cdots$$

$$0 > D_1 = -2 \ge D_3 \approx -7.876 \ge D_5 \approx -31.004 \ge \cdots$$

Finally, we examine the Poisson distribution, (12), for $k = 3$:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ e & -\frac{1}{2} & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ e & -\frac{1}{2} & -1 \\ -e & \sim 3.052 & \frac{1}{2} \end{pmatrix}, \cdots$$

$$0 < D_0 = 1 \leq D_1 = e \leq D_2 \approx 6.936 \leq D_3 \approx 17.646 \leq \cdots$$

$k = 4$:

$$0 < D_0 = 1 \leq D_2 \approx 7.273 \leq D_4 \approx 52.065 \leq \cdots$$

$$0 > D_1 = -e \geq D_3 \approx -19.46 \geq D_5 \approx -139.34 \geq \cdots$$

$k = 5$:

$$0 < D_0 = 1 \leq D_1 = e \leq D_2 \approx 7.366 \leq D_3 \approx 19.957 \leq \cdots$$

and $k = 6$:

$$0 < D_0 = 1 \leq D_2 \approx 7.385 \leq\approx 54.489 \leq D_6 \approx 402.175 \leq \cdots$$

$$0 > D_1 = -e \geq D_3 \approx -20.057 \geq D_5 \approx -147.998 \geq \cdots$$

In each examined case, and with each distribution, both parts of the conjecture can be calculated to hold.

Appendix

Presented here as an appendix is the Python code for several of the functions used in the calculations.

```
import numpy
import scipy.stats as stats

# Helper function to deal with floating point errors/rounding issues
def round2(n):
    n1 = round(n, 3)
    n2 = int(n1)
    if n1 == n2: return n2
    return n1

# Helper function to deal with floating point errors/rounding issues
def roundmat(m):
    m1 = m.round(3)
    m2 = m1.astype(int)
    if (m1 == m2).all(): return m2
    return m1
```

```
# Generates the individual sequences as columns to create
# the final matrix
def generate_column(column, length):
    data = []
    for row in range(length):
        # For the beginning of the sequence when n < k
        if row < k:
            if row == column:
                data.append(1)
                continue
            else:
                data.append(0)
                continue
        sum = 0
        for product_row in range(1, row):
            sum += probability(row - product_row) * data[product_row]
        new_value = (data[row - k] - sum) / probability(0)
        data.append(new_value)
    return data

# Combines the generated columns together into a k-width matrix
def generate_long_matrix(depth):
    columns = []
    for i in range(k):
        columns.append(generate_column(i, depth))
    return numpy.array(columns).T

# Main function
def main(prob):
    global probability
    global k
    probability = prob
    print(prob)
    print(numpy.array([prob(n) for n in range(20)]).round(3))
    for k in range(3, 7):
        print(f'\n\n\n\nk={k}')
        # Generate the long matrix at the appropriate depth
        long_matrix = roundmat(generate_long_matrix(5*2 + k + 4))
        for n in range(0, 5):
            print(f'\nn={n}')
            # For conjecture when k is even
            if k%2==0:
                mats = []
                dets = []
                for i in range(0,4):
```

```
            mats.append(long_matrix[2*n+i:2*n+i+k])
            dets.append(round2(numpy.linalg.det(mats[i])))
        print(f'\n2n={2*n}')
        print(mats[0])
        print(f'D={dets[0]}')
        print(f'\n2n+2={2*n+2}')
        print(mats[2])
        print(f'D={dets[2]}')
        if 0 < dets[0] <= dets[2]:
            print(f'0 < {dets[0]} <= {dets[2]}')
        else:
            print(f'err! 0, {dets[0]}, {dets[2]}')
        print(f'\n2n+1={2*n+1}')
        print(mats[1])
        print(f'D={dets[1]}')
        print(f'\n2n+3={2*n+3}')
        print(mats[3])
        print(f'D={dets[3]}')
        if 0 > dets[1] >= dets[3]:
            print(f'0 > {dets[1]} >= {dets[3]}')
        else:
            print(f'err! 0, {dets[0]}, {dets[2]}')
    # For conjecture when k is odd
    else:
        mat = long_matrix[n:n+k]
        det = round2(numpy.linalg.det(mat))
        print(mat)
        print(f'D={det}')
        if n == 0: continue
        previous_det = round2(numpy.linalg.det(long_matrix[n-1:n-1+k]))
        if 0 < previous_det < det:
            print(f'0 < {previous_det} < {det}')
        else:
            print(f'err! 0, {previous_det}, {det}')
```

References

[1] Andersen, E.S. On the collective theory of risk in case of contagion between the claims. *Trans XVth Int. Actuar.* **1957**, 2, 219-229.

[2] Grigutis, A.; Šiaulys, J. Recurrent Sequences Play for Survival Probability of Discrete Time Risk Mode. *Symmetry* **2020**, 12(12), 2111.

[3] Grigutis, A.; Jankauskas, J. On 2 × 2 determinants originating from survival probabilities in homogeneous discrete time risk model (preprint). *arXiv* **2021**, 2102.06987.