VILNIUS UNIVERSITY

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IMAGE ANALYSIS USING BAYES DISCRIMINANT FUNCTIONS

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VILNIAUS UNIVERSITETAS

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General Characteristic of the Dissertation

Topicality of the problem – image analysis is very important in medicine using computer diagnostic, in document analysis using letter and number recognition, and also important in such fields like defense, microscopy, remote sensing and others.

Method proposed in this dissertation is important classifying images, received with satellites, which are corrupted by noise, emerging from the natural phenomena such as fog, clouds or smoke during the fire. In some cases such noise can be modeled with Gaussian random fields.

Aim of the work – to classify optimally the observations of Gaussian random filed according to the dependency of the training sample. To propose analytic expressions of the error rates for the Bayes discriminant functions.

Tasks of the work

1. To apply the proposed supervised classification method in image analysis.
2. To examine the effectiveness of the proposed method experientially, comparing it with Bayes discriminant functions ignoring spatial dependency between the classifying point and the training sample. Also to use the unsupervised classification method based on gray level cooccurrence matrices for the comparison.
3. To examine the influence of the nearest neighbor number to the quality of the classification.
4. To examine numerically the dependences of the error rates for the Bayes discriminant functions on some statistical parameter values.

Scientific novelty

1. The original statistical supervised classification method, based on Bayes discriminant function (BDF) and spatial statistics elements, is proposed. This method is applied to image analysis.

2. The original Bayes error rate formulas are derived for the case, when features are modeled by the Gaussian random field, and class labels are modeled by the discrete Markov field.
3. The dependence of the derived error rates on some statistical parameter values is examined numerically.
4. The approximation of the proposed expected error rate, based on plug-in Bayes discriminant function, can be used as the spatial sampling design criterion.
5. The efficiency of the proposed method is examined experientially, using real image, image corrupted by the spatially correlated Gaussian random field noise with different spatial correlation range parameters and with image naturally corrupted by clouds.
6. The influence of the number of the nearest neighbors to the classification quality is examined experientially.
7. The proposed method is new, so it is not included in any statistical packages. This algorithm is realized in the environment of the statistical package R.

Methodology of research includes methods of probability theory, classical and spatial statistics. Several design and digital modeling methods are compared.

Practical value. Using the proposed method in practice, image classification results can be improved. This method is used for the cases, when noise in the classified data is with spatial dependency.

The realization of this method can be used as the example for the image classification.

The derived error rate expressions can be used as the measure for the effectiveness of the Bayes discriminant functions.

The approximation of the proposed expected error rate, based on the plug-in Bayes discriminant function, can be used as the spatial sampling design criterion.

Defended propositions

The original statistical supervised classification method is proposed. This method is based on Bayes discriminant function (BDF) and spatial statistics elements and can be used in image analysis.

The original Bayes error rate formula is derived, for the case, when features are modeled by Gaussian random field and class labels are modeled by the discreet Markov field.

The approximation of the expected error rate, based on plug-in Bayes discriminant function, is proposed and can be used as spatial sampling design criterion. Considering that features are modeled as Gaussian random field and class labels are modeled as discreet Markov field.

Expected Bayes error rate (EBER) is derived for the case of two classes and its dependency on the statistical parameter values is examined.

The scope of the scientific work. The scientific work consists of the general characteristic of the dissertation, 3 chapters, conclusions, list of literature and list of publication. The total scope of the dissertation – 117 pages, 61 equation, 30 pictures and 13 tables.

1. Spatial information and its use in image analysis

In this chapter spatial data and its sampling types are analyzed. Spatial population models and spatial relationships in populations are presented. Different neighborhood schemes are widely explained. The possibilities of applying spatial statistics in image analysis are discussed. At the end of this chapter its summery is presented and dissertation objectives are revised.

In order to apply the methods of spatial statistics, it is very important to know, how the data is arranged in the space. One of the main differences of spatial statistics from ordinary statistics is, that it is used to model not only for the spatial trend, but also for the spatial correlation.

Spatial data is any information about places, forms, relationships among them and geographical features. It covers remotely sensed data and also the map data. In short, spatial data is the result of observations performed in some space.

In order to have the knowledge about some characteristics from some space, the sample must be taken, because for the most cases there are no opportunities to do measurements in all points of the area. While creating the sample, as in all such situations, it is important that this sample was representative, evaluations must be as precise as possible and the sampling cost as little as possible. So, in this stage, the optimal sampling design is very important.

The main model in spatial statistics is the spatial data feature *Z* model:

$$Z(s) = \mu(s) + \varepsilon(s) \tag{1}$$

here $E(Z(s)) \equiv \mu(s)$ is called the mean function or the spatial trend and it can be constant or variable. $\varepsilon(s)$ is random error (noise) or spatial variance. It is also called the traditional additive data model, because as it is mentioned above it separates deterministic part from the random.

Data = spatial trend + spatial variance

The mathematical model of the random error in the area *D* is the random field $\{\varepsilon(s): s \in D\}$ with zero mean and covariance function *C*. According to the author Lopes H. F. (2008) the mostly used correlation functions are:

Exponential:

$$r_\alpha(h) = \exp\{-|h| / \alpha\} = e^{-\frac{|h|}{\alpha}} \tag{2}$$

Gaussian

$$r_\alpha(h) = \exp\{-h^2 / \alpha^2\} = e^{-\frac{h^2}{\alpha^2}} \tag{3}$$

and Mattern

$$r_{\alpha,\nu}(h) = \frac{1}{2^{\nu-1}\Gamma(\nu)}\left(2h\alpha\sqrt{\nu}\right)^{\nu} B_{\nu}\left(2h\alpha\sqrt{\nu}\right) \tag{4}$$

here h is the distance between two points, and α is the correlation range parameter. This parameter is very important, because it shows how far two points one from another are still related with each other.

To describe the variance of the spatial data covariance functions or semivariograms are used. Semivariograms are very important in geostatistics where they are used for kriging. Using semivariogram models in practice, it is possible to determine the dependency of the spatial data. According to the data, the empirical semivariogram is created and then the parametric model of the semivariogram can be fitted to it. Then this model can be used for predicting the unknown points of the area using kriging or in other cases.

Digital image is often a two dimensional number array. Every cell of the digital image is called a pixel and the number describing the intensity of this pixel is called a digital number (Liu and Mason 2009). The analysis of such images is used in different cases: military, astronomy, microscopy, remote sensing and others. Each of these cases have their own fields of applications with many specialized concepts and algorithms.

Text recognition is used very often. The scanned digital text information is transformed in to the text information. For this purpose the OCR (Optical Character Recognition) programs are used. But the methods of the spatial statistics can help here too. There are some cases when the scanned digital image is corrupted by some noise, and the recognition of such text looses its accuracy. If the noise which corrupted the text image can be modeled as the Gaussian random field, then spatial statistics methods proposed in this dissertation can be used to model this noise and to classify the image leaving only text information. When the noise is removed from the image, the text recognition with OCR programs becomes more accurate (Gupta et al. 2005), (Gupta et al. 2009).

The same principle can be used for the classification of the remotely sensed data, when in the real remotely sensed image some noise appears, which doesn’t let us to classify the information correctly. This can happen because of some natural phenomena such as smoke, fog or clouds. All these phenomena can be interpreted as spatially correlated field, and they can be modeled with Gaussian random fields.

The main objective of this dissertation is to propose the supervised classification method, based on Bayes discriminant functions, and used for image classification, when these images are corrupted by the spatially correlated noise.

2. Image, modeled by GRF, classification methods

In this chapter Bayes decision theory, discriminant functions and evaluation of classification are reviewed. The attention is focused on the supervised classification methods based on Bayes discriminant functions. Spatial dependency is incorporated into the classification problem. Feature observations are dependent and satisfy GRF model, and class labels satisfy discreet field model. Main subjects of this chapter are present in the papers of the author (Dučinskas and Stabingiene 2011) (Stabingiene and Dučinskas 2010) (Stabingiene and Dučinskas 2009).

If we consider, that classes are fully described and the prior class probabilities π_1, π_2, (π_1+π_2=1) are known and feature Z_0 is independent from training sample, then Bayes discriminant function (BDF) can be used for classification:

$$W_k(Z_0) = \left(Z_0 - \frac{1}{2}(\mu_1 + \mu_2) \right)(\mu_1 - \mu_2) / \sigma^2 + \gamma(k) \tag{5}$$

where $\gamma(k) = \ln\left(\pi_1(k)/\pi_2(k)\right)$, Z_0–feature of the observation to be classified.

In practice the situation with known population parameter values is very rare. In most cases these population parameters are unknown and they must be evaluated, and such functions are called plug-in Bayes discriminant functions PBDFI. For the case when Z_0 dos not depend on T=t the PBDFI used for classification is:

$$W_k\left(Z_0; \hat{\mu}; \hat{\sigma}^2\right) = \left(Z_0 - \frac{1}{2}\left(\hat{\mu}_1 + \hat{\mu}_2\right) \right)\left(\hat{\mu}_1 - \hat{\mu}_2\right)/\hat{\sigma}^2 + \gamma(k) \tag{6}$$

$$\hat{\mu} = (X_y' R^{-1} X_y)^{-1} X_y' R^{-1} Z = \begin{pmatrix} \hat{\mu}_1 \\ \hat{\mu}_2 \end{pmatrix}$$
$$\hat{\sigma}^2 = (Z - X_y \hat{\mu}) R^{-1} (Z - X_y \hat{\mu}) / (n-2)$$

here $Y = (Y(s_1),...,Y(s_n))'$ - label vector, $Z = (Z(s_1),...,Z(s_n))'$ - feature vector. $T' = (Z', Y')$ - training sample, X_y - design matrix, $\mu' = (\mu_1, \mu_2)$ and R – correlation matrix between the observation of training sample.

$$R = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{pmatrix}$$

here r_{ij} – correlation function between points s_i and s_j, $i, j=1,...,n$. In the realization part of this dissertation the exponential correlation function is used (eq. 2).

Very often training sample and classifying point are in the same correlated field, so it is important to incorporate the spatial dependency between classifying observation and the training sample (Dučinskas 2009), (Stabingienė et al. 2010).

BDF for classification of Z_0 dependent on $T=t$ ($\kappa=k$) is

$$W_{tk}(Z_0) = \left(Z_0 - \frac{1}{2}\left(\mu_{1t}^0 + \mu_{2t}^0\right)\right)\left(\mu_{1t}^0 - \mu_{2t}^0\right) / \sigma_{0t}^2 + \gamma(k) \quad (7)$$

$$\mu_{1t}^0 = E(Z_0 \mid T = t; Y(s_0) = 1) = \mu_1 + \alpha_0' \left(z_0 - X_y \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \right)$$

$$\mu_{2t}^0 = E(Z_0 \mid T = t; Y(s_0) = 2) = \mu_2 + \alpha_0' \left(z_0 - X_y \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \right)$$

$$\sigma_{0t}^2 = V(Z_0 \mid T = t; Y(s_0) = l) = \sigma^2 R_{0n}, \quad l = 1,2$$

where $\alpha_0' = r_0' R^{-1}$, $R_{0n} = 1 - r_0' R^{-1} r_0$. Here r_0–vector of correlations, described as $r_0' = (r_{01}\, r_{02} \cdots r_{0n})$, where r_{0i} is the value of the correlation function. And conditional distribution $Y(s_0)$ for known $\kappa=k$ is described only by the labels of the N_0, i.e.

$$\pi_1(k) = P(Y(s_0) = 1 \mid \kappa = k) = 1/(1 + \exp(-\lambda(k))$$

$$\pi_2(k) = 1 - \pi_1(k), \quad k = 0,..,K.$$

$$\lambda(k) = k\rho / K,$$

PBDF for the classification of the Z_0 dependent on $T=t$ ($\kappa=k$) (Dučinskas 2009) is:

$$W_{tk}\left(Z_0;\hat{\mu};\hat{\sigma}^2\right)=\left(Z_0-\frac{1}{2}\left(\hat{\mu}_{1t}^0+\hat{\mu}_{2t}^0\right)\right)\left(\hat{\mu}_{1t}^0-\hat{\mu}_{2t}^0\right)/\hat{\sigma}_{0t}^2+\gamma(k) \qquad (8)$$

$$\hat{\mu}_{1t}^0=E(Z_0\mid T=t;Y(s_0)=1)=\hat{\mu}_1+\alpha_0'\left(z_n-X_y\begin{pmatrix}\hat{\mu}_1\\ \hat{\mu}_2\end{pmatrix}\right)$$

$$\hat{\mu}_{2t}^0=E(Z_0\mid T=t;Y(s_0)=2)=\hat{\mu}_2+\alpha_0'\left(z_n-X_y\begin{pmatrix}\hat{\mu}_1\\ \hat{\mu}_2\end{pmatrix}\right)$$

$$\hat{\sigma}_{0t}^2=V(Z_0\mid T=t;Y(s_0)=l)=\hat{\sigma}^2R_{0n}.$$

The exact error rate EER for the Bayes discriminant function $W_{tk}\left(Z_0\right)$ is derived (Stabingienė and Dučinskas 2009):

$$P_0\left(S_n\right)=\pi_0\Phi\left(-\Delta_{0n}/2\right)+2\sum_{k=1}^{K}\sum_{l=1}^{2}\pi_k\pi_l(k)\Phi\left(-\Delta_{0n}/2+(-1)^l\lambda(k)/\Delta_{0n}\right)$$

Table 1. Values of the $P_0(S_n)$ with $\Delta_0=1$, $\pi_0=0.6$, $\pi_1=0.3$, $\pi_2=0.1$

ρ \ α	0.5	1	1.5	2	2.5	3
0	0.30237	0.26946	0.23562	0.20671	0.18229	0.16151
0.2	0.30180	0.26901	0.23526	0.20642	0.18205	0.16130
0.4	0.30011	0.26766	0.23419	0.20554	0.18131	0.16068
0.6	0.29735	0.26545	0.23243	0.20410	0.18011	0.15965
0.8	0.29358	0.26243	0.23002	0.20212	0.17845	0.15824
1	0.28891	0.25866	0.22700	0.19964	0.17638	0.15648
1.2	0.28343	0.25422	0.22343	0.19671	0.17391	0.15438
1.4	0.27726	0.24919	0.21938	0.19337	0.17111	0.15199
1.6	0.27052	0.24367	0.21491	0.18967	0.16800	0.14933
1.8	0.26334	0.23775	0.21009	0.18568	0.16462	0.14645
2	0.25581	0.23150	0.20499	0.18143	0.16104	0.14338

In table 1 the results of EER calculations are shown for the classes with high separation. From the results it is seen, that when the α parameter increases errors are getting smaller. The same situation is for the ρ parameter. The same is obtained for less separable classes too.

The expected error rate E_PER for the plug-in Bayes discriminant function PBDF is investigated (Stabingienė and Dučinskas 2010):

$$AEP_0 = \sum_y \sum_{l=1}^{2} \pi(y)\pi_l(y)\Phi\left(-\Delta_{0n}/2 + (-1)^l \gamma(y)/\Delta_{0n}\right) +$$
$$\sum_y \sum_l \pi_l(y)\pi(y)\phi(Q_1(y))(C(y) + 2\gamma^2(y)/(n-2))/\Delta_{0n}$$

here $\varphi(\cdot)$ is the density function of standard normal distribution and

$$C(y) = \Lambda' R_\mu \Lambda \Delta^2{}_{0n}/\rho_0, \; \Lambda = X'_y \alpha_0 - H/2 + \gamma(y)G/\Delta^2{}_{0n}.$$

Table 2. Values of the AEP_0 with Δ_0=0.2, π_4=0.5, π_3= π_5=0.15, π_2= π_6=0.1

ρ \ α	0.5	1	1.5	2	2.5	3
0	0.45957	0.45132	0.44314	0.43569	0.42891	0.42269
0.4	0.45462	0.44710	0.43959	0.43274	0.42651	0.42078
0.8	0.44256	0.43633	0.43023	0.42479	0.41992	0.41542
1.2	0.42771	0.42242	0.41769	0.41396	0.41094	0.40820
1.6	0.41229	0.40761	0.40408	0.40222	0.40138	0.40078
2	0.39718	0.39291	0.39056	.390786	0.39251	0.39447

Table 3. Values of the $P_{0n}$with Δ_0=0.2 and with different α and ρ values

ρ \ α	0.5	1	1.5	2	2.5	3
0	0.45877	0.45110	0.44271	0.43503	0.42805	0.42166
0.4	0.44698	0.44095	0.43394	0.42725	0.42100	0.41518
0.8	0.42014	0.41655	0.41191	0.40712	0.40240	0.39783
1.2	0.38851	0.38656	0.38372	0.38055	0.37725	0.37392
1.6	0.35633	0.35537	0.35374	0.35173	0.34950	0.34715
2	0.32529	0.32490	0.32405	0.32285	0.32140	0.31979

The results of numerical analysis in table 2 states, with higher cauterization of class labels and stronger spatial correlation between features of observations guarantees smaller spatial classification errors.

Expected Bayes error rate for the function $W_t(Z_0)$ was derived (Dučinskas and Stabingienė 2011):

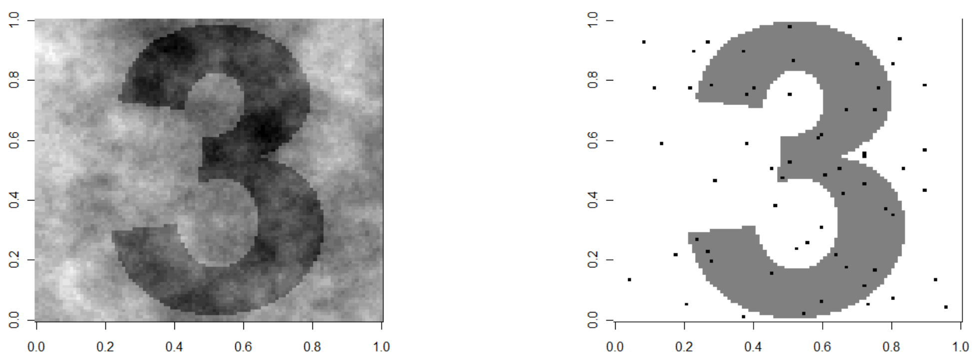
$$P_{on} = \sum_{j=0}^{n}\sum_{l=1}^{2}\pi_j \Phi\left(-\Delta_{0n}/2 + (-1)^l \rho(2j/n-1)/\Delta_{0n}\right)\Big/\left(1+\exp\left\{(-1)^l \rho(2j/n-1)\right\}\right)$$

From the table 3 can be seen, that EBER is monotonically decreasing when α and ρ values increases.

3. Image, modeled by GRF, classification methods

In this chapter the proposed method is applied to the images corrupted by spatially correlated noise. Here is done the reconstruction of black and white image corrupted by the spatially correlated noise. Also the classification of the remotely sensed image is presented. Main subjects of this chapter are present in the papers of the author (Stabingienė et al 2011), (Stabingienė et al 2010), (Dučinskas et al 2011). Also the classification of the real image naturally corrupted by clouds is presented.

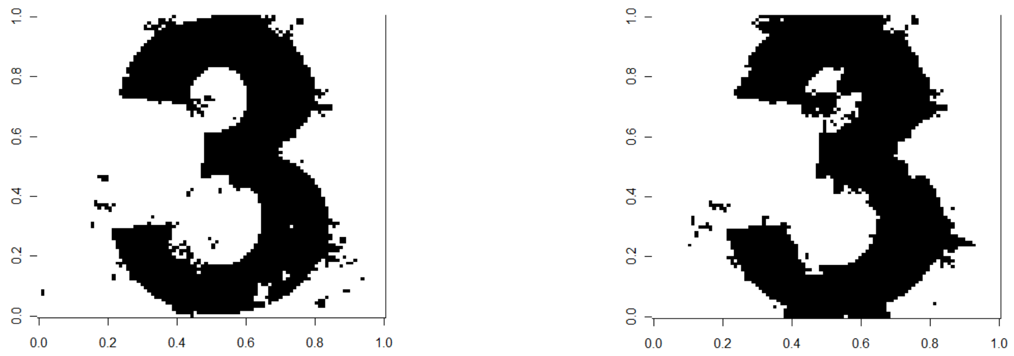
First experiment present the reconstruction of black and white image, corrupted by additive spatially correlated noise.



a) b)

Fig. 1. a) Black and white image of the number corrupted by the additive Gaussian random filed, b) points of the training sample used for classification.

As shown in Fig. 1a the image of number 3 is artificially covered by the noise, generated by Gaussian random filed. This field was created using statistical program R and its package geoR. In Fig. 1b the training sample used in classification of the image is presented. Training sample in this experiment consists of 60 points, with 30 points for each class.



a) b)

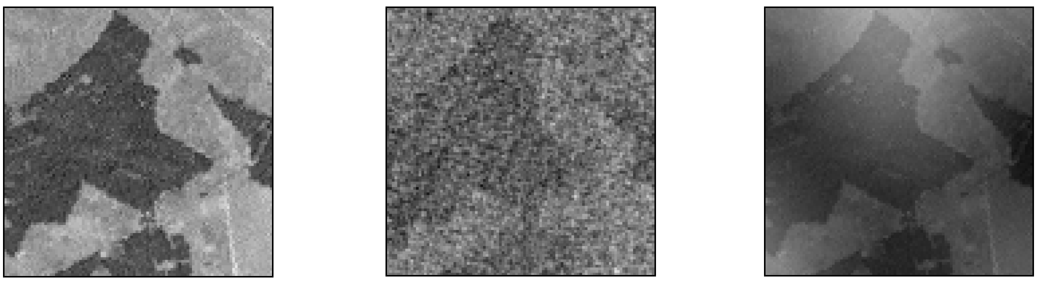
Fig. 2. a) Classification results for PBDF method, b) Classification results for PBDFI method.

Table 4. Empirical probabilities of misclassification for different classes (Stabingienė et. al. 2010).

Method	PBDF	PBDFI
P(2|1)	0.0206	0.0236
P(1|2)	0.0388	0.0771

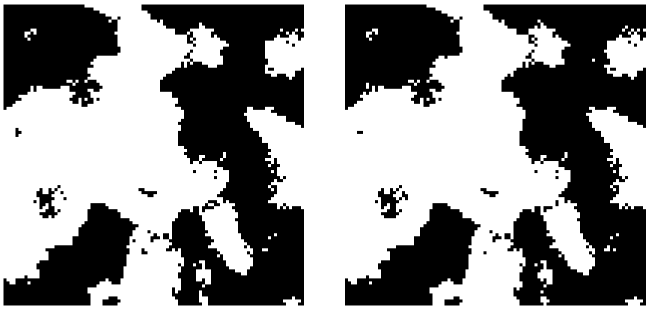
The results of classification are shown in Fig. 2 and in table 4. As it is seen from the results PBDF method performs better then the PBDFI method.

In the next example the remotely sensed image is used for the experiment. The image is acquired with Landsat7 satellite. The image is from Lithuania territory. As in the previous example the image is covered by the additive Gaussian random field which in this situation can be interpreted as cloud, fog or smoke cover. Fig. 3a shows the original image remotely sensed by the satellite. Fig. 3b and Fig. 3c are the images used for classification. Training sample consists of 60 points with 30 points for every class.



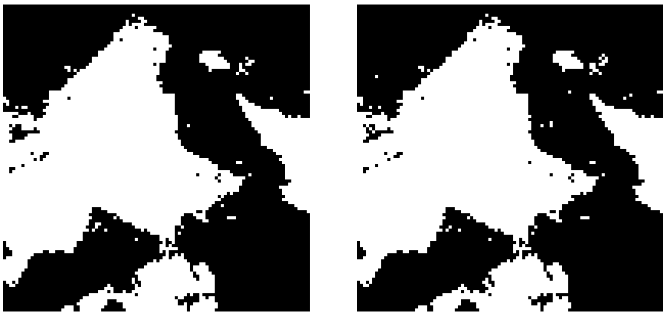
a) b) c)

Fig. 3. a) Real part of the remotely sensed image, b) remotely sensed image corrupted by Gaussian random field with α=10, c) remotely sensed image corrupted by Gaussian random field with α=50.



PBDF PBDFI

Fig. 4. Classification results of Fig. 3b image with different methods.



PBDF PBDFI

Fig. 5. Classification results of Fig. 3c image with different methods.

Table 5. Empirical probabilities of misclassification for different classes for different α values of Gaussian random field. (Stabingienė et al. 2011)

α	PBDF		PBDFI	
	$\hat{P}_{(2\|1)}$	$\hat{P}_{(1\|2)}$	$\hat{P}_{(2\|1)}$	$\hat{P}_{(1\|2)}$
10	0.217	0.073	0.222	0.077
50	0.150	0.065	0.154	0.067

The results presented in fig. 4, fig. 5 and in table 5 show that in this situation PBDF method performed slightly better then the PBDFI method. Also from the results can be noticed, that when correlation range parameter α grows, these both methods performs better.

General Conclusions

After the proposed PBDF method was applied to image analysis and compared with other methods, the conclusions were formulated:

1. In all presented experiments PBDF method was superior to the PBDFI method. It means that it is important to take into account the spatial dependency between classifying observation and the training sample during the classification.
2. The experiment results showed that when the spatial correlation range parameter grows, the results based on plug-in Bayes discriminant functions become more accurate. This doesn't hold for the other methods which are here compared with the PBDF and PBDFI methods.
3. From the investigation of the BDF error rate dependency on statistical parameter values, follows that bigger dependency between class labels and stronger spatial correlation between feature observations gives smaller error values. Also, observations with stronger spatial correlation can be classified more accurately (with the proposed method).
4. After the classification of the remotely sensed image naturally covered with clouds it can be stated that incorporation of the spatial dependency between classified observation and training sample produces better results. In this experiment the correlation range parameter was obtained $\alpha = 13.0305$, and it confirms, that in some cases, clouds are spatially correlated, so they can be modeled as Gaussian random field.

List of Published Works on the Topic of the Dissertation

In the reviewed scientific periodical publications

Dučinskas, K., Stabingienė, L. (2011). Expected Bayes error rate in supervised classification of spatial gaussian data. *Informatica*. Volume 22, No. 3, 371-381. ISSN 0868-4952.

Stabingienė, L., Stabingis, G., Dučinskas, K. (2011). Comparison of images modeled by Gaussian Random Fields. *Lietuvos matematikos rinkinys. LMD darbai*, Volume 52, 200-204. ISSN 0132-2818.

Stabingienė, L., Dučinskas, K. (2010). Error rates in spatial classification of Gaussian data with random labelling. *Lietuvos matematikos rinkinys. LMD darbai*, Volume 51, 426-430. ISSN 0132-2818.

Stabingienė, L., Stabingis, G., Dučinskas, K. (2010). Comparison of linear discriminant functions in image classification. *Lietuvos matematikos rinkinys. LMD darbai*, Volume 51, 227-231. ISSN 0132-2818.

In the reviewed scientific publications of conference proceedings

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About the author

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Graduated from Klaipėda University Faculty of Nature and Mathematics in 2005 acquiring Bachelor's Degree in Mathematics. Gained Master's Degree in Mathematics at Klaipėda University Faculty of Nature and Mathematics in 2007. Since 2007 works as assistant in Department of Statistics at Klaipėda University. Since 2010 also works as Junior Researcher in Department of Statistics at Klaipėda University. Since 2011 to 2012 was a external student at Vilnius University Institute of Mathematics and Informatics.

Vaizdų analizė naudojant Bajeso diskriminantines funkcijas

Mokslo problemos aktualumas – vaizdų analizė yra svarbi medicinoje taikant kompiuterinei diagnostikai, dokumentų vaizdų analizė naudojama raidžių ar skaičių atpažinimui, svarbi srityse, tokiose kaip gynyboje, mikroskopijoje, Žemės stebėjime iš palydovų ir pan.

Darbe siūloma metodika ypač aktuali klasifikuojant vaizdus, gautus iš palydovų, kurie yra sugadinti triukšmo, atsirandančio dėl gamtos sąlygų, reiškinių, tokių kaip rūkas, debesuotumas, dūmai gaisro metu. Tam tikrais atvejais toks triukšmas gali būti modeliuojamas Gauso atsitiktiniais laukais.

Darbo tikslas – optimaliai klasifikuoti Gauso atsitiktinio lauko stebinius priklausomus nuo mokymo imties. Pateikti analitines klaidų tikimybių išraiškas Bajeso diskriminantinėms funkcijoms.

Darbo uždaviniai:

1. Pasiūlytą klasifikavimo su mokymu metodiką pritaikyti vaizdų analizėje.
2. Pasiūlytos metodikos veiksmingumą patikrinti eksperimentų būdu, palyginant su Bajeso diskriminantinėmis funkcijomis, ignoruojančiomis erdvinę priklausomybę tarp klasifikuojamo taško ir mokymo imties. Palyginimui panaudoti klasifikavimo be mokymo metodą, paremtą pilkumo lygio pasikartojimų matricomis.
3. Ištirti artimiausių kaimynų skaičiaus įtaką vaizdo klasifikavimo kokybei.
4. Skaitiškai panagrinėti Bajeso diskriminantinių funkcijų klaidų tikimybių priklausomybes nuo tam tikrų statistinių parametrų reikšmių.

Mokslinis naujumas

1. Pasiūlytas originalus statistinio klasifikavimo su mokymu metodas, pagrįstas Bajeso diskriminantine funkcija (BDF) ir erdvinės statistikos elementais. Šis metodas pritaikomas vaizdų analizėje.
2. Atvejui, kai požymis modeliuojamas atsitiktiniu Gauso lauku, o klasės žymė modeliujama diskrečiu Markovo lauku, išvestos originalios formulės Bajeso klaidos tikimybėms.
3. Skaitmeniškai nagrinėjama išvestų klaidų tikimybių priklausomybė nuo tam tikrų statistinių parametrų reikšmių.
4. Pasiūlytos tikėtinos klaidos tikimybės, susijusios su Bajeso įterpta diskriminantine funkcija, aproksimacija, gali būti naudojama kaip erdvinių imčių plano kriterijus.
5. Pasiūlyto metodo veiksmingumas yra tikrinamas eksperimentų būdu, naudojant realų vaizdą ir vaizdus, sugadintus su erdvėje koreliuotu Gauso atsitiktinio lauko triukšmu su skirtingais erdvinės koreliacijos pločio parametrais.

6. Eksperimentų būdu yra tiriama panaudojamo artimiausių kaimynų skaičiaus (markoviškumo eilės) įtaka klasifikuojamo vaizdo kokybei.
7. Siūloma metodika yra nauja, todėl jokiuose statistiniuose paketuose, nėra įdiegta. Gana sudėtingas algoritmas yra realizuotas R sistemos aplinkoje.

Tyrimų metodika apima tikimybių teorijos, klasikinės ir erdvinės statistikos metodus, palyginimą iš kelių planų bei skaitmeninio modeliavimo metodus.

Praktinė vertė. Naudojant praktikoje pasiūlytą metodą yra pagerinami klasifikuojamo vaizdo rezultatai. Šis metodas ypač aktualus atveju, kada duomenyse esantis triukšmas pasižymi erdvine priklausomybe.

Pasiūlyto metodo realizacija gali būti pavyzdžiu klasifikuojant vaizdus.

Išvestos klaidų tikimybių išraiškos gali būti naudojamos kaip Bajeso diskriminantinių funkcijų veikimo matmuo.

Pasiūlytos tikėtinos klaidos tikimybės, susijusios su Bajeso įterpta diskriminantine funkcija, aproksimacija, gali būti naudojama kaip erdvinių imčių plano kriterijus.

Ginamieji teiginiai

1. Pasiūlytas originalus vaizdų analizėje taikomas statistinio klasifikavimo su mokymu metodas, pagrįstas Bajeso diskriminantine funkcija (BDF) ir erdvinės statistikos elementais.
2. Atvejui, kai požymis modeliuojamas atsitiktiniu Gauso lauku, o klasės žymė modeliujama diskrečiu Markovo lauku, išvesta originali formulė Bajeso klaidos tikimybei.
3. Pasiūlyta tikėtinos klaidos tikimybės, susijusios su Bajeso įterpta diskriminantine funkcija, aproksimacija, panaudojama kaip erdvinių imčių plano kriterijus. Laikant, kad požymis modeliuojamas atsitiktiniu Gauso lauku, o klasės žymė modeliujama diskrečiu Markovo lauku.
4. Išvesta vidutinė Bajeso klaidos tikimybė (EBER) dviejų klasių atveju ir ištirta jos priklausomybė nuo statistinių parametrų reikšmių.

Darbo rezultatų aprobavimas

Disertacijoje atliktų tyrimų rezultatai buvo paskelbti tarptautinėje ir nacionalinėse mokslinėse konferencijose:

Tarptautinėje taikomų stochastinių modelių ir duomenų analizės konferencijoje (ASMDA), 2009 m., Vilniuje;

Lietuvos matematikų draugijos konferencijoje, 2010 m., Šiauliuose.

Lietuvos matematikų draugijos konferencijoje, 2011 m., Vilniuje;

Lietuvos matematikų draugijos konferencijoje, 2012 m., Klaipėdoje.

Straipsniai disetacijos tema recenzuojamuose mokslo žurnaluose:

Dučinskas, K., Stabingienė, L. (2011). Expected Bayes error rate in supervised classification of spatial gaussian data. *Informatica*. Volume 22, No. 3, 371-381. ISSN 0868-4952.

Stabingienė, L., Stabingis, G., Dučinskas, K. (2011). Comparison of images modeled by Gaussian Random Fields. *Lietuvos matematikos rinkinys. LMD darbai*, Volume 52, 200-204. ISSN 0132-2818.

Stabingienė, L., Dučinskas, K. (2010). Error rates in spatial classification of Gaussian data with random labelling. *Lietuvos matematikos rinkinys. LMD darbai*, Volume 51, 426-430. ISSN 0132-2818.

Stabingienė, L., Stabingis, G., Dučinskas, K. (2010). Comparison of linear discriminant functions in image classification. Lietuvos *matematikos rinkinys. LMD darbai*, Volume 51, 227-231. ISSN 0132-2818.

Straipsniai disertacijos tema konferencijų pranešimų recenzuojamuose leidiniuose:

Dučinskas, K., Stabingienė, L., Stabingis, G. (2011). Image classification based on Bayes discriminant functions. In Proceedings of the 1th International Conference „Spatial Statistics 2011-Mapping Global Change“. Procedia Environmental Sciences, Volume 7, selected papers, 218-223. Elsevier. ISSN 1878-0296.

Stabingienė, L., Dučinskas, K. (2009). Exact error rates of the supervised classification based on Markov random fields. In Proceedings of the 13th International Conference „Applied Stochastic Models and Data Analysis“

(ASMDA-2009): selected papers, 120-123, Vilnius: Technika. ISBN 978-9955-28-463-5.

Darbo apimtis. Darbą sudaro bendra darbo charakteristika, 3 skyriai, išvados, literatūros sąrašas ir publikacijų sąrašas. Bendra disertacijos apimtis – 117 puslapių, 61 formulė, 30 iliustracijų, 13 lentelių.

Pirmame disertacijos skyriuje analizuojami erdvinių duomenų ir imčių tipai. Supažindinama su erdvinių populiacijų modeliais bei erdvinių sąryšių struktūromis populiacijoje. Plačiau pateiktos kaimynystės chemos. Aptariamos erdvinės statistikos taikymo galimybės vaizdų analizėje. Skyriaus pabaigoje pateikiamas apibendrinimas ir tikslinami disertacijos uždaviniai.

Antrame disertacijos skyriuje apžvelgiama Bajeso sprendimo teorija, diskriminantinės funkcijos, klasifikavimo vertinimas. Pagrindinis dėmesys skiriamas klasifikavimo su mokymu metodams, paremtiems Bajeso diskriminantinėmis funkcijomis. Į klasifikavimo problemą yra įvedama erdvinė priklausomybė. Požymių stebiniai priklausomi ir tenkina GRF modelį, o klasių žymės – diskretaus lauko modelį. Taip pat pateiktos klaidų tikimybių išraiškos Bajeso diskriminantinėms funkcijoms.

Trečiame disertacijos skyriuje yra taikoma pasiūlyta metodika vaizdams, sugadintiems erdvėje koreliuoto triukšmo. Čia atliekamas juodai balto vaizdo, sugadinto su erdvėje koreliuotu triukšmu, rekonstravimo pavyzdys. Taip pat pateiktas palydovinės nuotraukos vaizdo klasifikavimas. Papildomai yra atliktas klasifikavimas, nuotolinio stebėjimo vaizdo, natūraliai sugadinto debesimis.

Bendrosios išvados

Pasiūlytą PBDF metodą pritaikius vaizdų analizei ir palyginus jį su kitais metodais suformuluotos išvados:

1. Visų eksperimentų metu PBDF metodas buvo pranašesnis už PBDFI metodą. Tai reiškia, jog klasifikuojant naudinga atsižvelgti į klasifikuojamų stebinių erdvinę priklausomybę tarp klasifikuojamo stebinio ir mokymo imties.
2. Atliktų eksperimentų metu gauti rezultatai parodė, jog didėjant erdvinės koreliacijos pločiui duomenyse, Bajeso įterptomis diskriminantinėmis funkcijomis paremtų klasifikavimo metodų rezultatai tampa tikslesni. Tuo tarpu,

kitų metodų, kurie buvo lyginami su PBDF ir PBDFI, klasifikavimo tikslumas nesikeičia.

3. Ištyrus BDF klaidų tikimybių priklausomybę nuo statistinių parametrų reikšmių, gauta, jog didesnė priklausomybė tarp klasių žymių ir stipresnė erdvinė koreliacija tarp požymių stebinių, užtikrina mažesnes reikšmes. Taip pat galima teigti, kad stebiniai su stipresne erdvine priklausomybe gali būti klasifikuojami tiksliau (pagal pasiūlytą metodiką).
4. Atlikus realaus nuotolinio stebėjimo vaizdo, padengto debesimis, klasifikavimą, dar kartą galima teigti, jog erdvinės priklausomybės įvedimas tarp klasifikuojamo stebinio ir mokymo imties, leidžia gauti geresnį rezultatą. Kadangi, šiuo atveju koreliacijos pločio parametras buvo gautas α=13.0305, tai patvirtina, jog tam tikrais atvejais, debesims būdinga erdvinė koreliacija, todėl juos galima modeliuoti Gauso atsitiktiniu lauku.

Trumpos žinios apie autorių

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IMAGE ANALYSIS USING BAYES DISCRIMINANT FUNCTIONS

Summary of Doctoral Dissertation

Physical sciences (P 000)

Informatics (09 P)

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VAIZDŲ ANALIZĖ NAUDOJANT BAJESO DISKRIMINANTINES FUNKCIJAS

Daktaro disertacijos santrauka

Fiziniai mokslai (P 000)

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