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Pusiau reliatyvistinės radialinės Šriodingerio lygties su Saksono-Vudso potencialu sprendinių struktūros tyrimas

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Įvadas

Fundamentalūs gamtos dėsniai ir juos išreiškiančios lygtys užrašomos remiantis postulatais. Išsprendę svarbiausią kvantinės mechanikos lygtį, Šriodingerio lygtį, gauname dalelės ar dalelių sistemos būseną, aprašančią banginę funkciją. Tai, kad sprendimo rezultatas yra funkcija, o ne skaičius, rodo, jog lygtis aprašanti procesą turi būti diferencialinė, t.y. lygtyje turi būti įvertinta bent jau funkcijos išvestinė, kuri charakterizuoja greitį, antroji išvestinė charakteriruoja pagreitį. Paprastąja diferencialine lygtimi laikome tokią lygtį, į kurią įeina nepriklausomas kintamasis, jo funkcija ir tos funkcijos įvairių eilių išvestinės. Bendras n-tos eilės paprastosios diferencialinės lygties pavidalas yra

$$F\left(x, y^{'}(x), y^{''}(x), \ldots, y^{(n)}(x)\right) = 0,$$

čia x- nepriklausomas kintamasis, y(x) – ieškomoji funkcija, y', y'', ..., $y^{(n)}$– ieškomosios funkcijos išvestinės. Šioje lygtyje gali trūkti kai kurių kintamųjų, bet joje visada bus $y^{(n)}$.

Išspręsti diferencialinę lygtį, reiškia rasti tokią funkciją $y = \varphi(x)$, kurią įrašę į duotąją lygtį, gauname tapatybę [1].:

$$F\left(x, \varphi^{'}(x), \varphi^{''}(x), \ldots, \varphi^{(n)}(x)\right) \equiv 0$$

Diferencialinės lygties eile vadinama aukščiausios eilės išvestinės arba diferencialo, įeinančio į lygtį eilė. Funkcija, kurią įrašę į diferencialinę lygtį, gauname tapatybę, vadinama tos diferencialinės lygties sprendiniu. Diferencialinės lygties bendruoju sprendiniu vadinamas toks sprendinys, į kurį įeina tiek laisvųjų konstantų, kokia yra diferencialinės lygties eilė. Diferencialinės lygties atskiruoju sprendiniu vadinamas sprendinys, kuris gaunamas iš bendrojo sprendinio, parinkus konkrečias laisvųjų konstantų skaitines reikšmes. Laisvųjų konstantų reikšmėms rasti nurodomos argumento ir funkcijos pradinės reikšmės. Diferencialinės lygties atskirojo sprendinio radimo uždavinys vadinamas Koši uždaviniu.

Šriodingerio lygtis - aprašo kristalo atomų branduolių ir elektronų stacionarias būsenas. Ji nėra išvedama, o yra nusakoma postuluojant. Ja 1926 metais suformulavo austras Šriodingeris.

Darbe nagrinėsime ketvirtos eilės pusiau reliatyvistinę Šriodingerio lygtį, gausime šios diferencialinės lygties sprendinius, bangines funkcijas, kada lygties potencialas yra Saksono-Vudso potencialas. Paprastąją diferencialinę lygtį spręsime laipsninių eilučių metodu. Laipsnines eilutes, kurias naudosime darbe, dar vadiname apibendrintomis laipsninėmis eilutėmis, dėl papildomo nežinomo parametro.

Ketvirtos eilės pusiau reliatyvistinės Šriodingerio lygties sprendinių ieškosime tokių apibendrintų laipsninių eilučių sandauga.

Laipsninės eilutės neneigiamais kintamojo r laipsniais

$$\varphi(r)=\sum_{k=0}^{\infty} r^{k+\mu}\phi_k ,$$

ir laipsninės eilutės neneigiamais kintamojo $\frac{x}{\alpha}$ laipsniais

$$\varphi_0(x)=\sum_{k=0}^{\infty}\left(\frac{x}{\alpha}\right)^{k+\rho}\varphi_{0k} .$$

Pastarąsias laipsnines eilutes formaliai įrašysime į nagrinėjamą diferencialinę lygtį, nustatysime nežinomus parametrus, visus laipsninės eilutės koeficientus. Iš matematinės analizės žinoma, kad laipsninės eilutės jų konvergavimo intervalo viduje konverguoja absoliučiai ir tolygiai, todėl laipsnines eilutes galima panariui diferencijuoti, integruoti ir pereiti prie ribos panariui.

Laipsnines eilutes ir jų išvestinių išraiškas įrašysime į diferencialines lygtis. Jas sudauginsime Koši prasme (panariui) ir sulyginsime koeficientus prie vienodų nepriklausomo kintamojo laipsnių. Žinodami parametrų ρ ir μ reikšmes galėsime surasti visus, be galo daug, laipsninės eilutės koeficientus ir užrašysime jų radimo rekurentines formules.

Darbe spręsime tokį kraštinį uždavinį:

rasti ketvirtos eilės paprastosios diferencialinės lygties

$$C_1 DU_\alpha+\frac{d^2}{dr^2}U_\alpha+C_2 r\left(\frac{d}{dr}V\right)\left(\frac{d}{dr}\frac{U_\alpha}{r}\right)-\frac{l(l+1)}{r^2}U_\alpha+(CE_\alpha-CV-CV_{sl})U_\alpha=0 \qquad (1)$$

sprendinį, tenkinantį nulines kraštines sąlygas koordinačių pradžioje ir be galo nutolusiame taške.

Taigi suformuluoto kraštinio uždavinio sprendimas labai aktualus kvantinėje mechanikoje. Panašaus tipo uždavinius nagrinėja daug autorių [V.Vanagas (1930—1990), A. J. Janavičius, D. Jurgaitis, A. Kurtinaitis], o bendrų rezultatų nėra daug, dėl to, kad tokių uždavinių sprendimas sudėtingas ir klasikiniai matematinės analizės bei diferencialinių lygčių teorijos metodai čia daugeliu atvejų tobulintini. A. J. Janavičius tyrinėjo viendalelių branduolio būsenų energijų skaičiavimus ir parodė būtinumą įskaityti reliatyvistines pataisas masei. Pusiau reliatyvistinė ketvirtosios eiles diferencialinė lygtis nulio aplinkoje turi keturis tiesiškai nepriklausomus sprendinius, iš kurių trys turi fizikinę prasmę. Tie trys fizikiniai sprendiniai mažuose atstumuose gali būti sutapatinti su kvarkų spalvos indeksu, o atstumuose, didesniuose už vieną fermį arba hadrono matmenis, virsta vienu fzikiniu Šriodingerio lygties sprendiniu.

I. Uždavinio formulavimas

Rasti paprastosios ketvirtos eilės diferencialinės lygties (1) sprendinius ir ištirti tų sprendinių struktūrą.

$$C_1 DU_\alpha + \frac{d^2}{dr^2}U_\alpha + C_2 r\left(\frac{d}{dr}V\right)\left(\frac{d}{dr}\frac{U_\alpha}{r}\right) - \frac{l(l+1)}{r^2}U_\alpha + (CE_\alpha - CV - CV_{sl})U_\alpha = 0 \qquad (1)$$

čia

$$DU_\alpha = \frac{d^4}{dr^4}U_\alpha - \frac{2l(l+1)}{r^2}\frac{d^2}{dr^2}U_\alpha + \frac{4l(l+1)}{r^3}\frac{d}{dr}U_\alpha + \left(\frac{l^2(l+1)^2 - 6l(l+1)}{r^4}\right)U_\alpha \quad ,$$

$$C = \frac{2m}{h^2},\ C_1 = \left(\frac{h}{2mc}\right)^2,\ C_2 = \frac{1}{2mc^2},\ V_{sl}(r) = -\frac{K}{r}V'(r)$$

U_α — ieškomoji banginė funkcija, V_{sl} — sukinio potencialas, h — Planko konstanta, E_α — energija, m- masė, c, l, r, α, a — žinomi fizikiniai parametrai. Sprendinys turi tenkinanti kraštines sąlygas:

$$u(0)=0, \qquad (2^*)$$

$$u(\infty)=0. \qquad (2^{**})$$

Toks uždavinys paprastųjų diferencialinių lygčių teorijoje vadinamas kraštiniu uždaviniu, o (2*) ir (2**) sąlygos vadinamos kraštinėmis sąlygomis. (1) diferencialinės lygties nepriklausomas kintamasis r kinta intervale nuo nulio iki begalybės, taigi lygtis aprašo realią banginę funkciją.

(1) diferencialinę lygtį nagrinėjame su Saksono-Vodso potencialu.

$$V(r) = \frac{-V_0}{1 + e^{\frac{r-R_0}{a}}},$$

čia V_0 — fiksuota potencialo reikšmė, r — nepriklausomas radijinis kintamasis, R_0 — fiksuota radijinio kintamojo reikšmė, a — žinomas fizikinis parametras.

Sprendinių ieškosime laipsninių eilučių metodu. Trikdantįjį Saksono-Vodso potencialą išdėstysime laipsnine nepriklausomo kintamojo laipsnių eilute.

II. Uždavinio sprendimas

Nagrinėjame diferencialinę lygtį:

$$C_1 DU_\alpha + \frac{d^2}{dr^2}U_\alpha + C_2 r\left(\frac{d}{dr}V\right)\left(\frac{d}{dr}\frac{U_\alpha}{r}\right) - \frac{l(l+1)}{r^2}U_\alpha + (CE_\alpha - CV - CV_{sl})U_\alpha = 0\,, \qquad (1)$$

čia

$$DU_\alpha = \frac{d^4}{dr^4}U_\alpha - \frac{2l(l+1)}{r^2}\frac{d^2}{dr^2}U_\alpha + \frac{4l(l+1)}{r^3}\frac{d}{dr}U_\alpha + \left(\frac{l^2(l+1)^2 - 6l(l+1)}{r^4}\right)U_\alpha \quad ,$$

$$C = \frac{2m}{h^2}\,,\ C_1 = \left(\frac{h}{2mc}\right)^2,\ C_2 = \frac{1}{2mc^2}\,,$$

$$V_{sl}(r) = -\frac{K}{r}V'(r)\,.$$

Potencialas

$$V(r) = \frac{-V_0}{1 + e^{\frac{r-R_0}{a}}}\,. \qquad (3)$$

Įsivedame pažymėjimą

$$l(l+1) = A\,,$$

$$l^2(l+1)^2 = A^2\,,$$

$$2l(l+1) = 2A\,. \qquad (4)$$

ir (4) įrašę į (1) diferencialinę lygtį gauname, kad

$$C_1\frac{d^4}{dr^4}U_\alpha + \frac{d^2}{dr^2}U_\alpha - C_1\frac{2l(l+1)}{r^2}\frac{d^2}{dr^2}U_\alpha + C_1\frac{4l(l+1)}{r^3}\frac{d}{dr}U_\alpha + C_2\left[\left(\frac{d}{dr}V\right)\cdot\right.$$
$$\left.\cdot\left(\frac{d}{dr}U_\alpha\right)\right] + C_1\frac{l^2(l+1)^2 - 6l(l+1)}{r^4}U_\alpha - \frac{l(l+1)}{r^2}U_\alpha + (CE_\alpha - CV - CV_{sl})U_\alpha = 0. \qquad (5)$$

Dabar $\frac{d}{dr}U_\alpha$ pakeisime žymeniu u' ir turėsime

$$u' = \frac{d}{dr}U_\alpha\,,\quad u'' = \frac{d^2}{dr^2}U_\alpha\,,\quad u''' = \frac{d^3}{dr^3}U_\alpha\,,\quad u^{(4)} = \frac{d^4}{dr^4}U_\alpha\,. \qquad (6)$$

Pasinaudoję (6) pažymėjimais (5) perrašome kaip tokią diferencialinę lygtį:

$$C_1 u^{(4)} - C_1\frac{2A}{r^2}u'' + C_1\frac{4A}{r^3}u' + C_1\frac{A^2 - 6A}{r^4}u + u'' +$$

$$+C_2 r\left(\frac{d}{dr}V\right)\frac{1}{r}u' - \frac{A}{r^2}u + (CE_\alpha - CV - CV_{sl})u = 0. \qquad (7)$$

Surinksime koeficientus prie vienodos eilės išvestinių. Tai padarę turime

$$\begin{aligned} &C_1 u^{(4)} + \left(1 - C_1\frac{2A}{r^2}\right)u'' + \left(C_1\frac{4A}{r^3} + C_2\left(\frac{d}{dr}V\right)\right)u' + \\ &+\left(C_1\frac{A^2-6A}{r^4} - \frac{A}{r^2} + CE_\alpha - CV - CV_{sl}\right)u = 0. \end{aligned} \qquad (8)$$

(8) ketvirtos eilės diferencialinėje lygtyje įvedame naują nepriklausomą kintamąjį x, kadangi diferencialinės lygties nepriklausomas radijinis kintamasis r, yra ir potencialo vardiklyje, eksponentės laipsnyje. Sprendžiant reikės į diferencialinę lygtį įrašyti ir pirmąją potencialo išvestinę. Taigi, kad reiškinys būtų paprastesnis, pasirenkame x-ą tokį, kad skaičiuojant jėgos lauko ribą, ji sutaptų su riba, kuri yra skaičiuojant su r., t.y.

$$x = \alpha\left(e^{\frac{r}{a}} - 1\right). \qquad (9)$$

Patikrinkime ar tikrai keitinys nepakeičia potencialo ribos:

$$V(r) = \frac{-M_0}{1 + e^{\frac{r-R_0}{a}}} \qquad \lim_{r\to\infty} V(r) \to 0 \qquad \text{ir} \qquad \lim_{r\to 0} V(r) \to 0,$$

$$V(x) = \frac{-M_0}{x + \alpha + 1} \qquad \lim_{x\to 0} V(x) \to 0 \qquad \text{ir} \qquad \lim_{x\to 0} V(x) \to 0.$$

Pastebėkime, kad kai $r \to 0$, tai $x \to 0$, o kada $r \to \infty$, tai ir $x \to \infty$.

Kadangi keitinys geras, tai suskaičiavus (9) keturias išvestines gauname:

$$\begin{aligned} x'_r &= \frac{1}{a}(x+\alpha), \\ x''_r &= \frac{1}{a^2}(x+\alpha), \\ x'''_r &= \frac{(x+\alpha)}{a^3}, \\ x^{(4)}{}_r &= \frac{(x+\alpha)}{a^4}. \end{aligned} \qquad (10)$$

Bakalauro darbe nepavyko išspręsti lygties su keitiniu $u(r) = \psi(x)$, čia x yra aprašytas (10). Taigi toliau spręsime pasinaudodami nauju keitiniu, o ieškomąjį sprendinį U_α pakeisime (6) žymeniu.

Naujojo keitinio išraiška:

$$u(r)=\varphi(r)\varphi_0\left(x\right).$$

Suradę keturias išvestines

$$u'(r)=\varphi_r'(r)\varphi_0(x)+\varphi(r)\varphi_{0x}'(x)x_r',$$

$$u''(r)=\varphi_r''(r)\varphi_0(x)+2\varphi_r'(r)\varphi_{0x}'(x)x_r'+\varphi(r)\varphi_{0x}''(x)x_r'+\varphi(r)\varphi_0'(x)x_r'',$$

$$u'''(r)=\varphi_r'''(r)\varphi_0(x)+3\varphi_r''(r)\varphi_{0x}'(x)x_r'+3\varphi_r'(r)\varphi_{0x}''(x)x_r'+3\varphi_r'(r)\varphi_0'(x)x_r''+$$

$$+\varphi(r)\varphi_{0x}'''(x)x_r'+2\varphi'(r)\varphi_{0x}''(x)x_r''+\varphi(r)\varphi_0'(x)x_r''' \quad ,$$

$$u^{(4)}(r)=\varphi_r^{(4)}(r)\varphi_0(x)+\varphi_r'''(r)\varphi_{0x}'(x)x_r''+6\varphi_r''(r)\varphi_{0x}''(x)x_r'+6\varphi_r''(r)\varphi_{0x}'(x)x_r''+$$

$$+4\varphi_r'(r)\varphi_{0x}'''(x)x_r'+8\varphi_r'(r)\varphi_{0x}''(x)x_r''+4\varphi_r'(r)\varphi_{0x}'(x)x_r'''+\varphi(r)\varphi_{0x}^{(4)}(x)x_r'+$$

$$+3\varphi(r)\varphi_{0x}'''(x)x_r''+3\varphi(r)\varphi_{0x}''(x)x_r'''+\varphi(r)\varphi_{0x}'(x)x_r^{(4)}. \tag{11}$$

Pasinaudojame kintamojo x ir kintamojo r ryšiu, aprašytu (10) ir įrašę gautas išraiškas į ieškomosios funkcijos išvestinių išraiškas (11), gauname tokias ieškomosios funkcijos keturių išvestinių išraiškas:

$$u'(r)=\varphi_r'(r)\varphi_0(x)+\varphi(r)\varphi_{0x}'(x)\frac{1}{a}\left(x+\alpha\right),$$

$$u''(r)=\varphi_r''(r)\varphi_0(x)+2\varphi_r'(r)\varphi_{0x}'(x)\frac{1}{a}\left(x+\alpha\right)+\varphi(r)\varphi_{0x}''(x)\frac{1}{a}\left(x+\alpha\right)+\varphi(r)\varphi_0'(x)\frac{1}{a^2}\left(x+\alpha\right),$$

$$u'''(r)=\varphi_r'''(r)\varphi_0(x)+3\varphi_r''(r)\varphi_{0x}'(x)\frac{1}{a}\left(x+\alpha\right)+3\varphi_r'(r)\varphi_{0x}''(x)\frac{1}{a}\left(x+\alpha\right)+3\varphi_r'(r)\varphi_0'(x)\frac{1}{a^2}\cdot$$

$$\cdot(x+\alpha)+\varphi(r)\varphi_{0x}'''(x)\frac{1}{a}\left(x+\alpha\right)+2\varphi'(r)\varphi_{0x}''(x)\frac{1}{a^2}\left(x+\alpha\right)+\varphi(r)\varphi_0'(x)\frac{\left(x+\alpha\right)}{a^3},$$

$$u^{(4)}(r)=\varphi_r^{(4)}(r)\varphi_0(x)+4\varphi_r'''(r)\varphi_{0x}'(x)\frac{1}{a}\left(x+\alpha\right)+6\varphi_r''(r)\varphi_{0x}''(x)\frac{1}{a}\left(x+\alpha\right)+6\varphi_r''(r)\varphi_{0x}'(x)\frac{1}{a^2}\cdot$$

$$\cdot\left(x+\alpha\right)+4\varphi_r'(r)\varphi_{0x}'''(x)\frac{1}{a}\left(x+\alpha\right)+8\varphi_r'(r)\varphi_{0x}''(x)\frac{1}{a^2}\left(x+\alpha\right)+4\varphi_r'(r)\varphi_{0x}'(x)\frac{\left(x+\alpha\right)}{a^3}+\varphi(r)\cdot$$

$$\cdot\varphi_{0x}^{(4)}(x)\frac{1}{a}(x+\alpha)+3\varphi(r)\varphi_{0x}'''(x)\frac{1}{a^2}\left(x+\alpha\right)+3\varphi(r)\varphi_{0x}''(x)\frac{\left(x+\alpha\right)}{a^3}+\varphi(r)\varphi_{0x}'(x)\frac{\left(x+\alpha\right)}{a^4}. \tag{12}$$

Gautąsias išvestinių išraiškas (12) įrašę į sprendžiamą diferencialinę lygtį (8) ir turime, kad :

$$C_1\left[\varphi_r^{(4)}(r)\varphi_0(x)+4\varphi_r'''(r)\varphi_{0x}'(x)\frac{1}{a}\left(x+\alpha\right)+6\varphi_r''(r)\varphi_{0x}''(x)\frac{1}{a}\left(x+\alpha\right)+6\varphi_r''(r)\varphi_{0x}'(x)\frac{1}{a^2}\left(x+\alpha\right)+\right.$$

$$+4\varphi_r'(r)\varphi_{0x}'''(x)\frac{1}{a}\left(x+\alpha\right)+8\varphi_r'(r)\varphi_{0x}''(x)\frac{1}{a^2}\left(x+\alpha\right)+4\varphi_r'(r)\varphi_{0x}'(x)\frac{\left(x+\alpha\right)}{a^3}+\varphi(r)\varphi_{0x}^{(4)}(x)\cdot$$

$$\left.\cdot(x+\alpha)\frac{1}{\alpha}+3\varphi(r)\varphi_{0x}'''(x)\frac{1}{a^2}\left(x+\alpha\right)+3\varphi(r)\varphi_{0x}''(x)\frac{\left(x+\alpha\right)}{a^3}+\varphi(r)\varphi_{0x}'(x)\frac{\left(x+\alpha\right)}{a^4}\right]+\left(1-C_1\frac{2A}{r^2}\right)\cdot$$

$$\cdot\left(\varphi_r''(r)\varphi_0(x)+2\varphi_r'(r)\varphi_{0x}'(x)\frac{1}{a}(x+\alpha)+\varphi(r)\varphi_{0x}''(x)\frac{1}{a}(x+\alpha)+\varphi(r)\varphi_0'(x)\frac{1}{a^2}(x+\alpha)\right)+$$

$$+\left(C_1\frac{4A}{r^3}+C_2\left(\frac{d}{dr}V\right)\right)\left(\varphi_r'(r)\varphi_0(x)+\varphi(r)\varphi_{0x}'(x)\frac{1}{a}(x+\alpha)\right)+\left(C_1\frac{A^2-6A}{r^4}-\frac{A}{r^2}+CE_\alpha-\right.$$

$$\left.-CV-CV_{sl}\right)\varphi(r)\varphi_0(x)=0\,. \tag{13}$$

Toliau naudodamiesi kintamųjų x ir r ryšiu potencialą V(r) aprašyta (9) persirašome su nauju nepriklausomu kintamuoju

$$V=\frac{-V_0}{1+\alpha e^{\frac{r}{a}}}\ . \tag{14}$$

Randame potencialo išvestinę

$$V_r'=\frac{-M_0\alpha e^{\frac{r}{a}}}{a\left(1+\alpha e^{\frac{r}{a}}\right)^2}=\frac{V_0(x+\alpha)}{a(x+\alpha+1)^2}. \tag{15}$$

Ir potencialo, bei jo išvestines (14) ir (15) išraiškas įrašę į (13) lygtį gauname :

$$C_1\left[\varphi_r^{(4)}(r)\varphi_0(x)+4\varphi_r'''(r)\varphi_{0x}'(x)\frac{1}{a}(x+\alpha)+6\varphi_r''(r)\varphi_{0x}''(x)\frac{1}{a}(x+\alpha)+6\varphi_r''(r)\varphi_{0x}'(x)\frac{1}{a^2}(x+\alpha)+\right.$$

$$+4\varphi_r'(r)\varphi_{0x}'''(x)\frac{1}{a}(x+\alpha)+8\varphi_r'(r)\varphi_{0x}''(x)\frac{1}{a^2}(x+\alpha)+4\varphi_r'(r)\varphi_{0x}'(x)\frac{(x+\alpha)}{a^3}+\varphi(r)\varphi_{0x}^{(4)}(x)\frac{1}{a}\cdot$$

$$\left.\cdot(x+\alpha)+3\varphi(r)\varphi_{0x}'''(x)\frac{1}{a^2}(x+\alpha)+3\varphi(r)\varphi_{0x}''(x)\frac{(x+\alpha)}{a^3}+\varphi(r)\varphi_{0x}'(x)\frac{(x+\alpha)}{a^4}\right]+\left(1-C_1\frac{2A}{r^2}\right)\cdot$$

$$\cdot\left(\varphi_r''(r)\varphi_0(x)+2\varphi_r'(r)\varphi_{0x}'(x)\frac{1}{a}(x+\alpha)+\varphi(r)\varphi_{0x}''(x)\frac{1}{a}(x+\alpha)+\varphi(r)\varphi_0'(x)\frac{1}{a^2}(x+\alpha)\right)+$$

$$+\left(C_1\frac{4A}{r^3}+C_2\frac{V_0(x+\alpha)}{a(x+\alpha+1)^2}\right)\left(\varphi_r'(r)\varphi_0(x)+\varphi(r)\varphi_{0x}'(x)\frac{1}{a}(x+\alpha)\right)+\left(C_1\frac{A^2-6A}{r^4}-\frac{A}{r^2}+\right.$$

$$\left.+CE_\alpha-\frac{CV_0}{a(x+\alpha+1)}-\frac{CKV_0(x+\alpha)}{ra(x+\alpha+1)}\right)\varphi(r)\varphi_0(x)=0. \tag{16}$$

Dabar (16) lygtį padauginame iš r^4, kad vardiklyje nebeliktų kintamojo r, ir gauname tokią ketvirtos eilės paprastąją diferencialinę lygtį :

$$C_1r^4\left[\varphi_r^{(4)}(r)\varphi_0(x)+4\varphi_r'''(r)\varphi_{0x}'(x)\frac{1}{a}(x+\alpha)+6\varphi_r''(r)\varphi_{0x}''(x)\frac{1}{a}(x+\alpha)+6\varphi_r''(r)\varphi_{0x}'(x)\frac{1}{a^2}\cdot\right.$$

$$\cdot\left(x+\alpha\right)+4\varphi_r'(r)\varphi_{0x}'''(x)\frac{1}{a}\left(x+\alpha\right)+8\varphi_r'(r)\varphi_{0x}''(x)\frac{1}{a^2}\left(x+\alpha\right)+4\varphi_r'(r)\varphi_{0x}'(x)\frac{\left(x+\alpha\right)}{a^3}+$$

$$+\varphi(r)\varphi_{0x}^{(4)}(x)\frac{1}{a}\left(x+\alpha\right)+3\varphi(r)\varphi_{0x}'''(x)\frac{1}{a^2}\left(x+\alpha\right)+3\varphi(r)\varphi_{0x}''(x)\frac{\left(x+\alpha\right)}{a^3}+\varphi(r)\varphi_{0x}'(x)\frac{\left(x+\alpha\right)}{a^4}\Bigg]+$$

$$+\left(1r^4-r^2C_1 2A\right)\Bigg(\varphi_r''(r)\varphi_0(x)+2\varphi_r'(r)\varphi_{0x}'(x)\frac{1}{a}\left(x+\alpha\right)+\varphi(r)\varphi_{0x}''(x)\frac{1}{a}\left(x+\alpha\right)+\varphi(r)\varphi_0'(x)\cdot$$

$$\cdot\frac{1}{a^2}\left(x+\alpha\right)\Bigg)+\left(C_1 4Ar+C_2 r^4\frac{V_0(x+\alpha)}{a\left(x+\alpha+1\right)^2}\right)+\left(\varphi_r'(r)\varphi_0(x)+\varphi(r)\varphi_{0x}'(x)\frac{1}{a}\left(x+\alpha\right)\right)+$$

$$+\left(C_1(A^2-6A)-Ar^2+CE_\alpha r^4-r^4\frac{CV_0}{a(x+\alpha+1)}-\frac{r^3CKV_0(x+\alpha)}{a(x+\alpha+1)}\right)\varphi(r)\varphi_0\left(x\right)=0\,. \tag{17}$$

Randame r išraišką per x ir ji yra tokia:

$$e^{\frac{r}{a}}=\frac{x}{\alpha}+1. \tag{18}$$

Iš (18) turime, kad

$$\frac{r}{a}=\ln\left(\frac{x}{\alpha}+1\right),$$

$$r=a\ln\left(\frac{x}{\alpha}+1\right). \tag{19}$$

Pasinaudoję gautomis išraiškomis iš (19) gauname, kad tiriamoji ketvirtos eilės paprastoji diferencialinė lygtis (17) atrodo taip:

$$C_1a^4\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\Bigg[\varphi_r^{(4)}(r)\varphi_0(x)+4\varphi_r'''(r)\varphi_{0x}'(x)\frac{1}{a}\left(x+\alpha\right)+$$

$$+6\varphi_r''(r)\varphi_{0x}''(x)\frac{1}{a}(x+\alpha)+6\varphi_r''(r)\varphi_{0x}'(x)\frac{1}{a^2}\left(x+\alpha\right)+4\varphi_r'(r)\varphi_{0x}'''(x)\frac{1}{a}\left(x+\alpha\right)+8\varphi_r'(r)\varphi_{0x}''(x)\cdot$$

$$\cdot\frac{1}{a^2}\left(x+\alpha\right)+4\varphi_r'(r)\varphi_{0x}'(x)\frac{\left(x+\alpha\right)}{a^3}+\varphi(r)\varphi_{0x}^{(4)}(x)\frac{1}{a}\left(x+\alpha\right)+3\varphi(r)\varphi_{0x}'''(x)\frac{1}{a^2}(x+\alpha)+3\varphi(r)\cdot$$

$$\cdot\varphi_{0x}''(x)\frac{\left(x+\alpha\right)}{a^3}+\varphi(r)\varphi_{0x}'(x)\frac{\left(x+\alpha\right)}{a^4}\Bigg]+\left(a^4\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)-2a^2AC_1\cdot\right.$$

$$\left.\cdot\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\right)\Bigg(\varphi_r''(r)\varphi_0(x)+2\varphi_r'(r)\varphi_{0x}'(x)\frac{1}{a}\left(x+\alpha\right)+\varphi(r)\varphi_{0x}''(x)\frac{1}{a}\left(x+\alpha\right)+\varphi(r)\cdot$$

$$\cdot\varphi_0'(x)\frac{1}{a^2}\left(x+\alpha\right)\Bigg)+\left(C_1 4Aa\ln\left(\frac{x}{\alpha}+1\right)+C_2a^3\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\frac{V_0(x+\alpha)}{\left(x+\alpha+1\right)^2}\right)\cdot$$

$$\cdot\left(\varphi_r'(r)\varphi_0(x)+\varphi(r)\varphi_{0x}'(x)\frac{1}{a}\left(x+\alpha\right)\right)+\Bigg(C_1(A^2-6A)-Aa^2\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)+CE_\alpha a^4\cdot$$

$$\cdot\frac{CV_0}{(x+\alpha+1)}-\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\frac{a^2CKV_0(x+\alpha)}{(x+\alpha+1)}\Bigg)\varphi(r)\varphi_0\left(x\right)=0. \tag{20}$$

Paprastumo dėlei įsivedame pažymėjimą:

$$\varphi(r)=\varphi, \varphi_0(x)=\varphi_0 .$$

Pastarojoje lygtyje atskiriame kintamuosius ir gauname dvi ketvirtos eilės diferencialines lygtis ieškomosioms funkcijoms vienai ir kitai rasti. Tos lygtys yra tokios:

Pirmoji lygtis renkant narius prie φ:

$$\left[C_1 a^4 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0\right]\varphi^{(4)}+\left[4C_1 a^4 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\cdot\right.$$
$$\left.\cdot\ln\left(\frac{x}{\alpha}+1\right)\varphi_0'\right]\varphi'''+\left[6C_1 a^3 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0''(x+\alpha)+6C_1\ln\left(\frac{x}{\alpha}+1\right)\cdot\right.$$
$$\cdot\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)a^2\varphi_0'(x+\alpha)+a^4\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0-a^2\cdot$$
$$\left.\cdot\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)2C_1 A\varphi_0\right]\varphi''+\left[4C_1 a^3\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0'''(x+\alpha)+\right.$$
$$+8C_1 a^2\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0''(x+\alpha)+4a\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\cdot$$
$$\cdot\ln\left(\frac{x}{\alpha}+1\right)\varphi_0'(x+\alpha)+2a^3\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0'(x+\alpha)+4C_1 Aa\cdot$$
$$\left.\cdot\ln\left(\frac{x}{\alpha}+1\right)\varphi_0+C_2 a^3\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi_0\right]\varphi'+\left[C_1 a^3\cdot\right.$$
$$\cdot\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0^{(4)}(x+\alpha)+3C_1 a^2\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)+$$
$$\cdot\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0'''(x+\alpha)+3a\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0''(x+\alpha)+$$
$$+C_1\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0'(x+\alpha)+a^3\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\cdot$$
$$\cdot\ln\left(\frac{x}{\alpha}+1\right)\varphi_0''(x+\alpha)-a\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)C_1 2A\varphi_0''(x+\alpha)+a^2\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\cdot$$
$$\cdot\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0'(x+\alpha)-\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)C_1 2A\varphi_0'(x+\alpha)+4C_1 A\ln\left(\frac{x}{\alpha}+1\right)+$$
$$+C_2 a^2\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi_0'(x+\alpha)+C_1(A^2 6A)\varphi_0-$$
$$-Aa^2\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0+C_1 E_\alpha a^4\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi_0-$$
$$-a^3\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\frac{CV_0}{(x+\alpha+1)}\varphi_0-\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\cdot$$
$$\left.\cdot\ln\left(\frac{x}{\alpha}+1\right)\frac{a^2 CKV_0(x+\alpha)}{(x+\alpha+1)}\varphi_0\right]\varphi=0. \tag{21}$$

Antroji lygtis renkant narius prie φ_0 tokia:

$$\left[C_1 a^3 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi(x+\alpha)\right]\varphi_0^{(4)} + \left[4C_1 a^3 \ln\left(\frac{x}{\alpha}+1\right)\cdot\right.$$
$$\cdot\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi(x+\alpha) + 3C_1 a^2 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\cdot$$
$$\left.\cdot\ln\left(\frac{x}{\alpha}+1\right)\varphi(x+\alpha)\right]\varphi_0''' + \left[6C_1 a^3 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi''(x+\alpha)+\right.$$
$$+8C_1 a^2 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi'(x+\alpha) + 3C_1 a \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\cdot$$
$$\cdot\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi(x+\alpha) + a^3 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi(x+\alpha)-$$
$$\left.-a\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)2C_1 A\varphi(x+\alpha)\right]\varphi_0'' + \left[4C_1 a^3 \ln\left(\frac{x}{\alpha}+1\right) * \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\cdot\right.$$
$$\cdot\ln\left(\frac{x}{\alpha}+1\right)\varphi'''(x+\alpha) + 6C_1 a^2 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi''(x+\alpha)+$$
$$+4C_1 a \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi'(x+\alpha) + C_1 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\cdot$$
$$\cdot\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi(x+\alpha) + 2a^3 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi'(x+\alpha)-$$
$$-2a\ln\left(\frac{x}{\alpha}+1\right)\left(\frac{x}{\alpha}+1\right) - 2C_1 A\varphi'(x+\alpha) + a^2 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi(x+\alpha)-$$
$$-\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)2C_1 A\varphi(x+\alpha) + 4C_1 A\ln\left(\frac{x}{\alpha}+1\right)\varphi(x+\alpha) + C_2 a^2 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\cdot$$
$$\left.\cdot\ln\left(\frac{x}{\alpha}+1\right)\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi(x+\alpha)\right]\varphi_0' + \left[C_1 a^4 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi^{(4)}+\right.$$
$$+a^4 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi'' - a^2 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)2C_1 A\varphi''+$$
$$+4C_1 Aa\ln\left(\frac{x}{\alpha}+1\right)\varphi' + C_2 a^3 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi' + C_1(A^2-6A)\varphi-$$
$$-Aa^2 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi + CE_\alpha a^4 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi-$$
$$-a^3 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\frac{CV_0}{(x+\alpha+1)}\varphi-$$
$$\left.-\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\frac{a^2 CKV_0(x+\alpha)}{(x+\alpha+1)}\varphi\right]\varphi_0 = 0. \qquad (22)$$

Dabar iš pastarosios lygties imame 4 išvestinę ir narius su potencialu:

$$\left[C_1 a^3 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\varphi(x+\alpha)\right]\varphi_0^{(4)} + C_2 a^2 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\cdot$$

$$\cdot \ln\left(\frac{x}{\alpha}+1\right)\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi(x+\alpha)\varphi' + C_2 a^3 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi'\varphi_0 -$$

$$- a^3 \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\frac{CV_0}{(x+\alpha+1)}\varphi\varphi_0 - \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\cdot$$

$$\cdot \ln\left(\frac{x}{\alpha}+1\right)\ln\left(\frac{x}{\alpha}+1\right)\frac{a^2 CKV_0(x+\alpha)}{(x+\alpha+1)}\varphi\varphi_0 = H\,. \tag{23}$$

Pastarųjų diferencialinių lygčių sprendinių ieškosime laipsninių eilučių metodu [2]. (23) lygties sprendinių ieškosime laipsnine eilute kintamojo r laipsniais, o (22) lygties sprendinių ieškosime laipsnine eilute kintamojo $\frac{x}{\alpha}$ laipsniais. Tam, kad gauti ieškomų sprendinių išraiškas į (21) ir į (22) įeinančias logaritmines funcijas skleisime eilute kitamojo $\frac{x}{\alpha}$ laipsniais.

Pasinaudoję $\ln\left(1+\frac{x}{\alpha}\right)$ skleidiniu laipsnine eilute taško $x=0$ aplinkoje, konvergavimo srityje $\left|\frac{x}{\alpha}\right|\langle 1$, gauname tokius dėstinius:

$$\ln\left(1+\frac{x}{\alpha}\right) = \sum_{k=0}^{\infty}(-1)^k\left(\frac{x}{\alpha}\right)^{k-1}(k+1)^{-1},$$

$$\ln\left(1+\frac{x}{\alpha}\right)\ln\left(1+\frac{x}{\alpha}\right) = \sum_{k=0}^{\infty}\sum_{l=0}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^{k}(l+1)^{-1}(k-l+1)^{-1},$$

$$\ln\left(1+\frac{x}{\alpha}\right)\ln\left(1+\frac{x}{\alpha}\right)\ln\left(1+\frac{x}{\alpha}\right) = \sum_{k=0}^{\infty}\sum_{m=0}^{k}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^{k}(l+1)^{-1}(m-l+1)^{-1}(k-m+1)^{-1},$$

$$\ln\left(1+\frac{x}{\alpha}\right)\ln\left(1+\frac{x}{\alpha}\right)\ln\left(1+\frac{x}{\alpha}\right)\ln\left(1+\frac{x}{\alpha}\right) = \sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^{k}(l+1)^{-1}(n-m+1)^{-1}\cdot \tag{24}$$

Diferencialinės lygties (21) sprendinių ieškosime apibendrinta laipsnine eilute

$$\varphi(r) = \sum_{k=0}^{\infty} r^{k+\mu}\phi_k. \tag{25}$$

Pastarąją eilutę diferencijuojame keturis kartus ir gauname tokias išvestinių išraiškas:

$$\varphi'(r) = \sum_{k=0}^{\infty}(k+\mu)r^{k+\rho-1}\phi_k\,,$$

$$\phi''(r)=\sum_{k=0}^{\infty}(k+\mu)(k+\mu-1)r^{k+\rho-2}\phi_k,$$

$$\varphi'''(r)=\sum_{k=0}^{\infty}(k+\mu)(k+\mu-1)(k+\mu-2)r^{k+\rho-3}\phi_k,$$

$$\varphi^{(4)}(r)=\sum_{k=0}^{\infty}(k+\mu)(k+\mu-1)(k+\mu-2)(k+\mu-3)r^{k+\rho-4}\phi_k. \tag{26}$$

(24), (25), (26) išraiškas surašome į (21) lygtį :

$$\Bigg[\Bigg[C_1a^4\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi_0\Bigg]\cdot$$

$$\cdot\sum_{k=0}^{\infty}(k+\mu)(k+\mu-1)(k+\mu-2)(k+\mu-3)r^{k+\rho-4}\phi_k+$$

$$+\Bigg[4C_1a^4\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\Bigg]\varphi_0'\Bigg]\cdot$$

$$\cdot\sum_{k=0}^{\infty}(k+\mu)(k+\mu-1)(k+\mu-2)r^{k+\rho-3}\phi_k+\Bigg[6C_1a^3\sum\sum\sum\sum(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}\cdot$$

$$\cdot(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi_0''(x+\alpha)+6C_1a^2\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k\cdot$$

$$\cdot(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi_0'(x+\alpha)+$$

$$+a^4\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi_0-a^2\cdot$$

$$\cdot 2C_1A\sum_{k=0}^{\infty}\sum_{l=0}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(k-l+1)^{-1}\varphi_0\Bigg]\sum_{k=0}^{\infty}(k+\rho)(k+\rho-1)\left(\frac{x}{\alpha}\right)^{k+\rho-2}\varphi_k\left(\frac{1}{\alpha}\right)^2+$$

$$+\Bigg[4C_1a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi_0'''(x+\alpha)+$$

$$+8C_1a^2\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\cdot$$

$$\cdot\varphi_0''(x+\alpha)+4a\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}*\varphi_0'(x+\alpha)+$$

$$+2a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi_0'(x+\alpha)+$$

$$+4C_1Aa\sum_{k=0}^{\infty}(-1)^k\left(\frac{x}{\alpha}\right)^{k-1}(k+1)^{-1}\varphi_0+C_2a^3\cdot$$

$$\cdot\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi_0\Bigg]\cdot$$

$$\cdot\sum_{k=0}^{\infty}(k+\mu)r^{k+\rho-1}\phi_k+\Bigg[C_1a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}\cdot$$

$$\cdot (k-n+1)^{-1}\varphi_0^{(4)}(x+\alpha)+3C_1a^2\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}\cdot$$

$$\cdot (k-n+1)^{-1}\varphi_0'''(x+\alpha)+3a\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\cdot$$

$$\cdot \varphi_0''(x+\alpha)+C_1\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\cdot$$

$$\cdot \varphi_0'(x+\alpha)+a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi_0''(x+\alpha)-$$

$$-a\sum_{k=0}^{\infty}\sum_{l=0}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(k-l+1)^{-1}C_1 2A\varphi_0''(x+\alpha)+a^2\sum\sum\sum\sum(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}\cdot$$

$$\cdot (n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi_0'(x+\alpha)-\sum_{k=0}^{\infty}\sum_{l=0}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(k-l+1)^{-1}\cdot$$

$$\cdot C_1 2A\varphi_0'(x+\alpha)+4C_1A\sum_{k=0}^{\infty}(-1)^k\left(\frac{x}{\alpha}\right)^{k-1}(k+1)^{-1}+C_2a^2\sum_{k=0}^{\infty}\sum_{m=0}^{k}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}\cdot$$

$$\cdot (m-l+1)^{-1}(k-m+1)^{-1}\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi_0'(x+\alpha)+C_1(A^2-6A)\varphi_0-Aa^2\sum_{k=0}^{\infty}\sum_{l=0}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^k\cdot$$

$$\cdot (l+1)^{-1}(k-l+1)^{-1}\varphi_0+C_1E_\alpha a^4\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}\cdot$$

$$\cdot (k-n+1)^{-1}\varphi_0-a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\cdot$$

$$\cdot \frac{CV_0}{(x+\alpha+1)}\varphi_0-\sum_{k=0}^{\infty}\sum_{m=0}^{k}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(m-l+1)^{-1}(k-m+1)^{-1}\cdot$$

$$\cdot \left.\frac{a^2CKV_0(x+\alpha)}{(x+\alpha+1)}\varphi_0\right]\sum_{k=0}^{\infty}r^{k+\rho}\phi_k=0. \tag{27}$$

Dabar atlikę veiksmus (27) išraiškoje t.y. laipsnines eilutes sudauginę ir suprastinę gauname:

$$a^4C_1\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho-3}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}\cdot$$

$$\cdot (k-i+1)^{-1}(k-i+\mu)(k-i+\mu-1)(k-i+\mu)(k-i+\mu-1)(k-i+\mu-2)(k-i+\mu-3)\cdot$$

$$\cdot \varphi_0\phi_k+4C_1a^4\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho-3}(l+1)(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}\cdot$$

$$\cdot (k-i+\mu)(k-i+\mu-1)(k-i+\mu-2)\varphi_0'\phi_k+6C_1a^3\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho-2}(l+1)^{-1}\cdot$$

$$\cdot (m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}(k-i+\mu)(k-i+\mu-1)\varphi_0''(x+\alpha)\phi_k+$$

$$+6C_1a^2\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho-2}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}\cdot$$

$$\cdot(k-i+\mu)(k-i+\mu-1)\varphi_0'(x+\alpha)\phi_k + a^4\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho-2}(l+1)^{-1}(m-l+1)^{-1}\cdot$$

$$\cdot(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}(k-i+\mu)(k-i+\mu-1)\varphi_0\phi_k -$$

$$-2C_1Aa^2\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho-2}(l+1)^{-1}(i-l+1)^{-1}(k-i+1)^{-1}\cdot$$

$$\cdot(k-i+\mu)(k-i+\mu-1)\varphi_0\phi_k + 4C_1a^3\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho-1}(l+1)^{-1}(m-l+1)^{-1}\cdot$$

$$\cdot(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}(k-i+\mu)\varphi_0''(x+\alpha)\phi_k + 4a\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k\cdot$$

$$\cdot r^{k+\rho-1}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}(k-i+\mu)\varphi_0'(x+\alpha)\phi_k + 2a^3\cdot$$

$$\cdot\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho-1}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}\cdot$$

$$\cdot(k-i+\mu)\varphi_0'(x+\alpha)\phi_k + 4C_1Aa\sum_{k=0}^{\infty}\sum_{i=0}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho-1}(i+1)^{-1}(k-i+1)^{-1}(k-i+\mu)\varphi_0\phi_k +$$

$$+C_2a^3\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho-1}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}\cdot$$

$$\cdot(k-i+\mu)\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi_0\phi_k + C_1a^3\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(l+1)^{-1}(m-l+1)^{-1}\cdot$$

$$\cdot(n-m+1)^{*1}(i-n+1)^{-1}(k-i+1)^{-1}\varphi_0^{(4)}(x+\alpha)\phi_k + \sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(l+1)^{-1}\cdot$$

$$\cdot(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}\varphi_0'''(x+\alpha)\phi_k 3C_1a^2 + 3a\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\cdot$$

$$\cdot\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}\varphi_0''(x+\alpha)\phi_k + C_1\cdot$$

$$\cdot\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}\cdot$$

$$\cdot\varphi_0'(x+\alpha)\phi_k + a^3\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}\cdot$$

$$\cdot(k-i+1)^{-1}\varphi_0''(x+\alpha)\varphi_k - \sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(l+1)^{-1}(i-l+1)^{-1}(k-i+1)^{-1}\varphi_0''(x+\alpha)\phi_k +$$

$$+a^2\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}\cdot$$

$$\cdot\varphi_0'(x+\alpha)\phi_k - 2AC_1\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(l+1)^{-1}(i-l+1)^{-1}(k-i+1)^{-1}\varphi_0'(x+\alpha)\phi_k +$$

$$+4AC_1\sum_{k=0}^{\infty}\sum_{i}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(i+1)^{-1}(k-i+1)^{-1}\phi_k + C_2a^2\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{m=0}^{im}\sum_{l=0}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(l+1)^{-1}\cdot$$

$$\cdot(m-l+1)^{-1}(i-m+1)^{-1}(k-i+1)^{-1}\frac{V_0(x+\alpha)^2}{(x+\alpha+1)^2}\varphi_0'\phi_k + C_1(A+6A)\sum_{k=0}^{\infty}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho}\varphi_0\varphi_k -$$

$$- Aa^2\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(l+1)^{-1}(i-l+1)^{-1}(k-i+1)^{-1}\varphi_0\phi_k + C_1E_\alpha a^4\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k \cdot$$

$$\cdot\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(l+1)(m-l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}\varphi_0\phi_k -$$

$$- a^3\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}\cdot$$

$$\cdot\frac{CV_0}{(x+\alpha+1)}\varphi_0\phi_k - \sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{m=0}^{i}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k r^{k+\rho}(l+1)^{-1}(m-l+1)^{-1}(i-m+1)^{-1}(k-i+1)^{-1}\cdot$$

$$\cdot\frac{a^2CKV_0(x+\alpha)}{(x+\alpha+1)}\varphi_0\phi_k = 0. \tag{28}$$

Diferencialinės lygties (22) sprendinių ieškosime apibendrinta laipsnine eilute:

$$\varphi_0(x) = \sum_{k=0}^{\infty}\left(\frac{x}{\alpha}\right)^{k+\rho}\varphi_{0k}. \tag{29}$$

Pastarąją eilutę diferencijuojame keturis kartus ir gauname tokias išvestinių išraiškas:

$$\varphi_0'(x) = \sum_{k=0}^{\infty}(k+\rho)\left(\frac{x}{\alpha}\right)^{k+\rho-1}\varphi_{0k}\left(\frac{1}{\alpha}\right),$$

$$\phi_0''(x) = \sum_{k=0}^{\infty}(k+\rho)(k+\rho-1)\left(\frac{x}{\alpha}\right)^{k+\rho-2}\varphi_{0k}\left(\frac{1}{\alpha}\right)^2,$$

$$\varphi_0'''(x) = \sum_{k=0}^{\infty}(k+\rho)(k+\rho-1)(k+\rho-2)\left(\frac{x}{\alpha}\right)^{k+\rho-3}\varphi_{0k}\left(\frac{1}{\alpha}\right)^3,$$

$$\varphi_0^{(4)}(x) = \sum_{k=0}^{\infty}(k+\rho)(k+\rho-1)(k+\rho-2)(k+\rho-3)\left(\frac{x}{\alpha}\right)^{k+\rho-4}\varphi_{0k}\left(\frac{1}{\alpha}\right)^4. \tag{30}$$

(24), (29), (30) išraiškas surašome į (22) lygtį :

$$\left[C_1a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi(x+\alpha)\right]\cdot$$

$$\cdot\sum_{k=0}^{\infty}(k+\rho)(k+\rho-1)(k+\rho-2)(k+\rho-3)\left(\frac{x}{\alpha}\right)^{k+\rho-4}\varphi_{0k}\left(\frac{1}{\alpha}\right)^4 +$$

$$+\Bigg[4C_1a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi(x+\alpha)+$$

$$+3C_1a^2\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi(x+\alpha)\Big]\cdot$$

$$\cdot\sum_{k=0}^{\infty}(k+\rho)(k+\rho-1)(k+\rho-2)\left(\frac{x}{\alpha}\right)^{k+\rho-3}\varphi_{0k}\left(\frac{1}{\alpha}\right)^3+\Bigg[6C_1a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}\cdot$$

$$\cdot(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi''(x+\alpha)+8C_1a^2\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}\cdot$$

$$\cdot(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi'(x+\alpha)+\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}\cdot$$

$$\cdot(m-l+1)^{-1}(k-n+1)^{-1}3C_1a\varphi(x+\alpha)+a^3\sum_{k=0}^{\infty}\sum_{l=0}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(k-l+1)^{-1}\varphi(x+\alpha)-aC_1\cdot$$

$$\cdot 2A\sum_{k=0}^{\infty}\sum_{l=0}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(k-l+1)^{-1}C_1A\varphi(x+\alpha)\Bigg]\sum_{k=0}^{\infty}(k+\rho)(k+\rho-1)\left(\frac{x}{\alpha}\right)^{k+\rho-2}\varphi_{0k}\left(\frac{1}{\alpha}\right)^2+$$

$$+\Bigg[4C_1a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi'''(x+\alpha)+$$

$$+6C_1a^2\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi''(x+\alpha)+$$

$$+4C_1a\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi'(x+\alpha)+$$

$$+C_1\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi(x+\alpha)+$$

$$+2a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi'(x+\alpha)-$$

$$-2a\sum_{k=0}^{\infty}\sum_{l=0}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(k-l+1)^{-1}2C_1A\varphi'(x+\alpha)+$$

$$+a^2\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi(x+\alpha)-$$

$$-\sum_{k=0}^{\infty}\sum_{l=0}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(k-l+1)^{-1}2C_1A\varphi(x+\alpha)+4C_1A\sum_{k=0}^{\infty}(-1)^k\left(\frac{x}{\alpha}\right)^{k-1}(k+1)^{-1}\varphi(x+\alpha)+$$

$$+C_2a^2\sum_{k=0}^{\infty}\sum_{m=0}^{k}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(m-l+1)^{-1}(k-m+1)^{-1}\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi(x+\alpha)\Bigg]\cdot$$

$$\cdot\sum_{k=0}^{\infty}(k+\rho)\left(\frac{x}{\alpha}\right)^{k+\rho-1}\varphi_{0k}\left(\frac{1}{\alpha}\right)+\Bigg[C_1a^4\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}\cdot$$

$$\cdot(k-n+1)^{-1}\varphi^{(4)}+a^4\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi''-$$

$$-a^2\sum_{k=0}^{\infty}\sum_{l=0}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(k-l+1)^{-1}2C_1A\varphi''+4C_1Aa\sum_{k=0}^{\infty}(-1)^k\left(\frac{x}{\alpha}\right)^{k-1}(k+1)^{-1}\varphi'+$$

$$+C_2a^3\sum_{k=0}^{\infty}\sum_{m=0}^{k}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(m-l+1)^{-1}(k-m+1)^{-1}\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi'+$$

$$+C_1(A^2-6A)\varphi-Aa^2\sum_{k=0}^{\infty}\sum_{l=0}^{k}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(k-l+1)^{-1}\varphi+$$

$$+CE_\alpha a^4\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\varphi-$$

$$-a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\frac{CV_0}{(x+\alpha+1)}\varphi-$$

$$\left.-\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k\left(\frac{x}{\alpha}\right)^k(l+1)^{-1}(n-m+1)^{-1}(m-l+1)^{-1}(k-n+1)^{-1}\frac{a^2CKV_0(x+\alpha)}{(x+\alpha+1)}\varphi\right]\cdot$$

$$\cdot\sum_{k=0}^{\infty}\left(\frac{x}{\alpha}\right)^{k+\rho}\varphi_{0k}=0. \tag{30}$$

Dabar atlikę veiksmus (30) lygybėje t.y. laipsnines eilutes sudauginę ir suprastinę gauname:

$$C_1a^3\left(\frac{1}{\alpha}\right)^4\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho-4}(l+1)^{-1}(l-i+1)^{-1}(n-m+1)^{-1}(m-i+1)^{-1}\cdot$$

$$\cdot(k-n+1)^{-1}(k-i+\rho)(k-i+\rho-1)(k-i+\rho-2)(k-i+\rho-3)\varphi(x+\alpha)\varphi_{0k}+4C_1\left(\frac{1}{\alpha}\right)^3\cdot$$

$$\cdot a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho-3}(l+1)^{-1}(i-l+1)^{-1}(n-m+1)^{-1}(m-i+1)^{-1}(k-n+1)^{-1}(k-i+\rho)\cdot$$

$$\cdot(k-i+\rho-1)(k-i+\rho-2)\varphi(x+\alpha)\varphi_{0k}+3C_1a^2\left(\frac{1}{\alpha}\right)^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho-3}(l+1)^{-1}\cdot$$

$$\cdot(i-l+1)^{-1}(n-m+1)^{-1}(m-i+1)^{-1}(k-n+1)^{-1}(k-i+\rho)(k-i+\rho-1)(k-i+\rho-2)\cdot$$

$$\cdot\varphi(x+\alpha)\varphi_{0k}+6C_1a^3\left(\frac{1}{\alpha}\right)^2\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho-2}(l+1)^{-1}(i-l+1)^{-1}(n-m+1)^{-1}\cdot$$

$$\cdot(m-i+1)^{-1}(k-n+1)^{-1}(k-i+\rho)(k-i+\rho-1)\varphi''(x+\alpha)\varphi_{0k}+8C_1a^2\left(\frac{1}{\alpha}\right)^2\cdot$$

$$\cdot\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho-2}(l+1)^{-1}(l-i+1)^{-1}(n-m+1)^{-1}(m-i+1)^{-1}(k-n+1)^{-1}\cdot$$

$$\cdot(n-m+1)^{-1}(m-i+1)^{-1}(k-n+1)^{-1}(k-i+\rho)(k-i+\rho-1)\varphi'(x+\alpha)\varphi_{0k}+3C_1a\left(\frac{1}{\alpha}\right)^2\cdot$$

$$\cdot\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho-2}(l+1)^{-1}(l-i+1)^{-1}(n-m+1)^{-1}(m-i+1)^{-1}(k-n+1)^{-1}\cdot$$

$$\cdot(k-i+\rho)(k-i+\rho-1)\varphi(x+\alpha)\varphi_{0k}+a^3\left(\frac{1}{\alpha}\right)^2\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho-2}(l+1)^{-1}(i-l+1)^{-1}\cdot$$

$$\cdot\left(k-i+1\right)^{-1}(k-i+\rho)(k-i+\rho-1)\varphi(x+\alpha)\varphi_{0k}-2aAC_1\left(\frac{1}{\alpha}\right)^2\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{l=0}^{i}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho-2}\left(l+1\right)^{-1}\cdot$$

$$\cdot(i-l+1)^{-1}\left(k-i+1\right)^{-1}(k-i+\rho)(k-i+\rho-1)\varphi(x+\alpha)\varphi_{0k}+4C_1a^3\left(\frac{1}{\alpha}\right)\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}\left(-1\right)^k\cdot$$

$$\cdot\left(\frac{x}{\alpha}\right)^{k+\rho-1}\left(l+1\right)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}(n-m+1)^{-1}(k-n+1)^{-1}(k-i+\rho)\varphi'''(x+\alpha)\varphi_{0k}+$$

$$+6C_1a^2\left(\frac{1}{\alpha}\right)\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho-1}\left(l+1\right)^{-1}(i-l+1)^{-1}\left(n-m+1\right)^{-1}(m-i+1)^{-1}\cdot$$

$$\cdot(k-n+1)^{-1}(k-i+1)\varphi''(x+\alpha)\varphi_{0k}+4C_1a\left(\frac{1}{\alpha}\right)\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho-1}\left(l+1\right)^{-1}\cdot$$

$$\cdot(i-l+1)^{-1}\left(n-m+1\right)^{-1}(m-i+1)^{-1}(k-n+1)^{-1}(k-i+\rho)\varphi'(x+\alpha)\varphi_{0k}+C_1\left(\frac{1}{\alpha}\right)\cdot$$

$$\cdot\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho-1}\left(l+1\right)^{-1}(i-l+1)^{-1}\left(n-m+1\right)^{-1}(m-i+1)^{-1}(k-n+1)^{-1}\cdot$$

$$\cdot(k-i+\rho)\varphi(x+\alpha)\varphi_{0k}+2a^3\left(\frac{1}{\alpha}\right)\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho-1}\left(l+1\right)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}\cdot$$

$$\cdot(n-m+1)^{-1}(k-n+1)^{-1}(k-i+\rho)\varphi'(x+\alpha)\varphi_{0k}-2aAC_1\left(\frac{1}{\alpha}\right)\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{l=0}^{i}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho-1}\left(l+1\right)^{-1}\cdot$$

$$\cdot(i-l+1)^{-1}\left(k-i+1\right)(k-i+\rho)\varphi'(x+\alpha)\varphi_{0k}+a^2\left(\frac{1}{\alpha}\right)\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho-1}\left(l+1\right)^{-1}\cdot$$

$$\cdot(i-l+1)^{-1}\left(n-m+1\right)^{-1}(m-i+1)^{-1}(k-n+1)^{-1}(k-i+\rho)\varphi(x+\alpha)\varphi_{0k}-2AC_1\left(\frac{1}{\alpha}\right)\cdot$$

$$\cdot\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{l=0}^{i}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho-1}\left(l+1\right)^{-1}(i-l+1)^{-1}\left(k-i+1\right)^{-1}(k-i+\rho)\varphi(x+\alpha)\varphi_{0k}+4C_1A\left(\frac{1}{\alpha}\right)\cdot$$

$$\cdot\sum_{k=0}^{\infty}\sum_{l=0}^{k}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho-1}\left(l+1\right)^{-1}(k-l+1)^{-1}(k-i+\rho)\varphi(x+\alpha)\varphi_{0k}+C_2a^2\left(\frac{1}{\alpha}\right)\cdot$$

$$\cdot\sum_{k=0}^{\infty}\sum_{m=0}^{k}\sum_{i=0}^{m}\sum_{l=0}^{i}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho-1}\left(l+1\right)^{-1}(i-l+1)^{-1}\left(m-i+1\right)^{-1}\left(k-m+1\right)^{-1}(k-i+\rho)\cdot$$

$$\cdot\frac{V_0(x+\alpha)^2}{(x+\alpha+1)^2}\varphi\varphi_{0k}+C_1a^4\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho}\left(l+1\right)^{-1}(i-l+1)^{-1}\left(n-m+1\right)^{-1}\cdot$$

$$\cdot(m-i+1)^{-1}(k-n+1)^{-1}\varphi^{(4)}\varphi_{0\alpha}+a^4\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho}\left(l+1\right)^{-1}(i-l+1)^{-1}\cdot$$

$$\cdot\left(n-m+1\right)^{-1}(m-i+1)^{-1}(k-n+1)^{-1}\varphi''\varphi_{0k}-2Aa^2C_1\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho}(l+1)^{-1}(i-l+1)^{-1}\cdot$$

$$\cdot(k-i+1)^{-1}\varphi''\varphi_{0k}+4AaC_1\sum_{k=0}^{\infty}\sum_{l=0}^{k}\left(-1\right)^k\left(\frac{x}{\alpha}\right)^{k+\rho}(l+1)^{-1}(k-l+1)^{-1}\varphi'\varphi_{0k}+\sum_{k=0}^{\infty}\sum_{m=0}^{k}\sum_{i=0}^{m}\sum_{l=0}^{i}\left(-1\right)^k\cdot$$

$$\cdot\left(\frac{x}{\alpha}\right)^{k+\rho}\left(l+1\right)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}(k-m+1)^{-1}\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}C_2a^3\varphi'\varphi_{0k}+C_1(A^2-6A)\cdot$$

$$\cdot\sum_{k=0}^{\infty}\left(\frac{x}{\alpha}\right)^{k+\rho}\varphi\varphi_{0k}-Aa^2\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho}(l+1)^{-1}(i-l+1)^{-1}(k-i+1)^{-1}\varphi\varphi_{0k}+CE_{\alpha}a^4\cdot$$

$$\cdot\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho}(l+1)^{-1}(i-l+1)^{-1}(n-m+1)^{-1}(m-i+1)^{-1}(k-n+1)^{-1}\varphi\varphi_{0k}-$$

$$-a^3\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho}(l+1)^{-1}(i-l+1)^{-1}(n-m+1)^{-1}(m-i+1)^{-1}(k-n+1)^{-1}\cdot$$

$$\cdot\frac{CV_0}{(x+\alpha+1)}\varphi\varphi_{0k}-a^2CK\sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^k\left(\frac{x}{\alpha}\right)^{k+\rho}(l+1)^{-1}(i-l+1)^{-1}(n-m+1)^{-1}\cdot$$

$$\cdot(m-i+1)^{-1}(k-n+1)^{-1}\frac{V_0(x+\alpha)}{(x+\alpha+1)}\varphi\varphi_{0k}=0. \tag{31}$$

Atlikę skaičiavimus ir elementarius pertvarkius gauname, kad ieškomosios laipsninės eilutės (28) koeficientai randami iš šios rekurentinės formulės:

$$a^4C_1\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^k(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i+1)^{-1}(k-i+\mu)\cdot$$

$$\cdot(k-i+\mu-1)(k-i+\mu-2)(k-i+\mu-3)\varphi_0\phi_k+4C_1a^4\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-1}(l+1)\cdot$$

$$\cdot(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i)^{-1}(k-i+\mu-1)(k-i+\mu-2)(k-i+\mu-3)\varphi_0'\phi_{k-1}+$$

$$+6C_1a^3\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{l=0}^{n}(-1)^{k-1}(l+1)^{-1}(n-l+1)^{-1}(i-n+1)^{-1}(k-i)^{-1}\cdot$$

$$\cdot(k-i+\mu-1)(k-i+\mu-2)\varphi_0''(x+\alpha)\phi_{k-1}+6C_1a^2\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-1}(l+1)^{-1}\cdot$$

$$\cdot(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i)^{-1}(k-i+\mu-1)(k-i+\mu-2)\varphi_0'(x+\alpha)\phi_{k-1}+$$

$$+a^4\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-1}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i)^{-1}\cdot$$

$$\cdot(k-i+\mu-1)(k-i+\mu-2)\varphi_0\phi_{k-1}-2C_1Aa^2\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^{k-1}(l+1)^{-1}(k-i)^{-1}(k-i+\mu-1)\cdot$$

$$\cdot(k-i+\mu-2)\varphi_0\phi_{k-1}+4C_1a^3\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-2}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}\cdot$$

$$\cdot(i-n+1)^{-1}(k-i-1)^{-1}(k-i+\mu-2)\varphi_0''(x+\alpha)\phi_{k-2}+4a\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-2}\cdot$$

$$\cdot(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i-1)^{-1}(k-i+\mu-2)\varphi_0'(x+\alpha)\phi_{k-2}+2a^3\cdot$$

$$\cdot\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-2}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i-1)^{-1}\cdot$$

$$\cdot(k-i+\mu-2)\varphi_0'(x+\alpha)\phi_{k-2}+4C_1Aa\sum_{i=0}^{k}(-1)^{k-2}(i+1)^{-1}(k-i-1)^{-1}(k-i+\mu-2)\varphi_0\phi_{k-2}+$$

$$+C_2a^3\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-1}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i)^{-1}\cdot$$

$$\cdot (k-i+\mu-1)\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi_0\phi_{k-1}+C_1a^3\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-3}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}\cdot$$

$$\cdot (i-n+1)^{-1}(k-i-2)^{-1}\varphi_0^{(4)}(x+\alpha)\phi_{k-3}+3C_1a^2\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-3}(l+1)^{-1}\cdot$$

$$\cdot (m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i-2)^{-1}\varphi_0'''(x+\alpha)\phi_{k-3}+3a\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-3}\cdot$$

$$\cdot (l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i-2)^{-1}\varphi_0''(x+\alpha)\phi_{k-3}+C_1a^3\cdot$$

$$+a^3\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-3}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i-2)^{-1}\varphi_0''(x+\alpha)\varphi_{k-3}-$$

$$-\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^{k-3}(l+1)^{-1}(i-l+1)^{-1}(k-i-2)^{-1}\varphi_0''(x+\alpha)\phi_{k-3}+a^2\cdot$$

$$\cdot\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-3}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}(i-n+1)^{-1}(k-i-2)^{-1}\varphi_0'(x+\alpha)\phi_{k-3}-$$

$$-2AC_1\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^{k-3}(l+1)^{-1}(i-l+1)^{-1}(k-i-2)^{-1}\varphi_0'(x+\alpha)\phi_{k-3}+4AC_1\sum_{i=0}^{k}(-1)^{k-3}(i+1)^{-1}\cdot$$

$$\cdot (k-i-2)^{-1}\phi_{k-3}+C_2a^2\sum_{i=0}^{k}\sum_{m=0}^{i}\sum_{l=0}^{m}(-1)^{k-3}(l+1)^{-1}(m-l+1)^{-1}(i-m+1)^{-1}(k-i-2)^{-1}\cdot$$

$$\cdot\frac{V_0(x+\alpha)^2}{(x+\alpha+1)^2}\varphi_0'\phi_{k-3}+C_1(A+6A)(-1)^{k-3}\varphi_0\varphi_{k-3}-Aa^2\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^{k-3}(l+1)^{-1}(i-l+1)^{-1}\cdot$$

$$\cdot (k-i-2)^{-1}\varphi_0\phi_{k-2}+C_1E_\alpha a^4\sum_{k=0}^{\infty}\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{l=0}^{n}(-1)^{k-3}\left(\frac{x}{\alpha}\right)^{k-3}(l+1)^{-1}(n-l+1)^{-1}(i-n+1)^{-1}\cdot$$

$$\cdot (k-i-2)^{-1}\varphi_0\phi_{k-3}-a^3\sum_{i=0}^{k}\sum_{n=0}^{i}\sum_{m=0}^{n}\sum_{l=0}^{m}(-1)^{k-3}(l+1)^{-1}(m-l+1)^{-1}(n-m+1)^{-1}\cdot$$

$$\cdot (i-n+1)^{-1}(k-i-2)^{-1}\frac{CV_0}{(x+\alpha+1)}\varphi_0\phi_{k-3}-\sum_{i=0}^{k}\sum_{m=0}^{i}\sum_{l=0}^{m}(-1)^{k-3}(l+1)^{-1}(m-l+1)^{-1}(i-m+1)\cdot$$

$$\cdot (k-i-2)^{-1}\frac{a^2CKV_0(x+\alpha)}{(x+\alpha+1)}\varphi_0\phi_{k-3}=0. \tag{32}$$

Atlikę skaičiavimus ir elementarius pertvarkius gauname, kad ieškomosios laipsninės eilutės (31) koeficientai randami iš šios rekurentinės formulės:

$$C_1a^3\left(\frac{1}{\alpha}\right)^4\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^k(l+1)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}(n-m+1)^{-1}(k-n+1)^{-1}\cdot$$

$$\cdot (k-i+\rho)(k-i+\rho-1)(k-i+\rho-2)(k-i+\rho-3)\varphi(x+\alpha)\varphi_{0k}+4C_1a^3\left(\tfrac{1}{\alpha}\right)^3\cdot$$

$$\cdot\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^{k-1}\left(l+1\right)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}\left(n-m+1\right)^{-1}(k-n)^{-1}(k-i+\rho-1)\cdot$$

$$\cdot(k-i+\rho-2)\left(k-i+\rho-3\right)\varphi(x+\alpha)\varphi_{0k-1}+3C_1\left(\frac{1}{\alpha}\right)^3\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-1}\left(l+1\right)^{-1}(i-l+1)^{-1}\cdot$$

$$\cdot(m-i+1)^{-1}\left(n-m+1\right)^{-1}(k-n)^{-1}(k-i+\rho-1)(k-i+\rho-2)(k-i+\rho-3)a^2\varphi(x+\alpha)\varphi_{0k-1}+$$

$$+6C_1a^3\left(\frac{1}{\alpha}\right)^2\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-2}\left(l+1\right)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}(n-m+1)^{-1}(k-n-1)^{-1}\cdot$$

$$\cdot(k-i+\rho-2)(k-i+\rho-3)\varphi''(x+\alpha)\varphi_{0k-2}+8C_1a^2\left(\frac{1}{\alpha}\right)^2\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-2}\left(l+1\right)^{-1}\cdot$$

$$\cdot(i-l+1)^{-1}(m-i+1)^{-1}\left(n-m+1\right)^{-1}(k-n+1)^{-1}(k-i+\rho-2)(k-i+\rho-3)\varphi'(x+\alpha)\varphi_{0k-2}+$$

$$+3C_1a\left(\frac{1}{\alpha}\right)^2\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-2}\left(l+1\right)^{-1}*(i-l+1)^{-1}(m-i+1)^{-1}\left(n-m+1\right)^{-1}(k-n-1)^{-1}\cdot$$

$$\cdot(k-i+\rho-2)(k-i+\rho-3)\varphi(x+\alpha)\varphi_{0k-2}+\left(\frac{1}{\alpha}\right)^2\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^{k-2}\left(l+1\right)^{-1}(i-l+1)^{-1}\left(k-i-1\right)^{-1}\cdot$$

$$\cdot(k-i+\rho-2)(k-i+\rho-3)a^3\varphi(x+\alpha)\varphi_{0k-2}-2aAC_1\left(\frac{1}{\alpha}\right)^2\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^{k-2}\left(l+1\right)^{-1}(i-l+1)^{-1}\cdot$$

$$\cdot\left(k-l-1\right)^{-1}(k-i+\rho-2)(k-i+\rho-3)\varphi(x+\alpha)\varphi_{0k-2}+4C_1a^3\left(\frac{1}{\alpha}\right)\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-3}\left(l+1\right)^{-1}\cdot$$

$$\cdot(i-l+1)^{-1}(m-i+1)^{-1}(n-m+1)^{-1}(k-n-2)^{-1}(k-i+\rho-3)\varphi'''(x+\alpha)\varphi_{0k-3}+6C_1a^2\left(\frac{1}{\alpha}\right)\cdot$$

$$\cdot\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-3}\left(l+1\right)^{-1}\left(i-l+1\right)^{-1}(m-i+1)^{-1}\left(n-m+1\right)^{-1}(k-n-2)^{-1}(k-i-2)\cdot$$

$$\cdot\varphi''(x+\alpha)\varphi_{0k-3}+4C_1a\left(\frac{1}{\alpha}\right)\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-3}\left(l+1\right)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}(n-m+1)^{-1}\cdot$$

$$\cdot(k-n-2)^{-1}(k-i+\rho-3)\varphi'(x+\alpha)\varphi_{0k-3}+C_1\left(\frac{1}{\alpha}\right)\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-3}\left(l+1\right)^{-1}(i-l+1)^{-1}\cdot$$

$$\cdot(m-i+1)^{-1}(n-m+1)^{-1}(k-n-2)^{-1}(k-i+\rho-3)\varphi(x+\alpha)\varphi_{0k-3}+\left(\frac{1}{\alpha}\right)\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-3}\cdot$$

$$\cdot\left(l+1\right)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}(n-m+1)^{-1}(k-n-2)^{-1}(k-i+\rho-3)2a^3\varphi'(x+\alpha)\varphi_{0k-3}-$$

$$-2aAC_1\left(\frac{1}{\alpha}\right)\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^{k-3}\left(l+1\right)^{-1}(i-l+1)^{-1}\left(k-i-2\right)^{-1}(k-i+\rho-3)\varphi'(x+\alpha)\varphi_{0k-3}+$$

$$+a^2\left(\frac{1}{\alpha}\right)\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-3}\left(l+1\right)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}(n-m+1)^{-1}(k-n-2)^{-1}\cdot$$

$$\cdot(k-i+\rho-3)\varphi(x+\alpha)\varphi_{0k-3}-2AC_1\left(\tfrac{1}{\alpha}\right)\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^{k-3}\left(l+1\right)^{-1}(i-l+1)^{-1}\left(k-i-2\right)^{-1}\cdot$$

$$\cdot(k-i+\rho-3)\varphi(x+\alpha)\varphi_{0k-3}+4C_1A\left(\frac{1}{\alpha}\right)\sum_{l=0}^{k}(-1)^{k-3}(l+1)^{-1}\left(k-l-2\right)^{-1}(k-i+\rho-3)\cdot$$

$$\cdot \varphi(x+\alpha)\varphi_{0k-3}+C_2a^2\left(\frac{1}{\alpha}\right)\sum_{m=0}^{k}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^{k-3}(l+1)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}(k-m-2)^{-1}\cdot$$

$$\cdot (k-i+\rho-3)\frac{V_0(x+\alpha)^2}{(x+\alpha+1)^2}\varphi\varphi_{0k-3}+C_1a^4\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-4}(l+1)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}\cdot$$

$$\cdot (n-m+1)^{-1}(k-n-3)^{-1}\varphi^{(4)}\varphi_{0k-4}+a^4\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-4}(l+1)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}\cdot$$

$$\cdot (n-m+1)^{-1}(k-n-3)^{-1}\varphi''\varphi_{0k-4}-2Aa^2C_1\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^{k-4}(l+1)^{-1}(i-l+1)^{-1}(k-i-3)^{-1}\varphi''\varphi_{0k-4}+$$

$$+4AaC_1\sum_{l=0}^{k}(-1)^{k-4}(l+1)^{-1}(k-l-3)^{-1}\varphi'\varphi_{0k-4}+C_2a^3\sum_{m=0}^{k}\sum_{i=0}^{m}\sum_{l=0}^{i}(-1)^{k-4}(l+1)^{-1}(i-l+1)^{-1}\cdot$$

$$\cdot (m-i+1)^{-1}(k-m-3)^{-1}\frac{V_0(x+\alpha)}{(x+\alpha+1)^2}\varphi'\varphi_{0k-4}+C_1(A^2-6A)\varphi\varphi_{0k-4}-Aa^2\sum_{i=0}^{k}\sum_{l=0}^{i}(-1)^{k-4}(l+1)^{-1}\cdot$$

$$\cdot (i-l+1)^{-1}(k-l-3)^{-1}\varphi\varphi_{0k-4}+CE_\alpha a^4\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-1}(l+1)^{-1}(i-l+1)^{-1}(m-i+1)^{-1}\cdot$$

$$\cdot (m-i+1)^{-1}(n-m+1)^{-1}(k-n-3)^{-1}\varphi\varphi_{0-4}-a^3\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-1}(l+1)^{-1}(i-l+1)^{-1}\cdot$$

$$\cdot (m-i+1)^{-1}(n-m+1)^{-1}(k-n-3)^{-1}\frac{CV_0}{(x+\alpha+1)}\varphi\varphi_{0k-4}-a^2CK\sum_{n=0}^{k}\sum_{m=0}^{n}\sum_{i=0}^{m}\sum_{i=0}^{i}(-1)^{k-1}(l+1)^{-1}\cdot$$

$$\cdot (i-l+1)^{-1}(m-i+1)^{-1}(n-m+1)^{-1}(k-n-3)^{-1}\frac{V_0(x+\alpha)}{(x+\alpha+1)}\varphi\varphi_{0k-4}=0\,. \tag{33}$$

Gautąjį rezultatą suformuluosime kaip teoremą.

Teorema. Diferencialinės lygties

$$C_1DU_\alpha+\frac{d^2}{dr^2}U_\alpha+C_2r\left(\frac{d}{dr}V\right)\left(\frac{d}{dr}\frac{U_\alpha}{r}\right)-\frac{l(l+1)}{r^2}U_\alpha+(CE_\alpha-CV-CV_{sl})U_\alpha=0,$$

čia

$$DU_\alpha=\frac{d^4}{dr^4}U_\alpha-\frac{2l(l+1)}{r^2}\frac{d^2}{dr^2}U_\alpha+\frac{4l(l+1)}{r^3}\frac{d}{dr}U_\alpha+\left(\frac{l^2(l+1)^2-6l(l+1)}{r^4}\right)U_\alpha,$$

$$C=\frac{2m}{h^2},\ C_1=\left(\frac{h}{2mc}\right)^2,\ C_2=\frac{1}{2mc^2},\ V_{sl}(r)=-\frac{K}{r}V'(r)\,.$$

formalūs sprendiniai yra

$$U_\alpha=\sum_{k=0}^{\infty}r^{k+\mu}\phi_k\cdot\sum_{k=0}^{\infty}\left(\frac{x}{\alpha}\right)^{k+\rho}\varphi_{0k}\,.$$

Čia $\phi_{k, k=0.1...}$ *randami iš (32) rekurentinės formulės,* $\varphi_{0k, k=0.1...}$ *iš (33) rekurentinės formulės.*

Iš (32) lygties randame keturias nežinomo parametro μ *reikšmes:*

$$\mu = 0,\ \mu = 1,\ \mu = 2,\ \mu = 3.$$

Analogiškai iš (33) lygties randame keturias nežinomo parametro ρ *reikšmes:*

$$\rho = 0,\ \rho = 1,\ \rho = 2,\ \rho = 3.$$

Išvados

Nagrinėjamos ketvirtos eilės pusiau reliatyvistinės Šriodingerio lygties, kurios pavidalas yra toks

$$C_1 DU_\alpha + \frac{d^2}{dr^2}U_\alpha + C_2 r\left(\frac{d}{dr}V\right)\left(\frac{d}{dr}\frac{U_\alpha}{r}\right) - \frac{l(l+1)}{r^2}U_\alpha + (CE_\alpha - CV - CV_{sl})U_\alpha = 0,$$

čia

$$DU_\alpha = \frac{d^4}{dr^4}U_\alpha - \frac{2l(l+1)}{r^2}\frac{d^2}{dr^2}U_\alpha + \frac{4l(l+1)}{r^3}\frac{d}{dr}U_\alpha + \left(\frac{l^2(l+1)^2 - 6l(l+1)}{r^4}\right)U_\alpha,$$

$$C = \frac{2m}{h^2},\ C_1 = \left(\frac{h}{2mc}\right)^2,\ C_2 = \frac{1}{2mc^2},$$

$$V_{sl}(r) = -\frac{K}{r}V'(r).$$

Kada potencialas yra Saksono-Vudso:

$$V(r) = \frac{-V_0}{1 + e^{\frac{r-R_0}{a}}}$$

formalūs sprendiniai yra tokie:

$$u(r) = \sum_{k=0}^{\infty} r^{k+\mu}\phi_k \cdot \sum_{k=0}^{\infty}\left(\frac{x}{\alpha}\right)^{k+\rho}\varphi_{0k}.$$

Kadangi parametrų ρ ir μ galimos reikšmės yra sveikieji skaičiai, tai nagrinėjamos diferencialinės lygties sprendinys yra dviejų laipsninių eilučių sandauga ir kiekviena iš jų turi po laisvai parenkamą pirmąjį koeficientą. Vadinasi gautoji formaliųjų sprendinių šeima yra dviparametrinė.

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Research of the structure of solutions of semi-relativistic radial Shrodinger equation with Saxon-Woods potential

Summary

In this work Fourth order degenerate ordinary differential Schrödinger equation was studied. Methods of degree series is you solutions constructed. Structure of solutions and you number is explored.