Published for SISSA by Springer

Received: *December 22, 2023*
Accepted: *April 22, 2024*
Published: *May 15, 2024*

The oblique parameters from arbitrary new fermions

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Abstract: We compute the six oblique parameters S, T, U, V, W, X in a New Physics Model with an arbitrary number of new fermions, in arbitrary representations of $\mathrm{SU}(2) \times \mathrm{U}(1)$, and mixing arbitrarily among themselves. We show that S and U are automatically finite, but T is finite only if there is a specific relation between the masses of the new fermions and the representations of $\mathrm{SU}(2) \times \mathrm{U}(1)$ that they sit in. We apply our general computation to two illustrative cases.

Keywords: Electroweak Precision Physics, Higher Order Electroweak Calculations, Other Weak Scale BSM Models

ArXiv ePrint: 2312.09099

https://doi.org/10.1007/JHEP05(2024)190

JHEP05(2024)190

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1 Introduction

The oblique parameters (OPs) provide a convenient way of comparing the predictions of a New Physics Model (NPM) with those of the Standard Model (SM). The NPM is supposed to have the same gauge group as the SM, viz. $SU(2) \times U(1)$. The different particle content between the NPM and the SM must consist solely of extra fermions and/or scalars in the NPM. Those new fermions and scalars should preferably be in representations of the gauge group such that they cannot couple to the light fermions with which most experiments are performed; in that way, one ensures that their only effects are through their contributions to the vacuum polarizations, i.e. to the self-energies of the gauge bosons. One writes those new contributions, coming from loops[1] of the extra fermions and/or scalars, as

$$\Pi^{\mu\nu}_{VV'}(q) = g^{\mu\nu} A_{VV'}\left(q^2\right) + q^\mu q^\nu B_{VV'}\left(q^2\right), \tag{1.1}$$

where q^μ is the four-momentum of the gauge bosons and V and V' are the gauge bosons at hand, which may be either W^+ and W^-, or a photon γ and a Z^0, or two photons, or two Z^0's. Note that the functions $A_{VV'}(q^2)$ have mass-squared dimensions. Let us denote

$$A'_{VV'}\left(q^2\right) = \frac{\mathrm{d}A_{VV'}(q^2)}{\mathrm{d}q^2}, \tag{1.2a}$$

$$\widetilde{A}_{VV'}\left(q^2\right) = \frac{A_{VV'}(q^2) - A_{VV'}(0)}{q^2}. \tag{1.2b}$$

[1] We only consider the *one*-loop level vacuum polarizations.

Then the OPs are defined as[2,3]

$$S = \frac{4 s_W^2 c_W^2}{\alpha} \left[\widetilde{A}_{ZZ}\left(m_Z^2\right) + \frac{c_W^2 - s_W^2}{c_W s_W}\, A'_{\gamma Z}(0) - A'_{\gamma\gamma}(0) \right], \tag{1.3a}$$

$$T = \frac{1}{\alpha} \left[\frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2} \right], \tag{1.3b}$$

$$U = -S + \frac{4 s_W^2}{\alpha} \left[\widetilde{A}_{WW}\left(m_W^2\right) + \frac{c_W}{s_W}\, A'_{\gamma Z}(0) - A'_{\gamma\gamma}(0) \right], \tag{1.3c}$$

$$V = \frac{1}{\alpha} \left[A'_{ZZ}\left(m_Z^2\right) - \widetilde{A}_{ZZ}\left(m_Z^2\right) \right], \tag{1.3d}$$

$$W = \frac{1}{\alpha} \left[A'_{WW}\left(m_W^2\right) - \widetilde{A}_{WW}\left(m_W^2\right) \right], \tag{1.3e}$$

$$X = \frac{s_W c_W}{\alpha} \left[A'_{\gamma Z}(0) - \widetilde{A}_{\gamma Z}\left(m_Z^2\right) \right]. \tag{1.3f}$$

In eqs. (1.3), α is the fine-structure constant, s_W and c_W are the sine and the cosine, respectively, of the Weinberg angle θ_W, and m_Z and m_W are the masses of the Z^0 and $W^\pm$, respectively. At tree level

$$m_W = c_W m_Z \tag{1.4}$$

in both the NPM and the SM; this is because no neutral-scalar field is allowed to acquire a vacuum expectation value (VEV) unless it has either $J = Y = 0$ or $J = Y = 1/2$,[4] where J is the (total) weak isospin and Y is the weak hypercharge.

The comparison between the predictions of an NPM and the ones of the SM is done through formulas like, for instance,

$$\frac{m_W^{\mathrm{NPM}}}{m_W^{\mathrm{SM}}} = 1 + \alpha \left[\frac{S}{4\left(s_W^2 - c_W^S\right)} + \frac{c_W^2 T}{2\left(c_W^2 - s_W^2\right)} + \frac{U}{8 s_W^2} \right], \tag{1.5}$$

wherein the input observables in the renormalization of both the SM and the NPM are assumed to be α, m_Z, and the Fermi coupling constant G_F measured in muon decay;[5] the mass m_W is thought of as a *prediction* of either the SM or the NPM. Formulas analogous to eq. (1.5) exist for some twenty other measured observables [3].

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[2] In eqs. (1.3) we have used the sign conventions for s_W and c_W in ref. [1]. However, the formulas that we shall present for the OPs do not depend on those conventions.

[3] We adopt the definitions of the OPs in ref. [2]. Those definitions do not neglect the second derivatives of the $A_{VV'}\left(q^2\right)$ relative to q^2. For this reason, they produce extra parameters V, W, and X.

[4] A few other exceptional values of J and Y, like $J = 3$ and $Y = 2$, are permitted too.

[5] The angle θ_W is extracted from these input observables through

$$s_W^2 + c_W^2 = 1 \quad \text{and} \quad s_W c_W = \frac{\pi\alpha}{\sqrt{2} G_F m_Z^2}.$$

Equation (1.4) is *not* supposed to hold at loop level.

General formulas for the OPs when the new fermions of the NPM are placed in either singlets, doublets, or triplets of SU(2), and have some specific hypercharges, have been recently derived in ref. [4]. General formulas for the OPs when the new particles of the NPM are scalars in *any* representations of SU(2) × U(1) have been presented in ref. [5]. Here we generalize both papers by presenting general formulas for the OPs when the new particles of the NPM are fermions in *any* representations of SU(2) × U(1). We allow the new fermions to have arbitrary masses and to mix freely among themselves.[6] We do not specify the mechanism through which the fermion masses are generated. We implicitly assume the new fermions to be of Dirac type.[7]

This paper is organized as follows. In section 2 we introduce the functions in terms of which we are later going to write down the OPs. In section 3 we define the mixing matrices of the fermions and we prove some equations that apply to them. The formulas for the oblique parameters are displayed in section 4; we also demonstrate there the cancellation of the divergences of S and U, and we write down the equation that must be satisfied in order for the divergence of T to vanish too. In section 5 we consider the simple case of one vector-like multiplet of fermions, while in section 6 we analyse a model with two vector-like multiplets of fermions. We draw our conclusions in section 7. In appendix A we give formulas for the parameters S and U as they were defined in the original work by Peskin and Takeuchi [7].

2 Functions

In ref. [4] a few functions have been found to be relevant to the formulas for the OPs in an NPM with singlet, doublet, and triplet fermions. Now we have found that those functions are, indeed, all that one needs to write down the OPs when there are *any* new fermions. The functions were displayed in ref. [4] as linear combinations of the dispersive parts of various Passarino-Veltman functions (PVF) [8]. The PVF may be computed, for instance, by using the software `LoopTools` [9, 10]. However, it may be more convenient to present formulas for the functions that do not involve the PVF and that may be more immediately written in a code. That's what we do here. The functions are:

$$k\left(Q,I,J\right) = \frac{1}{3} - \frac{I+J}{4Q} - \frac{\left(I-J\right)^2}{2Q^2} + \frac{1}{4Q}\left[\frac{\left(I-J\right)^3}{Q^2} - \frac{I^2+J^2}{I-J}\right]\ln\frac{I}{J}$$
$$+\left[-I-J+\frac{\left(I+J\right)^2}{Q}+\frac{\left(I-J\right)^2\left(I+J\right)}{Q^2}-\frac{\left(I-J\right)^4}{Q^3}\right]\frac{f\left(Q,I,J\right)}{4}, \tag{2.1a}$$

$$j\left(Q,I,J\right) = -2 + \left[\frac{I-J}{Q} - \frac{I+J}{2\left(I-J\right)}\right]\ln\frac{I}{J}$$
$$+\left[-\frac{Q}{2}+\frac{3\left(I+J\right)}{2}-\frac{\left(I-J\right)^2}{Q}\right]f\left(Q,I,J\right), \tag{2.1b}$$

[6] We do not consider mixing between the NP fermions and the SM fermions. If this mixing is present, then one must do the computations of the OPs by following the recipe we give here both for the NPM and for the SM, and, afterwards, the true OPs are given by OP = $\mathrm{OP}^{\mathrm{NPM}} - \mathrm{OP}^{\mathrm{SM}}$.

[7] Various interesting sets of fermions that may be added to the SM have been identified in ref. [6]. Many of those sets contain Majorana neutrinos.

$$g\left(Q,I,J\right)=-\frac{\mathrm{div}}{3}+\frac{1}{6}\left(\ln\frac{I}{\mu^2}+\ln\frac{J}{\mu^2}\right)-\frac{5}{9}+\frac{I+J}{12Q}+\frac{\left(I-J\right)^2}{6Q^2}$$
$$+\frac{1}{4Q}\left[\frac{I^2+J^2}{I-J}-\frac{\left(I-J\right)^3}{3Q^2}\right]\ln\frac{I}{J}$$
$$+\left[-2Q+5\left(I+J\right)-\frac{3I^2+3J^2+2IJ}{Q}\right.$$
$$\left.-\frac{\left(I+J\right)\left(I-J\right)^2}{Q^2}+\frac{\left(I-J\right)^4}{Q^3}\right]\frac{f\left(Q,I,J\right)}{12},\tag{2.1c}$$
$$\hat{g}\left(Q,I,J\right)=1+\frac{1}{2}\left(\frac{I+J}{I-J}-\frac{I-J}{Q}\right)\ln\frac{I}{J}+\left[\frac{Q}{2}-I-J+\frac{\left(I-J\right)^2}{2Q}\right]f\left(Q,I,J\right),\tag{2.1d}$$
$$l\left(Q,I\right)=-\frac{5}{9}-\frac{4I}{3Q}+\left(-\frac{Q}{6}+\frac{I}{3}+\frac{4I^2}{3Q}\right)f\left(Q,I,I\right),\tag{2.1e}$$
$$h\left(I\right)=\frac{\mathrm{div}}{3}-\frac{1}{3}\ln\frac{I}{\mu^2},\tag{2.1f}$$
$$t\left(I,J\right)=\frac{I+J}{4}\left[\mathrm{div}-\frac{1}{2}\left(\ln\frac{I}{\mu^2}+\ln\frac{J}{\mu^2}\right)\right]+\frac{I+J}{8}-\frac{I^2+J^2}{8\left(I-J\right)}\ln\frac{I}{J},\tag{2.1g}$$
$$\hat{t}\left(I,J\right)=\mathrm{div}-\frac{1}{2}\left(\ln\frac{I}{\mu^2}+\ln\frac{J}{\mu^2}\right)+1-\frac{I+J}{2\left(I-J\right)}\ln\frac{I}{J}.\tag{2.1h}$$

In eqs. (2.1a)–(2.1e),

$$f\left(Q,I,J\right)=\begin{cases}\dfrac{1}{\sqrt{\Delta}}\ln\dfrac{I+J-Q+\sqrt{\Delta}}{I+J-Q-\sqrt{\Delta}} & \Leftarrow\ \Delta>0,\\ \dfrac{2}{\sqrt{-\Delta}}\left(\arctan\dfrac{Q+I-J}{\sqrt{-\Delta}}+\arctan\dfrac{Q+J-I}{\sqrt{-\Delta}}\right) & \Leftarrow\ \Delta<0,\\ \dfrac{1}{\sqrt{IJ}} & \Leftarrow \sqrt{Q}=\left|\sqrt{I}-\sqrt{J}\right|,\\ \dfrac{-1}{\sqrt{IJ}} & \Leftarrow \sqrt{Q}=\sqrt{I}+\sqrt{J},\end{cases}\tag{2.2}$$

where

$$\Delta=Q^2-2Q\left(I+J\right)+\left(I-J\right)^2.\tag{2.3}$$

Equations (2.1a)–(2.1d), (2.1g), and (2.1h) have been written assuming $I\neq J$. It is easy to find the fitting expressions for $I=J$:

$$k\left(Q,I,I\right)=\frac{1}{3}-\frac{I}{Q}+I\left(\frac{I}{Q}-\frac{1}{2}\right)f\left(Q,I,I\right),\tag{2.4a}$$
$$j\left(Q,I,I\right)=-3+\left(3I-\frac{Q}{2}\right)f\left(Q,I,I\right),\tag{2.4b}$$
$$g\left(Q,I,I\right)=-\frac{\mathrm{div}}{3}+\frac{1}{3}\ln\frac{I}{\mu^2}-\frac{5}{9}+\frac{2I}{3Q}-\left(Q-5I+\frac{4I^2}{Q}\right)\frac{f\left(Q,I,I\right)}{6},\tag{2.4c}$$
$$\hat{g}\left(Q,I,I\right)=2+\left(Q-4I\right)\frac{f\left(Q,I,I\right)}{2},\tag{2.4d}$$

$$t\,(I,I) = \frac{I}{2}\left(\mathrm{div} - \ln\frac{I}{\mu^2}\right), \tag{2.4e}$$

$$\hat{t}\,(I,I) = \mathrm{div} - \ln\frac{I}{\mu^2}. \tag{2.4f}$$

Some eqs. (2.1) and (2.4) depend on a dimensionless divergent quantity 'div' and on an arbitrary mass parameter μ; those two quantities are supposed to disappear from the formulas for any physical quantity like the OPs. We will soon see the way that happens in practice.

When the masses of the New Physics particles are much larger than the Fermi scale one may use, instead of expressions (2.1a)–(2.1e), their approximations for $Q \ll I, J$. Thus,

$$k\,(Q,I,J) = \begin{cases} Q\left[\frac{(I+J)\,(8IJ - I^2 - J^2)}{8\,(I-J)^4} - \frac{3I^2J^2}{2\,(I-J)^5}\ln\frac{I}{J}\right] + \mathrm{O}\left(Q^2\right) & \Leftarrow I \neq J, \\ -\frac{Q}{20I} + \mathrm{O}\left(Q^2\right) & \Leftarrow I = J, \end{cases} \tag{2.5a}$$

$$\frac{j\,(Q,I,J)}{Q} = \begin{cases} Q\left[\frac{10IJ + I^2 + J^2}{6\,(I-J)^4} - \frac{IJ\,(I+J)}{(I-J)^5}\ln\frac{I}{J}\right] + \mathrm{O}\left(Q^2\right) & \Leftarrow I \neq J, \\ \frac{Q}{60I^2} + \mathrm{O}\left(Q^2\right) & \Leftarrow I = J, \end{cases} \tag{2.5b}$$

$$g\,(Q,I,J) = \overline{g}\,(I,J) + k\,(Q,I,J)\,, \tag{2.5c}$$

$$\frac{\hat{g}\,(Q,I,J)}{Q} = \overline{\hat{g}}\,(I,J) + \frac{j\,(Q,I,J)}{Q}, \tag{2.5d}$$

$$l\,(Q,I) = -\frac{Q}{15\,I} + \mathrm{O}\left(Q^2\right), \tag{2.5e}$$

where

$$\overline{g}\,(I,J) = \begin{cases} -\frac{\mathrm{div}}{3} + \frac{1}{6}\left(\ln\frac{I}{\mu^2} + \ln\frac{J}{\mu^2}\right) + \frac{8IJ - I^2 - J^2}{9\,(I-J)^2} \\ \qquad + \frac{I^3 + J^3 - 3I^2J - 3IJ^2}{6\,(I-J)^3}\ln\frac{I}{J} & \Leftarrow I \neq J, \\ -\frac{\mathrm{div}}{3} + \frac{1}{3}\ln\frac{I}{\mu^2} + \frac{1}{6} & \Leftarrow I = J, \end{cases} \tag{2.6}$$

and

$$\overline{\hat{g}}\,(I,J) = \begin{cases} \frac{I+J}{2\,(I-J)^2} - \frac{IJ}{(I-J)^3}\ln\frac{I}{J} & \Leftarrow I \neq J, \\ \frac{1}{6I} & \Leftarrow I = J. \end{cases} \tag{2.7}$$

3 Mixing matrices

We put together in a set all the fermions that have the same chirality E (E may be either L—left—or R—right) and the same colour. If there are in the NPM any other non-SU(2) $\times$ U(1) conserved quantum numbers, then all the fermions in each set should have the same values of those quantum numbers too. Moreover, all the fermions in each set must have electric charges that differ among themselves by *integer* numbers; this means that, if any two fermions have electric charges that differ between themselves through a non-integer, then those two fermions must be placed in *different* sets. We emphasize that different sets must be treated separately, because they give separate contributions to each OP, just as new scalars in an NPM give separate contributions to the OPs from new fermions in the NPM.

We consider in turn each set of fermions with chirality E. In the set, the raising operator of weak isospin, viz. T_+, is represented by a matrix that we name $M_E \big/ \sqrt{2}$.[8] The lowering operator of weak isospin, i.e. T_-, is the Hermitian conjugate of T_+; therefore, it is represented by the matrix $M_E^\dagger \big/ \sqrt{2}$. Finally, the third component of weak isospin is

$$T_3 = [T_+, T_-] \tag{3.1}$$

and is represented by the matrix $H_E/2$,[9] where

$$H_E = \left[M_E, M_E^\dagger \right]. \tag{3.2}$$

We must take into account the weak-isospin commutation relation

$$[T_3, T_+] = T_+. \tag{3.3}$$

Since, as written in the previous paragraph, $T_+ \mapsto M_E \big/ \sqrt{2}$ and $T_3 \mapsto \left[M_E, M_E^\dagger \right] \big/ 2$, eq. (3.3) implies

$$M_E = M_E M_E^\dagger M_E - \frac{1}{2} \left(M_E^2 M_E^\dagger + M_E^\dagger M_E^2 \right). \tag{3.4}$$

Equation (3.4) implies

$$\operatorname{tr}\left(M_E M_E^\dagger \right) = \operatorname{tr}\left(M_E M_E^\dagger M_E M_E^\dagger \right) - \operatorname{tr}\left(M_E^2 {M_E^\dagger}^2 \right) \tag{3.5a}$$

$$= \frac{\operatorname{tr}\left(H_E^2 \right)}{2}. \tag{3.5b}$$

Equation (3.5) is separately valid for each set of fermions; in particular, it is valid for both $E = L$ and $E = R$.

We place the fermions of each set in a column vector, ordering them by decreasing electric charges. This means that the electric-charge operator is represented by the square matrix[10]

$$Q = \begin{pmatrix} Q_1 \times \mathbf{1}_{q_1} & \mathbf{0}_{q_1 \times q_2} & \mathbf{0}_{q_1 \times q_3} & \cdots \\ \mathbf{0}_{q_2 \times q_1} & Q_2 \times \mathbf{1}_{q_2} & \mathbf{0}_{q_2 \times q_3} & \cdots \\ \mathbf{0}_{q_3 \times q_1} & \mathbf{0}_{q_3 \times q_2} & Q_3 \times \mathbf{1}_{q_3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{3.6}$$

where $\mathbf{0}_{m \times n}$ denotes the $m \times n$ null matrix, $\mathbf{1}_n$ denotes the $n \times n$ unit matrix, q_n is the number of fermions in the set that have electric charge Q_n, and

$$Q_1 - Q_2 = Q_2 - Q_3 = \cdots = 1. \tag{3.7}$$

[8] The denominator $\sqrt{2}$ is purely conventional.

[9] The denominator 2 is just a convention.

[10] We implicitly assume that the electric charges of the left-handed fermions are the same as those of the right-handed fermions, so that all the fermions may acquire a Dirac mass.

When one adopts this ordering of the fermions in a set we see that, since T_+ connects the fermions of a given electric charge to the fermions with one unit less of electric charge, one must have

$$M_E = \begin{pmatrix} \mathbf{0}_{q_1\times q_1} & M_{E1} & \mathbf{0}_{q_1\times q_3} & \mathbf{0}_{q_1\times q_4} & \cdots \\ \mathbf{0}_{q_2\times q_1} & \mathbf{0}_{q_2\times q_2} & M_{E2} & \mathbf{0}_{q_2\times q_4} & \cdots \\ \mathbf{0}_{q_3\times q_1} & \mathbf{0}_{q_3\times q_2} & \mathbf{0}_{q_3\times q_3} & M_{E3} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{3.8a}$$

$$M_E^\dagger = \begin{pmatrix} \mathbf{0}_{q_1\times q_1} & \mathbf{0}_{q_1\times q_2} & \mathbf{0}_{q_1\times q_3} & \mathbf{0}_{q_1\times q_4} & \cdots \\ M_{E1}^\dagger & \mathbf{0}_{q_2\times q_2} & \mathbf{0}_{q_2\times q_3} & \mathbf{0}_{q_2\times q_4} & \cdots \\ \mathbf{0}_{q_3\times q_1} & M_{E2}^\dagger & \mathbf{0}_{q_3\times q_3} & \mathbf{0}_{q_3\times q_4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{3.8b}$$

where M_{En} is a $q_n \times q_{n+1}$ matrix.[11] Then,

$$H_E = \begin{pmatrix} M_{E1}M_{E1}^\dagger & \mathbf{0}_{q_1\times q_2} & \mathbf{0}_{q_1\times q_3} & \cdots \\ \mathbf{0}_{q_2\times q_1} & M_{E2}M_{E2}^\dagger - M_{E1}^\dagger M_{E1} & \mathbf{0}_{q_2\times q_3} & \cdots \\ \mathbf{0}_{q_3\times q_1} & \mathbf{0}_{q_3\times q_2} & M_{E3}M_{E3}^\dagger - M_{E2}^\dagger M_{E2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{3.9}$$

The Z^0 boson couples to $T_3 - Q s_W^2$. Since $T_3 \mapsto H_E/2$, it is convenient to define the matrix F_E through

$$F_E = H_E - 2 s_W^2 Q, \tag{3.10}$$

where Q is the diagonal, real matrix in eq. (3.6). The matrices F_E are Hermitian just as the matrices H_E.

Using eqs. (3.8) we see that

$$\mathrm{tr}\left(M_E M_E^\dagger\right) = \mathrm{tr}\left(M_{E1} M_{E1}^\dagger\right) + \mathrm{tr}\left(M_{E2} M_{E2}^\dagger\right) + \cdots. \tag{3.11}$$

Also, using eqs. (3.6) and (3.9),

$$\mathrm{tr}\left(Q H_E\right) = \left(Q_1 - Q_2\right) \mathrm{tr}\left(M_{E1} M_{E1}^\dagger\right) + \left(Q_2 - Q_3\right) \mathrm{tr}\left(M_{E2} M_{E2}^\dagger\right) + \cdots. \tag{3.12}$$

Utilizing eq. (3.7) we then conclude that

$$\mathrm{tr}\left(Q H_E\right) = \mathrm{tr}\left(M_E M_E^\dagger\right). \tag{3.13}$$

[11] For instance, it is well known that for a doublet of SU(2)

$$M_E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$$

while for a triplet of SU(2)

$$M_E = \begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}.$$

Equations (3.5) and (3.13) are crucial to demonstrate the finiteness of the oblique parameters S and U. Notice that those two equations depend neither on the masses of the fermions nor on the way that those masses are generated.

Notice that in this formalism we do not mention the hypercharge Y at all. In a weak basis each fermion has a well-defined T_3 and a well-defined Y. In the physical basis that we utilize this is not so: each physical fermion may be the superposition of various components with different T_3 and different Y. On the other hand, $Q = T_3 + Y$ has a well-defined value Q_f for each physical fermion f.

Using the covariant derivative [1]

$$D_\mu = \partial_\mu + ieA_\mu Q - i\,\frac{e}{s_W}\left(W_\mu^+ T_+ + W_\mu^- T_-\right) - i\,\frac{e}{s_W c_W}\,Z_\mu\left(T_3 - Q s_W^2\right), \tag{3.14}$$

where $e = \sqrt{4\pi\alpha}$ is the electric-charge unit, we may now write the gauge-kinetic Lagrangian for the fermions f_E in a set:

$$\begin{aligned}\mathcal{L}_{\rm gk} = {} & \frac{i}{2}\sum_f\left[\bar f_E\,\gamma^\mu\left(\partial_\mu f_E\right) - \left(\partial_\mu \bar f_E\right)\gamma^\mu\,f_E\right] \\ & - eA_\mu\sum_f Q_f\,\bar f_E\gamma^\mu f_E + \frac{e}{2 s_W c_W}\,Z_\mu\sum_{f,f'}\left(F_E\right)_{ff'}\bar f_E\gamma^\mu f'_E \\ & + \frac{e}{\sqrt{2}s_W}\sum_{f,f'}\left[W_\mu^+\left(M_E\right)_{ff'} + W_\mu^-\left(M_E^\dagger\right)_{ff'}\right]\bar f_E\gamma^\mu f'_E. \end{aligned} \tag{3.15}$$

4 Formulas for the OPs

Using the computations in ref. [4], we are now in a position to write the formulas for the various OPs.

The parameters V and W. One has

$$V = \frac{1}{8\pi s_W^2 c_W^2}\sum_{f,f'}\mathcal{F}\left[\left(F_L\right)_{ff'}, \left(F_R\right)_{ff'}, m_Z^2, m_f^2, m_{f'}^2\right], \tag{4.1a}$$

$$W = \frac{1}{4\pi s_W^2}\sum_{f,f'}\mathcal{F}\left[\left(M_L\right)_{ff'}, \left(M_R\right)_{ff'}, m_W^2, m_f^2, m_{f'}^2\right], \tag{4.1b}$$

where the sum runs over all the fermions f and f' in a set, m_f and $m_{f'}$ are the masses of f and f', respectively, and

$$\mathcal{F}\left(x, y, Q, I, J\right) = \left(|x|^2 + |y|^2\right) k\left(Q, I, J\right) - 2\,\mathrm{Re}\left(x y^*\right)\frac{j\left(Q, I, J\right)}{Q}\,\sqrt{IJ}. \tag{4.2}$$

It is worth pointing out that in eq. (4.1a), whenever $f \neq f'$, there are *two equal terms* in the sum, because the matrices F_L and F_R are Hermitian and

$$\mathcal{F}\left(x, y, Q, I, J\right) = \mathcal{F}\left(x^*, y^*, Q, J, I\right). \tag{4.3}$$

The parameter X. One has

$$X = \frac{1}{4\pi}\sum_f Q_f\left(F_L + F_R\right)_{ff}\,l\left(m_Z^2, m_f^2\right). \tag{4.4}$$

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The parameters S and U. One has

$$S = \frac{1}{2\pi} \sum_{f,f'} \mathcal{G}\left[(H_L)_{ff'}, (H_R)_{ff'}, m_Z^2, m_f^2, m_{f'}^2 \right] + \frac{1}{\pi} \sum_f Q_f \left(H_L + H_R \right)_{ff} h\left(m_f^2 \right) + \frac{2 s_W^2}{\pi} \sum_f Q_f \left[2 Q_f s_W^2 - \left(H_L + H_R \right)_{ff} \right] l\left(m_Z^2, m_f^2 \right), \tag{4.5a}$$

$$U = -S + \frac{1}{\pi} \sum_{f,f'} \mathcal{G}\left[(M_L)_{ff'}, (M_R)_{ff'}, m_W^2, m_f^2, m_{f'}^2 \right] + \frac{1}{\pi} \sum_f Q_f \left(H_L + H_R \right)_{ff} h\left(m_f^2 \right), \tag{4.5b}$$

where

$$\mathcal{G}\left(x, y, Q, I, J \right) = \left(|x|^2 + |y|^2 \right) g\left(Q, I, J \right) - 2 \operatorname{Re}\left(x y^* \right) \frac{\hat{g}\left(Q, I, J \right)}{Q} \sqrt{IJ}. \tag{4.6}$$

Note that in ref. [4] the function $\mathcal{G}$ was defined with the opposite sign.

Cancellation of the divergence in S. We remind that, according to eqs. (2.1),

$$g\left(Q, I, J \right) = -\frac{\widetilde{\text{div}}}{3} + \text{finite, } \mu\text{-independent terms}, \tag{4.7a}$$

$$h\left(I \right) = \frac{\widetilde{\text{div}}}{3} + \text{finite, } \mu\text{-independent terms}, \tag{4.7b}$$

and the functions $\hat{g}\left(Q, I, J \right)$ and $l\left(Q, I \right)$ do not contain $\widetilde{\text{div}}$, where $\widetilde{\text{div}} \equiv \text{div} + \ln \mu^2$ includes both the divergent quantity 'div' and the arbitrary mass μ. From eqs. (4.5a), (4.6), and (4.7) one sees that

$$S = -\frac{\widetilde{\text{div}}}{6\pi} \sum_{f,f'} \left[\left| (H_L)_{ff'} \right|^2 + \left| (H_R)_{ff'} \right|^2 \right] + \frac{\widetilde{\text{div}}}{3\pi} \sum_f Q_f \left(H_L + H_R \right)_{ff} + \text{finite, } \mu\text{-independent terms}. \tag{4.8}$$

But H_L and H_R are Hermitian matrices, therefore

$$S = \frac{\widetilde{\text{div}}}{6\pi} \left\{ -\operatorname{tr}\left[\left(H_L \right)^2 + \left(H_R \right)^2 \right] + 2 \operatorname{tr}\left[Q \left(H_L + H_R \right) \right] \right\} + \text{finite, } \mu\text{-independent terms}. \tag{4.9}$$

The terms in eq. (4.9) proportional to $\widetilde{\text{div}}$ vanish because of eqs. (3.5) and (3.13) (actually, they vanish separately for $E = L$ and $E = R$). Thus, S is both finite and μ-independent.

Cancellation of the divergence in U. Since S is $\widetilde{\text{div}}$-independent, eqs. (4.5b), (4.6), and (4.7) produce

$$U = \frac{\widetilde{\text{div}}}{3\pi} \left\{ -\sum_{f,f'} \left[\left| (M_L)_{ff'} \right|^2 + \left| (M_R)_{ff'} \right|^2 \right] + \sum_f Q_f \left(H_L + H_R \right)_{ff} \right\} + \text{finite, } \mu\text{-independent terms}. \tag{4.10}$$

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Therefore,

$$U = \frac{\widetilde{\mathrm{div}}}{3\pi} \left\{ -\mathrm{tr} \left(M_L M_L^\dagger + M_R M_R^\dagger \right) + \mathrm{tr} \left[Q \left(H_L + H_R \right) \right] \right\} + \text{finite, } \mu\text{-independent terms.} \tag{4.11}$$

The $\widetilde{\mathrm{div}}$-dependent terms in eq. (4.11) vanish because of eq. (3.13). Thus, U is both finite and μ-independent.

The parameter *T*. One has

$$T = \frac{1}{4\pi s_W^2 m_W^2} \sum_{f,f'} \left\{ 2\,\mathcal{H} \left[\left(M_L \right)_{ff'}, \left(M_R \right)_{ff'}, m_f^2, m_{f'}^2 \right] - \mathcal{H} \left[\left(H_L \right)_{ff'}, \left(H_R \right)_{ff'}, m_f^2, m_{f'}^2 \right] \right\}, \tag{4.12}$$

where

$$\mathcal{H} \left(x, y, I, J \right) = \left(|x|^2 + |y|^2 \right) t \left(I, J \right) - \mathrm{Re} \left(x y^* \right) \sqrt{IJ}\ \hat{t} \left(I, J \right). \tag{4.13}$$

We note that, because of eqs. (2.4e) and (2.4f),

$$\mathcal{H} \left(x, y, I, I \right) = \frac{I}{2}\ \hat{t} \left(I, I \right)\ |x - y|^2 . \tag{4.14}$$

In the second line of eq. (4.12) there are two equal terms in the sum whenever $f \neq f'$, because

$$\mathcal{H} \left(x, y, I, J \right) = \mathcal{H} \left(x^*, y^*, J, I \right). \tag{4.15}$$

Note that

$$2\,\mathcal{H} \left(x, y, I, J \right) - \mathcal{H} \left(x, y, I, I \right) - \mathcal{H} \left(x, y, J, J \right) = \left(|x|^2 + |y|^2 \right) \frac{\theta_+ \left(I, J \right)}{4} + \mathrm{Re} \left(x y^* \right) \frac{\theta_- \left(I, J \right)}{2}, \tag{4.16}$$

where $\theta_+ \left(I, J \right)$ and $\theta_- \left(I, J \right)$ are the functions that were defined in equations (12) and (13) of ref. [11].

Cancellation of the divergence in *T*. Because of eqs. (2.1g) and (2.1h),

$$t \left(I, J \right) = \frac{I + J}{4}\ \widetilde{\mathrm{div}} + \text{finite, } \mu\text{-independent terms,} \tag{4.17a}$$

$$\hat{t} \left(I, J \right) = \widetilde{\mathrm{div}} + \text{finite, } \mu\text{-independent terms.} \tag{4.17b}$$

Therefore,

$$\begin{aligned} T = {} & \frac{\widetilde{\mathrm{div}}}{16\pi s_W^2 m_W^2} \sum_{f,f'} \left\{ 2 \left[\left| \left(M_L \right)_{ff'} \right|^2 + \left| \left(M_R \right)_{ff'} \right|^2 \right] \left(m_f^2 + m_{f'}^2 \right) \right. \\ & - 8\,\mathrm{Re} \left[\left(M_L \right)_{ff'} \left(M_R^* \right)_{ff'} \right] m_f m_{f'} - \left[\left| \left(H_L \right)_{ff'} \right|^2 + \left| \left(H_R \right)_{ff'} \right|^2 \right] \left(m_f^2 + m_{f'}^2 \right) \\ & \left. + 4\,\mathrm{Re} \left[\left(H_L \right)_{ff'} \left(H_R^* \right)_{ff'} \right] m_f m_{f'} \right\} + \text{finite, } \mu\text{-independent terms} \end{aligned} \tag{4.18a}$$

$$\begin{aligned} = {} & \frac{\widetilde{\mathrm{div}}}{8\pi s_W^2 m_W^2} \left\{ \mathrm{tr} \left[\left(M_L M_L^\dagger + M_L^\dagger M_L + M_R M_R^\dagger + M_R^\dagger M_R \right) M^2 \right] \right. \\ & - 2\,\mathrm{tr} \left(M_L M M_R^\dagger M + M_R M M_L^\dagger M \right) - \mathrm{tr} \left[\left(H_L^2 + H_R^2 \right) M^2 \right] \\ & \left. + 2\,\mathrm{tr} \left(H_L M H_R M \right) \right\} + \text{finite, } \mu\text{-independent terms,} \end{aligned} \tag{4.18b}$$

where M is the mass matrix of the fermions. Thus, T is finite and μ-independent if

$$\mathrm{tr}\left[\left(M_L M_L^\dagger + M_L^\dagger M_L - H_L^2\right) M^2\right] \tag{4.19a}$$

$$+\mathrm{tr}\left[\left(M_R M_R^\dagger + M_R^\dagger M_R - H_R^2\right) M^2\right] \tag{4.19b}$$

$$+2\ \mathrm{tr}\left(H_L M H_R M - M_L M M_R^\dagger M - M_R M M_L^\dagger M\right) = 0. \tag{4.19c}$$

The oblique parameter T is *not* automatically finite, contrary to what happens with S and U. This should not surprise us. It is well known that T is divergent when the NPM does not obey eq. (1.4) at the tree level. In our case, the fermions may get masses either through bare mass terms, if they are in vector-like representations of SU(2) × U(1), or through their Yukawa couplings to neutral-scalar fields and the VEVs of those fields. Now, the VEVs may cause a violation of eq. (1.4) if the neutral-scalar fields do not feature $J(J+1) = 3Y^2$. If the fermion mass matrix M implicitly requires some scalar fields to have disallowed VEVs, then eq. (4.19) does not hold and T is divergent.[12]

5 One vector-like multiplet

We consider in this section the simple case of one vector-like multiplet of fermions with isospin J and hypercharge Y. All the $n = 2J + 1$ components of the multiplet have the same (bare) mass m, because there are, in general, no Yukawa couplings that can generate different masses for the different components of the multiplet. So, the only variables in this model are m and Y, which are continuous, and n, which is an integer.

The $n \times n$ matrices M_L and M_R are equal and they are given by

$$(M_L)_{rc} = (M_R)_{rc} = \delta_{c,r+1} \sqrt{r\,(n-r)}, \tag{5.1}$$

where the sub-index r stands for "row" and the sub-index c stands for "column" of a matrix. The $n \times n$ matrices H_L and H_R are equal and they are given by

$$(H_L)_{rc} = (H_R)_{rc} = \delta_{c,r}\,(n+1-2r)\,. \tag{5.2}$$

The electric-charge matrix is given by

$$Q_{rc} = \delta_{c,r}\, \frac{n+1-2r+2Y}{2}. \tag{5.3}$$

The $n \times n$ matrices F_L and F_R are equal and they are given by

$$(F_L)_{rc} = (F_R)_{rc} = \delta_{c,r}\left[(n+1-2r)\,c_W^2 - 2Y s_W^2\right]. \tag{5.4}$$

Because of eq. (4.14) and of the equalities between the matrices M_L and M_R and between the matrices F_L and F_R, the oblique parameter T vanishes. For the remaining OPs $O = S, U, V, W, X$ we obtain the general expression

$$O = \frac{n}{\pi}\left(A_O\, \frac{n^2-1}{3} + B_O Y^2\right), \tag{5.5}$$

[12] The fact that T may turn out divergent when one adds fermions to the SM and one gives arbitrary masses to those fermions had already been pointed out in ref. [12].

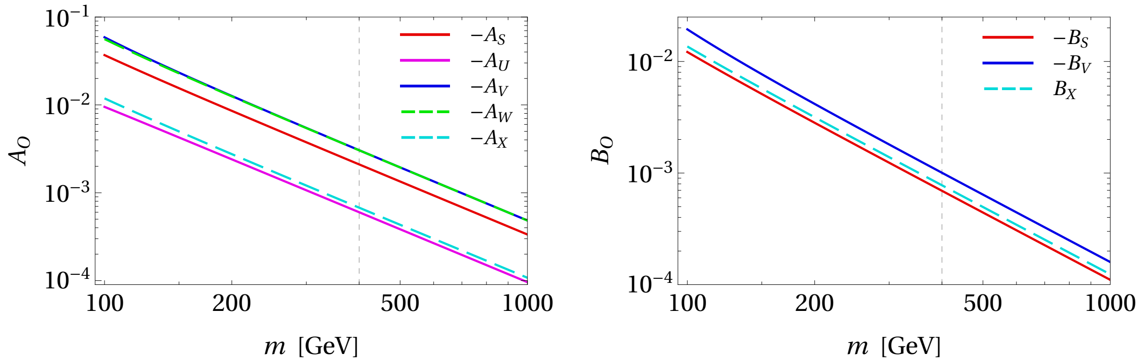

Figure 1. The coefficients A_O and B_O as functions of m according to eqs. (5.6) and (5.7). The dashed vertical line indicates the benchmark point value $m = 400\,\text{GeV}$. For large m, all the coefficients A_O and B_O vary as m^{-p} with p very close to 2.

where the coefficients A_O and B_O depend neither on n nor on Y; they only depend on m:

$$\begin{aligned} A_S &= g\left(m_Z^2, m^2, m^2\right) - \frac{m^2}{m_Z^2}\,\hat{g}\left(m_Z^2, m^2, m^2\right) \\ &\quad + h\left(m^2\right) - s_W^2\left(2 - s_W^2\right) l\left(m_Z^2, m^2\right) && \text{(5.6a)} \\ A_U &= g\left(m_W^2, m^2, m^2\right) - g\left(m_Z^2, m^2, m^2\right) \\ &\quad + m^2\left[\frac{\hat{g}\left(m_Z^2, m^2, m^2\right)}{m_Z^2} - \frac{\hat{g}\left(m_W^2, m^2, m^2\right)}{m_W^2}\right] \\ &\quad + s_W^2\left(2 - s_W^2\right) l\left(m_Z^2, m^2\right), && \text{(5.6b)} \\ A_V &= \frac{c_W^2}{4s_W^2}\left[k\left(m_Z^2, m^2, m^2\right) - \frac{m^2}{m_Z^2}\,j\left(m_Z^2, m^2, m^2\right)\right], && \text{(5.6c)} \\ A_W &= \frac{1}{4s_W^2}\left[k\left(m_W^2, m^2, m^2\right) - \frac{m^2}{m_W^2}\,j\left(m_W^2, m^2, m^2\right)\right], && \text{(5.6d)} \\ A_X &= \frac{c_W^2}{4}\,l\left(m_Z^2, m^2\right), && \text{(5.6e)} \end{aligned}$$

$$\begin{aligned} B_S = -B_U &= 4s_W^4\,l\left(m_Z^2, m^2\right) && \text{(5.7a)} \\ B_V &= \frac{s_W^2}{c_W^2}\left[k\left(m_Z^2, m^2, m^2\right) - \frac{m^2}{m_Z^2}\,j\left(m_Z^2, m^2, m^2\right)\right], && \text{(5.7b)} \\ B_X &= -s_W^2\,l\left(m_Z^2, m^2\right), && \text{(5.7c)} \end{aligned}$$

and $B_W = 0$. All the coefficients A_O and B_O are increasing functions of m, depicted in figure 1. Notice that, in general, the OPs V, W, and X may be as important as S and U.

Using eq. (2.5e) one finds that

$$B_S = -4s_W^4\,\frac{m_Z^2}{15m^2} + \mathrm{O}\left(\frac{m_Z^4}{m^4}\right). \tag{5.8}$$

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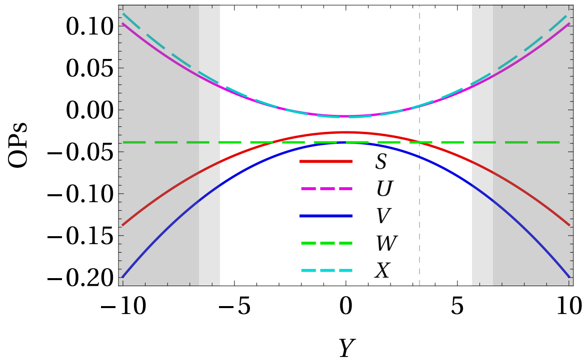

Figure 2. The oblique parameters as functions of Y, while $n = 5$ and $m = 400\,\text{GeV}$ are kept fixed. The dashed vertical line indicates the benchmark value $Y = 3.3$. The light-gray area indicates that the corresponding OPs lead to a fit to the observables with $\chi^2 > 17$; for the dark-grey area one has $\chi^2 > 20$.

Similarly, using eqs. (2.1f), (2.5c), (2.5d), (2.6), and (2.7) one finds that

$$A_S = -c_W^4 \frac{m_Z^2}{15m^2} + \mathrm{O}\left(\frac{m_Z^4}{m^4}\right). \tag{5.9}$$

Equations (5.8) and (5.9) are excellent approximations for the B_S and A_S, respectively, depicted in figure 1.

This New Physics Model gives a fit of the OPs which is just a little worse than letting the OPs vary freely. Indeed, by setting $V = W = X = 0$ and allowing S, T, and U to vary freely[13] we were able to accomplish a fit of all the relevant electroweak observables[14] with $\chi^2 = 14.201$; while in our NPM with $m = 400\,\text{GeV}$, $n = 5$, and $Y = 3.3$ we achieve $\chi^2 = 14.894$, which is not much worse.[15] We use the above values of m, n, and Y as our first benchmark point (BP1). Then,

- Keeping both n and m fixed at their BP1 values, we let Y vary and observe the variation of the OPs displayed in figure 2.
- Keeping both n and Y fixed at their BP1 values, we let m vary and observe the variation of the OPs displayed in figure 3.
- Keeping both Y and m fixed at their BP1 values, we let n vary and observe the variation of the OPs displayed in figure 4.

We also observe that there are approximate linear correlations between the parameters S and V, and between the parameters U and X, displayed in figure 5.

A more detailed description of the numerical analyses is given in subsection 6.4.

[13] Our best fit was obtained for $S = -1.2 \times 10^{-2}$, $T = 2.8 \times 10^{-2}$, and $U = 2.0 \times 10^{-3}$.

[14] We have used the following twenty observables, taken from ref. [13]: R_ℓ, R_b, R_c, A_ℓ, A_b, A_c, $A_{FB}^{(0,\ell)}$, $A_{FB}^{(0,b)}$, $A_{FB}^{(0,c)}$, $g_V^{\nu e}$, $g_A^{\nu e}$, $\bar{s}_\ell^2$ (three different values), m_W, Γ_W, Γ_Z, σ_{had}, $Q_W(\text{Cs})$, and $Q_W(\text{Tl})$.

[15] We perform a fit by defining $\chi^2 = RC^{-1}R^T$, where R is the row-vector of the residuals of the observables and C is the covariance matrix, which is evaluated according to the correlations among the observables [13–15].

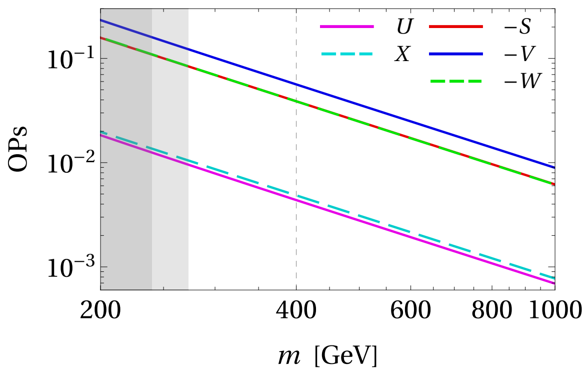

Figure 3. The oblique parameters as functions of m, while $n = 5$ and $Y = 3.3$ are kept fixed. The dashed vertical line indicates the benchmark value $m = 400\,\text{GeV}$. The meaning of the gray-shadowed bands is the same as in figure 2. For large m, all the oblique parameters are approximately proportional to m^{-2}.

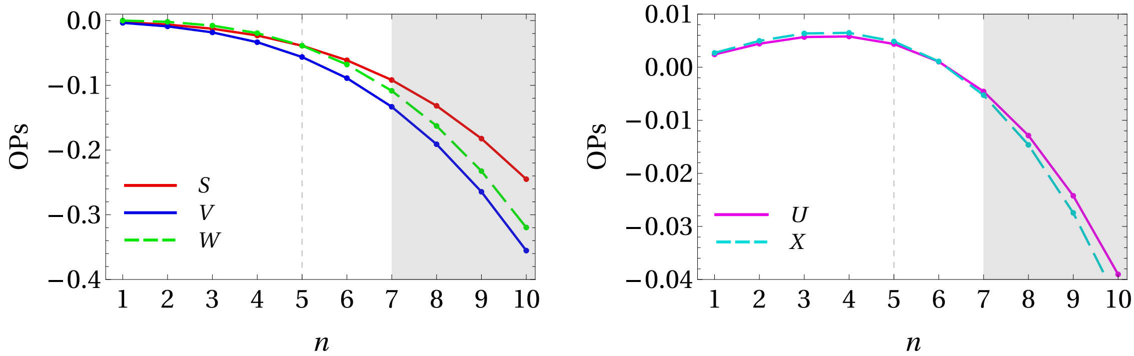

Figure 4. The oblique parameters as functions of n, while $m = 400\,\text{GeV}$ and $Y = 3.3$ are kept fixed. The dashed vertical line indicates the benchmark value $n = 5$. The dark-gray area means that the OPs lead to $\chi^2 > 20$ fit.

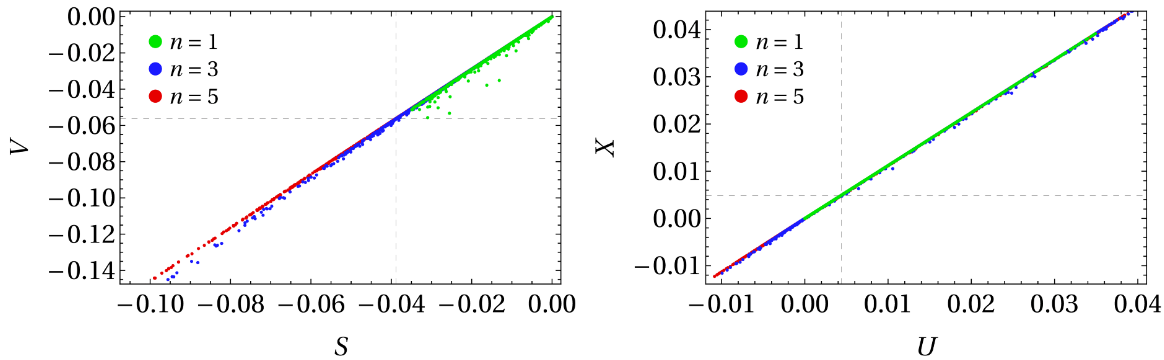

Figure 5. Correlation plots between oblique parameters for different values of n. The parameters S and V are distributed according to $V \approx 1.47\,S$ (left panel), while the parameters U and X obey $X \approx 1.12\,U$ (right panel). All points in the plots obey the restriction $\chi^2 \leq 20$. The dashed lines indicate the values of the oblique parameters at the benchmark point BP1.

6 Two vector-like multiplets

Since the formalism in section 3 may look a bit abstract, we give in this section the practical calculation of the mixing matrices in a specific NPM with *vector-like* (in order to avoid anomalies) fermions.[16] In our model all the fermion masses are justified either through bare mass terms or through $SU(2) \times U(1)$-invariant Yukawa couplings to the Higgs doublet of the SM; therefore, the oblique parameter T has no reason to feature an UV divergence and, indeed, it converges.

6.1 Description of the model

In the NPM that we suggest there are, besides all the fermion multiplets and scalar multiplets of the SM, the following multiplets of fermions:[17]

- One multiplet A_L of left-handed fermions with isospin J and hypercharge Y.
- One multiplet B_L of left-handed fermions with isospin $J + 1/2$ and hypercharge $Y + 1/2$.
- Two multiplets A_R and B_R of right-handed fermions with the same quantum numbers as those of A_L and B_L, respectively.

We define $n \equiv 2J + 1$. We write the multiplets of additional fermions as

$$A_E = \begin{pmatrix} a_{1,E} \\ a_{2,E} \\ \vdots \\ a_{n,E} \end{pmatrix}, \quad B_E = \begin{pmatrix} b_{0,E} \\ b_{1,E} \\ b_{2,E} \\ \vdots \\ b_{n,E} \end{pmatrix}. \tag{6.1}$$

There are bare-mass terms given by

$$\mathcal{L}_{\text{bare}} = -m_A \sum_{k=1}^{n} \overline{a_{k,R}}\, a_{k,L} - m_B \sum_{j=0}^{n} \overline{b_{j,R}}\, b_{j,L} + \text{H.c.} \tag{6.2}$$

The quantum numbers of the new fermion multiplets were chosen in such a way that they have $SU(2) \times U(1)$-invariant Yukawa couplings to the Higgs doublet of the SM $(\varphi^+,\ \varphi^0)^T$, which has isospin and hypercharge $1/2$. It is easy to convince oneself that the Yukawa couplings of φ^0 to the new fermions are given by

$$\mathcal{L}_{\text{Yukawa}} = \cdots - \varphi^0 \sum_{k=1}^{n} \sqrt{k} \left(y_R\, \overline{b_{k,R}}\, a_{k,L} + y_L\, \overline{b_{k,L}}\, a_{k,R} \right) + \text{H.c.}, \tag{6.3}$$

with Yukawa coupling constants y_R and y_L. Since the largest Yukawa couplings are $y_E \sqrt{n}$, we assume that

$$|y_R|, |y_L| < \frac{4\pi}{\sqrt{n}} \tag{6.4}$$

in order to respect unitarity.[18]

[16] The NPM that we deal with in this section has been recently suggested in ref. [16].

[17] For the sake of simplicity, we assume all the new fermions to be color singlets.

[18] Stronger unitarity constraints may exist, arising for instance from the scattering of fermions into gauge-boson pairs; see ref. [17] and, for the case of large *scalar* multiplets, see ref. [18].

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In eq. (6.3), note that $b_{0,L}$ and $b_{0,R}$ have no Yukawa couplings to φ^0. Together they form a Dirac fermion with electric charge $(n+1)/2+Y$. Its mass term is

$$-m_B\,\overline{b_{0,R}}\,b_{0,L}+\text{H.c.} \tag{6.5}$$

For $k=1,\ldots,n$, there are *two* Dirac fermions with electric charge $(n+1)/2+Y-k$. According to eqs. (6.2) and (6.3), their mass terms are given by

$$-\left(\overline{b_{k,R}},\,\overline{a_{k,R}}\right)\begin{pmatrix} m_B & \sqrt{k}\,m_C \\ \sqrt{k}\,m_D & m_A \end{pmatrix}\begin{pmatrix} b_{k,L} \\ a_{k,L} \end{pmatrix}. \tag{6.6}$$

In eq. (6.6), $m_C \equiv y_R v$ and $m_D \equiv y_L^* v^*$, where v is the VEV of φ^0, with $|v| \approx 174\,\text{GeV}$. According to eq. (6.4),

$$|m_C| < \frac{4\pi\,|v|}{\sqrt{n}}, \tag{6.7a}$$

$$|m_D| < \frac{4\pi\,|v|}{\sqrt{n}}, \tag{6.7b}$$

while $|m_A|$ and $|m_B|$ may be as large as one wishes. Our NPM has six real free parameters: Y, $\arg\left(m_A m_B m_C^* m_D^*\right)$, and

$$a \equiv |m_A|^2\,, \tag{6.8a}$$

$$b \equiv |m_B|^2\,, \tag{6.8b}$$

$$c \equiv |m_C|^2\,, \tag{6.8c}$$

$$d \equiv |m_D|^2\,. \tag{6.8d}$$

Additionally there is n, which is an integer.

For $k=1,\ldots,n$, we diagonalize the mass matrix in eq. (6.6) by making

$$\begin{pmatrix} b_{k,E} \\ a_{k,E} \end{pmatrix} = \mathcal{U}_{k,E}\begin{pmatrix} f_{k,E} \\ g_{k,E} \end{pmatrix}, \tag{6.9}$$

where the 2×2 matrices $\mathcal{U}_{k,E}$ are unitary and the physical fermions f_k and g_k have masses $m_{f,k}$ and $m_{g,k}$, respectively. We define

$$M_k = \begin{pmatrix} m_{f,k} & 0 \\ 0 & m_{g,k} \end{pmatrix}. \tag{6.10}$$

The matrices M_k are diagonal and real. The bi-diagonalization condition is

$$\begin{pmatrix} m_B & \sqrt{k}\,m_C \\ \sqrt{k}\,m_D & m_A \end{pmatrix} = \mathcal{U}_{k,R}\,M_k\,\mathcal{U}_{k,L}^\dagger. \tag{6.11}$$

It is convenient to write

$$\mathcal{U}_{k,E} = \begin{pmatrix} X_{k,E} \\ Y_{k,E} \end{pmatrix}, \tag{6.12}$$

where $X_{k,E}$ and $Y_{k,E}$ are 1×2 matrices. Thus, from eq. (6.9),

$$b_{k,E} = X_{k,E} \begin{pmatrix} f_{k,E} \\ g_{k,E} \end{pmatrix}, \tag{6.13a}$$

$$a_{k,E} = Y_{k,E} \begin{pmatrix} f_{k,E} \\ g_{k,E} \end{pmatrix}, \tag{6.13b}$$

$$\begin{pmatrix} f_{k,E} \\ g_{k,E} \end{pmatrix} = X^\dagger_{k,E} b_{k,E} + Y^\dagger_{k,E} a_{k,E}. \tag{6.13c}$$

The unitarity of $\mathcal{U}_{k,E}$ implies

$$X^*_{k,E} X^T_{k,E} = 1, \tag{6.14a}$$

$$Y^*_{k,E} Y^T_{k,E} = 1, \tag{6.14b}$$

$$X^*_{k,E} Y^T_{k,E} = Y^*_{k,E} X^T_{k,E} = 0, \tag{6.14c}$$

$$X^T_{k,E} X^*_{k,E} + Y^T_{k,E} Y^*_{k,E} = \mathbf{1}_2, \tag{6.14d}$$

where $\mathbf{1}_2$ is the 2×2 unit matrix. From eqs. (6.11) and (6.12),

$$m_B = X_{k,R}\, M_k\, X^\dagger_{k,L}, \tag{6.15a}$$

$$m_A = Y_{k,R}\, M_k\, Y^\dagger_{k,L}, \tag{6.15b}$$

$$\sqrt{k}\, m_C = X_{k,R}\, M_k\, Y^\dagger_{k,L}, \tag{6.15c}$$

$$\sqrt{k}\, m_D = Y_{k,R}\, M_k\, X^\dagger_{k,L}. \tag{6.15d}$$

Utilizing eq. (6.14d) and remembering that $M_k = M_k^\dagger$, one may derive from eqs. (6.15) that

$$b + kc = X_{k,R}\, M_k^2\, X^\dagger_{k,R}, \tag{6.16a}$$

$$a + kd = Y_{k,R}\, M_k^2\, Y^\dagger_{k,R}, \tag{6.16b}$$

$$b + kd = X_{k,L}\, M_k^2\, X^\dagger_{k,L}, \tag{6.16c}$$

$$a + kc = Y_{k,L}\, M_k^2\, Y^\dagger_{k,L}, \tag{6.16d}$$

where a, b, c, and d have been defined in eqs. (6.8).

6.2 The mixing matrices

We now apply our formalism to the model described in the previous subsection. Firstly, we put together all the physical fermions of each chirality in column vectors

$$V_E = \begin{pmatrix} b_{0,E} \\ f_{1,E} \\ g_{1,E} \\ \vdots \\ f_{n,E} \\ g_{n,E} \end{pmatrix}, \tag{6.17}$$

taking care *to order the fermions by their decreasing electric charges*. Indeed, the (diagonal) electric-charge matrix for the $2n+1$ physical fermions in V_E is

$$Q = \frac{1}{2} \begin{pmatrix} n+1 & 0_{1\times 2} & 0_{1\times 2} & \cdots & 0_{1\times 2} \\ 0_{2\times 1} & (n-1)\times \mathbf{1}_2 & 0_{2\times 2} & \cdots & 0_{2\times 2} \\ 0_{2\times 1} & 0_{2\times 2} & (n-3)\times \mathbf{1}_2 & \cdots & 0_{2\times 2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{2\times 1} & 0_{2\times 2} & 0_{2\times 2} & \cdots & (1-n)\times \mathbf{1}_2 \end{pmatrix} + Y \times \mathbf{1}_{2n+1}. \tag{6.18}$$

The (diagonal) mass matrix of the physical fermions in V_E is

$$M = \begin{pmatrix} m_B & 0_{1\times 2} & 0_{1\times 2} & \ldots & 0_{1\times 2} \\ 0_{2\times 1} & M_1 & 0_{2\times 2} & \ldots & 0_{2\times 2} \\ 0_{2\times 1} & 0_{2\times 2} & M_2 & \ldots & 0_{2\times 2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{2\times 1} & 0_{2\times 2} & 0_{2\times 2} & \cdots & M_n \end{pmatrix}, \tag{6.19}$$

where the matrices M_k have been defined in eq. (6.10).

We define the matrices $M_E \big/ \sqrt{2}$, which represent the action of the SU(2) operator T_+ on the fermions of V_E, through

$$V_E^T \frac{M_E}{\sqrt{2}} = \Big((T_+ b_{0,E}),\, (T_+ f_{1,E}),\, (T_+ g_{1,E}),\, \ldots,\, (T_+ f_{n,E}),\, (T_+ g_{n,E}) \Big). \tag{6.20}$$

Obviously, $T_+ b_{0,E} = 0$. Now, utilizing eqs. (6.13),

$$\sqrt{2}\, T_+ \begin{pmatrix} f_{1,E} \\ g_{1,E} \end{pmatrix} = \sqrt{2}\, T_+ \left(X_{1,E}^\dagger b_{1,E} + Y_{1,E}^\dagger a_{1,E} \right) \tag{6.21a}$$

$$= X_{1,E}^\dagger \sqrt{n}\, b_{0,E}, \tag{6.21b}$$

and, for $m = 1, \ldots, n-1$,

$$\sqrt{2}\, T_+ \begin{pmatrix} f_{m+1,E} \\ g_{m+1,E} \end{pmatrix} = \sqrt{2}\, T_+ \left(X_{m+1,E}^\dagger b_{m+1,E} + Y_{m+1,E}^\dagger a_{m+1,E} \right) \tag{6.22a}$$

$$= \sqrt{n-m} \left(X_{m+1,E}^\dagger \sqrt{m+1}\, b_{m,E} + Y_{m+1,E}^\dagger \sqrt{m}\, a_{m,E} \right) \tag{6.22b}$$

$$= \sqrt{n-m} \left(\sqrt{m+1}\; X_{m+1,E}^\dagger X_{m,E} + \sqrt{m}\; Y_{m+1,E}^\dagger Y_{m,E} \right) \begin{pmatrix} f_{m,E} \\ g_{m,E} \end{pmatrix}. \tag{6.22c}$$

Therefore,

$$M_E = \begin{pmatrix} 0 & M_{E,0} & 0_{1\times 2} & 0_{1\times 2} & \cdots & 0_{1\times 2} \\ 0_{2\times 1} & 0_{2\times 2} & M_{E,1} & 0_{2\times 2} & \cdots & 0_{2\times 2} \\ 0_{2\times 1} & 0_{2\times 2} & 0_{2\times 2} & M_{E,2} & \cdots & 0_{2\times 2} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{2\times 1} & 0_{2\times 2} & 0_{2\times 2} & 0_{2\times 2} & \cdots & M_{E,n-1} \\ 0_{2\times 1} & 0_{2\times 2} & 0_{2\times 2} & 0_{2\times 2} & \cdots & 0_{2\times 2} \end{pmatrix}, \tag{6.23}$$

where, from eq. (6.21),

$$M_{E,0} = \sqrt{n}\, X^*_{1,E} \tag{6.24}$$

is a 1×2 matrix, and the

$$M_{E,m} = \sqrt{n-m} \left(\sqrt{m+1}\ X^T_{m,E} X^*_{m+1,E} + \sqrt{m}\ Y^T_{m,E} Y^*_{m+1,E} \right) \quad (m = 1, \ldots, n-1) \tag{6.25}$$

are 2×2 matrices. Notice that M_E in eq. (6.23), just like Q in eq. (6.18) and M in eq. (6.19), is a $(2n+1) \times (2n+1)$ matrix, because there are $2n+1$ (new) Dirac fermions in our NPM.

For $k = 1, \ldots, n$ we define the 2×2 Hermitian, idempotent matrices

$$H_{k,E} \equiv X^T_{k,E} X^*_{k,E}, \tag{6.26a}$$

$$\left(H_{k,E}\right)^2 = H_{k,E}. \tag{6.26b}$$

(Equation (6.26b) follows from eq. (6.14a).) It is then easy to see that

$$M_E M_E^\dagger = \begin{pmatrix} M_{E,0} M^\dagger_{E,0} & 0_{1\times 2} & 0_{1\times 2} & \cdots & 0_{1\times 2} & 0_{1\times 2} \\ 0_{2\times 1} & M_{E,1} M^\dagger_{E,1} & 0_{2\times 2} & \cdots & 0_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 1} & 0_{2\times 2} & M_{E,2} M^\dagger_{E,2} & \cdots & 0_{2\times 2} & 0_{2\times 2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{2\times 1} & 0_{2\times 2} & 0_{2\times 2} & \cdots & M_{E,n-1} M^\dagger_{E,n-1} & 0_{2\times 2} \\ 0_{2\times 1} & 0_{2\times 2} & 0_{2\times 2} & \cdots & 0_{2\times 2} & 0_{2\times 2} \end{pmatrix} \tag{6.27}$$

has, because of eqs. (6.14),

$$M_{E,0} M^\dagger_{E,0} = n, \tag{6.28a}$$

$$M_{E,m} M^\dagger_{E,m} = (n-m) \left(m \times \mathbf{1}_2 + H_{m,E}\right) \qquad (m = 1, \ldots, n-1); \tag{6.28b}$$

while

$$M_E^\dagger M_E = \begin{pmatrix} 0 & 0_{1\times 2} & 0_{1\times 2} & \cdots & 0_{1\times 2} & 0_{1\times 2} \\ 0_{2\times 1} & M^\dagger_{E,0} M_{E,0} & 0_{2\times 2} & \cdots & 0_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 1} & 0_{2\times 2} & M^\dagger_{E,1} M_{E,1} & \cdots & 0_{2\times 2} & 0_{2\times 2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{2\times 1} & 0_{2\times 2} & 0_{2\times 2} & \cdots & M^\dagger_{E,n-2} M_{E,n-2} & 0_{2\times 2} \\ 0_{2\times 1} & 0_{2\times 2} & 0_{2\times 2} & \cdots & 0_{2\times 2} & M^\dagger_{E,n-1} M_{E,n-1} \end{pmatrix} \tag{6.29}$$

has

$$M^\dagger_{E,0} M_{E,0} = n \times H_{1,E}, \tag{6.30a}$$

$$M^\dagger_{E,m} M_{E,m} = (n-m) \left(m \times \mathbf{1}_2 + H_{m+1,E}\right) \qquad (m = 1, \ldots, n-1). \tag{6.30b}$$

Then, according to the definition (3.2),

$$H_E = M_E M_E^\dagger - M_E^\dagger M_E = \begin{pmatrix} H_{E,0} & 0_{1\times 2} & 0_{1\times 2} & \cdots & 0_{1\times 2} \\ 0_{2\times 1} & H_{E,1} & 0_{2\times 2} & \cdots & 0_{2\times 2} \\ 0_{2\times 1} & 0_{2\times 2} & H_{E,2} & \cdots & 0_{2\times 2} \\ \vdots & \vdots & \vdots & \ddots & \cdots \\ 0_{2\times 1} & 0_{2\times 2} & 0_{2\times 2} & \cdots & H_{E,n} \end{pmatrix}, \tag{6.31}$$

where

$$H_{E,0} = n, \tag{6.32a}$$
$$H_{E,k} = (n + 1 - 2k) \times \mathbf{1}_2 - H_{k,E} \qquad (k = 1, \ldots, n). \tag{6.32b}$$

Finally, the $(2n + 1) \times (2n + 1)$ Hermitian matrices F_E are given by eqs. (3.10), (6.18), and (6.31).

6.3 The finiteness of T

For completeness, in this subsection we explicitly demonstrate that eq. (4.19) holds in our NPM and that, therefore, the oblique parameter T is finite in it.

One may define

$$x_E \equiv \begin{cases} c \Leftarrow & E = R, \\ d \Leftarrow & E = L. \end{cases} \tag{6.33}$$

Then, from eqs. (6.16a) and (6.16c),

$$\mathrm{tr}\left(H_{k,E} M_k^2\right) = b + k x_E, \qquad (k = 1, \ldots n). \tag{6.34}$$

Also, eqs. (6.16a), (6.16b), and (6.14d) imply

$$\mathrm{tr}\left(M_k^2\right) = a + b + k\,(c + d) \qquad (k = 1, \ldots, n). \tag{6.35}$$

It is then easy to derive that

$$\mathrm{tr}\left(M_E M_E^\dagger M^2\right) = nb + \sum_{m=1}^{n-1} (n-m)\,\{m\,[a+b+m\,(c+d)]+b+m x_E\}\,, \tag{6.36a}$$
$$\mathrm{tr}\left(M_E^\dagger M_E M^2\right) = n\,(b+x_E) + \sum_{m=1}^{n-1} (n-m) \times \{m\,[a+b+(m+1)\,(c+d)]+b+(m+1)\,x_E\}\,, \tag{6.36b}$$
$$\mathrm{tr}\left(H_E^2 M^2\right) = n^2 b + \sum_{k=1}^{n} \left\{(n+1-2k)^2\,[a+b+k\,(c+d)] + (4k-2n-1)\,(b+k x_E)\right\}. \tag{6.36c}$$

Performing the sums over m by using

$$\sum_{m=1}^{n-1} 1 = n - 1, \tag{6.37a}$$
$$\sum_{m=1}^{n-1} m = (n - 1)\,\frac{n}{2}, \tag{6.37b}$$
$$\sum_{m=1}^{n-1} m^2 = (n - 1)\,\frac{n\,(2n - 1)}{6}, \tag{6.37c}$$
$$\sum_{m=1}^{n-1} m^3 = (n - 1)^2\,\frac{n^2}{4}, \tag{6.37d}$$

one finds that

$$\mathrm{tr}\left[\left(M_E M_E^\dagger + M_E^\dagger M_E - H_E^2\right) M^2\right] = 0 \tag{6.38}$$

for both $E = L$ and $E = R$. Therefore, each of the two lines (4.19a) and (4.19b) separately vanishes.

One also finds that

$$\begin{aligned}
\mathrm{tr}\left(H_L M H_R M\right) &= b H_{L,0} H_{R,0} + \sum_{k=1}^{n} \mathrm{tr}\left(H_{L,k} M_k H_{R,k} M_k\right) && \text{(6.39a)}\\
&= bn^2 + \sum_{k=1}^{n} \Big\{ (n+1-2k)^2\, \mathrm{tr}\left(M_k^2\right) \\
&\quad - (n+1-2k)\, \mathrm{tr}\left[\left(H_{k,L} + H_{k,R}\right) M_k^2\right] + \mathrm{tr}\left(H_{k,L} M_k H_{k,R} M_k\right)\Big\} && \text{(6.39b)}\\
&= bn^2 + \sum_{k=1}^{n} \Big\{ (n+1-2k)^2 \left[a + b + k\,(c+d)\right] \\
&\quad - (n+1-2k)\left[2b + k\,(c+d)\right] + b\Big\}\,, && \text{(6.39c)}\\
\mathrm{tr}\left(M_L M M_R^\dagger M\right) &= m_B \left(M_{L,0} M_1 M_{R,0}^\dagger\right) + \sum_{m=1}^{n-1} \mathrm{tr}\left(M_{L,m} M_{m+1} M_{R,m}^\dagger M_m\right) \\
&= m_B n \left(X_{1,L}^* M_1 X_{1,R}^T\right) \\
&\quad + \sum_{m=1}^{n-1} (n-m)\, \mathrm{tr}\Big[(m+1)\, X_{m,L}^T X_{m+1,L}^* M_{m+1} X_{m+1,R}^T X_{m,R}^* M_m \\
&\quad + m Y_{m,L}^T Y_{m+1,L}^* M_{m+1} Y_{m+1,R}^T Y_{m,R}^* M_m \\
&\quad + \sqrt{m\,(m+1)}\, X_{m,L}^T X_{m+1,L}^* M_{m+1} Y_{m+1,R}^T Y_{m,R}^* M_m \\
&\quad + \sqrt{m\,(m+1)}\, Y_{m,L}^T Y_{m+1,L}^* M_{m+1} X_{m+1,R}^T X_{m,R}^* M_m \Big] \\
&= nb + \sum_{m=1}^{n-1} (n-m)\left[(m+1)\, b + ma + m\,(m+1)\,(c+d)\right]. && \text{(6.39d)}
\end{aligned}$$

Therefore, once again performing the sums over m,

$$\mathrm{tr}\left(2\, M_L M M_R^\dagger M - H_L M H_R M\right) = 0 \tag{6.40}$$

and line (4.19c) is zero. Thus, in our model *each of the three lines of eq.* (4.19) *is separately zero.*

6.4 Numerical results

Our benchmark point 2 (BP2) has $|m_A| = |m_B| = 2000\,\mathrm{GeV}$, $|m_C| = |m_D| = 100\,\mathrm{GeV}$, $\arg\left(m_A^* m_B^* m_C m_D\right) = 1.5$, $Y = 3.3$, and $n = 5$. This yields a fit to the twenty electroweak observables with $\chi^2 = 14.214$, which is comparable to our best fit with null V, W, and X.

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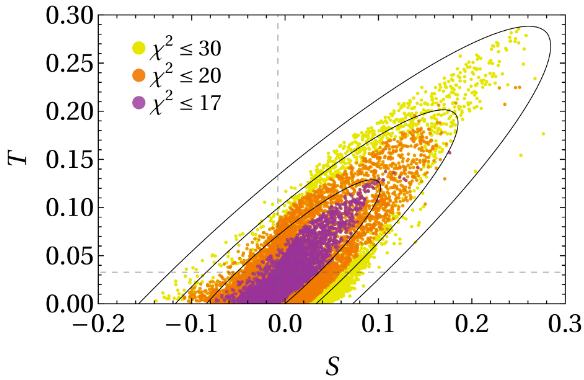

Figure 6. The correlation between the oblique parameters S and T in our New Physics Model, for $n = 5$. The black ellipses correspond to the 1σ, 2σ, and 3σ (2dof) allowed regions in the ST plane for a fit with $U = V = W = X = 0$ and completely free S and T. The dashed lines indicate the values of the oblique parameters at the benchmark point BP2.

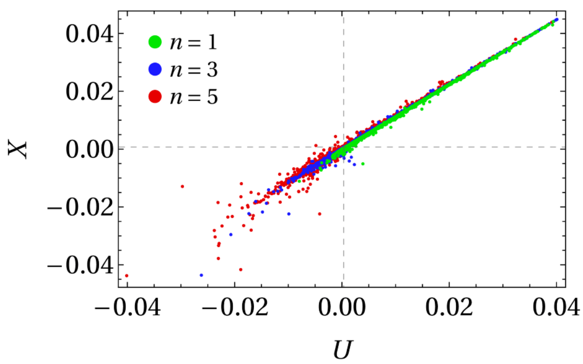

Figure 7. The correlation between the OPs U and X, for different values of n. All the points in this plot have $\chi^2 \leq 20$. The dashed lines indicate the values of the OPs in the BP2.

In order to explore the entire parameter space, we consider various integer values of n, we let the masses vary from 50 GeV to 3000 GeV but subject to the constraints (6.7), we let $\arg\left(m_A^* m_B^* m_C m_D\right)$ vary from 0 to 2π, and we let Y go from -10 to $+10$. We keep only the points that have χ^2 smaller than a certain number, which may be either 30, 20, or 17.[19] This differentiation of the points according to their χ^2 coincides well with the correlation between the S and T parameters in the electroweak fit, displayed in figure 6. This figure also shows that our NPM can only produce *positive* values for the parameter T.

In our NPM there is the approximate linear correlation between the oblique parameters U and X displayed in figure 7. The distribution of parameters is very similar to the one observed for the NPM of section 5, i.e. here too one has $X \approx 1.12\, U$.

[19]The pull of observable O is defined as $\left(O_{\text{fit}} - O_{\text{measured}}\right)/\,\delta^{\pm}_{\text{measured}}$, where O_{measured} is the central value and $\delta^{\pm}_{\text{measured}}$ is the error in the measurement of O. In practice, most pulls are always very small and only very few observables have large pulls. As a consequence, points with $\chi^2 < 30$ have all the pulls between -3 and $+3$; points with $\chi^2 < 20$ have pulls ranging from -2 to $+2$, except for the observables $A_{FB}^{(0,b)}$ and A_ℓ; and points with $\chi^2 < 17$ have pulls between -1 and $+1$, with the additional exceptions of R_ℓ and $Q_W(\text{Cs})$.

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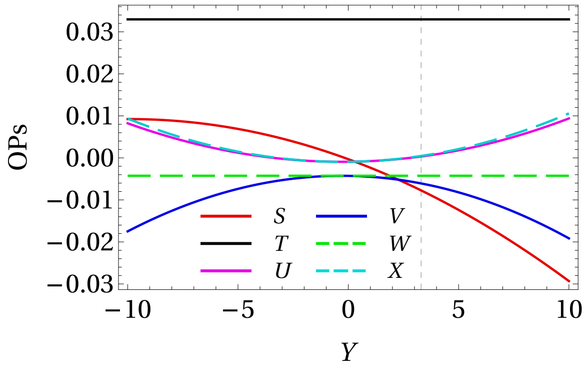

Figure 8. The OPs as functions of Y. The dashed vertical line indicates the BP2 value $Y = 3.3$.

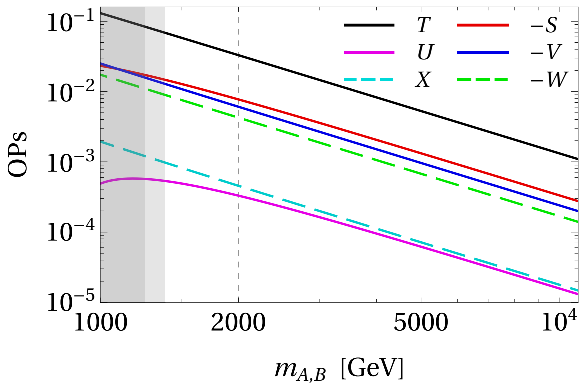

Figure 9. The OPs as functions of $m_A = m_B$. The dashed vertical line indicates the BP2 value $m_{A,B} = 2000\,\text{GeV}$. In the light-gray area, one obtains $\chi^2 > 17$; in the dark-gray area, $\chi^2 > 20$. For $m_A = m_B \gtrsim 2\,\text{TeV}$, all the OPs vary with m_A as m_A^{-2}, approximately.

In figure 8 the variation of the OPs with Y is displayed; all the other parameters of the model are kept fixed at their BP2 values. As ever, the OPs W and T are constant because eqs. (4.1b) and (4.12), respectively, do not depend on Y. It should be noted that in this NPM the impact of Y on χ^2 is weak, contrary to what happened in the model of section 5, *cf.* figure 2.

In figure 9 one observes that, as the value of $m_A = m_B$ increases, the absolute values of all the OPs decrease. Points with very low $m_A = m_B$ tend to have large χ^2; $\chi^2 \approx 14$ is minimal when $m_A = m_B = 2000\,\text{GeV}$ (i.e., at the BP2), and increases up to ≈ 16 for larger $m_A = m_B$.

When we keep all the mass parameters and Y fixed at their BP2 values, and we allow n to vary, we observe the variation of the OPs displayed in figure 10. The absolute values of all the OPs increase with n for $n > 4$, and eventually χ^2 becomes larger than at the BP2.

7 Conclusions

In this work we have presented general formulas for all six oblique parameters in an extension of the SM with additional fermions. The formulas are based on a formalism which defines matrices M_L and M_R that represent the action of the operator $T_+ \big/ \sqrt{2}$ on the *physical* left-

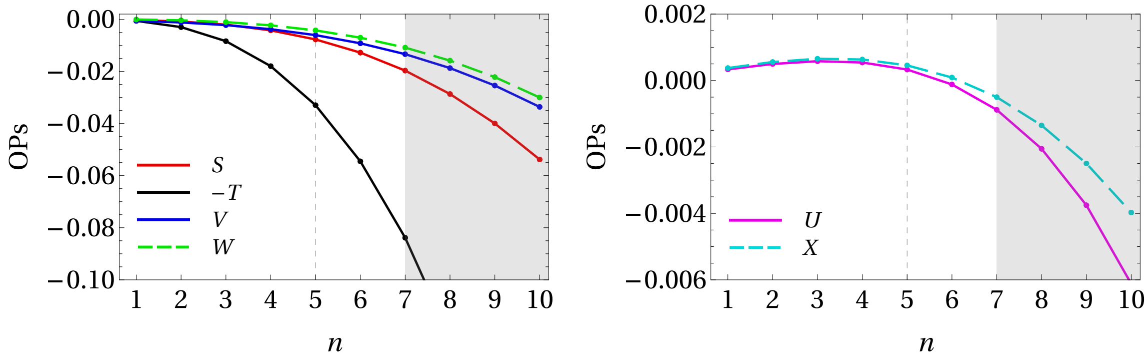

Figure 10. The OPs as functions of n. The dashed vertical line indicates the BP2 value $n = 5$. The gray area corresponds to fits with $\chi^2 > 20$.

and right-handed fermions, respectively; here, T_+ is the raising operator of gauge-SU(2). Starting from the matrices M_E $(E = L, R)$ one calculates the matrices $H_E \equiv \left[M_E, M_E^\dagger\right]$ and then the matrices $F_E \equiv H_E - 2Q s_W^2$, where Q is the electric-charge matrix. The formulas for the OPs are then eqs. (4.1), (4.4), (4.5), and (4.12), where one makes use of the functions $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ defined in eqs. (4.2), (4.6), and (4.13), respectively, and of functions defined in section 2.

We have applied our formulas to the cases of two models with new vector-like fermions in arbitrarily large representations of SU(2). Remarkably, in both models we have found that the oblique parameters V and W are usually of the same order of magnitude as S, while the oblique parameters X and U tend to be somewhat smaller; however, these features may be upended when one is dealing with fermion representations featuring either a large isospin $J \gtrsim 2$ or a large hypercharge $|Y| \gtrsim 5$.

It is worth remarking that, in the original formulation of the OPs (see appendix A), the parameters V, W, and X were set to zero and the parameters S and U had different definitions — S' and U', respectively. Our work demonstrates that, in general, that original formulation may lead to bad misjudgements, because neither V and W are necessarily smaller than S, nor necessarily $S \approx S'$ and $U \approx U'$ — as is shown through a simple example in appendix A.

Acknowledgments

L.L. thanks Heather Logan for pointing out to him ref. [17]. The work of F.A. was supported by grant UI/BD/153763/2022. F.A. and L.L. were supported by the Portuguese Foundation for Science and Technology through projects UIDB/00777/2020, UIDP/00777/2020, and CERN/FIS-PAR/0002/2021; L.L. was furthermore supported by CERN/FIS-PAR/0019/2021. D.J. was supported by the Lithuanian Particle Physics Consortium.

A The Peskin-Takeuchi approximation

In their original formulation by Peskin and Takeuchi [7], the OPs had different definitions. The parameters V, W, and X did not exist (or, equivalently, they were set to zero); the parameter T had the same definition as in eq. (1.3b); and the parameters S and U were

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instead defined to be S' and U', respectively, where

$$S' = \frac{4s_W^2 c_W^2}{\alpha}\left[A'_{ZZ}(0) + \frac{c_W^2 - s_W^2}{c_W s_W}\, A'_{\gamma Z}(0) - A'_{\gamma\gamma}(0)\right], \tag{A.1a}$$

$$U' = -S' + \frac{4s_W^2}{\alpha}\left[A'_{WW}(0) + \frac{c_W}{s_W}\, A'_{\gamma Z}(0) - A'_{\gamma\gamma}(0)\right]. \tag{A.1b}$$

It is clear that

$$S - S' = \frac{4s_W^2 c_W^2}{\alpha}\left[\widetilde{A}_{ZZ}\left(m_Z^2\right) - A'_{ZZ}(0)\right], \tag{A.2a}$$

$$(U+S) - (U'+S') = \frac{4s_W^2}{\alpha}\left[\widetilde{A}_{WW}\left(m_W^2\right) - A'_{WW}(0)\right]. \tag{A.2b}$$

In our NPM with additional fermions the Peskin-Takeuchi parameters S' and U' are given by

$$S' = \frac{1}{2\pi}\sum_{f,f'} \overline{\mathcal{G}}\left[(H_L)_{ff'}, (H_R)_{ff'}, m_f^2, m_{f'}^2\right] + \frac{1}{\pi}\sum_f Q_f\,(H_L + H_R)_{ff}\, h\left(m_f^2\right), \tag{A.3a}$$

$$U' = -S' + \frac{1}{\pi}\sum_{f,f'} \overline{\mathcal{G}}\left[(M_L)_{ff'}, (M_R)_{ff'}, m_f^2, m_{f'}^2\right] + \frac{1}{\pi}\sum_f Q_f\,(H_L + H_R)_{ff}\, h\left(m_f^2\right), \tag{A.3b}$$

where

$$\overline{\mathcal{G}}(x, y, I, J) = \left(|x|^2 + |y|^2\right)\overline{g}(I,J) - 2\,\mathrm{Re}\,(xy^*)\,\bar{\tilde{g}}(I,J)\sqrt{IJ}, \tag{A.4}$$

with functions $\overline{g}(I,J)$, $\bar{\tilde{g}}(I,J)$, and $h(I)$ given in eqs. (2.6), (2.7), and (2.1f), respectively.

For instance, in the model of section 5, wherein $M_L = M_R$, $H_L = H_R$, and all the extra fermions have equal mass,

$$S' = \frac{1}{\pi}\sum_f \left\{2Q_f\,(H_L)_{ff}\, h\left(m^2\right) + \left|(H_L)_{ff}\right|^2\left[\overline{g}\left(m^2, m^2\right) - m^2\,\bar{\tilde{g}}\left(m^2, m^2\right)\right]\right\}, \tag{A.5a}$$

$$S' + U' = \frac{2}{\pi}\left\{\sum Q_f\,(H_L)_{ff}\, h\left(m^2\right) + \sum_{f,f'}\left|(M_L)_{ff'}\right|^2\left[\overline{g}\left(m^2, m^2\right) - m^2\,\bar{\tilde{g}}\left(m^2, m^2\right)\right]\right\}. \tag{A.5b}$$

Now, because of eqs. (2.1f), (2.6), and (2.7),

$$\overline{g}\left(m^2, m^2\right) - m^2\,\bar{\tilde{g}}\left(m^2, m^2\right) = -h\left(m^2\right). \tag{A.6}$$

Hence,

$$S' \propto \sum_f \left[2Q_f\,(H_L)_{ff} - \left|(H_L)_{ff}\right|^2\right], \tag{A.7a}$$

$$S' + U' \propto \sum_f Q_f\,(H_L)_{ff} - \sum_{f,f'}\left|(M_L)_{ff'}\right|^2. \tag{A.7b}$$

Using eqs. (5.1)–(5.3) one easily concludes that $S' = U' = 0$ in that NPM. On the other hand, in the same NPM the OPs S and U are clearly nonzero — they are not even necessarily very small. So, it is clear that the Peskin-Takeuchi parameters S' and U' do not need to be, in general, good approximations to S and U, respectively.

Further dealing on the model of section 5, we note that in that model there is only one mass scale, viz. the mass m of the new fermions. Therefore, since the function $A_{ZZ}\left(q^2\right)$ has mass-squared dimensions,

$$A_{ZZ}\left(q^2\right) = am^2 + bq^2 + c\,\frac{q^4}{m^2} + d\,\frac{q^6}{m^4} + \mathrm{O}\left(\frac{q^8}{m^6}\right), \tag{A.8}$$

with numerical coefficients $a, b, c, d, \ldots$. Hence,

$$\widetilde{A}_{ZZ}\left(m_Z^2\right) - A'_{ZZ}\left(0\right) = c\,\frac{m_Z^2}{m^2} + d\,\frac{m_Z^4}{m^4} + \mathrm{O}\left(\frac{m_Z^6}{m^6}\right). \tag{A.9}$$

This explains the form of eqs. (5.8)–(5.9) in section 5.

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