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Optimizing the Piezoelectric Energy Harvester for the Second Eigen Frequency: Mathematical Modeling and Experimental Investigation of Optimal Beam Shapes

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Optimizing the Piezoelectric Energy Harvester for the Second Eigen Frequency: Mathematical Modeling and Experimental Investigation of Optimal Beam Shapes

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Abstract. This paper is dedicated to the optimization of the beam shape of a piezoelectric energy harvester (PEH) for maximum efficiency at the second eigenfrequency. A combination of mathematical modeling and experimental investigation is used to design and confirm an optimal shape for the cantilever, maximizing strain in the upper layers of the active element. The cantilever beam shape is nonlinear throughout the cross-section of it. The experimental results confirm that the optimal shape beam produces around 70% higher output power than the initial shape. The findings of this study can be used to guide the design of more efficient PEHs for a variety of applications.

1. Introduction

Piezoelectric energy harvesters have become increasingly important in recent years due to their ability to convert mechanical energy into electrical energy [1]. The efficiency of these devices is influenced by several factors, including the beam shape [2]. They could be used in many different applications, including being implanted in road [3], pavement [4], etc. To achieve optimal performance, the excitation frequency must be within the resonant frequency range of the harvester [5]. In the second eigenfrequency, strain is divided to a much broader surface of the beam allowing it to capture energy from a wider range of vibrations. In this study, we investigate the optimal beam shape for the second eigenfrequency of a piezoelectric energy harvester using both mathematical modeling and experimental investigation.

Firstly, a mathematical model was developed to predict the optimal beam shape for the second eigenfrequency of a piezoelectric energy harvester. The model involves solving the equations of motion for the beam and the piezoelectric material, considering the electrical and mechanical coupling between them. The optimization of the second eigenfrequency was carried out using Comsol Multiphysics software.

The optimization was carried out on the thickness of the beam, resulting in a nonlinear solution throughout all the piezoelectric energy harvesters' active elements. The confirmation of the results was executed by comparing the optimal shape active element to the initial one with constant geometry. Experimental research was conducted using a Polytec 3-dimensional laser scanning vibrometer machine and exciting the beam using a PVDF energy transducer. The experimental results confirmed the predictions of the mathematical model, showing that the optimal shape beam produced around 70 percent higher results than the initial shape.



2. Mathematical modelling

Starting mathematical modeling of the optimal shape cantilever, the most important part was setting restrictions to the geometric parameters so that the active element during the optimization process does not become thinned or thicken unreasonably. The minimum thickness was set to be at least 0.5mm, with other dimensions remaining the same as the initial shape cantilever – length 100mm, width 10mm. The optimal shape cantilever ended up having the same dimensions as the initial one, except having a non-linear cross-section, however with just a slight deviation of the initial thickness.

Comsol Multiphysics structural mechanics and optimization analysis modules were used to receive the mathematical solution and results are shown in Figure 1.

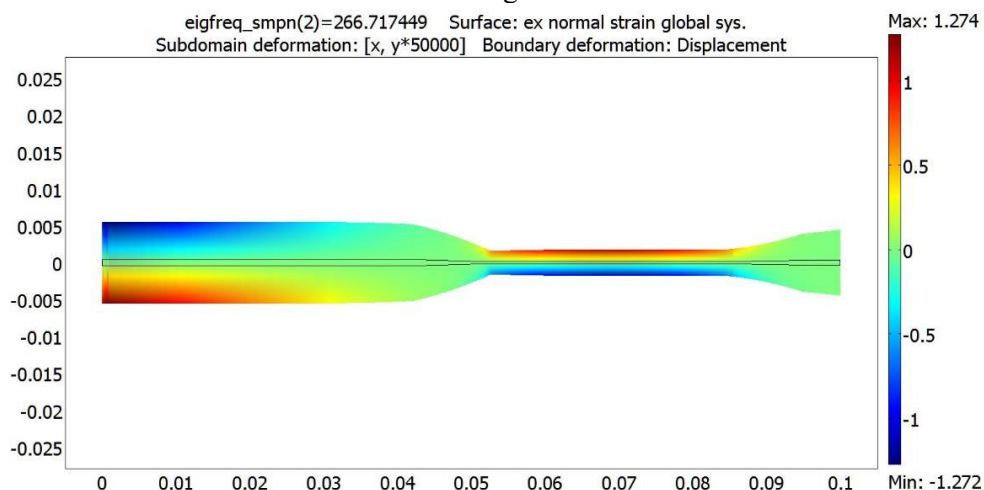


Fig. 1 - Optimal shape of the active element for the 2nd resonant mode w_2 . Field of normal strain

Figure 1 shows a modal analysis of the optimal shape cantilever, oscillating in 2nd resonant mode with its eigenfrequency being the same as an original one – around 267hz. The field of a normal strain is shown. With this optimal form two main regions where strain occur is obvious – first one positioned near the beginning of the cantilever at the fixing point, however this was prolonged compared to the initial shape and now lies between 0 and 0.25 of the cantilever length. Second through all center part of the cantilever, from 0.5 to 0.85 of the length.

Shape has three obvious parts, the left section is the linear cantilever, the central-active cantilever, which is loaded with most of the deformational energy, during w_2 resonant oscillations, and the third – inertial mass. This is close to the distribution of second-mode deformations of transverse oscillations in the active layer. In this case, the same resonant frequency w_2 is maintained and the resulting shape is uniform with respect to the mass distribution.

Strain values for optimal and initial shape cantilevers get integrated and a graph is formed to compare the results, which are shown in Figure 2.

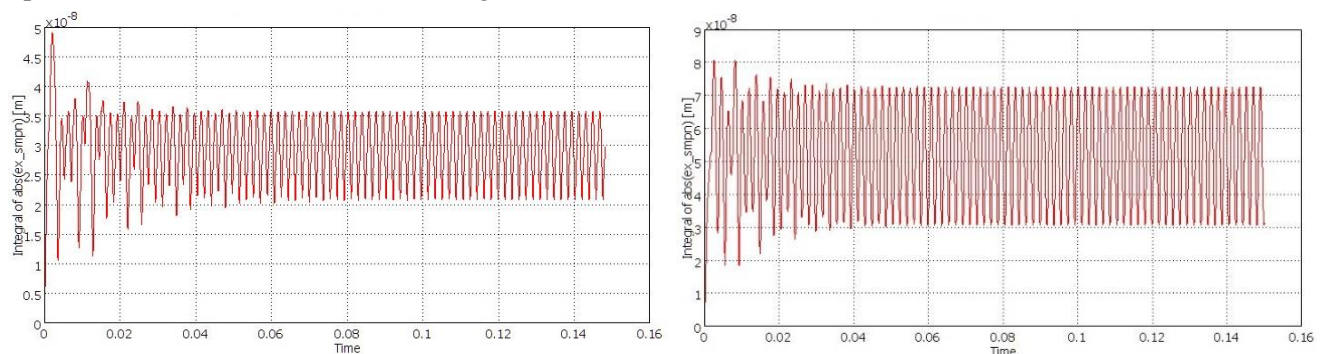


Fig. 2 – Integral values of strain. On the left initial shape cantilever, on the right – optimal shape

In graph named Figure 2 there is showed an integrated strain values for both cantilevers. On the left initial shape cantilever results are presented, and on the right – an optimally formed one.

This side-by-side comparison of constant cross section beam and optimal beam Integral values shows that the optimal shape beam oscillating in the w_2 mode generates about 70% higher deformation gradient value, and at the same time generates elastic energy.

3. Experimental review

The experimental investigation was executed using Polytec 3D laser machine. The cantilever was secured to the plate, its piezoelectric film was connected to the machine. There were short bursts of the full range of frequency thrown to the cantilever from 0 to 1000hz. This excited the cantilever and the laser could read displacement. The results are shown in Figure 3.

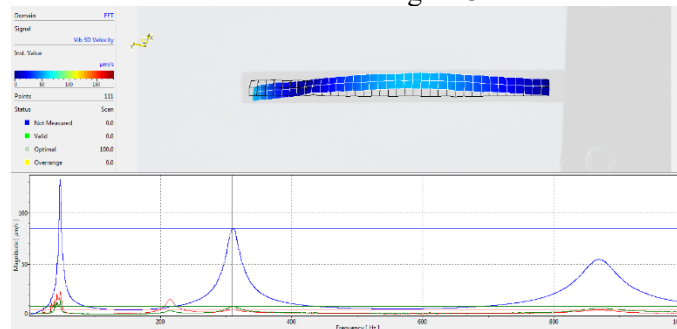


Fig. 3 – a working window of Polytec 3D laser machine with results of optimal shape cantilever loaded

Figure 3 shows the results of experimental analysis made with Polytec machine. The experimental results confirmed the predictions of the mathematical model, showing that the beam with the optimized shape produced around 70% higher output than the beam with the initial shape.

4. Conclusion

This paper presents a method for optimizing the beam shape of a PEH for maximum efficiency at the second eigenfrequency. Using a combination of mathematical modeling and experimental investigation, we designed an optimal beam shape that maximizes strain in the upper layers of the active element.

The experimental results demonstrate that the optimal shape beam produces around 70% higher output power than the initial shape. These findings can be used to guide the design of more efficient PEHs for a variety of applications.

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