VILNIUS UNIVERSITY

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POSSIBILITIES OF THE STATISTICAL SCORING MODELS' APPLICATION AT LITHUANIAN BANKS

Summary of Doctoral Disertation
Social Sciences, Economics (04 S)

Vilnius, 2013

The disertation was prepared at Vilnius university in 2008 – 2013.

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The dissertation is being defended in the public meeting of the Council of Scientific Field of Economics, in Audience No. 403 of Faculty of Economics, on 27 September 2013, at 14 o‘clock.
Address: Saulėtekio 9, Vilnius, Lithuania.
The dissertation summary was sent on 27 August, 2013.
It is possible to review the dissertation at the library of Vilnius University.

VILNIAUS UNIVERSITETAS

LAIMA DZIDZEVIČIŪTĖ

STATISTINIŲ VERTINIMO BALAIS MODELIŲ TAIKYMO LIETUVOS BANKUOSE GALIMYBĖS

Daktaro disertacijos santrauka
Socialiniai mokslai, ekonomika (04 S)

Vilnius, 2013

Disertacija rengta 2008 – 2013 Vilniaus universitete.

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Disertacija bus ginama viešame Ekonomikos mokslo krypties tarybos posėdyje 2013 m. rugsėjo mėn. 27 d. 14 val. Ekonomikos fakulteto 403 auditorijoje.
Adresas: Saulėtekio al. 9, Vilnius, Lietuva.
Disertacijos santrauka išsiuntinėta 2013 m. rugpjūčio mėn. 27 d.
Disertaciją galima peržiūrėti Vilniaus universiteto bibliotekoje.

CONTENTS

ABBREVIATIONS

EAD	–	exposure at default
CAP curve	–	cumulative accuracy profile curve
IRB approach	–	internal ratings based approach
IV	–	information value
LGD	–	loss given default
NPV	–	net present value
ODF	–	observed default frequency
PD	–	probability of default
ROC curve	–	receiver operating characteristic curve
WOE	–	weight of evidence

INTRODUCTION

The theme actuality. A bank granting a loan has to assume a credit risk. In order to make a decision to grant a loan or not, a bank shall have a credit risk assessment model in place. A bank cannot assess a credit risk of all of its debtors in the same way. Firstly, credit risk assessment criteria of various types of debtors differ. For example, creditworthiness of central governments and central banks depends on macroeconomic conditions, meanwhile, creditworthiness of a company is assessed taking into account its management, situation in a market, financial condition. Secondly, data sources of various debtor types differ, for example, a bank may use data of companies‘ financial reports to assess their credit risk, however, this is not possible assessing private persons. Thirdly, actual default frequencies of various debtor types differ. In the past a credit risk of both small/medium and large companies, as well as that of private persons, was being assessed individually at banks, an assignment of debtors to ratings was based on freely interpreted criteria. An expert had big freedom of choosing and weighting them, however, it took a lot of time for an expert to assign debtors to ratings. During the last decades a competition of banks and an aspiration to increase an income have increased considerably, and this induces to look for more efficient and accurate methods to assess a debtors‘ credit risk. In the 60s-70s of the last century, after the spread of credit cards, a need to automatize a loan granting process increased, so, scoring models began to spread. Scoring models can be expert, when criteria and their weights are determined by experts, statistical, when criteria and their weights are determined statistically, and mixed. Banks started to apply statistical scoring models for an assessment of a credit risk of small, medium companies and private persons more widely, they become more and more significant in the total context of all the credit risk assessment methods.

When applying statistical scoring models a loan granting process is automatized, it is possible, because of a decreased time and monetary cost and a more accurate credit risk assessment, to increase lending to “marginal debtors“, i. e. to applicants, that would otherwise not receive a loan. Because of that a bank‘s profit increases. More risky applications are being rejected, a first type error (when a bank grants a loan which becomes “bad“ later) and credit losses of a bank decrease. However, at the same time more non-risky applications are being accepted, a second type error (when a bank does not grant a loan, though it would have become “good“) decreases and a bank income increases. When an application assessment process is automatized, actual default frequencies are 15%-25% lower than those frequencies, when applications are being assessed individually, given the same reject rate of applications. Besides, a loan granting process becomes clearer, a credit risk assessment – more objective, and loan portfolio management improves. In Lithuania a necessity to apply statistical scoring models has especially increased after the transposition of the New Capital Adequacy Directive, prepared in accordance with the New Basel Capital Accord, into the national legal acts. According them, banks applying the internal ratings based approach are allowed to calculate their debtors‘ default probability themselves, however, they have to keep within the legal requirements (Bank of Lithuania 2006a). Besides, till the beginning of 2009 the portfolio of the loans granted by Lithuanian banks increased rapidly, in 2012 it started to increase again. The positive loan porfolio growth is also being expected in 2013. Because of the increased loan sample a possibility to apply statistical scoring models appeared for many banks, that had never applied such models before. So, it is very actual to analyse the application practice of statistical scoring models at Lithuanian banks, find the spheres, that need improvement, and develop appropriate models.

The essence of the scientific problem. Statistical scoring models are not being widely applied at Lithuanian banks (Dzidzevičiūtė 2010d). As there is lack of scientific works and articles about a development of these models, it is not clear for banks, how to assign debtors to “goods“ and “bads“, determine a period within which a debtor becomes “bad“, whether to develop one common model or separate models for different debtor types, which statistical technique to choose, how to construct a sample, whether to include rejected applications and, if yes, in what way, what input variables to use, how to assign debtors to ratings and calculate a default probability for ratings, especially for those of low default portfolios, etc.

Statistical scoring models are being applied when a bank has many loans of a concrete type. Besides, if a bank lacks data about actual “bad“ debtors of a concrete loan type, it is not possible to develop a valid model. Because of that statistical scoring models are being more seldom applied to assess a credit risk of companies than that of private persons. As features of debtors differ, it is difficult to develop a valid common model. Lithuanian banks not allways have enough data to develop their own statistical scoring models for companies, so, they seek for the other ways out: they develop expert models, apply statistical scoring models proposed in scientific articles, buy models or debtors‘ data sold by external loan registers, buy debtors‘ ratings determined with models developed by external loan registers. However, it is expensive to buy models or debtors‘ ratings form external loan registers. Statistical scoring models proposed by foreign authors are developed with data non-representing Lithuanian companies. Till now other Lithuanian authors have not proposed a statistical scoring model developed with a large Lithuanian companies‘ sample applying a default definition and suitable to assess companies of various economic sectors.

The dissertation goal – to develop the rating system of Lithuanian companies based on the statistical scoring model and assess the possibilities of its application at Lithuanian banks.

The dissertation object – rating systems based on statistical scoring models.

The dissertation exercises:

- To analyse development stages of statistical scoring models and a rating scale construction.
- To explore methods to calculate a default probability for ratings.
- To examine validation methods of rating systems based on statistical scoring models.
- To perform the survey of commercial Lithuanian banks and foreign bank branches operating in Lithuania in order to analyse the local development and application practice of scoring models, to determine the spheres of its improvement.
- To develop the rating system of Lithuanian companies based on the statistical scoring model using the data of Lithuanian companies received from the external loan register.
- To assess the possibilities of this rating system‘s application at Lithuanian banks.

The scientific newness. Other Lithuanian authors have not explored statistical scoring models, their development and application at Lithuanian banks, thoroughly. Till now, the biggest attention has been paid to the Altman models‘ suitability for a bankruptcy prognostication and a restructuring benefit assessment of Lithuanian companies (Mackevičius, Poškaitė 1999; Tvaronavičienė 2001; Bivainis, Tamošiūnas 2003; Mackevičius, Rakštelienė 2005; Mackevičius, Silvanavičiūtė 2006; Stundžienė, Boguslauskas 2006; Garškaitė 2008; Mackevičius, Sneidere 2010). Other statistical scoring models developed by foreign authors have not been widely analysed by Lithuanian scientists (see Mackevičius, Silvanavičiūtė 2006; Garškaitė 2008; Mackevičius, Sneidere 2010). Leipus and Valužis (2006) examined only structural (company‘s value) and reduced form (intensity) credit risk models, and not scoring models. Kamienas and Valvonis (2004), Valvonis (2006) examined theoretical aspects of scoring models together with other credit risk assessment methods, however, in their articles scoring models are examined only roughly, as one of the groups of credit risk assessment methods, and development stages of statistical scoring models are not analysed in detail. A majority of the methods analysed by them can be applied only assessing a credit risk of the companies, shares of which are listed on exchanges or which have issued debt securities. Scoring models are being applied to assess a credit risk of all other companies and private persons, so, they are the most important to Lithuanian banks. That is why development stages of statistical scoring models and their application at a bank are analysed in detail in this dissertation. Besides, till now other Lithuanian authors have not analysed a development of application and behavioural scoring models, reject inference methods and methods to calculate a default probability for ratings, especially for those of low default portfolios, separately. All this is analysed in detail in this dissertation.

Also, till now other authors have not performed a survey of Lithuanian banks about statistical scoring models. The information about that, whether Lithuanian banks apply statistical scoring models, what are their scale and character, is confidential; neither banks themselves nor the Bank of Lithuania supervising them publish such information in publicly available sources (print and online). Even two surveys were performed, with one of them related to the credit risk management organized at Lithuanian banks (Valvonis 2004) and the other – to the rating of large companies applied at Lithuanian banks (Savickaitė, Valvonis 2007), these articles did not provide with a credit risk assessment practice or scoring models of retail loans. For these reasons the dissertation‘s author performed the survey of Lithuanian banks in order to examine the application practice of retail application scoring models at Lithuanian banks (Dzidzevičiūtė 2010d). The results of this survey are analysed in detail in this dissertation.

Several Lithuanian authors (Grigaravičius 2003; Merkevičius and others 2006; Stoškus and others 2007; Mileris 2009, 2010; Buzius and others 2010; Mileris, Boguslauskas 2011) developed statistical company scoring models, however, these models were developed using small data samples and are suitable only for an assessment of specific types of companies. Grigaravičius (2003) developed the logistic regression model using the data of only 88 Lithuanian companies, the shares of which are listed on exchanges, so, the model suits for this company group mainly. Stoškus and others (2007) developing their discriminant analysis model explored only 13 companies: 5 bankrupt and 8 – working successfully. Merkevičius and others (2006) developing the artificial neural networks models used the data of only 742 Lithuanian companies, Mileris (2009, 2010), Mileris, Boguslauskas (2011) developing the discriminant analysis, the logistic regression and the artificial neural networks models – the data of only 100 Lithuanian companies. Besides, all the mentioned authors developing the models used only financial ratios, defining a “bad“ company used only a bankruptcy indication, and the majority of them didn‘t provide with the input variables included into the models and (or) their coefficients in their articles, so, banks couldn‘t apply their models in practice. The statistical model provided in this dissertation suits for an assessment of companies from all economic sectors, is developed using 22799 “company-years”, comprises both quantitative and qualitative input variables. A “bad“ company was defined using a default indication, so, the model may be applied calculating capital requirements in observance with the legal requirements of the Bank of Lithuania.

The practical significance. The proposed rating system of Lithuanian companies based on the statistical scoring model may be applied for different purposes not only at banks, but also at other companies at which a credit risk of companies is being assessed: consumer credit, small credit and leasing companies may apply it to assess a credit risk of debtors‘ employers, also at insurance companies, etc. It may also be being applied by companies, that want to assess their own creditworthiness. Though the proposed rating system is behavioural, and not application, it may also be applied as an application rating system. If a bank has enough data and can develop its own statistical company scoring model, it could make use of the performed analysis‘ results and the proposals provided in this dissertation: to choose the same input variables, that were included into the proposed model, group their values in the same way, apply the proposed methods, etc. Besides, the analyses provided in this dissertation may also be usefull developing and applying expert company scoring models or private person models.

The research methods. Examining development stages of statistical scoring models and a rating scale construction, as well as methods to calculate a default probability for ratings and validation methods, the theoretical literature and the documents were analysed, it was analysed logically, summarized, systematized, prescinded and concretized, also, a metaanalysis and a synthesis were performed. The questionnaire survey was performed to explore the development and application practice of retail application scoring models at Lithuanian banks. Grouping, comparison and graphical visualization methods were applied to summarize and systematize the gathered information, logical conclusions were made. The empirical data received to develop the rating system of Lithuanian companies based on the statistical scoring model were systematized, their statistical analysis and synthesis were performed. Also, developing the rating system, it was prescinded, modelled, systematized and analysed logically, summarizing conclusions were made. The information sources were the books and the scientific articles of foreign and Lithuanian authors, the recomendational documents of the international banking supervision institutions, the international seminars‘ material, the legal acts of European Union and Lithuania, the answers of Lithuanian banks to questionnaire‘s questions, the external loan register JSC “Creditinfo Lietuva“.

The research restrictions. Performing the research it was faced with the several restrictions:

- At the time of the survey performance 9 commercial banks and 7 foreign bank branches were operating in Lithuania. With all of them it was negotiated regarding a participation in the survey. However, only 8 commercial banks and 1 foreign bank branch agreed to participate. That is why the analysis of the development and application practice of retail application scoring models at Lithuanian banks is not full.

- Banks don‘t provide the external loan register JSC “Creditinfo Lietuva“ with the information about granting dates of concrete loans, so, it was not possible to develop an application model, when a default probability is forecasted for one year forward from a loan granting date. That is why only the behavioural scoring model of Lithuanian companies was developed.
- The default definition used to develop the statistical scoring model of Lithuanian companies was narrower than that required by the Bank of Lithuania (only a payment term delay more than 90 calendar days and a bankruptcy were included). The narrower definition was used taking into account that JSC “Creditinfo Lietuva” doesn’t gather information about a debt restructuring, loan value adjustments, an insolvency and other additional indications used to define a default.
- Not all of the input variables used in models proposed by foreign authors may be used in Lithuania because of accounting, economic, legal and other differences. The external loan register JSC “Creditinfo Lietuva“ gathers not all items of companies‘ financial reports, for example, the records about *Financing and investing activities*, etc. are not being gathered. So, developing the company model it was not possible to include some popular variables, for example, the ratio of *Profit (loss) before interest and tax* to *Total assets*. Besides, for this reason only the statistical scoring models developed by other authors, the variables of which may be calculated with the data gathered in this register, could be applied for the benchmarking analysis.
- Information about changes of companies‘ variables and exact dates of a becoming “bad“ was not received, so, it was not possible to apply the methods to calculate a default probability for ratings of low default portfolios proposed by some authors. Also, for the same reason it was not possible to develop a survival analysis model.

The aprobation of the dissertation results. Almost all the Parts of the dissertation and the research results are provided in the published articles (Parts 1.1-1.2, Parts 2.1-2.4, Part 3.1.1, Part 3.2.2). Five articles on the dissertation subject were published in the scientific magazines. One of the articles was presented at the conference *“International Conference of Economics and Management ICEM 2010”,* that was taking place in Riga on

22-23 April 2010. When the dissertation's author was working in the credit risk sphere at the Bank of Lithuania, she was the member of the working groups of several international banking supervision institutions and participated at several international seminars on a credit risk. Later the author worked at one of the commercial Lithuanian banks, at which some Parts of the dissertation were adapted.

The dissertation structure. Development and application peculiarities of rating systems based on statistical scoring models are described in the first Chapter, in the second – the results of the survey of Lithuanian banks performed by the dissertation's author are analysed, in the third – the rating system of Lithuanian companies based on the statistical scoring model is provided, the possibilities of its application at Lithuanian banks are assessed.

I. DEVELOPMENT AND APPLICATION PECULIARITIES OF RATING SYSTEMS BASED ON STATISTICAL SCORING MODELS

A rating system at a bank – a system comprising all of the methods, processes, controls, data collection and information technology systems, that support a credit risk assessment of debtors, an assignment of debtors to ratings and a calculation of credit risk parameters (Bank of Lithuania 2006a). Applying a rating system at a bank there are two processes: debtors are being assigned to ratings and credit risk parameters are being calculated for ratings. The New Basel Capital Accord (BCBS 2006) provides for three credit risk parameters: probability of default (thereinafter – PD), loss given default (thereinafter – LGD) and exposure at default (thereinafter – EAD). In this dissertation only one parameter – PD – is being analysed.

1.1. A DEVELOPMENT AND AN APPLICATION OF A STATISTICAL SCORING MODEL AND A RATING SCALE

When a rating system is based on a statistical scoring model, debtors are being assigned to ratings according to a result of such a model.

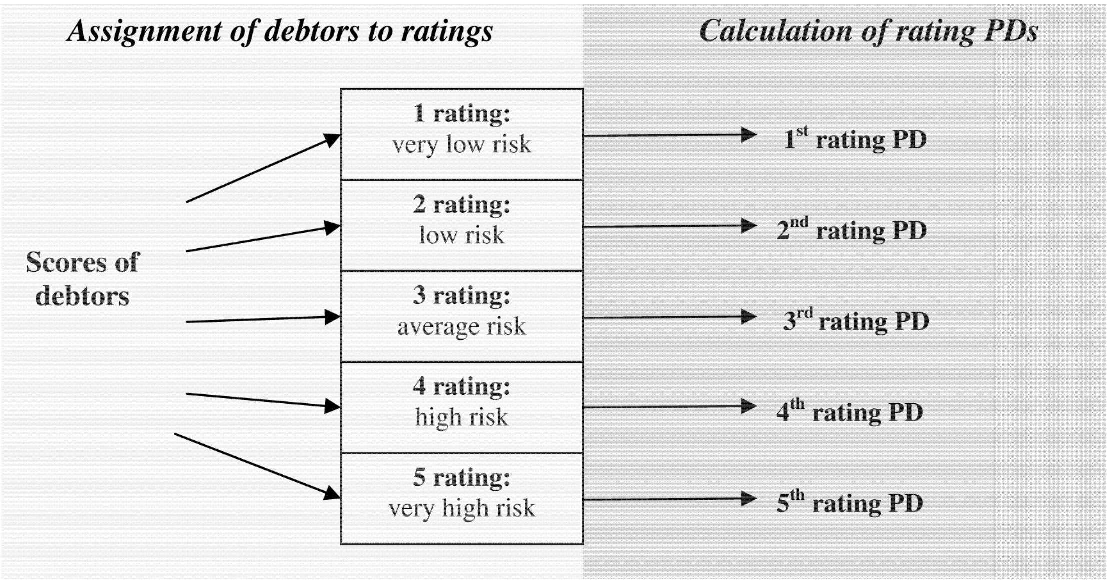

Figure 1. A rating system application at a bank

Source: compiled by the author.

A score of each debtor is being calculated with a statistical scoring model, based on which he is being assigned to one of the ratings. Then PD is being calculated for each rating (see Figure 1). That, what is being named as *a score*, depends on a result of a statistical scoring model. If a model result is individual PD of a debtor (for example, applying a logistic or a cloglog regression model), then *a score* is this individual PD of a debtor. However, if a model result is not PD, but a creditworthiness indicator (for example, applying a discriminant analysis), then this indicator is being treated as *a score*.

A statistical scoring model development – a complex process, that needs a thorough analysis of bank‘s data and a preliminary prevision of goals, that will be achieved applying such a model. Having summarized the literature of this sphere (Thomas and others 2002, Mays 2004, Siddiqi 2006, Anderson 2007), eight model development stages may be determined (see Figure 2).

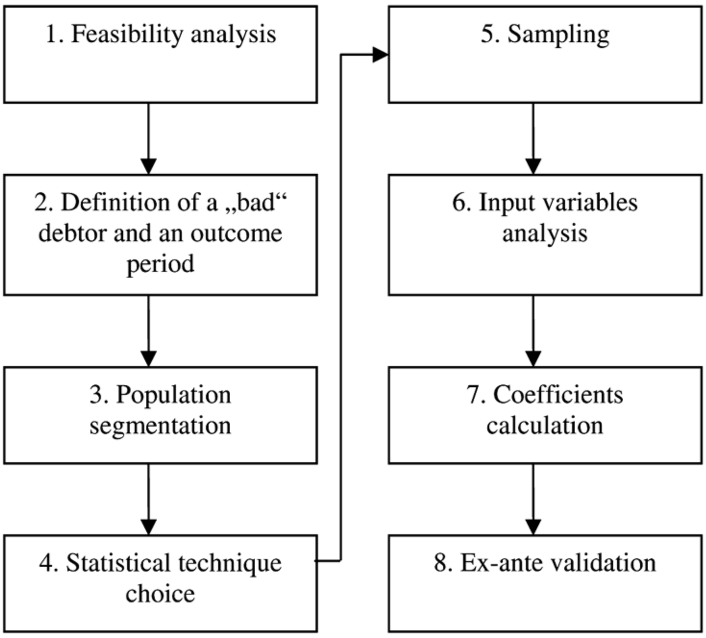

Figure 2. The model development stages

Source: compiled by the author in accordance with Thomas and others 2002; Mays 2004; Siddiqi 2006; Anderson 2007.

The first of them is to analyse a feasibility of a bank‘s project to develop and apply such a model, i. e. spheres of its application, etc. Scoring models may be applied calculating capital requirements, credit risk margins and loan value adjustments, in a reporting system, distributing capital, managing payment term delays, assessing bank‘s profitability and forming its strategy, performing a stress testing and making securitisation transactions, etc. (Thomas ir kt. 2002; Mays 2004; FIS, CEBS 2006b; DB, ONB 2007; Anderson 2007; Banque de France 2008). After a feasibility analysis a “bad“ debtor[1] and a period applied

[1] Statistical scoring models may be developed on a debtor or a loan level. Thereinafter, models developed on a debtor level (if it is not indicated otherwise) will be kept in mind for the simplicity purposes.

to determine, whether a debtor became “bad“ or not, that may be called an outcome period, are defined. Statistical company scoring models may be based on different definitions of a “bad“ company – a bankruptcy, a financial distress (an insolvency), a debt restructuring, loan value adjustments, a payment term delay, a default. Recently banks applying the internal ratings based (thereinafter – IRB) approach assigning their debtors to a group of “bads“ have to apply a default definition. However, even banks not applying the IRB approach defining a “bad“ company often use a default indication. Having analysed 60 articles written by the authors from various countries of the world during 12 years (from 1999 to 2011), one could see that defining a “bad“ company an indication of a bankruptcy was chosen almost so often as that of a default; a financial distress (an insolvency) indication was chosen more seldom (see Figure 3). A default definition was sometimes narrowed to a payment term delay more than 90 calendar days, and a bankruptcy or any other indication were not included (Aragon 2004; Mircea 2007; Luppi and others 2008).

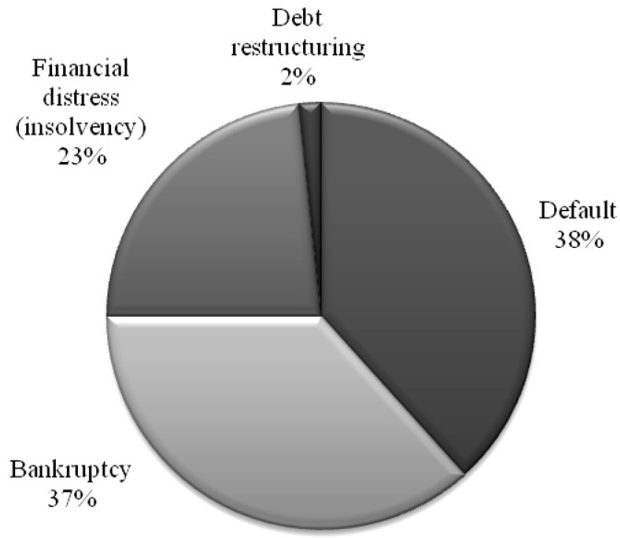

Figure 3. The choice frequency of a “bad“ company definition

Source: compiled by the author in accordance with 2; 3; 7; 8; 9; 11; 14; 23; 25; 30; 31; 34; 35; 36; 38; 39; 45; 48; 49; 51; 57; 59; 61; 62; 66; 71; 73; 74; 75; 79; 80; 81; 83; 86; 87; 89; 90; 97; 104; 105; 108; 118; 121; 122; 123; 124; 127; 128; 129; 131; 132; 133;135; 138; 140; 147; 148;149; 154; 164.

Banks applying the IRB approach calculating their capital requirements should rely on a default indication which is stricter, however, in other cases it could be relied on an indication of a bankruptcy or an insolvency. Applying the stricter definition, i. e. a default indication, there may occur more technical defaults, when a debtor, who is past due more than 90 calendar days, pays a payment.

Developing private person scoring models a “bad“ debtor is being mostly defined as a debtor, who is past due more than 90 calendar days. However, various other definitions are possible – a debtor, who is delaying to pay two payments in succession, past due six months, etc. (Greene 1992; Thomas and others 2000; Andreeva 2006; Thomas 2007; Kočenda, Vojtek 2009).

The other important action, that must be performed in the second model development stage, is an outcome period definition, i. e. a period, within which a debtor becomes “good“ or “bad“. For example, if a debtor gets a loan on 2 October 2008 (i. e. on a T_0 date) and a bank defining a “bad“ debtor applies an indication of a default in one year, then it developing a new model assigns this debtor to a group of “bads“, if he defaults at least once till 2 October 2009. This one year period from a loan granting date is being called an outcome period; debtor PD is calculated for this period (see Figure 4). It is mostly of 1 year (Greene 1992; Hayden 2003). Such an outcome period is also provided in the legal acts of European Union and Lithuania prepared in accordance with the New Basel Capital Accord (BCBS 2006; EP 2006; Bank of Lithuania 2006a).

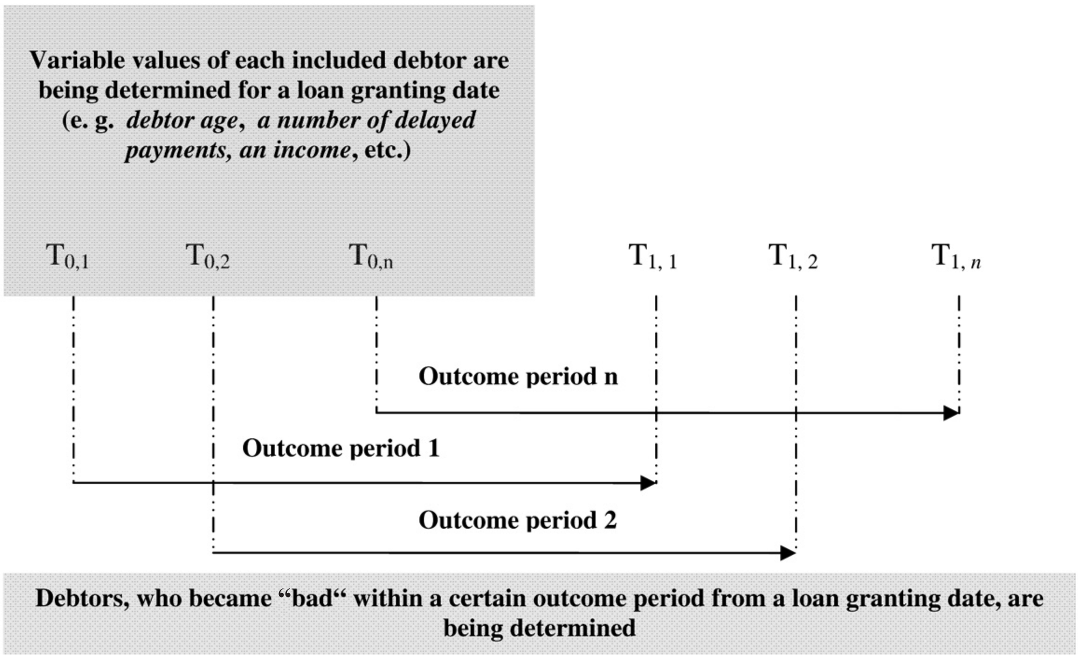

Figure 4. An application model development*

Source: compiled by the author. *The assumption was made that an application assessment date concurs with a loan granting date. However, in practice these dates often differ. Then, if input variable values change during a period from an application assessment to a loan granting, these new values can be used.

When there are very few or no actual “bad“ debtors, developing a statistical private person or company model, debtors of the worst rating of “good“ debtors may be being treated as “bad“ debtors (BCBS 2005b), also another stricter definition of a “bad“ debtor may be applied or a longer outcome period may be used.

Having defined a “bad“ debtor[2] and chosen an outcome period, it is being decided, what debtor groups a developed model will be applied for. Data used developing a model should be representative in respect of debtors, who will be assessed with this model. In other words, a developed model should be applied in the same country and for debtors of the same type as actual data. Scoring models may be being differentiated according to that, whether they are being applied to assess new applications (so called *application models*, see Figure 4), or behaviour of already granted loans (so called *behavioural models*, see Figure 5).

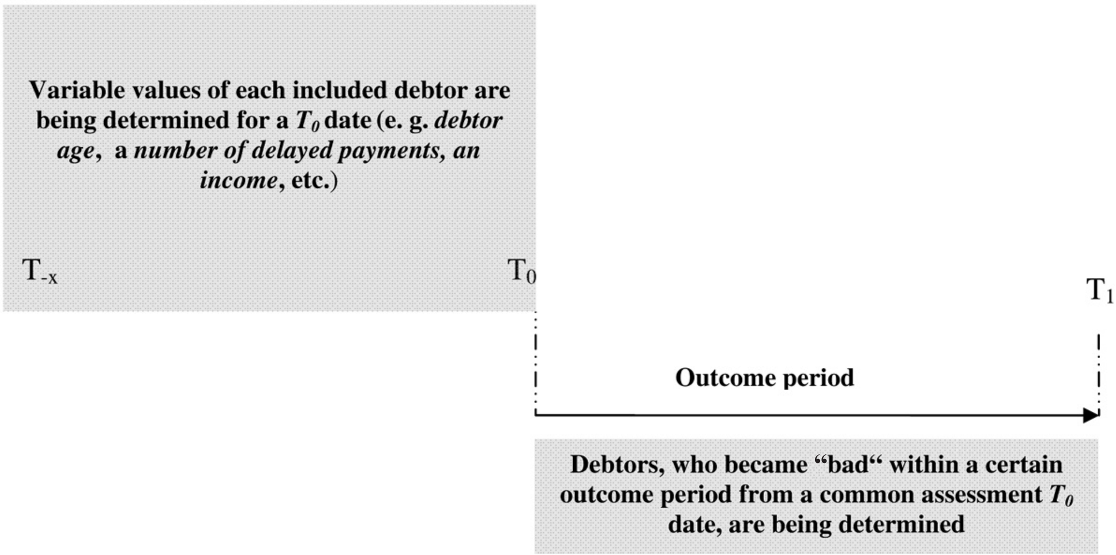

Figure 5. A behavioural model development
Source: compiled by the author.

The author of this dissertation proposed 4 methods to develop application and behavioural models, that are based on her practice working at the Bank of Lithuania and at one commercial Lithuanian bank (see Table 1).

Having assessed discriminatory power of both a common model and separate models, i. e. a model capacity to separate “bad“ debtors from “good“, a bank has to choose a model or several models with the biggest discriminatory power. If more models were developed, a bank would have to validate not one, but several models, that is why a time, information technology and wage cost would increase.

[2] Thereinafter in this dissertation only one conception is being used – “a “bad“ debtor“, which may mean a defaulted debtor or a debtor characterized by other features according to a used “bad“ debtor definition.

Table 1. The development methods of application and behavioural models

Method	New debtors application models	Existing debtors new application models	Behavioural models
1st	**X**		**X**
2nd	**X**	**X**	**X**
3rd	**X**	**X**	
4th	**X** (common model)		

Source: compiled by the author.

Having determined, what debtor groups a model would be developed for, a statistical technique is being chosen, a sample is being constructed, input variables are being analysed, coefficients of input variables included into a model are being calculated, an ex-ante validation is being performed. If it were necessary, one would come back to any of the earlier stages, and a process would be repeated, for example, a bank may decide to develop one model to assess all private person loans, however, analysing input variables a bank may notice that there are a lot of unpredictive input variables, so a bank may come back to the earlier stage – a population segmentation – and narrow a debtor group (for example, choose only data of private person mortgage loans).

A statistical technique choice depends on several things: on a data structure, a type of used input variables (only quantitative, only qualitative or of both types), a development purpose of a statistical scoring model (a reduction of a percentage of incorrectly predicted debtors, a reduction of a missclasification cost, etc.) and on individual discriminatory power of input variables. If input variable values do not differ very much in groups of "goods" and "bads", applying very flexible techniques – *k* nearest neighbour methods or artificial neural networks – an overfitting risk occurs, so a bank needs to use a bigger number of the nearest neighbours and so on. If a bank wants to calculate individual PDs of debtors, it is usefull to choose a logistic or a cloglog regression as well as other methods allowing to calculate individual PDs (a probit, a linear, a tobit regression, a survival analysis). If a bank wants to calculate probabilities of becoming "bad" within periods of a various duration and the most expected time of becoming "bad", it is usefull to choose a survival analysis. The research results showed that validity of models developed applying various different statistical techniques did not differ very much, differences in model validity were much bigger using different debtors data samples (see Thomas and other 2002; Andreeva 2006; Anderson 2007). Earlier statistical company scoring models *(Altman, Lis, Tafler, Springate, Fulmer)* were mostly developed applying a discriminant analysis, however, in 1974 Chesser, and in 1980 Ohlson proposed a logistic regression (see Altman 2000; Mackevičius, Silvanavičiūtė 2006; Garškaitė 2008). Aziz, Dar (2004) performed the survey in order to determine a statistical technique mostly applied by article authors. The survey results showed that from 1968 to 1996 a discriminant analysis was mostly applied (even 45 per cent of all the examined cases) (see Figure 6). However, this dissertation‘s author, having analysed newer 89 articles written by the authors from various countries of the world during 15 years (from 1996 to 2011), determined that a logistic regression was mostly applied (even 37 per cent of all the examined cases), a popularity of a discriminant analysis was already significantly smaller.

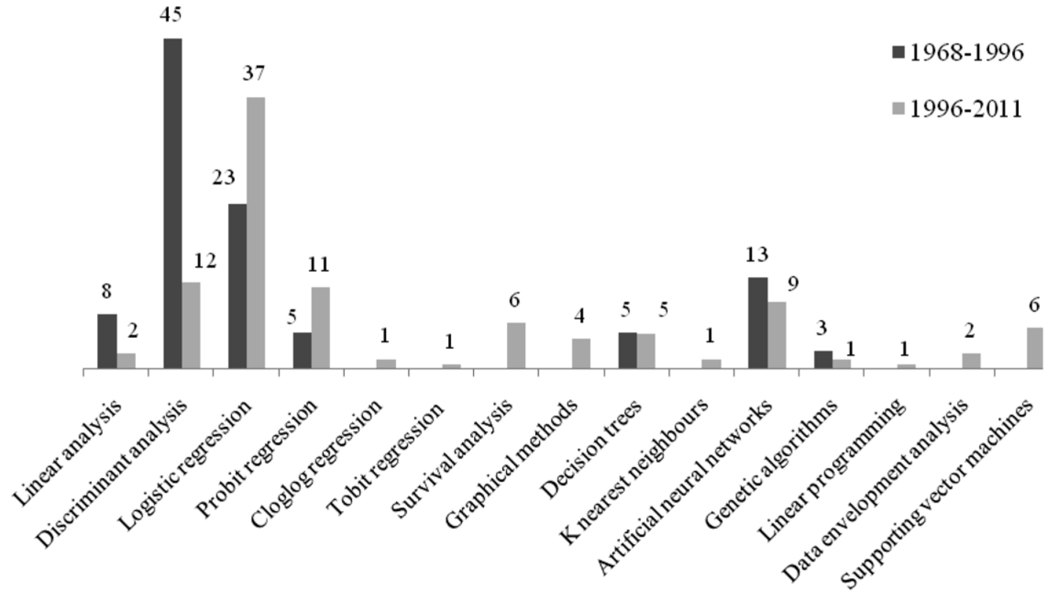

Figure 6. The statistical technique application frequency, in %.

Source: compiled by the author in accordance with 2; 3; 5; 7; 8; 9; 10; 11; 12; 14; 23; 25; 26; 29; 30; 31; 33; 34; 35; 36; 38; 39; 45; 48; 49; 51; 55; 57; 59; 60; 61; 62 ; 66; 70; 71; 72; 73; 74; 75; 76; 79; 80; 81; 82; 83; 85; 86; 87; 88; 89; 90; 94; 96; 97; 98; 99; 102; 104; 105; 108; 113; 118; 121; 122; 123; 124; 127; 128; 129; 131; 132; 133; 135; 138; 140; 145; 146; 147; 148; 149; 152; 154; 155; 157; 163; 164; 168; 170; 172.

Recently, namely a logistic regression is also being most widely applied at banks, standartized computer packages are developed, though, also decision trees and a survival analysis are becoming more popular, especially a *Cox* regression (Stepanova, Thomas 2002; Tong and others 2005; Bellotti, Crook 2007; Anderson 2007; SAS 2008, 2009). Though, in recent years, a number of researches related to machine learning and programming methods of a new generation – artificial neural networks and supporting vector machines – has increased (see Danėnas, Garšva 2009; Buzius and others 2010; Lahsasna and others 2010; Mileris, Boguslauskas 2010, 2011), however, these methods are still being in a research phase, their suitability to assess a debtor‘s credit risk has not been examined thoroughly, there are no standartized computer packages. Besides, the research results showed that, though in most cases model sensitivity[3] applying these methods was bigger than that applying pure statistical methods, however, a percentage of incorrectly predicted debtors was similar.

Constructing a model sample a bank should take into account several aspects: a sample size, an imbalance of “goods“ and “bads“, optimality of a number of debtors, a breakdown of a data set, a reject inference (Hand, Vinciotti 2002; Crone, Finlay 2002, 2012; ONB 2004; Verstraeten, Van den Poel 2005; SAS 2008, 2009; Burez, Van den Poel 2008; Marquez 2008; Menardi 2009).

When there are very few debtors, a bank may include debtors‘ records several times, use data of several loan types with a similar risk or even several banks, use several different reference dates of an outcome period, etc. When there are very few or no actual “bad“ debtors, a bank may increase a number of actual “bad“ debtors including each record of “bad“ debtors (or only some randomly chosen records of “bad“ debtors) several times, decrease a number of actual “good“ debtors, use several different reference dates of an outcome period, treat debtors of the worst rating of “good“ debtors as “bad“ debtors, use another stricter definition of a “bad“ debtor or a longer outcome period, use data of several loan types with a similar risk or even several banks, etc. (BCBS 2005b, Bank of Lithuania 2006a).

If developing a model rejected applications are not being included, i. e. if only data of the debtors, who have got a loan, are being used, improper input variables may be being included into a model and (or) inaccurate coefficients may be being calculated. A bank seeks to ensure a model suitability for a rejection of the riskiest debtors. However, a model developed only with data of the debtors, who have got a loan, would not be representative. Including rejected applications a bank does not know, whether a debtor, who had been refused a loan, would have become “bad“ or not. There are various reject inference methods: methods, based on real behaviour of rejected applicants, an augmentation, an assignment of all rejected applications to “bad“ loans, an extrapolation, a clasterization and others. In Part 1.1.1.7 of this dissertation the reject inference techniques examined by various authors are analysed and the developed models‘ validity is compared.

Developing a conceptually sound rating system it is very important to decide what a credit rating should indicate, i. e. a rating philosophy. A bank should decide, whether it wants its rating system to grade debtors according to their current condition (point-in-time) or their expected condition over a cycle (through-the-cycle), because a rating philosophy influences many aspects such as a credit approval, a loan pricing, an early warning of defaults, procyclicality of regulatory and economic capital and, as a result,

[3] Model sensitivity shows a share of actual “bad” debtors also assigned to “bad” debtors by a model.

bank‘s profitability and its competitive position. Developing point-in-time models, macroeconomic variables and (or) other variables, that are sensitive to an economic cycle, are being included, so, in worsening macroeconomic conditions, a debtors‘ risk increases *ceteris paribus*. Meanwhile, a debtor score determined with a through-the-cycle model is not changing when an economic cycle is changing. It is being recommended for banks to develop models of both types: through-the-cycle models – for an assessment of long-term loans, a development of a bank operating strategy or for other purposes, point-in-time models – for an assessment of short-term loans and so on (Löffler 2004; BCBS 2005; Bank of Japan 2005). The following macroeconomic variables are being mostly used for models: a change of a real gross domestic product, an unemployment rate, interest rates, a real estate price index (Bunn, Redwood 2003; Dionne and others 2006; Malik, Thomas 2006; Castro 2008; Bonfim 2009). Lithuanian banks developing point-in-time models could include the variables predetermining the Lithuanian bussiness cycle and (or) the sovereign credit rating of Lithuania: the government budget balance, as a percentage of the gross domestic product, the current account balance, as a percentage of the gross domestic product, the quarterly change of net foreign assets, the short-term external debt and others (see Pačebutaitė 2011, Proškutė 2012). However, not only macroeconomic, but also other variables may be sensitive to an economic cycle, for example, a debtor‘s income, a payment history, revolving loan usage on an assessment day or during a short period till an assessment day. If a bank wanted its model to be more through-the-cycle, it should not include input variables, that are sensitive to an economic cycle, at all or should adjust them (however, in such a case data of a longer period should be included[4]). Table 2 provides with the input variables mostly included into scoring models, sorted by a frequency.

Often internal payment history variables are the most important to a credit risk assessment. A debtor applying to a bank for a loan for the first time should be assessed *ceteris paribus* not so favourably as a debtor, who has already received loans at this bank and allways paid payments in time. Banks might model internal payment history variables separately, and a modeling result might be being used as a separate input variable of a main model. Banks applying expert models might also model internal payment history variables separately. Then a modeling result might be being used as a separate input variable of an expert model, its concrete weight might be determined expertly. However, then a bank would have to validate an additional model: this model should be being assessed analysing input variables of an expert model and as a separate model.

4 Monetary – credit cycles continue at least 7 years (Paliulytė 2004). Developing a through-the-cycle model one should have data of at least two cycles. So, one should gather dynamic rows of input variable values, that are sensitive to an economic cycle, for at least 14 years and deduct periodically changing cyclicality (and seasonality, if data are quarterly) components leaving only a trend and random deviations.

Table 2. The input variables mostly included into scoring models

Input variables of private person models	Input variables of company models
Age	Net profit (loss) / Total assets
Data about a living place (whether a debtor has his own dwelling, rents it and so on)	Sales revenue / Total assets
Length of service at a current working place	Liabilities / Total assets
Length of living at a current address	Current assets / Current liabilities
Other information related to a job of a debtor (a manager, a self-employed person, a specialist, a pensioner, a student, etc.)	Working capital / Current assets
Marital status	Retained earnings (losses) / Total assets
Fact of having children	Profit (loss) before interest and tax / Total assets
Family income	Profit (loss) before tax / Interest expenses
Number of financially dependent persons	Net profit (loss) / Equity
Length of being a bank client	Change of an increase in Sales revenue
How long information about loans of a debtor is in a data basis of an external loan register	Age
Number of credit accounts of a debtor	Number of employees
Average revolving loan usage in 12 or 6 months	Economic sector
Number of inquiries about a debtor in the last 12 months	Geographical region
Information about at least one payment term delay in the last 12 months was received from external loan registers	Total assets (a logarithmic transformation)
Number of loans of a debtor, that are past due 30 days or more (information from external loan registers)	Sales revenue (a logarithmic transformation)

Source: compiled by the author in accordance with Shumway (1999); Altman (2000); Stepanova, Thomas (2001, 2002); Bunn, Redwood (2003); Grigaravičius (2003); McNab, Wynn (2003); Aragon (2004); Lykke (2004); Fernandes (2005); Siddiqi (2006); Dionne and others (2006); Malik, Thomas (2006); Mircea (2007); Castro (2008); Ciampi, Gordini (2008); Luppi and others (2008); Marquez (2008); Bonfim (2009); Psillaki and others (2009); Kočenda, Vojtek (2009); Chung-Hua Shen and others (2010); Hörkkö (2010).

Having developed a statistical scoring model, performed its ex-ante validation and received acceptable results, a new rating scale is being constructed at a bank or a developed model is being adapted to a rating scale already operating at a bank. For example, if an expert model and a rating scale of 5 ratings were applied at a bank, then, having developed a statistical model allowing to calculate individual PDs, this rating scale may be left, however, debtors have to be being assigned to these ratings according to individual PDs determined with a new statistical model. An optimal number of ratings has to be being determined at a bank, and then debtors have to be being assigned to these ratings according to individual PDs or creditworthiness indicators determined with a new statistical model. Scientific articles provide for a varying number of ratings – from 5 to

16 (see Grigaravičius 2003; Savickaitė, Valvonis 2007; Bandyopadhyay 2007; Mileris, Boguslauskas 2011, Dzidzevičiūtė 2012). When a bank applies the IRB approach for a capital requirements calculation, a company rating scale shall have at least seven ratings for "good" companies and one for companies that have actually become "bad" (Bank of Lithuania 2006a). However, when loans are being assigned to a retail loan group, a minimum of ratings is not determined.

It is being recommended for a bank to differentiate low risk debtors more, to determine narrower score intervals for better ratings, and wider – for worse. A ranking should be monotone, i. e. a number of "bads" in a rating, as a share of all debtors in that rating, and a number of "bads" in a rating, as a share of all "bads", should increase, when ratings are worsening. A bank should avoid an undue concentration and an excessive heterogeneity of debtors in a rating, however, a number of such debtors should not be too small because in such a case a bank could not calculate rating PD and assess its validity (Bank of Lithuania 2006a). A bank may determine concentration limits. A debtors' distribution should be unimodal and close to normal, i. e. the greatest percentage share of debtors should be in a middle rating. Also, a bank may compare a χ^2 value applying a χ^2 goodness of fit test and an information value of different rating scales and choose a rating scale with the biggest values (FIS, CEBS 2006a; SAS 2008, 2009).

Having developed a rating scale, a rating PD calculation method is being chosen (see Part 1.2). Banks applying or intending to apply the IRB approach for a capital requirements calculation have to calculate their debtors‘ PDs themselves (BCBS 2006; EP 2006; Bank of Lithuania 2006a). This dissertation‘s author is of the opinion that even banks not applying the IRB approach should calculate rating PDs, they might use such PDs in various internal processes (for example, to calculate internal capital requirements, loan value adjustments, credit risk margins and so on).

1.2. METHODS TO CALCULATE A DEFAULT PROBABILITY FOR RATINGS

Rating PD may be being calculated applying one of the several methods (see Table 3). It is being recommended for banks to apply an arithmetical average of individual PDs or an arithmetical average of one-year actual "bad" rates. When debtors are being assigned to ratings according to a result of an expert model or that of a statistical model not allowing to calculate individual PDs, only the last method (i. e. PD(4)) may be being applied.

Applying an arithmetical average of individual PDs, rating PDs are very unstable, they depend on debtors' individual PDs very much and need to be periodically recalculated.

Table 3. The rating PD calculation methods

Method	**Formula**	**Comments**
Arithmetical average of individual PDs	$PD(1)_{rating} = \dfrac{\sum_{i=1}^{n} PD_i}{n}$, where: PD_i means individual PD of the i^{th} debtor assigned to that rating; i changes from 1 to n; n means a number of debtors assigned to that rating.	The methods may be being applied only when debtors are being assigned to ratings according to their individual PD
Arithmetical average of PD interval boundaries	$PD(2)_{rating} = \dfrac{PD_{lower} + PD_{upper}}{2}$, where: PD_{lower} means a lower PD interval boundary; PD_{upper} means an upper PD interval boundary.	
Geometrical average of PD interval boundaries	$PD(3)_{rating} = \sqrt{(PD_{lower} \cdot PD_{upper})}$	
Arithmetical average of one-year actual "bad" rates	$PD(4)_{reitingo} = \dfrac{\sum_{t=1}^{T} ODF_t}{T}$; $ODF_t = \dfrac{B_t}{N_t}$, where: ODF_t means an actual "bad" rate in a year t; B_t means a number of actual "bad" debtors of a rating in a year t; N_t means a number of rating's debtors at a beginning of a year t; t varies from 1 to T; T means a number of years used to calculate PD(4).	The method may be being applied when there are statistical models of all types and expert models

Source: compiled by the author in accordance with Blochwitz and others (2004); BCBC (2005a); Fritz and others (2006); Bank of Lithuania (2006a); Dzidzevičiūtė (2010c, 2012).

However, this method is very accurate because each debtor is being taken into account. When a point-in-time rating system is being applied, rating PD(1) would stay more or less stable in worsening macroeconomic conditions[5], however, credit risk capital requirements would increase at a bank applying the IRB approach because of a debtors' migration from better ratings to worse ones.

Applying arithmetical and geometrical averages of PD interval boundaries, rating PDs depend on a rating scale construction (i. e. on lower and upper boundaries). However, applying these methods (the same as calculating PD(1)) no difficulties arise when there are too few or no actual "bad" debtors in a rating within a year and it is not possible to

[5] Meanwhile, debtors' individual PDs would increase.

calculate an accurate PD(4). Besides, when a point-in-time rating system is being applied, PD(2) and PD(3) remain stable in worsening macroeconomic conditions (though capital requirements increase). It is more conservative to apply an arithmetical average of PD interval boundaries than a geometrical one because PD(2) *ceteris paribus* exceeds PD(3).

Applying the fourth method rating PDs are being calculated as an arithmetical average of one-year actual "bad" rates (thereinafter – ODF). Difficulties arise when there are too few or no actual "bad" debtors in a rating within a year. In such a case the special rating PD calculation methods for low default portfolios should be being applied (see Part 1.2.2 of the dissertation). Also, difficulties arise when there are short-term loans. In the last case a sum of time of each debtor being assessed by a concrete rating during a year may be being used as a denominator of the ODF_t formula, for example, if a loan matures in June, a debtor gets the weight of 0,5, and not that of 1, because in that year this debtor was being assessed by this rating only half a year. It is convenient to apply this method when a new statistical model was developed using data of a period shorter than 5 years[6].

At banks applying the IRB approach rating PD shall be at least 0.03% (Bank of Lithuania 2006a)[7]. If in the best ratings there are very few or no actual "bad" debtors and because of that annual ODFs are very small (calculating PD(4)) or in the best rating there are very few riskier debtors (calculating PD(1)) and because of that calculated rating PD is lower than 0.03%, then banks applying the IRB approach shall equate PD of such ratings with 0.03%.

Not only debtors' common rating PDs, but also debtors' individual PDs may be being applied to calculate capital requirements at banks applying the IRB approach and for internal purposes at all banks. They can be being calculated only applying a statistical scoring model allowing to calculate individual PDs. A bank applying such a model may calculate and use PDs of both types. They may be being used for different purposes, e. g. individual PDs may be being used assigning debtors to ratings and calculating capital requirements, and rating PDs – calculating value adjustments and for other purposes because, applying individual PDs for those purposes, work of bank's information technology systems would be more difficult.

When in a rating there are very few or no actual "bad" debtors, international and national banking supervision institutions recommend to join several ratings, to calculate one-year PDs from multi-year PDs, to add conservatism margins, to use external PDs (i. e.

6 When a bank applying the IRB approach calculates PD, a length of an underlying historical observation period used shall be at least 5 years. However, banks implementing the IRB approach in some cases are being allowed to reduce this period to 2 years (Bank of Lithuania 2006a).

7 This limit is valid when exposures are assigned to a group of institutions, company or retail exposures. An exposure means an asset or off-balance sheet item.

mapping internal rating scale with a rating scale of one of the international rating agencies or with a rating scale of one of the external loan registers and so on), to use data of several banks or several loan types with a similar risk, etc. Also, as it was mentioned earlier, it is possible to equate PDs of the ratings, initial calculated PDs of which are less than 0.03%, with 0.03% (BCBS 2005b, FSA 2005; CEBS 2006; Bank of Lithuania 2006a). However, in the later case the difficulties arise when PDs of more than one rating have to be equated with 0.03%. The difficulties also arise choosing the other options. When a rating scale is already being applied at a bank, a bank may not want to change it joining several ratings, besides, such a new rating scale may not comply with the principles of an optimal rating scale construction (see Part 1.1). Calculating multi-year PDs, there may be lack of "bad" debtors even during the longer period, it is more difficult when there are short-term loans, besides, having calculated one-year PDs from multi-year PDs even in several ratings these PDs may be less than 0.03%. It may be not clear for a bank how to determine conservatism margins. Also, there simply may be no appropriate external PDs. Using data of several banks or several loan types with a similar risk, calculated PDs are non-representative in respect of one concrete bank or one concrete loan type, besides, there may be no loan types with a similar risk at a bank or different banks may not want to provide their own data to other banks. For these reasons special rating PD calculation methods for low default portfolios are becoming more and more popular (see Part 1.2.2 of the dissertation).

1.3. A RATING SYSTEM VALIDATION

Before a bank starts to apply a rating system in its activity, its ex-ante validation has to be performed. Later, having started to apply a rating system in a bank‘s activity, a regular ex-post validation has to be being performed (Bank of Lithuania 2006b). An ex-ante validation consists of two stages: a suitability verification of a received equation (an equation correspondence to empirical data) and a verification of other rating system aspects applying the methods provided in Figure 7.

Applying a logistic regression, a suitability verification of an equation comprises the following stages (Kleinbaum, Klein 2002; Čekanavičius, Murauskas 2004; Fernandes 2005; Pukėnas 2005, 2009; Hand 2009):

- an economic logic verification of coefficient algebraic signs (for example, when profitability increases, debtor PD has to decrease *ceteris paribus* and *vice versa*);
- an analysis of the regression equation coefficient inequality to 0 applying Omnibus and Wald tests;

- an analysis of coefficients‘ standard errors;
- an analysis of a logistic regression equation correspondence with empirical data applying a Hosmer-Lemeshow test;
- a determination and a multicolinearity analysis;
- a correct classification analysis (see Table 4).

Table 4. A classification table

<table>
<tr><th colspan="2" rowspan="3">Observed</th><th colspan="3">Predicted</th></tr>
<tr><th colspan="2">Number of debtors</th><th rowspan="2">Percentage correct, %</th></tr>
<tr><th>“Good”</th><th>“Bad”</th></tr>
<tr><td rowspan="2">Number of debtors</td><td>“Good”</td><td>True negatives (TN)</td><td>False positives (FP)</td><td>$\frac{TN}{TN+FP}$</td></tr>
<tr><td>“Bad”</td><td>False negatives (FN)</td><td>True positives (TP)</td><td>$\frac{TP}{FN+TP}$</td></tr>
<tr><td colspan="4">Overall percentage, %</td><td>$\frac{TN+TP}{TN+FP+FN+TP}$</td></tr>
</table>

Source: Engelmann and others (2003); Čekanavičius, Murauskas (2004); Stein 2005; Pukėnas 2005, 2009; Hand (2009).

If results of an equation suitability verification are not satisfactory, a bank may come back to any of the earlier model development stages, for example, group and (or) code input variable values in another way, choose other (or include additional) input variables, develop a model on the other level, narrow a debtor group, choose another statistical technique and so on.

When results of an equation suitability verification are satisfactory, model discriminatory power has to be being assessed. Only later, if a model discrimination is at least acceptable, a bank should construct a rating scale, calculate rating PDs, assess calibration accuracy (see Figure 7). Only after that rating system stability and input variables should be analysed using a validation sample and a benchmarking with external benchmarks should be performed.

Having analysed 89 articles written by the authors from various countries of the world during 15 years (from 1996 to 2011), one can see that the most widely applied validation methods are CAP and ROC curves and a correct classification analysis, however, entropy-based methods are also becoming more popular, especially the information value method, also the Brier score method (see Figure 8). These methods are also being applied at Lithuanian banks (see Chapter II of the dissertation), the Bank of Lithuania also recommends to apply them (Bank of Lithuania 2006b).

Scoring model and rating scale validation		***Rating PD validation***	
Discriminatory power analysis	1. CAP, ROC curves 2. Brier score method 3. Bayes error rate 4. I and II type errors 5. Information value (and other entropy-based methods, e. g. CIER, MIE*)	**PD calibration accuracy assessment**	1. Normal tests 2. Binomial tests 3. Hosmer-Lemeshow test 4. Traffic light approach 5. Reliability diagrams
Rating scale analysis	1. Score intervals 2. Monotonicity 3. Concentration in ratings 4. Debtors' distribution 5. Discrimination	**Stability analysis**	1. PD calibration accuracy changes 2. Changes of data, input variables, assumptions, calculation methods and others
Stability analysis	1. Discriminatory power stability 2. Rating transition matrices 3. Rating scale stability 4. Input variable stability	**Benchmarking**	Internal PDs are compared with external PDs
Input variables analysis**	1. Univariate 2. Bivariate 3. Multivariate		
Benchmarking	Ratings' comparison with external ratings		

Figure 7. A rating system validation

Source: compiled by the author in accordance with Stein (2002); Blochwitz and others (2004); ONB (2004); BCBS (2005); Fritz and others (2006); Bank of Lithuania (2006b); Banca D'italia (2006); Tasche (2006); Lingo, Winkler (2008); Medema and others (2009); **CIER* – conditional information entropy ratio; *MIE* – mutual information entropy; ***univariate analysis* – an analysis of missings, outliers, etc.; *bivariate analysis* – an analysis of individual discriminatory power, a connection with an output variable or separate input variables, etc.; *multivariate analysis* – a multicollinearity analysis (e. g. a variance inflation factor (VIF) and others), etc.

Applying both an ex-ante and an ex-post validation it is being recommended to follow the following course: a discriminatory power assessment → a PD calibration accuracy assessment → a rating system stability assessment (ONB 2004; BCBS 2005; Banca D'italia 2006). It is recommended to perform an input variable analysis and a benchmarking in the

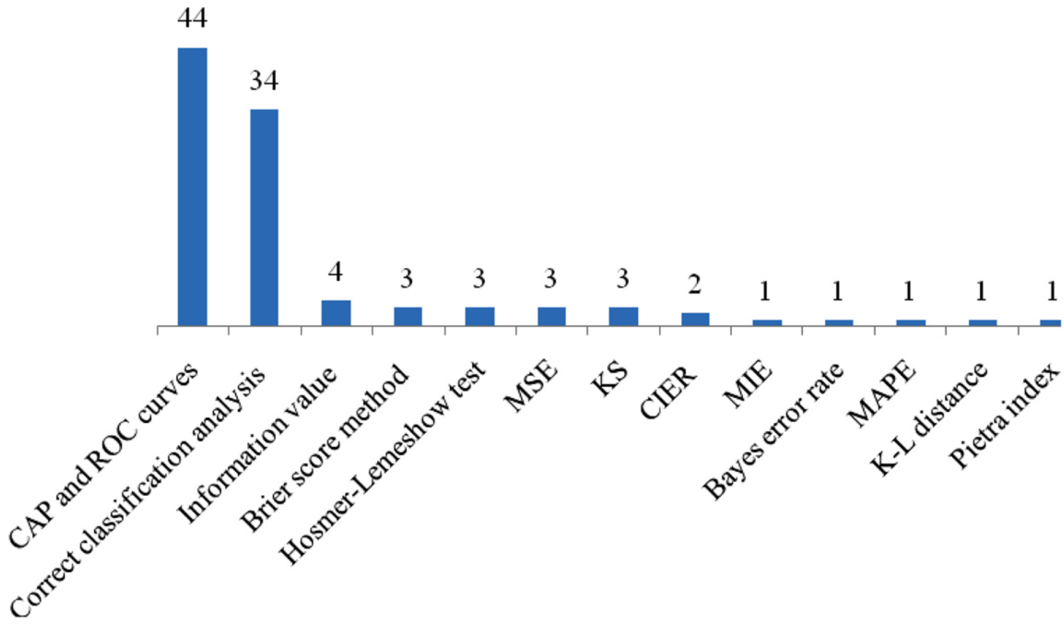

Figure 8. The validation methods[8] application frequency, in %.

Source: compiled by the author in accordance with 2; 3; 5; 7; 8; 9; 11; 12; 14; 23; 25; 26; 29; 30; 31; 33; 34; 35; 36; 38; 39; 45; 48; 49; 51; 55; 57; 59; 60; 61; 62 ; 66; 70; 71; 72; 73; 74; 75; 76; 79; 80; 81; 82; 83; 85; 86; 87; 88; 89; 90; 94; 96; 97; 98; 99; 102; 104; 105; 108; 113; 118; 121; 122; 123; 124; 127; 128; 129; 131; 132; 133; 135; 138; 140; 145; 146; 147; 148; 149; 152; 154; 155; 157; 163; 164; 168; 170; 172.

end, their results may help finding the spheres of a rating system, that need improvement. However, a benchmarking may be performed also as a separate part of an assessment of discriminatory power and PD calibration accuracy (i. e. a backtesting + a benchmarking[9]). When a bank does not have enough historical data to backtest, discriminatory power and PD calibration accuracy may be being assessed only performing a benchmarking. The Bank of Lithuania requires banks applying the IRB approach to validate their rating systems regularly (at least once a year). Even banks not applying the IRB approach shall *mutatis mutandis* keep within the validation requirements of the Bank of Lithuania (Bank of Lithuania 2006b, 2008).

[8] The validation methods applied by the authors mostly comprise only methods of an equation suitability verification and a discriminatory power assessment, because the authors did not construct a rating scale and did not calculate rating PDs. Abbreviations: *MSE* – mean squared error; *KS* – Kolmogorov-Smirnov method; *CIER* – conditional information entropy ratio; *MIE* – mutual information entropy; *MAPE* – mean absolute percentage error; *K-L* distance – Kullback Leibler distance.

[9] A backtesting is a group of validation methods when forecasted internal values of a bank are compared with actual internal values, and a benchmarking is a group of validation methods when forecasted internal values are compared with external values.

II. THE APPLICATION OF STATISTICAL SCORING MODELS AT LITHUANIAN BANKS

In order to elucidate a scale and peculiarities of a scoring models application for a retail application assessment, the survey of commercial banks and foreign bank branches operating in the country was performed in January-November 2008. It was performed by e-mail, the standardized questionnaire made by the author was e-mailed to credit risk management specialists of all banks. Nine banks (eight commercial banks and one foreign bank branch) voluntarily agreed to participate.

Performing the survey, the development, application and validation aspects of not only statistical, but also those of expert and mixed scoring models, that might be usefull developing statistical scoring models, were examined. As it is more important to banks not to grant loans to risky debtors than to regularly reassess a risk of already granted loans, i. e. they pay more attention to application, and not to behavioural models, this survey was oriented only to application models. However, its results may be also usefull to banks developing behavioural models. Performing the banks' survey, it was aimed at an examination of the following:

- what criteria and how many of them were being applied assigning loans to a retail loan group;
- whether a bank was applying statistical scoring models, what scale was;
- what models were being applied and what loan types for; if statistical models were not being applied, then – what reasons for;
- what data and input variables were being used developing statistical scoring models;
- whether expert and mixed scoring models were being applied at a bank, what scale was;
- if expert and (or) mixed scoring models were being applied at a bank, then – what input variables were being used. In such a way it was aimed at a determination, what criteria a bank was paying attention to assessing a debtor (or a loan), and at an adaptation of that experience developing the statistical scoring model of Lithuanian companies;
- what level – of a loan or a debtor – scoring models were being applied on;
- whether statistical scoring models, data, etc. applied at a controlled bank were the same as ones applied at a parent bank;
- what validation methods of scoring models were being applied at a bank;
- what bank activity spheres developed models were being applied in;
- what changes of the scoring models‘ application were being foreseen.

The results showed that statistical application scoring models were being applied at four banks, at two of them – only for an assessment of private person applications (see Table 5). Meanwhile, the previous survey‘s results received examining the credit risk management of banks operating in the country (see Valvonis 2004) showed that at least till 2004 statistical models of a loan risk assessment had not been applied at any of the six banks that participated in that survey.

Table 5. The scoring models application practice

Model type	**Company loans**	**Private person loans**	**In total** $^{+}$
Statistical	2 banks	4 banks	4 banks
Mixed	1 bank	2 banks	2 banks
Expert	6 banks	6 banks	6 banks

Source: compiled by the author. $^{+}$A bank might apply statistical, mixed and expert models at the same time.

Both banks, that were applying statistical company application models, were applying them for all company application types without a differentiation of them. However, banks inclined to a bigger differentiation of private person application models: they distributed loans into groups by types and developed separate models to assess loans of different types. Such models were quite different by input variables and their number as well as by their values‘ groups and coefficients (having the same input variable). Besides, the results of the other researches showed a differing risk of differrent private person loan types (see Part 1.1.1.3 of the dissertation), so, if a bank has enough data, in the future it should develop a separate model for each loan type.

The statistical models of one of the four banks were developed applying a logistic regression, those of the other – applying a discriminant analysis, two other banks, that were applying statistical models, did not indicate a chosen statistical technique. One of the banks as a reason indicated the fact that the model had been developed by the parent bank on the group level using the whole group‘s data that is why the controlled bank did not know, what statistical technique had been applied.

Performing the survey banks were asked, whether the same statistical scoring models were being applied at a controlled bank operating in Lithuania and in a whole banking group. Two banks, that were applying statistical models, did not have parent banks. At one of the other two the models developed by the parent bank using the whole group‘s data were being applied, some models of the other bank were developed using the local bank data, other models – using the whole group‘s data, because the local bank‘s data sample was too small to develop a separate model. At that bank all the models were developed by the parent bank applying the same statistical technique – a logistic regression. Having compared the models of the same loan type at the banks of that banking group operating in different countries, that were based only on the local data of one or the other country, it was clear that there were significant differences in model input variables and their number as well as in their values‘ groups and coefficients (having the same input variable). This only supports the research results of various authors that models of the same loan type developed with the data of different countries are very different (see Part 1.1.1.3 of the dissertation) and show that it is better to develop models with local data: debtors‘ peculiarities of different countries differ very much and local bank data represent debtors of that country significantly better than whole group‘s data.

Nevertheless, an insufficient data sample induced a development of statistical models dedicated to several banks. When a loan sample of a local bank was too small, seeking for bigger model validity, data of several banks of a banking group were being used. Besides, models applied by Lithuanian banks at the time of the survey performance were developed using data of the economic boom period (2004–2007) that is why a number of actual "bad“ debtors would have been too small to develop models of some loan types. After an inclusion of data of later periods comprising the economic recession years, this problem will not be so actual.

Two banks, that were applying statistical models, used not only internal, but also external data received from external loan registers, debt collection companies, etc. in the development of them. More information (a company registration date, a number of employees, financial data, an external payment history) was bought developing company models. Developing private person models only external payment history data were bought. The models of the third bank, that was applying statistical models, were developed by the parent bank with the whole group‘s data, so, the local bank simply did not know, whether the external register data of the parent bank‘s country or the other countries had been used. Nevertheless, having analysed the model input variables of that bank, one could see that external loan register information had not been used. At the fourth bank, that was applying statistical models, external loan register data had not been used. At three banks statistical models were developed and being applied on a debtor level, i. e. a risk of a debtor and not that of a loan was being assessed. At the fourth bank only the company models were developed and being applied on a debtor level, and the private person models – on a loan level.

All the banks, that were not applying statistical models, indicated that a past data collection period would have been too short to develop a statistical model. The banks also mentioned the other reasons:

- an insufficient sample of debtors (loans) (2 banks);
- new loan types, data of which had not been collected at a bank (2 banks);

- an insufficiency of information about variables (2 banks);
- unreliable past data (1 bankas);
- an expert assessment applying statistical models would be insufficient (2 banks).

Table 6. The foreseen changes of the scoring models‘ application

Change	**Number of banks**
New statistical models were planned to be applied	2 banks
New mixed models were planned to be applied	–
New expert models were planned to be applied	–
There were plans to adjust models currently applied	5 banks
including local data	2 banks
including external loan register data	3 banks
including group data	–
Other changes were planned	–
No changes were planned	3 banks

Source: compiled by the author.

Though statistical models were not spread at Lithuanian banks, not all the banks, that were not applying them, were intending to implement them (see Table 6). The majority of the banks were intending to adjust their applied models including additional internal and external data. At Lithuanian banks a need of data bought from external loans registers has increased. Even three banks were planning to adjust their applied models including such data. Also, it may happen so that two banks, that were planning to develop new statistical models, will include external loan register information in the development of them[10].

So, taking into account the fact that statistical scoring models were not being widely applied at Lithuanian banks and not all the banks, that were not applying them, were intending to implement them, the rating system of Lithuanian companies based on the statistical scoring model was developed.

[10] See Dzidzevičiūtė (2010d) for more about the results of this survey.

III. THE RATING SYSTEM OF LITHUANIAN COMPANIES BASED ON THE LOGISTIC REGRESSION MODEL

In this Chapter the last two exercises of the research are being solved. Part 3.1 of the dissertation provides the development of the statistical scoring model and the rating scale of Lithuanian companies. In Part 3.2 the calculation of company rating PDs is described, the PD calculation methods are assessed. In Part 3.3 the application possibilities of the developed rating system are assessed.

3.1. THE DEVELOPMENT OF THE STATISTICAL SCORING MODEL AND THE RATING SCALE OF LITHUANIAN COMPANIES

In Part 3.1.1. of the dissertation the data used in modeling are described, the detailed modeling process description comprising all the following stages is provided: the definition of a “bad“ company and an outcome period, the choice of a company group and a statistical technique, the sample construction, the input variables analyses, the coefficients calculation and the ex-ante validation.

The data. The data of Lithuanian companies from all economic sectors for 2005-2008 were received from the external loan register JSC “Creditinfo Lietuva” which collects and stores companies’ information about their age, locality, legal status and legal form, economic sector, annual turnover, number of employees, managers, members of a board, subsidiaries and branches, claims, arrests and legal processes, bankruptcies, debts, changes of names and addresses, public rating, shares, inquiries about them and the other information from banks, leasing, telecommunication and public utility companies, public registers, etc.

Each company was attributed to one of the two possible groups: to “goods“ or to “bads“. A default indication was used to define a status of a “bad“ company. A default was defined as a company status when company payments to at least one bank were delayed more than 90 calendar days or a bankruptcy process was started[11]. When a company had

[11] The default definition provided by the Bank of Lithuania is slightly different, i. e. *“A default shall be considered to have occurred with regard to a particular debtor when either or both of the two following events has taken place: 1) the debtor is past due more than 90 days on any material credit obligation to the bank, the parent bank or any of its controlled financial undertakings, excluding the cases when the exposure amount balance does not exceed LTL 100, or another amount which the bank considers insignificant; 2) a bank considers that the debtor is unlikely to pay its credit obligations to the bank, parent bank or any of its controlled financial undertakings in full, without recourse by the bank to actions such as realising collateral (if held).”* (Bank of Lithuania, 2006a). As the information about an unlikeliness to pay (except only a bankruptcy procedure) is not being collected at JSC “Creditinfo Lietuva”, the definition used in the dissertation was narrower.

defaulted at least once within one year from the respective year-end date, i. e. from the T_0 reference date, it was attributed to “bads“ (see Figure 9).

The outcome period of 1 year was chosen taking into account the legal acts of European Union and Lithuania prepared in observance with the New Basel Capital Accord, besides, an outcome period of such a duration was chosen in most cases (see Part 1.1.1.2 of the dissertation).

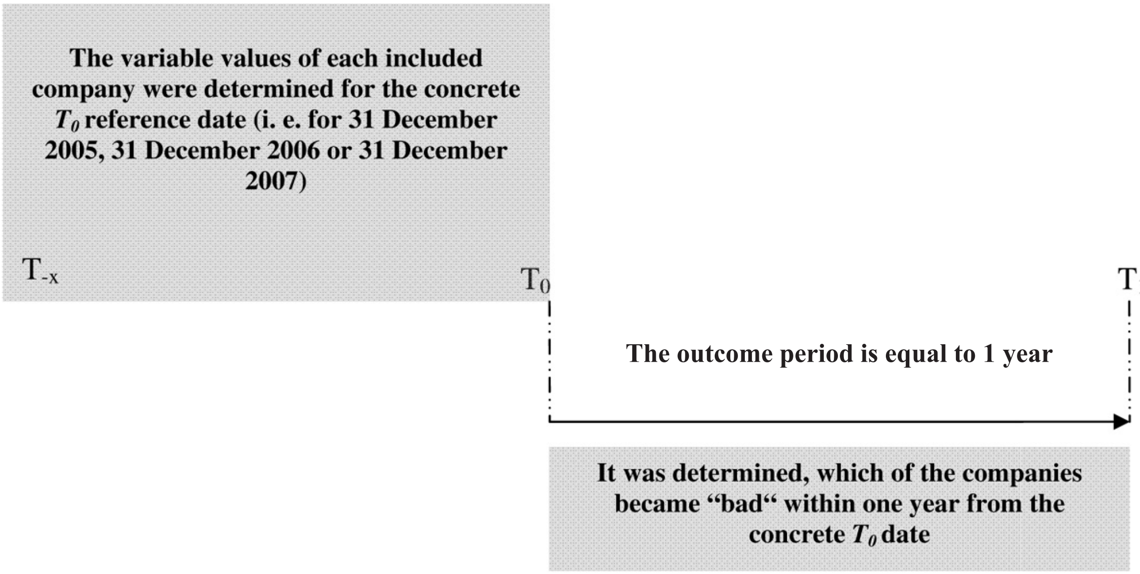

Figure 9. The company data gathering scheme

Source: formed by the author.

3 reference points were used: 31 December 2005, 31 December 2006 and 31 December 2007. Those companies, that were “bad“ on the concrete reference date, were not included. The values of input variables characterizing company creditworthiness were determined for the concrete T_0 date, however, they might be momental (e. g. financial ratios from balance sheets) or for the period x from T_{-x} to T_0 (e. g. the information about delayed payments during one year before the reference date). For example, input variables of the company ABC were taken for 31 December 2007, i. e. the reference date was 31 December 2007. Then it was assessed, whether within one year period from 31 December 2007 until 31 December 2008 the ABC had defaulted at least once for at least one bank. If yes, then forming the data array it would have been attributed to “bads“ and the output variable 1 would have been assigned. However, if ABC had not defaulted within this one year period, then this company would have been attributed to “goods“, and the output variable 0 would have been assigned.

The data of each separate year were joined into one common data array and a “company-year“ was used for the further analysis; when data on a concrete company had been given for all three years, the data of such a company were "tripled" and used as data of three separate companies. In total, the data array of 29597 rows ("company-years") was obtained.

The population segmentation. The proposed company scoring model is generic (external) because the external loan register data comprising information of many banks were used. As companies from all economic sectors were included, the model is being recommended to assess a risk of various companies and it is not industry-specific.

Also, one should notice that the model is behavioural (portfolio), i. e. it is being recommended for banks to apply it for regular reassessments of already existing debtors. JSC "Creditinfo Lietuva" did not gather the information about granting dates of concrete loans, so, it was not possible to develop an application scoring model. The outcome period developing the proposed model was determined starting from the respective year-end and not from the loan granting date[12]. However, even if the model is behavioural (and not application), it is also possible to apply it in a decision-making process deciding to grant a loan or not.

The model was developed on a company (and not on a loan) level, i. e. it is being dedicated to an assessment of companies, and not to that of loans. Besides, the model may be being applied to assess companies, that take various types of loans (investment loans, working capital financing loans, credit lines, etc.).

The statistical technique. A logistic regression was chosen taking into account its advantages if compared to other statistical techniques (see Part 1.1.1.5 of the dissertation). The other alternative was also considered – a survival analysis, however, as there was no information about the dates when concrete companies had become "bad", a survival analysis was rejected. Besides, the data of only three years were received, and applying a survival analysis one should use data of a longer period. Besides, having included only the companies, the information of which had been received for all three years, the data sample would have been too small.

The sampling. Upon joining the data of three years into one common data array, 29597 rows ("company-years") were obtained. This total data array was divided into the development sample consisting of 19193 rows (64.85% of the total data array), and the validation sample consisting of 10404 rows (35.15% of the total data array). The rows of the validation sample were chosen randomly. Developing a logistic regression model "bad" companies should compose not less than 20% of all companies, so, from the total data sample 1683 actual "bads" were chosen randomly and included into the validation sample. The other rows of the validation sample were the actual "goods", that were also

[12] See Dzidzevičiūtė (2010a) for the further comparison of application and behavioural scoring models.

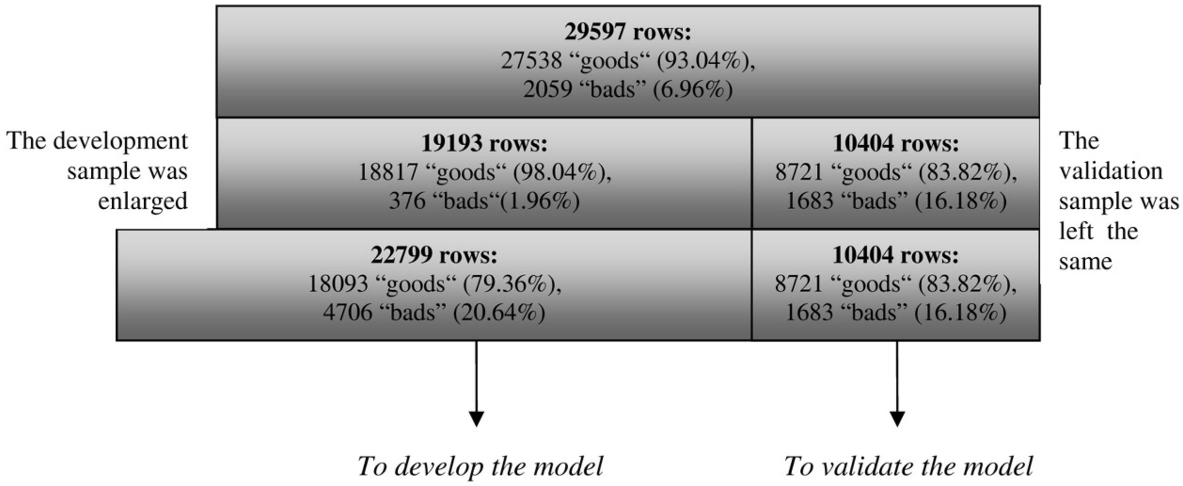

Figure 10. The development and the validation samples

Source: the calculations of the author.

chosen randomly (see Figure 10). The validation sample was not adjusted any more and was used to validate the developed model.

The initial development sample consisted of 19193 rows ("company-years"), 376 of them were assigned to "bads", 18817 – to "goods". To adjust the initial development sample several methods were applied:

1) the calculated needed sample size was compared with the initial development sample size;
2) the structure of actual "goods" and "bads" was analysed and the optimal structure was derived.

The following formula was applied to calculate the needed sample size (SAS 2009; Dzidzevičiūtė 2010b, c):

$$n = \left(\frac{\Phi^{-1}(\frac{\alpha}{2}) \cdot \sqrt{PD_{MAX}(1 - PD_{MAX})}}{\Delta PD}\right)^2,$$

where: PD_{MAX} means maximum PD, that can be determined by bank experts analyzing a historical experience of companies; α means the significance level equal to 0.05; $\Phi^{-1}()$ means the inverse standard normal distribution function (it is possible to calculate it applying the MS Excel function NORMSINV()); ΔPD means a PD error, e. g. if a bank chooses the 95% confidence level and the 0.20% PD error, it wants to be 95% confident that an arithmetical average of individual PDs calculated by a model will be no more than 20bp off PD_{MAX}.

In the initial development sample ODF was 1.96%, however, in order to be conservative, slightly higher maximum PD had to be used for the calculation of the needed sample size (e. g. 2.4%). Let's say, the dissertation's author wanted to be 95% confident that the arithmetical average of individual PDs calculated by the model would be no more than 20bp off this PD_{MAX}. Then the needed sample size calculated according to the formula above was equal to 22496. One could notice that the calculated needed sample size exceeded the initial development sample, i. e. there were only 19193 rows ("company-years") and 22496 rows were needed.

Besides, the initial proportions of "goods" and "bads" were 98.04% and 1.96% in the development sample. Meanwhile, as previously mentioned, developing a logistic regression it is recommended to use at least 20% of "bads" and 80% of "goods". To achieve such proportions, the mixture of undersampling and oversampling techniques was used, i. e. the number of "goods" was reduced (every 26th row was deleted) and the number of "bads" was increased (each row was repeated 12.516 times) to reach 20% in the total structure. After the adjustment, the number of "goods" was 18093 (79.36%) and the number of "bads" – 4706 (20.64%), in total 22799 rows. This number was even bigger than the one calculated using the formula above.

The input variables' analysis, the coefficients' calculation. The input variables, that are in the final model, were chosen in 3 cycles:

1) in the first cycle based on an expert judgment, 57 input variables presented in Appendix 1 were determined.
2) in the second cycle, 48 input variables (from 57) were chosen taking into account several criteria (economic logic, monotonicity, individual discriminatory power of a variable);
3) in the third cycle, 22799 rows composed of 48 input variables' dummies and output variables (0 or 1) were inputted into the SPSS program and the logistic regression equation consisting of 19 input variables was created applying the forward stepwise (Wald) procedure (see Appendix 2).

The first cycle. Initially, 57 input variables characterizing all company features were determined (see Appendix 1): the financial ratios, the variables related to external delayed payments, age, a legal form, a county and an economic sector of a company, information about company management, a change of its address and name, negative facts about a company, claims from external debt collection companies, etc. The values of all the quantitative input variables were joined into 10 groups by percentiles (in some cases negative values were used as a separate group, e. g. for *Total assets/Equity* because the negative values of this ratio indicate a very risky situation of a company and small positive values, on the contrary, indicate a non-risky situation, so they could not be mapped into one group). The initial groups for the input variables *Annual turnover* and *Number of* *employees* were determined based on the external loan register's grouping, and not by percentiles. The initial groups for the input variables *Age* and *Total number of delayed payments during the last year* were determined based on an expert judgment. As all the values of quantitative input variables were grouped, an outliers' analysis was not made.

To code the values, the weight of evidence (thereinafter – WOE) method was applied because dummies assigned applying this method accurately reflect a risk of a concrete group (Dzidzevičiūtė 2010b,c):

$$WOE_i = \ln(\frac{G_i}{B_i}),$$

where: WOE_i means WOE of the *i*-th group; G_i means a proportion of "goods" in the *i*-th group, % from all "goods"; B_i means a proportion of "bads" in the *i*-th group, % from all "bads".

Table 7 provides the dummies' calculation of the input variable *County*. The higher WOE, the lower the risk of the concrete group. When the percentage proportion of "goods" in the respective group exceeds the percentage proportion of "bads" in that group, WOE will be more than 0 and *vice versa*. As one could notice, the riskiest county is Panevėžys, as its WOE is the lowest if compared with the other counties[13].

The initial groups were adjusted taking into account:

- economic logic, i. e. a risk of groups has to reflect expectations of an expert before modeling, e. g. group WOE of negative values of *Total assets/Equity* had to be very low because negative values indicate a very risky situation of a company;
- monotonicity, i. e. ODF has to decrease or increase monotonically when a value of a quantitative input variable increases (at least, to a certain level, for example, a distribution can be U-shaped);
- micronumerocity, i. e. if a number of debtors in a concrete group is very small, it is better to assign them to one of the other groups with similar ODF. Developing this model, missings were put into a separate group, however, in the case of micronumerocity, they were assigned to one of the other groups with similar ODF. When there were no actual "bad" companies in a group, such a group was joined to one of the other groups;
- individual discriminatory power, i. e. an information value (thereinafter – IV) of various grouping alternatives was compared and the alternative with the

[13] Only 10 values of the qualitative variable *County* were possible, so WOE was calculated for each county separately. However, when there are a lot of values of a qualitative variable measured using a nominal scale, all values may be sorted in an ascending order of ODF and then grouped.

Table 7. The analysis of the input variable *County*

County	**Alytus**	**Kaunas**	**Klaipėda**	**Marijampolė**	**Panevėžys**	**Šiauliai**	**Tauragė**	**Telšiai**	**Utena**	**Vilnius**	**Total**
"Goods"	519	3921	2315	467	996	1070	235	498	398	7674	18093
"Bads"	143	1053	533	247	546	312	52	130	156	1534	4706
Total	662	4974	2848	714	1542	1382	287	628	554	9208	22799
ODF	21.60%	21.17%	18.71%	34.59%	35.41%	22.58%	18.12%	20.70%	28.16%	16.66%	20.64%
WOE	-0.0576	-0.0320	0.1220	-0.7097	-0.7456	-0.1143	0.1617	-0.0036	-0.4100	0.2633	
										IV	**0.100**
										χ^2 value	**410.33**

Source: the calculations of the author.

Table 8. The analysis of the input variable *Net profit (loss) / Total assets*

Percentiles	**0.1 percentile < -16.7%**	**0.2-0.4 percentile (-16.69%-1.43%)**	**0.5 percentile (1.44%-3.34%)**	**0.6-0.7 percentile (3.35%-9.96%)**	**0.8 percentile (9.97%-15.52%)**	**0.9-1 percentile >15.52%**	**Missings**	**Total**
"Goods"	1384	4788	1749	3897	2036	4157	82	18093
"Bads"	884	2015	520	637	234	377	39	4706
Total	2268	6803	2269	4534	2270	4534	121	22799
ODF	38.98%	29.62%	22.92%	14.05%	10.31%	8.31%	32.23%	20.64%
WOE	-0.8984	-0.4812	-0.1337	0.4645	0.8167	1.0536	-0.6035	
							IV	**0.429**
							χ^2 value	**1506.13**

Source: the calculations of the author.

biggest IV was chosen, the unpredictive input variables were totally excluded from the further analysis (see Appendix 1). Also, a χ^2 goodness-of-fit test was applied[14].

Table 8 provides the adjustment of the initial values grouping of the input variable *Net profit (loss) / Total assets*. It is clear that some initial groups were joined (see percentiles from 0.2 to 0.4, from 0.6 to 0.7 and from 0.9 to 1) to reach the monotonously decreasing ODF and increasing WOE, i. e. the bigger the input variable values in a group, the smaller group's ODF and the bigger group's WOE. IV and χ^2 values of this grouping alternative are the biggest if compared with the other alternatives.

The second cycle. From the initial 57 input variables, based on their individual discriminatory power, economic logic and monotonicity, 48 input variables were chosen and further used in the modeling. IV was calculated using the following formula (e. g. 0.1 in Table 7 for the input variable *County)* (SAS 2009):

$$IV_{\mathrm{var}iable} = \sum_{i=1}^{n} (G_i - B_i) \cdot WOE_i ,$$

where: $IV_{variable}$ means IV of an input variable; G_i means a proportion of "goods" in the *i*-th group, % from all "goods"; B_i means a proportion of "bads" in the *i*-th group, % from all "bads"; WOE_i means WOE of the *i*-th group; *n* means a number of groups.

Interpreting the meaning of IV, the following explanations were used: <0.020 – an unpredictive input variable; [0.020-0.100) – weak input variable predictiveness; [0.100-0.300) – medium input variable predictiveness; >=0.300 – strong input variable predictiveness. As one could notice from Tables 7 and 8 above, the predictiveness of the input variable *County* is medium, whereas the predictiveness of the input variable *Net profit (loss) / Total assets* is strong. Appendix 1 provides IV of all the analysed input variables.

The third cycle. In the third cycle, 48 chosen input variables were further analysed. The WOE values of input variables and actual output variables (0 or 1) were inputted into SPSS program. Applying the forward stepwise (Wald) procedure, input variables having a strong relationship with the output variable were included step-by step into the regression equation, and after that some input variables were excluded from the equation. In total, 21 steps were made, 19 input variables were left (see Appendix 2). According to the developed model, individual PD of a company is determined applying the formulas below:

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14 When there was the same number of groups, $\chi 2$ values were compared (the bigger, the better), and when the number of groups differed, *p* values with *k-1* degrees of freedom were compared (the smaller, the better).

$$PD_i = \frac{1}{1+e^{-Z_i}};$$

$$Z_i = \ln(\frac{PD_i}{1-PD_i}) = -1.352 - 0.677 \cdot X_{1i} - 0.958 \cdot X_{2i} - 0.821 \cdot X_{3i} - 0.831 \cdot X_{4i} - 0.423 \cdot X_{5i} - 0.755 \cdot X_{6i} - 0.911 \cdot X_{7i} -$$
$$- 0.135 \cdot X_{8i} - 0.145 \cdot X_{9i} - 0.164 \cdot X_{10i} - 0.403 \cdot X_{11i} - 0.460 \cdot X_{12i} - 0.257 \cdot X_{13i} - 0.336 \cdot X_{14i} - 0.155 \cdot X_{15i} -$$
$$- 0.774 \cdot X_{16i} - 0.668 \cdot X_{17i} - 0.330 \cdot X_{18i} - 0.561 \cdot X_{19i},$$

where: PD_i means a probability that the *i*-th company will default within one year after an assessment date; $X_{1i}, \ldots X_{ni}$ mean the dummies of the input variables, i. e. WOE of the concrete group indicated in Appendix 2; Z_i means a natural logarithm of the odds ratio of the *i*-th company, also called logit.

Appendix 2 provides the groups of input variables and their dummies (WOE), also, shows the step when the concrete input variable was included into the logistic regression equation. One could notice that the input variables left in the final cycle characterize all company features: *age*, *a size* (annual turnover, a number of employees and, to some extent, natural logarithms of net profit and non-current amounts payable and liabilities as bigger companies generate relatively bigger absolute net profit amounts and take relatively bigger loans), *a financial condition* (even eight financial ratios were included), *a locality* (companies were grouped by the counties), *an economic sector* (companies were grouped according to the NACE 2 classificator), *external delayed payments* (a total number of delayed payments to banks, leasing, telecomunication, public utility companies and other companies and an average duration of all these delayed payments during the last year before a scoring date), *negative facts about a company and claims from external debt collection companies*.

The ex-ante validation. The coefficient algebraic signs comply with economic logic, all the coefficients are statistically significantly unequal to 0. The percentages correct, overall and in each category, are big, the model is compliant with the empirical data, the input variables are not too multicolinear, the determination coefficients are quite big. The model discriminatory power is excellent (for the further details see Part 3.1.1.3 of the dissertation).

The rating scale construction. In Part 3.1.2. of the dissertation the rating scale of Lithuanian companies is provided. 22799 "company-years" were assigned to 9 ratings for three scoring dates according to individual PDs estimated by the logistic regression model (see Table 9). The several different rating scales were analysed. However, the chosen rating scale is the most optimal from all the analysed because it was constructed in compliance with the principles of *acceptable score intervals* (i. e. PD intervals for better ratings should be narrower than for worse), *monotonicity* (i. e. ODF and a number of “bads”, % from all “bads”, in the worse rating must allways be higher than in the better one), *an acceptable concentration* (there should be enough ratings to avoid undue concentrations of companies), *a companies' distribution* (a distribution should be close to normal, the greatest share of companies should be in the middle rating), *a discrimination* (a χ^2 goodness-of-fit test was applied and the rating system showing the best discrimination of companies was chosen[15]).

The 1st rating indicates the lowest company risk, the 9th – the highest risk. The 10th rating is dedicated to the companies, that have already become “bad”.

Table 9. The rating scale of Lithuanian companies

Rating	Lower PD boundary	Upper PD boundary	Companies	“Bads“	ODF	“*Bads*“, from *all* “*bads*“	*Rating companies*, from *all companies*	χ^2 value
1	0.01%	1.00%	568	13	2.29%	0.28%	2.49%	116.79
2	1.01%	2.20%	1543	40	2.59%	0.85%	6.77%	306.86
3	2.21%	3.70%	2023	55	2.72%	1.17%	8.87%	396.70
4	3.71%	8.00%	4439	203	4.57%	4.31%	19.47%	699.66
5	8.01%	16.00%	4675	573	12.26%	12.18%	20.51%	200.64
6	16.01%	28.00%	3529	697	19.75%	14.81%	15.48%	1.71
7	28.01%	40.50%	2288	811	35.45%	17.23%	10.04%	306.14
8	40.51%	61.00%	2075	1085	52.29%	23.06%	9.10%	1268.75
9	61.01%	99.99%	1659	1229	74.08%	26.12%	7.28%	2892.28
10	Actual “bads” (PD=100%)							
		Total	**22799**	**4706**		**100.00%**		**6189.53**

Source: the calculations of the author.

The analysis‘ results of the rating scale stability and the input variables as well as those of the benchmarking also showed that this rating system could be applied at banks (for the further details see Part 3.1.2 of the dissertation).

15 When two rating scales with the same number of ratings were compared, $\chi 2$ values were analysed and the rating scale with the bigger $\chi 2$ value was chosen. When two rating scales with the differring number of ratings were compared, *p* values with *k-1* degrees of freedom were analysed and the rating scale with the smaller *p* value was chosen.

3.2. THE DEFAULT PROBABILITY CALCULATION FOR COMPANY RATINGS

Rating PD may be calculated applying various methods (see Table 3). Three methods (i. e. PD(1), PD(2) or PD(3)) require individual PDs of companies and the fourth method requires one-year rating ODFs. 10404 “company-years” of the validation sample were divided into three parts for three dates (31 December 2005, 31 December 2006, 31 December 2007) and assigned to ratings according to their individual PDs. Then rating PDs were calculated for 31 December 2007 (see Table 10).

One could notice that the values of PD(1), PD(2) and PD(3) are very similar, especially for ratings 2-7. However, the values of PD(4) are significantly smaller than the values of PD(1), PD(2) or PD(3).

PD(4) was calculated as an arithmetical average of annual ODFs in 2006 and 2007, respectively, however, there were not many actual defaults in 2006-2007. In ratings 1-3, both in 2006 and 2007, there were no more than 20 defaults. In 2007, also in rating 7 there were no more than 20 defaults. The especially severe problem was the rating 1 as there were no actual defaults either in 2006 or in 2007, so, PD(4) for rating 1 is equal to 0%. This PD could not be used for a capital requirements calculation at a bank applying the IRB approach. Such a bank should change this PD to the minimum value determined by the Bank of Lithuania – 0.03%. However, even in such a case the concern of the Bank of Lithuania would be that credit risk capital requirements might be underestimated as a result of a default scarcity.

Taking this into account PD(4) for ratings 1-3 and 7 was recalculated applying several techniques recommended for low default portfolios (see Part 3.2.2 of the dissertation and Table 11). The calculation results showed that the Pluto, Tasche (2005) technique without a correlation could be easily implemented at banks. However, if an ordinal ranking of debtors is incorrect, this technique doesn’t ensure monotonicity of PDs in low default portfolios. The same problem exists in the Kiefer (2006) technique. The Forrest (2005) technique without a correlation ensures monotonicity and conservatism of PDs, however, it requires programming skills, otherwise an iterative recalculation of PDs will be very time-consuming. PDs estimated under these three techniques passed almost all the validation tests.

The PDs estimated under the Burgt (2007) and the Tasche (2009) techniques are too low for the better ratings, these PDs didn‘t pass the validation tests.

If it is not possible to extract information about rating transitions during a year and an exact default date, it makes no sence to apply the techniques based on rating transition matrices; in any case, they are quite time-consuming. However, some supervisors (e. g. the Bank of Lithuania) require the banks to estimate rating transition matrices, so, at the same time the LDP problem would be solved.

Applying the Forrest (2005) and the Pluto, Tasche (2005) techniques with a correlation, conservative PDs may be too high, thus calculated credit risk capital requirements may not satisfy banks as well as their supervisors taking into account that the IRB approach in Basel II should ensure not an over-conservative, but an accurate calculation of capital requirements.

The multi-period techniques proposed by Pluto, Tasche (2005) and Wilde, Jackson (2006) give either too high or too low PDs; in some cases assumptions are unrealistic and cannot be fulfilled in practice.

The technique based on unemployment rates proposed by Sabato (2006) is appropriate only to calculate PDs for private persons. Modifications of the technique to estimate PDs for companies wouldn't allow deriving reasonable PDs. Besides, the technique is appropriate only to calculate PDs for specific sub-groups of age, education, etc., but not for ratings.

All PDs were validated using several tests[16] and the results were compared. The data about defaults in 2008 (i. e. ODF_{2008}) were used for validation purposes. The values of PD(1) seems to be the most appropriate, they passed all the validation tests. The results of the stability analysis and the benchmarking are also good. So, PD(1) was chosen to be applied further.

3.3. THE APPLICATION OF THE DEVELOPED RATING SYSTEM AT A BANK

The rating system application granting loans. The rating system proposed in this dissertation may also be applied as an application rating system. Choosing a cut-off rating from which applications of companies applying for a loan should be rejected, the following aspects were analysed:

- the portfolio ODF dependence upon the application reject rates;
- the ratio of the change in cumulative actual "goods" to the change in cumulative actual "bads";
- the net present portfolio value;
- the distributions of actual "good" and actual "bad" companies and ratings' ODF.

16 Hosmer-Lemeshow, binomial, normal tests, Brier score.

Table 10. The rating PDs of Lithuanian companies

Rating	Lower PD boundary	Upper PD boundary	2006			2007			2008			PD(1)	PD(2)	PD(3)	PD(4)
			Companies on 31 December 2005	Defaulted till 31 12 2006	ODF_{2006}	Companies on 31 December 2006	Defaulted till 31 12 2007	ODF_{2007}	Companies on 31 December 2007	Defaulted till 31 12 2008	ODF_{2008}				
A	*B*	*C*	*D*	*E*	*F=E/D*	*G*	*H*	*I=H/G*	*J*	*K*	*L=K/J*	M	N=(B+C)/2	O=√(B*C)	P=(F+I)/2
1	0.01%	1.00%	99	0	0.00%	222	0	0.00%	369	2	0.54%	0.58%	0.51%	0.10%	0.00%
2	1.01%	2.20%	292	3	1.03%	554	7	1.26%	706	10	1.42%	1.50%	1.61%	1.49%	1.15%
3	2.21%	3.70%	344	0	0.00%	259	8	3.09%	361	11	3.05%	2.98%	2.96%	2.86%	1.54%
4	3.71%	8.00%	732	31	4.23%	660	33	5.00%	889	49	5.51%	5.48%	5.86%	5.45%	4.62%
5	8.01%	16.00%	726	44	6.06%	278	36	12.95%	464	52	11.21%	11.50%	12.01%	11.32%	9.51%
6	16.01%	28.00%	568	78	13.73%	254	47	18.50%	326	68	20.86%	20.76%	22.01%	21.17%	16.12%
7	28.01%	40.50%	333	90	27.03%	38	9	23.68%	50	17	34.00%	34.61%	34.26%	33.68%	25.36%
8	40.51%	61.00%	275	64	23.27%	412	259	62.86%	482	272	56.43%	53.45%	50.76%	49.71%	43.07%
9	61.01%	99.99%	151	70	46.36%	184	144	78.26%	376	279	74.20%	76.25%	80.50%	78.10%	62.31%
		Total	**3520**	**380**	**10.80%**	**2861**	**543**	**18.98%**	**4023**	**760**					

Source: the calculations of the author.

Table 11. The comparison of PD(4) for ratings

Rating	Burgt (2007) CAP curve technique			Tasche (2009) ROC curve technique			Forrest (2005) technique			Pluto, Tasche (2005) technique			Kiefer (2006) Bayes technique		
	PD_{2006}	PD_{2007}	PD(4)*	PD_{2006}	PD_{2007}	PD(4)*	PD_{2006}	PD_{2007}	PD(4)*	PD_{2006}	PD_{2007}	PD(4)*	PD_{2006}	PD_{2007}	PD(4)*
1 LDP	**0.28%**	**0.09%**	**0.19%**	**0.06%**	**0.002%**	**0.03%**	**0.85%**	**1.49%**	**1.17%**	**0.35%**	**1.07%**	**0.71%**	**0.60%**	**1.51%**	**1.06%**
2 LDP	**0.37%**	**0.26%**	**0.32%**	**0.18%**	**0.05%**	**0.12%**	**0.879%**	**1.78%**	**1.33%**	**0.40%**	**1.36%**	**0.88%**	**0.68%**	**1.89%**	**1.29%**
3 LDP	**0.61%**	**0.78%**	**0.70%**	**0.80%**	**0.73%**	**0.77%**	**0.88%**	**3.63%**	**2.26%**	**0.43%**	**2.91%**	**1.67%**	**0.44%**	**2.91%**	**1.68%**
4	4.23%	5.00%	4.62%	4.23%	5.00%	4.62%	4.23%	5.00%	4.62%	4.23%	5.00%	4.62%	4.23%	5.00%	4.62%
5	6.06%	12.95%	9.51%	6.06%	12.95%	9.51%	6.06%	12.95%	9.51%	6.06%	12.95%	9.51%	6.06%	12.95%	9.51%
6	13.73%	18.50%	16.12%	13.73%	18.50%	16.12%	13.73%	18.50%	16.12%	13.73%	18.50%	16.12%	13.73%	18.50%	16.12%
7 LDP	27.03%	**28.06%**	**27.54%**	27.03%	**34.63%**	**30.83%**	27.03%	**38.64%**	**32.83%**	27.03%	**17.31%**	**22.17%**	27.03%	**24.57%**	**25.80%**
8	23.27%	62.86%	43.07%	23.27%	62.86%	43.07%	23.27%	62.86%	43.07%	23.27%	62.86%	43.07%	23.27%	62.86%	43.07%
9	46.36%	78.26%	62.31%	46.36%	78.26%	62.31%	46.36%	78.26%	62.31%	46.36%	78.26%	62.31%	46.36%	78.26%	62.31%

Source: the calculations of the author. * PD(4) was calculated as an arithmetical average of PD_{2006} and PD_{2007}.

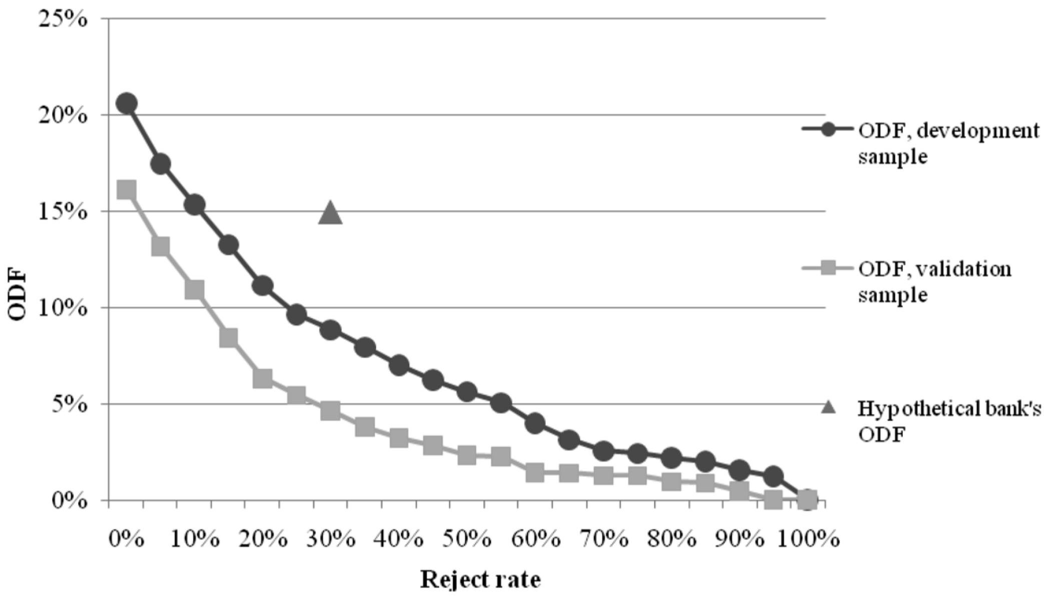

Figure 11. The portfolio ODF dependence upon the application reject rates
Source: the calculations of the author.

The portfolio ODF dependence upon the application reject rates. Portfolio ODF of the development sample companies was equal to 20.64%, however, if banks had not granted loans to the riskiest companies (to 10% of all companies) in 2005-2007, i. e. if the riskiest applications had been rejected (reject rate = 10%)[17], ODF would have decreased till 15.35%, etc. The triange in the Figure 11 depicts company portfolio ODF and the reject rate of the hypothetical bank, that would choose to apply this rating system. Let‘s say, last year this bank applied its own rating system and rejected 30% of all company applications, and its portfolio ODF of the last year was 15%. Before starting to apply the proposed rating system, the bank should determine a cut-off rating. The bank could choose a point below the triange, keeping the same reject rate level, but reducing ODF, or a point to the left from the triangle, keeping the same ODF, but reducing a reject rate.

From Figure 12 it is clear that if applications had been accepted only from the 5th rating, the reject rate in the development sample would have been 41.89%, and in the validation sample – 33.15%. If applications had been accepted from the 6th rating, the reject rate in the development sample would have been 26.41%, and in the validation sample – 22.12%. So, the mentioned bank could choose the 7th or the 6th rating as a cut-off rating, i. e. accept applications only from the 6th or the 5th rating. Of course, the bank could adjust the proposed rating system at its discretion determining other boundaries of

[17] For the simplicity purposes the assumption was made that in each year each company applied for a loan only one time, i. e. the rating system is applied on a loan (and not on a debtor) level.

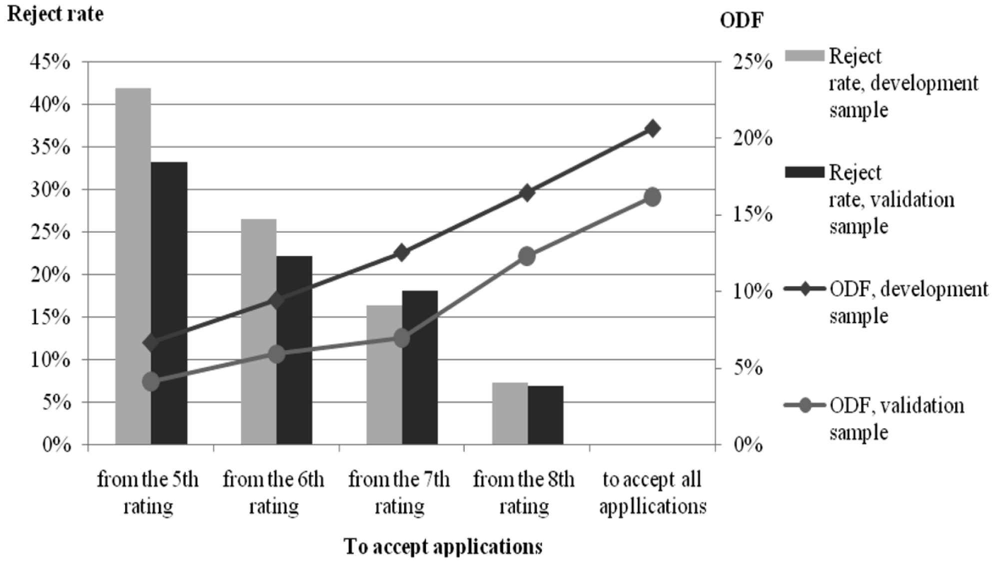

Figure 12. The cut-off rating choice
Source: the calculations of the author.

individual PDs or even changing the number of ratings. Then it could determine a cut-off rating taking into account its own individual needs. Besides, it should also perform the analysis presented in Figure 12 with its own data.

However, one should have in mind that developing the proposed rating system the external loan register data were used, so, they represent both accepted and rejected applications because a company, that did not get a loan at one bank, could apply to other bank and get it there. Meanwhile, a single bank performing the analysis with its own data would include only information about accepted applications and move a reject rate hypothetically upwards. Of course, this bank could also include rejected applications, having assigned them to "goods" or "bads", applying the reject inference techniques provided in Parts 1.1.1.7 and 3.3.3 of the dissertation.

The ratio of the change in cumulative actual "goods" to the change in cumulative actual "bads". As was mentioned in Part 1.1.1.1 of the dissertation, a rating, rejecting applications from which this ratio is from 5:1 to 3:1, is usually chosen as a cut-off rating.

From Table 12 it is clear that using the development sample data such a ratio is achieved choosing the 7th rating as a cut-off rating (i. e. accepting applications only from the 6th rating).

Meanwhile, using the validation sample data such a ratio is achieved choosing the 7th or the 8th rating as a cut-off rating (i. e. accepting applications only from the 6th or the 7th rating.

Table 12. The cut-off rating using the development sample

A	B	C	D	E	F=D/E
Cut-off rating	**Cummulative “goods“**	**Cummulative “bads“**	**Change in Cummulative “goods“**	**Change in Cummulative “bads“**	**Ratio of changes**
1	0	0	-	-	-
2	555	13	555	13	43
3	2058	53	1503	40	38
4	4026	108	1968	55	36
5	8262	311	4236	203	21
6	12364	884	4102	573	7
7	15196	1581	2832	697	4
8	16673	2392	1477	811	2
9	17663	3477	990	1085	1

Source: the calculations of the author.

The net present portfolio value. As was mentioned in Part 1.1.1.1 of the dissertation, a rating, rejecting applications from which a net present value (thereinafter – NPV) of a portfolio is the biggest, may be chosen as a cut-off rating. Applying this method more factors are taken into account than applying two previous methods: not only a number of actual “goods“ and “bads“ in a rating, but also EAD, LGD, a loan interest rate, a risk-free interest rate used to discount cash flows, a loan duration, a payment schedule and other factors. Because of that the conclusion could be made that this method suits for a cut-off rating choice more than the first two.

Let’s say, a bank would have decided not to grant a loan to any of the development sample companies. In such a case this bank would have suffered an alternative cost (i. e. it would not have got a certain income, if it had not granted loans to companies which would have become “good”) and would have got an alternative benefit (it would have avoided a certain cost, if it had not granted loans to the companies which would have become “bad”), however, it would not have suffered any actual cost and would not have got any actual benefit. As one can see from Figure 13, if a bank had not granted any loans, its alternative cost would have exceeded an alternative benefit, so, its portfolio NPV would have been negative. A bank should not choose the options giving negative portfolio NPV. It is the best for a bank to choose the option giving the biggest portfolio NPV. The biggest NPV of the development sample companies’ portfolio would be achieved choosing the 7th rating as a cut-off rating (i. e. accepting applications only from the 6th rating). Meanwhile, the

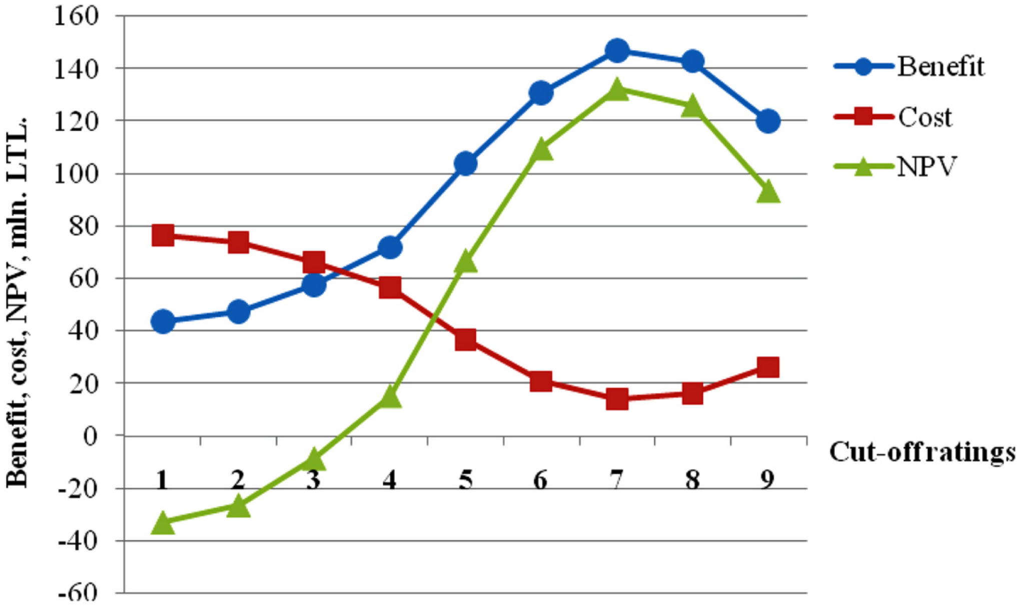

Figure 13. The NPV analysis (the development sample)

Source: the calculations of the auhor.

biggest NPV of the validation sample companies' portfolio would be achieved choosing the 8th rating as a cut-off rating (i. e. accepting applications only from the 7th rating).

The distributions of actual "good" and actual "bad" companies and ratings' ODF. From Figure 14 it is clear that in the development sample the "good" companies' share

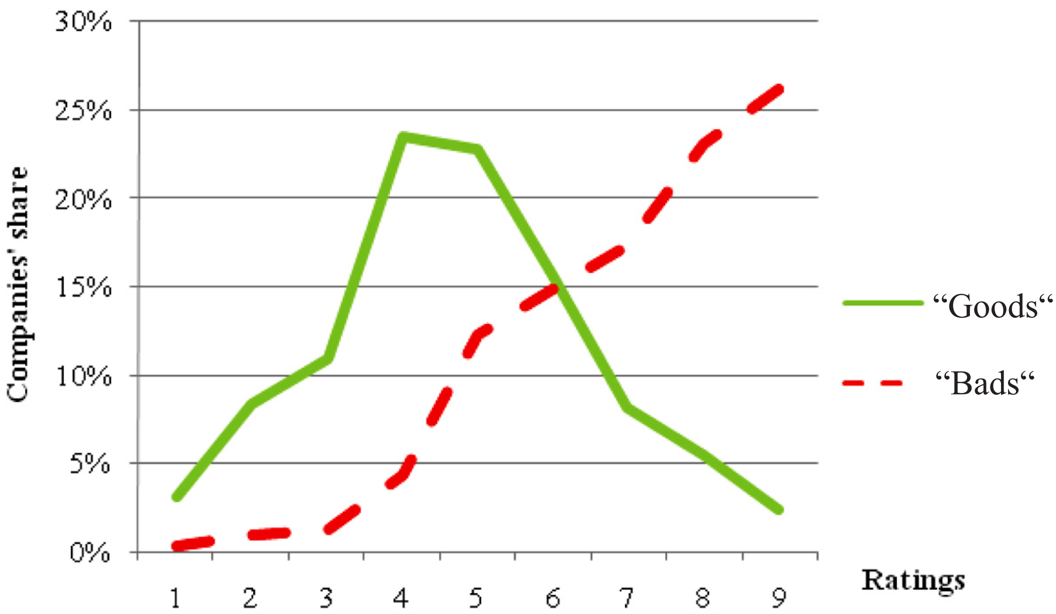

Figure 14. The distributions of "good" and "bad" companies in the development sample

Source: the calculations of the author.

(from all “good“ companies) in the 6th rating is almost the same as the “bad“ companies‘ share (from all “bad“ companies) in the 6th rating.

Till the 6th rating the “good“ companies‘ share was bigger than the “bad“ companies‘ share and from the 6th rating – lower. The same thing is in the validation sample.

From Table 9 it is clear that ratings’ ODF starts to increase significantly from the 7th rating.

So, having performed the analyses, it is clear that the 6th, the 7th or the 8th rating should be a cut-off rating. It was decided to treat the 8th rating as a “hard“ cut-off rating, i. e. it is being proposed to reject applications of companies assigned to the 8th and the 9th ratings right away. The 6th rating was chosen to be a “soft“ cut-off rating. This means that a bank should accept applications of companies assigned to ratings from the 1st to the 5th, meanwhile, applications of companies assigned to the 6th and the 7th ratings should be additionally assessed (e. g. a loan can be granted only applying additional credit risk mitigants and so on). The rating system should also have the 10th rating which would not be used while granting new loans, but companies, to which loans had already been granted earlier and that have already become “bad”, would be assigned to this rating (see Table 13).

Table 13. The rating system application granting loans

Rating	Lower PD boundary	Upper PD boundary	Risk description	Actions granting loans
1	0.01%	1.00%	Excellent condition	To accept an application
2	1.01%	2.20%	Very good condition	
3	2.21%	3.70%	Good condition	
4	3.71%	8.00%	Moderate risk	
5	8.01%	16.00%	Satisfactory risk	
6	16.01%	28.00%	Monitoring is needed	“Grey zone“: an additional assessment is needed
7	28.01%	40.50%	Higher than average risk	
8	40.51%	61.00%	High risk	To reject an application
9	61.01%	99.99%	The highest risk	
10	**100.00%**		**Actual “bads“**	Not applied

Source: compiled by the author.

This rating system is more suitable to assesss the companies, loans of which are assigned to a retail loan group at a bank, because a company assignment to ratings is based not on freely interpreted criteria, but on the input variables, that are defined in advance.

However, bank employees, taking into account additional information about *a company* (its position in a market, its management, stockholders, a risk of a whole company group to which a company belongs and so on), *a loan* (its amount, maturity, purpose and so on) and *collaterals,* could override company ratings determined by this rating system and, if needed, adjust them or a decision to grant/not to grant a loan. Then this rating system would suit for an assessment of companies, loans of which are not assigned to a retail loan group, particularly, taking into account that the ratings' number complies with the requirements of the Bank of Lithuania in such a case (i. e. at least seven ratings for "good" companies and one rating for "bad" companies).

Applying the proposed rating system a loan granting process should be fully or partially automatized. If an additional assessment (of the companies of the "grey zone" or those assigned to the other ratings) were performed in bank information technology systems using criteria defined in advance, then a loan granting process would be fully automatized and a company would get the answer, whether it would get a loan or not, right away. However, if an additional assessment (of the companies of the "grey zone" or those assigned to the other ratings) were performed by bank experts and (or) a higher bank body, then a loan granting process would be only partially automatized.

A bank might determine loan amount limits in accordance with a company rating, i. e. an amount exceeding a certain limit would not be granted to companies with a certain rating. However, a bank choosing to apply this rating system should perform the analysis described in this Part with its own data.

The proposed rating system may be applied not only as the main rating system granting loans, but also as an override tool of a rating determined by another rating system applied by a bank or as a benchmarking tool.

The rating system application in a pricing process. Loan interest rates have to be determined in accordance with a risk. For each company, taking into account its rating, a credit risk margin has to be added to an initial interest rate. In Figure 15 the credit risk margins calculated using the development sample data and several different LGDs (10%, 45% and 100%) are provided.

One could notice that the worse the company rating and the bigger the loss when a company becomes "bad", the bigger the credit risk margin that should be added to an initial interest rate.

An impact of LGD especially increases from the 7^{th} rating. Besides, even if it is being recommended applying the proposed rating system not to grant loans to companies assigned to the 8^{th} and the 9^{th} ratings, however, if a bank with big risk tolerance decided to grant loans even to such companies, very big credit risk margins would be added to initial interest rates (especially, when LGD is 100%).

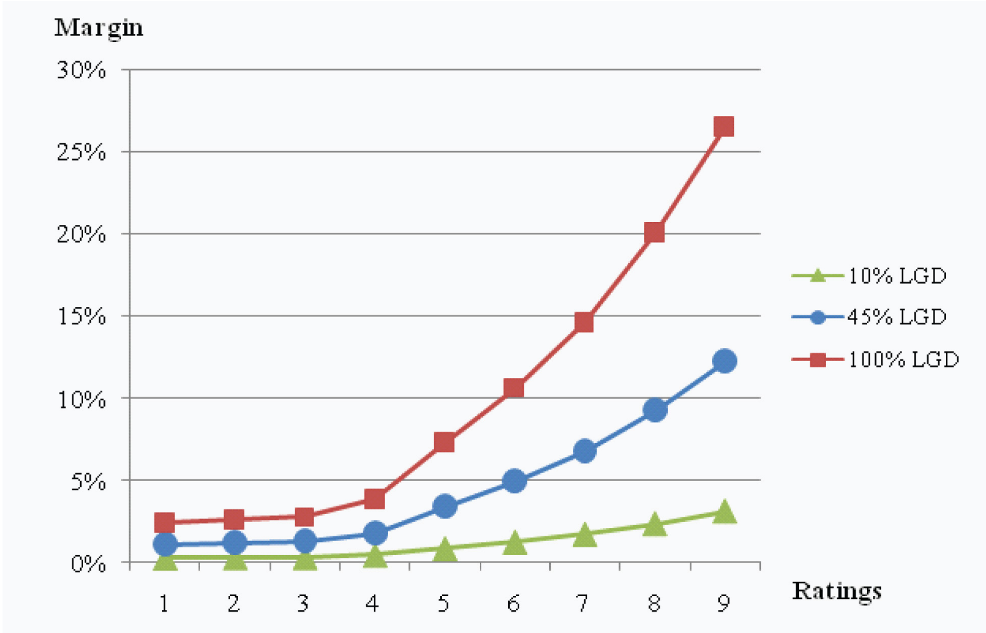

Figure 15. The credit risk margins (the development sample)

Source: the calculations of the author.

The rating system application in other bank activity spheres. This rating system may be applied for a capital requirements calculation at banks applying the IRB approach. It is more suitable for a calculation of credit risk capital requirements of the companies, that are assigned to a retail loan group in accordance with the requirements of the Bank of Lithuania. However, all banks, even those applying the standardized approach, could apply this rating system calculating internal capital requirements, rating PD(1) could be used applying internal portfolio risk assessment methods. Also, this rating system could be applied performing a stress testing. In Part 3.3.1 of the dissertation the exemplary calculations are provided.

The rating system may be applied not only as an application rating system, but also as a behavioural rating system. In such a case the 8^{th} and the 9^{th} ratings could be used. Though loans would not be granted to companies assigned to these ratings, however, companies of better ratings could get into them later, already having got a loan. A review frequency of companies‘ ratings should also be related to their ratings: behaviour of companies in worse ratings should be assessed more frequently, e. g. quarterly.

The rating system may also be applied calculating loan value adjustments (company ratings may be used calculating NPV), in a reporting system (it is being proposed to provide reports about a debtors' distribution and a stability index, ratings' ODF changes, a debtors' migration across ratings, a portfolio ODF dependence upon a reject rate, credit risk margins, stress testing results and so on), forming a bank's strategy (determining target debtors' markets, etc.). However, one should have in mind that this rating system is more point-in-time than through-the-cycle, so, in worsening macroeconomic conditions company ratings will deteriorate, and in improving macroeconomic conditions ratings will improve. That is why forming a long-term bank's activity strategy the other, through-the-cycle, rating system would suit more.

Not only the whole rating system, but also its composite parts may be being applied at banks: the logistic regression model of Lithuanian companies, the methods applied at each development stage of this model, input variables and so on (see Table 14). For example, a bank developing its own statistical company model might apply the rating scale consisting of nine ratings proposed in this dissertation. However, if it decided to construct its own rating scale, it might use the construction principles of an optimal rating scale. At that bank the logistic regression model of Lithuanian companies and the rating PDs provided in this dissertation might be being applied for benchmarking purposes.

The reject inference research. When a bank gets unsatisfactory validation results and decides to develop a new statistical scoring model, it has to decide, whether to include rejected applications data or not, and, if yes, in what way. So, the purpose of the research provided in this Part was, having analysed various reject inference techniques and the researches performed by other authors, to develop 34 statistical scoring models based on the data of Lithuanian companies in various ways including rejects or ignoring them at all, and compare the models' discriminatory power as well as the appropriateness of reject inference techniques.

The author of the dissertation applying one of the Shumway logistic regression models (including Zmijewski variables) rejected or accepted applications hypothetically. Two reject rates were determined: low (10.92%) and high (50%). The logistic regression model of Lithuanian companies described above was applied as a proxy model. Then 34 new logistic regression models were developed in various ways including rejects or ignoring them at all. Several reject inference techniques were applied: an augmentation, an assignment of all rejects to "bads", seven extrapolation modifications. The discriminatory power of the models was assessed applying four indicators (see Appendix 3).

The research results showed that rejecting 10.92 per cent of all applications the model developed with only accepts' data discriminated better than any of the models developed in one or another way including rejects. When a reject rate was 50 per cent, according to two indicators the discriminatory power of even seven models developed including rejects was bigger than that of the model developed with only the accepts' data. The higher the reject rate, the smaller the discriminatory power of models applying the same reject inference technique. There are more missing values, so, any of the techniques applicable to recover this information is less reliable.

Table 14. The possibilities of the rating system application at Lithuanian banks

		Rating scale of Lithuanian companies	Rating scale construction principles	Logistic regression model of Lithuanian companies	Input variables	Reject inference techniques	Other methods applied at the separate model development stages	Rating PDs	Rating PDs of low default portfolios	Rating PD calculation methods	Rating PD calculation methods of low default portfolios	Ex-post validation scheme
Company models	At a bank developing its own statistical company model	X	X	X	X	X	X	X	X	X	X	X
	At a bank applying its own statistical company model	X	X	X	X	–	–	X	X	X	X	X
	At a bank developing its own expert company model	X	X	X	X	–	–	X	X	X	X	X
	At a bank applying its own expert company model	X	X	X	X	–	–	X	X	X	X	X
Private person models	At a bank developing its own statistical private person model	–	X	–	–	X	X	–	–	X	X	X
	At a bank applying its own statistical private person model	–	X	–	–	–	–	–	–	X	X	X
	At a bank developing its own expert private person model	–	X	–	–	–	–	–	–	X	X	X
	At a bank applying its own expert private person model	–	X	–	–	–	–	–	–	X	X	X

Source: compiled by the author.

Reject rate

Rejects' data quality		<=3%	>3%
	Good	To assign all rejects to „bads"	Extrapolation
	Bad	Not to include rejects at all	Augmentation

Figure 16. The reject inference technique choice

Source: compiled by the author.

Having analysed the reject inference techniques and the discriminatory power of the developed models, the scheme of a reject inference technique choice was prepared (see Figure 16).

When a reject rate exceeds 3 per cent[18] and rejects' data quality is bad (e. g. bank's employees did not input a part of rejects' information into bank information technology systems), an augmentation is being recommended. Besides, an augmentation will provide a benefit particularly when there is a large number of accepts judged by a proxy model to be worthy of a rejection and these cases have a distinctly poor performance.

When a reject rate exceeds 3 per cent and rejects' data quality is good, a bank should choose one of the extrapolation techniques, let's say, a stratified-fuzzy or a stratified-random parcelling as the discriminatory power of the models applyging these techniques is the biggest. The higher the reject rate, the bigger the appropriateness of stratified parcelling techniques. So, when a reject rate is high, it is better to parcel rejects on a stratified basis applying a proxy model. Meanwhile, when a reject rate is 10.92% or lower, a parceling for the entire reject region, let's say, a hard cut-off technique, may also allow achieving an excellent discrimination. Though the research results showed that, rejecting 10.92 per

[18] An applications' reject rate depends on several factors: on a loan type, a target debtors' population, bank's risk tolerance, economic conditions, debtors' expectations. Reject rates of different loan types and at different banks differ very much (in the scoring literature both very low (2.2%, 3%) and very high (83%) reject rates are mentioned; the reject rate of 30% is most often mentioned (Hand, Henley 1997; Siddiqi 2006; Puri and other 2011)). When a reject rate is very low, the assumption that all rejects would have become "bad" can be made with some confidence. However, when a reject rate is not very low (i. e. exceeds 3%), based on information gathered via external loan registers' files and also on random override studies conducted by issuers over the years, one could argue that a certain portion of rejects would have become "good" (Siddiqi 2006).

cent of all applications, the model developed with only accepts‘ data discriminated better than any of the models developed in one or another way including rejects, the difference in discriminatory power indicators‘ values was insignificant, so, it is being recommended to apply reject inference techniques.

There are several situations when it is being recommended to ignore rejects at all while developing a new statistical scoring model. One case may be when going forward, plans are to increase a reject rate of a population in a significant fashion. While estimations on a rejected population may be weak, these applications will still most likely continue to be rejected. The other situation is when a current strategy and a decision-making process appear random in nature. If a current model doesn‘t have discriminatory power, it may be assumed that accepts are close enough to a random sample. However, in this case the random supplementation technique[19] should also be considered. When a reject rate does not exceed 3 per cent, it is plausible that all rejects would have become “bad“. So, when a reject rate does not exceed 3 per cent and rejects‘ data quality is bad, it is also being recommended to ignore rejects at all. However, if rejects‘ data quality satisfies a bank, they all should be assigned to “bads“.

It is being recommended to include rejects not only developing a new model, but also validating a newly developed model or an old model that is already applied.

[19] Some high-risk applications, which would otherwise be rejected, are accepted. Then their performance is known, not inferred, and can be used directly developing a new model (see part 1.1.1.7 of the dissertation).

CONCLUSIONS AND PROPOSALS

1. Applying a rating system based on a statistical scoring model at a bank there are two processes: debtors are being assigned to ratings according to a model result and rating PDs are being calculated. Developing a statistical scoring model there are several stages: at first, a project feasibility is being analysed, then a “bad“ debtor and an outcome period are being defined, a population is being segmented and a statistical technique is being chosen, a sample is being constructed, input variables are being analysed and coefficients are being calculated, finally, an ex-ante validation is being performed. Having analysed the statistical company scoring models developed by other authors it is clear that defining a “bad“ company a bankruptcy indication is chosen almost so often as a default indication, and a financial distress (an insolvency) indication is chosen more seldom. Developing private person scoring models a “bad“ debtor is being mostly defined as a debtor, that is past due more than 90 calendar days. However, various other definitions are possible – a debtor, who is delaying to pay two payments in succession, past due six months, etc. Authors developing statistical scoring models mostly apply a logistic regression, a popularity of a dicriminant analysis is significanlty smaller. Though in recent years a number of researches related to machine learning and programming methods of a new generation (i. e. artificial neural networks and supporting vector machines) has increased, however, these methods are still being in a research phase, their suitability to assess a debtor‘s credit risk has not been examined thoroughly, there are no standartized computer packages. Besides, the research results showed that, though in most cases model sensitivity applying these methods was bigger than that applying pure statistical methods, however, a percentage of incorrectly predicted debtors was similar.
2. It is difficult for banks to choose an appropriate reject inference technique. It is expensive to buy information from external loan registers. Besides, different definitions of a “bad“ loan may be being applied at banks, different crediting conditions may be being determined, etc. When a bank decides to use internal data about other loans granted to a rejected applicant, problems also arise: a risk of different loan types as well as dates of a loan granting and an application rejection may differ significantly, a part of rejected applicants may not have loans at the same bank. Because of these reasons the reject inference techniques, when rejects are distributed into groups of “goods“ and “bads“ hypothetically, are so popular. Augmentation and extrapolation techniques are perhaps mostly applied by banks. However, the research results of various authors showed that models developed applying an augmentation were

usually less valid than those developed with only accepts‘ data. Meanwhile, models developed applying an extrapolation were usually more valid than those developed with only accepts‘ data.

3. An optimal number of ratings has to be being determined at a bank and debtors have to be being assigned to these ratings according to their individual PDs or creditworthiness indicators. Scientific articles provide for a number of ratings varying from 5 to 16. A bank constructing a rating scale should take into account several factors: intervals of individual PDs or creditworthiness indicators, monotonicity, a debtors’ concentration and a distribution, a discrimination.
4. Rating PD may be being calculated both from individual PDs and from one-year actual “bad“ rates. However, it is possible to calculate rating PDs from individual PDs only applying a statistical scoring model allowing to calculate individual PDs (for example, a logistic, a cloglog regression or survival analysis models).
5. Having analysed the validation methods applied by other authors, one can see that the most widely applied validation methods are CAP and ROC curves and a correct classification analysis, however, entropy-based methods are also becoming more popular, especially the information value method, also the Brier score method. The most popular methods of a PD calibration accuracy assessment are Hosmer-Lemeshow, binomial, normal tests, a traffic light approach. Many of these methods were being applied at Lithuanian banks, they are also recommended by the Bank of Lithuania.
6. The results of the survey performed by the author showed that:
    - at Lithuanian banks statistical scoring models of retail applications were not being widely applied, only four banks were applying them. Statistical scoring models of Lithuanian banks were developed applying a logistic regression and a dicriminant analysis. All the banks, that were not applying statistical models, indicated that a past data collection period would have been too short to develop a statistical model. Mixed models were being applied only at two banks, these banks were also applying statistical models. Expert models were being most widely applied at banks, only two banks from those applying these models were planning to develop new statistical models in the future. Company application models were mostly developed on a debtor, an not on a loan, level, besides, common models for all company retail loan types were developed. Private person application models were developed both on a debtor and on a loan level, banks distributed private person loans into groups by types and developed separate models for these types.

- a need of data, that may be bought from external loan registers, has increased. Though banks were hindered by a big cost of a purchase of such information, even three banks were planning to adjust their applied models including external loan registers data. It is plausible that even those two banks, that were planning to develop new statistical scoring models, will buy and regularly use external loan registers information. Lithuanian banks developing statistical, mixed and expert models used almost the same input variables, that were mostly used in models provided by other authors. However, Lithuanian banks did not include the variables of an assets and revenue logarithmic transformation into models, and the ratio of net profit (loss) to sales revenue, that was usually included by them, was not one of the mostly used in models developed by other authors. A variety of expert models‘ input variables was bigger than that of statistical models‘ input variables because developing expert models it is not necessary to have gathered past data.
- when a bank had a parent bank in a foreign country, statistical models were sometimes developed with only local data, and, when a bank lacked local data, common models with whole group‘s data were developed. Input variables and their number as well as their values‘ groups and coefficients of models of the same loan type developed for different countries differed. So, models developed using whole group‘s data do not represent Lithuanian banks‘ debtors. The conclusion could be made that, when a bank lacks local data to develop its own statistical scoring model, it is better for it to buy an external model developed using Lithuanian data or debtors‘ ratings determined by an external model than to develop a model using parent bank group‘s data. The other way out could be usage of data of several banks operating in Lithuania or a mixed model development, for example, a bank could develop a statistical model with only those input variables about which it has enough reliable information and use a statistical model result as a separate input variable of an expert model comprising more input variables about which a bank lacks reliable information.

7. Taking into account the fact that statistical company models were not spread at Lithuanian banks, the rating system of Lithuanian companies based on the logistic regression model was developed. The model consists of 19 input variables comprising varied company information: a financial condition, an external payment history, age, a size, a county, an economic sector, records of negative information about a company at the external loans register, claims of debt collection companies. The optimal rating scale consisting of 10 ratings (nine ratings for “good“ companies and one rating for

the companies, that have already become “bad”) was constructed, the “hard“ cut-off and the “soft“ cut-off ratings were determined. Rating PDs were calculated applying four methods. However, for the further analysis only one of them was chosen – the method when rating PD is calculated as an arithmetical average of individual PDs of companies assessed with that rating. These PDs were accurate applying all the methods of a PD calibration accuracy assessment.

8. The ex-ante validation of the rating system was performed applying the most popular methods recommended by many banking supervision institutions, the favourable results were received. Also, the scheme of an ex-post validation, already having started to apply the proposed rating system at a bank, was constructed proposing tolerated limits of validation indicators‘ values and bank actions when these limits are broken.
9. The proposed rating system of Lithuanian companies may be being applied for an assessment of companies of various economic sectors and companies taking various loan types (investment loans, working capital financing loans an so on) and in various bank proceses (granting loans, calculating credit risk margins and credit risk capital requirements, performing a stress testing and in many of the other proceses). The dissertation provides the practical examples of such an application. This rating system complies with the legal acts of European Union and Lithuania prepared in accordance with the New Basel Capital Accord that is why it can be being applied at banks, that are applying or are planning to apply the IRB approach. However, before starting to apply it, a bank should validate it with its own data. Besides, this rating system may be being applied not only at banks, but also at other companies, that grant loans or assess a company risk, e. g. at consumer credit and small credit companies assessing a risk of debtors‘ employers, at insurance companies providing guaranty services, etc. It may also be being applied by companies, that want to assess their own creditworthiness. Not only the whole rating system, but also its composite parts may be being applied at Lithuanian banks: the logistic regression model of Lithuanian companies, the methods applied at each development stage of this model, input variables and so on. The analysis provided in this dissertation may also be usefull developing and applying expert company scoring models and private person scoring models.
10. Till now Lithuanian authors have not analysed reject inference techniques. That is why the reject inference research was performed. Using the Lithuanian companies data, 34 logistic regression models were developed in various ways including rejects or ignoring them at all and the models‘ discriminatory power was compared. The

research results showed that, when a reject rate was 10.92%, the models, developed in any of the ways including rejects, differentiated applications worse than the model developed with only the accepts' data. When a reject rate was 50%, according to two indicators the discriminatory power of even seven models, developed including rejects, was bigger than that of the model developed with only the accepts' data. According to the results of this research, the scheme of a reject inference technique choice, that may be adapted at banks, was prepared.

11. It is being recommended for Lithuanian banks developing or planning to develop statistical scoring models:
    - to develop separate models for companies, loans of which are assigned to a retail loan group. Assigning company loans to a retail loan group not only a total loan amount, but also additional criteria should be being used. Determining these additional criteria banks could use net sales revenue, assets and employees‘ number indicators, that are being used deciding, whether a company is allowed to present condensed financial reports. The calculations‘ results showed that at banks, applying the IRB approach, capital requirements are very sensitive to changes of criteria applied assigning loans to a retail loan group, that is why these criteria should be being chosen carefully at banks, bigger attention to that may also be being paid by the Bank of Lithuania.
    - defining a “bad“ debtor to use a default indication and an outcome period of one year, to choose a logistic regression because it has many advantages if compared to the other statistical techniques, to develop and apply for different purposes both point-in-time and through-the-cycle models. For a new model to be more through-the-cycle, a bank should gather dynamic rows of input variable values, that are sensitive to an economic cycle, for at least 14 years and deduct periodically changing cyclicality (and seasonality, if data are quarterly) components leaving only a trend and random deviations.
    - when there is enough data, to develop separate private person models for different loan types on a loan level, determining as many as possible loan types, because the scientific articles‘ analysis and the Lithuanian banks‘ survey results showed a differring risk in different private person loan types, i. e. to develop separate mortgage, consumer, credit cards, leasing, etc. models. This would allow including specific input variables, statistical models would be more accurate and representative. However, at first, a bank should analyse its internal data and determine, whether a debtors‘ risk of different loan types differs or not. Meanwhile, it is better to develop a company model common for all loan

types. In any case, a bank should check various alternatives and choose the one giving the best ex-ante validation results. Besides, a bank should also assess a time, information technology, employees‘ wage cost.

- o to develop separate application and behavioural models. A separate application model might also be developed for existing bank debtors applying for a new loan. A bank should gradually go from an application score to a behavioural score, i. e. a weight given to an application score should decrease as time passes, and a weight given to a behavioural score – increase.
- o to include not only ratios, but also absolute financial variables, their logarithmic transformations. Besides, a bank should gather information about expert models variables‘ values in the past, it might be used developing new statistical models. Often internal payment history variables are the most important to a credit risk assessment, that is why it is being recommended for banks to use the payment history variables provided in this dissertation. Internal payment history variables might be being modelled separately, and a result of such modeling might be being used as a separate input variable of a main model. Banks might model internal payment history variables separately not only applying statistical, but also expert or mixed models.
- o to choose an appropriate reject inference technique. In certain cases it is enough to use only accepts‘ data developing a statistical scoring model. Rejects may be ignored at all when a bank foresees to apply a more conservative credit strategy, i. e. when there are plans to increase a reject rate of a population in a significant fashion or a current strategy and a decision-making process appear random in nature (if a current model doesn‘t have discriminatory power, it may be assumed that accepts are close enough to a random sample). It is not expedient to include rejects and then, when a reject rate does not exceed 3% and rejects' data quality is bad. When rejects' data quality is bad, but a reject rate is bigger than 3%, it is being proposed to apply an augmentation. When rejects' data quality is good, it is proposed, taking into account a reject rate, to assign all rejects to "bads" or apply one of the extrapolation or clusterisation techniques. Rejects should be being included not only developing a new model, but also validating a new or an earlier developed model.
- o to construct an optimal rating scale applying the principles provided in this dissertation (debtors of the same rating should not be too heterogeneous, however, there should be enough debtors to calculate rating PD and to validate it, etc.). All banks, even those not applying the IRB approach, should determine,

what debtors are being treated as defaulted, and dedicate at least one rating for such debtors, besides, all banks should calculate rating PDs.

- when there are very few or no actual “bad“ debtors, to apply special rating PD calculation methods of low default portfolios. The calculation results showed that the Pluto, Tasche (2005) technique without a correlation could be easily implemented at banks. However, if an ordinal ranking of debtors is incorrect, this technique doesn’t ensure monotonicity of PDs in low default portfolios. The same problem exists in the Kiefer (2006) technique. The Forrest (2005) technique without a correlation ensures monotonicity and conservatism of PDs, however, it requires programming skills, otherwise, an iterative recalculation of PDs will be very time-consuming. So, when bank employees (or independent external researchers) do not have programming skills, it is better to choose the Pluto, Tasche (2005) technique without a correlation and, when PDs are non-monotonic, to smooth their values exponentially.
- to choose tolerated limits of validation indicators‘ values received performing ex-ante and ex-post validation, after a breach of which a rating system would be adjusted. It is being recommended to apply methods comprising all the validation spheres, even then, when expert, mixed or statistical models not allowing to calculate individual PDs are being applied and the IRB approach is not being applied. Gathered validation results, for example, information about “bad“ debtors, might be used developing new statistical scoring models.
- to apply scoring models not only granting loans, calculating credit risk margins or loan value adjustments, but also in other initial proceses, for example, performing a loan collection, distributing capital, performing a stress testing and so on. This is especially actual for banks applying or planning to apply the IRB approach.

12. The Bank of Lithuania and the Associacion of Lithuanian banks might help banks to develop statistical scoring models. Developing such models it would be possible to use the loan risk data base of the Bank of Lithuania. Banks, that have their own internal models, might apply external models for benchmarking. External models would be usefull for the Bank of Lithuania itself: they would allow to compare a debtors‘ risk of differrent banks, assess a debtors‘ risk of a conrete bank and that of the whole banking sector. Besides, when banks choose a rating PD calculation method of low default porftolios themselves, at different banks PDs calculated for ratings of the same loan type with the same number of “good“ and “bad“ debtors may differ significantly, i. e. banks may choose not only different methods, but also

different parameters applying the same method (confidence levels, a correlation and so on). The Bank of Lithuania or the Associacion of Lithuanian banks might develop a single common rating PD calculation method of low default portfolios and prepare comparative PD tables, observing which banks would adjust their calculated rating PDs of low default portfolios.

13. The possible further directions of researches are the following:
    - Information technology specialists should construct standardised computer packages in order to develop statistical scoring models of a new generation.
    - Having longer period's data of Lithuanian companies, it would be possible to develop survival analysis models.
    - It's possible to develop statistical scoring models of Lithuanian private persons for different loan types.
    - Having needed data, it's possible to examine other special methods of the PD calculation for LDP portfolios.

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APPENDIXES

Appendix 1

THE INDIVIDUAL DISCRIMINATORY POWER OF INPUT VARIABLES

No.*	Input variable**	Information value	Predictiveness	Chosen in the the 2nd cycle (Yes/No)	Input variables in the final model (Yes/No)
X26	**Total assets / Equity**	**0.528**	**Strong**	**Yes**	**Yes**
X16	Equity / Amounts payable and liabilities	0.514	Strong	Yes	No
X17	Profit (loss) before tax / Amounts payable and liabilities	0.505	Strong	Yes	No
X35	Net profit (loss) / Amounts payable and liabilities	0.492	Strong	Yes	No
X12	Profit (loss) before tax / Total assets	0.443	Strong	Yes	No
X11	**Net profit (loss) / Total assets**	**0.429**	**Strong**	**Yes**	**Yes**
X18	**Current assets / Amounts payable and liabilities**	**0.428**	**Strong**	**Yes**	**Yes**
X32	Profit (loss) before tax / Current amount payable and liabilities	0.426	Strong	Yes	No
X36	Profit (loss) from ordinary activities / Amounts payable and liabilities	0.426	Strong	Yes	No
X31	Net profit (loss) / Current amount payable and liabilities	0.417	Strong	Yes	No
X19	Amounts payable and liabilities / Total assets	0.416	Strong	Yes	No
X47	**Average delay duration during the last year (in days)**	**0.409**	**Strong**	**Yes**	**Yes**
X44	Total weighted delay duration during the last year (in days) Comments: As the T_0 reference date was allways the last day of the year (December 31), each delayed payment was weighted going backwards from the end of the respective year. A delay duration of each month is multiplied by the special weight, i. e. if a delay begins in December, the weight is 1 (*12/12*), if in November, the weight is 0,917 (*11/12*), if in October, the weight is 0,833 (*10/12*), etc.	0.384	Strong	Yes	No
X37	Gross profit (loss) / Amounts payable and liabilities	0.370	Strong	Yes	No
X46	**Total number of delayed payments during the last year**	**0.355**	**Strong**	**Yes**	**Yes**
X25	**Cash and cash equivalents / Current amount payable and liabilities**	**0.344**	**Strong**	**Yes**	**Yes**

X10	**Profit (loss) before tax / Sales revenue**	**0.339**	**Strong**	**Yes**	**Yes**
X28	Profit (loss) from ordinary activities / Total assets	0.338	Strong	Yes	No
X20	**Ln (Net profit)**	**0.335**	**Strong**	**Yes**	**Yes**
X23	Net working capital / Amounts payable and liabilities	0.330	Strong	Yes	No
X41	Ln (Profit before tax)	0.327	Strong	Yes	No
X45	Weighted number of delayed payments during the last year Comments: As the T_0 reference date was allways the last day of the year (December 31), each delayed payment was weighted going backwards from the end of the respective year. A delay duration of each month was multiplied by the special weight.	0.322	Strong	Yes	No
X9	Net profit (loss)/Sales revenue	0.316	Strong	Yes	No
X48	Weighted average delay duration during the last year Comments: As the T_0 reference date was allways the last day of the year (December 31), each delayed payment was weighted going backwards from the end of the respective year. A delay duration of each month was multiplied by the special weight.	0.316	Strong	Yes	No
X14	Retained earnings (losses) / Total assets	0.315	Strong	Yes	No
X33	Profit (loss) from ordinary activities / Current amount payable and liabilities	0.300	Strong	Yes	No
X34	Gross profit (loss) / Current amount payable and liabilities	0.296	Medium	Yes	No
X49	Cash and cash equivalents / Total assets	0.259	Medium	No	-
X22	Ln (Retained earnings)	0.227	Medium	Yes	No
X15	Retained earnings (losses) / Sales revenue	0.209	Medium	Yes	No
X50	Ln (Cash and cash equivalents)	0.209	Medium	No	-
X39	(Current assets – Inventories, prepayments and contracts in progress) / Current amount payable and liabilities	0.206	Medium	Yes	No
X27	Profit (loss) from ordinary activities / Sales revenue	0.203	Medium	Yes	No
X29	**Gross profit (loss) / Total assets**	**0.202**	**Medium**	**Yes**	**Yes**
X40	Ln (Profit from ordinary activities)	0.195	Medium	Yes	No
X21	Ln (Equity)	0.185	Medium	Yes	No
X13	Net working capital / Total assets	0.180	Medium	Yes	No
X3	**Economic sector**	**0.178**	**Medium**	**Yes**	**Yes**

X24	Current assets / Current amount payable and liabilities	0.170	Medium	Yes	No
X4	**Company age (in years)**	**0.154**	**Medium**	**Yes**	**Yes**
X6	**There‘re/there‘re no records from debt collection companies about claims to a company during the last year**	**0.146**	**Medium**	**Yes**	**Yes**
X39	**Current amount payable and liabilities / Total assets**	**0.130**	**Medium**	**Yes**	**Yes**
X30	Sales revenue / Total assets	0.115	Medium	Yes	No
X42	**Ln (Non-current amounts payable and liabilities)**	**0.102**	**Medium**	**Yes**	**Yes**
X5	**There‘re/there‘re no records of negative information about a company during the last year in the external loan register**	**0.101**	**Medium**	**Yes**	**Yes**
X2	**County**	**0.098**	**Weak**	**Yes**	**Yes**
X1	**Annual turnover (in th. LTL)**	**0.096**	**Weak**	**Yes**	**Yes**
X7	**Number of employees**	**0.091**	**Weak**	**Yes**	**Yes**
X43	**Sales revenue / Current assets**	**0.089**	**Weak**	**Yes**	**Yes**
X51	Gross profit (loss) / Sales revenue	0.085	Weak	No	-
X8	There‘re/there‘re no records about unreliability of company managers during the last year	0.074	Weak	Yes	No
X52	Ln (Amounts payable and liabilities)	0.071	Weak	No	-
X53	Ln (Amounts receivable within one year)	0.058	Weak	No	-
X54	Legal form	0.011	Unpredictive	No	-
X55	Company at least once has changed a name	0.010	Unpredictive	No	-
X56	Company at least once has changed a name from 2000	0.009	Unpredictive	No	-
X57	Company at least once has changed an adress	0.002	Unpredictive	No	-

Source: compiled by the author. *Input variables are marked according to the row determined in the first cycle.**Input variables are sorted by their discriminatory power in a descending order.

Appendix 2

THE INPUT VARIABLES OF THE LOGISTIC REGRESSION MODEL

Notation in the regression equation*	Input variable	WOE	Inclusion into the regression equation
	Anuall turnover (in th. LTL)		
X1	*(0-10]*	0.7423	**Step 12**
	(10-100]	0.6225	
	(100-200]	-0.3791	
	(200-1000]	-0.2636	
	(1000-2000]	-0.1054	
	(2000-7000]	-0.0756	
	(7000-10 000]	-0.0633	
	(10 000-20 000]	0.4853	
	(20 000-100 000]	0.6751	
	>100 000 + missings	2.1215	
	County		
X2	Alytus	-0.0576	**Step 5**
	Kaunas	-0.0320	
	Klaipėda	0.1220	
	Marijampolė	-0.7097	
	Panevėžys + missings	-0.7456	
	Šiauliai	-0.1143	
	Tauragė	0.1617	
	Telšiai	-0.0036	
	Utena	-0.4101	
	Vilnius	0.2633	

Economic sector (according NACE 2)			
X3	Agriculture, forestry and fishing (Section A)	0.0755	**Step 3**
	Manufacturing industry, mining, quarrying and other industry (Sections B, C, D, E)	-0.3917	
	Construction (Section F)	-0.4294	
	Wholesale and retail trade, transportation and storage, accommodation and food service activities (Sections G, H, I)	0.2072	
	Information and communications (Section J)	0.4560	
	Real estate operations (Section L)	0.0533	
	Professional, scientific and technical activity, administration and services (Sections M, N)	1.1056	
	Public administration and defence, education, human health services and social work activities (Section O, P, Q)	1.2143	
	Finance and insurance activity and other services (Section R,S,T,U and K)	0.7455	
Company age (in years)			
X4	<=1	0.3561	**Step 9**
	(1-2]	-0.3124	
	(2-3]	-0.4151	
	(3- 4]	-0.6345	
	(4-5]	-0.4952	
	(5-6]	-0.2218	
	(6-7]	-0.0585	
	(7-9]	0.2315	
	(9-10]	0.2647	
	>10	0.4076	
There‘re / there‘re no records of negative information about a company during the last year in the external loan register**			
X5	Yes	-2.4501	**Step 14**
	No	0.0414	

There‘re / there‘re no records from debt collection companies about claims to a company during the last year* **			
X6	Yes	-2.4520	**Step 6**
	No	0.0602	
Number of employees			
X7	<=2	0.5735	**Step 7**
	[3-29]	-0.0567	
	[30-39]	-0.5763	
	[40-69]	-0.0435	
	[70-99]	0.2521	
	[100-149]	0.3587	
	>=150 + missings	0.8388	
Profit (loss) before tax / Sales revenue			
X8	< - 15.70%	-0.8263	**Step 20**
	-15.69% – 1.24%	-0.5411	
	1.25% – 2.82%	0.5197	
	>2.82%	0.5493	
	Missings	0.7843	
Net profit (loss) / Total assets			
X9	< -16.70%	-0.8984	**Step 17**
	- 16.69% – 1.43%	-0.4812	
	1.44% – 3.34%	-0.1337	
	3.35% - 9.96%	0.4645	
	9.97% - 15.52%	0.8167	
	>15.52%	1.0536	
	Missings	-0.6035	

Current assets / Amounts payable and liabilities			
X10	<0.2554	-0.7301	**Step 18**
	0.2555 – 0.4460	-0.5630	
	0.4461 – 0.9471	-0.2989	
	0.9472 – 1.0741	-0.1627	
	1.0742 – 1.2633	0.2489	
	1.2634 – 1.6287	0.9182	
	1.6288 – 2.6630	1.1514	
	>2.6630	1.5442	
	Missings	0.1358	
Ln (Net profit)			
X11	<8.8968	-0.3980	**Step 4**
	8.8969 – 9.8999	-0.3597	
	9.9000 – 10.5798	-0.0986	
	10.5799 – 11.6622	0.2004	
	11.6623 – 12.2063	0.7272	
	12.2064 – 12.7400	0.8123	
	>12.7400	0.8983	
	Missings	-0.5742	
Cash and cash equivalents / Current amount payable and liabilities			
X12	<0.0055	-1.0490	**Step 8**
	0.0056 – 0.0190	-0.5456	
	0.0191 – 0.0374	-0.4332	
	0.0375 – 0.1122	0.1621	
	0.1123 – 0.1807	0.2429	
	0.1808 – 1.108	0.5672	
	>1.108	0.8585	
	Missings	-0.5417	

Total assets / Equity			
X13	<1.2847 + missings	1.7303	**Step 2**
	1.2848 – 1.9445	0.8725	
	1.9446 – 2.9413	0.5192	
	2.9414 – 3.8115	0.4375	
	3.8116 – 7.6253	-0.2744	
	7.6254 – 15.3320	-0.4712	
	>15.3320 + negative values	-0.8341	
Gross profit (loss) / Total assets			
X14	<5.028%	-0.7371	**Step 13**
	5.029% – 22.142%	-0.5062	
	22.143% – 30.901%	-0.0100	
	>30.901%	0.3168	
	Missings	0.9766	
Current amount payable and liabilities / Total assets			
X15	<11.2460%	0.7322	**Step 21**
	11.2461% - 20.4192%	0.3893	
	20.4193% - 28.4675%	0.1006	
	28.4676% - 63.8380%	0.0566	
	63.8381% - 76.5530%	-0.0404	
	76.5531% - 93.2751%	-0.2669	
	>93.2751%	-0.7336	
	Missings	-0.1932	

Ln (Non-current amounts payable and liabilities)			
X16	<10.3983	0.6535	**Step 11**
	10.3984 – 11.1692	0.2362	
	11.1693 – 11.7500	-0.1410	
	11.7501 – 13.3798	-0.2158	
	13.3799 – 13.9489	-0.2784	
	13.9490 – 14.6272	-0.3210	
	>14.6273	-0.3432	
	Missings	0.3342	
Sales revenue / Current assets			
X17	<0.8229	-0.5463	**Step 16**
	0.8230 – 1.3839	-0.4201	
	1.3840 – 2.2739	-0.1449	
	>2.2740 + missings	0.2381	
Total number of delayed payments during the last year****			
X18	There were no delayed payments during the last year	0.3285	**Step 15**
	1 delayed payment	-0.5268	
	2 delayed payments	-0.8548	
	3-4 delayed payments	-1.0295	
	5-8 delayed payments	-1.4815	
	9-14 delayed payments	-1.6886	
	>=15 delayed payments	-1.7467	

Average delay duration during the last year, in days****			
X19	There were no delayed payments during the last year	0.3285	**Step 1**
	<6.14	0.2679	
	6.15 – 8.50	-0.3468	
	8.51 – 12.06	-0.4916	
	12.07 –14.78	-0.5005	
	14.79 – 17.87	-1.3155	
	>17.87	-1.5708	

Source: the calculations of the author. * Input variables are marked according to the row determined in the third cycle. ** All the negative facts about a company, that are registered at JSC "Creditinfo Lietuva", e. g. negative media information, etc. *** Only the records registered at JSC "Creditinfo Lietuva" were used. **** Delayed payments to banks, leasing, telecomunication, public utility companies and other companies registered at JSC "Creditinfo Lietuva".

Appendix 3

THE REJECT INFERENCE RESEARCH RESULTS

Table 1. The models' discriminatory power (the reject rate is 10.92%)*

	AR	**AUC**	**IV**	**Brier score**
All data (proxy model)	**72.23%**	**86.11%**	**2.460**	**0.0922**
Only accepts' data	**69.22%**	**84.61%**	**2.366**	**0.0993**
Augmentation	68.72%	84.36%	2.252	0.0998
Hard cut-off (705)	68.32%	84.16%	2.293	0.1009
Stratified-fuzzy parceling (705)	68.28%	84.14%	2.282	0.1010
Hard cut-off (1109)	68.26%	84.13%	2.055	0.1015
Stratified-polarised parceling (705)	68.26%	84.13%	2.122	0.1024
Fuzzy parceling for the entire reject region (705)	68.20%	84.10%	2.152	0.1020
Stratified-random parceling (705)	68.09%	84.05%	2.126	0.1009
Stratified-polarised parceling (1109)	68.09%	84.05%	2.146	0.1027
Stratified-fuzzy parceling (1109)	67.95%	83.98%	2.016	0.1025
Fuzzy parceling for the entire reject region (1109)	67.77%	83.89%	2.064	0.1039
Stratified-random parceling (1109)	67.72%	83.86%	2.038	0.1028
Random parceling for the entire reject region (705)	67.61%	83.80%	2.208	0.1028
Reclassification (705)	67.53%	83.77%	2.024	0.1046
Random parceling for the entire reject region (1109)	67.46%	83.73%	1.998	0.1047
Reclassification (1109)	65.48%	82.74%	1.883	0.1122
Aassignment of all rejects to "bads"	61.50%	80.75%	1.718	0.1379

Source: the calculations of the author. **AR* – accuracy ratio, *AUC* – area under ROC curve measure, *ROC* – receiver operating characteristic curve, *IV* – information value. The models are sorted in a descending order by AR and AUC. These two measures in essence show the same, their values are linearly related. The bigger AR, AUC, IV and the smaller the Brier score, the bigger the model discriminatory power. The number of rejects assigned to "bads" is 705 or 1109.

Table 2. The models' discriminatory power (the reject rate is 50%)

	AR	**AUC**	**IV**	**Briero balas**
All data (proxy model)	**72.23%**	**86.11%**	**2.460**	**0.0922**
Stratified-fuzzy parceling (3607)	66.39%	83.20%	2.152	0.1070
Stratified-random parceling (3607)	66.07%	83.04%	2.035	0.1064
Augmentation	66.05%	83.02%	2.013	0.1088
Stratified-polarised parceling (1860)	66.04%	83.02%	2.142	0.1095
Stratified-fuzzy parceling (1860)	65.57%	82.79%	2.128	0.1126
Straified-polarised parceling (3607)	65.38%	82.69%	2.099	0.1071
Fuzzy parceling for the entire reject region (1860)	65.22%	82.61%	2.106	0.1170
Only accepts' data	**65.16%**	**82.58%**	**2.339**	**0.1102**
Stratified-random parceling (1860)	65.13%	82.57%	2.134	0.1129
Random parceling for the entire reject region (3607)	64.31%	82.16%	1.918	0.1115
Hard cut-off (3607)	64.05%	82.03%	1.883	0.1115
Reclassification (1860)	62.98%	81.49%	2.026	0.1673
Random parceling for the entire reject region (1860)	62.52%	81.26%	1.963	0.1184
Hard cut-off (1860)	62.28%	81.14%	1.755	0.1139
Fuzzy parceling for the entire reject region (3607)	61.73%	80.87%	1.781	0.1139
Reclassification (3607)	60.86%	80.43%	1.916	0.2153
Assignment of all rejects to "bads"	53.07%	76.54%	1.062	0.3247

Source: the calculations of the author. *The number of rejects assigned to "bads" is 1860 or 3607.

THE LIST OF THE AUTHOR‘S PUBLICATIONS ON THE DISSERTATION TOPIC

1. Dzidzevičiūtė L. 2010: Application and Behavioural Statistical Scoring Models. – *Economics and Management* 15, 1046–1056.
2. Dzidzevičiūtė L. 2010: Statistical Scoring Model of Lithuanian Companies. – *Ekonomika* 2010 89 (4), 96–115.
3. Dzidzevičiūtė L. 2010: Statistinių vertinimo balais modelių kūrimo ir taikymo ypatumai. – *Pinigų studijos* 1, 35–54.
4. Dzidzevičiūtė L. 2010: Statistinių vertinimo balais modelių taikymas Lietuvos bankuose. – *Pinigų studijos* 2, 69–85.
5. Dzidzevičiūtė L. 2012: Estimation of Default Probability for Low Default Portfolios. – *Ekonomika* 2012 91(1), 132-156.

THE INFORMATION ABOUT THE AUTHOR OF THE DISSERTATION

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10/2008 - 09/2012	Doctoral studies at Vilnius University, Faculty of Economics, Quantitative Methods and Modeling Department
09/2003 - 06/2005	Vytautas Magnus University, Faculty of Economics and Management: Master of Finance and Banking
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10/2008 - 01/2010	Expert at SC "SEB bankas", Risk Control Department, Credit Risk Control Unit
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REZIUMĖ

STATISTINIŲ VERTINIMO BALAIS MODELIŲ TAIKYMO LIETUVOS BANKUOSE GALIMYBĖS

Šios disertacijos tikslas – sukurti statistiniu vertinimo balais modeliu pagrįstą Lietuvos įmonių reitingų sistemą ir įvertinti šios sistemos taikymo Lietuvos bankuose galimybes.

Darbas susideda iš trijų skyrių. Pirmajame skyriuje yra aprašyti statistiniais vertinimo balais modeliais pagrįstų reitingų sistemų kūrimo ir taikymo bankuose ypatumai. Kuriant statistinį vertinimo balais modelį banke, iš pradžių yra analizuojamos galimybės įgyvendinti projektą, tada apibrėžiamas „blogas“ skolininkas ir stebėjimo laikotarpis, pasirenkama skolininkų grupė ir statistinis metodas, sudaroma imtis, analizuojami įvesties kintamieji ir apskaičiuojami koeficientai, atliekamas išankstinis modelio patikimumo vertinimas. Autorė nuodugniai išanalizavo šios srities literatūrą, tarptautinių bankų priežiūros institucijų rekomendacinius dokumentus, su tuo susijusius teisės aktus, kitų autorių sukurtus įmonių ir fizinių asmenų vertinimo balais modelius.

Antrajame skyriuje yra išanalizuoti šios disertacijos autorės atliktos šalyje veikiančių komercinių ir užsienio bankų skyrių apklausos rezultatai. Šios apklausos tikslas buvo išsiaiškinti vertinimo balais modelių taikymo mažmeninių paskolų paraiškoms vertinti mastą ir ypatumus. Rezultatai parodė, kad Lietuvos bankuose statistiniai mažmeninių paraiškų modeliai nebuvo plačiai taikomi, juos pasirinko tik keturi bankai. Statistiniai modeliai Lietuvos bankuose buvo sukurti taikant logistinę regresiją ir diskriminantinę analizę. Dažniausiai bankų paminėta statistinių modelių nepopuliarumo priežastis – nepakankamas praeities duomenų kaupimo laikotarpis. Mišrius modelius buvo pasirinkę tik du bankai, jie taip pat taikė ir statistinius modelius. Plačiausiai bankų buvo taikomi ekspertiniai modeliai, iš juos taikančių bankų tik du planavo ateityje kurti naujus statistinius modelius.

Kadangi įmonių statistiniai modeliai Lietuvos bankuose nebuvo paplitę, buvo sukurta logistinės regresijos modeliu pagrįsta Lietuvos įmonių reitingų sistema. Ši sistema yra pateikta trečiajame skyriuje, jame taip pat įvertintos jos taikymo Lietuvos bankuose galimybės. Lietuvos įmonių logistinės regresijos modelis tinka visų ekonominės veiklos rūšių įmonėms vertinti, yra sukurtas naudojant net 22 799 „įmonės-metų“ įrašus, jį sudaro tiek kiekybiniai, tiek kokybiniai įvesties kintamieji. „Blogai“ įmonei apibūdinti buvo taikytas įsipareigojimų neįvykdymo apibrėžimas, todėl modelis ir juo pagrįsta reitingų sistema gali būti taikomi skaičiuojant kapitalo poreikį vadovaujantis Lietuvos banko teisės aktų reikalavimais. Buvo sudaryta optimali reitingų skalė, susidedanti iš dešimties reitingų (devynių „gerų“ įmonių reitingų ir vieno reitingo tokioms įmonėms, kurios jau faktiškai tapo „blogos“), nustatyti „kieto“ ir „minkšto“ lūžio reitingai. Logistinės regresijos modeliu pagrįsta Lietuvos įmonių reitingų sistema gali būti taikoma ne tik bankuose įvairiais tikslais, bet ir kitose įstaigose, kuriose tenka vertinti įmonių kredito riziką: vartojimo kreditų, smulkiųjų vartojimo kreditų ir išperkamosios nuomos įmonės gali jį taikyti skolininkų darbdavių kredito rizikai vertinti, taip pat laidavimo paslaugas teikiančiose draudimo įmonėse ir pan. Ją taip pat gali taikyti įmonės, norinčios įvertinti savo pačių kreditingumą. Nors pasiūlyta reitingų sistema yra elgsenos (angl. *behavioural*), o ne paraiškų (angl. *application*), ją galima naudoti ir kaip paraiškų reitingų sistemą.

Jeigu bankas turi pakankamai duomenų ir gali sukurti savo statistinį įmonių vertinimo balais modelį, jis galėtų pasinaudoti šioje disertacijoje pateiktais atliktos analizės rezultatais ir pasiūlymais: pasirinkti tuos įvesties kintamuosius, kurie buvo įtraukti į siūlomą modelį, tokiu pačiu būdu sugrupuoti jų reikšmes, taikyti tuos pačius metodus ir t. t. Nors šioje disertacijoje pateikti pasiūlymai yra skirti Lietuvos bankams, kuriantiems ar planuojantiems kurti statistinius įmonių vertinimo balais modelius, jie gali būti naudingi ir kuriant bei taikant ekspertinius įmonių modelius ar fizinių asmenų modelius.

Beveik visos disertacijos dalys ir tyrimų rezultatai yra pateikti paskelbtuose straipsniuose. Mokslo žurnaluose yra publikuoti penki straipsniai disertacijos tema. Vienas jų buvo pristatytas 2010 m. balandžio 22–23 d. Rygoje vykusioje konferencijoje *„International Conference of Economics and Management ICEM 2010“.* Dirbdama Lietuvos banke, kredito rizikos srityje, disertacijos autorė buvo kelių tarptautinių bankų priežiūros institucijų darbo grupių narė, dalyvavo keliuose su kredito rizika susijusiuose tarptautiniuose seminaruose. Vėliau autorė dirbo viename iš Lietuvos komercinių bankų, kuriame buvo pritaikytos kai kurios disertacijos dalys.