Turinys

1. Įvadas

Nagrinėsime padangų nusidėvėjimo modelį, atsižvelgiant į natūralųjį nusidėvėjimą ir padangų gedimus. Tarkime, kad nusidėvėjimo procesas yra tiesinis. Padangų gedimai suskirstomi į keturis pagrindinius tipus:

1) protektoriaus dilimo gylis viršija nustatytą normą;
2) gamybos defektai;
3) mechaniniai gedimai;
4) padangos sudedamųjų dalių gedimai.

Gedimai 2-4 tipų vadinsime trauminiais.
Padangų gedimo intensyvumas 2-4 gedimų tipų priklauso nuo laiko ir padangos nusidėvėjimo lygio.

Padangos protektorius dyla sąlyčio su keliu vietoje veikiamas spaudimo, mašinai slystant keliu. Priklausomai nuo kelio sąlygų protektoriaus nusidėvėjimo schema gali skirtis. *Abrazyvinis nusidėvėjimas* – pasireiškia, kai padanga eksploatuojama nelygiame paviršiuje, esant aukštam trinties koeficientui. *Nusidėvėjimas nuo riedėjimo* pasireiškia esant aukštam trinties koeficientui eksploatuojant padangas palyginus lygiame paviršiuje. Tokį nusidėvėjimą sąlygoja kylanti temperatūra, kuri mažina gumos atsparumą plyšimui. Gumos paviršiuje susidaro slydimo krypčiai statmeni plyšiai. *Nusidėvėjimą dėl terminės destrukcijos* sąlygoja aukšta temperatūra sąlyčio su keliu vietoje, kai mašina ilgai slysta.

Geros gamybos principų taikymas sumažina padangų gedimų skaičių ir dėl gamybos defektų iš eksploatacijos išimama maža dalis padangų, bet jų rida būna nedidelė, todėl jos gali daryti įtaką patikimumo funkcijai, t. y. tikimybei, kad pravažiavus tam tikrą kilometrų skaičių padanga dar bus eksploatuojama.

Šio darbo tikslas yra aprašyti padangų nusidėvėjimo modelį, užrašyti H. Dinges transformacijas patikimumo funkcijai $S(t)$ ir $P^{(k)}(t)$ – k-tojo tipo gedimo tikimybei intervale $[0;t]$ ir sudaryti programą, kuri generuotų padangų gedimo momentus ir apskaičiuotų šių funkcijų įverčius.

2. Modelis

Pažymime t – padangos ridą kilometrais. Tarkime, kad padanga nudilo $Z(t)$ dydžiu pravažiavus t kilometrų. Tarkime, kad nusidėvėjimo procesas aprašomas tokia lygtimi

$$Z(t) = \frac{t}{A}, \quad t \geq 0 \tag{2.1}$$

A – atsitiktinis dydis, kurio pasiskirstymo funkciją žymėsime π. Padanga išimama iš eksploatavimo dėl natūralaus susidėvėjimo, kai likęs protektoriaus vidutinis gylis yra mažesnis už kritinį protektoriaus gylį z_0 arba dėl vieno iš 2-4 trauminių gedimų tipų, kurių skaičių žymėsime q. Padanga išimama iš eksploatavimo dėl natūralaus susidėvėjimo momentu

$$T^{(0)} = z_0 A \tag{2.2}$$

Dabar galime apibrėžti patikimumo funkciją $S_0(t)$ ir jos vidurkį e_0

$$S_0(t) = \mathrm{P}\{T^{(0)} > t\} = \pi\left(\frac{t}{z_0}\right) \tag{2.3}$$

ir

$$e_0 = \mathrm{E}T^{(0)} = z_0 \int_0^\infty a\, d\pi(a) \tag{2.4}$$

Trakime, kad trauminių gedimų momentai $T^{(k)}$ (k = 1,…,q) yra nepriklausomi, su sąlyga, kad $A = a$ ir jų gedimų intensyvumas priklauso tik nuo susidėvėjimo lygio, t. y. atsitiktinio dydžio $T^{(k)}$ sąlyginė patikimumo funkciją yra lygi

$$S_k\langle t | a\rangle = \mathrm{P}\{T^{(k)} > t | A = a\} = \exp\left\{-\int_0^t \lambda_k\left(\frac{s}{a}\right) ds\right\} = \exp\left\{-a\Lambda_k\left(\frac{t}{a}\right)\right\}, \tag{2.5}$$

kur λ_k tam tikra teigiama išmatuojama funkcija ir

$$\Lambda_k(z) = \int_0^z \lambda_k(y)dy .$$

Iš (2.5) lygybės išplaukia, kad

$$\mathrm{P}\{t < T^{(k)} \leq t + \Delta | T^{(k)} > t, Z(t) = z\} = \lambda_k(z)\Delta + o(\Delta),$$

kai $\Delta \to 0$. Reiškia, kad intensyvumas $\lambda_k(z)$ proporcingas sąlyginei tikimybei k – tojo tipo trauminio gedimo per trumpą laiko intervalą, su sąlyga, kad intervalo pradžioje padanga dar funkcionavo ir jos protektoriaus susidėvėjimo gylis buvo z.

Kai (2.5) teisinga, atsitiktinio dydžio $T^{(k)}$ sąlyginis tankis yra lygus

$$f_k\left(t \;\middle|\; a\right) = \lambda_k\left(\frac{t}{a}\right) e^{-a\Lambda_k\left(\frac{t}{a}\right)}. \tag{2.6}$$

3. Pagrindinės padangų patikimumo charakteristikos

Besąlyginės patikimumo charakteristikos

Jeigu T žymi padangos gedimo momentą, tai

$$T = \min\left(T^{(0)}, T^{(1)}, ..., T^{(q)}\right). \tag{3.1}$$

Tarkime

$$\Lambda_{.}(z) = \sum_{k=1}^{q} \Lambda_k(z),$$

Tada patikimumo funkcija ir atsitiktinio dydžio T vidurkis yra lygūs

$$S(t) = \mathrm{P}\{T > t\} = \int_{\frac{t}{z_0}}^{\infty} e^{-a\Lambda_{.}\left(\frac{t}{a}\right)} d\pi(a),$$

ir

$$e = \int_0^{\infty} S(t)dt = \int_0^{\infty} d\pi(a) \int_0^{az_0} e^{-a\Lambda_{.}\left(\frac{t}{a}\right)} dt = \int_0^{\infty} a d\pi(a) \int_0^{z_0} e^{-a\Lambda_{.}(z)} dz .$$

Vidinį integralą suintegruojame dalimis ir gauname

$$\int_0^{z_0} e^{-a\Lambda_{.}(z)} dz = -\int_0^{z_0} e^{-a\Lambda_{.}(z)} d(z_0 - z) = z_0 - a\int_0^{z_0} (z_0 - z) e^{-a\Lambda_{.}(z)} d\Lambda_{.}(z).$$

Pasinaudojame (2.4) lygybe ir gauname

$$e = e_0 - \int_0^{\infty} a^2 d\pi(a) \int_0^{z_0} (z_0 - z) e^{-a\Lambda_{.}(z)} d\Lambda_{.}(z). \tag{3.3}$$

Pažymime V gedimo tipo indikatorių

$$V = \begin{cases} 0, & kai \quad T = T^{(0)}, \\ 1, & kai \quad T = T^{(1)}, \\ ... & \\ q, & kai \quad T = T^{(q)}. \end{cases} \tag{3.4}$$

Svarbios patikimumo charakteristikos yra $P^{(k)}(t)$ – k - tojo gedimo tipo tikimybė intervale [0;t] ir jos ribinė reikšmė $P^{(k)} = P^{(k)}(\infty)$.

Jeigu k = 1,..., q, tai pasinaudoję (2.6) formule gauname

$$P^{(k)}(t) = \mathrm{P}\{T = T^{(k)} \le t\} = \int_0^{\infty} d\pi(a) \int_0^{(az_0)\wedge t} f_k\left(s \;\middle|\; a\right) \prod_{l \ne k} S_l\left(s \;\middle|\; a\right) ds =$$

$$= \int_0^{\infty} d\pi(a) \int_0^{(az_0)\wedge t} \lambda_k\left(\frac{s}{a}\right) e^{-a\Lambda_{.}\left(\frac{s}{a}\right)} ds$$

$$= \int_0^{\infty} a d\pi(a) \int_0^{z_0 \wedge \left(\frac{t}{a}\right)} e^{-a\Lambda_{.}(z)} d\Lambda_k(z), \tag{3.5}$$

čia $x \wedge y = \min(x, y)$.

Gauname

$$P^{(k)} = \int_0^{\infty} a d\pi(a) \int_0^{z_0} e^{-a\Lambda_{.}(z)} d\Lambda_k(z). \tag{3.6}$$

Sumuojant pagal k gauname trauminių gedimų tikimybes $P^{(tr)}(t) = \mathrm{P}\left(T \le t,\ V \ne 0 \right)$ ir $P^{(tr)} = P^{(tr)}(\infty)$

$$P^{(tr)}(t) = \int_0^{\infty} a d\pi(a) \int_0^{z_0 \wedge \left(\frac{t}{a}\right)} e^{-a\Lambda_{.}(z)} d\Lambda_{.}(z) = 1 - \int_0^{\infty} e^{-a\Lambda_{.}\left(z_0 \wedge \left(\frac{t}{a}\right)\right)} d\pi(a), \tag{3.7}$$

$$P^{(tr)} = 1 - \int_0^{\infty} e^{-a\Lambda_{.}(z_0)} d\pi(a). \tag{3.8}$$

Taigi, iš (3.2) ir (3.7) gauname, kad

$$P^{(0)}(t) = 1 - S(t) - P^{(tr)}(t) = \int_0^{\frac{t}{z_0}} e^{-a\Lambda_{.}(z_0)} d\pi(a) \tag{3.9}$$

ir

$$P^{(0)} = \int_0^{\infty} e^{-a\Lambda_{.}(z_0)} d\pi(a).$$

4. Neparametrinis kumuliacinių intensyvumų vertinimas

Tarkime, kad bandoma n padangų, kiekvienai padangai stebime gedimo momentą T_i, gedimo tipą V_i, padangos dilimo dydį Z_i gedimo momentu T_i. Tada

$$Z_i = T_i / A_i, \tag{4.1}$$

kur $\frac{1}{A_i}$ yra i – tosios padangos susidėvėjimo greitis. Darome prielaidą, kad

$$(T_1, Z_1, V_1), \dots, (T_n, Z_n, V_n) \tag{4.2}$$

yra nepriklausomi vienodai pasiskirstę atsitiktiniai vektoriai. Iš formulės (4.1) išplaukia, kad A_i dydžiai žinomi. Todėl galime sakyti, kad mūsų duomenys yra

$$(A_1, Z_1, V_1), \dots, (A_n, Z_n, V_n).$$

Vektoriai yra nepriklausomos atsitiktinio vektoriaus (A, T / A, V) kopijos, A, T, ir V apibrėžti (2.1), (3.1) ir (3.4) formulėmis.

Geriausias pasiskirstymo funkcijos π neparametrinis įvertis yra empirinė pasiskirstymo funkcija

$$\hat{\pi}(a) = \frac{1}{n} \sum_{i=1}^{n} 1_{\{A_i \le a\}}.$$

Žinoma, kad šis įvertis yra tolygiai tolydus , t. y.

$$\sup_a |\hat{\pi}(a) - \pi(a)| \xrightarrow[n \to \infty]{} 0.$$

Be to, jeigu π tolydi, tai atsitiktinė funkcija

$$\sqrt{n}(\hat{\pi} - \pi)$$

artėja į normalųjį pasiskirstymą W_0 su vidurkiu 0 ir kovariacijos funkcija

$$\mathrm{E}\{W_0(a) W_0(a')\} = \pi(a \wedge a') - \pi(a)\pi(a'). \tag{4.3}$$

Kadangi neturime duomenų apie $Z_i > z_0$, negalime rasti neparametrinį kumuliacinių intensyvumų $\lambda_k(z)$ įvertį, kai $z > z_0$. Todėl toliau turėsime omenyje, kad $z \in [0; z_0]$

5. Neparametriniai padangų patikimumo charakteristikų įverčiai

Pasinaudosime V. Bagdonavičiaus knygoje “Estimation of tire wear and traumatic failure intensities” gautais nepaslinktais įverčiais:

$\tilde{S}_n(t) = \frac{1}{n}\sum_{i=1}^{n} 1_{\{T_i > t\}}$ – patikimumo funkcijos $S(t)$ įvertis;

$\tilde{e}_n = \frac{1}{n}\sum_{i=1}^{n} T_i$ – atsitiktinio dydžio T vidurkio e įvertis;

$\tilde{P}_n^{(k)}(t) = \frac{1}{n}\sum_{i=1}^{n} 1_{\{V_i = k, T_i \le t\}}$ – k -tojo tipo gedimo tikimybės intervale $[0;t]$ įvertis;

$\tilde{P}_n^{(k)} = \frac{1}{n}\sum_{i=1}^{n} 1_{\{V_i = k\}}$ – k -tojo tipo gedimo tikimybės intervale $[0;t]$ ribos įvertis;

$\tilde{P}_n^{(tr)}(t) = \frac{1}{n}\sum_{i=1}^{n} 1_{\{V_i \ne 0, T_i \le t\}}$ – trauminio tipo gedimo tikimybės intervale $[0;t]$ įvertis;

$\tilde{P}_n^{(tr)} = \frac{1}{n}\sum_{i=1}^{n} 1_{\{V_i \ne 0\}}$ – trauminio tipo gedimo tikimybės intervale $[0;t]$ ribos įvertis.

$\frac{1}{n}\sum_{i=1}^{n} 1_{\{T_i > t\}}$ yra suma nepriklausomų vienodai pasiskirsčiusių Bernulio atsitiktinių dydžių. Sakome, kad ξ_i ~Bernulio atsitiktinis dydis, t. y.

$$P\{\xi_i = 1\} = p \text{ ir } P\{\xi_i = 0\} = q = 1 - p .$$

Čia (5.1)

$$p = P\{T > t\} = S(t),\ q = 1 - S(t).$$

Suma $\tilde{S}_n(t) = \frac{1}{n}\sum_{i=1}^{n} 1_{\{T_i > t\}}$ įgyja reikšmes:

$$\frac{0}{n}, \frac{1}{n}, \ldots, \frac{m}{n}, \ldots, \frac{n}{n},$$

su tikimybėmis:

$$P\left\{\tilde{S}_n(t) = \frac{m}{n}\right\} = C_n^m p^m q^{n-m} = C_n^m (S(t))^m (1 - S(t))^{n-m} = I_{1-S(t)}(n-m, m+1),$$

kur

$$I_{1-S(t)}(n-m, m+1) = \frac{1}{\mathrm{B}(n-m, m+1)} \int_0^{1-S(t)} u^{n-m-1}(1-u)^m\, du ,$$

$$\mathrm{B}(n-m, m+1) = \int_0^1 u^{n-m-1}(1-u)^m\, du ,$$

čia n – nepriklausomų bandymų skaičius, p – sėkmės tikimybė.
Pažymime μ – sėkmingų bandymų skaičius. Visiems m = 0, 1, …, n teisinga lygybė

$$P\langle \mu \le m | n, p \rangle = \sum_{k=0}^{m} C_n^k p^k (1-p)^{n-k} = I_{1-p}(n-m, m+1)$$

Yra žinoma, kad jeigu $p \to 0$, kai $n \to \infty$, tai $y \sim np$, kur y – Binominio ats. d. vidurkis, tai Binominį pasiskirstymą galima aproksimuoti Puasono pasiskirstymu su nuo argumento *m*, *n* ir *p* priklausomu parametru:

$$P\langle \mu \le m | n, p \rangle = 1 - I(y, m+1) + R_n(m) = \sum_{k=0}^{m} \frac{y^k}{k!} e^{-y} + R_n(m), \tag{5.2}$$

Liekamasis narys $R_n(m)$ yra mažėjantis, kai m = const ir $M = 2n - m \xrightarrow[n\to\infty]{} \infty$, Tuomet įvertis $R_n(m)$ yra pakankamai geras visoms *p* reikšmėms iš intervalo $0 < p < 1$

$$R_n(m) = \begin{cases} O\left(\frac{1}{M^2}\right), & kai \quad y = y_1 = \frac{pM}{2-p} \\ O\left(\frac{1}{M^4}\right), & kai \quad y = \frac{y_1}{1 + \left[m(m+2) + my_1 - 2y_1^2\right] / 6M^2} \end{cases} \tag{5.3}$$

Jeigu $p \to 0$, $np \to \lambda$, tuomet $y \sim np$, kai $n \to \infty$. Šiuo atveju (5.2) formulė yra tradicinė, kai $y = np$, kitais atvejais (5.2) ir (5.3) turi mažesnę paklaidą negu žinomos aproksimacijos Puasono dėsniu.

Kolmogorovo pasiūlytas Puasono aproksimacijos tikslinimas iki dydžių n^{-2} eilės, yra paremtas $P\{\mu = k\} = C_n^k p^k (1-p)^{n-k}$ tikimybių aproksimacijų tikslinimu, tuo tarpu (5.2) ir (5.3) iškart aproksimuoja tikimybinį skirstinį $P\{\mu \le m\}$; $I(y, m+1)$ dydžius galima apskaičiuoti iš χ^2 lentelės su $2m+2$ laisvės laipsniais.

$$I(y, m+1) = P\{\chi^2_{2m+2} < 2y\} = \int_0^{2y} \frac{1}{2^{\frac{n}{2}} \Gamma\left(\frac{n}{2}\right)} x^{\frac{n}{2}-1} e^{-\frac{x}{2}} dx,$$

Pastebėkime, kad (5.3) įvertis tolygus visiems *p* iš intervalo (0,1), tai reiškia, kad (5.2) ir (5.3) galima naudoti ne tik retiems įvykiams, o bet kokiems $p \in (0,1)$. Mūsų uždavinyje yra teisinga

$$P\left\{\sqrt{n}\left(\tilde{S}_n(t) - S(t)\right) \le x\right\} \to \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{x}{\sqrt{S(t)(1-S(t))}}} e^{-\frac{u^2}{2}} du,$$

kai $n \to \infty$, $nS(t)(1 - S(t)) \to \infty$.

Kadangi $\tilde{S}_n(t)$ yra suma nepriklausomų Bernulio atsitiktinių dydžių, tai jos tikimybinį dėsnį galima užrašyti taip

$$P\left\{\sqrt{n}\left(\tilde{S}_n(t) - S(t)\right) \le x\right\} \sum_{\substack{m \in \left\{m: \sqrt{n}\left(\frac{m}{n} - S(t)\right) \le x\right\}, \\ m = 0,1,\ldots,n}} C_n^m (S(t))^m (1 - S(t))^{n-m} = \sum_{k \le \left(\frac{x}{\sqrt{n}} + S(t)\right)n} C_n^k (S(t))^k (1 - S(t))^{n-k}$$

Jei *x* parenkame taip, kad $m = \left(\frac{x}{\sqrt{n}} + S(t)\right)n$ būtų sveikas skaičius, tuomet

$$P\left\{\sqrt{n}\left(\tilde{S}_n(t) - S(t)\right) \le x\right\} = \sum_{k=0}^{m} C_n^k (S(t))^k (1 - S(t))^{n-k},$$

Dabar galime pasinaudoti L. N. Bolševo teorema ir gauname, kad

$$P\left\{\sqrt{n}\left(\tilde{S}_n(t) - S(t)\right) < x\right\} = \sum_{k=0}^{m} \frac{y^k}{k!} e^{-y} + R_n(m),$$

kur

$$R_n(m) = \begin{cases} O\left(\dfrac{1}{M^2}\right), & kai \quad y = y_1 = \dfrac{S(t)M}{2 - S(t)} \\ O\left(\dfrac{1}{M^4}\right), & kai \quad y = \dfrac{y_1}{1 + \left[m(m+2) + my_1 - 2y_1^2\right] / 6M^2} \end{cases},$$

čia $M = 2n - m$.

6. Lygybės binominėms tikimybėms

Pažymime $b(k;\, n,\, p) = C_n^k p^k (1-p)^{n-k}$, $0<p<1$. Tarkime $\alpha' = \frac{k}{n}$, kai k = 1, 2,..., n-1 tuomet turime lygybę

$$b(k;\, n,\, p)\exp\left\{-S^{(n)}(\alpha')\right\} = \frac{1}{\sqrt{2\pi(n+1)\alpha'(1-\alpha')}}\exp\left\{-\frac{n+1}{2}A^2(\alpha',\, p)\right\}. \tag{6.1}$$

Čia $\alpha'' = \frac{k+1}{2(n+1)}$, kai k = 0, 1,..., n gauname

$$b(k; n,\, p)\exp\left\{-T^{(n)}(\alpha'')\right\} = \frac{1}{\sqrt{2\pi(n+1)p(1-p)}}\exp\left\{-\frac{n+1}{2}A^2(\alpha'',\, p)\right\}, \tag{6.2}$$

kur $A(\alpha,\, p)$ apibrėžiama formulėmis

$$A(\alpha,\, p) = \frac{\alpha - p}{\sqrt{pq}}\sqrt{1+\pi(\alpha,\, p)},$$

$$A(0,\, p) = -\sqrt{2\ln\frac{1}{q}},\ A(1,\, p) = \sqrt{2\ln\frac{1}{p}}. \tag{6.3}$$

Lengva apskaičiuoti A pagal formulę

$$\frac{1}{2}A^2(\alpha,\, p) = \alpha\ln\left(\frac{\alpha}{p}\right) + (1-\alpha)\ln\frac{1-\alpha}{1-p}. \tag{6.4}$$

Kiekvienam $a \in R$, $0<\alpha<1$ ir $0<p<1$ pažymėkime

$$\tilde{D}(\alpha,\, p) = \sqrt{\frac{\alpha\beta}{pq}}\left[1+\pi(\alpha,\, p)\right]^{\frac{1}{2}} = \frac{\sqrt{\alpha\beta}}{(\alpha-p)}A(\alpha,\, p)$$

$$D(\alpha, a) = \tilde{D}(\alpha,\, p), \tag{6.5}$$

kur $a = A(\alpha,\, p)$.

<u>*Dinges lema*</u> Kiekvienam $\alpha \in (0,1)$ $D(\alpha,\cdot)$ yra teigiama iškili funkcija tenkinanti lygybes, kai $D(\alpha,0) = 1$

$$\lim_{\alpha\to-\infty}\frac{D(\alpha, a)}{a} = -\sqrt{\frac{\alpha}{1-\alpha}},\ \lim_{\alpha\to\infty}\frac{D(\alpha, a)}{a} = \sqrt{\frac{1-\alpha}{\alpha}},$$

$$\frac{dD(\alpha, a)}{da} = -\sqrt{\alpha\beta}\,\frac{\pi(\alpha,\, p)}{\alpha-p}. \tag{6.6}$$

Kai $\alpha = \frac{1}{2}$, imkime simetrines funkcijas

$$D\left(\frac{1}{2}, a\right) = \frac{|a|}{\sqrt{1-\exp\{-a^2\}}} \tag{6.7}$$

Funkcijos $A(\cdot,\cdot)$ ir $D(\cdot,\cdot)$ bus naudojamos binominiam pasiskirstymui nagrinėti.

7. Dinges transformacijų taikymas patikimumo funkcijai *S(t)*

Teorema 1 Kadangi $\tilde{S}_n(t)$ turi binominį pasiskirstymą su parametrais $(n, S(t))$, tuomet

$$P\{\tilde{S}_n(t) \le k\} = \psi^{(n)}\left(\frac{k+1}{n+1}, S(t)\right), \tag{7.1}$$

kai k=0,1,2,… . Čia tikimybės $S(t)$ apibrėžtos lygybe (5.1)

Išvada 1 Kadangi $\tilde{S}_n \sim B(n, S(t))$, bei $E\tilde{S}_n = nS(t)$, $D\tilde{S}_n = nS(t)(1-S(t))$, Tai, kai sandauga $nS(t)(1-S(t))$ yra didelė mes galime aproksimuoti

$$P\{\tilde{S}_n(t) \le k\} \sim \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\sqrt{n+1}A\left(\frac{k+1}{n+1}, S(t)\right)} e^{-\frac{v^2}{2}} dv, \tag{7.2}$$

Kai $\tilde{a}_k = A\left(\frac{k+1}{n+1} S(t)\right)$ nelabai didelis, tai

$$P\left\{A\left(\frac{\tilde{S}_n(t)+1}{n+1}\right) \le \tilde{a}_k\right\} \sim P\{Z \le \tilde{a}_k\sqrt{n+1}\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\tilde{a}_k\sqrt{n+1}} e^{-\frac{v^2}{2}} dv,$$

Teorema 2 Kadangi $\tilde{S}_n$ turi binominį pasiskirstymą su parametrais $(n, S(t))$, tuomet

$$L\left(A\left(\frac{\tilde{S}_n}{n}, S(t)\right)\right) \sim N\left(0, \frac{1}{n}\right), \tag{7.3}$$

arba

$$P\left\{A\left(\frac{\tilde{S}_n+1}{n+1}, S(t)\right) < x\right\} \approx \frac{1}{\sqrt{2\pi\frac{1}{n+1}}} \int_{-\infty}^{x} e^{-\frac{u^2}{2\frac{1}{n+1}}} du.$$

Išvada 2 Statistikos $\tilde{S}_n(t)$ tikimybinis skirstinys tenkina nelygybes

$$P\{\tilde{S}_n(t) \ge k\} = P\left\{A\left(\frac{\tilde{S}_n(t)}{n}, S(t)\right) \ge A\left(\frac{k}{n}, S(t)\right)\right\} \ge 1 - \Phi\left(\sqrt{n}A\left(\frac{k}{n}, S(t)\right)\right), \tag{7.4}$$

$$P\left\{\tilde{S}_n(t) \le k\right\} = P\left\{A\left(\frac{\tilde{S}_n(t)}{n}, S(t)\right) \ge A\left(\frac{k}{n}, S(t)\right)\right\} \ge \Phi\left(\sqrt{n}A\left(\frac{k}{n}, S(t)\right)\right) \tag{7.5}$$

Šie grubūs įvertinimai duoda tam tikrą informaciją apie tam tikrus klasikinius priartėjimus Muavre-Laplace teoremoje.

Dabar galime padaryti tokias išvadas:

a) srityje $\dfrac{\pi\left(\frac{k}{n}, S(t)\right)}{\frac{k}{n} - S(t)} \le 0$ turime nelygybę

$$P\left\{\tilde{S}_n \ge k\right\} \ge 1 - \Phi\left(\frac{k - nS(t)}{\sqrt{nS(t)(1 - S(t))}}\right), \tag{7.6}$$

b) srityje $\dfrac{\pi\left(\frac{k}{n}, S(t)\right)}{\frac{k}{n} - S(t)} \ge 0$ turime nelygybę

$$P\left\{\tilde{S}_n \le k\right\} \ge \Phi\left(\frac{k - nS(t)}{\sqrt{nS(t)(1 - S(t))}}\right). \tag{7.7}$$

Gautus tvirtinimus (7.2) ir (7.7) galime užrašyti įprastiniais pavidalais:

$$P\left\{\tilde{S}_n(t) \le k\right\} \ge \Phi\left(\frac{k - nS(t)}{\sqrt{nS(t)(1 - S(t))}}\left[1 + \pi\left(\frac{k}{n}, S(t)\right)\right]^{\frac{1}{2}}\right),$$

$$P\left\{\tilde{S}_n(t) \le k\right\} \sim \Phi\left\{\frac{k + 1 - (n+1)S(t)}{\sqrt{(n+1)S(t)(1 - S(t))}}\left[1 + \pi\left(\frac{k+1}{n+1}, S(t)\right)\right]^{\frac{1}{2}}\right\},$$

$$P\left\{\tilde{S}_n(t) \le k\right\} \sim \Phi\left(\frac{k + \frac{2}{3} - \left(n + \frac{1}{3}\right)S(t)}{\sqrt{n + \frac{S(t)(1 - S(t))}{6}}}\left[1 + \pi\left(\frac{k + \frac{1}{2}}{n}, S(t)\right)\right]^{\frac{1}{2}}\right), \tag{7.8}$$

arba bendriau

$$P\left\{\tilde{S}_n(t)\le k\right\}\sim\Phi\left(\frac{k+\frac{2}{3}+a-\left(n+\frac{1}{3}\right)S(t)}{\sqrt{n+\frac{S(t)(1-S(t))}{6}}}\left[1+\pi\left(\frac{k+\frac{1}{2}}{n},S(t)\right)\right]^{\frac{1}{2}}\right), \tag{7.9}$$

kai $a=b_1\left(\frac{1-S(t)}{k+1}-\frac{S(t)}{n-k}\right)+b_2\frac{1-2S(t)}{2(n+1)}$.

8. Dinges transformacijų taikymas patikimumo charakteristikai $P^{(k)}(t)$

$\tilde{P}_n^{(k)}(t)=\frac{1}{n}\sum_{i=1}^{n}1_{\{V_i=k,T_i\le t\}}$ yra suma nepriklausomų vienodai pasiskirsčiusių Bernulio atsitiktinių dydžių. Pagal (3.5) formulę turime

$$P^{(k)}(t)=\mathrm{P}\left\{T=T^{(k)}\le t\right\}=\int_0^\infty ad\pi(a)\int_0^{z_0\wedge\left(\frac{t}{a}\right)}e^{-a\Lambda.(z)}d\Lambda_k(z),$$

atsitiktinio dydžio $T^{(k)}$ sąlyginis tankis apibrėžtas (2.6) lygybe.

<u>*Teorema 3*</u> Kadangi $\tilde{P}_n^{(k)}(t)$ turi binominį pasiskirstymą su parametrais $\left(n,P^{(k)}(t)\right)$, tuomet

$$P\left\{\tilde{P}_n(t)\le k\right\}=\psi^{(n)}\left(\frac{k+1}{n+1},P^{(k)}(t)\right), \tag{8.1}$$

kai k=0,1,2,… . Čia tikimybės $P^{(k)}(t)$ apibrėžtos aukščiau.

<u>*Išvada 3*</u> Kadangi $\tilde{P}_n^{(k)}(t)\sim B\left(n,P^{(k)}(t)\right)$, tai, kai sandauga $nP^{(k)}(t)\left(1-P^{(k)}(t)\right)$ yra didelė mes galime aproksimuoti

$$P\left\{\tilde{P}_n^{(k)}(t)\le k\right\}\sim\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\sqrt{n+1}A\left(\frac{k+1}{n+1},S(t)\right)}e^{-\frac{v^2}{2}}dv. \tag{8.2}$$

Kai $\tilde{b}_k=A\left(\frac{k+1}{n+1}P^{(k)}(t)\right)$ nelabai didelis, tai

$$P\left\{A\left(\frac{\tilde{P}_n^{(k)}(t)+1}{n+1}\right)\le\tilde{b}_k\right\}\sim P\left\{Z\le\tilde{b}_k\sqrt{n+1}\right\}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\tilde{b}_k\sqrt{n+1}}e^{-\frac{v^2}{2}}dv,$$

<u>*Teorema 4*</u> Kadangi $\tilde{P}_n^{(k)}(t)$ turi binominį pasiskirstymą su parametrais $\left(n,P^{(k)}(t)\right)$, tuomet

$$L\left(A\left(\frac{\tilde{P}_n^{(k)}(t)}{n}\right),P^{(k)}(t)\right)\sim N\left(0,\frac{1}{n}\right), \tag{8.3}$$

arba

$$P\left\{A\left(\frac{\widetilde{P}_n^{(k)}(t)+1}{n+1}, P^{(k)}(t)\right) < x\right\} \approx \frac{1}{\sqrt{2\pi\frac{1}{n+1}}}\int_{-\infty}^{x} e^{-\frac{u^2}{2\frac{1}{n+1}}}du .$$

Išvada 4 Statistikos $\widetilde{P}_n^{(k)}(t)$ tikimybinis skirstinys tenkina nelygybes

$$P\left\{\widetilde{P}_n^{(k)}(t) \geq k\right\} = P\left\{A\left(\frac{\widetilde{P}_n^{(k)}(t)}{n}, P^{(k)}(t)\right) \geq A\left(\frac{k}{n}, P^{(k)}(t)\right)\right\} \geq 1-\Phi\left(\sqrt{n}A\left(\frac{k}{n}, P^{(k)}(t)\right)\right), \tag{8.4}$$

$$P\left\{\widetilde{P}_n^{(k)}(t) \leq k\right\} = P\left\{A\left(\frac{\widetilde{P}_n^{(k)}(t)}{n}, P^{(k)}(t)\right) \geq A\left(\frac{k}{n}, P^{(k)}(t)\right)\right\} \geq \Phi\left(\sqrt{n}A\left(\frac{k}{n}, P^{(k)}(t)\right)\right). \tag{8.5}$$

Dabar galime padaryti tokias išvadas:

a) srityje $\dfrac{\pi\left(\frac{k}{n}, P^{(k)}(t)\right)}{\frac{k}{n} - P^{(k)}(t)} \leq 0$ turime nelygybę

$$P\left\{\widetilde{P}_n^{(k)}(t) \geq k\right\} \geq 1-\Phi\left(\frac{k-nP^{(k)}(t)}{\sqrt{nP^{(k)}(t)\left(1-P^{(k)}(t)\right)}}\right), \tag{8.6}$$

b) srityje $\dfrac{\pi\left(\frac{k}{n}, P^{(k)}(t)\right)}{\frac{k}{n} - P^{(k)}(t)} \geq 0$ turime nelygybę

$$P\left\{\widetilde{P}_n^{(k)}(t) \leq k\right\} \geq \Phi\left(\frac{k-nP^{(k)}(t}{\sqrt{nP^{(k)}(t\left(1-P^{(k)}(t)\right)}}\right). \tag{8.7}$$

Gautus tvirtinimus (8.2) ir (8.7) galime užrašyti įprastiniais pavidalais:

$$P\left\{\widetilde{P}_n^{(k)}(t) \leq k\right\} \geq \Phi\left(\frac{k-nP^{(k)}(t)}{\sqrt{nP^{(k)}(t)\left(1-P^{(k)}(t)\right)}}\left[1+\pi\left(\frac{k}{n}, P^{(k)}(t)\right)\right]^{\frac{1}{2}}\right),$$

$$P\left\{\widetilde{P}_n^{(k)}(t) \leq k\right\} \sim \Phi\left\{\frac{k+1-(n+1)P^{(k)}(t)}{\sqrt{(n+1)P^{(k)}(t)\left(1-P^{(k)}(t)\right)}}\left[1+\pi\left(\frac{k+1}{n+1}, P^{(k)}(t)\right)\right]^{\frac{1}{2}}\right\},$$

$$P\left\{\widetilde{P}^{(k)}{}_n(t)\le k\right\}\sim\Phi\left(\frac{k+\frac{2}{3}-\left(n+\frac{1}{3}\right)P^{(k)}(t)}{\sqrt{n+\frac{P^{(k)}(t)\left(1-P^{(k)}(t)\right)}{6}}}\left[1+\pi\left(\frac{k+\frac{1}{2}}{n},P^{(k)}(t)\right)\right]^{-\frac{1}{2}}\right), \qquad (8.8)$$

arba bendriau

$$P\left\{\widetilde{P}^{(k)}{}_n(t)\le k\right\}\sim\Phi\left(\frac{k+\frac{2}{3}+a'-\left(n+\frac{1}{3}\right)P^{(k)}(t)}{\sqrt{n+\frac{P^{(k)}(t)\left(1-P^{(k)}(t)\right)}{6}}}\left[1+\pi\left(\frac{k+\frac{1}{2}}{n},P^{(k)}(t)\right)\right]^{-\frac{1}{2}}\right), \qquad (8.9)$$

kai $a'=b'_1\left(\frac{1-P^{(k)}(t)}{k+1}-\frac{P^{(k)}(t)}{n-k}\right)+b'_2\,\frac{1-2P^{(k)}(t)}{2(n+1)}$.

9. Duomenų generavimas

Programos aprašymas

Tarkime, kad bandoma *n* padangų, kiekvienai padangai stebime gedimo momentą T_i, gedimo tipą V_i, padangos dilimo dydį Z_i gedimo momentu T_i, kur *i* = 1,...,n. Dydžiai $\left(T^{(1)}, T^{(2)}, T^{(3)}, T^{(4)}\right)$ yra sąlyginai nepriklausomi.

$$P\{T_1 > t_1, \ldots T_4 > t_4 | A = a\} = P\{T_1 > t_1 | A = a\} \cdot \ldots \cdot P\{T_4 > t_4 | A = a\},$$

$$\lambda_k(z) = \left(\frac{z}{\theta_k}\right)^{\alpha_k},$$

$$\Lambda_k(z) = \int_0^z \frac{y^{\alpha_k}}{\theta_k^{\alpha_k}} dy = \frac{z^{\alpha_k + 1}}{\theta_k^{\alpha_k}(\alpha_k + 1)},$$

kur *k* =1,2,3.

Tada dydžių $\left(T^{(1)}, T^{(2)}, T^{(3)}, T^{(4)}\right)$ sąlyginis pasiskirstymas yra Veibulo su pasiskirstymo funkcija

$$P\{T^{(k)} > t | A = a\} = e^{-a\Lambda_k\left(\frac{t}{a}\right)} = e^{-a\frac{\left(\frac{t}{a}\right)^{\alpha_k+1}}{(\alpha_k+1)\theta_k^{\alpha_k}}},$$

Tarkime, kad funkcija *A* yra pasiskirsčiusi pagal eksponentinį dėsnį:

$$A \sim E(\lambda_a),$$

t. y.

$$\pi(a) = 1 - e^{-\lambda_a a},$$

kai $a \geq 0$.

Jeigu $\omega \sim F(x) = R(0,1)$, tai $X = F^{-1}(\omega) \sim \pi$. Todėl atsitiktinius dydžius A_i generuojame pagal formulę:

$$1 - e^{-\lambda_a A_i} = \omega_i,$$

kur ω_i, *i* = 1,2,...,*n*, yra nepriklausomi vienodai pasiskirstę atsitiktiniai dydžiai $\omega_i \sim R(0,1)$.

$$A_i = -\frac{1}{\lambda_a}\ln(1 - \omega_i),$$

$$1 - e^{\frac{-A_i\left(\frac{T_i^{(k)}}{A_i}\right)^{\alpha_k+1}}{(\alpha_k+1)\theta_k^{\alpha_k}}} = \omega_{ki},$$

čia *k* = 1, 2, 3, *i* = 1,2,..., *n*.

$$\frac{\left(T_i^{(k)}\right)^{\alpha_k+1}}{A_i^{\alpha_k}(\alpha_k + 1)\theta_k^{\alpha_k}} = -\ln(1 - \omega_{ki}),$$

$$T_i^{(k)} = \left(-A_i^{\alpha_i}(\alpha_k + 1)\theta_k^{\alpha_k}\ln(1 - \omega_{ki})\right)^{\frac{1}{\alpha_k+1}} = \left(-\left(A_i\theta_k\right)^{\alpha_k}(\alpha_k + 1)\ln(1 - \omega_{ki})\right)^{\frac{1}{\alpha_k+1}},$$

Kritinis padangos nusidėvėjimo lygis $z_0 = 0{,}16$ cm,

$$T_i^{(0)} = z_0 A_i \,,$$

$T_i^{(0)}$ pasiskirstymas yra eksponentinis.
Natūralaus padangos susidėvėjimo vidurkis yra

$$ET_i^{(0)} = z_0 EA_i = \frac{z_0}{\lambda_a} = 100000 \text{ km},$$

$$\lambda_a = \frac{z_0}{ET_i^{(0)}} = \frac{1{,}6 \cdot 10^{-6}}{10^5} = 1{,}6 \cdot 10^{-11} \,.$$

Pažymime

$$\lambda_a^* = \lambda_a \cdot 10^{11} = 1{,}6 \,.$$

Tada

$$A^* = A \cdot 10^{-11},$$

$$A^* \sim E(\lambda_a^*) = E(1.6),$$

$$A \sim E\left(1{,}6 \cdot 10^{-11}\right),$$

t. y.

$$P\{A \le a\} = P\{A^* \cdot 10^{11} \le a\} = P\{A^* \le 10^{-11} \cdot a\} = 1 - e^{-1{,}6 \cdot 10^{-11} a} \,.$$

Turime

$$T_i^{(0)} = z_0 A_i = z_0 A_i^* 10^{11} = 1{,}6 \cdot A_i^* \cdot 10^5 \,.$$

Kadangi $A_k^* \sim E(1{,}6)$, tai

$$T_i^{(k)} = \left(-\left(A_i^* 10^{11} \theta_k\right)^{\alpha_k} (\alpha_k + 1) \ln(1 - \omega_k)\right)^{\frac{1}{\alpha_k + 1}} .$$

Pasirenkame parametrus ω_k ir α_k, ir apskaičiuosime parametrų θ_k reikšmes:

$$ET_i^{(k)} = \left[\left(\frac{10^{11}}{1{,}6}\theta_k\right)^{\alpha_k} (\alpha_k + 1) \cdot \ln(1 - \omega_k)\right]^{\frac{1}{\alpha_k + 1}} = 150000 \text{ (km)},$$

$$\left(-\left(\frac{10^{11}}{1{,}6}\theta_1\right)^{\alpha_k} (\alpha_k + 1) \ln(1 - \omega_k)\right)^{\frac{1}{\alpha_k + 1}} = 150000 \,,$$

$$\theta_k = \left(\frac{1.5 \cdot 10^5}{(\alpha_k + 1)^{\frac{1}{\alpha_k + 1}} \left(\ln(1 - \omega_k)\right)^{\frac{1}{\alpha_k + 1}}}\right)^{\frac{\alpha_k + 1}{\alpha_k}} \cdot \frac{1.6}{10^{11}} \,,$$

Programa sugeneruoja $n \times (q + 5)$ matricą $\left(T^{(0)}, T^{(1)}, T^{(2)}, T^{(3)}, T, V, Z, A\right)$ ir apskaičiuoja patikimumo funkcijos $S(t)$ ir $P^{(k)}(t)$ – k-tojo tipo gedimo tikimybės intervale $[0; t]$ įverčius.

$T = \min\left(T^{(0)}, T^{(1)}, \ldots, T^{(q)}\right).$

Kadangi neturime duomenų apie $Z_i > z_0$, negalime rasti neparametrinį kumuliacinių intensyvumų $\lambda_k(z)$ įvertį, kai $z > z_0$. Todėl turėsime omenyje, kad $z \in [0; z_0]$.

10. Programos tekstas

```
data parametrai_A;
z_0 = 1.6*10**5;
n=300;
output;
run;
data parametrai_B;
input alpha_1 alpha_2 alpha_3;
cards;
0.5 0.52 0.55
;
run;
data parametrai_C;
input teta1 teta2 teta3;
cards;
49953 35119 5428
;
run;
proc iml;
/*nuskaitome parametrus*/
use parametrai_A;
read all into PA;
use parametrai_B;
read all into PB;
use parametrai_C;
read all into PC;
/*generuojame*/

n = PA[2];
q = ncol(PB);
z_0 = PA[1];
sample = j(n,q+5,0);
cols = 0:q;
cols = char(cols);
cols = concat(j(1,q+1,'T'),cols) || {'T'} || {'V'} || {'Z'} || {'A'}  ;
seed = 0;
lambda_A = 1.6;
do i = 1 to n;
      A = (1/lambda_A)*ranexp(seed);
      sample[i,1] = z_0*A;
      sample[i,q+2]=sample[i,1];
      sample[i,q+3] = 0;
      do k = 2 to q + 1;
            sample[i,k] = ((((A*10**11)*PC[k-1])**PB[k-1])*(PB[k-1]+1)*(-
1)*log(1-uniform(seed)))**(1/(PB[k-1]+1));
            if (sample[i,k]<sample[i,q+2]) then
            do;
                  sample[i,q+2] = sample[i,k];
                  sample[i,q+3] = k-1;
            end;
      end;
      sample[i,q+4] = (sample[i,q+2]/A)*10**(-11);
      sample[i,q+5] = A;
end;

/*isvedame duomenis i lentele*/
CREATE sampleSet FROM sample[c = cols];
APPEND from sample;
quit;
proc sql;
```

```
create table iverciai
 as select t, v from sampleset
  where t > 80000;
quit;
data iverciai;
set iverciai;
retain SumS(0);
SumS = SumS+1;
run;
/*apskaičiuosime Sn(t)-patikimumo f-jos S(t) įvertį */
data iverciai;
set iverciai;
n =300;
Sn = (1/n)*SumS;
run;
/* paskutine Sn reiksme*/
proc sql;
create table Sn_ivertis
as select Sn, SumS from iverciai
where SumS = ( select max(SumS) from iverciai );
quit;
proc print data=Sn_ivertis;
   id Sn SumS;
   by Sn SumS;
run;
proc print data=Sampleset;
id  T V Z A;
run;

/*Apkskaičiuojame k-tojo tipo gedimo tikimybės intervale [0,t] įverčius,
Pn^k(t)*/
proc freq data=iverciai order=internal;
tables V/chisq nocum ;
run;

proc means data=sampleset;
var T Z;
run;
```

11. Rezultatai

- Patikimumo funkcijos $S(t)$ įvertis

```
The SAS System           12:39 Monday, May 29, 2006  31

  Sn     SumS

 0.26     78
```

- Generuojami duomenys

T	V	Z	A	T_0	T_1	T_2	T_3
50836.05	1	.000001286	0.39522	63235.67	50836.05	378076.04	230242.27
50150.63	3	.000001574	0.31864	50982.12	100533.65	115042.32	50150.63
26840.87	0	.000001600	0.16776	26840.87	53956.65	286004.37	79529.99
6872.02	1	.000000128	0.53618	85788.90	6872.02	45494.37	343118.16
48834.87	0	.000001600	0.30522	48834.87	350859.60	235350.35	245788.99
127023.46	3	.000001158	1.09721	175553.01	149918.17	163575.90	127023.46
17797.52	0	.000001600	0.11123	17797.52	108624.95	135883.22	85762.69
11732.31	1	.000000775	0.15138	24220.77	11732.31	34573.48	87336.16
34436.01	0	.000001600	0.21523	34436.01	55648.54	181072.17	161443.76
56400.84	0	.000001600	0.35251	56400.84	141758.64	108741.62	59388.53
55483.67	0	.000001600	0.34677	55483.67	322780.78	104033.95	167915.37
29103.74	0	.000001600	0.18190	29103.74	54816.57	205902.53	151746.73
50497.69	1	.000000656	0.76977	123163.50	50497.69	385300.01	56835.77
38387.42	0	.000001600	0.23992	38387.42	204657.19	44676.93	55353.01
147505.89	2	.000000668	2.20777	353243.93	309544.22	147505.89	528090.27
62264.15	1	.000001179	0.52832	84530.73	62264.15	339608.64	86741.20
115246.15	0	.000001600	0.72029	115246.15	242263.79	137585.12	431397.65
22423.49	0	.000001600	0.14015	22423.49	151909.61	63628.94	88189.03
10940.91	0	.000001600	0.06838	10940.91	42813.88	41282.46	158594.20
211227.20	0	.000001600	1.32017	211227.20	222747.55	470169.05	211368.65
2171.64	0	.000001600	0.01357	2171.64	56825.85	25318.45	5288.60
164258.79	0	.000001600	1.02662	164258.79	311467.27	723319.02	184780.69
5362.55	0	.000001600	0.03352	5362.55	52124.38	44076.15	35800.87
74765.42	0	.000001600	0.46728	74765.42	138031.98	345347.37	116376.94
64256.82	0	.000001600	0.40161	64256.82	198211.29	205883.68	84175.57
79494.81	1	.000000695	1.14413	183061.53	79494.81	278033.76	138033.63
46597.57	0	.000001600	0.29123	46597.57	215175.51	279540.17	184641.68
163625.99	2	.000001416	1.15532	184850.46	206468.29	163625.99	366282.97
149760.57	1	.000001597	0.93751	150002.20	149760.57	291902.38	253793.94
60762.05	1	.000000371	1.63684	261894.48	60762.05	127442.16	423536.67
76102.10	0	.000001600	0.47564	76102.10	314405.62	195223.59	148715.53
72044.95	1	.000000932	0.77288	123661.12	72044.95	218015.88	156679.98
140757.27	3	.000000957	1.47136	235417.15	276754.34	237576.96	140757.27
19581.13	0	.000001600	0.12238	19581.13	134803.85	125647.29	55953.15
148542.71	1	.000001293	1.14902	183842.55	148542.71	192154.84	148593.45
288370.62	3	.000001249	2.30869	369390.09	294604.56	586778.28	288370.62
116001.07	2	.000000544	2.13240	341183.28	508947.93	116001.07	263158.78
50198.33	1	.000001566	0.32061	51298.01	50198.33	86389.63	98145.29
21257.40	1	.000000180	1.18204	189126.87	21257.40	223377.39	167643.28
23587.80	0	.000001600	0.14742	23587.80	31224.38	245876.63	167285.75
93362.42	1	.000001291	0.72297	115675.33	93362.42	203699.70	208966.20
9623.05	0	.000001600	0.06014	9623.05	151108.08	38169.20	48907.40
28697.96	0	.000001600	0.17936	28697.96	118534.92	179730.46	194336.31
37721.21	0	.000001600	0.23576	37721.21	141982.83	107934.51	49974.69
102516.47	3	.000001380	0.74277	118843.27	295181.47	227062.49	102516.47
89201.10	2	.000000625	1.42698	228317.60	160730.63	89201.10	572235.21
20692.42	3	.000000928	0.22310	35695.34	90440.98	58345.96	20692.42
96531.16	1	.000001418	0.68092	108947.09	96531.16	235051.79	360687.50
27243.30	0	.000001600	0.17027	27243.30	67958.43	245155.42	62913.58

T	V	Z	A	T_0	T_1	T_2	T_3
12364.16	0	.000001600	0.07728	12364.16	109092.64	19013.20	100688.41
43545.27	0	.000001600	0.27216	43545.27	301225.62	87849.44	108254.41
3139.64	0	.000001600	0.01962	3139.64	79676.63	32200.52	115585.23
11530.42	0	.000001600	0.07207	11530.42	232103.06	36972.60	188251.27
10290.88	0	.000001600	0.06432	10290.88	13656.92	68643.79	35846.14
120802.19	2	.000001257	0.96095	153751.65	305938.04	120802.19	251666.98
35781.64	2	.000001212	0.29520	47232.79	69520.85	35781.64	36046.49
15633.62	3	.000000513	0.30503	48805.02	43139.80	121674.03	15633.62
8178.12	0	.000001600	0.05111	8178.12	38679.99	162608.10	94796.68
9116.40	1	.000000394	0.23119	36989.72	9116.40	92685.49	233061.39
4543.93	3	.000000175	0.25898	41436.55	47820.45	119072.24	4543.93
96298.73	3	.000001002	0.96152	153843.24	207642.02	278808.76	96298.73
72221.44	1	.000000906	0.79751	127601.80	72221.44	187149.55	343712.91
64497.03	3	.000000658	0.98059	156894.06	196928.32	306201.82	64497.03
16151.45	1	.000000935	0.17278	27644.44	16151.45	182586.63	159323.46
85617.79	0	.000001600	0.53511	85617.79	99638.10	271825.84	232747.79
49088.50	3	.000000843	0.58226	93161.18	340103.73	132853.82	49088.50
41634.76	1	.000001495	0.27848	44556.46	41634.76	47691.46	271341.54
70027.66	1	.000000323	2.16627	346603.10	70027.66	288597.27	279963.94
28177.05	2	.000000390	0.72304	115685.88	116974.66	28177.05	32618.89
1023.42	0	.000001600	0.00640	1023.42	17548.30	33704.49	57433.38
41500.75	0	.000001600	0.25938	41500.75	224811.38	169417.63	180441.94
22017.24	0	.000001600	0.13761	22017.24	74410.07	131743.12	26709.36
72195.17	0	.000001600	0.45122	72195.17	107357.99	264443.30	121191.70
154782.49	0	.000001600	0.96739	154782.49	193545.84	187294.58	304028.54
26223.62	3	.000000473	0.55401	88641.31	172216.04	144557.21	26223.62
4382.64	0	.000001600	0.02739	4382.64	109328.42	137232.01	14833.45
67040.95	0	.000001600	0.41901	67040.95	158571.75	197617.25	91939.06
16441.21	2	.000000388	0.42394	67830.70	22863.49	16441.21	33660.46
96950.93	0	.000001600	0.60594	96950.93	238185.55	284263.35	160985.90
7255.84	3	.000001044	0.06947	11114.97	39304.94	25868.06	7255.84
18479.45	0	.000001600	0.11550	18479.45	32582.29	215278.72	22378.35
129214.35	1	.000001441	0.89689	143502.34	129214.35	226539.82	230913.18
8503.66	0	.000001600	0.05315	8503.66	45883.39	114122.58	21929.83
124858.95	0	.000001600	0.78037	124858.95	334116.09	435645.31	455445.22
116218.63	3	.000001223	0.95026	152041.10	135111.62	162288.11	116218.63
13410.48	1	.000000886	0.15129	24206.05	13410.48	84410.72	141690.41
52555.18	1	.000000953	0.55123	88196.57	52555.18	351785.44	193949.19
4583.86	0	.000001600	0.02865	4583.86	69700.78	35224.35	53428.15
14751.56	2	.000000451	0.32703	52325.47	99312.28	14751.56	75597.15
47954.02	0	.000001600	0.29971	47954.02	125313.34	189338.54	55933.56
69782.81	0	.000001600	0.43614	69782.81	240249.61	112717.16	156912.80
21699.68	0	.000001600	0.13562	21699.68	136098.94	282545.56	39230.75
168712.96	0	.000001600	1.05446	168712.96	281345.47	459816.81	189834.23
26366.72	2	.000001128	0.23369	37391.18	126492.11	26366.72	215543.50
7987.11	0	.000001600	0.04992	7987.11	125018.97	108376.40	59365.61
10700.83	0	.000001600	0.06688	10700.83	182060.15	86576.72	140877.35
83188.91	3	.000000689	1.20659	193054.03	256944.96	529749.84	83188.91
108968.76	3	.000001445	0.75395	120632.46	267926.80	124292.35	108968.76

T	V	Z	A	T_0	T_1	T_2	T_3
16145.83	0	.000001600	0.10091	16145.83	215222.02	25105.73	96744.83
51669.07	0	.000001600	0.32293	51669.07	157634.03	173180.96	278105.15
60924.39	0	.000001600	0.38078	60924.39	290598.78	193906.32	214477.78
46990.88	0	.000001600	0.29369	46990.88	396384.06	211651.11	79436.45
2977.38	0	.000001600	0.01861	2977.38	15890.21	34061.16	44054.24
3564.24	0	.000001600	0.02228	3564.24	24131.90	81724.70	100046.27
18549.45	3	.000000105	1.77203	283524.78	78044.29	333633.67	18549.45
41217.12	1	.000000480	0.85780	137247.87	41217.12	87185.51	242820.68
97602.50	2	.000000529	1.84340	294943.87	360935.73	97602.50	270429.18
12945.19	3	.000000933	0.13870	22192.37	127468.57	189631.71	12945.19
70138.77	3	.000000372	1.88417	301466.53	335884.85	369465.89	70138.77
16243.28	1	.000000912	0.17813	28501.14	16243.28	135658.70	132023.62
62502.96	0	.000001600	0.39064	62502.96	279481.98	104745.81	281843.04
22463.87	3	.000000239	0.94136	150617.77	158365.72	105080.07	22463.87
12711.77	1	.000000225	0.56531	90449.59	12711.77	153659.16	181263.43
107135.77	0	.000001600	0.66960	107135.77	157573.35	370422.73	169772.56
19129.44	1	.000000897	0.21324	34117.90	19129.44	232319.77	100177.87
23265.79	2	.000000287	0.81146	129833.61	44043.67	23265.79	65064.52
20493.98	3	.000000172	1.18879	190205.88	102646.78	565486.91	20493.98
19629.41	0	.000001600	0.12268	19629.41	33793.62	237112.28	139869.40
11675.08	2	.000000334	0.34984	55974.61	50108.01	11675.08	182528.51
15889.90	0	.000001600	0.09931	15889.90	248752.38	47833.76	109578.56
56065.35	3	.000000869	0.64553	103284.37	154123.02	125834.02	56065.35
74038.47	1	.000000186	3.98823	638117.23	74038.47	410767.85	443995.92
5712.93	0	.000001600	0.03571	5712.93	50523.77	56832.43	58315.40
302232.27	0	.000001600	1.88895	302232.27	320982.02	767945.08	507447.74
24877.94	0	.000001600	0.15549	24877.94	34779.71	58502.54	28129.42
112636.18	0	.000001600	0.70398	112636.18	112820.50	638716.42	122517.13
105559.04	1	.000001141	0.92544	148069.71	105559.04	264018.82	213907.39
6692.98	0	.000001600	0.04183	6692.98	18141.04	142666.38	93251.07
53459.83	0	.000001600	0.33412	53459.83	173417.19	327202.27	63492.51
51566.41	0	.000001600	0.32229	51566.41	70513.98	72124.15	190855.09
126376.56	0	.000001600	0.78985	126376.56	392684.03	263657.18	141750.36
8713.23	0	.000001600	0.05446	8713.23	38761.71	16956.72	16818.88
145578.61	2	.000001509	0.96500	154400.12	379688.62	145578.61	275662.09
18136.26	0	.000001600	0.11335	18136.26	58168.00	65130.15	81794.99
252197.37	2	.000001434	1.75887	281419.23	256833.40	252197.37	473439.07
118503.94	2	.000000617	1.92069	307311.06	170253.31	118503.94	566670.81
71820.43	1	.000001376	0.52187	83499.30	71820.43	396936.23	93815.15
53418.89	3	.000001228	0.43504	69605.69	165762.56	148814.93	53418.89
227052.81	0	.000001600	1.41908	227052.81	230946.24	287664.14	391020.70
81055.75	1	.000000561	1.44425	231079.43	81055.75	168768.85	121869.77
21885.16	2	.000000394	0.55496	88794.03	51321.11	21885.16	34828.23
20844.40	2	.000000987	0.21115	33783.52	163978.34	20844.40	191387.79
196943.82	3	.000001279	1.54037	246459.30	477659.14	257968.00	196943.82
20613.08	0	.000001600	0.12883	20613.08	110593.24	130817.52	55462.43
20219.94	0	.000001600	0.12637	20219.94	44883.85	87131.16	65630.38
32167.61	0	.000001600	0.20105	32167.61	87310.54	318208.01	97532.22
13949.13	1	.000000186	0.75142	120226.77	13949.13	154252.53	275840.12

T	V	Z	A	T_0	T_1	T_2	T_3
99561.69	3	.000000776	1.28229	205166.82	233675.31	124078.98	99561.69
56358.57	1	.000001474	0.38246	61193.43	56358.57	217612.37	186279.45
71010.91	0	.000001600	0.44382	71010.91	434787.07	214005.92	78171.36
23649.00	2	.000000171	1.38694	221911.02	114463.27	23649.00	150566.56
54800.82	0	.000001600	0.34251	54800.82	327921.53	90341.67	270520.18
2221.43	0	.000001600	0.01388	2221.43	26530.95	81104.47	27052.00
111653.30	0	.000001600	0.69783	111653.30	351970.31	288353.05	167822.75
31690.23	0	.000001600	0.19806	31690.23	77798.27	108932.93	147042.54
11562.18	3	.000000407	0.28376	45401.18	93855.32	103060.19	11562.18
59029.03	0	.000001600	0.36893	59029.03	190827.76	61685.93	71942.13
66616.26	0	.000001600	0.41635	66616.26	339048.01	345527.05	165331.55
135599.10	1	.000000625	2.16814	346902.98	135599.10	489997.59	513256.06
7939.91	0	.000001600	0.04962	7939.91	111299.22	94371.25	22310.97
29023.78	0	.000001600	0.18140	29023.78	256141.81	40459.17	104056.90
58917.96	0	.000001600	0.36824	58917.96	190193.61	406510.44	125071.56
68194.82	2	.000000778	0.87625	140199.56	245199.88	68194.82	99622.39
14642.05	2	.000000278	0.52698	84316.57	41783.93	14642.05	412101.77
3767.30	0	.000001600	0.02355	3767.30	51406.55	181431.50	69674.57
69724.74	0	.000001600	0.43578	69724.74	199067.62	70924.48	103419.56
44941.74	3	.000001145	0.39246	62794.05	169618.59	72023.36	44941.74
52975.32	0	.000001600	0.33110	52975.32	53373.73	216987.78	210152.69
27775.74	2	.000000293	0.94863	151781.40	697659.51	27775.74	54573.44
26750.38	1	.000001480	0.18072	28914.99	26750.38	170317.52	100934.74
44436.86	0	.000001600	0.27773	44436.86	82777.25	316567.14	71060.68
9045.48	1	.000000973	0.09300	14879.59	9045.48	153793.64	39829.30
22663.02	0	.000001600	0.14164	22663.02	261339.27	128532.20	104620.91
27784.74	1	.000000297	0.93557	149691.58	27784.74	83105.59	102353.47
53700.43	3	.000000501	1.07096	171353.87	371259.91	459768.81	53700.43
86293.33	1	.000001234	0.69907	111851.84	86293.33	379750.02	160724.62
54271.21	0	.000001600	0.33920	54271.21	194223.62	163793.27	288294.93
158722.61	2	.000000709	2.23881	358209.70	519820.09	158722.61	257140.54
26680.10	0	.000001600	0.16675	26680.10	106748.91	229108.41	215183.54
68329.87	0	.000001600	0.42706	68329.87	352516.48	93238.18	170680.74
131926.78	2	.000001062	1.24228	198764.37	206893.62	131926.78	147259.54
9786.86	1	.000000687	0.14246	22793.00	9786.86	115943.15	28483.79
105638.70	2	.000001053	1.00309	160495.12	328130.73	105638.70	298813.17
25525.77	0	.000001600	0.15954	25525.77	49127.24	91721.54	92498.08
45542.35	0	.000001600	0.28464	45542.35	158354.97	73313.54	128988.19
9301.90	0	.000001600	0.05814	9301.90	127037.43	39919.66	52221.43
10361.82	0	.000001600	0.06476	10361.82	102621.78	82906.08	43963.38
39666.94	0	.000001600	0.24792	39666.94	248333.27	504373.07	57670.71
52489.83	0	.000001600	0.32806	52489.83	136820.55	211960.81	79429.02
132966.00	1	.000001366	0.97368	155789.48	132966.00	156338.25	241775.12
96688.56	3	.000000480	2.01376	322201.19	452770.45	994290.60	96688.56
26173.42	0	.000001600	0.16358	26173.42	83326.12	108730.71	69550.86
25304.73	3	.000000460	0.55007	88010.49	357178.41	212422.52	25304.73
39604.79	3	.000000825	0.47989	76783.05	74772.93	89266.39	39604.79
140021.04	2	.000000779	1.79840	287744.79	253102.52	140021.04	251794.89
105791.46	3	.000000762	1.38854	222166.08	152769.67	414431.71	105791.46

T	V	Z	A	T_0	T_1	T_2	T_3
16711.16	0	.000001600	0.10444	16711.16	100948.27	151253.98	197770.08
17089.05	0	.000001600	0.10681	17089.05	55515.62	153933.37	134100.68
2507.93	3	.000000025	0.99199	158718.87	319277.95	342724.64	2507.93
16962.22	1	.000000217	0.78162	125059.57	16962.22	177912.89	441624.73
2456.22	0	.000001600	0.01535	2456.22	68797.24	71340.37	57153.15
10422.01	0	.000001600	0.06514	10422.01	66155.52	125402.97	19108.35
25302.82	0	.000001600	0.15814	25302.82	122040.52	203987.36	191220.60
86415.91	0	.000001600	0.54010	86415.91	371735.38	102454.66	201793.57
5298.94	2	.000000244	0.21687	34699.27	229998.81	5298.94	110609.76
1780.42	3	.000000074	0.23929	38285.76	16827.49	81778.70	1780.42
71434.29	0	.000001600	0.44646	71434.29	346555.54	449388.17	77694.66
60691.12	0	.000001600	0.37932	60691.12	113297.23	107821.93	156083.52
129579.06	1	.000000860	1.50622	240994.83	129579.06	483232.53	216379.85
42931.32	1	.000000721	0.59544	95270.11	42931.32	459100.51	225640.10
77550.64	2	.000001482	0.52323	83717.31	358278.41	77550.64	174338.75
4695.33	0	.000001600	0.02935	4695.33	94445.87	41365.84	36832.83
10765.80	0	.000001600	0.06729	10765.80	104626.07	39408.07	123023.19
21760.86	0	.000001600	0.13601	21760.86	36083.46	57007.20	93877.48
37616.92	2	.000001244	0.30241	48385.02	73138.82	37616.92	83464.24
240293.01	3	.000001188	2.02290	323664.33	281552.76	809791.69	240293.01
122799.45	1	.000001394	0.88117	140987.77	122799.45	629057.67	270194.13
5894.82	2	.000000043	1.36245	217991.23	286080.65	5894.82	120641.68
24018.48	0	.000001600	0.15012	24018.48	117953.55	67896.74	208049.13
113346.60	0	.000001600	0.70842	113346.60	193682.55	858930.14	356619.87
116039.68	0	.000001600	0.72525	116039.68	155400.37	224460.66	284003.00
36332.73	0	.000001600	0.22708	36332.73	120158.72	122271.26	61346.16
32478.31	0	.000001600	0.20299	32478.31	321964.77	198057.37	106276.04
292263.01	0	.000001600	1.82664	292263.01	471950.18	945371.57	375337.38
67283.16	1	.000000405	1.66252	266003.92	67283.16	221030.14	295850.19
19669.96	1	.000000679	0.28978	46364.70	19669.96	134000.73	165628.63
9623.73	3	.000000401	0.24000	38399.31	56404.83	60882.75	9623.73
20394.54	1	.000000343	0.59544	95269.66	20394.54	78367.69	329377.16
12083.01	0	.000001600	0.07552	12083.01	42148.04	54549.75	39609.52
27438.16	3	.000001139	0.24095	38551.82	92307.52	163546.06	27438.16
207011.73	1	.000000967	2.14000	342400.14	207011.73	224584.69	233687.05
191657.25	0	.000001600	1.19786	191657.25	452977.25	778871.04	383208.96
32107.63	0	.000001600	0.20067	32107.63	108211.03	381201.51	108760.66
36727.51	0	.000001600	0.22955	36727.51	115784.20	273652.24	65101.75
68062.22	0	.000001600	0.42539	68062.22	153575.76	130771.34	186266.02
35822.54	1	.000001239	0.28916	46266.14	35822.54	92398.06	215262.26
103153.38	0	.000001600	0.64471	103153.38	272605.75	782998.30	139559.97
46865.12	3	.000000840	0.55809	89294.91	85526.20	234854.88	46865.12
80102.09	1	.000000489	1.63762	262019.61	80102.09	798493.83	630092.66
51425.07	2	.000001575	0.32658	52252.11	87253.40	51425.07	160763.96
64066.99	0	.000001600	0.40042	64066.99	167504.70	405402.85	341382.23
73025.87	0	.000001600	0.45641	73025.87	247194.33	266568.64	98878.29
92403.00	3	.000001295	0.71353	114164.22	326156.75	258182.35	92403.00
70113.78	1	.000000493	1.42351	227762.16	70113.78	479322.29	144437.55
142714.16	3	.000000821	1.73808	278092.24	513279.26	372526.95	142714.16

T	V	Z	A	T_0	T_1	T_2	T_3
148052.71	3	.000001245	1.18899	190238.79	387042.97	312466.99	148052.71
11124.61	0	.000001600	0.06953	11124.61	68526.83	15627.38	106161.75
64653.18	2	.000001347	0.47982	76771.81	195111.04	64653.18	103873.76
64500.93	3	.000001079	0.59755	95607.75	267266.26	645060.52	64500.93
125923.16	0	.000001600	0.78702	125923.16	287157.28	177161.21	295850.76
3067.29	0	.000001600	0.01917	3067.29	59707.49	60298.10	70331.71
59040.53	2	.000001077	0.54816	87705.36	445473.64	59040.53	205768.74
42706.40	2	.000000600	0.71137	113818.49	242899.85	42706.40	153724.29
51871.06	0	.000001600	0.32419	51871.06	173708.75	225102.14	221898.32
120632.10	0	.000001600	0.75395	120632.10	143910.45	248793.81	166868.99
91506.80	3	.000000692	1.32283	211652.92	910772.08	473043.45	91506.80
75634.40	2	.000001031	0.73395	117431.99	130079.07	75634.40	149267.11
34656.31	2	.000000518	0.66940	107103.80	196749.21	34656.31	264646.38
18372.25	0	.000001600	0.11483	18372.25	25213.60	54323.66	96447.57
1070.67	0	.000001600	0.00669	1070.67	33902.53	69939.81	50255.73
12789.99	2	.000001065	0.12010	19216.01	44916.94	12789.99	123547.35
38543.27	0	.000001600	0.24090	38543.27	94728.73	172094.48	134044.21
102707.10	2	.000000731	1.40458	224732.95	188121.84	102707.10	358205.85
17240.01	3	.000000885	0.19487	31179.59	200064.52	96138.85	17240.01
25823.49	0	.000001600	0.16140	25823.49	40042.83	104285.91	113428.78
14064.17	3	.000000380	0.36984	59174.85	110160.63	296798.11	14064.17
27671.68	1	.000001528	0.18113	28980.94	27671.68	248726.22	104227.91
18097.37	0	.000001600	0.11311	18097.37	52072.10	150333.00	105165.79
50394.14	0	.000001600	0.31496	50394.14	93073.25	307582.20	140191.62
105386.23	1	.000001363	0.77321	123714.10	105386.23	471781.25	210514.54
17869.22	0	.000001600	0.11168	17869.22	64820.50	94777.79	166847.11
177296.22	2	.000001143	1.55073	248116.03	409937.31	177296.22	201134.41
70622.84	1	.000001175	0.60091	96146.32	70622.84	204428.65	136664.72
32275.88	3	.000000228	1.41797	226874.52	215472.62	421344.90	32275.88
6354.13	0	.000001600	0.03971	6354.13	172447.47	171836.86	132991.22
20407.84	0	.000001600	0.12755	20407.84	89052.18	285619.42	101263.83
164512.05	0	.000001600	1.02820	164512.05	324452.73	304637.28	261139.55
29604.75	3	.000000757	0.39089	62542.78	108081.57	257036.44	29604.75
30413.35	0	.000001600	0.19008	30413.35	264564.51	99448.47	109388.60
124996.52	1	.000001134	1.10215	176343.23	124996.52	197537.87	286500.28
99234.72	1	.000000849	1.16943	187108.55	99234.72	118728.39	106065.94
64190.06	2	.000001116	0.57532	92051.36	199581.10	64190.06	297638.83
89406.07	3	.000000693	1.29096	206553.41	360226.75	160728.55	89406.07
48774.68	1	.000000581	0.83927	134283.71	48774.68	90253.71	89561.68
42040.22	0	.000001600	0.26275	42040.22	60041.16	160241.35	165230.80
83161.45	0	.000001600	0.51976	83161.45	132506.21	129231.51	187783.03
140617.57	2	.000001282	1.09668	175468.54	317107.60	140617.57	505726.01
91333.17	2	.000001034	0.88361	141377.64	182663.53	91333.17	264356.18
159394.64	3	.000001096	1.45411	232657.56	162723.81	387311.60	159394.64
65170.34	3	.000001163	0.56019	89630.05	286501.15	312308.26	65170.34
93168.10	0	.000001600	0.58230	93168.10	200410.51	197739.54	148019.28
21093.27	0	.000001600	0.13183	21093.27	44598.41	161934.08	101054.04
50735.68	0	.000001600	0.31710	50735.68	186119.18	322873.49	205153.41
5428.12	3	.000000371	0.14641	23425.42	29527.47	78707.43	5428.12

T	V	Z	A	T_0	T_1	T_2	T_3
36610.73	1	.000000416	0.88097	140955.24	36610.73	86759.34	120092.58
22837.03	1	.000000289	0.79038	126460.50	22837.03	193117.02	32597.22
18435.71	0	.000001600	0.11522	18435.71	53746.50	127197.78	126183.33
60146.66	0	.000001600	0.37592	60146.66	148281.24	216869.48	272571.74
33064.37	2	.000000764	0.43264	69222.97	70718.78	33064.37	277918.53
7473.88	2	.000000793	0.09430	15088.05	143892.97	7473.88	112861.58

- $P^{(k)}(t)$ – k -tojo tipo gedimo tikimybės intervale $[0;t]$ įverčiai

```
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                               The FREQ Procedure

                           V    Frequency     Percent
                           ffffffffffffffffffffffffff
                           0          25       32.05
                           1          17       21.79
                           2          17       21.79
                           3          19       24.36


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                              The MEANS Procedure

Variable      N            Mean         Std Dev         Minimum         Maximum
ffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffff
T           300        59153.95        54507.92         1023.42       302232.27
Z           300    1.2058643E-6    4.8921239E-7    2.5281781E-8          1.6E-6
ffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffff
```

12. Išvados

- Gautos statistikų $\tilde{S}_n(t)$ ir $\tilde{P}_n^{(k)}(t)$ H. Dinges tipo transformacijos :

$$P\left\{\tilde{S}_n(t) \le k\right\} \sim \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\sqrt{n+1}A\left(\frac{k+1}{n+1}, S(t)\right)} e^{-\frac{v^2}{2}} dv,$$

ir

$$P\left\{A\left(\frac{\tilde{S}_n+1}{n+1}, S(t)\right) < x\right\} \approx \frac{1}{\sqrt{2\pi\frac{1}{n+1}}} \int_{-\infty}^{x} e^{-\frac{u^2}{2\frac{1}{n+1}}} du.$$

- Pasinaudojant L. N. Bolševo teorema yra gauta lygybė :

$$P\left\{\sqrt{n}\left(\tilde{S}_n(t) - S(t)\right) < x\right\} = \sum_{k=0}^{m} \frac{y^k}{k!} e^{-y} + R_n(m),$$

kur

$$R_n(m) = \begin{cases} O\left(\frac{1}{M^2}\right), & kai \quad y = y_1 = \frac{S(t)M}{2-S(t)} \\ O\left(\frac{1}{M^4}\right), & kai \quad y = \frac{y_1}{1+\left[m(m+2)+my_1-2y_1^2\right]/6M^2} \end{cases},$$

čia $M = 2n - m$.

- Sudaryta programa su SAS paketu, kuri generuoja padangų gedimo momentus ir apskaičiuoja patikimumo funkcijos $S(t)$ ir $P^{(k)}(t)$ – k-tojo tipo gedimo tikimybės intervale $[0;t]$ įverčius, laikant, kad trauminiai gedimo momentai $\left(T^{(1)}, T^{(2)}, T^{(3)}\right)$ pasiskirstę pagal Veibulo dėsnį, natūralaus susidėvėjimo gedimo momentai $T^{(0)}$ pasiskirstę pagal eksponentinį dėsnį.

13. Summary

The work represents a tire exploiting model including both tire degradation and tire failures. The estimators of reliability characteristics: reliability function $S(t)$ and $P^{(k)}(t)$ – the probability of k traumatic failure type were analysed. Natural tire wear and traumatic failure time data are analysed with SAS software.

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