DOI: 10.46300/9109.2024.18.2 Volume 18, 2024

Introducing Computational Thinking and Algebraic Thinking in the European Educational Systems

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Received: February 27, 2023. Revised: December 16, 2023. Accepted: February 19, 2024. Published: March 28, 2024.

Abstract—Computational Thinking is part of the new curriculum in many countries and this new competence is often combined with Algebraic Thinking. Both types of thinking are part of the core of Mathematics and Computer Science. Algebraic Thinking is linked to acquiring the ability to represent and generalize patterns in any application area. Furthermore, the ability to communicate a mathematical argument, using the necessary language and symbolism, is a skill that is dependent on training in this type of thinking. Although Algebraic Thinking can be developed at different levels, and it is also developed at university levels, more and more countries see it as a basic mode of thought that should be encouraged from early childhood education. Algebraic Thinking has also a close relationship with Computational Thinking, and they are currently united in different situations, such as the international PISA student evaluation tests. We argue in this paper that this is a transversal competence that can be practiced in any subject and at any age. Sometimes combined with the process of teaching Mathematics. It is essential, in our opinion, to strengthen the inclusion of strategies that encourage students to reflect deeply on the concepts, theories, and applications they are learning, giving rise, among others, to number sense and abstraction. In this paper, we present the implementation of these two types of thinking, algebraic and computational, in the pre-university curriculum, particularly in Spain, within a

European project. In this project, we seek to create more appropriate learning approaches for those who are often disadvantaged and help them to take advantage of Computational Thinking and Algebraic Thinking and, therefore, STEM knowledge, helping to a stronger and more equal society. We analyze its status and its relationship with the concepts taught in the different courses, although focusing on the subject of Mathematics.

Keywords— Algebraic Thinking, Computational Thinking, Mathematics, cross-curricular competence, digital competence, STEAM, technology, learning.

I. Introduction

EDUCATIONAL systems are increasingly dynamic in the STEM areas. One aspect of this process is the adaptation of the study plans and the teaching and learning process. Due to these changes, students, from an early age, have to begin to manage skills that allow them to function effectively in an environment rich in information, very dynamic, and in constant change, both from the social and economic point of view. All of these changes combine both the digital challenge in daily life and the development of basic skills that must continue to be studied and that can be applied to any work environment, [1].

At the beginning of the 21st century, the concept of Computational Thinking was rescued in USA, [2], relating it to Computer Science and digital change. Subsequently, it has taken root in the most pioneering schools and countries and is now part of the PISA (Program for International Student Assessment) educational tests, [3]. Algebraic Thinking began to gain prominence in 1989 and connected with continuous technological advances, [4], although it is based on mathematical thinking that had been applied in schools long before.

On the other hand, Computational Thinking has gained ground in education systems and has established itself in recent years as a new competence to be introduced into the curricula of the most advanced countries. Its introduction into the curriculum in many countries has taken disparate approaches, with different speeds of action and with different visions of its implementation. In certain countries, this thinking is part of the educational system as part of Digital Technologies, [5], and can be introduced as a transversal competence, [6]. Specifically, in Spain, the new education law, which came into effect in the 2022-2023 academic year in the first phase, establishes the Computational Thinking (CT) as part of the Mathematics subject. Within the same subject, the Spanish government has joined the modeling concept, although both of them, computational thinking and modeling, should not be interpreted as an exclusive part of Mathematics, but must also be implemented in the rest of the blocks of the curricula, [7].

Before Computational Thinking, Algebraic Thinking (AT) began to be forged, with two new currents, Pre-Algebra and Early-Algebra, which promoted starting Algebraic Thinking in Primary Education. This way of thinking proposed activities that develop the ability to generalize and recommend working with patterns and the study of their regularities. In this way, algebraic thinking could be developed by students of Primary Education, and, in general, at any early age, [8], [9], being algebra in the heart of mathematics, [10]. Thus, some researchers observed that Primary and Secondary students could consider arithmetic operations as functions, algebraically elaborate and symbolize conjectures about basic arithmetic relationships, use algebraic representations to solve problems, use letters as variables to represent quantities, etc. For this reason, it has also been proposed that, in the early grades, algebraic thinking can be introduced with tasks and exercises that "include relationships between quantities, identification of structures, generalization, problem-solving, modeling, justification, testing, and prediction", [11].

The formalization of patterns, functions, and generalizations is one of the ways to develop algebraic thinking in students according to the National Council of Teachers of Mathematics (NCTM) in the USA, [12], which includes it in its principles and standards. And it is these concepts that are precisely closely related to Computational Thinking. To develop Algebraic Thinking, the NCTM proposes that mathematics programs in the early years of schooling should be oriented to enable students to understand patterns, relationships, and functions.

Being part of an international project to create an analytics-based learning environment, that promotes Computational Thinking along with mathematical problem solving, we try to implement computational thinking and algebraic thinking in schools. Thus, with this paper we try to present the implementation of both types of thinking, first presenting each thought and then how they are being developed in the national curricula, to end with the contribution of the research we carry out offering new tools to teachers, and participating in their training.

This approach of researching both thoughts, along with the use of data analytics, can improve the digital skills and self-awareness of pre-university students based on learning analytics. Furthermore, the use of interactive tasks as a tool to implement the learning of certain skills in students can improve the development of computational thinking as well as avoid a move away from mathematical concepts.

There is a significant overlap between AT and CT and the integration of these two areas of thinking into the math curriculum can promote students' competency, [13]. In the literature several articles relate the term computational thinking to the area of mathematics; although if these articles are read in depth, it can be seen that the studies were, in general, not related to any specific topic of mathematics. In the majority of these studies that relate both concepts, there is only a generic relationship and no mathematical topic or concept is delved into, [14].

In studies that relate computational thinking with mathematics, although they are carried out at different levels of education, the students who participate are, in most of these studies, primary school students. Furthermore, there are no experiences that have been developed for the training of teachers in these fields. But in cases in which a mathematical concept is referred to, it is usually related to algebra or two-dimensional geometry, [15], [16]. In this sense, our research aims to fill that gap that still exists in terms of mathematical skills, objectives, and capacities that have barely been explored in scientific research.

II. Algebraic Thinking

Recently, there has been a shift from teaching algebra, which is characterized through specific content areas, to promoting algebraic thinking as process, [17]. Algebraic Thinking does not have a single definition, and, although they share similar characteristics, several authors have tried to define this type of thinking. One of these definitions comes from the year 2000 [18], where authors point out that Algebraic Thinking involves representing, generalizing, and formalizing patterns and regularities in any aspect of mathematics. These authors say that as this reasoning develops, progress is made in the use of language and symbolism necessary to support and communicate Algebraic Thinking, and they point particularly to equations, variables, and functions.

Another author who has tried to contribute to this field is Radford, who at the beginning of the 21st century, characterized Algebraic Thinking through three interrelated elements, [19]: the sense of indeterminacy (unknowns, variables, and parameters), analyticity (as a way of working with indeterminate objects), and the symbolic designation of its objects. Other authors, such as Rojas and Vergel, also raised the need to explicitly reflect on the relationship between the development of Algebraic Thinking and generalization processes, [20].

In Spain, and according to the new education law in force since the 2022-23 academic year (but not fully applied), Algebraic Thinking provides the language with which mathematics is communicated. Furthermore, it enables a shift from the particular to the general in terms of recognizing patterns and dependency relationships between variables and expressing them through different representations. The new regulations also say that the modeling of mathematical or real-world situations can be developed with symbolic expressions that are fundamental characteristics of algebra, [7].

Although the formulation, representation, and resolution of problems using computing tools and concepts are characteristics of Computational Thinking, for organizational reasons Spanish legislation incorporates two sections called computational thinking and mathematical modeling into algebraic thinking. The law itself recognizes that they are not exclusive to Algebraic Thinking and that, therefore, they must be worked on transversally throughout the entire teaching process of the subjects.

III. Computational Thinking

The origin of the term Computational Thinking is attributed to Seymour Papert, widely known for the development of the educational software Logo, [21]. Mainstream awareness of the concept is, however, largely due to [2]. In her paper, the author, initially, linked Computational Thinking (CT) directly to Computer Science, [2]. However, subsequently, she made attempts to link it to any area of knowledge. This author described CT as "problem-solving, system design, and understanding human behavior based on the fundamental concepts of computer science", [2]. CT is a type of analytical thinking that uses mathematical and engineering thinking to understand and solve complex problems within the constraints of the real world. This discussion is elaborated by Tedre and Denning, who also link the concept to many already pre-existing ideas of abstraction, decomposition, data representation, and algorithms and their design, [22].

Other authors perceived Computational Thinking in a similar but different way, as a challenging and potentially problematic term, [23], [24], but it has also contributed to bringing Computer Science content into the school curriculum and to foster the computing education movement. On the other hand, the concept of Computational Thinking can encompass several terms or classes of skills, such as cognitive skills, as decomposition, pattern recognition, abstraction, and algorithmic design, and also non-cognitive variables and related soft skills such as persistence, self-confidence, tolerance to ambiguity, creativity, and teamwork, [25]. Computational Thinking is fundamentally about problem-solving using concepts and strategies most closely related to Computer Science. Both Algebraic Thinking and Computational Thinking are strongly associated with problem-solving and generalization of solutions.

A deeper historical development of Computational Thinking and the intellectual ideas for development can be found in the work of Tedre and Denning, with what they name "impenetrable fog of interrelated concepts", [22], [26].

Several definitions appear in the literature for Computational Thinking, but these can be classified into two main categories: first, generic definitions that focus on Computational Thinking as a thought process, and second, operational or model definition that describes what Computational Thinking entails.

Through the use of computational thinking, people activate logical data analysis, abstraction, modeling, and approach the calculation or detection of the possible solution(s). If we take these practices to school and implement them in the classroom, students will be better able to understand how the world works. In addition, we will have been able to equip students with skills that are considered essential to solve complex problems. The main skills of Computational Thinking are now broadly considered to include: abstraction, decomposition, data collection, analysis and representation, algorithmic thinking, transferability, and evaluation and adjustment.

Therefore, we define Computational Thinking as a type of analytical thinking that uses mathematical and engineering thinking to understand and solve complex problems within the constraints of the real world, [27].

IV. Computational Thinking and Algebraic Thinking in the Current Curriculum

A. *Computational Thinking in the current curriculum*

One of the first countries to introduce compulsory computing to develop CT was Slovakia in 2008 for children from the age of 7, through a subject called Computing. Later, in September 2014, England introduced Computational Thinking into the curriculum. A new curriculum subject, Computer Science, was created, which included programming and other elements of the curriculum. And it did so, this being a novelty, for pupils from the age of 5. The subject being replaced was Information and Communication Technologies (ICT), which, as in other countries, focused almost exclusively on end-user application skills, simply using word processing, spreadsheets, etc., [28].

In these new curricula, Computational Thinking is placed at the center of Computer Science. According to the Department for Education of England, a high-quality computing education prepares school students to develop Computational Thinking and creativity to understand and change the world, [29].

However assessment and training are a fundamental part of any new curriculum and, in the implementation of the new computing subject, very few details were provided to teachers on how to assess the expected learning outcomes of the new national curriculum. At GCSE (General Certificate of Secondary Education, an academic qualification for different subjects in England and other countries), assessment was not delineated. Thus, examination boards included in the definitions of the assessment system a non-exam assessment (NEA) part, which accounts for 20% of the final grade. In this part, students have to carry out a programming project individually, in response to a detailed problem provided to them. In addition, the A-level coursework is much more open-ended: in this case, students have to develop a computer project of their own choice, in which they are asked to demonstrate their skills.

In other countries, such as Australia, Computational Thinking is part of Digital Technologies within their curriculum. However, the Australian government itself says that CT skills can be taught in all subjects [5], even with so-called unplugged assignments. According to Bower and Falkner, “there are many opportunities to apply computational thinking to humanities and social science disciplines. As Jeanette Wing opines, computational thinking is for everyone, everywhere”, [30].

More about Computational Thinking integration in compulsory education in Europe are provided in the report done by the Joint Research Center of the European Commission [25], [31].

In contrast to educational systems where new subjects have been introduced, we can cite the curricula of Sweden and Finland which have chosen to integrate Computational Thinking skills into existing subject curricula. In these countries, several subjects are used to incorporate the contents, skills, and integrated competencies of the CT; specifically, those subjects are Mathematics, Physics, Languages, and Technology. We can find a more complete analysis of these curricula in references [32] and [33].

In the Spanish System, Computational Thinking appears as a specific competence for Primary education, specifically, the fourth competence out of a total of eight, [7]. This fourth competence is defined as using Computational Thinking, organizing data, decomposing into parts, recognizing patterns, generalizing and interpreting, modifying and creating algorithms in a guided way, to model and automate everyday life situations. This competence has two evaluation criteria in the system: model everyday life situations using, in a patterned way, using basic principles of computational thinking; and use appropriate technological tools in research and problem solving.

This implementation of computational thinking in Primary School extends to Secondary Education in a very similar way, taking into account the regulations published by the Spanish government, [7]. The Spanish government defines a specific competence related to Computational Thinking as follows: use the principles of computational thinking by organizing data, decomposing it into parts, recognizing patterns, interpreting, modifying, and creating algorithms, to model situations and solve problems effectively.

In this way, computational thinking is directly related to problem-solving and procedural thinking, using abstraction to identify the most relevant aspects and decomposing them into simpler tasks to reach a solution to the problem that can be executed by a system. IT. Bringing computational thinking to everyday life involves relating the fundamental aspects of computing to the needs of students.

The development of this competence involves the creation of abstract models of everyday situations, their automation and modeling, and their coding in a language that is easy to interpret by a computer system.

This specific competence is connected to some descriptors, such as:

- mathematical competence and science, technology, and engineering competence (STEM competence), which involves understanding the world using scientific methods, mathematical thinking, and representation, technology and engineering methods to transform the environment;
- environment in a committed, responsible, and sustainable way;
- digital competence, which involves the safe, healthy, sustainable, critical, and responsible use and interaction of digital technologies for learning, work, and participation in society; and
- entrepreneurial competence, which involves developing a vital approach to acting on opportunities and ideas, using

the specific knowledge necessary to generate results of value for others.

B. *Algebraic Thinking in the current curriculum*

Algebraic Thinking is developed as basic knowledge. The skills, attitudes, and simplest knowledge with which we can form the contents of an area constitute its basic knowledge. Learning them is necessary for the acquisition of specific skills. Algebraic Thinking is divided in the Spanish curriculum, for Primary, into four sections, [7].

The first section is dedicated to patterns, where some concepts are introduced: strategies for identification, representation (verbally or through tables, graphs, and invented notations), and reasoned prediction of terms from regularities in a collection of numbers, figures, or images; and creating recurring patterns from regularities or other patterns using numbers, shapes or images.

The second section is quite brief and related to mathematical modeling, that is, the modeling process based on everyday problems through mathematical representations.

In the third section, relationships and functions are developed, with relationships of equality and inequality and the use of their mathematical symbols. Determination of unknown data (represented by a letter or symbol) in simple expressions related by these signs and the = and ≠ symbols.

The fourth section is dedicated to Computational Thinking. It is planned to develop strategies for the interpretation, modification, and creation of simple algorithms (sequences of ordered steps, schemes, simulations, repetitive patterns, loops, nested and conditional instructions, computational representations, block programming, educational robotics...).

It is interesting to note that in the Spanish curriculum, CT is being quoted as part of the Algebraic Thinking knowledge. This intersection of Computational Thinking and Algebraic Thinking is discussed by several authors, as we can see in the reference [34].

Differences in definition affect how Computational Thinking concepts are taught and assessed in compulsory education, and on the positioning of Computational Thinking skills in the curriculum. Different approaches to integrating Computational Thinking skills in school curricula include 1) embedding Computational Thinking across the curriculum as a transversal theme/skill set, 2) integrating Computational Thinking as a separate (new) subject, and 3) incorporating Computational Thinking skills within other subjects such as mathematics and technology; combinations of these approaches are also adopted.

V. Analysis of the Spanish Curricula for Primary Education

In this section, we analyze the curricula for Primary Education of the Mathematics subject, what includes the both concepts of Computational Thinking and Algebraic Thinking.

We have selected several items to characterize two important fields, such as Learning itself, which should be behind any subject, and the field of Mathematics combined with CT. This union between CT and AT is carried out by the government, since it wants to introduce CT as part of the Mathematics subject. For the field of Learning, these items have been Learning itself, Cognitive development, Education, and Social attitudes. For the field of Mathematics, the items of Math, CT, analysis, and data have been selected.

The rooting of all these items can be seen in Fig. 1. As we can see, the Computational Thinking (CT) item is the smallest of all, an issue that may be logical because it is a new concept, which is beginning to be introduced, and which is also simply a part of the subject. It is also striking that pure mathematics is not the one that has the greatest roots within the curriculum, although it can be understood that sometimes it may be intrinsically introduced.

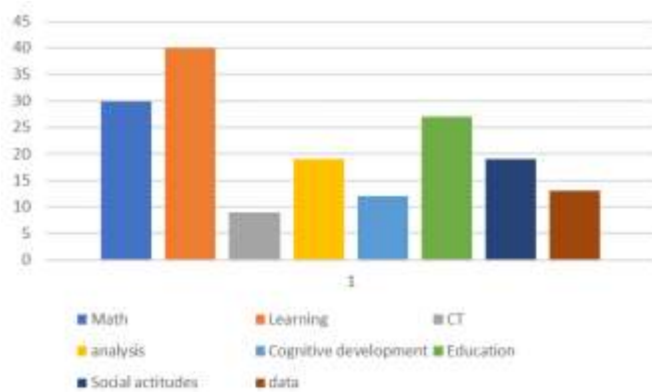

Fig. 1 Rooting of concepts in the Spanish curriculum

One-way analysis of variance (One-way ANOVA) is a statistical method for examining differences in the means of various groups. This method allows the means of a quantitative dependent variable to be compared between several groups defined by a single independent variable or factor. The objective is to test the hypothesis that the means are equal between the groups. The analysis can also provide an estimate of the effect size, which measures the magnitude of the difference between the means.

In Table I, we present a one-way Anova for the aforementioned concept groups, Mathematics and Learning.

Analyzing these results, we can conclude that the two groups are similar in terms of their average and that the curriculum is balanced in that aspect.

In Table II, you can see the summary of the main characteristics of the two groups.

TABLE I. ONE-WAY ANOVA

Origin of variations	*Sum of squares*	*Degrees of freedom*	*Mean squares*	*F*	*Probability*	*Critical value for F*
Among groups	91.12	1	91.12	0.7996	0.4056	5.9873
Within groups	683.75	6	113.95			
Total	774.87	7				

TABLE II. MAIN CHARACTERISTICS OF THE GROUPS

Groups	*Count*	*Sum*	*Mean*	*Variance*
Mathematics	4	71	17.75	83.58
Learning	4	98	24.5	144.33

However, if we perform a two-way Anova, and we also take into account the sub-items of each group, as shown in Table III, we can see that the balance is broken when delving deeper into each of the two large fields.

This fact is in line with what is seen in Fig. 1, where the different sub-groups were seen to be unbalanced in Spanish legislation. We can see how the CT, analysis, and data subgroups are the first, fourth, and third subgroups, respectively, with less roots in the text.

TABLE III. TWO-WAY ANOVA

Origin of variations	*Sum of squares*	*Degrees of freedom*	*Mean squares*	*F*	*Probability*	*Critical value for F*
Groups	670.37	3	223.45	50.1214	0.0046	9.2766
Sub-groups	91.12	1	91.12	20.4392	0.0202	10.1279
Error	13.37	3	4.45			
Total	774.87	7				

If we create a text cloud representing these roots, but where the concept can be seen through a keyword, it is at least striking that the three concepts with the greatest casuistry (life, “vida” in Spanish; situation, “situación”; and strategy, “estrategia”) are not directly related to Mathematics (the subject), nor with Computational Thinking (a new competence that has been introduced with this subject), nor with Learning or, in general, the teaching-learning process. This is shown in Fig. 2.

In a similar way, Fig. 3 shows the text cloud but with the concepts related to CT, analysis, and data. In this case, the concept of number (“número”) and the expressions that have to do with it (natural numbers, collection of numbers, etc.) are of great importance, as well as data (“dato”) (data analysis, data organization, etc.) and interpretation (“interpretación”) (description and interpretation, interpretation techniques, etc.). That is to say, in the general context, the part corresponding to mathematics itself and computational thinking do not seem to have as much importance as they should. However, if we focus on the concepts that should be key, those related to these groups appear.

Fig. 2. Text cloud taking into account all groups

Fig. 3. Text cloud with the concepts related to CT, analysis, and data

VI. COLLABORATING FOR A BETTER IMPLEMENTATION OF TWO WAYS OF THINKING IN SCHOOLS

There is growing European concern about worsening student performance, particularly PISA results, [35], and the provision of scientific and engineering skills to boost industrial competitiveness. The countries involved in this debate have also been concerned about the provision of digital and engineering skills for some time, [36]. To address the situation, several proposals have been made, including DigCompEdu 2.0, [37], which sets digital goals for Europe 2030 and establishes guidelines for education.

Making careers that require mathematical profiles in compulsory education more accessible and attractive to young people is a key element to improving access to STEM careers. Leveraging transformative digital technologies, the Computational Thinking and Mathematical Problem Solving, an Analytics-Based Learning Environment (CT&MathABLE) project, funded by the European Union's Erasmus+ program, provides teachers with new approaches to develop Algebraic and Computational Thinking competencies. Thinking individually, adapted to the student.

The project provides a learning analytics-based framework to support individualized learning trajectories for students, primarily, aged 9-14 across Europe, improving the digital skills and self-awareness of school students based on learning analytics and improving Computational Thinking and

Algebraic Thinking through interactive tasks.

The introduction of these types of thinking in the curricula of educational systems is positive for the preparation of students, whatever their subsequent academic or professional career; no matter if they are introduced in as cross-curricular way or incorporated within other subjects, including Computer Science, [38], [39].

However, the introduction of any new subject or concept in educational systems must be preceded, and accompanied, by certain elements of additional support so that guarantee that the implementation is fast, effective, and of high quality. Thus, aspects such as teacher training and the development of materials are fundamental for teaching Computational Thinking and Algebraic Thinking.

If excellence is sought in any subject, the teachers who teach those subjects must also be excellent. About Computer Science, where until recently computational thinking was covered and still is in several countries, a deficit persists in the number of teachers trained to teach computer science in secondary education, [40], [41]. The amount of primary school teachers that have some professional or academic training related to Computer Science is very low, normally because it is not included in their preparation at the university to be teachers, [42], [43].

Regarding material resources, almost a decade ago, when other types of materials appeared that could replace textbooks, Oates defended the importance of textbooks and other teaching resources within the teaching-learning process, contributing, According to him, some vital characteristics: clear delimitation of content with a precise description, focus on key concepts and knowledge, coherent learning progressions within the subject, stimulation and support of student reflection, and varied application of concepts and principles, [44]. Some recent progress has been made in this direction in terms of access to teaching and learning resources in CT. The EU project TeaEdu4CT, [45], has produced models and resources for the introduction of CT into multiple contexts in compulsory education. These resources are also available in a large number of European Languages.

Similar resources and approaches for the area of algorithmic thinking are still lacking, and addressing this is a major concern. CT&MathABLE creates digital, learning analytics-driven, interactive, and responsive resources that are uniquely tailored to each individual. This process consists of individually tailored learning trajectories that are dynamically re-designed based on current performance.

VII. Conclusion

Computational Thinking has been forced into the curricula of many countries and is increasingly becoming a matter of necessity. That is, those countries that do not begin to develop this thinking will remain in an educational system at a slower speed than others. Just as Algebraic Thinking is already a reality in almost all modern educational systems, typically within Mathematics, Computational Thinking is still making its way, being included in some countries within the subject of computer science, in others within mathematics, and in others. within ICT.

With this scenario on the table, international collaborations are being launched through projects, such as CT&MathABLE, that allow the analysis of Computational Thinking, the way it is implemented in the countries' curricula, the generation of support material for their work by teachers, and, in general, a way for it to reach schools in optimal conditions for teaching students. In this sense, and regardless of the model to follow, future development depends largely on the rapid and effective action of governments and, in the case of the European Union, also on the coordination of all its members.

Not only is research important, which can provide definitions of thoughts, modes of implementation, and generation of useful material in classes for teachers and students, but teacher training is considered essential for a good and rapid introduction to computational thinking. in the schools. Neglecting these opportunities or falling behind in the development of Computational Thinking and Algebraic Thinking can represent a risk or delay for the education of students in the coming years and, therefore, for the economic prosperity of countries.

Acknowledgment

This work has been funded through the EU ERASMUS+ Programme – KA220-SCH - Cooperation partnerships in school education, Project Reference: 2022-1-LT01-KA220-SCH-000088736. All information about the project is open on the CT&MathABLE website at https://www.fsf.vu.lt/en/ct-math-able.

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Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

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Sources of funding for research presented in a scientific article or scientific article itself

This work has been funded through the EU ERASMUS+ Programme – KA220-SCH - Cooperation partnerships in school education, Project Reference: 2022-1-LT01-KA220-SCH-000088736. All information about the project is open on the CT&MathABLE website at https://www.fsf.vu.lt/en/ct-math-able.

Conflicts of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.