# THE PEAK MODEL FOR FINITE RANK
# SUPERSINGULAR PERTURBATIONS

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*Abstract.* In its original form the peak model for rank one supersingular perturbations of class $\mathfrak{H}_{-4}$ or higher of a nonnegative self-adjoint operator requires that the Gram matrix of the model should be diagonal. Here we remove the restriction on the Gram matrix. In particular we explain the origin of the Krein $Q$-function associated with the Gram matrix.

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