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CONFERENCE PROGRAMME & ABSTRACTS

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*Nonlocal differential problems and their applications**
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*This mini-symposium is supported by the Research Council of Lithuania (LMTLT), agreement No. S-MIP-23-43.

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Abstracts of MMA2024, May 28 – May 31, 2024, Pärnu, Estonia

LOCAL FRACTAL FUNCTIONS OF HIGHER ORDER AND SYMMETRIES

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In this talk we prove the existence of local fractal functions of the Orlicz-Sobolev class of order $m \geq 0$. The graph of a local fractal function coincides with the attractor of an appropriate iterated function system (ifs), whose construction is fairly standard. Local fractal functions appear naturally as the fixed points of the Read-Bajraktarević operator when restricted to a suitable Orlicz-Sobolev space. Our results extend some of the outcomes obtained by Massopust on Lebesgue and Sobolev spaces to higher order, dimension and function spaces (where the role of the norm is now played by a Young function). As an application, we discuss the existence of local fractal functions on analytic spaces and we highlight the existence of symmetries arising from the complex conjugation.

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ON INTERACTION OF SETS OF PERIODIC SOLUTIONS

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A class of autonomous ordinary differential equations of the second order is considered, which is characterized by the existence of several period annuli (the sets of periodic solutions).

The number of solutions to the Neumann boundary value problem is studied. The bifurcation diagrams are constructed. The estimates of the number of solutions are obtained.

Perturbations of the autonomous equations with period annuli by external periodic force are studied. The chaotic behavior of solutions to the perturbed equation is observed. Visual evidences of chaotic behavior are presented. The Lyapunov exponents are used for the analysis of the sensitive dependence of solutions on the initial data.

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EXPLICIT TIME INTEGRATION OF SOURCE-FREE DYNAMICAL SYSTEMS

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The flow ϕ_t of a source-free dynamical system $\dot{x} = f(x)$, i.e., with a divergence-free vector field: $\mathrm{div} f(x) = 0$, is phase volume-preserving: $\det \frac{\partial \phi_t(x)}{\partial x} = 1$ for all t and x, a desirable property to be maintained by a numerical time integration method. Feng and Shang have shown in [1] that every divergence-free vector field $f : \mathbb{R}^n \to \mathbb{R}^n$ can be written as the sum of $n-1$ vector fields $f = f_{1,2} + f_{2,3} + \cdots + f_{n-1,n}$, where each of $f_{k,k+1}$ is Hamiltonian in the variables (x_k, x_{k+1}), i.e., there exist functions $H_{k,k+1} : \mathbb{R}^n \to \mathbb{R}$ such that $f_{k,k+1} = \left(0, \ldots, 0, -\frac{\partial H_{k,k+1}}{\partial x_{k+1}}, \frac{\partial H_{k,k+1}}{\partial x_k}, 0, \ldots, 0\right)^T$. Considering this decomposition, a phase volume-preserving numerical method is obtained by applying a splitting method with symplectic substeps, which are volume-preserving. Different splitting approaches allow the construction of symmetric as well as higher-order volume-preserving methods [2]. In general, the obtained methods are one-dimensionally implicit unless $\frac{\partial f_i(x)}{\partial x_i} = 0$ for all $i = 1, \ldots, n$.

To address the question of fully explicit symplectic integration of general (nonseparable) Hamiltonian dynamics, Tao in [3] proposed symplecticity-preserving time integration methods in an extended phase space by introducing a mechanical restraint that binds together two copies of the original phase space. In [3], good long-time numerical simulation properties were demonstrated, and an error bound for integrable dynamics was derived. In this work, we propose explicit time integration methods of source-free dynamical systems in an extended phase space, similarly to [3], by considering the augmented Hamiltonians: $\bar{H}_{k,k+1}(x_1, \ldots, x_n, y_1, \ldots, y_n) = H^A_{k,k+1}(x_1, \ldots, x_{k-1}, y_k, x_{k+1}, \ldots, x_n) + H^B_{k,k+1}(x_1, \ldots, x_k, y_{k+1}, x_{k+2}, \ldots, x_n) + \frac{1}{2}\omega\left((x_k - y_k)^2 + (x_{k+1} - y_{k+1})^2\right)$, where ω is a binding constant of system's variable x with its copy y. With augmented Hamiltonians, we obtain vector fields

$$\bar{f}_{k,k+1} = \Bigg(0, \ldots, 0, -\frac{\partial H^A_{k,k+1}}{\partial x_{k+1}} - \omega(x_{k+1} - y_{k+1}), \frac{\partial H^B_{k,k+1}}{\partial x_k} + \omega(x_k - y_k), 0, \ldots, 0, \\ 0, \ldots, 0, -\frac{\partial H^B_{k,k+1}}{\partial y_{k+1}} + \omega(x_{k+1} - y_{k+1}), \frac{\partial H^A_{k,k+1}}{\partial y_k} - \omega(x_k - y_k), 0, \ldots, 0\Bigg)^T,$$

which allow the construction of explicit splitting methods for general source-free dynamical systems. The properties of proposed explicit numerical methods are explored and numerically demonstrated.

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ON THE LAPLACE TRANSFORM OF THE RIEMANN ZETA-FUNCTION II

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The present talk is devoted to the Laplace transform

$$\mathfrak{L}\left(s, |\zeta|^{2k}\right) = \int_0^\infty \left|\zeta\left(\frac{1}{2} + ix\right)\right|^{2k} e^{-sx} dx$$

of the Riemann zeta-function $\zeta(s)$ with complex variable s, where $k \in \mathbb{N}$. In [1], only the case $k = 2$ has been considered. In [2] we had the case with the odd k. This report is a more general case of the transform for arbitrary k using the $\mathfrak{G}$ Meijer function.

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INVESTIGATION OF A DISCRETE STURM–LIOUVILLE PROBLEM WITH TWO-POINT NONLOCAL BOUNDARY CONDITION

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We investigate SLP with one classical Dirichlet Boundary Condition (BC) or Neumann BC:

$$-u'' = \lambda u, \quad t \in (0,1), \quad \lambda \in \mathbb{C}, \tag{1}$$

$$u(0) = 0 \quad \text{or} \quad u'(0) = 0, \tag{$2_{d,\,n}$}$$

and another two-points Nonlocal Boundary Condition (NBC)

$$u(1) = \gamma u(\xi), \qquad u'(1) = \gamma u'(\xi), \qquad u(1) = \gamma u'(\xi) \quad \text{or} \quad u'(1) = \gamma u(\xi), \tag{$3_{1,2,3,4}$}$$

where NBC's parameter $\gamma \in \mathbb{R}$ and $\xi \in [0,1]$.

We introduce a uniform grids in $[0,1]$: $\overline{\omega}^h = \{t_j = jh, j = \overline{0,n}\}$, $\omega^h = \{t_j = jh, j = \overline{1,n-1}\}$ with stepsizes $h_j \equiv h$ and $\omega^h_{1/2} = \{t_{j+1/2} = (t_j + t_{j+1})/2, j = \overline{0,n-1}\}$ with stepsizes $h_{j+1/2} = t_{j+1/2} - t_{j-1/2} \equiv h$. Additionally, we use a nonuniform grid $\overline{\omega}^h_{1/2} = \omega^h_{1/2} \cup \{t_{-1/2} = 0, t_{n+1/2} = n\}$ where stepsizes $h_{1/2} = t_{1/2} - t_{-1/2} = h/2$, $h_{n+1/2} = t_{n+1/2} - t_{n-1/2} = h/2$. We make an assumption that $\xi = m/n$ is located on the grid $\overline{\omega}^h$. We approximate differential SLP (1)—($3_{1,2,3,4}$) by the discrete SLP, using natural approximation of derivative $\overline{\delta}$:

$$-\delta^2 U = \lambda U, \quad t \in \omega^h, \quad \lambda \in \mathbb{C}_\lambda, \tag{4}$$

$$U_0 = 0 \quad \text{or} \quad (\overline{\delta} U)_0 = 0, \tag{$5_{d,\,n}$}$$

$$U_n = \gamma U_m, \qquad (\overline{\delta} U)_n = \gamma (\overline{\delta} U)_m, \qquad U_n = \gamma (\overline{\delta} U)_m \quad \text{or} \quad (\overline{\delta} U)_n = \gamma U_m, \tag{$6_{1,2,3,4}$}$$

We investigate discrete SLP and analyze how complex eigenvalues of this problem depend on the parameters of the two-points NBC. Some results for the both SLP were presented in [1; 3].

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HIGH ORDER SECOND DERIVATIVE DIAGONALLY IMPLICIT MULTISTAGE INTEGRATION METHODS FOR ODES

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For the numerical solution of the m-dimensional system of ordinary differential equations (ODEs)

$$y'(x) = f\big(y(x)\big), \quad y(x_0) = y_0, \quad x \in [x_0, \bar{x}],$$

we consider the class of second derivative diagonally implicit multistage integration methods (SDIMSIMs). These methods are a specific subclass of second derivative general linear methods, which are represented in the following form:

$$\begin{aligned} Y_i^{[n]} &= h\sum_{j=1}^{s} a_{ij} f\big(Y_j^{[n]}\big) + h^2 \sum_{j=1}^{s} \overline{a}_{ij} g\big(Y_j^{[n]}\big) + \sum_{j=1}^{r} u_{ij} y_j^{[n-1]},\ i = 1, 2, \ldots, s, \\ y_i^{[n]} &= h\sum_{j=1}^{s} b_{ij} f\big(Y_j^{[n]}\big) + h^2 \sum_{j=1}^{s} \overline{b}_{ij} g\big(Y_j^{[n]}\big) + \sum_{j=1}^{r} v_{ij} y_j^{[n-1]},\ i = 1, 2, \ldots, r, \end{aligned}$$

where $g(\cdot) = f'(\cdot)f(\cdot)$.

In [1] methods of order $p \leq 4$ were investigated In this talk, we aim to extend this research by investigating methods of order $p \geq 5$ as presented in the [2]. Developing methods of higher order requires establishment of conditions based on the parameters of the methods. These conditions, in the form of a system of polynomial equations, cannot be obtained and solved through symbolic manipulation tools. Therefore, we propose an approach for constructing implicit and explicit SDIMSIM with Runge–Kutta stability property using a Fourier series method variation, previously utilized for constructing high-order general linear methods. Examples of fifth and sixth order explicit and implicit SDIMSIMs, suitable for both non-stiff and stiff differential systems in a sequential computing environment, are provided. Additionally, the effectiveness of the newly derived methods is validated through numerical experiments.

This is a joint work with Ali Abdi, Mohammad Sharifi and Gholamreza Hojjati from University of Tabriz, Iran.

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RECENT ADVANCES IN THE NUMERICAL SOLUTION OF ORDINARY AND FRACTIONAL DIFFERENTIAL EQUATIONS

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Recently, the class of Runge–Kutta methods named *Hamiltonian Boundary Value Methods (HBVMs)* has been introduced for the efficient numerical solution of Hamiltonian problems [5; 6]. This is a class of energy-conserving, low-rank R–K methods, which relies on the expansion of the vector field along the Legendre polynomial basis. In this respect, the methods can be also regarded as spectral methods in time and, in fact, they are able to reach a spectral accuracy [1]. Later on, this approach has been extended to cope with different kinds of differential problems [3], including fractional differential equations [7]. Concerning these latter equations, however, only recently, the basis for the expansion has been tailored for the problem at hand [2], thus resulting into a much more efficient procedure [4].

In this talk, the main facts concerning this approach will be recalled, along with the more recent application for solving fractional differential equations.

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HYERS-ULAM STABILITY OF IMPLICIT VOLTERRA EQUATIONS ON TIME SCALES

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Consider nonlinear k-th order Volterra integrodifferential equation on an arbitrary time scale $\mathbb{T}$

$$x^{\Delta^k}(t) = f\left(t, x(t), x^{\Delta}(t), \dots, x^{\Delta^k}(t), \int_{t_0}^{t} k(t, s, x(s), x^{\Delta}(s), \dots, x^{\Delta^k}(s))\Delta\, s\right) \tag{1}$$

with initial conditions

$$x^{\Delta^i}(t_0) = x_i, \quad i = 0, 1, 2, \dots, k-1, \quad t_0, t \in I_{\mathbb{T}} = [t_0, +\infty) \cap \mathbb{T}.$$

We reduce equation (1) to implicit Volterra integral equation

$$z(t) = F\left(t, z(t), \int_{t_0}^{t} K(t, s, z(s))\,\Delta\, s\right), \quad t_0, t \in I_{\mathbb{T}} = [a, +\infty) \cap \mathbb{T}, \tag{2}$$

where $z\colon I_{\mathbb{T}} \to \mathbb{R}^{n(k+1)}$ is the unknown function, $K\colon I_{\mathbb{T}} \times I_{\mathbb{T}} \times \mathbb{R}^{n(k+1)} \to \mathbb{R}^{n(k+1)}$ be rd-continuous in its first and second variable, $L\colon I_{\mathbb{T}} \to \mathbb{R}$ be rd-continuous, $\gamma > 1$, $\beta = L(s)\gamma$,

$$|K(t, s, z) - K(t, s, z')| \leq L(s)|z - z'|, \quad (z, z') \in \mathbb{R}^{n(k+1)}, \quad s < t,$$

$$|F(t, z, w) - F(t, z', w')| \leq M(|z - z'| + |w - w'|),\ m = \sup_{t \in I_{\mathbb{T}}} \frac{1}{e_\beta(t, t_0)} \left| F(t, 0, \int_{t_0}^{t} K(t, s, 0)\Delta\, s \right| < \infty.$$

If $M(1 + 1/\gamma) < 1$, then the integral equation (2) has a unique solution $z \in C_\beta^k(I_{\mathbb{T}}; \mathbb{R}^{n(k+1)})$, where $C_\beta^k(I_{\mathbb{T}}; \mathbb{R}^{n(k+1)})$ be the Banach space of rd-continuous functions such that

$$\sup_{t \in I_{\mathbb{T}}} \frac{\max_{0 \leq i \leq k} |x^{\Delta^i}(t)|}{e_\beta(t, t_0)} < \infty.$$

We also prove that equation (1) is Hyers-Ulam stable.

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LONG STANDING AND NEVER OLD QUESTION: WHO IS A BOSS (STABILITY OR APPROXIMATION)

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Adaptive grids in space and time are used to fit the grid points to the dynamics of the solution and to minimize the approximation/truncation error.

At the same time uniform grids are used in many recent big data projects due to two very important properties: high order approximations can be constructed directly on uniform grids and the obtained structure of grids is well suited for parallel computing techniques. As a consequence different modifications and new discrete schemes are proposed which try to preserve the uniformity of the grids as close as possible. The well known NFL theorem predicts that some additional costs should be expected. In our talk we consider some new interesting discrete schemes and show that the stability analysis should be tailored to each source of approximation errors.

A linear hyperbolic problem is approximated by the following implicit symmetrical three-level scheme

$$\begin{aligned} &\frac{U^{n+1} - 2U^n + U^{n-1}}{\tau^2} + \beta \frac{U^{n+1} - U^{n-1}}{2\tau} + A_h \frac{U^{n+1} + U^{n-1}}{2} = F^n, \\ &U^0 = u_0, \quad U^1 = u_0 + \tau v_0. \end{aligned} \tag{1}$$

In [1], this scheme is modified for the specific quasi-uniform time grid when at some grid points the lengths of grid steps are doubled or halved. The most valuable property of this scheme is that the approximation is still done on uniformly distributed grid points, thus basic advantages of such discrete schemes are preserved.

By making a full stability analysis of interpolation errors introduced by the proposed algorithm we prove that the cases of doubling and reducing twice the time steps lead to totally different error accumulation rates. Results of computational experiments are presented and they confirm the accuracy of theoretical error estimates.

We also investigate the difference in the stability of the backward Euler (BE) finite difference scheme and the discontinuous Galerkin (DG) finite element scheme when both schemes are used to solve one dimensional parabolic problem on dynamically shifted uniform space grids. The theoretical analysis proves that the accumulation of interpolation and projection approximation errors are quite different.

Finally we ask the question if adaptive grids are really required and are more efficient than uniform grids?

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MODELLING OF BACTERIAL PLUME FORMATION IN A CIRCULAR CONTAINER

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Microorganisms such as bacteria *Escherichia coli* move around either towards attractants (e.g. food) or away from repellants (e.g. toxins). This process is called chemotaxis, and it allows populations of bacteria to self-organize and form patterns [1]. One of the most widely used methods to model chemotaxis is the Keller-Segel approach [2].

E. coli pattern formation can be modelled using a system of reaction-diffusion-chemotaxis equations, representing dynamics of bacteria, self-excreted chemoattractant, and oxygen [3]. In this work, the model was coupled with Navier-Stokes equations to improve the modelling of plume formation [4]. The dimensionless 2D in space model is governed by these equations:

$$\frac{\partial n}{\partial t} + \left(\vec{u}\cdot\vec{\nabla}\right) n = D_n \Delta n - \chi \nabla (n \nabla c) + \alpha n \left(1 - \frac{n}{o}\right) \tag{1}$$

$$\frac{\partial c}{\partial t} + \left(\vec{u}\cdot\vec{\nabla}\right) c = D_c \Delta c + \frac{n}{1+\beta n} - c \tag{2}$$

$$\frac{\partial o}{\partial t} + \left(\vec{u}\cdot\vec{\nabla}\right) o = D_o \Delta o - \lambda n \tag{3}$$

$$\frac{\partial \omega}{\partial t} + \left(\vec{u}\cdot\vec{\nabla}\right) \omega = \nu \Delta \omega - \kappa \frac{\partial n}{\partial x} \tag{4}$$

$$\Delta \Psi = -\omega, \qquad (x,y) \in (0,l)\times(0,h), \quad t>0, \tag{5}$$

where $n(x,y,t)$, $c(x,y,t)$, and $o(x,y,t)$ represent cell density, chemoattractant, and oxygen respectively, $\omega(x,y,t)$ is the vorticity, and $\Psi(x,y,t)$ is the stream function. Together with appropriate initial and boundary conditions, the governing equations form a boundary-value problem, which was solved using finite difference technique.

The aim of this work is to investigate the effects of model parameters on bacteria pattern formation, with special emphasis on plume formation.

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FAST NUMERICAL ALGORITHMS FOR THE EVALUATION OF RIEMANN-LIOUVILLE INTEGRALS

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In many applications, it is necessary to compute numerical solutions of one or many ordinary differential equations of fractional order over many time steps. At the core of such numerical solvers, we typically find schemes for the approximate evaluation of Riemann-Liouville integrals. Due to the inherent memory properties of the associated differential and integral operators, such computations require a significant amount of computer memory and a relatively large runtime if straightforward disctretization approaches are used. More sophisticated techniques, such as algorithms based on the FFT, can reduce the runtime significantly but not the memory requirements. Other concepts like, e.g., nested meshes reduce the memory demands but come with a significant administrative overhead. The goal of this talk is to present a class of algorithms that simultaneously address both the runtime and the memory footprint issues.

Specifically, our methods are based on so-called diffusive representations of the Riemann-Liouville integral operator. This allows us to handle the process memory in an implicit manner, thus immediately reducing the algorithm's required computer memory to an order of magnitude comparable to that of an algorithm for local problems. Many different analytic variants of such representations exist, and each of them allows multiple combinations of algorithms for its numerical evaluation. It is therefore necessary to identify special cases that admit a particularly accurate numerical approximation with a small number of arithmetic operations, thus additionally addressing the runtime matter in a satisfactory way.

The work described in this talk is performed within a joint project with Renu Chaudhary and Afshin Farhadi (THWS) and André Schmidt and Paul E. Haacker (Institute of Nonlinear Mechanics, Universität Stuttgart). This project is supported by the German Federal Minsitry of Education and Research (BMBF) under Grant 05M22WHA.

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CO AND CO_2 CONCENTRATION IN BIOMASS COMBUSTION PROCESS DEPENDING ON BIOMASS POROSITY

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To simulate thermal decomposition of biomass we define every biomass as three organic compounds. Organic compound thermal decomposes in volatile part and carbon part [2]. Coal combustion are described with equation

$$C + \theta O_2 \rightarrow 2(1-\theta)CO + (2\theta - 1)CO_2. \tag{1}$$

Reactions are modeled using Arrhenius kinetics. To determine changes off biomass that microwave pretreatment gives we estimate these changes using porosity. Our objective is to determine how porosity changes in biomass affects CO and CO_2 concentration in mathematical model. Gases we model using the Darcy law, ideal gas law and mass balance equation [1]. Numerical solutions were found using finite deference scheme and finite volume method in program MatLab.

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A GLOBAL APPROXIMATION METHOD FOR SECOND-KIND VOLTERRA-FREDHOLM EQUATIONS

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This talk deals with the numerical treatment of the following mixed Volterra-Fredholm integral equation

$$(I + \mu V K)f = g,$$

where $\mu \in \mathbb{R} \setminus \{0\}$, f is the unknown function, g is a given right-hand side, I is the identity operator, V is the Volterra operator given by

$$(Vf)(y) = \int_{-1}^{y} h(x, y) f(x)(y - x)^{\rho}(1 + x)^{\sigma} dx, \qquad \rho, \sigma > -1, \qquad y \in [-1, 1],$$

with h an assigned kernel, and K is the Fredholm operator defined as

$$(Kf)(y) = \int_{-1}^{1} k(x, y) f(x)(1 - x)^{\alpha}(1 + x)^{\beta} dx, \qquad \alpha, \beta > -1, \qquad y \in [-1, 1],$$

with k a known kernel.

A global approximation method of Nyström type based on a mixed Gauss-product cubature formula is developed to approximate the solution f in suitable weighted spaces equipped with the uniform norm. Stability and convergence results are discussed and numerical tests are given to show the good performance of the method.

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INTEGRATING FRACTIONAL DIFFERENTIAL EQUATIONS WITH NEURAL NETWORKS FOR DYNAMICAL SYSTEMS MODELLING

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This study integrates Fractional Differential Equations (FDE) with Neural Networks for the modelling of dynamical systems. Inspired by concepts like Neural Ordinary Differential Equations (Neural ODE) [1] and fractional calculus within neural systems [2], we propose the Neural FDE [3]. In this neural network architecture, parameterised by $\boldsymbol{\theta}$ (representing weights and biases), the objective is to determine the function $\boldsymbol{f_\theta}$ of a Fractional Differential Equation of order α (equation (1)), ensuring that the solution to (1) accurately fits provided data (a time series), by adjusting the weights and biases within $\boldsymbol{f_\theta}$ [3],

$$ {}_0^C D_t^\alpha \boldsymbol{h}(t) = \boldsymbol{f_\theta}(t, \boldsymbol{h}(t)) \text{ with } \boldsymbol{h}(t_0) = \boldsymbol{h}_0, \quad \alpha = \alpha_{\boldsymbol{\phi}}. \tag{1} $$

Here, ${}_0^C D_t^\alpha \boldsymbol{h}(t)$ denotes the Caputo fractional derivative [4], where $\boldsymbol{h}(t)$ represents the state of the dynamical system at time t, and $\boldsymbol{h}(t_0) = \boldsymbol{h}_0$ is the initial condition. As this model introduces an additional parameter (α) compared to the Neural ODE, the order of the Fractional Differential Equation, α, is also learned by another neural network, $\alpha_{\boldsymbol{\phi}}$, with parameters $\boldsymbol{\phi}$.

This hybrid fractional differential equation can then be used for making future predictions or predicting missing data. The solution of (1) is obtained numerically, with the *values* of $\boldsymbol{f_\theta}(t, \boldsymbol{h}(t))$ provided by the neural network.

We limit our consideration to $\alpha \in (0, 1)$, as it finds applicability in various scenarios and simplifies the formulation.

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MODELLING AND SIMULATION OF SYSTEMS WITH COMPLEX CHARACTERISTICS: CERTAINTIES, UNCERTAINTIES AND COMPROMISES

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In this talk we discuss the link between real world systems and the development of appropriate model equations. Our focus is on problems that involve a history, delay or after-effect, and we begin with a discussion of how these phenomena can be reflected in relatively simple model equations, applying classical results from [2] and [4]. This leads to a discussion of characteristic values and eigenfunctions and an understanding of the characteristics of the solution space and the limitations imposed by modelling choices. Here we apply more recent insights from [6].

We discuss approaches to model selection and parameter estimation and we consider how to match the purpose of the model with its formulation. We give examples that show how different modelling paradigms can be necessary to develop effective models in different circumstances (sse, for example, [3] for further details on this theme).

We use the recent COVID-19 pandemic as a case study where model equations were widely used but poorly understood (see [5] and [1]). We give examples that show why the conclusions drawn from the model may be quite unhelpful to the application and we provide some ideas about how to avoid these problems arising in future modelling projects.

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A NONLINEAR BACKWARD DIFFUSION ALGORITHM FOR IMAGE RESTORATION

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We discuss the recent image processing methodology introduced in [2] in the context of nonlinear diffusive PDEs (see also [1; 3; 4; 5; 7]). A corrupted image u is represented in a continuous framework and used as initial guess for the evolution equation:

$$\frac{\partial u}{\partial t} = \sqrt{\frac{\|\nabla u\|^2}{1 + \|\nabla u\|^2}} \, \Delta u, \tag{1}$$

where ∇ and Δ denote the gradient an the Laplace operator, respectively. Equation (1) is defined on a rectangle and Neumann conditions are imposed on the boundaries. Contrary to the classical approaches, descending for example from the pioneering work of Perona & Malik [6], the nonlinear diffusion coefficient depends locally on the magnitude of $\|\nabla u\|$ and grows proportionally to this quantity. An ad-hoc discretization scheme uses centered finite-differences scheme for the space variable and it is implemented on two interlaced grids. Euler's scheme is applied for time discretization going backwards in time. The aim is to smooth out the regions of almost uniform shade and emphasize discontinuities. The proposed methodology is quite efficient as satisfactory results are obtained after only one time step of appropriate size. The algorithm is well-suited for edge detection and segmentation problems. We show some applications in the case of images affected by blur and noise.

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ON DISCRETE SHIFTS OF THE MELLIN TRANSFORM OF THE RIEMMAN ZETA-FUNCTION

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We consider the Mellin transform $\mathcal{Z}(s)$, $s = \sigma + it$, of the square of the Riemann zeta-function $\zeta(s)$, i.e.,

$$\mathcal{Z}(s) = \int_1^\infty |\zeta(1/2 + ix)|^2 x^{-s}\,\mathrm{d}x,$$

and the approximation of analytic functions by discrete shifts $\mathcal{Z}(s + ikh)$ with $h > 0$ and $k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$. Let $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$, and $H(D)$ denote the space of analytic functions on D equipped with the topology of uniform convergence on compacta.

In the report, we will discuss the following theorem, for details, see [1].

Theorem 1. *For $h > 0$, there exists a non-empty closed set F_h such that, for every compact set $K \subset D$, $f(s) \in F_h$ and $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{N+1} \# \left\{ 0 \leqslant k \leqslant N : \sup_{s\in K} |\mathcal{Z}(s + ikh) - f(s)| < \varepsilon \right\}.$$

Moreover, the limit

$$\lim_{T\to\infty} \frac{1}{N+1} \# \left\{ 0 \leqslant k \leqslant N : \sup_{s\in K} |\mathcal{Z}(s + ikh) - f(s)| < \varepsilon \right\}$$

exists and is positive for all but at most countably many $\varepsilon > 0$.

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NUMERICAL APPROACHES TO SOLVE AN INVERSE PROBLEM FOR A FRACTIONAL DIFFUSION-WAVE EQUATION WITH MULTI-TERM FRACTIONAL POWERS OF MINUS LAPLACIAN

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Two numerical approaches designed using MATLAB to recover the unknowns of the following fractional diffusion-wave equation with multi-term fractional powers of minus Laplacian

$$D_t^{\alpha} u(x,t) = \sum_{j=1}^{n} b_j (-\Delta)^{\beta_j} u(x,t) + f_0(x,t), \tag{1}$$

are considered. The first approach is based on the minimization of the error of the Laplace transform of the observation function minus the Laplace transform of the observation function with the unknowns as variables and the second approach uses location of poles of Laplace transform of observation function. Finally, a comparison of the two approaches is presented.

Uniqueness for the solution is proved theoretically, also by means of location of poles.

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JOINT MIXED LIMIT THEOREM FOR EPSTEIN AND HURWITZ ZETA-FUNCTIONS

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In the talk, we will present some known results on the joint asymptotic behaviour of zeta-functions having and having no Euler's product over prime numbers. Moreover, we will discuss a new result for the Epstein and Hurwitz zeta-functions, more precisely, we will prove the joint probabilistic limit theorem in terms of the weak convergence of probability measures on $\mathbb{C}^2$ defined by means of these zeta-functions.

We recall the definitions of the mentioned functions.

The Epstein zeta-function $\zeta(s;Q)$ was introduced by P. Epstein in [1]. Let Q be a positive definite quadratic $n \times n$ matrix and $Q[\underline{x}] = \underline{x}^{\mathrm{T}} Q \underline{x}$ for $\underline{x} \in \mathbb{Z}^n$. The Epstein zeta-function $\zeta(s;Q)$, $s = \sigma + it$, is defined, for $\sigma > \frac{n}{2}$, by the series $\zeta(s;Q) = \sum_{\underline{x} \in \mathbb{Z}^n \setminus \{\underline{0}\}} (Q[\underline{x}])^{-s}$, and can be continued analytically to the whole complex plane, except for a simple pole at the point $s = \frac{n}{2}$ with residue $\frac{\pi^{n/2}}{\Gamma(n/2)\sqrt{\det Q}}$.

The Hurwitz zeta-function $\zeta(s,\alpha)$ was introduced in [2]. The function $\zeta(s,\alpha)$ with a fixed parameter α, $0 < \alpha \leq 1$, is defined, for $\sigma > 1$, by $\zeta(s,\alpha) = \sum_{m=1}^{\infty} (m+\alpha)^{-s}$, and has analytic continuation to the whole complex plane, except for a simple pole at the point $s = 1$ with residue 1.

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FRACTIONAL CALCULUS OF ZETA FUNCTIONS

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This talk outlines fractional calculus of zeta functions. The main results are based on a complex generalization of the Grünwald-Letnikov fractional derivative. The functional equations can be rewritten in a simplified form. Thus, we reduce their computational cost. Moreover, for the case of the Riemann ζ function we propose a quasisymmetric form of the corresponding functional equation. The second part of this talk deals with the link with distribution of prime numbers. In particular, we show some results on free-zero regions for this class of functions. Finally, we discuss the representation of these functions in terms of Euler products.

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LOCAL ERROR ESTIMATION FOR IMPLICIT-EXPLICIT GENERAL LINEAR METHODS

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We investigate implicit-explicit methods for differential systems with stiff and non-stiff parts. Stage order and order conditions are formulated and estimation of local discretization errors in fixed and variable stepsize environments is discussed. We also describe the construction of such methods with desirable accuracy and stability properties. This is a joint work with Angela Cardone, University of Salerno, and Giuseppe Izzo, University of Naples.

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APPROXIMATION OF ANALYTIC FUNCTIONS BY SHIFTS OF ABSOLUTELY CONVERGENT DIRICHLET SERIES RELATED TO PERIODIC ZETA-FUNCTIONS

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Let $\mathfrak{a} = \{a_m : m \in \mathbb{N}\}$ and $\mathfrak{b} = \{b_m : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\}$ be two periodic sequences of complex numbers, $0 < \alpha \le 1$ and $\theta > \frac{1}{2}$ are fixed parameters, $v_u(m) = \exp\left\{-\left(\frac{m}{u}\right)^\theta\right\}, m \in \mathbb{N}$, and $v_u(m,\alpha) = \exp\left\{-\left(\frac{m+\alpha}{u}\right)^\theta\right\}, m \in \mathbb{N}_0$, with $u > 0$. Let $s = \sigma + it$ denote a complex variable. We consider the Dirichlet series

$$\zeta_u(s;\mathfrak{a}) = \sum_{m=1}^{\infty} \frac{a_m v_u(m)}{m^s} \text{ and } \zeta_u(s,\alpha;\mathfrak{b}) = \sum_{m=0}^{\infty} \frac{b_m v_u(m,\alpha)}{(m+\alpha)^s}$$

which are absolutely convergent in any half-plane $\sigma > \sigma_0$ with finite σ_0. We obtain the following results:

1. Approximation of a class of analytic functions by continuous shifts $\zeta_{u_T}(s+i\tau;\mathfrak{a}), \tau \in \mathbb{R}$, with multiplicative sequence $\mathfrak{a}$, and $u_T \to \infty$ and $u_T \ll T^2$.
2. Approximation of a class of analytic functions by discrete shifts $\zeta_{u_N}(s+ikh;\mathfrak{a}), h > 0, k \in \mathbb{N}_0$, with multiplicative sequence $\mathfrak{a}$, and $u_N \to \infty$ and $u_N \ll N^2$.
3. Joint approximation of a class of pairs of analytic functions by continuous shifts $\big(\zeta_{u_T}(s+i\tau;\mathfrak{a}), \zeta_{u_T}(s+i\tau,\alpha;\mathfrak{b})\big), \tau \in \mathbb{R}$, with multiplicative sequence $\mathfrak{a}$, and $u_T \to \infty$ and $u_T \ll T^2$.
4. Joint approximation of a class of pairs of analytic functions by discrete shifts $\big(\zeta_{u_N}(s+ikh_1;\mathfrak{a}), \zeta_{u_T}(s+ikh_2,\alpha;\mathfrak{b})\big), h_1 > 0, h_2 > 0, k \in \mathbb{N}_0$, with multiplicative sequence $\mathfrak{a}$, and $u_N \to \infty$ and $u_N \ll N^2$.

More precisely, let K_1, K_2 be compact sets of the strip $D = \{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$ with connected complements, $f_1(s)$ continuous, nonvanishing and $f_2(s)$ continuous functions on K_1 and K_2, and analytic in interior of K_1, K_2, respectively.

Suppose that the set $\{(h_1 \log p : p \in \mathbb{P}), (h_2 \log(m+\alpha) : m \in \mathbb{N}_0), 2\pi\}$ is linearly independent over the field of rational numbers. Then the limit

$$\lim_{N\to\infty} \frac{1}{N+1} \#\Big\{0 \le k \le N : \sup_{s\in K_1} |\zeta_{u_N}(s+ikh_1;\mathfrak{a}) - f_1(s)| < \varepsilon_1, \\ \sup_{s\in K_2} |\zeta_{u_N}(s+ikh_2,\alpha;\mathfrak{b}) - f_2(s)| < \varepsilon_2\Big\}$$

exists and is positive for all but at most countably many $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$. The above results form PhD thesis of the author.

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REGULARITY RESULTS FOR VERY WEAK TIME-PERIODIC POISEUILLE-TYPE SOLUTION WITH MINIMALLY REGULAR FLOW RATE

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In this presentation, we provide a summary of our results concerning the time-periodic very weak solutions of the heat equation with a non-local additional condition of the prescribed flux

$$F(t) = \int_{\sigma} U(x,t)\mathrm{d}x$$

and their regularity properties. Specifically, we present an example of a function $F(t)$ that belongs to $L^2(-\pi,\pi)$ but $F \notin W^{\beta,2}(-\pi,\pi)$ for $0 < \beta < 1$. The very weak solution corresponding to such flux F, has the regularity as stated in the definition (see [1],[2]) and it is not better. Furthermore, we discuss results that suggest a correlation between the improvement of solutions regularity and the increase in regularity of the given function F.

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NOTES ON SIMULTANEOUS APPROXIMATION BY THE CLASS OF ZETA-FUNCTIONS

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The theory of universality focuses on the approximation of analytic functions by the shifts of certain zeta-functions. In a much more complicated situation, we touch on studying approximation by the wide classes of zeta-functions. One such class is the so-called Selberg-Steuding class $\widetilde{\mathcal{S}}$, which was introduced by A. Selberg in [3] and modified by J. Steuding in [4], and is defined by Dirichlet series $L(s)$ satisfying certain axioms.

The first approximation result related to the class $\widetilde{\mathcal{S}}$ was obtained by J. Steuding in [4] and later improved by H. Nagoshi and J. Steuding in [2]. R. Kačinskaitė, A. Laurinčikas and B. Žemaitienė have obtained [1] the joint universality theorem for L-functions belonging to the Selberg-Steuding class $\widetilde{\mathcal{S}}$. More precisely, we have shown a result about simultaneous approximation of a collection of analytic functions $\big(f_1(s), \dots, f_r(s)\big)$ in the strip $\big\{s \in \mathbb{C} : \sigma_L < \sigma < 1\big\}$ by a collection of shifts $\big(L(s + ia_1\tau), \dots, L(s + ia_r\tau)\big)$, $L(s) \in \widetilde{\mathcal{S}}$, where $\sigma_L > \frac{1}{2}$ is a number depending on L, and real algebraic numbers $a_1, \dots, a_r$ are linearly independent over the field of rational numbers $\mathbb{Q}$.

In the talk, we discuss two results on the investigation of joint universality property for $L(s) \in \widetilde{\mathcal{S}}$, i.e., we present the joint functional independence and denseness for the above-mentioned collection.

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STALLING IN QUEUING SYSTEMS WITH HETEROGENEOUS CHANNELS

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In the talk, a model of stalling in the queueing system (QS) with any number of different capacities heterogeneous servers will be discussed and some new results published in [1] will be presented. The model of stalling in QSs with two heterogeneous servers has been considered in [2], where the explicit probabilities of steady states were derived.

We will discuss the optimization of a stalling buffer as well, and we will show that stalling helps us to solve the slow server problem under an appropriate choice of stalling buffer size, making the slow servers usable under various values of system load. Moreover, some applications of the developed model in heterogeneous server clusters and in work productivity modelling for forest harvesting applications will be presented.

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A NONLINEAR MATHEMATICAL MODEL OF THE COURSE AND THERAPY OF RHEUMATOID ARTHRITIS AND ITS REALIZATION

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Mathematical models of immune mediated disorders provide an analytic platform in which we can address specific questions concerning disease immune dynamics to dictate the choice of treatment. Rheumatoid arthritis is a systemic autoimmune disease characterized by the joint inflammation and the cartilage destruction. Autoreactive B lymphocytes represent the integral elements of the pathophysiology of rheumatoid arthritis. Immune balance between the effector and the regulatory T cell subsets guide the autoreactive B cell fate and play a cardinal role in disease severity. Using non-linear differential equations, we developed a novel mathematical model that describes the immunopathogenesis of rheumatoid arthritis [1; 2]. The model explores the functional dynamics of cartilage destruction during disease progression, in which a system of differential equations deciphers the interactions between autoreactive B lymphocytes and T helper cells. As the further task, we present here the refined model of the disease course in which the immunomodulatory effect of IL-6, – a molecule that drives the cross-talk of pro-inflammatory and regulatory subsets of T lymphocytes, is explained. IL-6 targeting is also taken into consideration in the disease treatment model, in which the modalities of treatment with methotrexate and tocilizumab in a separate or combined scheme are addressed. For such treatment model, the corresponding Cauchy problem is posed and its solution is found. In conclusion, we propose a novel mathematical model that best describes the readouts of the course and treatment outcomes of rheumatoid arthritis and, therefore, may take a rapid pace towards its implementation in biomedical and clinical research.

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LEARNING HAMILTONIAN DYNAMICS WITH STRUCTURE-PRESERVING NEURAL NETWORKS AND DIMENSIONALITY REDUCTION

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Data-driven approaches employing structure-preserving algorithms, such as symplectic neural networks known as *SympNets* [1], have gained recognition for learning Hamiltonian systems' dynamics. Despite their promising results, the challenge of high dimensionality persists. In this work, we investigate dimensionality reduction techniques to model Hamiltonian systems effectively in lower-dimensional subspaces, thereby reducing training times while preserving prediction accuracy.

We focus on learning nonlinear localized wave solutions in a one-dimensional crystal lattice model, as the lattice can be of an arbitrary dimension and, as we show, the problem lends well to being modeled in a lower-dimensional subspace. That is done by employing dimensionality reduction techniques, such as the non-symplectic *Proper Orthogonal Decomposition* (POD) [2] and geometric structure-preserving *Proper Symplectic Decomposition* (PSD) [3].

Moreover, we extend our previous work [4] by imposing symplectic neural networks architecture's map to be symmetric, i.e., equal to its adjoint map, as proposed in [5], a characteristic inherent to Hamiltonian system flows. Our results demonstrate that incorporation of the additional flows' property in symplectic neural networks together with structure-preserving dimensionality reduction enhances model predictions and their *valid prediction time* (VPT) even further.

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ON CONVERGENCE CONDITIONS IN SELF-REGULARIZATION OF ILL-POSED PROBLEMS BY PROJECTION METHODS

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We consider an operator equation

$$Au = f, \quad f \in R(A) \neq \overline{R(A)},$$

where $A \in L(E, F)$ is the linear continuous operator between Banach spaces E and F. Instead of the exact right-hand side f we have only an approximation $f^\delta \in F$ satisfying condition $\|f - f^\delta\| \leq \delta$ with known δ. We consider projection methods. Let $E_n \subset E$, $Z_n \subset F^*$, $n \in \mathbb{N}$, be finite-dimensional nontrivial subspaces which have the role of approximating the spaces E and F^*, respectively. Let Q_n be the linear operator defined by

$$Q_n : F \to Z_n^* \quad \forall g \in F,\, z_n \in Z_n \,:\, \langle Q_n g, z_n \rangle_{Z_n^*, Z_n} = \langle z_n, g \rangle_{F^*, F} \,. \tag{1}$$

Then the finite-dimensional approximation $u_n \in E_n$ to the solution u_* can be found from the equation

$$Q_n A u_n = Q_n f^\delta. \tag{2}$$

Under conditions $\dim(E_n) = \dim(Z_n)$, $\mathcal{N}(Q_n A) \cap E_n = \{0\}$ the operator $A_n := Q_n A|_{E_n} : E_n \to Z_n^*$ has an inverse. In case of exact data ($\delta = 0$) the convergence $\|u_* - u_n\|_E \to 0$ as $n \to \infty$ is guaranteed, if there exists a sequence of approximations $(\hat{u}_n)_{n \in \mathbb{N}}$, $\hat{u}_n \in E_n$, satisfying the conditions

$$\|u_* - \hat{u}_n\|_E \to 0 \text{ as } n \to \infty, \qquad \|A_n^{-1} Q_n A(u_* - \hat{u}_n)\| \to 0 \text{ as } n \to \infty.$$

Under two conditions

$$\exists \tau < \infty \colon \sup_{v_n \in E_n, v_n \neq 0} \frac{\|A v_n\|_F}{\|Q_n A v_n\|_{Z_n^*}} \leq \tau \quad \forall n \in \mathbb{N}, \qquad \|A_{n+1}^{-1}\| \operatorname{dist}(f, AE_n) \to 0 \text{ as } n \to \infty$$

the self-regularization $\|u_* - u_n\|_E \to 0$ as $\delta \to 0$ is guaranteed by choice of the dimension $n(\delta)$ from the discrepancy principle as the first index such that $\|Au_n - f^\delta\|_F \leq b\delta, \quad b > \tau + 1$.

In general, estimation of τ may be complicated. We will consider some applications on the collocation method for Volterra integral equations of the first kind.

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TIME PERIODIC INVERSE PROBLEM WITH NONLOCAL CONDITION

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In a bounded domain σ we consider the time periodic boundary value problem for the heat equation:

$$\begin{gathered} U_t(x,t) - \nu \Delta U(x,t) = q(t), \\ U(x,t)\big|_{\partial\sigma} = 0, \\ U(x,0) = U(x,2\pi), \\ \int\limits_{\sigma} U(x,t)\,dx = F(t), \quad F(0) = F(2\pi), \end{gathered} \tag{1}$$

where U and q are the unknown functions while F is a given function, i.e. for given F we find the right hand side q such that the solution U satisfies the additional nonlocal condition $\int\limits_{\sigma} U(x,t)\,dx = F(t)$. Problem (1) can be interpreted as an inverse parabolic problem. Using the concept of a very weak solution introduced in paper [1] for the initial boundary value problem, we prove the existence of a unique weak solution of time periodic problem (1) under the assumption that function F has minimal regularity, i.e function F belongs only to $L^2(0,2\pi)$ (see [2]).

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PERIODIC CUBIC SPLINE HISTOPOLATION

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For a given grid of points $a = x_0 < x_1 < \ldots < x_n = b$ and given real numbers z_i, $i = 1, \ldots, n$, consider the problem of histopolation, i.e., the problem of finding a function S from some class of functions such that

$$\int_{x_{i-1}}^{x_i} S(x)dx = z_i h_i, \quad i = 1, \ldots, n, \tag{1}$$

where $h_i = x_i - x_{i-1}$, $i = 1, \ldots, n$.

It is known that cubic spline histopolation problem with classical boundary conditions and arbitrary placement of histopolation knots is uniquely solvable [1]. This is not true for periodic spline histopolation. In case of periodic spline histopolation we look for a solution to the problem from the class of mth degree polynomial splines satisfying boundary conditions

$$S^{(j)}(a) = S^{(j)}(b), \quad j = 0, 1, \ldots, m-1.$$

It is known that for m even, n arbitrary and also for m odd, n odd problem (1) has a unique solution [2]. For m odd, n even it is known that in case of uniform grid there exist values z_i, $i = 1, \ldots, n$, such that periodic histopolation problem does not have a solution [2; 3]. In case of non-uniform grid the negative result is proven for $m = 1$, n even and m odd, $n = 2$ (see [2]) leaving other cases open.

We restrict ourselves to the cubic spline case $m = 3$, even number of subintervals $n = 2k$, $k = 2, 3, \ldots$ and arbitrary placement of knots; analyse different representations of the histospline and discuss solvability issues.

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POINTWISE-IN-TIME A-PRIORI AND A-POSTERIORI ERROR CONTROL FOR TIME-FRACTIONAL PARABOLIC EQUATIONS

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An initial-boundary value problem with a Caputo time derivative of fractional order $\alpha \in (0, 1)$ is considered, solutions of which typically exhibit a singular behaviour at an initial time. For this problem, building on some ideas from [1], we give a simple and general numerical-stability analysis using barrier functions, which yields sharp pointwise-in-time error bounds on quasi-graded temporal meshes with arbitrary degree of grading. This approach is employed in the error analysis of the L1 and Alikhanov L2-1_σ fractional-derivative operators [2], as well as an L2-type discretization of order $3 - \alpha$ in time [3]. This methodology is also generalized for semilinear fractional parabolic equations [4]. In particular, our error bounds accurately predict that milder (compared to the optimal) grading yields optimal convergence rates in positive time. The theoretical findings are illustrated by numerical experiments.

Furthermore, pointwise-in-time a posteriori error bounds will be given in the spatial L_2 and L_∞ norms. Hence, an adaptive mesh construction algorithm is applied for the L1 method, which yields optimal convergence rates $2 - \alpha$ in the presence of solution singularities [5; 6; 7].

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CURVE ESTIMATION WITH MODIFIED COMPLETE SPLINES AND EXPONENTIAL PARAMETERIZATION

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We discuss the issue of fitting data points $\mathcal{Q}_m = \{q_i\}_{i=0}^m$ in arbitrary Euclidean space $\mathbb{E}^n$ (see [1]). It is also assumed, that the knots $\mathcal{T}_m = \{t_i\}_{i=0}^m$ are unknown and as such they need to be replaced by some $\hat{\mathcal{T}}_m = \{\hat{t}_i\}_{i=0}^m$. For $\mathcal{Q}_m$ *dense* the issue of convergence rate of a given interpolation scheme $\hat{\gamma}$ (based on $\hat{\mathcal{T}}_m$) in approximating γ (with $\gamma(t_i) = q_i$) has been extensively studied (as for classical case with $\mathcal{T}_m$ given - see e.g. [1; 7]). A possible remedy to substitute $\mathcal{T}_m$ with $\hat{\mathcal{T}}_m$ is to apply *an exponential parameterization*:

$$\hat{t}_0 = 0, \qquad \hat{t}_{i+1} = \|q_{i+1} - q_i\|^\lambda \tag{1}$$

with $i = 0, \ldots, m-1$ and $\lambda \in [0, 1]$ - see [7]. The convergence issue (and its rate) of $\hat{\gamma}$ to γ has been tackled for (1) and $\hat{\gamma}$ forming either *piecewise Lagrange quadratics* or *cubics* or *modified Hermite interpolants* (see [2; 3; 4]). We examine here the asymptotics in $\gamma \approx \hat{\gamma} = \hat{\gamma}^{MC}$ forming *a modified complete spline* based on $\mathcal{Q}_m$ and (1) - see [5]. We establish *a linear convergence rate* (in terms of $\delta_m = \max_{i\in\{0,\ldots,m-1\}}\{t_{i+1} - t_i\}$) in estimating γ by $\hat{\gamma}^{MC}$ for $\lambda \in [0, 1)$. The latter supplements the case of $\lambda = 1$ yielding a faster quartic rate. Lastly we report on numerical tests confirming the above asymptotics and *its sharpness*. Finding the knots for *sparse reduced data* (here $m << \infty$) can be dealt by solving a relevant optimization task (see [6]).

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BLOOD FLOW MODELLING IN HUMAN HEART USING FSI

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The goal of this talk is to present the FSI (fluid-structure interaction) CFD (computational fluid dynamics) simulations of the blood flow in the human heart left atrial appendage for a patient-specific geometry. These simulations are important for the medical doctors decision making for the patients with atrial fibrillation. In order to compute blood flow velocity we use Navier-Stokes equations coupled with Shell mechanics Uflyand-Mindlin model. The FSI CFD simulations in the heart is a challenging problem: the existing softwares are not too robust for real life Reynolds numbers and often do not converge to the solution of the Navier-Stokes equations for the blood coupled with the elasticity equations of the wall. That is why we first provide the CFD computations with the rigid wall when the codes are more stable. Using this solution as the reference velocity and pressure, then we provide the FSI computations which become much more robust (see [1]).

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Abstracts of MMA2024, May 28 – May 31, 2024, Pärnu, Estonia

ABOUT THE NUMERICAL SOLUTION OF SINGULAR FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS

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Due to various new applications in physics, chemistry, and other fields of science, the interest in fractional derivatives and equations containing them has increased significantly over last century (see for example [1; 3]). In [2] a class of singular fractional integro-differential equations was investigated.

We now consider singular fractional integro-differential equations of the form

$$(M^{\beta} D_0^{\beta} u)(t) = a_1 (M^{\beta_1} D_0^{\beta_1} u)(t) + a_0 (V_{\varphi} u)(t) + f(t),\ 0 < t \le T. \tag{1}$$

By $C^m[0,T]$ ($m \in \mathbb{N}_0$) we denote the space of m times continuously differentiable functions u on the closed interval $[0,T]$; $C^0[0,T] = C[0,T]$, and by $L^1(0,1)$ we denote the Banach space consisting of real or complex valued functions φ defined on $(0,1)$ such that $\|\varphi\|_{L^1(0,1)} = \int_0^1 |\varphi(x)| dx < \infty$. In equation (1), the operator M^{β} ($\beta \in \mathbb{R}$) is defined as $(M^{\beta} u)(t) = t^{\beta} u(t)$ ($0 < t \le T$) for $u \in C[0,T]$,

$$(V_{\varphi} u)(t) = \int_0^t \frac{1}{t} \varphi\left(\frac{s}{t}\right) u(s) ds = \int_0^1 \varphi(x) u(tx) dx, \quad 0 \le t \le T, \quad u \in C[0,T],$$

i.e. $V_{\varphi} : C[0,T] \to C[0,T]$ is a cordial Volterra integral operator with core $\varphi \in L^1(0,1)$, and

$$\beta, \beta_1, a_0, a_1 \in \mathbb{R}, \quad q < \beta < q+1, \quad \beta > \beta_1 \ge 0, \quad f \in C^q[0,T].$$

The fractional differential operator D_0^{β} (of the order $\beta \in [0,\infty)$) in equation (1), is defined as the inverse of the Riemann-Liouville integral operator $J^{\beta} : C[0,T] \to C[0,T]$ on the space $J^{\beta} C[0,T]$, i.e. $D_0^{\beta} v = (J^{\beta})^{-1} v$, where v belongs to the range $J^{\beta} C[0,T]$ of J^{β}, $\beta \ge 0$.

We study the unique solvability of equations of the form (1) and discuss the numerical solution of such equations.

This is a joint work with Arvet Pedas.

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APPROXIMATION BY BEURLING'S ZETA-FUNCTIONS

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The system $\mathcal{P}$ of real numbers $1 < p_1 \leqslant p_2 \leqslant \cdots \leqslant \cdots$, $p_n \to \infty$ as $n \to \infty$, is called generalized prime numbers. Using the system $\mathcal{P}$, the system $\mathcal{N}_\mathcal{P}$ of generalized integers

$$m = p_1^{\alpha_1} \cdots p_r^{\alpha_r}, \quad r \in \mathbb{N},\ \alpha_j \in \mathbb{N}_0 = \mathbb{N} \cup \{0\},\ j = 1, \ldots, r,$$

is constructed. Generalized prime numbers were introduced by Beurling [1], he also obtained the first results on the asymptotic behaviour as $x \to \infty$ for

$$\pi_\mathcal{P}(x) = \sum_{p \leqslant x,\, p \in \mathcal{P}} 1 \qquad \text{and} \qquad \mathcal{N}_\mathcal{P}(x) = \sum_{m \leqslant x,\, m \in \mathcal{N}_\mathcal{P}} 1,$$

and relations among them. To study $\pi_\mathcal{P}(x)$, Beurling introduced zeta-functions $\zeta_\mathcal{P}(s)$ defined in some half-planes by

$$\zeta_\mathcal{P}(s) = \prod_{p \in \mathcal{P}} \left(1 - \frac{1}{p^s}\right)^{-1} \qquad \text{or} \qquad \zeta_\mathcal{P}(s) = \sum_{m \in \mathcal{N}_\mathcal{P}} \frac{1}{m^s}.$$

Now, the functions $\zeta_\mathcal{P}(s)$ are called Beurling zeta-functions. Various authors developed analytic theory for the functions $\zeta_\mathcal{P}(s)$. Note that each case of $\mathcal{P}$ requires a separate studying.

In report, we consider the approximation problem of analytic functions by shifts $\zeta_\mathcal{P}(s + i\tau)$ for some classes of systems $\mathcal{P}$. We assume the estimation

$$\mathcal{N}_\mathcal{P}(x) = ax + O\left(x^\delta\right), \quad 0 \leqslant \delta < 1,\ a > 0,$$

and suppose that there exists $\widehat{\sigma} < 1$, such that

$$\widehat{\sigma} = \inf\left\{\sigma : \int_0^T |\zeta_\mathcal{P}(\sigma + it)|^2 \,\mathrm{d}t \ll_\sigma T,\ \sigma > \delta\right\}.$$

Under the above hypotheses, we obtain [2] that there exists a set of analytic functions, approximated by shifts $\zeta_\mathcal{P}(s + i\tau)$. Identification of that set requires new restrictions for the system $\mathcal{P}$.

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ON THE NUMERICAL SOLUTION OF THE NEUMANN PROBLEM FOR LAPLACE'S EQUATION IN PLANAR DOMAINS WITH CORNERS

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A new boundary integral equation (BIE) method for the numerical solution of the exterior Neumann problem for the Laplace equation in planar domains with corners is proposed. Using the single layer representation of the potential, the differential problem is reformulated in terms of a BIE whose solution has singularities at the corners.

A "modified" Nyström type method based on a Gaussian type quadrature formula is proposed for its approximation.

Convergence and stability results for the proposed method are proved in proper weighted spaces of continuous functions. Moreover, the use of a smoothing transformation allows to increase the regularity of the solution and, consequently, the order of convergence of the method.

The efficiency of the proposed method is shown by illustrating some numerical tests.

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ON CENTRAL PART INTERPOLATION APPROXIMATIONS FOR SYSTEMS OF FRACTIONAL DIFFERENTIAL EQUATIONS

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We consider an initial value problem for a system of fractional differential equations of the form

$$(\mathrm{D}^{\alpha}_{Cap} y_i)(t) + \sum\nolimits_{j=1}^{n} a_{ij}(t) y_j(t) = f_i(t), \quad 0 \le t \le 1, \quad i = 1, \dots, n, \quad n \in \mathbb{N} := \{1, 2, \dots\}, \tag{1}$$

$$y_i(0) = y_{0i}, \quad y_{0i} \in \mathbb{R} := (-\infty, \infty), \quad i = 1, \dots, n, \tag{2}$$

where $0 < \alpha < 1$, and the functions a_{ij}, f_i are continuous: $a_{ij}, f_j \in C[0,1]$ for $i, j = 1, \dots, n$. Here $\mathrm{D}^{\alpha}_{Cap} y$ is an α-order Caputo fractional derivative of a function $y = y(t)$. Then it turns out that problem (1)-(2) possesses a unique solution $y_1, \dots, y_n$ such that $y_i \in C[0,1]$, $\mathrm{D}^{\alpha}_{Cap} y_i \in C[0,1]$, $i = 1, \dots, n$. However, we cannot generally expect (see [1]) that $y_1, \dots, y_n$ belong to $C^q[0,1]$ for $a_{ij}, f_i \in C^q[0,1]$, $i, j = 1, \dots, n$, $q \in \mathbb{N}$. Instead, we can show that, for $i, j = 1, \dots, n$, if $a_{ij}, f_i \in C^{q,\mu}(0,1]$, $q \in \mathbb{N}$, $\mu \in (0,1)$, then y_i and its derivatives $\mathrm{D}^{\alpha}_{Cap} y_i$ belong to $C^{q,\nu}(0,1]$, where $\nu = \max\{1-\alpha, \mu\}$. Here, by $C^{q,\mu}(0,1]$ ($q \in \mathbb{N}$, $0 < \mu < 1$) we denote the set of functions $y \in C[0,1] \cap C^q(0,1]$ such that

$$|y^{(k)}(t)| \le c\, t^{1-\mu-k}, \quad 0 < t \le 1, \quad k = 1, \dots, q,$$

where c is a positive constant independent of t.

We propose a high-order method for solving (1)-(2) based on improving the boundary behavior of the exact solution with the help of a change of variables, and on central part interpolation by polynomial splines on the uniform grid. The central part interpolation approach was used for solving Fredholm integral equations of the second kind and it has shown accuracy and numerical stability advantages compared to standard piecewise polynomial collocation methods, including collocation at Chebyshev points [2]. We apply this approach for solving (1)-(2) and derive global error estimates for the approximate solution analogous to [3].

This work is my joint collaboration with Arvet Pedas and Mikk Vikerpuur.

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MATHEMATICAL MODELING OF NEURAL FIELDS WITH DIFFUSION

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We are concerned with the analytical and numerical solution of the following integro-differential equation

$$\begin{aligned} \partial_t v(x,\xi,t) &= \nu \partial^2_{\xi\xi} v(x,\xi,t) - \gamma v(x,\xi,t) \\ &\quad + \int_\Omega W(x,x',\xi,\xi') S(v(x',\xi',t)) dx' d\xi' + G(x,\xi,t), \end{aligned} \tag{1}$$

for $(x,\xi,t) \in \Omega_T = \Omega \times [0,T]$, where $\Omega = \mathbb{T}^n \times U$, $U = (0,L)$ and $\mathbb{T}^n$ is the $n-$dimensional torus.

We search for a solution v of (1), being a periodic function on x, satisfying Neumann boundary conditions with respect to ξ and an initial condition.

Our analysis, which is described with detail in [2], relies on perturbing weak solutions to the diffusion-less problem (with $\nu = 0$), that is, a standard neural field, for which weak problems have not been studied to date. We find rigorous asymptotic estimates for the problem with and without diffusion, and prove that the solutions of the two models stay close, in a suitable norm,on finite time intervals. Using a computational method that is described in [1], we provide numerical evidence of our perturbative results.

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SOME GENERALIZED LIMIT THEOREMS FOR THE EPSTEIN ZETA-FUNCTION

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Let Q be a positive definite quadratic $n \times n$ matrix and $Q[\underline{x}] = \underline{x}^{\mathrm{T}} Q \underline{x}$ for $\underline{x} \in \mathbb{Z}^n$. The Epstein zeta-function $\zeta(s;Q)$, $s = \sigma + it$, is defined, for $\sigma > \frac{n}{2}$, by the series $\zeta(s;Q) = \sum_{\underline{x} \in \mathbb{Z}^n \setminus \{\underline{0}\}} (Q[\underline{x}])^{-s}$, and can be continued analytically to the whole complex plane, except for a simple pole at the point $s = \frac{n}{2}$ with residue $\frac{\pi^{n/2}}{\Gamma(n/2)\sqrt{\det Q}}$. The function $\zeta(s;Q)$ was introduced by P. Epstein in [1], its value-distribution was investigated by various authors; for example, an extensive survey of the results for the function $\zeta(s;Q)$ is given in [2].

In the talk, some generalized continuous and discrete results of joint works with A. Laurinčikas on the value-distribution for $\zeta(s;Q)$ with even $n \geq 4$ and integers $Q[\underline{x}]$ will be presented. More precisely, we will show that, for a special differentiable function $\varphi(t)$ with a monotonic derivative,

$$\frac{1}{T} \text{meas} \{t \in [0,T] : \zeta(\sigma + i\varphi(t);Q) \in A\}, \quad A \in \mathcal{B}(\mathbb{C}),$$

converges weakly to an explicitly given probability measure on $(\mathbb{C}, \mathcal{B}(\mathbb{C}))$ as $T \to \infty$ [3]. Here meas$A$ denotes the Lebesgue measure of a measurable set $A \subset \mathbb{R}$, and $\mathcal{B}(\mathbb{C})$ – the Borel σ-field of the space $\mathbb{C}$. Moreover, we will discuss that, for a certain increasing differentiable function $\varphi(t)$ with a continuous monotonic bounded derivative and with an additional condition for the sequence $\{\varphi(k)\}$, on $(\mathbb{C}, \mathcal{B}(\mathbb{C}))$, there exists an explicitly described probability measure $P_{Q,\sigma}$ such that

$$\frac{1}{N} \# \{N \leqslant k \leqslant 2N : \zeta(\sigma + i\varphi(k);Q) \in A\}, \quad A \in \mathcal{B}(\mathbb{C}),$$

converges weakly to $P_{Q,\sigma}$ as $N \to \infty$ [4]. Also, a few examples of the function $\varphi(t)$ will be given.

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ON MATHEMATICAL MODELLING AND SIMULATION OF IMMUNE SYSTEM'S RESPONSE IN TUMOR TREATMENT

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We address the intricacies surrounding the efficacy of oncolytic viruses and the infiltration of healthy cells to combat malignant cells. Similar models and treatment of experimental data for the process also explored in [1].

We consider PDE systems (1) with special right hand sides modelling the different amounts of injections:

$$\begin{cases} \dfrac{\partial C}{\partial t} - D\triangle C = \sum \gamma\delta(x - x_p) + f(x),\ x \in \Omega, \\ D\dfrac{\partial C}{\partial n} + \beta C = 0,\ x \in \partial\Omega, \end{cases} \tag{1}$$

where $\delta(x)$ is the dirac delta function, C is the concentration, D is the diffusivity, γ is the injection rate and x_p is the injection position.

These are discretized by finite element techniques.

Consideration of simpler models for similar problems in the field such as ODE systems has been explored in [2] and main features are discussed here as well. We present the graphical results of the obtained fields and discuss main aspects of several underlying modelling stages. Moreover we discuss the presence of experimental data and the identification of parameters for these models.

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ON GENERALIZED SHIFTS OF THE LERCH ZETA-FUNCTION

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Let $s = \sigma + it$ be a complex variable, and $0 < \lambda \leqslant 1$, $0 < \alpha \leqslant 1$ two parameters. The Lerch zeta-function $L(\lambda, \alpha, s)$ is defined, for $\sigma > 1$, by the Dirichlet series

$$L(\lambda, \alpha, s) = \sum_{m=0}^{\infty} \frac{\mathrm{e}^{2\pi i\lambda m}}{(m+\alpha)^s},$$

and has a meromorphic continuation to the whole complex plane with a simple pole at the point $s = 1$ when $\lambda = 1$.

In the report, we consider the approximation of analytic functions by generalized shifts $L(\lambda, \alpha, s + i\varphi(\tau))$. The real-valued function $\varphi(\tau)$ is defined for $\tau \geqslant \tau_0 > 0$, is increasing to $+\infty$ and has a monotonic derivative such that

$$\varphi(2\tau)\left(\min(\varphi'(\tau), \varphi'(2\tau))\right) = O(\tau), \quad \tau \to \infty.$$

We present the following results.

THEOREM 1. *Suppose that the set $\{\log(m+\alpha) : m \in \mathbb{N}_0\}$ is linearly independent over the field of rational numbers, K is a compact subset of the strip $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$ with connected complement, and $f(s)$ is a continuous on K function, and analytic in the interior of K. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{T} \mathrm{meas}\left\{\tau \in [0, T] : \sup_{s\in K} |L(\lambda, \alpha, s + i\varphi(\tau)) - f(s)| < \varepsilon\right\} > 0. \tag{1}$$

Moreover, "$\liminf$" can be replaced by "$\lim$" for all but at most countably many $\varepsilon > 0$.

THEOREM 2. *Suppose that the parameters λ and α are arbitrary. Then there exists a non-empty closed set $F_{\lambda,\alpha,\varphi} \subset D$ such that, for every compact set $K \subset D$, $f(s) \in F_{\lambda,\alpha,\varphi}$ and $\varepsilon > 0$, inequality* (1) *is valid, and the same assertion as in Theorem 1 with "$\lim$" is true..*

Theorems 1 and 2 generalize the results of [1] and [2] obtained for $\varphi(\tau) = \tau$.

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STABILITY ANALYSIS OF STATIONARY MOTION OF THE 3D SWINGING ATWOOD'S MACHINE

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The swinging Atwood machine under consideration consists of two masses m_1, m_2 attached to opposite ends of a massless inextensible thread wound round two massless frictionless pulleys of negligible radius (see [1]). The mass m_1 is allowed to swing in two dimensions and its behaves like a spherical pendulum of variable length while the mass m_2 is constrained to move only along a vertical. Such a system has three degrees of freedom and its equations of motion may be written in the form

$$r\ddot{\theta} = -g\sin\theta - 2\dot{r}\dot{\theta} + \frac{p_\varphi^2\cos\theta}{r^3\sin^3\theta}, \qquad r^2\sin^2\theta\dot{\varphi} = p_\varphi = const,$$
$$(2+\varepsilon)\ddot{r} = r\dot{\theta}^2 - g(1+\varepsilon-\cos\theta) + \frac{p_\varphi^2}{r^3\sin^2\theta}. \tag{1}$$

Here the variables r, θ, φ describe geometrical configuration of the system, g is a gravity constant, $\varepsilon = (m_2 - m_1)/m_1$, and p_φ is an integral of motion determined from the initial conditions.

In the case of $p_\varphi = 0$ the mass m_1 oscillates on a vertical plane and we obtain the swinging Atwood machine which may demonstrate a periodic motion (see [2; 3]).

One can easily check that there exists an exact particular solution to equations (1) of the form

$$\varphi(t) = \sqrt{\frac{g(1+\varepsilon)}{r_0}}t + \varphi_0,\ r(t) = r_0,\ \theta(t) = \theta_0 = \arccos(1/(1+\varepsilon)). \tag{2}$$

Solution (2) describes a uniform motion of the mass m_1 in a horizontal plane on a circular orbit of radius $r_0\sin\theta_0$. Simulation of the system motion shows that small variation of the initial conditions results only in small perturbation of the mass m_1 orbit. Analyzing the Hamiltonian function of the system and applying the stability theory, we have proved that solution (2) is stable with respect to the variables $r, \dot{r}, \theta, \dot{\theta}, \dot{\varphi}$.

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ON THE CONVERGENCE OF THE DIFFERENCE SCHEME FOR NONLINEAR PARABOLIC EQUATION WITH INTEGRAL BOUNDARY CONDITION

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We analyze the two dimensional nonlinear parabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - f(x,y,u), \quad (x,y) \in \Omega = \{0 < x < 1, 0 < y < 1\},\ t \in (0,T],$$

with given initial condition and boundary conditions

$$\begin{aligned} &u(x,y,0) = \phi(x,y), \quad (x,y) \in \Omega, \\ &u(x,0,t) = \mu_1(x,t), \quad u(x,1,t) = \mu_2(x,t), \quad u(0,y,t) = \mu_3(y,t) \end{aligned}$$

and with nonlocal integral condition on one boundary of rectangular domain

$$u(1,y,t) = \gamma \int_0^1 \int_0^1 u(x,y,t)dxdy + \mu_4(y,t).$$

We write down the finite difference method and estimate the error of the solution based on the properties of M-matrices, and then prove the convergence of the differential scheme. The main result of the investigation is that the majorant function is constructed and establishing the stability and convergence of the differential scheme.

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SOLITARY SOLUTIONS TO THE HEPATITIS C MODEL WITH THE PROLIFERATION OF INFECTED AND UNINFECTED HEPATOCYTES

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It is shown in [1] that the model of the Hepatitis C model with the proliferation of infected and uninfected hepatocytes [2] can be expressed in the form of two Riccati equations coupled with the multiplicative and diffusive terms:

$$\begin{cases} \frac{dx}{dt} = a_0 + a_1 x + a_2 x^2 + a_3 xy + a_4 y, \\ \frac{dy}{dt} = b_0 + b_1 y + b_2 y^2 + b_3 xy + b_4 x, \end{cases} \tag{1}$$

where $a_k, b_k \in \mathbb{R}$; $k = 0, 1, ..., 4$. The standard form of a solitary solution reads:

$$x(t) = \sigma \frac{\prod_{k=1}^{n} (\exp(\eta(t-t_0)) - x_k)}{\prod_{k=1}^{n} (\exp(\eta(t-t_0)) - t_k)}, \tag{2}$$

where $n \in \mathbb{N}$ is the order of the solitary solution; σ, η, t_0, x_k, $t_k \in \mathbb{R}$.

It is a common feature of nonlinear dynamical systems that a solitary solution form a separatrix in the space of system parameters and initial conditions [3]. Therefore, the existence of solitary solutions may help to understand the global dynamics of the system. The necessary condition for the existence of the order–1 solitary solutions to Eq.(1) is derived in [1]:

$$a_3 = b_2; b_3 = a_2. \tag{3}$$

However, the limit transition from Eq.(1) to the system of Riccati equations coupled with the diffusive terms yields a system of degenerate Riccati equations [4]. Moreover, the fact that the Riccati system coupled with the diffusive terms admits non-deformed order-1 solitary solutions, serves as a proof for the structural instability of the hepatitis C model with the proliferation of infected and uninfected hepatocytes.

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NEW HEURISTIC PARAMETER CHOICE FOR THE CLASS OF REGULARIZATION METHODS

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We consider an operator equation

$$Au = f_*, \quad f_* \in R(A) \neq \overline{R(A)},$$

where $A \in L(H,F)$ is the linear continuous operator between real Hilbert spaces H and F. Instead of the exact right-hand side f_* we have only an approximation $f \in F$. To get the regularized solution we consider the following class of regularization methods

$$u_r = (I - A^*Ag_r(A^*A)u_0 + g_r(A^*A)A^*f,$$

where u_0 is the initial approximation, r is the regularization parameter and the generating function $g_r(\lambda)$ satisfies for $r \geq 0$ the conditions

$$\sup_{0\leq\lambda\leq\|A^*A\|} |g_r(\lambda)| \leq \gamma r, \qquad \sup_{0\leq\lambda\leq\|A^*A\|} \lambda^p \left|1 - \lambda g_r(\lambda)\right| \leq \gamma_p r^{-p}, \quad 0 \leq p \leq p_0.$$

Well-known heuristic rule for choosing regularization parameter is the quasioptimality principle, where the parameter is chosen as the global minimum point of the function

$$\psi_Q(r) = r\left\|A^*B_r^2(Au_r - f)\right\|, \qquad B_r = (I - AA^*g_r(AA^*))^{1/2p_0},$$

on the set of parameters $\Omega = \{r_j : r_j = qr_{j-1}, j = 1,2,...,M, q > 1\}$. Unfortunately, this rule is unstable in this sense that it often fails in case of heat-type problems.
To get stable parameter choice rule we introduce modified quasioptimality criterion function $\psi_{\mathrm{MQ}}(r)$ in the form:

$$\psi_{\mathrm{MQ}}(r_0) = \psi_{\mathrm{Q}}(r_0),$$

$$\psi_{\mathrm{MQ}}(r_j) = \max\{\psi_{\mathrm{Q}}(r_j), (d_{MD}(r_j)/d_{MD}(r_{j-1}))^{2/(2+1/p_0)}\psi_{\mathrm{MQ}}(r_{j-1})\}, j > 0,$$

where the function $d_{MD}(r) = \|B_r(Au_r - f)\|$. Choosing global minimum point of the function $\psi_{\mathrm{MQ}}(r)$ as regularization parameter gives us stable heuristic rule, but to get more accuracy we present the following algorithm. Let r_{MQ} be the global minimum point of the function $\psi_{\mathrm{MQ}}(r)$ on the set of local minimum points of the function $\psi_Q(r)$.
Heuristic rule. For the regularization parameter choose the parameter

$$r_H = \min\{r_Q, \max\{r_{MQ}, r_{HR}\}\},$$

where r_Q and r_{HR} are the global minimum points of the functions $\psi_Q(r)$ and $\psi_{HR}(r) = r^{1/2}d_{MD}(r)$, respectively.

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A POLYNOMIAL COLLOCATION METHOD FOR THE LIGHTHILL'S PROBLEM

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Lighthill [1] derived a nonlinear integral equation which describes the temperature distribution of the surface of a projectile moving through a laminar layer at high Mach numbers, which is given by

$$2t^{1/3}u^4(t) + \Gamma(2/3)\left(J^{2/3}u'\right)(t) = 0, t > 0, \tag{1}$$

$$u(0) = 1, \tag{2}$$

where J^α, $0 < \alpha < 1$, is the Riemann-Liouville operator defined by

$$\left(J^\alpha v\right)(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}v(s)ds, \quad t \geq 0.$$

In [2] was proved that (1)-(2) posseses a unique continuous solution, $u^\star$, such that

$$0 < u^\star(t) < 1, \quad t > 0; \quad u^\star(t) \longrightarrow 0 \,\text{as}\, t \to +\infty,$$

with the following unimprovable, for large t, estimate

$$u^\star(t) \leq \frac{\sin(\pi/3)}{\sqrt{3}}t^{-1/6} \sim 0.841t^{-1/6}.$$

Then, the convergence $u^\star(t) \longrightarrow 0 \,\text{as}\, t \to +\infty$ is slow.

This work is concerned with the numerical solution of (1). Indeed we consider a numerical method based on polynomial collocation method, with several choices for the mesh, and compare the numerical results obtained.
Some considerations about the convergence of the proposed method are made.

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DISCRETE VERSION OF THE MISHOU THEOREM FOR ABSOLUTELY CONVERGENT DIRICHLET SERIES RELATED TO PERIODIC ZETA-FUNCTIONS

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In [3], Mishou obtained a joint universality theorem for the Riemann and Hurwitz zeta-functions. A discrete version of Mishou's theorem was given in [2].
Let $\mathfrak{a} = \{a_m : m \in \mathbb{N}\}$ and $\mathfrak{b} = \{b_m : m \in \mathbb{N}_0\}$ be two periodic sequences of the complex numbers. Moreover, for a fixed $\theta > \frac{1}{2}$, $v_u(m) = \exp\left\{-\left(\frac{m}{u}\right)^\theta\right\}$ and $v_u(m,\alpha) = \exp\left\{-\left(\frac{m+\alpha}{u}\right)^\theta\right\}$, where $0 < \alpha \leq 1$ and $u > 0$. Define two series $\zeta_u(s;\mathfrak{a}) = \sum_{m=1}^\infty \frac{a_m v_u(m)}{m^s}$ and $\zeta_u(s,\alpha;\mathfrak{b}) = \sum_{m=0}^\infty \frac{b_m v_u(m,\alpha)}{(m+\alpha)^s}$. Then the later series are absolutely convergent in any half-plane $\sigma > \sigma_0$. These series are certain convolutions with the periodic and periodic Hurwitz zeta-functions, respectively. Let, for positive h_1 and h_2, $L(\mathbb{P};\alpha,h_1,h_2,\pi) = \{(h_1 \log p : p \in \mathbb{P}), (h_2 \log p : p \in (m+\alpha)), 2\pi)\}$, where $\mathbb{P}$ is the set of all prime numbers. In the report, we will discuss approximation of a pair of analytic functions by shifts of the functions $\zeta_{u_N}(s;\mathfrak{a})$ and $\zeta_{u_N}(s,\alpha;\mathfrak{b})$ with $u_N \to \infty$ as $N \to \infty$. Let $\mathcal{K}$ be be the class of compact sets of the strip $\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$ with connected complements, and $H_0(K)$ and $H(K)$ with $K \in \mathcal{K}$ the classes of continuous nonvanishing and continuous functions on K, respectively, that are analytic in the interior of K. Then the following statement is valid [1].

Theorem 1. *Suppose that the sequence $\mathfrak{a}$ is multiplicative, the set $L(\mathbb{P};\alpha,h_1,h_2,\pi)$ is linearly independent over the field of rational numbers, and $u_N \to \infty$ and $u_N \ll N^2$ as $N \to \infty$. Let $K_1, K_2 \in \mathcal{K}$ and $f_1(s) \in H_0(K_1)$, $f_2(s) \in H(K_2)$. Then, for every $\epsilon > 0$, the limit*

$$\lim_{N\to\infty} \frac{1}{N+1} \#\left\{0 \leq k \leq N : \sup_{s\in K_1} |\zeta_{u_N}(s+ikh_1;\mathfrak{a}) - f_1(s)| < \epsilon_1, \sup_{s\in K_2} |\zeta_{u_N}(s+ikh_2,\alpha;\mathfrak{b}) - f_2(s)| < \epsilon_2\right\}$$

exists and is positive for all but at most countably many $\epsilon_1 > 0$ and $\epsilon_2 > 0$.

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A GLOBAL APPROXIMATION METHOD FOR SECOND-KIND NONLINEAR INTEGRAL EQUATIONS

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A global approximation method of Nyström type is explored for the numerical solution of a class of nonlinear integral equations of the second kind of the type:

$$f(y) - \int_{-1}^{1} k_1(x,y)f(x)dx - \int_{-1}^{1} k_2(x,y)h(x,f(x))dx = g(y), \qquad y \in [-1,1],$$

Integral equations of this type have wide applications in problems involving nonlinearities such as heat radiation, heat transfer, acoustic, elasticity, and electromagnetic problems; see [1; 2].

Some of these models are mathematically formulated in terms of boundary value problems having nonlinear boundary conditions, which can be reformulated as the equation under consideration [3; 4] .

In applicative contexts, the kernels of the equation are smooth and/or weakly singular. Therefore cases of smooth and weakly singular kernels are both considered. Firstly the solvability of the equation is studied in the space of continuous functions and the smoothness of the solution is stated according to the smoothness of the known functions. In the case of regular kernels, the proposed numerical method uses a Gauss-Legendre rule, whereas in the second case of weakly singular kernels, the method resorts to a product rule based on Legendre nodes. Stability and convergence are proved in subspaces of Sobolev and Zygmund type, equipped with the uniform norm.

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ON SENSITIVE DEPENDENCE OF SOLUTIONS ON THE INITIAL DATA IN ORDINARY DIFFERENTIAL EQUATIONS

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A sensitive dependence of solutions on the initial data is an indication of chaotic behavior. This phenomenon was extensively studied for three-dimensional systems. It appears and plays an important role in many processes of real life. Examples in biology, chemistry, financial mathematics are well known.

In our talk several less-known examples are considered, starting from Duffing-type equations, and including formally very simple ordinary differential equations of order three and four.

Examples are provided, and illustrative materials are supplied.

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ON THE MEAN SQUARE OF THE HURWITZ ZETA-FUNCTION IN SHORT INTERVALS

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Let $s = \sigma + it$ be a complex variable, and $0 < \alpha \leqslant 1$ a fixed parameter. The Hurwitz zeta-function $\zeta(s, \alpha)$ is given, for $\sigma > 1$, by Dirichlet series

$$\zeta(s, \alpha) = \sum_{m=0}^{\infty} \frac{1}{(m+\alpha)^s},$$

and has analytic continuation to the whole complex plane, except for a simple pole $s = 1$.

In the theory of zeta-functions, mean square estimates play an important role. For example, they are the main ingredient in proofs of universality theorems on approximation of analytic functions. A problem of effectivization of universality theorems requires mean square estimates in short intervals.

Our report is devoted to a mean square estimate of the function $\zeta(s, \alpha)$ in short intervals.We will announce the following theorem

THEOREM 1. *Suppose that $\alpha \neq 1, 1/2$, and $1/2 < \sigma < 7/12$ is fixed. Let $T^{27/82} \leqslant H \leqslant T^{\sigma}$. Then, uniformly in H, the estimate*

$$\int_{T-H}^{T+H} |\zeta(\sigma + it, \alpha)|^2 \, \mathrm{d}t \ll_{\sigma,\alpha} H$$

is valid.

Theorem 1 extends a known results for the Riemann zeta-function [1], i.e., for $\zeta(s, 1)$. Note that the case of $\zeta(s, \alpha)$ is different from that of $\zeta(s, 1)$ because $\zeta(s, \alpha)$ has no Euler's product over primes.

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ON ZETA-FUNCTION OF CERTAIN CUSP FORMS

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Let $\mathcal{F}(z)$ be a cusp form of weight κ for the full modular group with the Fourier series expansion $\mathcal{F}(z) = \sum_{m=-\infty}^{\infty} c(m)\mathrm{e}^{2\pi imz}$. The corresponding zeta-function $\zeta(s,\mathcal{F})$, $s = \sigma + it$, is defined for $\sigma > (\kappa+1)/2$, by the series

$$\zeta(s,\mathcal{F}) = \sum_{m=1}^{\infty} \frac{c(m)}{m^s},$$

and has analytic continuation to an entire function. We suppose additionally that $\mathcal{F}(z)$ is a normalized Hecke-eigen cusp form. Thus, $\zeta(s,\mathcal{F})$ has the Euler product over prime numbers.

Our report is devoted to universality of the function $\zeta(s,\mathcal{F})$. Let $D_{\mathcal{F}} = \{s \in \mathbb{C} : \kappa/2 < \sigma < (\kappa+1)/2\}$. Then it is known [1] that the shifts $\zeta(s+i\tau,\mathcal{F})$, $\tau \in \mathbb{R}$, approximate any non-vanishing analytic function defined on $D_{\mathcal{F}}$. In the report, we discuss the density of approximating shifts in short intervals. For this, we use the following conjecture: there exists $\sigma_0 \in (\kappa/2, (\kappa+1)/2)$ and $0 < \delta < 1$ such that, for $\sigma \in (\kappa/2, \sigma_0]$ and $T^\delta \leqslant H \leqslant T$, uniformly in H the estimate

$$\int_{T-H}^{T+H} |\zeta(s+i\tau;\mathcal{F})|^2 \,\mathrm{d}t \ll_\sigma H$$

is valid.

Let $\mathcal{K}_{\mathcal{F}}$ be the class of compact sets of $D_{\mathcal{F}}$ with connected complements, and $H_{0\mathcal{F}}(K)$, $K \in \mathcal{K}$, the class of continuous non-vanishing functions on K that are analytic in the interior of K.

Theorem 1. *Suppose that all above conditions for the form $\mathcal{F}(z)$ are valid. Let $K \in \mathcal{K}_{\mathcal{F}}$, $f(s) \in H_{0\mathcal{F}}(K)$, and $T^\delta \leqslant H \leqslant T$. Then, for any $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{H} \mathrm{meas}\left\{\tau \in [T, T+H] : \sup_{s\in K} |f(s) - \zeta(s+i\tau;\mathcal{F})| < \varepsilon\right\} > 0.$$

Moreover, "$\liminf$" can be replaced by "$\lim$" for all but at most countably many $\varepsilon > 0$.

Proof of Theorem 1 will appear in [2].

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MATHEMATICS OF EXTREMES IN COASTAL AND MARINE SCIENCE

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Marine-driven hazards are often created by a variety of components that have not only different nature, temporal and spatial scales, but also exhibit radically different statistical properties of their extremes. For example, extreme water levels are jointly created by (i) storm surges that follow an exponential distribution, (ii) the volume of excess water in the Baltic Sea that follows a normal distribution, and (iii) wave-generated local water level setup that often obeys an inverse Gaussian (Wald) distribution.

The most dangerous water levels follow a generalized extreme value (GEV) or (generalized) Pareto distribution. The properties of both extreme water levels and these distributions vary substantially in different coastal segments of the eastern Baltic Sea, providing not only delicate information about a variety of driving mechanisms of waves and water level but also unique opportunities for insight into future of marine and coastal extremes.

I intend to narrate a story about how extreme water levels are born in the Baltic Sea [1], how it is possible to separate their drivers and identify which distributions they follow [2], which important message is provided via the simplest (trend) analysis of increasing water level maxima [3] and how these trends coexist with the contradicting narratives of (non)stationarity, changing climate [4] and human impacts [5]. Further on I delve deeper into the present and future of extremely high and low water levels on the shores of Estonia, Latvia and Lithuania [6]. Finally, I intend to provide a small selection of unexpected features of extremes of various kinds in our coastal regions [7].

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COLLOCATION BASED APPROXIMATIONS FOR FRACTIONAL BOUNDARY VALUE PROBLEMS

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We construct a high order method for the numerical solution of fractional weakly singular integro-differential equations in the form

$$(D^{\alpha}_{\mathrm{Cap}}y)(t) + h(t)y(t) + \int_0^t L_\kappa(t,s)y(s)ds = f(t),\ 0 \le t \le b,\ 0 < b < \infty, \tag{1}$$

subject to the conditions

$$a_{11}y(0) + a_{12}y(b_1) = \gamma_1; \quad a_{21}y'(0) + a_{22}y(b_1) = \gamma_2. \tag{2}$$

The problem is reformulated as an integral equation of the second kind with respect to $z = D^{\alpha}_{\mathrm{Cap}}y$, the Caputo fractional derivative of y of order α, with $1 < \alpha < 2$, where y is the solution of the original problem. Using this reformulation, the regularity properties of both y and its Caputo derivative z are studied. Based on this information a piecewise polynomial collocation method is developed for finding an approximate solution z_N of the reformulated problem. Using z_N, an approximation y_N for y is constructed and a detailed convergence analysis of the proposed method is given. In particular, the attainable order of convergence of the proposed method for appropriate values of grid and collocation parameters is established. To illustrate the performance of our approach, results of some numerical experiments are presented.

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DISCRETE STURM–LIOUVILLE PROBLEM FOR TWO-DIMENSIONAL ELLIPTIC EQUATION WITH THE MULTIPLE INTEGRAL IN NONLOCAL CONDITION

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We consider the following Sturm-Liuoville Problem (SLP) with nonlocal boundary condition (NBC) involving multiple integral [2]:

$$L(u) := -\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = \lambda u, \quad (x,y) \in \Omega \subset \mathbb{R}^2, \tag{1}$$

$$u(x,y) = \int_\Omega k(x,y,\xi,\eta)u(\xi,\eta)d\xi d\eta, \quad (x,y) \in \partial\Omega, \tag{2}$$

where $\Omega := \Omega_x \times \Omega_y$, $\Omega_x := \{x\colon 0 < x < a\}$, $\Omega_y := \{y\colon 0 < y < b\}$. We will assume that the function k satisfies the condition $\int_\Omega |k(x,y,\xi,\eta)| d\xi d\eta \le \varrho < 1$. The finite difference method for the linear two-dimensional parabolic equation with the NBC (2) in the square domain $\Omega = [0,1]^2$ was studied in [1].

Let us write the Finite-Difference Scheme for SLP (1)–(2):

$$\begin{aligned} L^h(U) : &= -\delta_x^2 U - \delta_y^2 U = \lambda U, \quad (x_i, y_j) \in \omega^h, \\ U_{ij} &= [K_{ij}, U]^{\mathrm{tr}}, \quad (x_i, y_j) \in \partial\overline{\omega}^h, \end{aligned}$$

and we use two-dimensional trapezoidal rule $[K_{ij}, U]^{\mathrm{tr}}$ where $K_{ij}^{kl} = k(x_i, y_j, x_k, y_l)$, $(x_i, y_j) \in \partial\overline{\omega}^h$, $(x_k, y_l) \in \overline{\omega}^h$ for approximation of integral in BC. We suppose that $\max_{x_{ij} \in \partial\overline{\omega}^h} [|K_{ij}|, 1]^{\mathrm{tr}} \le \rho < 1$.

The main aims of our study are the investigation SLP for various cases of kernel K. The main difficulty of this problem is that in the non-classical case we cannot use the method of separation of variables and decompose the problem into one-dimensional problems. Numerical experiments show that the spectrum of this problem is similar to the spectrum of corresponding one-dimensional problems. For example, all eigenvalues are real.

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FINITE DIFFERENCE METHOD FOR ELLIPTIC EQUATION WITH TWO MULTIPLE INTEGRAL CONDITIONS

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We consider a finite difference approximation to the solution of the nonlocal boundary value problem for two-dimensional Poisson equation

$$-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = f(x,y), \quad (x,y) \in \Omega \subset \mathbb{R}^2$$

with nonlocal conditions including multiple integrals

$$\begin{aligned} u(x,0) &= \gamma_0 \int_\Omega u(\xi,\eta)d\xi d\eta + g_0(x), \quad x \in \Omega_x, \\ u(x,L_y) &= \gamma_1 \int_\Omega u(\xi,\eta)d\xi d\eta + g_1(x), \quad x \in \Omega_x, \\ u(0,y) &= g_2(y), \quad u(L_x,y) = g_3(y), \quad y \in \Omega_y, \end{aligned}$$

where $\Omega := \Omega_x \times \Omega_y$, $\Omega_x := \{x\colon 0 < x < L_x\}$, $\Omega_y := \{y\colon 0 < y < L_y\}$.

We evaluate double integrals using Trapezoidal rule in two dimensions. The Laplace operator is approximated in a standard way on a five-point stencil.

The main aim is to find the conditions for the parameters γ_0 and γ_1 under which unique solution exists.

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ON SIMULATION WITH PARTICLE MOTION TRACKING FOR BIOMASS EXTRACTION PROCESS

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The extraction process of active components from biomass is based on filtration of the solvent through a filter cake of biomass. We focus on the simulation of the evolution of the filter cake. The computational domain is separated into two parts: the fluid part $\Omega_F(t)$ and the filter cake part $\Omega_C(t)$ with a time dependent interface separating the two. The filter cake can grow by adhesion of new biomass particles carried by the flow in Ω_F.

As a result, one obtains a coupled system of Navier-Stokes and Darcy equations

$$u_t + u \cdot \nabla u = \nu \triangle u - \nabla p + f, \quad \Omega_F;$$

$$u = -K \nabla p + f, \quad \Omega_C;$$

$$\nabla \cdot u = 0, \quad \Omega_F \cup \Omega_C.$$

On the interface Beavers-Joseph conditions are imposed [1].

The particles are assumed to be round and influenced by hydrodynamic drag force and gravity. The particles are assumed to stick to the filter cake upon collision. As a result, the filter cake domain grows in time.

The equations are implemented using a finite volume scheme on a voxel mesh extending a previous solver [2]. The interface between the Darcy and Navier-Stokes domains is not resolved fully and changes in discrete steps corresponding to the voxel size.

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A GENERAL COLLOCATION ANALYSIS FOR WEAKLY SINGULAR VOLTERRA INTEGRAL EQUATIONS WITH VARIABLE EXPONENT

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Piecewise polynomial collocation of weakly singular Volterra integral equations (VIEs) of the second kind has been extensively studied in the literature, where integral kernels of the form $(t-s)^{-\alpha}$ for some constant $\alpha \in (0,1)$ are considered. Variable-order fractional-derivative differential equations currently attract much research interest, and in Zheng and Wang SIAM J. Numer. Anal. 2020 such a problem is transformed to a weakly singular VIE whose kernel has the above form with variable $\alpha = \alpha(t)$, then solved numerically by piecewise linear collocation, but it is unclear whether this analysis could be extended to more general problems or to polynomials of higher degree. In the present paper the general theory (existence, uniqueness, regularity of solutions) of variable-exponent weakly singular VIEs is developed using novel techniques. These results then underpin an error analysis of collocation methods where piecewise polynomials of any degree can be used. This error analysis is also novel — it makes no use of the usual resolvent representation, which is a key technique in the error analysis of collocation methods for VIEs in the current research literature. Furthermore, all the above analysis for a scalar VIE can be extended to certain nonlinear VIEs and to systems of VIEs. The sharpness of the theoretical error bounds obtained for the collocation methods is demonstrated by numerical examples.

This is joint work with Professor Hui Liang.

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ON DERIVATIVE SAMPLING AND CONVERGENCE IN VARIATION OF GENERALIZED SHANNON SAMPLING SERIES

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I this presentation we consider generalized operators for approximating derivatives and also in the case of functions with bounded variation.

We take the formula, connecting generalized sampling operators $G_w^{\overline{\chi}}$ and sampling Kantorovich operators K_w^{χ}, presented in [2] by D. Costarelli and G. Vinti in form

$$(K_w^{\chi} f)(x) = (G_w^{\overline{\chi}} F)' \left(x + \frac{1}{2w} \right) \tag{1}$$

and give it a more simpler form. To do that, we use generalized Kantorovich-type (or Durrmeyer) sampling operators, we introduced in [4] and related kernels, introduced in [3]. This approach allows us to study derivative sampling. We give estimates of order of approximation in terms of modulii of smootness.

An overview about sampling Kantorovich operators in BV-spaces is given in [1]. We use generalized Kantorovich-type sampling operators in BV-spaces an give estimates of order of approximation in terms of corresponding modulii of smootness.

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APPROXIMATION OF ANALYTIC FUNCTIONS BY SHIFTS OF THE PERIODIC ZETA-FUNCTIONS

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Let $s = \sigma + it$ be a complex variable, and $\mathfrak{a} = \{a_m : m \in \mathbb{N}\}$ a periodic sequence of complex numbers with minimal period $q \in \mathbb{N}$. The periodic zeta-function $\zeta(s; \mathfrak{a})$ is defined, for $\sigma > 1$, by the series

$$\zeta(s; \mathfrak{a}) = \sum_{m=1}^{\infty} \frac{a_m}{m^s},$$

and is meromorphically continued to the whole complex plane with a possible simple pole at the point $s = 1$. In the report, we consider approximation of analytic functions by shifts $\zeta(s + i\varphi(t); \mathfrak{a})$ with a certain function $\varphi(t)$ and multiplicative sequence $\mathfrak{a}$.

We will discuss the following problems.

1. Approximation of analytic functions by discrete shifts $\zeta(s + iht_k; \mathfrak{a})$, $h > 0$, $k \in \mathbb{N}$, where $\{t_k\}$ is a sequence of Gram points [4].

2. Joint approximation of collections of analytic functions by shifts $(\zeta(s + i\gamma_1(\tau); \mathfrak{a}_1), \dots, \zeta(s + i\gamma_r(\tau); \mathfrak{a}_r))$, where $\gamma_1(\tau), \dots, \gamma_r(\tau)$ are increasing to infinity continuously differentiatiable functions [1].

3. Joint approximation of collections of analytic functions by shifts $(\zeta(s + ih_1\gamma_k; \mathfrak{a}_1), \dots, \zeta(s + ih_r\gamma_k; \mathfrak{a}_r))$, $h_j > 0$, where $\{\gamma_k : k \in \mathbb{N}\}$ is a sequence of non-trivial zeros of the Riemann zeta-function [2].

4. Approximation of analytic functions by shifts of compositions $F(\zeta(s + ih_1\gamma_k; \mathfrak{a}_1), \dots, \zeta(s + ih_r\gamma_k; \mathfrak{a}_r))$, where $F : H^r(D) \to H(D)$ is a certain continuous operator, and $H(D)$ denotes the space of analytic functions on $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$ [3].

The above results form the doctoral dissertation of the author.

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TIME-FREQUENCY ANALYSIS IN EUCLIDEAN SPACES

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Time-frequency analysis can be described as Fourier analysis simultaneously both in time and in frequency. Its origins are in quantum mechanics, in signal processing, and in pseudo-differential operators. A time-frequency transform is a sesquilinear mapping from a family of test functions in time to functions in the time-frequency plane. The class of time-frequency transforms is further restricted by imposing conditions stemming from basic transformations of signals and those which an idealized energy density could satisfy. In [1] with Vesa Vuojamo and Heikki Orelma, we characterized time-frequency transforms in Euclidean spaces in terms of the corresponding pseudo-differential operator quantizations and integral kernel conditions. In the talk, I will also show how to generalize time-frequency analysis to those locally compact groups that allow a nice-enough Fourier transform: wide families of topological groups can be treated, including all the compact groups and all the locally compact commutative groups, and more.

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THE FUČÍK TYPE PROBLEM WITH NONLOCAL TWO-POINT BOUNDARY CONDITIONS

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Let us investigate the Fućik problem

$$x'' = -\mu x^+ + \lambda x^-, \tag{1}$$

with one classical

$$x(0) = 0, \tag{2}$$

and other nonlocal two-point boundary condition

$$x(1) = \gamma x(\xi), \tag{3}$$
$$x(1) = \gamma x'(\xi), \tag{4}$$

here $\xi = \frac{m}{n} \in (0,1)$, $\gamma \in \mathbb{R}$, m and n $(0 < m < n)$ are positive coprime integer numbers.

Also we analyze the Fućik problem (1) with condition

$$x'(0) = 0 \tag{5}$$

in the left boundary, as well as with nonlocal conditions (3) – (4) in the right boundary. The aim of investigation is to analyze main properties of the nonlocal problem spectrum and compare them with classical Fućik spectrum. The idea of boundary conditions was taken from the work [1], where the Sturm-Liouville equation

$$x'' = -\lambda x$$

was analyzed with boundary conditions (2) - (5).

Some of the results are the logical continuation and generalization of previous authors' investigations [2; 3; 4].

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OPERATORS OF FUZZY MATHEMATICAL MORPHOLOGY

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Mathematical morphology arose from the needs of practical geology in the 1970s. However soon, fuzzy morphology theory found important applications in other fields, particularly in image processing, and is now widely studied and applied by many researchers. Mathematical morphology theory within fuzzy sets was firstly introduced in the paper by De Baets et al. [1]. As in "classical" approach, the fuzzy mathematical morphology was based on two operators - dilation $\mathcal{D}$ and erosion $\mathcal{E}$ are induced on the linear space by a fuzzy structuring element B.

A lot of work has been done in the study of fuzzy morphological operators on Euclidean space and many applications have been found in the study of different practical problems. Taking as a basis a pair of operators that behave like dilation and erosion lead to various abstract, in particular, algebraic approaches to the subject of mathematical morphology. In turn, this made it possible to develop a categorical view of mathematical morphology, see, e.g. [2], [3], [4] et al. However, the disadvantage of these abstract approaches is that the role of the structural element is practically lost.

As far as we know, there was no much work done to develop "classical"(fuzzy) mathematical morphology theory within the framework of category theory. This in turn essentially restricts the possibility of considering relations between (fuzzy) morphological spaces, their transformations, products and direct sums of (fuzzy) morphological spaces, etc.

The purpose of our talk is to present a category containing "classical" (fuzzy) morphological spaces realized in the spirit of the article [1]. This provides us the flexibility to deal with transformations of fuzzy morphological spaces, which in turn can enrich the use in practical applications. As the basis for this category, we take structured additive groups $(X, S, +_X, 0_X)$ and $(Y, T, +_Y, 0_Y)$ as objects and certain L-fuzzy relations $R : X \times Y \to L$ between them as morphisms.

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APPROXIMATE SOLUTIONS TO LINEAR FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS

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We consider a class of fractional weakly singular integro-differential equations

$$(D^{\alpha_2}_{Cap}y)(t) + d_1(t)\,(D^{\alpha_1}_{Cap}y)(t) + d_0(t)y(t) + \int_0^t (t-s)^{-\kappa_0} K_0(t,s)y(s)ds + \int_0^t (t-s)^{-\kappa_1} K_1(t,s)(D^{\theta}_{Cap}y)(s)ds = f(t), \quad 0 \le t \le b, \tag{1}$$

subject to boundary conditions

$$a_i\, y^{(i)}(0) + b_i\, y^{(i)}(b) = \gamma_i, \quad a_i, b_i, \gamma_i \in \mathbb{R}, \quad i = 0, \ldots, n-1. \tag{2}$$

Here D^{δ}_{Cap} is the Caputo differential operator of order $\delta > 0$ and $n := \lceil \alpha_2 \rceil$ is the smallest integer greater or equal to the fractional order α_2. We assume that

$$0 < \alpha_1 < \alpha_2 \le n, \quad \theta \in (0, \alpha_2), \quad \kappa_0, \kappa_1 \in [0, 1)$$

and that the given functions d_0, d_1, K_0, K_1 and f are continuous on their respective domains.

On the basis of [1] we study the existence, uniqueness and regularity of the solution y to problem (1)–(2) and show that under suitable conditions this problem can be reformulated as a Volterra integral equation of the second kind with respect to the fractional derivative $D^{\alpha_2}_{Cap}y$. We regularize the solution of this integral equation with the use of a suitable smoothing transformation and construct a numerical solution to the transformed integral equation by applying a piecewise polynomial collocation method on a mildly graded or uniform grid. We show the convergence of the proposed algorithm and present global superconvergence results for a class of specific collocation parameters. Finally, we complement the theoretical results with some numerical examples.

This is a joint work with Arvet Pedas.

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ON A SOLVABILITY OF A THREE-PARAMETER PROBLEM ARISING IN HEAT TRANSFER

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A convective flow in a tall vertical annulus of radii R and 1, in a case when the walls of the annulus are maintained at the same temperature, can be descsribed by the nonlinear boundary value problem

$$T'' + \frac{1-\alpha}{r} T' + F\,e^{T} = 0, \quad T(R) = 0 = T(1), \tag{1}$$

where α is a real number, F is a positive number and $R \in (0,\ 1)$. We study the existence of the positive solutions $T(r)$ of problem (1) depending on the values of the parameters α, F and R.

By using the technique described in [1] and based on the Krasnosels'kiĭ-Guo fixed point theorem of cone expansion and compression, we prove the following theorem.

THEOREM 1. *Suppose that α and R are real numbers and $R \in (0,\ 1)$. There exists a positive number F^* such that for every $F \in (0;\ F^*)$ the problem* (1) *has two positive solutions $\underline{T}$ and $\overline{T}$ with $\|\underline{T}\| < 1 < \|\overline{T}\|$.*

The results obtained allow us to formulate a number of statements about solvability of the problem (1) depending on the values of one of the parameters α, F or R, if other two parameters are fixed.

The results of present study can be used for design of bioreactor systems.

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