VILNIUS UNIVERSITY

ANDŽELA MIALIK

LOGISTIC METHOD TO INDICATE UNSUSTAINABLE GROWTH SITUATIONS

Summary of doctoral dissertation

Physical Sciences, Informatics (09P)

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This doctoral dissertation was written at Vilnius University in 2009-2013.

Scientific supervisor:
Prof. dr. Stasys Girdzijauskas (Vilnius University, Social Sciences, Economics – 04 S)

Consultant:
Prof. dr. Virgilijus Sakalauskas (Vilnius University, Physical Sciences, Informatics – 09 P)

The dissertation will be defended before the Board of Sciences of Informatics:

Chair:
Prof. habil dr. Gintautas Dzemyda (Vilnius University, Physical Sciences, Informatics – 09 P).
Members:
prof. habil dr. Edmundas Kazimieras Zavadskas (Vilnius Gediminas Technical University, Social Sciences, Economics – 04 S);
prof. dr. Romas Baronas (Vilnius University, Physical Sciences, Informatics – 09 P);
prof. dr. Audrius Lopata (Vilnius University, Physical Sciences, Informatics – 09 P);
prof. dr. Vitalijus Denisovas (Klaipėda University, Physical Sciences, Informatics – 09 P).

Opponents:
prof. habil. dr. Rimvydas Simutis (Kaunas Technological University, Physical Sciences, Informatics – 09 P);
prof. (HP) dr. Arūnas Andziulis (Klaipėda University, Technological Sciences, Engineering of Informatics – 07 T)

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Address: Muitinės st. 12, LT-44280, Kaunas, Lithuania.

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ANDŽELA MIALIK

LOGISTINIS NETVARIŲ AUGIMO SITUACIJŲ ATPAŽINIMO METODAS

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Mokslinis vadovas:
prof. dr. Stasys Girdzijauskas (Vilniaus universitetas, socialiniai mokslai, ekonomika – 04 S)

Konsultantas:
prof. dr. Virgilijus Sakalauskas (Vilniaus universitetas, fiziniai mokslai, informatika – 09 P)

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Pirmininkas – prof. habil. dr. Gintautas Dzemyda (Vilniaus universitetas, fiziniai mokslais, informatika – 09P).

Nariai:
prof. habil dr. Edmundas Kazimieras Zavadskas (Vilniaus Gedimino technikos universitetas, socialiniai mokslai, ekonomika – 04S);
prof. dr. Romas Baronas (Vilniaus universitetas, fiziniai mokslai, informatika – 09P);
prof. dr. Audrius Lopata (Vilniaus universitetas, fiziniai mokslai, informatika – 09P);
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prof. (HP) dr. Arūnas Andziulis (Klaipėdos universitetas, technologiniai mokslai, informatikos inžinerija – 07T).

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SUMMARY OF DOCTORAL DISSERTATION

INTRODUCTION

Everything that rises must converge
/Flannery O'Connor/

The relevance of the work

While analysing the models of growth processes, more and more attention is paid to the logistic growth. Logistic growth models are applied in various scientific fields: biology, chemistry, physics, medicine, economics, informatics, etc [1, 2, 3, 4]. Scholars all around the world search for, discover and propose new models [5, 6, 7], examine their efficiency and apply them to model growth processes. In economics, the models of growth processes are employed to explore economic expansion, capital investing, to analyse and predict financial crises and to make other economic decisions (such as withdrawing from the market or changing it, creating new investment strategies, etc.). In such cases, the capital growth is modeled. A growth model is intended to be simple, thus attention is rarely paid to various growth obstacles and restrictions [3, 2]. For this reason in many models the capital is defined as of unrestricted growth which is not a precise evaluation of the capital.

Systems are created to model economic processes. They allow to recognize and predict financial crises, look for the appropriate solution in particular situations. Prediction is a complex process which is mostly carried out on the basis of historical data [8]. It is difficult to precisely determine concrete risk points in the process of logistic modeling, therefore, in order to determine the limit, the evaluation of error probability and designation of floating risk range are only considered.

The growth process is managed more efficiently when the logistic model based on real statistical data is applied. A user of such system can operatively react to the changes of a growth trajectory. This allows to evaluate the threats in time, to easier survive economic declines and avoid huge losses for both businesses and natural persons. The tools that now exist in the market are mostly based on typical logistic models widely used in biology [9, 10].

The model introduced in the present research is dedicated to predict growth limit. It requires determining the tendencies of product development and designating unsustainable economic situations as well as evaluating the approaching deviation of the growth process.

Overview of scholarly investigation of the problem

The research of population growth intensified in the 19th century. Galileo Galilei (1564 – 1642), Thomas Robert Malthus (1766 – 1834) and other scholars were the first to notice and investigate the growth tendencies and prediction possibilities. In 1825, Benjamin Gompertz (1779 – 1865) published a study where he discussed the works of T. R. Malthus and introduced a new growth model. In 1838, Pierre-Francois Verhülst offered a new function of logistic growth applicable to examine the growth of population. The differential equation offered by P. F. Verhülst was a differential form of Malthus's equation supplied with the factor expressed as linearly decreasing function [2]. Afterwards scholars carried out many similar researches. Biochemist Brailsford Robertson (1884 – 1930) suggested determining the growth process of an individual in relation with the logistic function. Raymond Pearl used an improved logistic function (1879 – 1940) to designate the population growth. He also considered the factor of the human age. Alfred J. Lotka and Vito Volterra applied the generalized growth equations to quantitative model of competition between different species. They created the equations for the system predator-prey. Scholar Zvi Griliches was one of the first to initiate the technological change in research (substitution models) who described the process of growth by means of S-functions [11]. In 1979, Vijay Mahajan, Eitan Muller applied Bass and Gompertz's logistic growth functions in the research of a product growth [1].

The application of S-function to predict technological advancement has been applied since the beginning of 1960. Scholarly literature describes and analyses the models created by Zvi Griliches and Edwin Mansfield in 1961 [11, 12]. The scholars discussed the factors following which companies had to rationalize their development. Everett M. Rogers (1931 – 2004) drew an assumption that innovations spread in society according to the S-function tendencies [13].

The application of the growth models, initially targeted at the examination of biological processes, expanded to physics, chemistry, sociology and other fields [2, 14, 6, 15]. By means of logistic analysis, Arnulf Grubler and Nebojsa Nakicenovic [16] prove that, from the point of view of empirical argument, the growth of the economic process (fluctuation of prices, development of infrastructure, technological advancement) is neither homogeneous nor sustainable. The structure of S-function model that involves the saturation parameter is the main object analysed in the processes of economic dispersion. The authors claim that economic development is determined by various stages, many interrelated groups of technologies and that phasing-out period from one competitor to another complies with Kondratiev's long-wave model [16]. By means of S-function logistic growth model, it is possible to determine the tendency of growth and predict future events. Dmitry Kucharavy and Roland De Guio noted that in case the function is adjusted according to the statistical data, the technique of adjustment provides the opportunity to obtain two out of three parameters of the logistic function. However, the essence of the problem lies in the unsustainable and undetermined nature of the upper limit of growth. It is not sufficient to indicate a wide spectrum of growth while making a prediction. It is important to determine and evaluate separate parameters of growth. The S-function prediction study relies on the assumption that logistic function reflects the natural growth law [17]. The scholarly works of recent 2-3 decades have introduced the problem of the dispersion of various innovations, substitutes of infrastructures and energy sources. The results of the analysis of the technologies and the global technology changes were presented by A. Grubler in 1986, 1989 [18]. The scholar explored the dispersion of infrastructures including the channels, railway, motor-ways and air-lines and revealed that their dispersion complies with the logistic principle. Economic, social and technological development occurs due to two reasons: essentially new ideas (the main innovations) and renewal and improvement of existing methods and systems (products and processes) [19, 16].

A wider dispersion of logistic economic models began only two decades ago, especially after the introduction of the general interest model proposed by Girdzijauskas in 2002. The application of the logistic interest model in economy is analysed in the works of D. Štreimikienė, V. Moskaliova, E. Jurkonytė, R. Mackevičius, D. Grundey,

M. Dubnikovas, V. Boguslauskas, J. Čepinskis, A. Pikturna, F. Ivanauskas, G. Garšva, E. Merkevičius, E. Jociūtė and others [20, 21, 22, 23, 24, 25, 26]. It should be noted that Carlota Perez used the logistic function to illustrate the long business cycle (Kondratiev). She stressed the following peculiarities of economic cycles: the beginning of the technological period is considered as an abrupt invasion, the rise as commotion, the development as synergy and the saturation as maturity [27].

The evaluation of the growth process saturation is crucial in economic research. The main factor of the logistic model is the aim to determine the growth parameters as accurately as possible. The most difficult parameter to determine, due to its relatively frequent changing, is the **growth limit** parameter. To determine the parameters of the model, the method of least squares is widely applied [6]. However, it fails in the logistic growth model when the growth limit is unknown. The fuzzy method is applied in many spheres including business and economics. The fuzzy method is used in a variety of economic fields: marketing (purchase analysis, fraud indication, evaluation of the service quality), finances (the schemes of share market prediction, portfolio selection, risk management, loan evaluation systems) and e-business (e-trade solutions, personalization, e-trade risk analysis) and others [28].

The **object** of the research is the product (population) growth in a limited space.

The **aim** of the research is to create the method based on a logistic interval growth model that would enable to identify unsustainable situations in the economic development.

For the aim of the research to be achieved the following tasks have been set:

- to explore growth modeling systems (Loglet, LSM);
- to determine the advantages of logistic growth models and designate their relevance;
- to create a new LogMod logistic modeling method:
    - to designate methods of digital analysis appropriate in a logistic growth model;

    - to evaluate market capacity limit by means of the fuzzy set method;
- to experimentally compare the accuracy of the Loglet and LogMod methods by indicating the growth of real estate market in Lithuania;
- to examine the LogMod method of market capacity determination and evaluate its appropriateness by indicating the growth of the real estate market index in the USA.

The assumptions of the dissertation

- General (logistic) interest model is the most appropriate logistic growth model to model economic situations.
- The method of least squares is insufficient to determine the market saturation parameter K_p.
- It is beneficial to use the fuzzy set method to determine the saturation parameter in a logistic model.
- Market saturation is a source of risk in the case of leveraged investment.
- LogMod model is appropriate to indicate unsustainable economic situations of markets' growth.

The novelty of the research

Logistic models (including the pseudo-logistic model whose co-author is the author of the dissertation) have been analysed in the present study. It has been determined that general (logistic) interest model is the most suitable to examine economic populations. Further researches lead to the construction of a new transformed model considering truncated mean of statistical data from 10 to 90 percent of the potential capital value. Such evaluation reduces deviation of the logistic model results. The created expert *fuzzy* system helps to determine the value of the saturation parameter K_p and use it to predict critical situations. Having applied the method of least squares to the statistical data series, the rest parameter of the model determining the speed of growth was calculated. The errors of the parameter K_p prediction are corrected by means of the bootstrap method. The results of the application of these tools allow to evaluate the sustainability of market development. It is important to note that, in this methodology, the prediction of K_p does not depend on statistical data.

The author of the dissertation participated in the creation of a new leveraged investment model. The model shows that the longer the investment period and the greater saturation of the market, the higher the investment risk. By means of general (logistic) interest model it was designated that the application of investment leverage in saturated markets may cause the debt trap effect.

Practical application of the work

Application of the logistic growth model provides the possibility to indicate the formation of an unsustainable economic market state, hence, the opportunity to make rational decisions. The expert's knowledge is replaced by the expert *fuzzy* model designed for that particular market. The newly created model will allow the monitoring of market saturation in its heat phase. It is important for banks to know that the usage of leveraged investment in saturated markets can cause the debt trap effect.

The methodology of the research

The following methods were applied in the present research: systemic and comparative analysis of scholarly literature, mathematic modeling, economic logistic analysis and synthesis, statistical analysis, modeling of algorithms. The MatLab software environment, the tool of fuzzy set modeling Matlab Fuzzy Logic Toolbox, and ready-made MagicDraw UML architectural models of graphic processes were used to create the model. Statistical data used in the created model was processed by the Microsoft Excel calculator.

The structure of the work

The first part embraces the analysis of scholarly literature where the focus is laid on the overview of growth models. Classical and logistic growth models are explored. Growth models are generalized, classified and systematized. The analysis of the growth modeling systems is provided.

The second part presents the transformed interval logistic growth model. The new methodology of market growth evaluation is created. The algorithm of modeling and the methods of digital analysis applied in the study are described. The automatic generation of the rules applied in the *fuzzy* system is provided. The methodology of the expert *fuzzy* system used to calculate the saturation parameter is defined. The leveraged investment model is discussed.

The third part presents the following research according to the designed methodology: the real estate market in Lithuania, investment fund and the index of real estate in the USA were examined. The results of the new growth model application are given. The case of leveraged investment modeling that shows the debt trap phenomenon is analysed. It is revealed that when the saturation level grows, investment risk occurs and rapidly increases.

LOGISTIC MODELS OF A PRODUCT GROWTH

The research and application of logistic growth models increases every year: different logistic models are used in different countries [29, 30, 31]. The notion of **saturation** is spread in natural and social sciences as well as in the field of technologies. It is the 'keystone' in the study of unsustainable economic situations too. An intensive dispersion of the theme "Logistic theories" began several decades ago in biology (popular model offered by P. F. Verhülst), chemistry, physics, social sciences (F. Bass, B**.** Gompertz growth functions). These theories are now widely applied in economics. Logistic models become more popular among the scholars from various fields. Having proved the expediency of the logistic models, digital methods of analysis were adapted in these models. This allows automatization of calculations, hence, more accurate results.

Saturation is important not only in biology, chemistry, physics, medicine but also economics. There are many known saturation values. For instance, in medicine, oxygen saturation in blood; in physics, magnetic saturation of materials; in biology, saturation of population in an area, etc. Saturation of solutions is interesting. A solution in which a particular material no longer dissolves is considered to be saturated. It is important to note that the characteristics of a saturated and unsaturated system differ greatly [32, 33].

In order to saturate a market it must be either a captive or a closed market. The capacity of the market determines its size. It might be assumed that an infinite market cannot be saturated because its capacity is infinite, thus the market is never filled to the full extent [33]. The most common definition of the market is the sphere of exchange where, under the laws of demand and supply, the process of buying and selling takes place. It is a particular sphere where (beside various mediators) two participants, the seller and the buyer, act. From the point of view of the investor, every product group or even and individual product has its own market. A market may also be conceived of as a sum of several individual markets. Markets can be **captive**, **open**, or **semi-captive** (this type makes the biggest number of markets). All markets have their own capacity (the capacity of an infinite market is infinite) [34, 25, 35].

Financial bubbles have a great impact on economic stability. The classical definition of a financial bubble was offered by Kindleberger in 1978 [36]. However, his definition is rather inaccurate (or, to be more precise, old-fashioned), therefore, it had to be modified by indicating the essential reason of the bubble formation, i.e. the finite market size. It should be stressed that a **bubble** (**financial bubble**) is a constant increase of the price when the initial increase of the price occurs due to the capital saturation within a captive or semi-captive market which causes great internal rate of return. As a consequence, the expectation of further continuous growth of income is built up which additionally attracts new short-term investors (as well as speculators) and thus cause overproduction [32, 37, 38, 39].

The amount of capital which might be effectively absorbed in a concrete market (concrete investment environment) is conceived of as a market capacity [34, 25]. It should not be forgotten that the most important characteristic of market capacity is the potential capital. Market capacity is the largest theoretically possible amount of sales of a particular product which might be achieved by all companies in the market during a set period of time. Frequently, due to increasing production and sales of goods and services of the same market, the market is gradually filled and market **saturation** is reached. Saturation typically refers to fullness, filling to some extent, complete fullness, satiety [34, 25, 37]. Market saturation occurs when market capacity turns from the infinite into measurable or when investments grow faster than the market itself expands (i.e., when the supply grows faster than the demand).

Scholars pay more attention to market saturation [2, 40, 41, 42], yet they consider only the external – surface – aspect of saturation [2, 43].

During the last decade deep logistic studies carried out in Lithuania surpassed the expectations: it was revealed that saturation not only exists in the sphere of finances but manifests itself in a paradoxical way, hence it is one of the main factors causing economic cataclysms [44, 45, 32, 46].

The theory of economics defines **capital** as the value or assets of financial and physical funds. In economic models, capital is conceived of as a certain (abstract) value dependent on time. Money put into the bank for interest or invested into production, i.e., 'employed' and creating the added value, turn into capital. Logistic management of capital is the administration of the capital (in other words, 'employed money' or other assets – real, financial or intellectual) within a market of limited capacity, i.e., when "the space of activity is partially or fully closed" [32, 47, 48].

Economic growth takes place cyclically, the sequence of changes is reoccurring, yet irregular. When the capital or the niche shrinks, the economic slowdown, decline or crisis might be caused by not only economic bubbles but *credit traps*. The effect was found in the centre of Economic logistic analysis in Kaunas Faculty of Humanities, Vilnius university, and still requires a thorough analysis, precision and detailed practical examination. The application of informatics and economics in the study of logistic models allowed obtaining more accurate results. During the common researches, the efficiency of the logistic interest model was revealed [32, 47, 49].

The generalization of growth models described in the theoretical part of the work is provided in Table 1. In 2002, a professor of Vilnius university, S. A. Girdzijauskas restructured P. F. Verhülst model: he suggested expressing the parameter $\boldsymbol{m}$ by the interest rate $\boldsymbol{i}$. In such a way the limits of the model's applicability were expanded. A generalized logistic interest model was created on the basis of P. F. Verhülst model. The new model creates a platform for not only the P. F. Verhülst model and exponential models but also many other specialized growth models.

Table 1. Growth models

No.	Model	Differential form of growth models	Growth model	Coefficient values
1	Generalised logistic model	$\frac{dK}{dt} = m \cdot K^{\alpha} \cdot \left[1 - \left(\frac{K}{K_p}\right)^{\beta}\right]^{\gamma}$	K – the size of population at the time moment t, K_p – the size of potential population, m – growth coefficient; α, β, γ – growth parameters.	Initial conditions: when $t = 0$, $K = K_0$
2	Malthus's (simple interest)	$\frac{dK}{dt} = m$	$K(t) = K_0 + m \cdot t$	$\alpha = 0; \beta \neq 0; \gamma = 0$
3	Malthus's (compound interest). Exponential growth	$\frac{dK}{dt} = m \cdot K$	$K(t) = K_0 \cdot e^{m \cdot t}$	$\alpha = 1; \beta \neq 0; \gamma = 0$
4	Linear-limited. Pseudo-logistic function	$\frac{dK}{dt} = m \cdot \left(1 - \frac{K}{K_p}\right)$	$K(t) = K_p + \left(K_0 - K_p\right) \cdot e^{-m \cdot t / K_p}$	$\alpha = 0;\ \beta = \gamma = 1$
5	Verhulst equation – logistic growth model	$\frac{dK}{dt} = m \cdot K \cdot \left(1 - \frac{K}{K_p}\right)$	$K(t) = \frac{K_p}{1 + \left(K_p / K_0 - 1\right) \cdot e^{-m \cdot t}}$	$\alpha = \beta = \gamma = 1$
6	Von Bertalanffy's growth function	$\frac{dK}{dt} = m \cdot K^{\frac{2}{3}} \cdot \left[1 - \left(\frac{K}{K_p}\right)\right]^{\frac{1}{3}}$	$K(t) = K\left[1 + \left[1 - \left(\frac{K_0}{K}\right)^{\frac{1}{3}}\right] e^{-\frac{1}{3} \cdot m \cdot K^{-\frac{1}{3_t}}}\right]^3$	$\alpha = \frac{2}{3},\ \beta = \frac{1}{3},\ \gamma = 1$
7	General growth function	$\frac{dK}{dt} = m \cdot K^{1+\beta \cdot (1-\gamma)} \cdot \left[1 - \left(\frac{K}{K_p}\right)^{\beta}\right]^{\gamma}$	$K(t) = \frac{K}{\left[1 + \left[(\gamma - 1) \cdot \beta \cdot m \cdot K^{\beta \cdot (1-\gamma)} t + \left[\left(\frac{K}{K_0}\right)^{\beta} - 1\right]^{1-\gamma}\right]^{\frac{1}{1-\gamma}}\right]^{\frac{1}{\beta}}}$	$\alpha = 1 + \beta \cdot (1 - \gamma)$
8	Richard's growth function	$\frac{dK}{dt} = m \cdot K \cdot \left[1 - \left(\frac{K}{K_p}\right)^{\beta}\right]$	$K(t) = K\left[1 - e^{-\beta \cdot m \cdot t}\left[1 - \left(\frac{K_0}{K}\right)^{-\beta}\right]\right]^{\frac{1}{\beta}}$	$\alpha = \gamma = 1$

9	Gompertz's growth function	$\frac{dK}{dt} = m \cdot K \cdot \left[\ln\left(\frac{K_p}{K} \right) \right]$	$K(t) = K_0 \cdot e^{-\beta \cdot (e^{\alpha \cdot t} - 1)}$	$\gamma = 1,\ \beta \to 0.$
10	Weibull distribution		$K(t) = K_p \cdot e^{-\left(\frac{t}{\beta}\right)^{\alpha}}$	α, β > 0, when α=2, is known as Rayleigh distribution
11	Heinz von Foerster coalition growth model	$\frac{dK}{dt} = m \cdot K^{1+h}$	$K(t) = \frac{1}{(h \cdot m(T - t))^{1/h}}$	h,m positive constants
12	O.C.Ferreira logistic model	$\frac{dK}{dt} = m \cdot K \cdot (K_p - K)$	$K(t) = \frac{K_p}{\left(1 + \left(\frac{K_p}{K_0} - 1\right) e^{-m \cdot K_p \cdot t}\right)}$	

Source: the table of growth models based on P. F. Verhülst model [50], supplemented by the author of the dissertation.

All growth functions are expressed according to one variable ***m*** in order to reveal the importance of the parameter and to facilitate the transition to the logistic growth model because namely this model is related to the interest rate which is the most important factor in economics. The given models are widely applied to analyse economic phenomena because the interest rate ***i*** is not directly expressed in historical models.

Reformation of logistic models and indication of general (logistic) interest caused a qualitative spurt in the research. It occurred after the growth parameter was expressed by interest substitution $m = \ln(1+i)$. Having applied discounting and regarding the market saturation, the possibility arose to describe the mechanism of the formation of unsustainable economic situations. The main links of the mechanism are as it follows: **paradoxes** of **increasing profitability** (manifested as an economic bubble) and **credit trap** (manifested as unbearable burden of loans). They were discovered by applying the current value and discounting procedures. *It is practically impossible to do it by means of standard logistic functions* because they are not suitable for discounting. The detected paradoxes are very important (frequently appear as the determinants) in the formation of unsustainable economic situations [47, 32, 37].

From the variety of existing growth models the ones that might be applied to analyse economic processes in one way or another are selected. It appeared that these models might be classified according to certain features (Table 2). According to the type of interest growth: simple, compound, nominal, continuous and pseudo-simple interest. According to universality (generality): selected and general (logistic) forms. General forms might be supplemented by current values and expressions of interest rate. The dependence of the latter on the saturation (ratio K/K_p) obviously reveals the existence of the increasing profitability paradox [47].

Table 2. Base interest matrix (system)

No.	Model	Separate (common) interest	General (logistic) interest	Current value of general interest	General interest rate
1	Simple interest	$K = K_0 \cdot (1 + i \cdot t)$	$K = \frac{K_p \cdot K_0 \cdot (1 + i \cdot t)}{(K_p - K_0) + K_0 \cdot (1 + i \cdot t)}$	$K_0 = \frac{K_p \cdot K}{K_p + (K_p - K) \cdot i \cdot t}$	$i = \frac{K - K_0}{K_0 \cdot (1 - K/K_p)} \cdot t^{-1}$
2	Compound interest	$K = K_0 \cdot (1 + i)^t$	$K = \frac{K_p \cdot K_0 \cdot (1 + i)^t}{(K_p - K_0) + K_0 \cdot (1 + i)^t}$	$K_0 = \frac{K_p \cdot K}{K + (K_p - K) \cdot (1 + i)^t}$	$i = \sqrt[t]{\frac{K}{K_0} \cdot \frac{1 - K_0/K_p}{1 - K/K_p}} - 1$
3	Nominal interest	$K = K_0 \cdot \left(1 + \frac{j}{k}\right)^{k \cdot t}$	$K = \frac{K_p \cdot K_0 \cdot \left(1 + \frac{j}{k}\right)^{k \cdot t}}{(K_p - K_0) + K_0 \cdot \left(1 + \frac{j}{k}\right)^{k \cdot t}}$	$K_0 = \frac{K_p \cdot K}{K + (K_p - K) \cdot \left(1 + \frac{j}{k}\right)^{t \cdot k}}$	$j = k \cdot \left(\sqrt[t \cdot k]{\frac{K}{K_0} \cdot \frac{1 - K_0/K_p}{1 - K/K_p}} - 1\right)$
4	Continuous interest	$K = K_0 \cdot e^{j \cdot t}$	$K = \frac{K_p \cdot K_0 \cdot e^{j \cdot t}}{(K_p - K_0) + K_0 \cdot e^{j \cdot t}}$	$K_0 = \frac{K_p \cdot K}{K + (K_p - K) \cdot e^{j \cdot t}}$	$j = \ln\left(\sqrt[t]{\frac{K}{K_0} \cdot \frac{1 - K_0/K_p}{1 - K/K_p}}\right)$
5	Pseudo-simple interest	$K = K_0 + t \cdot \ln(1 + i)$	$K = K_p + (K_0 - K_p) \cdot (1 + i)^{-t/K_p}$	$K_0 = K_p + (K - K_p) \cdot (1 + i)^{t/K_p}$	$i = \left(\frac{K_p - K_0}{K_p - K}\right)^{\frac{K_p}{t}} - 1$

Source: [47], supplemented by the autor of the dissertation.

The values of variables in Table 2:

K_p is the potential (limit, maximum) value of the invested capital;

K_0 is the initial value of investment;

K is the accumulated investment sum during the period t;

i, j are the interest rates;

t is the period of investment or the number of investment periods (calculated in the same units of time as interest rate);

k is the number of recalculations in one interest period.

Despite the successful application of P. F. Verhülst model in both academic research and practice and its suitability for a variety of products, it was acknowledged that the model is not convenient to analyse certain growth processes. It is commonly accepted to use the growth coefficient (usually marked as $\boldsymbol{r}$) expressed by the percent (interest) rate $\boldsymbol{i}$, that is, to use the expression $\boldsymbol{r} = 1+\boldsymbol{i}$ for solving both theoretical and practical (especially financial economic) tasks. Such form of the percent (interest) rate is entrenched in simple and compound percent (interest), therefore it is desirable to use this form in other growth models. It is also important to note that the mentioned expression of the interest rate should not only exist in growth models but that these models should be somehow related to the corresponding formulas of simple and compound interest. In other words, it is important to find out whether it is possible to obtain simpler forms out of the complex ones by changing individual parameters of the formula. An extensive research revealed that some forms of logistic models have such relations, and transformations are possible [50, 51]. For instance, general (logistic) interest turn into compound interest if the potential product value $\boldsymbol{K_p}$ is increased infinitely, whereas simple interest of limited growth (logistic) turn into simple interest. The comparison of the interest models is provided in Table 3 [33, 52, 53].

Table 3. Relations between growth models and interest

No.	Model	Future value	Current value	Interest rate, growth parameter
1	Compound interest	$K(t) = K_0(1+i)^t$	$K_0 = \frac{K}{(1+i)^t}$	$i = \frac{K - K_0}{K_0}$
2	**General (logistic) interest (Compound interest of limited growth)**	$K(t) = \frac{K_p \cdot K_0(1+i)^t}{(K_p - K_0) + K_0(1+i)^t}$	$K_0 = \frac{K \cdot K_p}{K + (K_p - K) \cdot (1+i)^t}$	$i = \sqrt[t]{\frac{K}{K_0} \cdot \frac{1 - K_0/K_p}{1 - K/K_p}} - 1$
3	Simple interest	$K(t) = K_0(1 + i \cdot t)$	$K_0 = \frac{K}{1 + i \cdot t}$	$i = \frac{K - K_0}{K_0}$
4	Simple interest of limited growth (logistic)	$K(t) = \frac{K_p \cdot K_0(1 + i \cdot t)}{(K_p - K_0) + K_0(1 + i \cdot t)}$	$K_0 = \frac{K_p \cdot K}{K_p + (K_p - K) \cdot i \cdot t}$	$i = \frac{K - K_0}{K_0 \cdot (1 - K/K_p)} \cdot t^{-1}$

Source: [33], supplemented by the autor of the dissertation.

The values of variables in the formulas in Table 3:

- *K(t)* is the *size of a product* (sometimes capital) after t time moments;
- K_p is the *maximum* (potential, limit) *product value* (market capacity);
- K_0 is the *initial product value* (initial investment);
- *i* is the *interest rate*;
- *t* is the investment period or a sum of investment periods (measured in the same units of time as for the interest rate);

The graphs of the models are provided in Fig. 1. The graphs demonstrate the specificity of growth: graphs of simple and compound interest indicate infinite growth, whereas the logistic models have limited growth while their graphs appear to be part of S-function. Considering the fact that every modeled real growth process has a limit, the general (logistic) interest will be treated as the main model of the research.

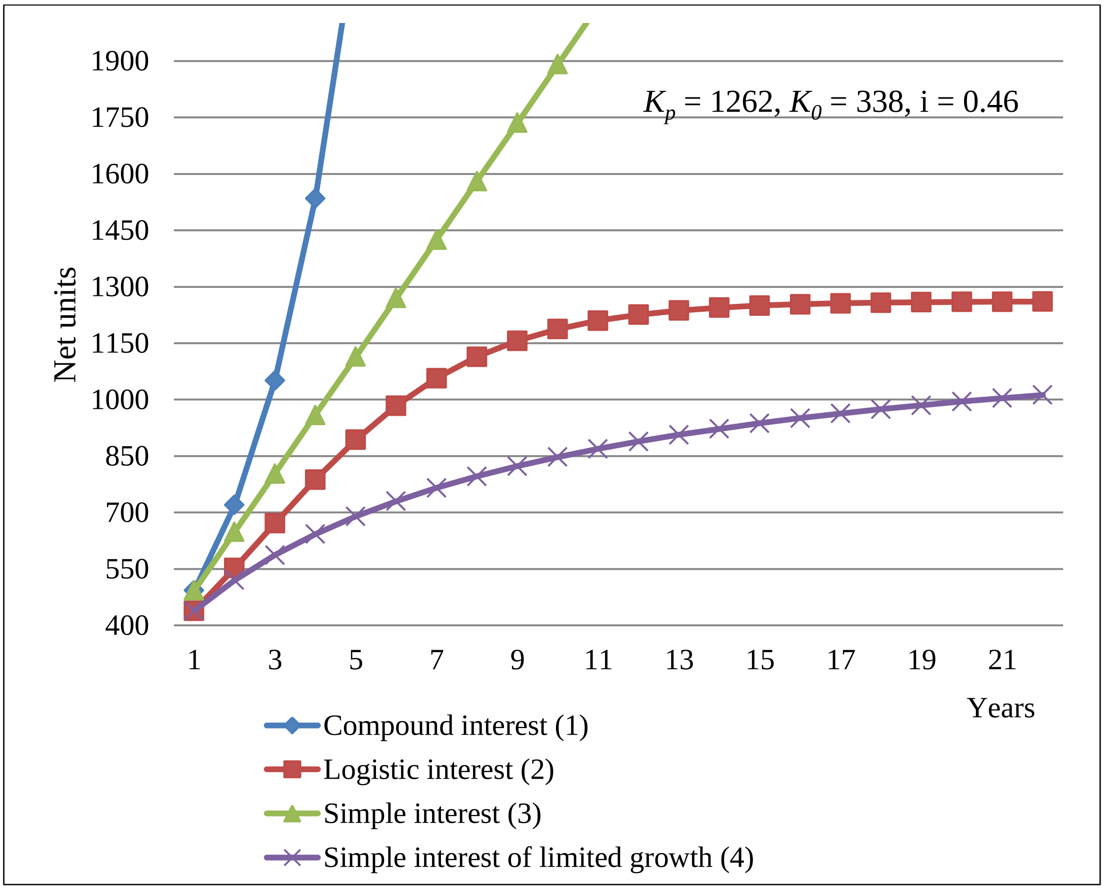

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Fig. 1. Functions of product growth models

After the overview of scholarly literature and analysis of known growth functions targeted at the prediction of a product or a cell growth, population size, birth- and death-rates, it was decided to modify the model of logistic interest proposed by S. A. Girdzijausas in 2002 [7] by evaluating the truncated mean (from 10 to 90 percent) of statistical data.

The profitability of a product (investment) is one of the most important features of a market and one of the most important economic characteristics. However, this characteristic has not been investigated sufficiently so far. The dependence of profitability on saturation has been ignored for a long time; the relation between profitability and inflation, over-production, etc. has not been studied. It happened so because profitability was explored without the application of logistic interest models and, at the same time, the market saturation was not considered. Later on, after applying general interest the paradox of increasing profitability was discovered, yet it remained unclear if it might be detected by means of other logistic models.

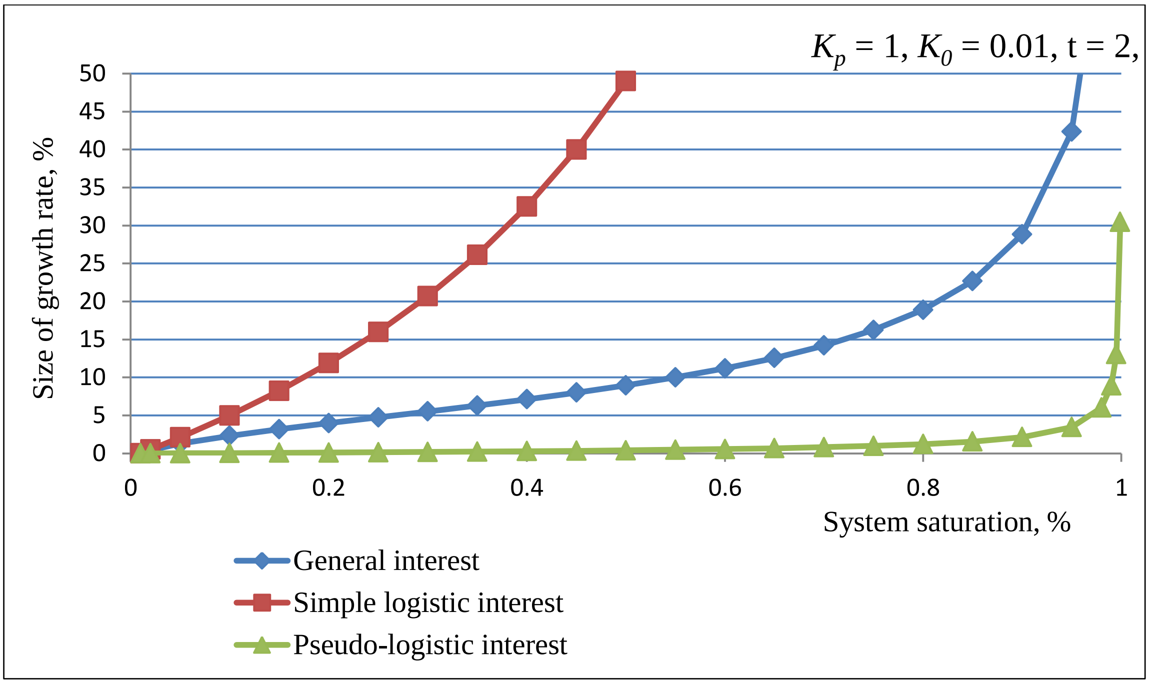

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Fig. 2. Dependence of interest rate on the market saturation level

After modeling interest rate (Fig. 2) with different growth (interest) models, the existence of the increasing profitability paradox has been approved. Increasing profitability is indicated not only by the general (logistic) interest but simple logistic and pseudo-logistic interest. It should be noted that in all cases the increase of profitability spurts up when saturation approaches 90%, whereas if saturation approaches 100% it becomes practically uncontrollable. Previously authors only determined this characteristic for the general interest. The research reveals that the effect of growing profitability is typical of other logistic models as well.

Generalization of the first part

After the examination of various sources the conclusion was made that saturation is one of the most important characteristics of growing systems. Economics makes no exception – market saturation affects the development of many spheres.

Logistic analysis of models revealed that the most appropriate model for economic research is the model of general (logistic) interest as it has best (direct) relation with the formula of compound interest. Pseudo-logistic interest (created together with the author of the dissertation) might also be applied to model slowly changing economic processes.

It was known that the effect of increasing profitability is detected by applying the general (logistic) interest model. Expanded research proved that the effect of increasing profitability is typical of other logistic models as well.

The overview and analysis of scholarly literature lead to the conclusion that the logistic growth model proposed by S. A. Girdzijauskas in 2002 should be modified by additionally evaluating the truncated mean.

EXAMINATION OF THE LOGISTIC INTEREST MODEL. TRANSFORMATION

Recent researches of economic growth demonstrate that the examination of capital (product) growth should consider saturation [54, 47, 55]. This requires setting the limit of growth. In economic growth, just as in nature, social life and historical data, occasional processes point to tendencies of market economy. As Elliott, a researcher of economic waves wrote, every phenomenon has its own causes and everyone has a duty to try to find them [56, 57].

Transformation of the logistic interest growth model

Considering the logistic analysis, it might be assumed that saturation is one of the most important market factors. The increasing level of saturation causes growing profitability, hence growing risk. Logistic model enabled to detect unusual cases, unsustainable growth phases. In order to obtain more accurate results, the logistic growth model will be modified by evaluating the truncated mean as well as applying the methods of digital analysis, fuzzy logics.

Various logistic models might be used in different spheres. The general (logistic) interest model of S. A. Girdzijauskas is expressed by the following equation [7]:

$$K(t)=\frac{K_p\cdot K_0\cdot(1+i)^t}{(K_p-K_0)+K_0\cdot(1+i)^t} \tag{1}$$

Here, K_p is the potential (limit, maximum) value of a product (capital); K_0 is the initial size of a product (capital); $K(t)$ is the size of a product (capital) accumulated in t time moments, i is the interest rate (growth parameter), t is the period, time moment.

The mentioned model will be modified by using the truncated mean. Truncated mean is such a mean which is calculated considering not a series of variables but rather its central part acquired by rejecting an equal amount of the values of the feature at the low and the high ends (for instance, the truncated mean of 80% is calculated by rejecting 10% of the lowest and 10% of the highest values of the sample). Thus, the truncated mean is more resistant to the influence of extreme values and is applicable when these values seem to be unreliable (in the given case, due to the market fluctuation).

The growth period in the logistic model is called a ***typical period*** and is marked by the symbol $\boldsymbol{\Delta t}$. In the present case, typical period is calculated by considering the trajectory growth from 10% to 90 % of the potential capital $\boldsymbol{K_p}$ value. The expression obtained by applying the truncated mean is the following:

$$\Delta t = \frac{ln81}{\ln(1+i)} \tag{2}$$

By inserting it into the logistic interest equation (1) and taking that $K_0 = 0.1 \cdot K_p$, a new expression of interval growth model is obtained:

$$K(t) = \frac{K_p}{1 + 9 \cdot \left(81^{-1/\Delta t}\right)^t} \tag{3}$$

This new model (3) will be called the transformed interval logistic model. Its advantage is that the lowest and highest values of statistical data are eliminated thus reducing the deviation of the growth limit determination. Consequently, the obtained results of the analysis are more accurate. The empirical research in the third part will be based on this model. The efficiency of the model will be designated.

Leveraged investment growth model

A new model of leveraged investment in saturated markets was created on the basis of logistic (general) interest. The created leveraged investment model is comprised of two parts: investment and loan. The operation of the model is based on the fact that the dynamics of own and borrowed capitals are different [32, 47]:

$$K(t) = (1+s)\cdot\frac{K_p \cdot K_0 \cdot (1+j)^t}{(K_p - K_0) + K_0(1+j)^t} + s\cdot\frac{K_p \cdot S_0 \cdot (1+i)^t}{(K_p - S_0) + S_0(1+i)^t} \quad (4)$$

Here $\boldsymbol{K_p}$ is the limit, maximum value of the invested capital, $\boldsymbol{K_0}$ is the initial investment, $\boldsymbol{S_0}$ is the loan equal to the initial investment, $\boldsymbol{K(t)}$ is the sum of the investment accumulated in $\boldsymbol{t}$ periods, $\boldsymbol{i}$ is the interest rate, $\boldsymbol{j}$ is the planned yield of the investment, $\boldsymbol{t}$ is the investment period or investment time moments.

This model enables the calculation of the investment after $\boldsymbol{t}$ time moments and after the return of the initial credit. Since the model is based on logistic interest, the calculation of the investment efficiency requires consideration of the saturation of the market in which the investment will be made [32].

Logistic growth modeling method

Frequently, classical growth models are applied to solve relevant problems of product growth modeling. The present work introduces the methodology of a growth process modeling and describes the mechanism of the parameters' calculation.

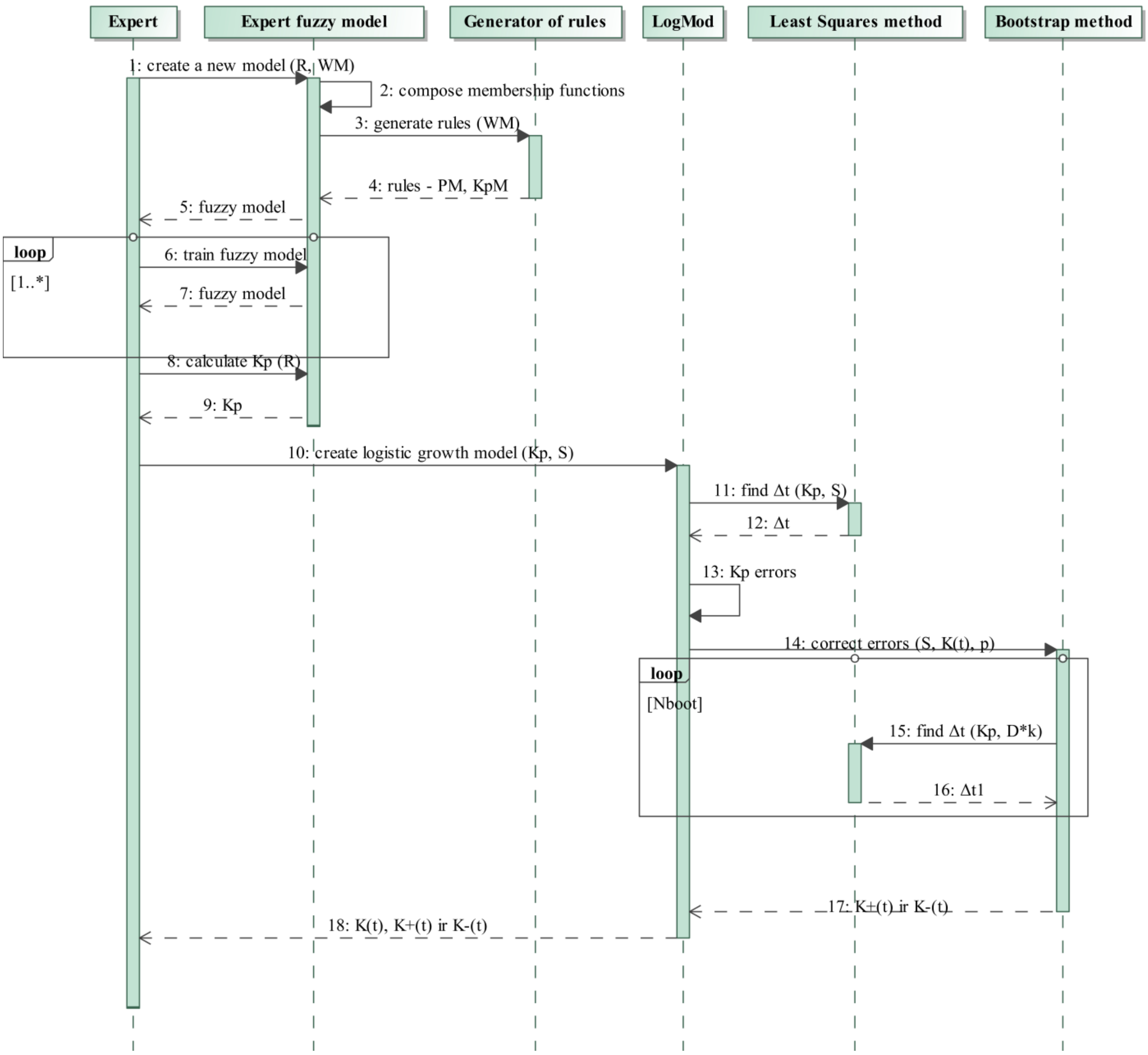

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Fig. 3. Diagram of modeling sequences

Fig. 3 one of the main roles in the given diagram of sequences is undertaken by a highly qualified expert of the market. The generator of rules creates rules necessary for the expert fuzzy system according to the values of parameters given by the expert. Membership functions calibrated by the expert in the fuzzy environment must match certain requirements. The determination of market capacity, the parameter $\boldsymbol{K_p}$, is one of the most problematic tasks of modeling. The modeling mechanism embraces several known and widely applied methods: the least squares and bootstrap methods. It also includes the principle of error calculation.

The diagram of logistic modeling activities is given in Fig. 4.

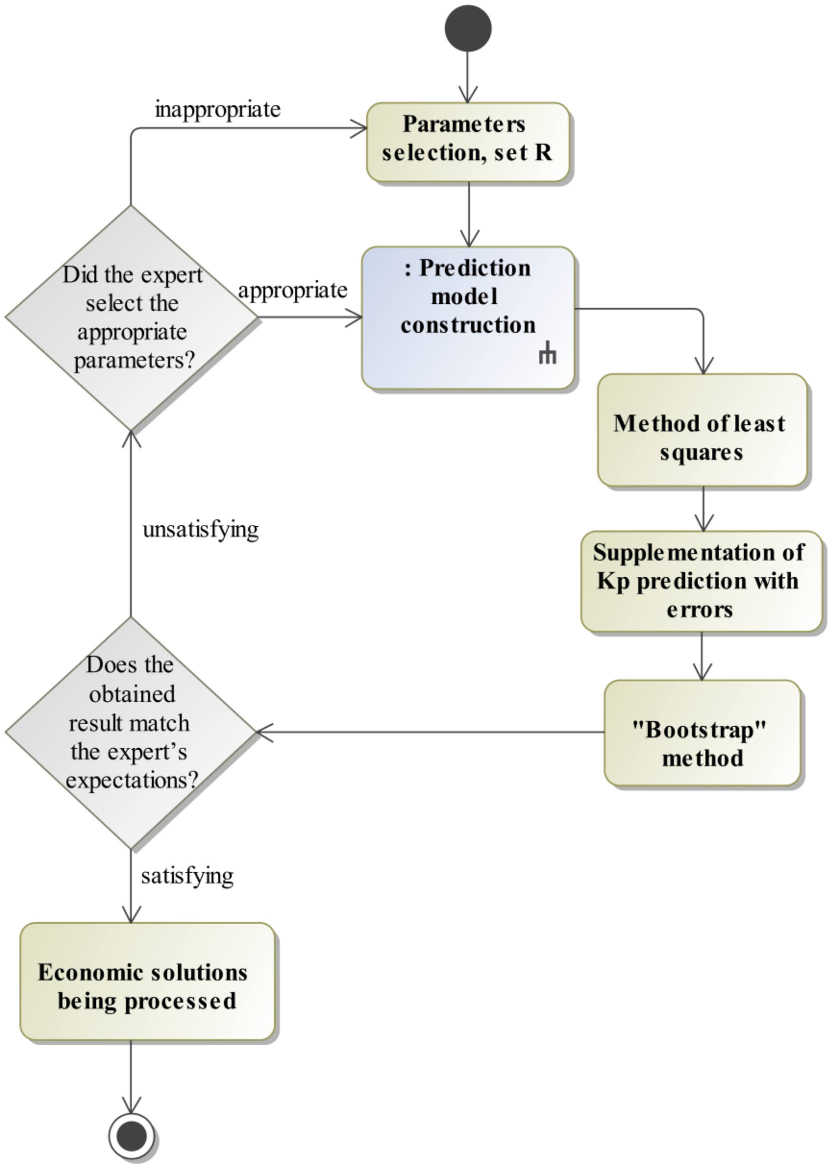

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Fig. 4. Diagram of growth activities

The modeling scheme shows the sequence of actions to reach the desired result. At the beginning, the expert selects parameters which have an impact on the market being analysed. Afterwards, the expert fuzzy model is created to predict K_p (saturation parameter). When the value of K_p is obtained, the least squares method is applied to calculate the parameter Δt. The errors of K_p prediction are determined. The bootstrap method is used to rectify the errors of K_p prediction. Later on, the expert, after the evaluation of the results, must decide if the model matches his/her expectations. In case it does, the model might be used to determine K_p. If it does not, it is necessary to check whether the parameters were selected appropriately, whether the fuzzy expert model was properly created.

Prediction of the saturation parameter K_p by means of fuzzy sets

Prediction of the saturation parameter K_p is carried out in the Matlab fuzzy environment. Fuzzy sets logics is a mathematical formalism dedicated to process indefinite knowledge [58].

Fuzzy sets

The set and element are the main notions of the set theory. Fuzzy set is defined by enumerating its elements. The dependence level of each element may vary from 0 (does not belong to the set) to 1 (belongs to the set completely). The dependence level might be based on certain statistical data or expression of human experience. The form of function is chosen on the grounds of the expert's experience or it might be determined by analysing experimental data of a process or phenomenon. There are four graphic and analytic expressions of the most popular membership functions: triangle, trapezoid, Gaus and $\cos^2$. It is rational to use Gaus's functions to reflect human experience because they contain two parameters (average and dispersion), whereas their physical significance is obvious. Besides, these functions are smooth which is a clear advantage while integrating fuzzy set systems into further procedures of modeling and optimization [58].

The realization of the fuzzy modeling environment embraces several steps: fuzzification, generation of rules, de-fuzzification (elimination of fuzziness).

After the analysis of scholarly literature it is possible to claim that the determination of growth limit is either entrusted to a user or the least squares method is applied. The limit determined by the user is not permanent, each time the user has to set market capacity for a certain time moment. The method of least squares does not consider the logic meaning of market capacity because it is related directly to statistical data, hence does not rely on the knowledge of either the user or an expert. The proposed expert fuzzy set method accumulates expert knowledge in the expert fuzzy system and thus allows to calculate $\boldsymbol{K_p}$, the value of saturation parameter, according to the parameters of the market being analysed.

Composition of the expert fuzzy model

The principle of the realization of fuzzy methodology serves as a basis for the expert K_p prediction model. Created expert system will enable to determine K_p, the saturation parameter, hence show the true limit of the market development.

The diagram of activities of the expert K_p prediction model composition is given in Fig. 5.

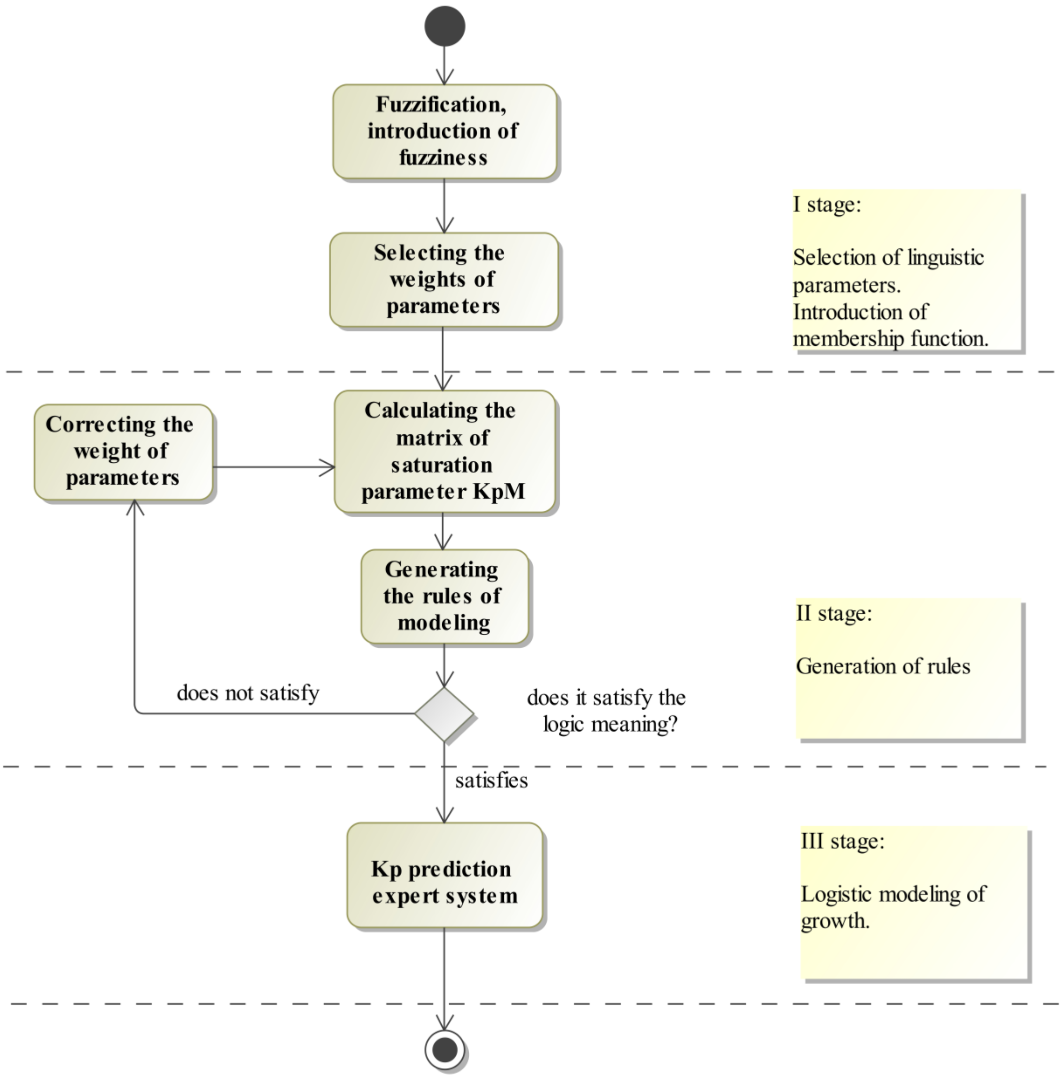

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Fig. 5. Diagram of K_p prediction activities

K_p prediction model is created with the parameters selected by the expert. Limits are introduced for parameters and membership functions are selected. Linguistic terms and their digital values are entered. Having determined the weights of parameters, rules are generated and applied in the fuzzy model. If the rules satisfy the logic meaning, the assumption is made that $\boldsymbol{K_p}$ prediction model is created. In case there is no logic meaning, the weights of parameters are changed and rules generated anew.

The sequence of growth modeling is divided into three stages. After experts evaluate the specific data of the sphere or market being analysed, knowledge and experience, parameters that have an impact on the growth are selected and membership functions of the fuzzy system are generated. Their set is determined in the following way: $\boldsymbol{R}=\{r_1, ..., r_j\}$, $\boldsymbol{j} \in [1..\infty)$.

The first stage – *fuzzification*

The first stage of the logistic growth model composition is fuzzification of variables whose linguistic variables are "Saturation" and growth parameters are selected from the set $\boldsymbol{R}$. In the present case, the constant values of linguistics terms are the following ones: "Decreases", "Stable", "Increases". Maximum and minimal values, limits, are determined for each parameter. Limits of parameters may change, therefore, provided higher or lower value will occur in future in comparison with the current value, the upper and the bottom limits will have to be changed respectively.

The second stage – the structure of the generation of rules and the matrix of saturation growth

The fuzzy model uses rules which are generated automatically. The matrix K_pM is composed. According to the compatibility of the selected parameters and the impact of each of them on the development, the basis embracing 3^n rules is formed.

The element of K_pM, K_p (saturation) growth matrix is calculated according to the function:

$$f_i(a_{i1} \cdot w_{11}, \cdots, a_{in} \cdot w_{nn}) = \begin{cases} -1, kai - \max(w_e) > \sum_{j=1}^{n} a_{ij} w_{jj} \\ 0, when - \max(w_e) \leq \sum_{j=1}^{n} a_{ij} w_{jj} \leq \max(w_e) \\ 1, kai \max(w_e) < \sum_{j=1}^{n} a_{ij} w_{jj} \end{cases} \quad (5)$$

The function demonstrates how K_pM matrix depends on the weight matrix (*WM*) and linguistic terms (*PM*). Linguistic terms refer to the following digital values: $a_{ij} \in \{-1, 0, 1\}$, $i \in \{1, ..., 3^n\}$, o $j \in \{1, ..., n\}$. Every line of the matrix *PM* comprises a unique combination of the expressions of the selected parameters' growth. In order to record the rule of the model, the respective series from the matrices *PM* and K_pM are taken, then the i^{th} rule would be as it follows:

$$(a_{i1}, \cdots, a_{in}, f_i(a_{i1} \cdot w_{11}, \cdots, a_{in} \cdot w_{nn}))$$

The fuzzy expert model is composed according to the methodology proposed by Mamdami (*Ebrahim Mamdani*) [58]. The expert can regulate the influence of the parameters' weight at any time. In the view of experts, if automatic generation seems to be inaccurate, it is possible to change the value of saturation behaviour in the fuzzy model itself.

The third stage – calculation of the K_p parameter

Fig. 6 demonstrates the expert K_p prediction system. Statistical data is taken and processed by applying the created fuzzy model. Thus the digital value of the saturation parameter K_p is obtained. A user has a clear and understandable result and can see the limit of the true value. It enables to make decisions regarding the attractiveness of a particular market for investments and expediency of the leverage application.

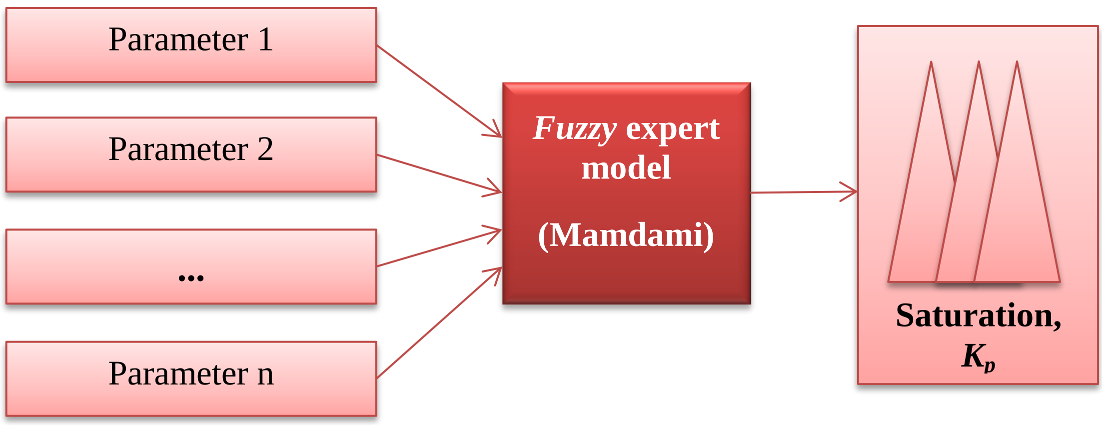

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Fig. 6. General scheme of the fuzzy logistic growth model

Digital methods

The method of ***least squares*** is applied to attract the statistical points to the mathematical function, here, logistic model. The expression of the least squares method in the present case is:

$$\min_{\Delta t}\sum_{i=1}^{m}(F(t_i,\Delta t)-y_i)^2=\min_{\Delta t}\sum_{i=1}^{m}\left(\frac{K_p}{1+9\cdot(81^{\frac{-1}{\Delta t}})^t}-y_i\right)^2 \qquad (6)$$

By applying this method with the saturation value $\boldsymbol{K_p}$ the second parameter $\boldsymbol{\Delta t}$ of the interval logistic growth model was discovered.

The fuzzy model created by an expert may lack completeness, therefore, additional percentage error is introduced to predict the parameter $\boldsymbol{K_p}$. The parameter with the error is marked as K_p^+ ir K_p^- (positive and negative increase respectively). The least squares method is applied for the errors. When the calculation of the parameter $\boldsymbol{K_p}$ error is introduced in the logistic model, the graphic expression depicted in Fig. 7 is obtained. Here, the blue curves indicate the designated interval.

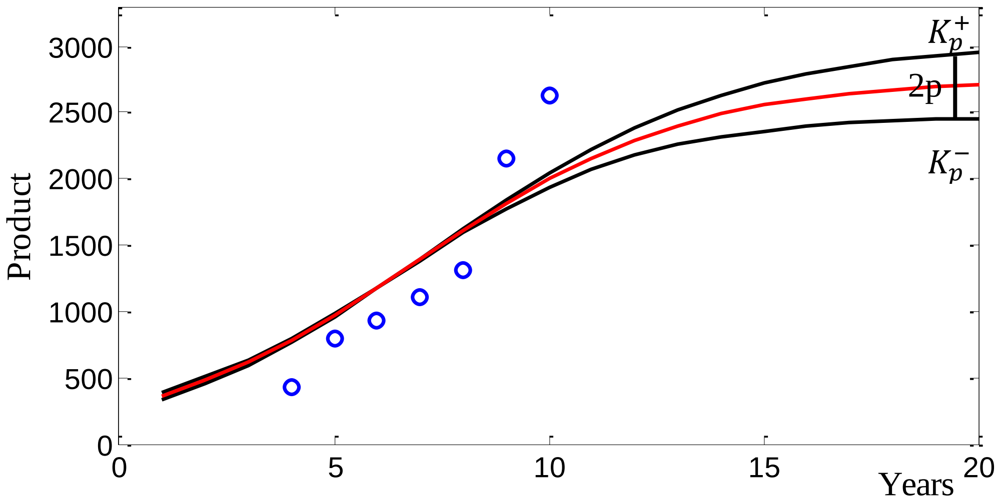

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Fig. 7. Calculation of the parameter K_p error

Bootstrap method is an intensive computer method for sample selection which is widely applicable and allows reducing prediction errors [56]. The method is used for $K^+(t)$ and $K^-(t)$ functions. By means of the bootstrap method, distributions $\boldsymbol{\Delta t}$ are acquired. In the $K^+(t)$ percentage value is 5%, whereas in the $K^-(t)$ percentage value is 95%. In such a way the bootstrap methods corrects the errors of the predicted saturation parameter K_p in a logistic model.

Generalization of the second part

A new transformed logistic model is offered. The modification deals with the evaluation of the truncated mean of statistical data and elimination of accidental data thus providing a more accurate view of market development.

A new modeling method of saturated systems is created. It is based on the transformed logistic method. The problem of the determination of the saturation parameter was solved by applying the fuzzy set expert system and bootstrap method. It allowed to increase the accuracy of the market saturation parameter K_p.

The newly created method opens new possibilities to examine market heating, formation of financial bubbles, recognize approaching decline and unsustainable situations. When the error of market capacity determination is reduced, the results of the growing system analysis become more accurate.

Rectification of the errors of the saturation parameter K_p leads to a more accurate final result. A user can make respective economic and management decisions.

A new leveraged investment methodology is created by applying the general (logistic) interest model.

THE EXAMINATION OF THE TRANSFORMED LOGISTIC MODEL APPLICATION

In this chapter, the methodology constructed in the second chapter is employed. The following studies of the offered methodology were carried out:

- the examination of the real estate market, i.e., pecuniary sums of buying-selling deals, in the biggest cities of Lithuania;
- the examination of the fuzzy model fund organization;
- the exploration of a randomly chosen investment fund;
- the study of the real estate market index in the USA;
- leveraged investment modeling.

The speed of economic growth depends on the saturation of the market where the investments are made. In a saturated market, the growth is more rapid, whereas in case saturation overcomes 90% the process becomes especially fast. Experts of economics are interested in the disproportionally rapid growth phases. Unnaturally rapid growth causes hidden overproduction and threats economic stability. The research will help to indicate approaching phases of rapid growth. On the grounds of the acquired information it will be possible to designate whether a concrete investment has reached (transgressed) the determined limit. There will be a possibility to make necessary investment decisions according to the changes of market growth and thus avoid impending declines.

It should be stressed that market saturation is closely related with the overproduction occurring in the same markets (most often, overproduction is hidden). The importance of saturation in financial markets will be indicated by the modeled examples. The comparison with the results obtained with the Loglet model will be carried out.

The assumption of the real estate markets and fund research

According to the changes of parameters and statistical data, the fuzzy model is trained regarding the assumption that the growth of $\boldsymbol{S_t}$ (statistical value of a respective time moment) was very close to that of $\boldsymbol{K_{pt}}$ (growth limit at a respective time moment). Hence the equation:

$$K_{pt+1} = \frac{-S_t}{\Delta K_{pt+1} - 1} \quad (7)$$

The change $\boldsymbol{\Delta K_{pt+1}}$ is calculated according to the created fuzzy model. The acquired percentage change is recalculated regarding the value of statistical data at a respective time moment following the equation (7) given above.

The research of the real estate market in Lithuania

Real estate market in Lithuania is young and has neither deep historical experience nor thorough examination. On the other hand, it has already suffered a relatively severe financial crisis. In order to avoid harsh consequences in the future that would affect the economic of the whole country and finances of each of us, it must be systematically examined. The present study aims at the prediction of market saturation by evaluating the pecuniary sum of buying-selling deals in the biggest cities of Lithuania.

The expert fuzzy model for a particular case in the real estate market in Lithuania is created following the given methodology. The sequence of growth modeling is divided into three stages. Having evaluated the knowledge and experience of the specialists of the analysed sphere, four parameters that have an impact on the growth of the real estate market are chosen. According to the survey of Lithuanian real estate market experts carried out by P. Simukaitis in 2013 [59], the main economic parameters that affect the formation of a real estate bubble in Lithuania were designated. They are as it follows: wages, state's GDP, inflation and VILIBOR.

Considering the anticipated market capacity at a respective time moment, the growth tendency is explored. ***Loglet*** and the transformed logistic interest model ***LogMod*** are used in the modeling.

A reliable fuzzy model was created in the study. It was shown that the model can recognize the formation of a bubble and stabilization of the market after the burst of the bubble. Insufficiency of the Loglet model to examine economic market was revealed. It is so because the model is dedicated to the analysis of biological processes.

The examination of a randomly chosen investment fund

A randomly chosen investment fund of the SEB bank is recorded in the fund list on the website [60]. The title of the fund is *Seb Technology Fund*; its investments are mostly targeted at the companies of USA market. Therefore, the expert has chosen the following parameters typical of the USA market: national average wages (NAW), state's GDP, USA share market index (SP500 – Standard & Poor's 500), and consumer prices index (CPI) [61, 62, 63, 64].

To analyse the fund, the methodology described in the second part of the work was applied and the expert fuzzy system was trained. The created expert fuzzy model complied with all the expectations of the expert. After the analysis of the periods of statistical data that embrace the periods of crisis and ascent, the following conclusions are made:

- the composition and improvement of the *fuzzy* model is a process which must be developed and renewed by constantly involving new data;

- the model is appropriate to examine markets only in the periods of ascent;
- small amount of data and great fluctuations have a negative effect on the performance of the model, i.e., the method of least squares provides poor results regarding the data with great fluctuation in the market;
- the examined statistical data matches with the limits set by the LogMod model.

The examination of the real estate market index in the USA

The given graph (Fig. 8) demonstrates that USA market of real estate has suffered three economic declines.

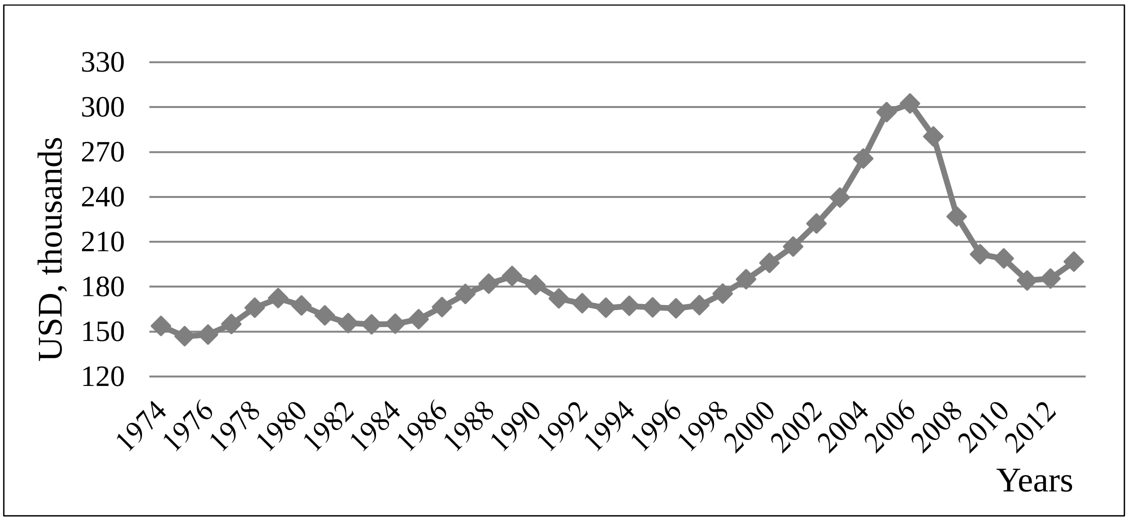

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Fig. 8. Data of real estate market in the USA

Before creating the expert fuzzy model, the statistical data was divided into three sections according to the three economic declines. The first section, 1974-1985 period, embraces the stage of training the fuzzy expert model. The correctness of the created expert model is tested by using the statistical data of the periods of 1983-1992 and 1996-2013.

Following the given methodology, the expert fuzzy model is created for the case of USA real estate market. Four parameters that have an impact on the growth were chosen after the evaluation of the knowledge of USA real estate market specialists [61, 66, 67]. The parameters are: national average wages (NAW), state's GDP, current unemployment rate (CUR) and consumer price index (CPI).

The expert fuzzy model created according to the data of the first crisis period, 1974-1985, was later adapted to another period: 1983-1992, during which, in 1989, a burst of a bubble was indicated. In such a way the reliability of the model and the possibility to recognize the threat of approaching decline were approved.

To generalize the sequence of modeling and the obtained results, it is possible to state that LogMod modeling was successful. The model enables to identify the threat of approaching decline and indicate the period of the economic bubble formation. During the experiment it was determined that the expert fuzzy model created for the first period recognized bubble formation in the second and third periods. LogMod method obtained more accurate results, thus the effectiveness and precision of the created LogMod model was shown.

Leveraged investment case

A leveraged investment model is made of two parts: investment, i.e., capital, and a loan. Modeling is carried out following the function (4) described in the second part.

The difference between the ordinates of the functions might be considered to be a natural expression of investment risk (Fig. 9) [47].

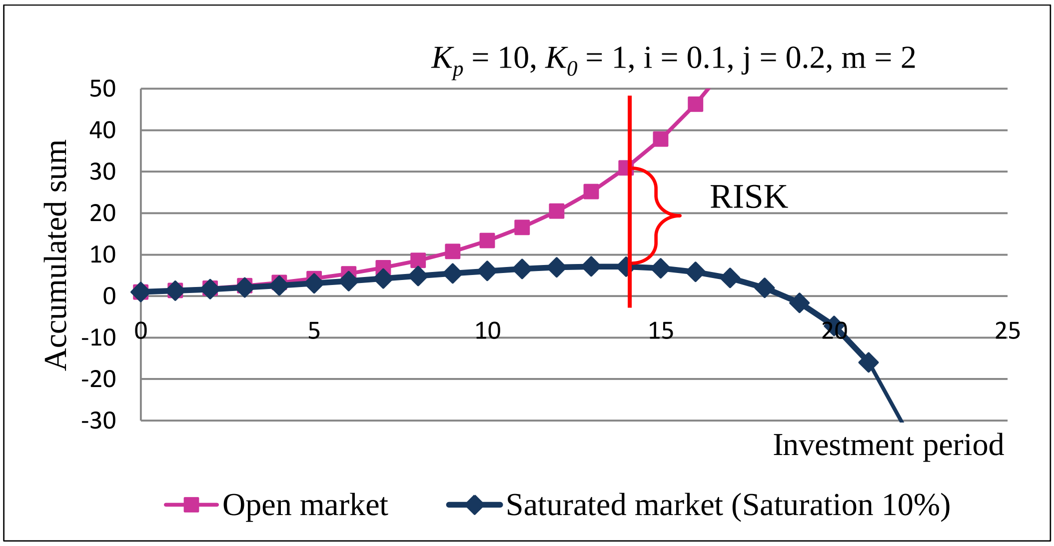

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Fig. 9. Leveraged investment in an unsaturated and a saturated market

The functions depicted in Fig. 9 demonstrate that market saturation essentially influences the conditions of leveraged investment. When two equal “leverages” (equally structured investment portfolios) are invested in unsaturated (open) and slightly saturated (10%)

markets, their growth in the markets would be similar only in the very beginning. As it is shown in the diagram, later on, the graphs separate. The growth in the saturated market turns into an uncontrollable decline since the fourteenth time moment (period). When saturation is increased, the decline begins much earlier.

Different bend of the curves indicates that the same investment may gain absolutely contrary results depending on the market state: in the open (unsaturated) market it gains great profit, whereas in the saturated market even greater loss. The difference between the ordinates of the functions might be perceived of as the risk of leveraged investment. The longer the investment period and the higher the saturation of the market in which the investment is made, the higher the investment risk. The curves in Fig. 10 clearly illustrate the processes. It presents the graphs of an investment portfolio growth under the conditions of saturation from zero (absolutely unsaturated market) to 99% market saturation. The impact of market saturation on the investment growth is obvious: increase of saturation "pushes" the trajectory down thus creating the impression of the "trap effect".

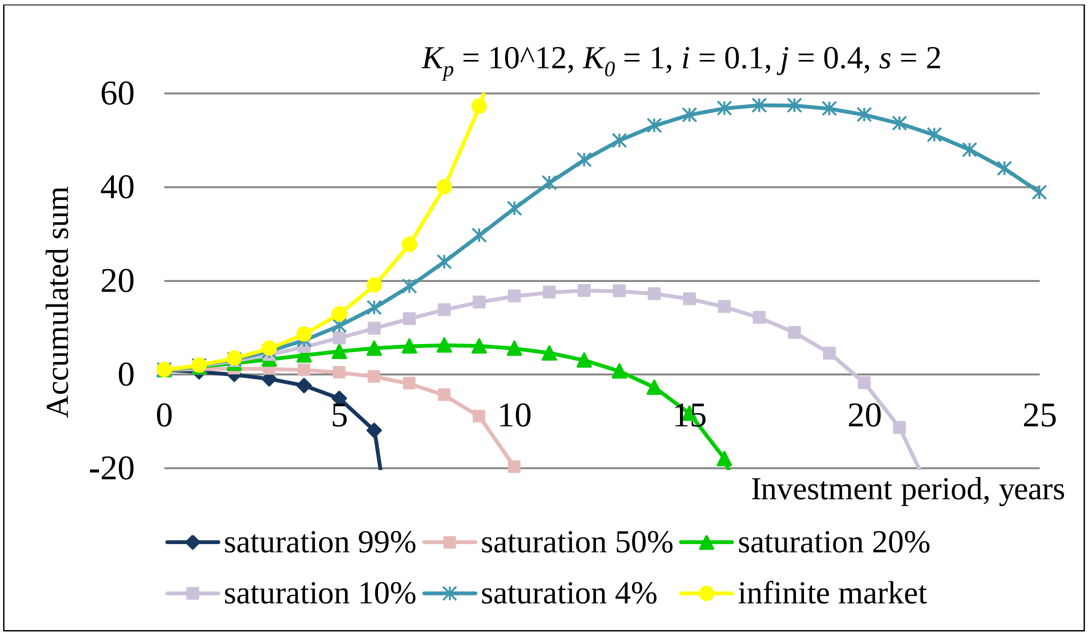

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Fig. 10. Leveraged investment in markets of various saturation

The lack of knowledge and understating as well as misjudgment of market situation and its internal processes are the major causes of leveraged investment risk. The leveraged trade in a saturated market is a potential threat to experience a financial crisis [32].

The generalization of the third part

The modeling method offered in the second part serves as a basis for the experiment which revealed that the fuzzy model is reliable and effective. It enables to recognize the formation of a bubble and stabilization of the market after the burst of a bubble. The experiment also demonstrated the advantage of the ***LogMod*** model in comparison with the known ***Loglet*** model.

After the leveraged investment modeling was carried out it was designated that the impact of market saturation on the investment growth is obvious. The increase of saturation in the investment environment raises the level of risk and may cause a “debt trap” effect.

CONCLUSIONS AND SUGGESTIONS

After the investigation of logistic identification of unsustainable growth situations which is based on the fuzzy set method and interval logistic growth model, the following conclusions might be made:

1. The analysis of various sources led to the conclusion that saturation is one of the most important characteristics of growing economic systems. It was determined that saturation level is related to sustainability of economic systems. Too high saturation of markets (over 90%) threats sustainability.
2. General (logistic) interest model is most suitable for economic research since it is best related to the formula of compound interest. Pseudo-logistic interest (created together with the author of the dissertation) might also be applied to model slowly changing economic processes. It was known that the effect of growing profitability is detectable by means of general (logistic) interest modeling. The research proved that the effect of growing profitability is typical of other logistic models (including the newly created pseudo-logistic model).
3. A new method of saturated system modeling, based on the transformed interval logistic model, was created. The problem of the saturation parameter

determination was solved by applying the expert *fuzzy* set system, targeted at the statistical data period embracing the bubble period, and the *bootstrap* method. This allowed to increase the accuracy of the saturation parameter K_p.

4. The newly created method opens the possibilities to investigate market heating, formation of financial bubbles, recognize an approaching decline and unstable situation. Reduced error of market capacity determination helps to obtain more accurate results of the growing system analysis.
5. An effective *fuzzy* model of Lithuanian real estate market was created. It recognizes bubble formation and market stabilization after the burst of a bubble. The experiment approved the advantage of the created *LogMod* model in comparison with the known *Loglet* model based on the method of least squares.
6. The experiment showed that the *fuzzy* model created for the USA real estate market index is reliable and effective. The model revealed that such methodology of modeling is efficient and able to recognize approaching unstable situations. Expert models of USA real estate market and fund matched the expectations of the expert, whereas present statistical data complies with the values set by the *LogMod* model.
7. Market saturation has a direct impact on the investment profitability. By applying general (logistic) interest model, a new methodology of leveraged investment was created and investment risk unknown so far was discovered. It was shown that the same investment may gain contrary results due to market conditions: a great profit might be gained in an unsaturated market, whereas in a saturated market, investment may gain even greater loss. Logistic analysis exposed that saturation is the main market risk factor. Increasing saturation in the investment environment may cause the "debt trap" effect.

DAKTARO DISERTACIJOS SANTRAUKA

Viskas kas auga turi konverguoti
/Flannery O'Connor/

Darbo aktualumas

Analizuojant augimo procesų modelius vis daugiau dėmesio skiriama logistiniam[1] augimui. Logistiniai augimo modeliai taikomi įvairiuose mokslo srityse: biologijoje, chemijoje, fizikoje, medicinoje, ekonomikoje, informatikoje ir kitur [1, 2, 3, 4]. Mokslininkai visame pasaulyje ieško, randa ir siūlo naujus modelius [5, 6, 7], tiria jų efektyvumą ir taiko juos augimo procesų modeliavimui. Ekonomikoje augimo procesų modeliai taikomi nagrinėjant ūkio plėtrą, investuojant kapitalą, analizuojant bei prognozuojant finansines krizes, o taip pat priimant ir kitus ekonominius sprendimus (traukiantis iš rinkos ar ją keičiant, kuriant naujas investavimo strategijas ir pan.). Šiuo atveju modeliuojamas kapitalo augimas. Kuriant augimo modelį stengiamasi jį supaprastinti, todėl dažniausiai neatsižvelgiama į įvairius augimo trukdžius ir apribojimus [3, 2]. Būtent todėl daugelyje modelių kapitalas yra laikomos neriboto augimo, tačiau toks kapitalo vertinimas nėra tikslus.

Ekonominių procesų modeliavimui yra kuriamos sistemos, kurios leidžia atpažinti bei prognozuoti finansines krizes, ieškoti susidariusių situacijų tinkamo sprendimo. Prognozavimas yra sudėtingas procesas, ir jis atliekamas, dažniausiai, remiantis istoriniais duomenimis [8]. Atliekant logistinį modeliavimą yra sudėtinga tiksliai nustatyti konkrečius rizikos taškus, todėl apibrėžiant ribą apsiribojama paklaidos galimybės įvertinimu ir svyruojančio rizikos intervalo nustatymu.

Augimo procesas efektyviau valdomas, jei taikysime logistinį modelį, pagrįstą realiais statistiniais duomenimis. Tokios sistemos vartotojas gali operatyviai reaguoti į augimo trajektorijos pakitimus. Tai leidžia laiku įvertinti gresiančius pavojus, lengviau išgyventi ekonominius nuosmukius bei išvengti didelių nuostolių tiek verslui, tiek ir fiziniams asmenims. Šiuo metu rinkoje egzistuojantys įrankiai dažniausiai sudaryti įprastų logistinių modelių, plačiai taikomų biologijoje, pagrindu [9, 10].

[1] „Logistikos“ terminas šiame kontekste neturi nieko bendro su transporto logistika. Jis yra siejamas su logistine, t. y. *S* pavidalo funkcija.

Darbe pristatomas modelis yra taikomas augimo ribos prognozavimui. Tam reikia nustatyti produkto raidos dėsningumus ir apibrėžti netvariąsias ekonomines situacijas, o taip pat įvertinti artėjantį augimo proceso nukrypimą.

Problemos ištyrimo lygmuo

Populiacijų augimo tyrimai suintensyvėjo XIX amžiuje. Augimo tendencijas ir prognozavimo galimybes pirmieji pasaulyje pastebėjo ir pradėjo analizuoti Galileo Galilei (1564 – 1642), Thomas Robert Malthus (1766 – 1834) ir kiti mokslininkai. 1825 metais Benjamin Gompertz (1779 – 1865) paskelbė darbą, kuriame buvo plėtojami T. R. Malthus darbai ir paskelbtas naujas augimo modelis. 1838 m. Pierre-Francois Verhülst pasiūlė naują logistinio augimo kreivę, tinkamą gyventojų skaičiaus augimo tyrimui. P. F. Verhülst pasiūlyta produkto augimo diferencialinė lygtis buvo Malthus lygties diferencialinė forma, papildyta daugikliu, turinčiu tiesiškai mažėjančios funkcijos pavidalą [2]. Vėlesnių kartų mokslininkai taip pat atliko nemažai panašių tyrimų. Biochemikas Brailsford Robertson (1884 – 1930), remdamasis logistine funkcija, pasiūlė apibrėžti individo augimo procesą. Raymond Pearl patobulintą logistinę funkciją (1879 – 1940) naudojo gyventojų skaičiaus augimui apibūdinti. Šiuo atveju taip pat buvo vertinant žmogaus amžius. Alfred J. Lotka ir Vito Volterra apibendrintas augimo lygtis pritaikė kiekybiniam konkurencijos tarp skirtingų rūšių modeliavimui. Jie sukūrė sistemos plėšrūnas-grobis lygtis. Mokslininkas Zvi Griliches buvo vienas iš pirmųjų savo technologijų pasikeitimo (pakaitinio modeliavimo) tyrimuose, aprašęs augimo procesą, panaudojant S-kreives [11]. 1979 m. mokslininkų Vijay Mahajan, Eitan Muller produkto augimo tyrime buvo pritaikytos Bass ir Gompertz logistinio augimo kreivės [1].

S-kreivės taikymas technologijų inovatyvumo prognozavimui taikomas nuo 1960 metų pradžios. Mokslinėje literatūroje pristatomi ir analizuojami Zvi Griliches ir Edwin Mansfield 1961 m. sukurti modeliai [11, 12]. Ten buvo aprašyti rodikliai, pagal kuriuos įmonės turėjo siekti vystymosi racionalumo. Prielaidą, kad naujovės visuomenėje plinta pagal S-kreivės tendenciją kėlė ir Everett M. Rogers (1931 – 2004) [13].

Augimo modelių taikymas prasidėjęs nuo biologinių procesų, vis labiau plėtėsi į fizikos, chemijos, medicinos, sociologijos ir kitas mokslo sritis [2, 14, 6, 15]. Atlikdami logistinę analizę, mokslininkai Arnulf Grubler ir Nebojsa Nakicenovic [16] parodo, kad empiriškai argumentuojant, ekonominio proceso augimas (kainų svyravimas, infrastruktūros plėtra, technologiniai pokyčiai) nėra tolygūs ir tolydūs. S-formos modelio struktūra, turinti prisotinimo parametrą, yra pagrindinis objektas, analizuojamas ekonominės sklaidos procesuose. Autoriai parodo, kad ekonominę plėtrą lemia įvairūs etapai, daugybė tarpusavyje susijusių technologijos grupių, ir kad perėjimo laikotarpis, nuo vieno dominuojančio konkurento prie kito, atitinka ilgųjų bangų Kondratjevo modelį [16]. S-formos logistiniu augimo modeliu galima atpažinti augimo tendenciją ir galima prognozuoti, kas vyks ateityje. Dmitry Kucharavy ir Roland De Guio pastebėjo, kad, jei kreivę pritaikome pagal statistinius duomenis, pritaikymo technika suteikia galimybę gauti du iš trijų logistinės kreivės parametrus. Tačiau problemos esmę sudaro viršutinės augimo ribos nepastovumas ir neapibrėžtumas. Prognozuojant neužtenka parodyti didelį augimo diapazoną. Svarbu tinkamai nustatyti ir įvertinti atskirus augimo parametrus. Visa S- kreivės studijų prognozavimo esmė remiasi prielaida, kad logistinė kreivė atitinka natūralaus augimo dėsnį [17]. Pastarųjų 2-3 dešimtmečių augimo tyrinėtojų darbuose pristatyta įvairių naujovių, infrastruktūrų ir energijos šaltinių pakaitalų sklaidos problema. Technologiniai ir globalūs technologinio pasikeitimo analizės rezultatai buvo pristatyti A. Grubler 1986, 1989 metais [18]. Mokslininkas tyrė infrastruktūrų sklaidą, įskaitant kanalus, geležinkelius, autostradas ir oro linijas ir parodė, kad jų sklaida atitinka logistinį principą. Ekonominis, socialinis ir technologinis vystymasis vyksta dėl dviejų priežasčių: dėl iš esmės naujų idėjų (pagrindinių naujovių) pasirodymo ir esamų metodų bei sistemų (produktų ir procesų) atnaujinimo ir tolesnio tobulinimo [19, 16].

Platesnė logistinių ekonominių modelių sklaida prasidėjo tik prieš porą dešimtmečių, ypač, kai 2002 m. Girdzijausko buvo paskelbtas bendrųjų palūkanų modelis. Logistinių palūkanų modelio taikymas ekonomikoje yra analizuojamas D. Štreimikienės, V. Moskaliovos, E. Jurkonytės, R. Mackevičiaus, D. Grundey, M. Dubnikovo, V. Boguslausko, J. Čepinskio, A. Pikturnos, F. Ivanausko, G. Garšvos, E. Merkevičiaus, E. Jociūtės ir kitų mokslininkų tiriamuose darbuose [20, 21, 22, 23, 24, 25, 26]. Pažymėtina, kad Carlota Perez pasinaudojo logistine kreive norėdama iliustruoti ilgąjį (Kondratievo) verslo ciklą. Ji išryškino tokius ekonominių ciklų ypatumus: technologinio laikotarpio pradžia traktuojama kaip staigus įsiveržimas, pakilimas – kaip subruzdimas, plėtra – kaip sinergija ir prisotinimas – kaip branda [27]. D. Kucharavy, E.

Schenk ir R. De Guio ir toliau vysto savo tyrimus naujų technologijų ilgalaikių prognozių pagrindu [68].

Augimo proceso prisotinimo įvertinimas yra esminis ekonomikos tyrimuose. Pagrindinė logistinio modeliavimo esmė yra siekis kaip įmanoma tiksliau apibrėžti augimo parametrus iš kurių sunkiausiai nustatomas yra sąlyginai dažnai kintantis **augimo ribos** parametras. Modelio parametrų nustatymui taikomas mažiausių kvadratų metodas [6]. Tačiau logistiniame augimo modelyje, kai nežinoma augimo riba, jis nepasiteisina. Neraiškios aibės metodas taikomas daugelyje sryčių tarp kurių yra verslo ir ekonomikos sritis. Šioje srityje neraiškių aibių metodas taikomas nuo marketingo (pirkimo analizė, sukčiavimo nustatymas, paslaugų kokybės įvertinimas) iki finansų (akcijų rinkos prognozavimo schemos, portfelio parinkimas, rizikų valdymas, paskolų įvertinimo sistemos) ir e-verslo (e-prekybos sprendimai, personalizavimas, rizikos analizė e-prekyboje) ir kitose [28].

Darbo objektas – produkto (populiacijos) augimas ribotoje erdvėje.

Darbo tikslas – logistinio intervalinio augimo modelio pagrindu sukurti metodą, leidžiantį atpažinti netvariąsias ekonominės raidos situacijas.

Tikslui pasiekti iškelti tokie ***uždaviniai***:

- Atlikti augimo modeliavimo sistemų analizę (Loglet, LSM).
- Išanalizuoti logistinių augimo modelių privalumus bei nusakyti jų aktualumą.
- Sukurti naują LogMod logistinio modeliavimo metodą:
  - Logistiniam augimo modeliui nustatyti tinkamus skaitmeninės analizės metodus.
  - Rinkos talpos ribą įvertinti neraiškių aibių metodu.
- Eksperimentiškai palyginti Loglet ir LogMod metodų tikslumą, nustatant Lietuvos NT rinkos augimą.

- Ištirti rinkos talpos nustatymo LogMod metodą ir įvertinti modelio tinkamumą, nustatant JAV NT rinkos indekso augimą.

Ginamieji disertacijos teiginiai

- Iš visų logistinių augimo modelių tinkamiausias ekonominių sistemų modeliavimui yra bendrųjų (logistinių) palūkanų modelis.
- Mažiausių kvadratų metodas nepakankamas rinkos prisotinimo parametrui K_p, nustatyti.
- Logistinio modelio prisotinimo parametro nustatymui tikslinga naudoti neraiškių aibių metodą.
- Rinkos prisotinimas yra rizikos šaltinis, investuojant su svertu.
- LogMod modelis tinkamas atpažinti netvariąsias ekonomikos rinkų augimo situacijas.

Darbo mokslinis naujumas

Išanalizuoti logistiniai augimo modeliai (įskaitant ir pseudologistinį modelį, sukurtą dalyvaujant autorei) ir nustatyta, kad ekonominių populiacijų tyrimams geriausiai tinka bendrųjų (logistinių) palūkanų modelis. Tolimesnių tyrimų rezultate gautas naujas, transformuotasis modelis, įvertinant statistinių duomenų nupjautąjį vidurkį, nuo 10 iki 90 procentų potencialiojo kapitalo reikšmės. Toks vertinimas sumažina logistinio modelio rezultatų iškraipymus. Sukurta *fuzzy* ekspertinė sistema padeda nustatyti prisotinimo parametro $\boldsymbol{K_p}$ reikšmę ir ją panaudoti kritinių situacijų prognozei. Pritaikius statistinių duomenų eilutei mažiausių kvadratų metodą, apskaičiuojamas likęs modelio parametras, lemiantis augimo greitį. Parametro $\mathbf{K_p}$ prognozės paklaidos pataisomos taikant saviranko (angl. bootstrap) metodą. Šių priemonių naudojimo rezultatai leidžia vertinti rinkos raidos tvarumą. Svarbu ir tai, kad šioje metodikoje $\boldsymbol{K_p}$ prognozė nepriklauso nuo statistinių duomenų.

Dalyvaujant autorei sukurtas naujas investavimo su svertu modelis. Jis parodo, kad didėjant investavimo trukmei ir rinkos, į kurią yra investuojama, prisotinimo laipsniui, didėja investavimo rizika. Bendrųjų (logistinių) palūkanų modeliu nustatyta, kad investavimo sverto naudojimas prisotintose rinkose gali sukelti skolos spąstų efektą.

Praktinė nauda

Modeliuojant augimą pritaikius logistinį augimo modelį, atsiras galimybė nustatyti netvariąsios ekonomikos rinkos būsenos formavimąsi. Dėl to šiuo laiko momentu atsiras galimybė priimti racionalius sprendimus. Eksperto žinios pakeičiamos rinkai sudarytu fuzzy ekspertiniu modeliu. Naujai sukurtas metodas leis pastebėti rinkos prisotinimą dar jos kaitimo fazėje. Svarbi žinia bankams, kad sverto naudojimas investuojant prisotintose rinkose gali sukelti skolos spąstų efektą.

Tyrimo metodika

Disertacijoje atliekamam tyrimui panaudoti tokie metodai: sisteminė ir lyginamoji mokslinių šaltinių analizė, matematinis modeliavimas, ekonominė logistinė analizė ir sintezė, statistinė analizė, algoritmų modeliavimas. Kuriant modelį naudota MatLab programavimo aplinka. Taip pat panaudotas neraiškių aibių Matlab Fuzzy Logic Toolbox modeliavimo įrankis. MagicDraw UML paruošti grafinių procesų architektūriniai modeliai. Statistiniai duomenys, naudojami sukurtame modelyje, apdoroti Microsoft Excel skaičiuokle.

Darbo struktūra

Pirmoji dalis apima mokslinės literatūros analizę, kur didžiausias dėmesys skiriamas augimo modelių apžvalgai. Išnagrinėti klasikiniai ir logistiniai augimo modeliai. Augimo modeliai apibendrinti, suklasifikuoti ir susisteminti. Pristatyta augimo modeliavimo sistemų analizė.

Antrojoje dalyje pristatytas transformuotasis intervalinis logistinis augimo modelis. Sukurta nauja rinkos augimo vertinimo metodika. Aprašytas modeliavimo algoritmas. Aprašyti taikomi skaitmeninės analizės metodai. Pateiktas automatinis fuzzy sistemoje naudojamų taisyklių generavimas. Aprašyta ekspertinės fuzzy sistemos sudarymo metodika, kuri taikoma prisotinimo parametro apskaičiavimui. Pristatytas investavimo su svertu modelis.

Trečiojoje dalyje, remiantis sudaryta metodologija, atlikti šie tyrimai: ištirta Lietuvos NT rinka, investicinis fondas ir JAV NT rinkos indeksas. Pateikti naujojo augimo modelio taikymo tyrimo rezultatai. Išanalizuotas investicijos su svertu modeliavimo atvejis, kuriame pasireiškia skolos spąstų reiškinys. Parodyta, kad, didėjant prisotinimo laipsniui, atsiranda ir sparčiai auga investavimo rizika.

IŠVADOS IR PASIŪLYMAI

Išnagrinėjus logistinį netvarių augimo situacijų identifikavimą, paremtą neraiškių aibių metodu ir intervaliniu logistiniu augimo modeliu, galima daryti tokias išvadas:

1. Išnagrinėjus įvairius šaltinius buvo prieita išvados, kad viena svarbiausių augančių ekonominių sistemų charakteristikų yra prisotinimas. Nustatyta, kad prisotinimo laipsnis yra susietas su ekonominių sistemų tvarumu. Pernelyg didelis rinkų prisotinimas (daugiau nei 90%) kelia grėsmę tvarumui.
2. Ekonominiams tyrimams tinkamiausias yra bendrųjų (logistinių) palūkanų modelis, kaip geriausiai susietas su sudėtinių procentų formule. Pseudologistinės palūkanos (sukurtos dalyvaujant autorei) taip pat gali būti taikomos lėtai kintantiems ekonominiams procesams modeliuoti. Buvo žinoma, kad didėjančio pelningumo efektas aptinkamas modeliuojant bendrųjų (logistinių) palūkanų modeliu. Tyrimai parodė, kad minėtas didėjančio pelningumo efektas yra charakteringas ir kitokiems logistiniams modeliams (įskaitant ir naujai sukurtąjį pseudologistinį).
3. Sukurtas naujas prisotinamųjų sistemų modeliavimo metodas, paremtas transformuotuoju intervaliniu logistiniu modeliu. Prisotinimo parametro nustatymo problema išspręsta pritaikius neraiškių aibių *fuzzy* ekspertinę sistemą, sudarytą statistinių duomenų periodui apimančiam burbulo laikotarpį, ir *saviranko*s metodą. Tai leido padidinti rinkų prisotinimo parametro K_p nustatymo tikslumą.
4. Remiantis naujai sukurtu metodu atsiranda galimybė tirti rinkų kaitimą, finansinių burbulų susiformavimą, atpažinti artėjantį nuosmukį bei nestabilią situaciją. Sumažinus rinkos talpos nustatymo paklaidą, augančios sistemos analizės rezultatai yra tikslesni.
5. Sudarytas efektyvus Lietuvos NT rinkos *fuzzy* modelis, kuriuo galima atpažinti burbulo formavimąsi bei rinkos stabilizavimą po burbulo sprogimo.

Eksperimentas parodė sukurto *LogMod* modelio pranašumą, lyginant jį su žinomu mažiausių kvardratų metodu paremtu *Loglet* modeliu.

6. Eksperimentas parodė, kad sudarytas, JAV NT rinkos indekso, *fuzzy* modelis yra patikimas ir efektyvus. Šis modelis parodė, jog tokia modeliavimo metodika yra veiksminga ir sugeba atpažinti artėjančias nestabilias situacijas. JAV NT rinkos ir fondo ekspertiniai modeliai atitiko eksperto lūkesčius, o šiuo metu statistiniai duomenys atitinka *LogMod* modelio nustatytas reikšmėms.
7. Rinkų prisotinimas tiesiogiai veikia investicijos pelningumą. Pritaikius bendrųjų (logistinių) palūkanų modelį, sukurta investavimo su svertu metodika ir nustatyta iki šiol nežinoma investavimo rizika. Parodyta, kad ta pati investicija, priklausomai nuo rinkos sąlygų, gali duoti visiškai priešingus rezultatus: neprisotintojoje rinkoje galimas didelis pelnas, o prisotintojoje rinkoje – dar didesnis nuostolis. Logistinė analizė parodė, jog pagrindinis rinkos rizikos veiksnys yra prisotinimas. Prisotinimo didinimas investicijos aplinkoje gali sukelti „skolos spąstų“ efektą.

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INFORMACIJA APIE DISERTACIJOS AUTORĘ

Vardas, Pavardė: Andžela Mialik

Gimimo data: 1984 07 26

Išsilavinimas:

Aukštoji mokykla	Baigimo metai	Įgyta kvalifikacija arba specialybė
Vilniaus universitetas	2007	Informatikos bakalauro kvalifikacinis laipsnis
Vilniaus universiteto Kauno humanitarinis fakultetas	2009	Vadybos ir verslo administravimo magistro kvalifikacinis laipsnis (verslo informacijos sistemų programa)

Mokslinė veikla:

Metai	Aprašymas
2009 m.	Veiklos is0902 COST projekto dalyvių paraiškos ruošimas.
2010 m.	Projekto paraiškos ruošimas mokslinkų grupei Lietuvos mokslo tarybai.
2011 m.	Projekto paraiškos ruošimas: „Support for Frontier Research – ERC Advanced Grant”. Proposal: “Study of economic paradoxes causing economic crises”.

Atliktų tyrimų paskelbtos mokslinės publikacijos:

Mokslinės informacijos instituto (ISI) pagrindinio sąrašo leidiniuose

GIRDZIJAUSKAS, S. A., STREIMIKIENE, D., MIALIK, A. (2012) *Failures of Capitalism and Unknown Economic Paradoxes*. Sustainability, 4 (11), 2818-2837 pp.

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