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Exploring the Distribution of Zeros of the Prime Zeta Function: New Insights Through 3D Visualization and Statistical Analysis

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This study focuses on the prime zeta function, an under-explored function in analytic number theory. Special attention is devoted to its zero-free region and the distribution of the zeros. Our research identified over 10000 zeros within the complex rectangle $(0.1,1) \times (0,10^4)$. Using statistical analysis, we validated some conjectures regarding zeros' distribution patterns (both within and beyond the critical strip), offering new insights into their underlying structure.

A significant aspect of this study is the utilization of 3D visualizations, which allows us to understand better the behaviour of the prime zeta function and the spatial distribution of its zeros. These visual representations provide a more precise, intuitive grasp of complex surfaces and singularities associated with the prime zeta function. The 3D models depict the intersections of real and imaginary surfaces and highlight zero-plane isolines, aiding in detecting the patterns previously challenging to observe.

The findings reveal a concentration of zeros within the interval $(0.3 < \sigma < 0.7)$, with a prominent peak in the neighbourhood of the line $\sigma = 0.6$. Additionally, the density of zeros diminishes significantly outside the critical strip, $\sigma > 1$. It has been proved that the prime zeta function $\zeta_{\mathbb{P}}(s)$ has no zeros in the half-plane $\sigma > \sigma_0$, where $\sigma = 1.77954465354699$... is the root of the equation $\zeta_{\mathbb{P}}(\sigma) = 2^{1-\sigma}$. Note that by our calculations,

$$\sigma_{\max} = \max_{|t| \le 2: 10^5} \{ \sigma \mid \zeta_{\mathbb{P}}(s) = 0 \} = 1.682628788045196...$$

Summing up, these results, supported by graphical and numerical data, contribute to the broader understanding of the dynamics of the prime zeta function and pave the way for future explorations through advanced visualization techniques. These insights will be useful for researchers seeking to explore the prime zeta function's intricate behaviour further.