

Aristotelian Roots of Contemporary Tense Logic

Živilė PABIJUTAITE*, Pranciškus GRICIUS**

ABSTRACT. Tense logic is a branch of contemporary logic which includes formal devices that allow us to deal with the temporal relations between propositions. The aim of our paper is threefold: 1) to reveal how Aristotelian philosophical ideas about time, truth, possibility and necessity were reinterpreted by the founder of contemporary tense logic Arthur Prior; 2) to discuss what novel solutions to the classical problem of future contingents are available using Priorean invention; 3) to describe how the tools of tense logic have transcended their original theoretical purposes.

Keywords: Tense Logic, Sea-Battle Paradox, Future Contingents, Arthur Prior, Aristotle

Introduction

The 20th century is renowned as a period of flourishing for various non-classical logics. In the early 1920s the Polish logician Jan Łukasiewicz introduced one of the first systems of many-valued logic that includes more than two traditional truth-values (Łukasiewicz 1970 [1920]). It was followed by quantum logics created by Garrett Birkhoff and John von Neumann in 1936, who supplemented classical logic with tools necessary for describing quantum phenomena. The branch of non-classical logics central to this paper is modal logic, the development of which has gained significant momentum in the mid-20th century with the works of Saul Kripke (1959, 1963). This group of logics extends classical logic by introducing various non-truth-functional operators in order to express concepts of necessity, possibility,

* Faculty of Philosophy, Universiteto St. 9/1, Vilnius University, Vilnius, LT-01513, Lithuania. e-mail: zivile.pabijutaite@fsf.vu.lt

** University of Oxford, Vilnius University St Hilda's College, Cowley Pl., Oxford OX4 1DY, United Kingdom. e-mail: pranciskus.gricius@st-hildas.ox.ac.uk



knowledge, belief, moral obligation and right, and more. The need for such an extension arises from the observation that everyday language includes a great variety of simple inferences and arguments which are intuitively valid, but may not be adequately captured by classical logic alone. Here we provide a concise example of such argument:

(1) Today it is the case that p .

(2) Tomorrow it will be the case that p happened yesterday (a day before tomorrow).

This argument, consisting of only one premise (1) and a conclusion (2), is clearly valid since both the premise and the conclusion obviously state the same information, with the difference between them being only that of tense. However, classical propositional logic does not provide us the tools necessary to capture this temporal difference – here (1) and (2) would be formalized as two logically unrelated propositions, A and B . Tense logic is a branch of modal logic which includes formal devices that allow us to deal with such temporal relations between propositions and enable us to prove the conclusions in the arguments which consist of tensed propositions.¹ It was developed by a New-Zealand born logician and philosopher Arthur Prior (1914–1969) in his seminal works *Time and Modality* (1957) and *Past, Present and Future* (1967). Seeking to adequately formalize propositions that express temporal information, Prior introduced tense-logical operators P (*at some time in the past*), F (*at some time in the future*), H (*always in the past*) and G (*always in the future*) which supplement the syntax of classical propositional and predicate logic. These operators, which function in a similar way alethic modal operators \diamond (*possibly*) and \square (*necessarily*) do, and their combinations allow us to properly express all grammatical tenses used in everyday natural languages. Using these tools, the aforementioned argument should be formalized as:

¹ It is important to note that today in the English-speaking world there exist two different ways of naming this branch of logic – *tense* and *temporal* logic. Although these terms are often used synonymously, the choice between them can also be motivated by different ontological commitments: *tense*, which in everyday language is often understood as a grammatical category pertaining to sentences, is a choice more common between the proponents of the A-theory of time where the categories of *past*, *present* and *future* are deemed to be objective and irreducible to other categories (Bigelow 1996, Bourne 2006, Forrest 2004, Markosian 2004, 2010, Tooley 1997 and others), while *temporal* logic is the name usually preferred by the B-theorists who think that grammatical tense does not have any metaphysical grounding in reality and that the temporal relations between the propositions should be defined using the concepts of *earlier than/ simultaneously with/ later than*, avoiding the talk about past, present and future (amongst others, Dyke 2002, Mellor 1991, 1998, Russell 2015, Smart 2008). Prior himself prioritized the term *tense logic* and was an avid supporter of the A-theory of time without using the term *A-theory* which was first coined by Gale (1966). However, in this paper we use the words *tense* and *temporal* synonymously.

(1) p (2) Fp

– where the relation between the two statements now can be formally demonstrated.²

1. *Time, truth, and necessity in Aristotle's On interpretation IX*

Prior was noted not only for his contributions to the field of logic, but also as a prominent historian of ancient and medieval philosophy. The two interests he had were not orthogonal to one another as Prior was inspired by some classical logical and philosophical paradoxes that vexed both ancient and medieval thinkers. One such puzzle is The Sea-Battle Paradox formulated by Aristotle in his *On interpretation IX*.³ The essence of this paradox lies in the tension between the two common intuitions. On the one hand, it is widely believed that the future is not yet determined, that there is nothing at the present moment which would compel us towards one future path rather than the other, and that we are free agents with objective ability to choose. On the other hand, the principle of bivalence, one of the most fundamental principles in classical logic, requires that every proposition be either true or false at a given time. Assuming all propositions are either true or false, and that the future is not yet determined, what is the status of propositions such as *There will be a sea-battle tomorrow*?

Although today the question about the truth-value of future-tense propositions is considered an independent major problem in contemporary analytic philosophy⁴, Aristotle discussed this issue merely as a digression from a more general topic⁵ – the square of opposition, a scheme which depicts the logical relations between the different forms of propositions. In Aristotelian logic, the most basic element is a term – a unit of language which represents an object or a concept and is itself neither true nor false. By substituting different terms in the places of a subject and a predicate and connecting them with a copula *is (not) / are (not)*, we get a proposition (ἀπόφανσις) which expresses a certain state of affairs and has a

² The notion of a proposition which is the most common in contemporary tense logic and which is also embraced in this paper, diverges significantly from the default notion prevalent in the other branches of contemporary logic. The classical tradition of A. N. Prior traces its roots in the ancient and medieval logical systems where it was common to use the propositions which, although being tensed, either did not provide any direct temporal references, or relied on indexical references tied to the circumstances of their utterance. Such propositions were not seen as incomplete or in need of further specification. In the classical-Priorean framework, the example of a standard proposition is not *P is happening at the moment T* with a stable truth-value, but rather *P is happening [now]* with a changing truth-value.

³ Most extensively discussed by Prior in chapter 7 of his 1967.

⁴ Belnap and Green 1994, 2001, Borghini & Torrenzo 2013, Greenough 2008, MacFarlane 2003, Malpass and Wauer 2012, Thomason 1970.

⁵ For a more detailed discussion of the historical and philosophical context see Gaskin 1995.

truth-value *true* or *false* (DI 4 16b 29-30).⁶ The Aristotelian classification of propositions is based on three criteria: (1) the type of a subject term (singular terms such as *Socrates* refer to some particular individual, while general terms such as *human* can refer to more than one object); (2) the quantity of a subject term (propositions with a subject such as *all men* assert something about all members of a class, while a proposition with a subject like *some men* expresses information about some part of that class); (3) the quality of the copula (propositions with the copula *is/ are* are called affirmative (κατάφασις) and those that deny the relationship between the subject and predicate with a copula *is not/ are not* are called negative (ἀπόφασις)). Focusing his attention on propositions consisting of general terms, Aristotle distinguishes four basic types of propositions (general affirmative and general negative, particular affirmative and particular negative) and provides a scheme that depicts logical relations between them (DI 17a37-18a12)⁷:

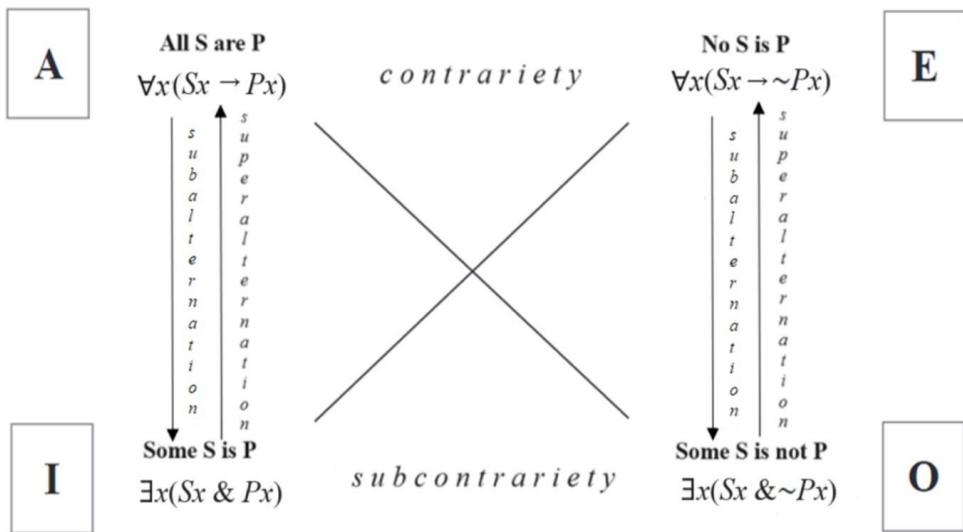


FIGURE 1. Square of opposition

⁶ It should be noted that Aristotle is constructing his categorical propositions not only with the verb *to be* but also with the verb *ὑπάρχειν*, which translates as *to belong/ to pertain*.

⁷ It is important to note that the graphical scheme of the square of opposition was not provided by Aristotle himself who merely described the relations. The graphical representation is dated back to the 2nd century AD. The square of opposition with formalization in modern logical notation is discussed extensively in Parsons 2014. Here and elsewhere the schemes were created by the authors of this paper.

Relations of contrariety, subcontrariety, contradictoriness, subalternation and superalternation define a specific distribution of the truth-values between the opposing propositions: contrary propositions cannot be both true, subcontrary ones cannot both be false⁸, subalternation allows us to infer the truth of a particular proposition if a corresponding universal proposition of the same quality is true, in the case of superalternation the falsity of a universal proposition is implied by the falsity of a corresponding particular proposition of the same quality, and contradictory statements must always have different truth-values.

The square of opposition, together with syllogistic theory, constitutes the essence of Aristotelian logic and was later severely criticised by the proponents of modern logic who deemed that most of the relations of the square become invalid when considering the propositions with empty subject terms.⁹ In fact, in his *On interpretation* Aristotle himself discussed several group of statements which he considered to be potential exceptions to the relations depicted by the square. One such class is contingent propositions about the future, expressing some future state of affairs which is neither impossible nor necessary. Considering a pair of contradictory propositions such as *Tomorrow there will be a sea battle* and *It is not the case that tomorrow there will be a sea-battle*¹⁰ (and assuming that neither of the corresponding state of affairs is determined to happen), is it still possible to claim that at the present moment the truth-values are distributed in such way that one of the propositions is true and the other false? Aristotle presents The Sea-Battle paradox as an argument with a fatalistic conclusion and provides two slightly different versions of it (the conclusion of both of these arguments can be generalized to all contingent state of affairs) (*DI* 19a23-19a39):

⁸ In *On interpretation*, the relation of subcontrariety is not explicitly discussed and is only hinted but not named.

⁹ From the viewpoint of modern predicate logic, universal affirmative and negative propositions with empty terms are vacuously true, while particular affirmative and negative propositions are considered to have existential import and, when their subject term is empty, are ascribed a truth-value *false*. This results in the invalidation of the relations of contrariety, subcontrariety, subalternation and superalternation, leaving only the relation of contradictoriness intact.

¹⁰ Although neither of these statements include the quantifier-words such as *all* or *some* (as in more typical examples that Aristotle provides) the relation between them is clearly that of contradictoriness as it is not possible for them both to be true or false at the same time.

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|---|---|
| <p>1) Every proposition is either true or false;
 2) today it is possible for someone to claim that tomorrow there will be a sea-battle and for someone else to deny it;
 3) (only) one of the two contradictory propositions must be true;
 4) today one (and one only) of the propositions mentioned in (2) must be true;
 5) if some proposition is true, a corresponding state of affairs which makes it true must obtain.
 ∴ Today it is necessary that tomorrow there will be a sea-battle.</p> | <p>1) Every proposition is either true or false;
 2) if today there is a sea-battle, yesterday it was true to say that it would happen tomorrow;
 3) everything in the past is unchangeable and therefore necessary;
 4) the proposition <i>There will be a sea-battle tomorrow</i> asserted yesterday was necessarily true.
 ∴ It is necessary that today there is a sea-battle.</p> |
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Originally meant to highlight the tension between the principle of bivalence and the open future intuition, this paradox was reconsidered through a theological prism during the medieval period, trying to reconcile God's foreknowledge of future events with the Christian doctrine of free will. Unlike Aristotle's First Mover, understood as an abstract metaphysical principle whose contemplation is not directed towards the physical world (*Met.* 1072a19-b30), the Christian God possesses attributes of goodness and justice and actively participates in the life of the created universe. One of the first theological versions of the paradox can be found in Boethius' *Consolation of Philosophy* (book V) and his second commentary on Aristotle's *On interpretation* (225,10-226,14).¹¹ Boethius and later medieval thinkers noticed the apparent inconsistency between the idea that we must have free will in order to be praised or punished for our actions, and the fact that God infallibly knows not only past and present but also all future facts, including our future choices even before they are made. Boethius expresses this tension with the question:

How God can know in advance that these things will happen if they are uncertain. [...] If the mind of God cannot be uncertain, then those things that he knows will happen absolutely must happen. And if that is true, then human thoughts and actions have no freedom about them at all, because the mind of God sees all things in advance and can never be led astray, which means that his certainty compels all thoughts and actions to happen. (*Consolatio philosophiae*, book V, chapter III, translation by David R. Slavitt)

The theological version of the aforementioned fatalistic argument could be presented this way (where for p we can substitute any future contingent proposition):

¹¹ For an earlier discussion of this version of the paradox see Augustine's *City of God*, book V, chapters 9 and 10.

- 1) There exists an omniscient Being knowing all past, present and future facts;
 - 2) everything in the past is unchangeable and therefore necessary;
 - 3) if someone knows that p , then p ;
 - 4) yesterday an omniscient Being knew that p ;
 - 5) today it is necessary that yesterday an omniscient Being knew that p .
- \therefore Today it is necessary that p .

2. Prior on The Sea-Battle paradox

Philosophically, the argument from the existence of the omniscient God to determinism can be reformulated in such a way that it does not rest on the assumption that there is such a being as God, since whatever God knows is true, and whatever is true is known by God; thus, it is enough to just talk about what is, was or will be and to drop God's knowledge from the picture. Looking through the lens of tense logic developed by Prior it is possible to discern five principles that are at play in the arguments for determinism¹²:

- (P1) $\varphi \rightarrow P_n F_n \varphi$
 (P2) $\Box(P_n F_n \varphi \rightarrow \varphi)$
 (P3) $P_n \varphi \rightarrow \Box P_n \varphi$
 (P4) $\Box(\varphi \rightarrow \psi) \rightarrow (\Box \varphi \rightarrow \Box \psi)$
 (P5) $F_n \varphi \vee F_n \sim \varphi$

(where subscript letters n and m specify some period of time, e.g., one day). The first principle says that if φ , then it was the case n ago that it will be after n that φ . For example, if a dog is barking, then yesterday it was the case that the dog will bark tomorrow. According to the second principle, it is necessary that if n ago it was the case that after n it will be the case that φ , then it is φ . For example, it is necessary that if yesterday it was the case that the dog will bark tomorrow, then the dog is barking today. The third states that the past is necessary: the fact that the dog barked n moments ago is now necessary: once something took place, it is impossible to change that. The fourth is a well-known modal principle, called axiom K, an axiom which is included in any normal modal logic. It states that given that an implication is necessary, if its antecedent is necessary, then its consequence is also necessary. Roughly speaking, if in every possible world it is the case that if φ , then ψ , then if φ is true in all possible worlds, then ψ is true in all possible worlds. The final principle is the law of future excluded middle: it states that for any period of time n , either it will be after n that φ or it will be after n that $\sim \varphi$.

¹² In the presentation we follow, with slight modifications, Goranko 2023: 32–35.

Note that these principles seem to be uncontroversial: each one seems to be intuitively correct. These principles, however, are all that is needed to secure the deterministic conclusion that if something will happen, it will happen by necessity, and if it won't happen, it won't happen by necessity.

1. $F_m p \rightarrow P_n F_n F_m p$ (by P1, when $\varphi / F_m p$)
2. $P_n F_n F_m p \rightarrow \Box P_n F_n F_m p$ (by P3, when $\varphi / F_n F_m p$)
3. $F_m p \rightarrow \Box P_n F_n F_m p$ (by 1 and 2 by transitivity of \rightarrow)
4. $\Box(P_n F_n F_m p \rightarrow F_m p)$ (by P2, when $\varphi / F_m p$)
5. $\Box(P_n F_n F_m p \rightarrow F_m p) \rightarrow (\Box P_n F_n F_m p \rightarrow \Box F_m p)$ (by P4, when $\phi / P_n F_n F_m p$, $\psi / F_m p$)
6. $\Box P_n F_n F_m p \rightarrow \Box F_m p$ (by 4 and 5 by *modus ponens*)
7. $F_m p \rightarrow \Box F_m p$ (by 3 and 6 by transitivity of \rightarrow)
8. $F_m \sim p \rightarrow \Box F_m \sim p$ (repeating the argument from 1-7 with $\sim p$ in place of p)
9. $F_m p \vee F_m \sim p$ (by P5)
10. $\Box F_m p \vee \Box F_m \sim p$ (by 7, 8, 9)

The conclusion reached states that determinism holds.

If indeterminist worldview is to be defended, then one of the five principles must be rejected. Perhaps the best-known answer to the above argument is Ockhamist semantics – a formal semantic theory specifying truth conditions for future statements, that was developed by Prior (1968: 122–127). This semantics shows how the third principle – the principle stating that whatever is past is necessary – can be falsified. Ockhamist semantics is based on the branching time structure¹³, which Saul Kripke suggested to Prior in a letter written in 1958. Kripke wrote:

Now in an indetermined system, we perhaps should not regard time as a linear series, as you have done. Given the present moment, there are several possibilities for what the next moment may be like – and for each possible next moment, there are several possibilities for the next moment after that. Thus the situation takes the form, not of a linear sequence, but of a “tree”¹⁴.

Kripke here contrasts two ways of thinking about the structure of time. The linear sequence that Kripke is referring to is the structure of time that Prior worked with in his book *Time and Modality* (1957): time is represented as a line stretching from the past through the present to the future.

¹³ There is another well-known semantics, called Peircean, which is also based on branching-time structures (see Prior 1968: 128–134). It will not be discussed here due to space limitations. Also, note that it is a separate question whether the historical William of Ockham endorsed what is here called Ockhamist semantics. Some think he did not, see Øhrstrøm 1984: 217.

¹⁴ Quoted in Øhrstrøm and Hasle 2020: §3.

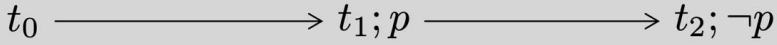


FIGURE 2. Linear time model

Now it is obvious that this way of thinking about time codifies determinism: from any point in time, there is a unique future awaiting. What Kripke suggests to Prior is a branching or tree-like structure of time, where each moment has a set of alternative possible futures associated with it (therefore, it fits the indeterminist worldview much better):

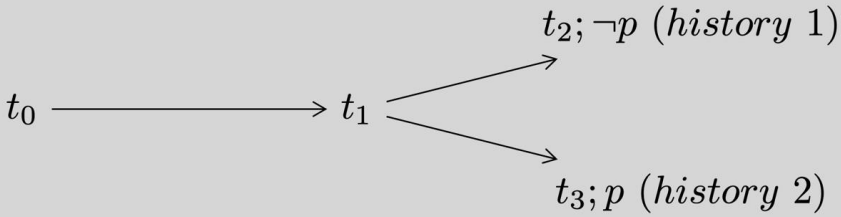


FIGURE 3. Branching time model

However, the branching-time structure raises new questions. If linear time framework is adopted, then the truth value of a statement about the future, say $F_m p$, at time t , depends on the truth value of p at moment $t + m$: if p is true there, then $F_m p$ is true at t ; false otherwise. So if m is one day and p is the proposition *the dog is barking*, then it is today true that tomorrow the dog will bark, if and only if tomorrow it is true that the dog is barking. Thus, in the above diagram of linear time, at t_0 the statement $F_n p$ is true but $F_{n+m} p$ is not (where n is the distance between t_0 and t_1 , and m is the distance between t_1 and t_2). So if p and m are interpreted as before and n is also one day, then in the above diagram at t_0 it is true that the dog will bark tomorrow but false that the dog will bark on the day after tomorrow. However, there is no straightforward way of answering the question of whether $F_m p$ is true at t_1 in the branching-time diagram above (where m is the distance between t_1 and t_2 , and the distance between t_1 and t_3).

The Ockhamist semantics provides one specific answer regarding the evaluation of such statements. Essentially, Ockhamist semantics says it depends on

the history that is selected (where history is a maximal path through the branching structure). For example, if history 2 is selected, then $F_m p$ is true at t_1 , while if history 1 is selected, then $F_m p$ is false at t_1 . Thus, according to the Okchamist semantics, statements are evaluated not only relative to a time moment but also relative to some *history*. Importantly, modal operators \diamond (possible) and Υ (necessary) function as quantifiers over histories, so that *necessarily there will be a sea battle tomorrow* is today true if and only if in every history (that passes the present moment) tomorrow there is a sea battle¹⁵. The number of different histories depends on the number of contingent propositions.

Ockhamist semantics invalidates P3 which is needed for the derivation of deterministic conclusion. P3 says that $P_n \varphi \rightarrow \Box P_n \varphi$, that is, if n ago φ holds, then it is necessary that n ago φ holds. Now consider the case of P3 where $F_n F_m p$ is substituted for φ , $P_n F_n F_m p \rightarrow \Box P_n F_n F_m p$ (where n is the distance between t_0 and t_1). If it was the case yesterday that after two days the dog will bark, then it necessarily was the case yesterday that after two days the dog will bark. Now, in our diagram above, is $P_n F_n F_m p \rightarrow \Box P_n F_n F_m p$ true at t_1 relative to history 2? The antecedent $P_n F_n F_m p$ is true at t_1 relative to history 2, since $F_n F_m p$ is true at t_0 relative to history 2; the latter holds because p is true at t_3 relative to history 2. On the other hand, the consequent $\Box P_n F_n F_m p$ is *false* at t_1 relative to history 2, since if it were true, then $P_n F_n F_m p$ would have to hold relative to any history, and hence in particular relative to history 1. However, $P_n F_n F_m p$ is *false* at t_1 relative to history 1, as can be easily checked. The implication $P_n F_n F_m p \rightarrow \Box P_n F_n F_m p$ is thus false at t_1 since it has a true antecedent and a false consequent. The general principle that what is past is necessary is falsified. One of the premises needed in the derivation of determinism is no longer true, and thus the argument for determinism halts to a stop.

Analysis of the problem of the future of contingents with the tools of formal logic makes progress on the issue, for the principles causing the problem are singled out perspicuously, the possible solutions are discerned, and the merits of these solutions can thus be evaluated by abductive methodology.

¹⁵ Here we do not provide the formal Ockhamist semantics for reasons of space. Note that there are two ways of doing it. As it was first developed by Prior (1968: §7), the semantics rested on having two notions of a formula: one includes all formulas and the other only those that are not about the future. On this approach, we first assign values to statements that are not about the future relative to moments of time, and then assign prime-facie assignments relative to histories and moments of time, and then modal operators are interpreted as saying that the formula to which it is attached is true on all or some (as the case may be) prime-facie assignments. Alternatively, it is possible to work with one sorted notion of a formula. Then an interpretation of them assigns a subset of the cartesian product of the set of times and the set of histories. See Thomasson 1984 and Goranko 2023 for discussion.

3. *Other applications of Prior's tense logic*

The advances that Prior made are not limited, however, to the original problem of future contingents. Once time is analysed in a formal system, many fundamental questions about it can be given a rigorous formulation. For example, precise formulations can be given of the view that the world has no beginning or an end, that time is dense and not discrete, and other important metaphysical views. Additionally, and philosophically much more interestingly, the language of propositional tense logic can be enriched with individual terms, quantifiers, and predicates, and then one can raise questions about whether individuals exist temporarily or eternally, about how individuals persist through time: are individuals wholly present at every moment of their existence, as endurantists would have it, or do individuals persist by being made out of a series of time slices, as perdurantists hold?

More generally, Prior's work in tense logic paved the way for other important developments in analytic philosophy, especially in metaphysics and philosophical logic¹⁶. Note that in Prior's models of tense logic, the set of time moments is considered as an element of the model. This suggests that time is an objective phenomenon about which hypotheses in the formal object language can be formulated. It is to be expected, thus, that the correct logic of time will codify some substantial metaphysical principles. This seems quite a simple and uncontroversial idea when the subject matter is time; when the subject matter is possibility and necessity, however, it is less so. In the first half of the XX century, the dominant view of modality was metalinguistic; for example, Carnap (1947) held that what is necessary is that whose truth follows from some semantic rules for the language. The major development in modal logic was Kripke's (1963) work on possible worlds semantics, which takes the set of possible worlds to be in the model, exactly in the same way as the set of time moments is considered in Prior's models of tense logic. Kripke's formal work in the model theory of modal logics suggested that modal discourse is concerned with objective phenomena and not with the language one speaks. Similarly, it is to be expected, thus, that the correct logic of modality will codify some substantial metaphysical principles. Possible worlds semantics and its philosophical surroundings had a major impact on the development of metaphysics, and analytic philosophy more generally. It is reasonable to think that Kripke took at least some inspiration from Prior's work in tense logic, which in turn, as already noted, took inspiration from Ancient authors, in particular Aristotle.

¹⁶ In what follows, the main points are Williamson's' (2014: 9).

The influence of Prior's work is not confined to tense logic and analytic philosophy: the formal systems he developed turned out to be immensely useful in computer science and artificial intelligence, something that Prior himself had foreseen¹⁷. Starting already in the 70s, most notably with the work of Pnueli (1977), temporal logics became an important part of computer science. Later in the 80s, further work dealing with temporal databases (Dean and McDermott 1987) and reasoning about temporal processes (McDermott 1982) were an integral segment of research in artificial intelligence (see Fisher et al. 2005).¹⁸

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¹⁷ See Øhrstrøm and Hasle 1993: 48, where Prior's unpublished manuscript *A Statement of Temporal Realism* is quoted.

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