VILNIUS UNIVERSITY

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COMPUTERIZED MODELING TECHNOLOGY OF UNIQUE FOOTWEAR SURFACE MANUFACTURING DESIGN

Summary of Doctoral Thesis

Technological Sciences, Informatics Engineering (07 T)

Vilnius, 2017

Doctoral thesis was written in 2012–2016 at the Vilnius University Institute of Mathematics and Informatics.

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The thesis will be defended at the public session of the Scientific Council of the Scientific Field of Informatics Engineering in the auditorium number 203 at Vilnius University at 10 a. m. on the 26th of May 2017.
The summary of the doctoral thesis was distributed on the 25th of April 2017.
A copy of the doctoral thesis is available for review at the Library of Vilnius University or on this website: www.vu.lt/lt/naujienos/ivykiu-kalendorius.

VILNIAUS UNIVERSITETAS

MARTYNAS SABALIAUSKAS

NESTANDARTINĖS AVALYNĖS GAMYBOS FORMŲ PAVIRŠIŲ KOMPIUTERINIO MODELIAVIMO TECHNOLOGIJA

Daktaro disertacijos santrauka

Technologijos mokslai, informatikos inžinerija (07 T)

Vilnius, 2017

Disertacija rengta 2012–2016 metais Vilniaus universiteto Matematikos ir informatikos institute.

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Disertacija bus ginama viešame Vilniaus universiteto Informatikos inžinerijos mokslo krypties tarybos posėdyje 2017 m. gegužės 26 d. 10 val. Vilniaus universitete 203 auditorijoje.
Adresas: Akademijos g. 4, LT-08663 Vilnius, Lietuva.
Disertacijos santrauka išsiuntinėta 2017 m. balandžio 25 d.
Su disertacija galima susipažinti Vilniaus universiteto bibliotekoje ir VU interneto svetainėje adresu: www.vu.lt/lt/naujienos/ivykiu-kalendorius.

1 Introduction

1.1 Research area

Invention of 3D scanning technology played an important role in new fields of application research and development regarding 3D data analysis. Fast-growing mass customization markets require new fields of research to improve manufacturing efficiency regarding unique products (those produced under special orders). Digital drawings and three-dimensional models provide 70 % of technical data that is needed for clear manufacturing, however, there are still some unused applications where 3D data can be applied to operate with complete efficiency. Innovative solutions are needed to develop an automatic system that will flatten the surface of the foot's complex geometry.

Research scope of this thesis is method analysis of non-linear projection, their application and improvement methods.

1.2 Relevance of the problem

Currently in custom footwear production, flattening of individual lasts is a manual job. There are a number of decisions in the market relating to the automated manufacturing of footwear. These decisions are implemented in separate systems or plug-ins. Orthopedic specialists are using below-listed programs as additional tools for non-standard footwear modeling:

1. *ShoeMaster* is an automated shoe modeling software. Solutions and modeling tools, that are realized in *ShoeMaster*, are not compatible with the individual foot scan data. The program is developed to work with standardized foot models.

2. *Blender* and *3D Coat* are freeform digital modeling systems. They have a quite fast surface flattening tool, however, the result is inaccurate comparing to manual flattening output. *Blender* and *3D Coat* unwrapping tool are adapted for texture mapping to an object. The program does not have an output function as well.

3. *Terra Flat* is tested by orthopedic specialists and is an extension of *Rhinoceros* program. Foot scan surface mesh consists of 30,000 to 50,000 triangles connected through the vertex. *Terra Flat* plug-in operates with 2–3 times lower quantity of polygons that is in initial foot scan mesh surface. Considering to mesh reduction foot scan geom-

etry becomes approximated. Flattened shells compared with manual output is 30 % incorrect.

Currently, no modeling system present in the market of the manufacturing process complies with the parameterization precision requirements. Digital flattening system of digital models of nonstandard 3D organic nature object (forms) would help to automate the process of mass-production unit for 3D surface flattening creation.

If a method was designed that would automatically form a digital mold from a digital last model, then the following manual individual footwear manufacturing processes could be avoided:

1. Segmentation of surface lasts.

2. Flattening using wax foil.

3. The design of drawings according to the contour of wax foil.

4. Digitization of the received technical data.

This technological solution would help to reduce the model production preparation time of about 30 %. Additionally, in centralized production situation cases of ordering of services in remote production facilities would help to reduce model delivery costs and save time. Delivery costs account for a big share of production time and consumption costs, as all manufacturing models of massive individual production are different. Developed method would provide solutions to solve similar problems in the industry, ensure the creation of parallel systems adapted to other industries, which need to obtain transcripts of 3D objects.

Another acute problem – is speeding up of algorithm parameterization. Each digital model is defined by adjacency matrix whose dimension complies with the number of vertices of this digital model. Parameterization process solves optimization problem thereby calculating inverse adjacency matrix, therefore the complexity of algorithm becomes polynomial. The fact is that adjacency matrix is a sparse matrix which can be processed by an algorithm of direct complexity. This processing is possible by using Cholesky decomposition or other factorizations aiming to speed-up linear system solving process.

1.3 Research objectives

The research object of the thesis is processing and formation of triangular and quadrangular surfaces of 3D digital model algorithms: surface segmentation, parameterization, triangulation, smoothing and simplification of the surface, construction of parametric surface and graph search algorithms. To investigate and implement these algorithms the thesis analyses global optimization, matrix decomposition and computational geometry tasks.

1.4 The aim and tasks of the research

The aim of this thesis is to develop a fast and economically effective, expert knowledge based digital flattening system for 3D individual models of lasts.

To reach the aim the following tasks were solved:

1. To analyze 3D digital model surface formation and processing algorithms.
2. To analyze parametric surface formation methods, that could be applied by forming parametric surfaces from point clouds (digital models) received from scanned lasts.
3. To analyze which parameterization methods best minimize conformal and authalic deformations.
4. To analyze processing methods of sparse matrices that are used for speeding up parameterization algorithms.
5. To evaluate which of standard parameterization methods (authalic, conformal, isometric) best complies with manual mold used in individual footwear production.
6. To design automatic digital last model segmentation method used for parameterization of scanned last surfaces.
7. To compare the results of manual and automated digital last surface segmentation.
8. To design software used for unique footwear surface segmentation design and parameterization of these segments.

1.5 Research methods

Tasks of the thesis are solved by analytic and experimental methods. By analyzing scientific and experimental results in digital model surface segmentation and parameterization methods, information search, systematization, analysis and generalization methods were used. Theoretic analysis methods were used to analyze algorithm complexity and their speeding up. Generalization method is used to evaluate the results of parameterization received through statistic methods and comparing it with the results received in individual footwear production.

1.6 Scientific novelty

The following results were obtained in the thesis:

1. Comparative statistic analysis was performed in the thesis, aimed at evaluating last molds designed on the basis of different mathematic methods in comparison with the molds, received during individual footwear production process.

2. The analytic solution of 2×2 singular value decomposition was used for local ARAP method optimization phase. The analytic solution of SVD is locally integrated into ARAP method optimization phase and Cholesky decomposition were applied for the calculation of inverse Laplace matrix in global ARAP method optimization phase. All these processes enabled the speeding up of parameterization of isometric ARAP method.

3. The new digital last model surface segmentation method was designed which allows the compliance of digital parameterization molds up to 98 % in comparison with the molds used in individual footwear production.

4. A new lasts modeling algorithm for approximation of Bezier C^2 surface of a quadrangular surface was proposed.

1.7 Practical significance

An accurate and a fast flattening system of digital surface models and surface fragments would provide significant benefits for many industrial sectors: footwear industry, textile industry, automotive industry, furniture industry and medical products industry. Fast surface flattening method of virtual models would provide solutions for similar problems in the industry, as well as the creation of parallel systems adapted to other industries, which need to obtain transcripts of mass customization and unique 3D objects.

The digital last model segmentation method together with isometric surface parameterization method can be used in designing 2D drawings for individual footwear production.

1.8 Statements to be defended

1. The composition of isometric ARAP parameterization method and proposed digital last model segmentation method gives a better parameterization result than the composition of manual segmentation and conformal ABF++ and LSCM parameterization methods.

2. Using isometric ARAP parameterization method the general areal and conformal deformation is minimized as possible, differently from conformal ABF++ and LSCM parameterization methods.

3. Bezier surface approximation algorithm for quadrangular surface proposed in the thesis has several characteristics:

   a) the vertices in featured areas distribute evenly,

   b) the detail parameter $n = 1, 2, 3, \ldots$ of parametric surface can be freely chosen.

4. Proposed analytical solution of singular value decomposition $\mathbf{U\Sigma V}$ of a 2×2 real matrix $\mathbf{A}$ is suitable to calculate triangular surface deformations in local ARAP parameterization phase.

1.9 Approbation and publications of the research

The main results of the dissertation were published in 10 research papers: four papers are published in periodicals, reviewed scientific journals; two papers are published in conference proceedings; four papers are published in conference abstracts. The main results have been presented and discussed at 9 national and 2 international conferences.

Practical application methods designed in this thesis received the highest award in the 6th Lithuanian junior researchers conference "Interdisciplinary research in physical and technological science" of *INFOBALT* association.

1.10 Outline of the Thesis

The thesis is comprised of 5 chapters, literature list and appendices. The chapters of the thesis are as follows: Introduction, A review of mesh application and processing methods, New methods for mesh processing, Mesh parameterization experiments, General conclusions. The general scope of the thesis — 130 pages, 50 numbered formulas, 36 figures, 7 tables and 7 algorithms. The literature list is comprised of 111 sources.

2 Segmentation and flattening of 3D objects

The segmentation of 3D digital models is applied with the aim to cluster 3D object surface according to the color, form, vertices density and other criteria. The segmentation most often used in practice aims at separating such surface areas where texture mapping in 3D space would be as much as possible reduced and deformed in the plane. The benefit of such segmentation would be more exact 3D object parameterized fragments, higher quality 2D images in the 2D plane and size of textures taking less memory. These clustering algorithms are based on digital model separation according to gradient direction or surface splitting according to normal directions.

To compute a flattened surface of a given 3D object (or graph) means to construct an isomorphic graph on the plane. The surface mapping or parameterization is used to put the surface into one-to-one correspondence with an image, stored in the 2D domain. One of the main applications of that construction is texture mapping.

2.1 Manual segmentation and flattening of 3D lasts

In this section, we are going present the manual segmentation and flattening of lasts, which is used in orthopedics to produce individual products. The manual lasts segmentation performed by experts is comprised by:

1. The composition of the contour of the last's surface and sole.

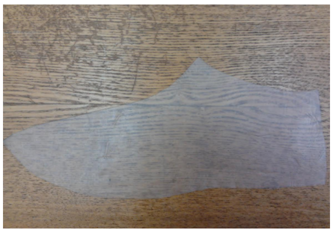
2. Last splitting by plane lengthwise.

The upper and the lower contours of the lasts are made up by eliminating sole and their upper part having regard to the arched curve in the surface which marks the transition of the last to its sides. Then segregation of the lasts by plane is performed according to this rule: there is a point marked in the last which corresponds to the middle line of the finger next to big toe. Then a straight line is drawn through this point towards the arithmetic average of the last's upper surface and the final drawing goes into the direction of the most distant point of the heel. Finally, lateral segments according to the designed last surface splitting lines are detached and serve as the basis for production scheme. Figure 1 shows the compilation of the manufacturing drawings using wax foil.

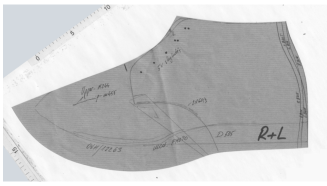
(a) Segmented individual last

(b) Surface of the last is covered with heated wax foil

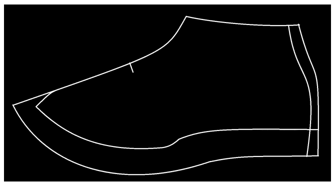
(c) According to the segment lines, the wax foil is chopped and removed



(d) Wax foil is heated again and flattened



(e) Making manufacturing molds according to the wax foil contour



(f) Digitization of molds

Figure 1: Making manufacturing drawings using wax foil

2.2 Parameterization methods

To compute a parameterization of an object means to construct a bijection between vertices of 3D graph and 2D domain vertices respective (Figure 2).

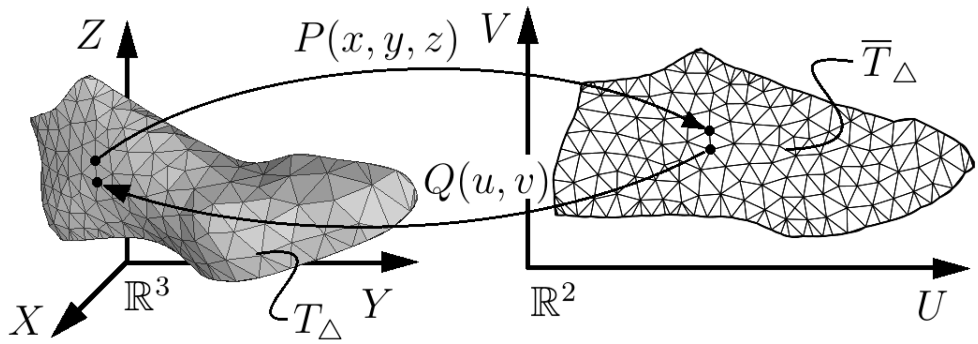


Figure 2: Surface mapping

According to Maxwell-Tutte theorem if $G = (V, E)$ is 3-connected planar (triangular mesh) then any barycentric drawing is a valid embedding. Therefore each $T_\triangle$ surface can be projected to plane α inside the non-self-intersecting closed contour. This projection is called parameterization

$$f : T_\triangle \leftrightarrow \overline{T}_\triangle, T_\triangle \subset \mathbb{R}^3, \overline{T}_\triangle \subset \mathbb{R}^2,$$

where $P(x, y, z) : T_\triangle \to \overline{T}_\triangle$ and $Q(u, v) : \overline{T}_\triangle \to T_\triangle$ are relations.

There are three methods to make a projection $\overline{T}_\triangle$ of given triangular surface $T_\triangle$:

1. Length-preserving or isometric mapping.

2. Angle-preserving or conformal mapping.

3. Area-preserving or authalic mapping.

In this thesis, isometric ARAP mapping and two conformal ABF++ and LSCM mappings were investigated.

2.3 Distortion analysis

Distortion measures show deformation of the parameterization result. There are 2 most popular types of measuring area and angle deformation. D_{angle} shows the general conformal deformation including local authalic deformation of each triangle

$$D_{angle} = \frac{1}{\sum_{i=1}^{T} S_i} \sum_{j=1}^{T} S_j \Big(\frac{\sigma_{1,1}^j}{\sigma_{2,2}^j} + \frac{\sigma_{2,2}^j}{\sigma_{1,1}^j} \Big), \tag{1}$$

D_{area} shows the general authalic deformation including local authalic deformation of each triangle

$$D_{area} = \frac{1}{\sum_{i=1}^{T} S_i} \sum_{j=1}^{T} S_j \left(\sigma_{1,1}^j \sigma_{2,2}^j + \frac{1}{\sigma_{1,1}^j \sigma_{2,2}^j} \right)^{\Theta}, \tag{2}$$

where $\sigma_{1,1}^j$ and $\sigma_{2,2}^j$ are singular values of singular value decomposition of 2×2 Jacobian transformation $\mathbb{R}^3 \to \mathbb{R}^2$ matrix of j-th triangle, S_t is area of t-th triangle, $\triangle A_t B_t C_t \in T_{\triangle}$, $t = 1, \ldots, T$, $\Theta \in (0, \infty)$ is weight parameter. A product of D_{angle} and D_{area} shows the general isometric deformation.

2.3.1 Singular value decomposition

In linear algebra, the singular value decomposition (SVD) is a factorization $\mathbf{U\Sigma V}^{\intercal}$ of given matrix $\mathbf{A}$, where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices and $\mathbf{\Sigma}$ is a rectangular diagonal matrix with non-negative real numbers on the diagonal. For the 2×2 case, $\mathbf{A}$ matrix satisfies a condition:

$$\begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix} = \begin{pmatrix} u_{1,1} & u_{1,2} \\ u_{2,1} & u_{2,2} \end{pmatrix} \begin{pmatrix} \sigma_{1,1} & 0 \\ 0 & \sigma_{2,2} \end{pmatrix} \begin{pmatrix} v_{1,1} & v_{2,1} \\ v_{1,2} & v_{2,2} \end{pmatrix}, \tag{3}$$

where

$$\mathbf{UU}^{\intercal} = \mathbf{VV}^{\intercal} = \mathbf{I}, \quad \sigma_{1,1}, \sigma_{2,2} \geqslant 0. \tag{4}$$

The thesis proposed fast and robust algorithm for 2×2 SVD calculation (algorithm 1).

Algorithm 1: singular value decomposition of a 2 × 2 real matrix

Input : $\mathbf{A}$

Output: $\mathbf{U}, \mathbf{\Sigma}, \mathbf{V}$

$\sigma_{1,2}, \sigma_{2,1} \leftarrow 0$;

$\sigma_{1,1} \leftarrow \left(\sqrt{(a_{1,1}-a_{2,2})^2+(a_{1,2}+a_{2,1})^2}+\sqrt{(a_{1,1}+a_{2,2})^2+(a_{1,2}-a_{2,1})^2}\right)/2$;

$\sigma_{2,2} \leftarrow \left|\sigma_{1,1}-\sqrt{(a_{1,1}-a_{2,2})^2+(a_{1,2}+a_{2,1})^2}\right|$;

if $\sigma_{1,1} > \sigma_{2,2}$ **then** $v_{1,1} \leftarrow \sqrt{(\sigma_{1,1}^2-a_{1,2}^2-a_{2,2}^2)/(\sigma_{1,1}^2-\sigma_{2,2}^2)}$ **else** $v_{1,1} \leftarrow 1$ **end**;

if $a_{1,1}a_{1,2} < -a_{2,1}a_{2,2}$ **then** $v_{1,2} \leftarrow \sqrt{1-v_{1,1}^2}$ **else** $v_{1,2} \leftarrow -\sqrt{1-v_{1,1}^2}$ **end**;

$v_{2,1} \leftarrow -v_{1,2}$, $v_{2,2} \leftarrow v_{1,1}$;

if $\sigma_{1,1} \neq 0$ **then** $u_{1,1} \leftarrow (a_{1,1}v_{1,1}+a_{1,2}v_{2,1})/\sigma_{1,1}$ **else** $u_{1,1} \leftarrow 1$ **end**;

if $\sigma_{1,1} \neq 0$ **then** $u_{2,1} \leftarrow (a_{2,1}v_{1,1}+a_{2,2}v_{2,1})/\sigma_{1,1}$ **else** $u_{2,1} \leftarrow 0$ **end**;

if $\sigma_{2,2} \neq 0$ **then** $u_{1,2} \leftarrow (a_{1,1}v_{1,2}+a_{1,2}v_{2,2})/\sigma_{2,2}$ **else** $u_{1,2} \leftarrow -u_{2,1}$ **end**;

if $\sigma_{2,2} \neq 0$ **then** $u_{2,2} \leftarrow (a_{2,1}v_{1,2}+a_{2,2}v_{2,2})/\sigma_{2,2}$ **else** $u_{1,1} \leftarrow u_{2,2}$ **end**;

Algorithm 1 is suitable for regular, singular and zero 2 × 2 real input matrices. Using analytical solutions $\sigma_{1,1}$, $\sigma_{2,2}$ from algorithm 1 the calculation of D_{angle} and D_{area} measures is much faster and simpler than the usage of standard programming libraries for SVD calculation.

2.4 Processing of sparse matrices

An $n \times n$ matrix is called sparse matrix in which the number of non-zero elements asymptotically equal to $O(n)$. By designing surface parameterization, optimization task is solved and the objective function is minimized. According to the digital model adjacency matrix, linear equation system is constructed, therefore to solve it efficient sparse matrix processing methods saving computer memory and conducting only the necessary calculations are important. One of such methods analyzed in the thesis is Cholesky decomposition of sparse Laplacian matrix $\mathbf{L}$:

$$\mathbf{L} = \mathbf{M}\mathbf{D}\mathbf{M}^{\intercal}.$$

Parameterization optimization problem can be solved by using inverse matrix method to solve linear equations

$$\mathbf{L}^{-1} = (\mathbf{M}^{\intercal})^{-1}\mathbf{D}^{-1}\mathbf{M}^{-1}.$$

This factorization is fixed throughout parameterization algorithm and allows to reduce RAM usage during the optimization.

3 Proposed methods for mesh processing

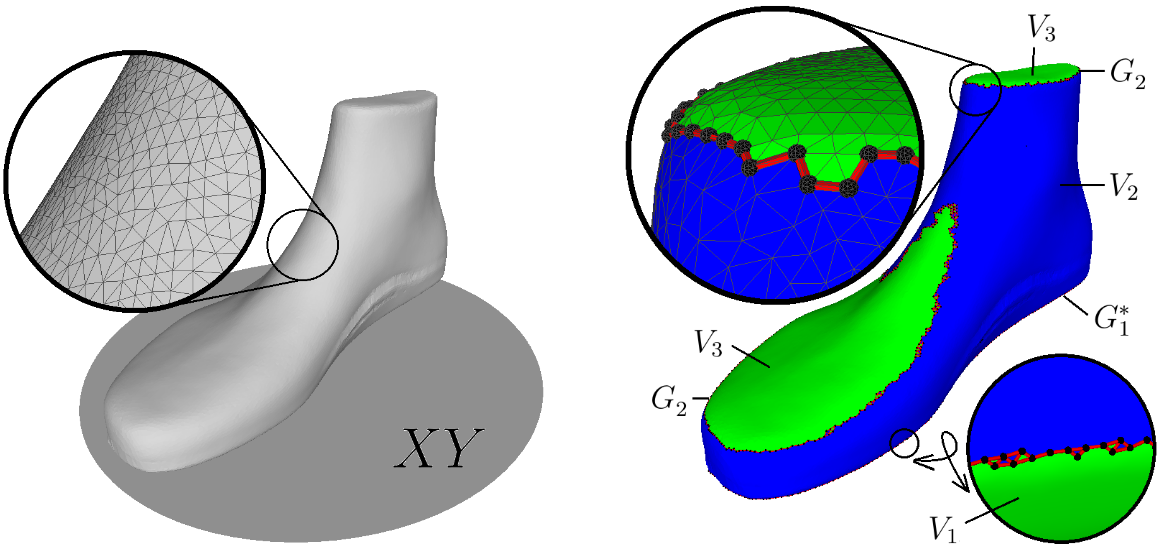
The thesis proposed a new method for segmentation of 3D lasts and well known Catmull-Clark surface subdivision method was improved. The new surface subdivision method enables to construct quadrangular approximation with chosen precision of C^2 surface from a quadrangular input mesh.

3.1 New 3D digital last segmentation method

In this section, we are going present the manual segmentation of lasts, which is used in orthopedics to produce individual products. Main steps of this method:

Step 1: Orientation of 3D lasts

In the digitization process of the lasts with the 3D scanner, they are placed vertically so that sole is upside down, therefore their digital models respectively stand on XY plane (Figure 3a) of Cartesian coordinate system. So we maintain that if the last is differently orientated it should be the basic condition to place it vertically.



(a) The last is placed on XY plane

(b) The intersection of the last segments performed by normal directions

Figure 3: Making last segments

Step 2: The composition of the upper and lower contours

Because the last stands on XY plane its normal triangle sole directions are directed downwards, i. e. XY plane and angles which make up normal directions with that plane belong to interval $[-\frac{\pi}{2}; -\frac{\pi}{4}]$. Suppose $\triangle ABC$ is any triangle of digital last model whose vertices are positioned clockwise outwards of the digital model. Suppose $\overrightarrow{a}(a_1, a_2, a_3) = \overrightarrow{BC}$, $\overrightarrow{b}(b_1, b_2, b_3) = \overrightarrow{CA}$, then $\triangle ABC$ normal equal to

$$\overrightarrow{n}_{\triangle ABC}(n_1, n_2, n_3) = \overrightarrow{n}_{\triangle ABC}(a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1). \quad (5)$$

An angle between normal $\overrightarrow{n}_{\triangle ABC}(n_1, n_2, n_3)$ and normal $\overrightarrow{n}_{XY}(0, 0, 1)$ of XY plane equal to

$$\alpha_{\triangle ABC} = \arcsin\left(\frac{n_3}{\sqrt{n_1^2 + n_2^2 + n_3^2}}\right). \quad (6)$$

Suppose $G = (V, E)$ is the basic graph corresponding to triangulated digital last model. Let's mark the subsets $V_1, V_2, V_3 \subset V$:

$$V_1 = \{v \subset V : A, B, C \in v, -\pi/2 \leqslant \alpha_{\triangle ABC} \leqslant -\pi/4\}, \quad (7)$$

$$V_2 = \{v \subset V : A, B, C \in v, -\pi/4 < \alpha_{\triangle ABC} < \pi/4\}, \quad (8)$$

$$V_3 = \{v \subset V : A, B, C \in v, \pi/4 \leqslant \alpha_{\triangle ABC} \leqslant \pi/2\}. \quad (9)$$

Let's mark $Adj[v]$ the set of neighbor vertices of vertex v and $|Adj[v]|$ – the number of the elements of the set. Respectively by eliminating edges $\{u_1, v_1\}$, $\{u_2, v_2\}$ out of graphs (Figure 3b):

$$G_1 = \Big(V_1 \cap V_2, \big\{\{u, v\} \in E : u, v \in V_1 \cap V_2\big\}\Big), \quad (10)$$

$$G_2 = \Big(V_2 \cap V_3, \big\{\{u, v\} \in E : u, v \in V_2 \cap V_3\big\}\Big) \quad (11)$$

so that $|Adj[u_1]| = 2$ ir $|Adj[u_2]| = 2$ by realizing shortest path Dijkstra's algorithm between vertices $u_1, v_1 \in V_1 \cap V_2$ and $u_2, v_2 \in V_2 \cap V_3$, we receive 2 new paths $u_1 \rightsquigarrow v_1$, $u_2 \rightsquigarrow v_2$. Respectively by adding eliminated edges $\{u_1, v_1\}$, $\{u_2, v_2\}$ to these paths, we receive graphs

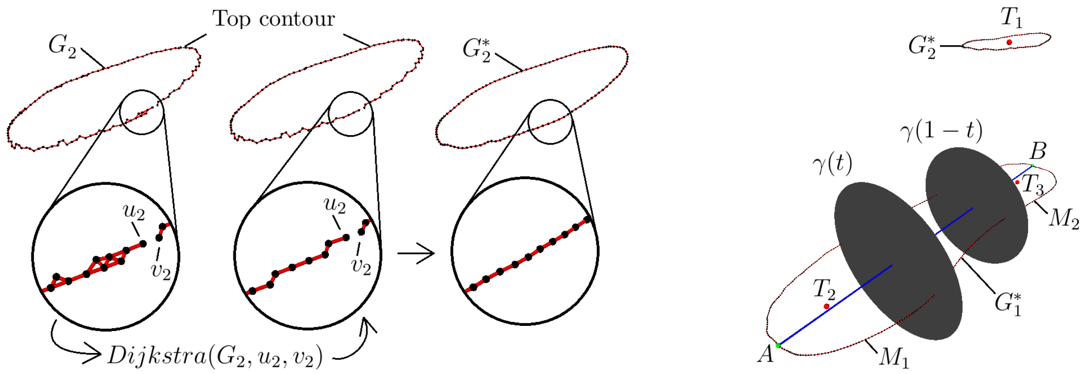
$$G_1^* = \Big(V_1^*, \big\{\{u, v\} \in u_1 \rightsquigarrow v_1 \cup \{u_1, v_1\}\big\}\Big), \quad (12)$$

$$G_2^* = \Big(V_2^*, \big\{\{u, v\} \in u_2 \rightsquigarrow v_2 \cup \{u_2, v_2\}\big\}\Big), \quad (13)$$

complying with the top and the lower contours (Figure 4a).

Step 3: Last segregation by plane

Plane in space is defined by 3 non-collinear points. The main task of segregating the last lengthwise is to receive 2 segments, which when segregated would be least deformed or in other words would be equal in area. On the basis of the experiment and the last



(a) Smoothing of the last's contours (b) Calculation of the lasts plane segregation

Figure 4: Smoothing contours and calculating segregation points

segregation lengthwise performed by experts it was determined that the first from the 3 plane points complies with the arithmetic average of the upper G_1^* contour points. The other 2 points belong to the straight line, which passes lengthwise the lower contour and best complies with it (correlates). One of the possible methods to find such 2 points is based on the composition of the segmentation of the lower contour as well as the calculation of arithmetic average of the points belonging to these segments. Suppose $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$ are 2 points which are furthest apart from each other in respect to lower contour and $C(x_C, y_C, z_C)$ which is any point of the lower contour. Suppose $t \in [0, 1]$ is the parameter showing where AB section would be intersected by plane $\gamma(t)$

$$\gamma(t) : \sum_{k \in \{x,y,z\}} \Big(k - k_A(1-t) - k_B t\Big)\Big(k_A - k_B\Big) = 0,$$

which is orthogonal to primary straight line AB. Then point C is orientated on the left or right side of the plane $\gamma(t)$ by a sign of $\delta(t, C)$ value:

$$\delta(t, C) = \sum_{k \in \{x,y,z\}} \Big(k_C - k_A(1-t) - k_B t\Big)\Big(k_A - k_B\Big).$$

Mark the sets of lower contour G_1^* vertices:

$$M_1 = \Big\{v \in V_1^* : \delta(t, v) > 0\Big\}, \tag{14}$$

$$M_2 = \Big\{v \in V_1^* : \delta(1-t, v) < 0\Big\}. \tag{15}$$

According to heuristics of experiments, the best way is to divide section AB into 3 equal parts and assign $t = \frac{1}{3}$.

M_1 set belongs to subspace restricted by plane $\gamma(t)$, closer to point A, M_2 set belongs to subspace restricted by plane $\gamma(1-t)$, closer to point B (Figure 4b). Therefore the last is lengthwise segregated by plane defined by 3 points $T_j(x_{T,j}, y_{T,j}, z_{T,j}), j = 1, 2, 3$:

$$T_1 = \frac{1}{|V_1^*|} \sum_{v \in V_1^*} v, \quad T_2 = \frac{1}{|M_1|} \sum_{v \in M_1} v, \quad T_3 = \frac{1}{|M_2|} \sum_{v \in M_2} v. \tag{16}$$

Finally the last contour G_3^* we need to find is to be projected in to plane α:

$$\alpha : \begin{vmatrix} x - x_{T,1} & y - y_{T,1} & z - z_{T,1} \\ x_{T,2} - x_{T,1} & y_{T,2} - y_{T,1} & z_{T,2} - z_{T,1} \\ x_{T,3} - x_{T,2} & y_{T,3} - y_{T,2} & z_{T,3} - z_{T,2} \end{vmatrix} = ax + by + cz + d = 0,$$

out of here

$$a = y_{T,1}z_{T,2} - y_{T,1}z_{T,3} - z_{T,1}y_{T,2} + z_{T,1}y_{T,3} + y_{T,2}z_{T,3} - z_{T,2}y_{T,3}, \tag{17}$$

$$b = -x_{T,1}z_{T,2} + x_{T,1}z_{T,3} + z_{T,1}x_{T,2} - z_{T,1}x_{T,3} - x_{T,2}z_{T,3} + z_{T,2}x_{T,3}, \tag{18}$$

$$c = x_{T,1}y_{T,2} - x_{T,1}y_{T,3} - y_{T,1}x_{T,2} + y_{T,1}x_{T,3} + x_{T,2}y_{T,3} - y_{T,2}x_{T,3}, \tag{19}$$

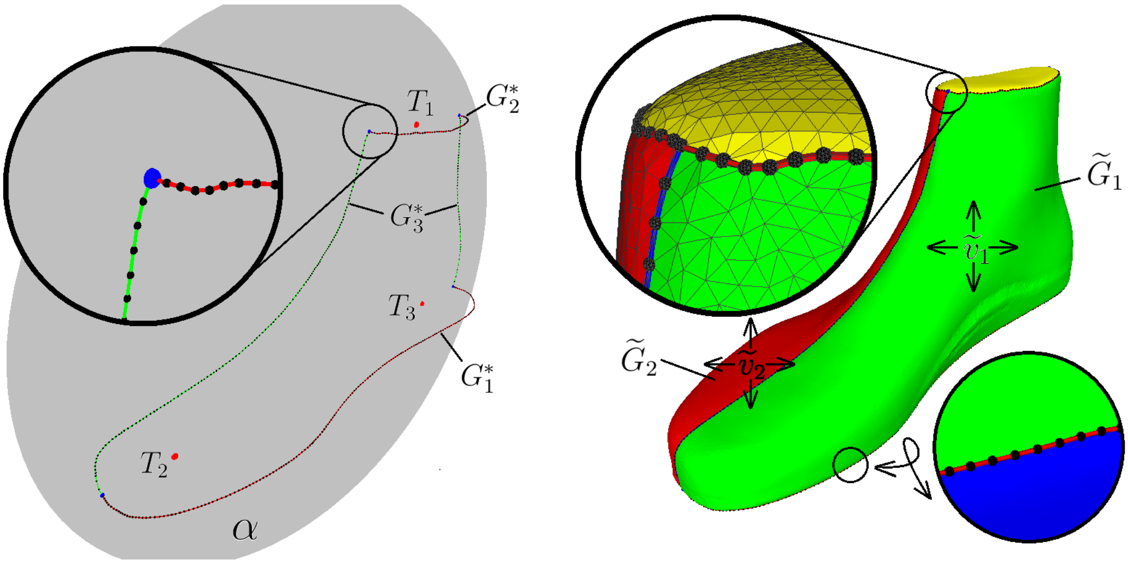
$$d = -x_{T,1}a - y_{T,1}b - z_{T,1}c. \tag{20}$$

Step 4: Making contour of intersection between last and plane

The first vertex v_0^*, belonging to contour $G_3^*(V_3^*, E_3^*)$, is assigned to $v \in V$, which is closest to the plane $\gamma(t)$ according to $\min_{v \in V} d(\alpha, v)$. Here $d(\alpha, v)$ is the Euclidean distance from the point representing the vertex v to the plane. The projection of this point $P_0(x_0, y_0, z_0)$ representing the vertex P_0^* in plane α equals

$$P_0^*(x_0 + ta, y_0 + tb, z_0 + tc), \quad \text{where} \quad t = -\frac{ax_0 + by_0 + cz_0 + d}{a^2 + b^2 + c^2}, \tag{21}$$

here a, b, c, d are parameters of plane α. The next step is to find the closest vertex v_1^* to plane α which is next to vertex v_0^*, and the point P_1, representing vertex v_1^* is projected to α according to (21). Graph G_3^* is comprised from vertices v_0^* and v_1^* so that $G_3^* = (\{v_0^*, v_1^*\}, \{\{v_0^*, v_1^*\}\})$. The following vertices $v_2^*, v_3^*, v_4^* \ldots$ are inserted into graph G_3^* by ith step. The idea of this algorithm is to construct contour G_3^* by adding each time one vertex v_i^* and maintaining the former vertex positioning direction of the vector $\overrightarrow{v_{i-2}^* v_{i-1}^*}$ and taking the nearest vertex v_i^* to the vertex v_{i-1}^*. When v_{i-1}^* and v_0^* coincides, course of algorithm run is suspended and contour G_3^* is deduced (Figure 5a). Each contour needs start and end vertices which are selected by projecting nearest vertices of contours G_1^* and G_2^* into plane α. This algorithm needs to be repeated until the ending vertex is attached to the contour G_3^*.



(a) Last segregation by plane (b) The composition of the final last segments

Figure 5: Making segmentation of 3D last

Step 5: Smoothing of segment contours and result deduction

Aiming to position points gradually from the contours G_1^*, G_2^*, G_3^*, it is possible to apply the formula of moving average value. In general case, it is possible to calculate the coordinates of the vertices using formula

$$P_v^* = \frac{1}{2}P_v + \frac{1}{4}\sum_{u \in Adj[v]} P_u, \tag{22}$$

where v is the vertex of any graph the coordinates P_v, P_v^* of which are recalculated coordinates of a vertex P_v by smoothing contour using moving average formula. Using this method all the coordinates of contour vertices are recalculated and the newly received vertex coordinates are assigned to primary contours G_1^*, G_2^*, G_3^*. By repeating this algorithm several times the contour will be more smoothed out.

Finally, we can get output results. Contours G_1^*, G_2^*, G_3^* restrict the left and right sides of the last which correspond to the molds. It is enough to find vertices $\widetilde{v}_1, \widetilde{v}_2$ on every side belonging to each mold then realize breadth first search algorithm with contour G_1^*, G_2^*, G_3^* restrictions to get segments $\widetilde{G}_1, \widetilde{G}_2$ (Figure 5b).

3.2 New subdivision method for quadrangular mesh

Freeform modeling is used in computer-aided design (CAD) to describe the surface of the 3D geometric object. Most systems today use nonuniform rational B-spline (NURBS) mathematics for modeling. In CAD systems high-quality C^2 surface is defined as a quadrangular surface where the patches are curvature continuous to one another. One of the ways to construct C^2 surface is Catmull–Clark fast and robust surface subdivision method (Figure 6). However using this method impossible to get a result of chosen precision.

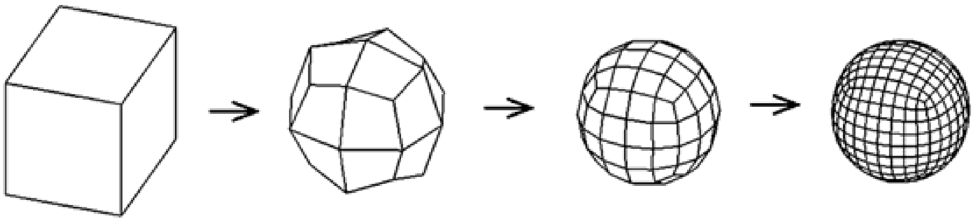


Figure 6: Catmull–Clark surface subdivision

This thesis proposes a new method for quadrangular approximation of C^2 surface where the detail parameter of quadrangular output corresponds to the natural number (Figure 7)

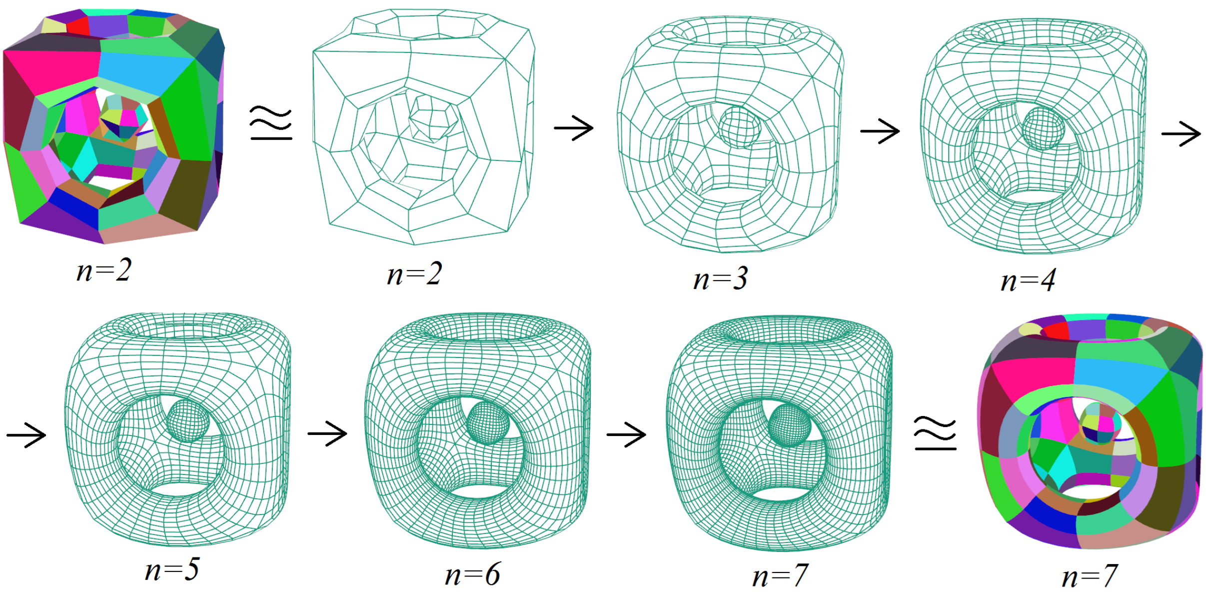


Figure 7: Proposed subdivision

The proposed subdivision method has been tested successfully using different input models and reflection lines of quadrangular output (Figure 8).

Proposed method can be applied for real-time modeling of individual lasts.

4 Experimental investigation

In this section experimental investigation has been made by parameterizing segments of the lasts using isometric and conformal methods. The results were compared with original molds used in the individual footwear industry.

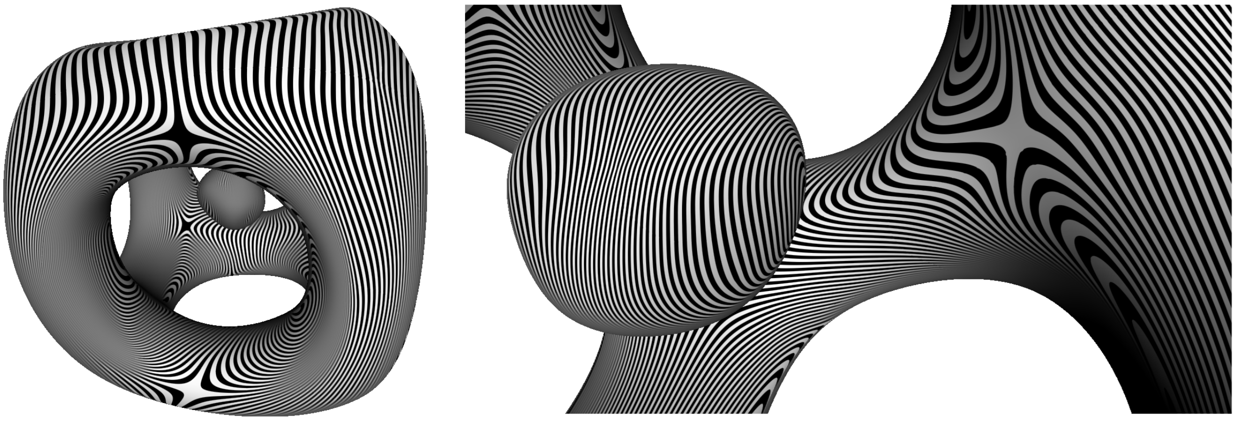


Figure 8: Reflection lines of quadrangular output

4.1 Comparison of molds of the lasts obtained by theoretical and experimental methods

To test flattening algorithms we have chosen five different digitized pairs of shoetrees with correspondent to molds which made by wax foil. After appreciation of correspondent molds of shoetrees, each shoetree was processed by cutting down a sole and head and then cut straight from the middle of a heel to the middle of the second foot's finger. In total 20 fragments were flattened by ABF++, LSCM and ARAP algorithms (table 1).

An error of each fragment was calculated by $\frac{2C}{A+B}$, where A is an area of original mold, B is an area of flattened half-shoetree, C is the maximum area which can be obtained by the intersection of original mold and flattened half-shoetree. A, B and C values were calculated by counting colored pixels of the intersection of molds. We've measured also differences between corner points, called margins (figure 9). The dotted line corresponds to original mold, the solid line corresponds to boundary edges of the flattened surface of shoetree. The values in Margins column in Table 1 coincides to the difference in millimeters between corner points, numbered from 1 to 3 in Figure 9.

Experimental results have shown that average values of relative similarity of flattened half-shoetrees equal to 97.49 %, 97.36 %, 97.70 %, which were obtained by ABF++, LSCM and ARAP algorithms, respectively. According it is advisable to use ARAP algorithm for shoetree flattening. However, algorithms were compared interdependently and according to results there still wasn't suitable evolvents for shoe production. The main thing is that difference between corner points fluctuates middling about 8–9 mm, though the permissible error is only 2 mm. Therefore, future works on this problem can be testing of mixed parameterization algorithms or flattening by the 3D contour.

Table 1: Flattening results using ARAP, ABF++ and LSCM methods

Method			ABF++		LSCM		ARAP	
Shoetrees			$\frac{2C}{A+B}$	Margins	$\frac{2C}{A+B}$	Margins	$\frac{2C}{A+B}$	Margins
1st pair	L.	L.	98.23 %	(6, 4, 7)	98.26 %	(2, 4, 6)	96.40 %	(17, 16, 9)
		R.	93.63 %	(16, 10, 18)	95.25 %	(9, 6, 8)	98.13 %	(13, 14, 4)
	R.	L.	98.37 %	(12, 5, 7)	98.23 %	(8, 13, 4)	96.82 %	(9, 6, 7)
		R.	96.87 %	(3, 19, 4)	97.93 %	(12, 8, 7)	97.46 %	(9, 16, 9)
2nd pair	L.	L.	97.98 %	(4, 12, 3)	97.45 %	(7, 17, 8)	98.82 %	(4, 17, 7)
		R.	96.67 %	(10, 23, 21)	97.92 %	(9, 8, 3)	97.94 %	(14, 10, 13)
	R.	L.	96.02 %	(13, 9, 10)	97.24 %	(3, 13, 15)	96.49 %	(15, 3, 11)
		R.	97.57 %	(16, 16, 12)	96.62 %	(6, 16, 5)	98.35 %	(12, 5, 2)
3rd pair	L.	L.	97.36 %	(6, 16, 3)	98.17 %	(10, 6, 8)	97.60 %	(8, 4, 2)
		R.	98.19 %	(5, 4, 5)	94.00 %	(15, 11, 26)	98.01 %	(12, 13, 28)
	R.	L.	98.47 %	(2, 3, 6)	97.72 %	(12, 9, 10)	98.81 %	(7, 7, 4)
		R.	96.69 %	(21, 6, 7)	96.30 %	(9, 10, 16)	94.50 %	(17, 14, 21)
4th pair	L.	L.	97.30 %	(7, 5, 5)	98.64 %	(8, 8, 6)	97.18 %	(5, 2, 7)
		R.	97.56 %	(7, 5, 15)	97.03 %	(16, 5, 19)	98.42 %	(18, 11, 9)
	R.	L.	98.24 %	(9, 6, 3)	95.92 %	(7, 13, 2)	97.52 %	(25, 6, 25)
		R.	96.88 %	(8, 4, 21)	96.46 %	(22, 11, 25)	96.50 %	(9, 2, 9)
5th pair	L.	L.	99.09 %	(5, 7, 2)	98.82 %	(2, 9, 8)	98.70 %	(8, 4, 6)
		R.	98.59 %	(5, 9, 4)	98.09 %	(13, 10, 13)	98.57 %	(11, 13, 15)
	R.	L.	97.36 %	(9, 7, 3)	98.95 %	(2, 3, 6)	98.90 %	(7, 4, 7)
		R.	98.81 %	(8, 4, 3)	98.26 %	(6, 2, 2)	98.80 %	(5, 2, 9)
Average:			97.49 %	(9, 9, 8)	97.36 %	(9, 9, 10)	97.70 %	(11, 8, 10)

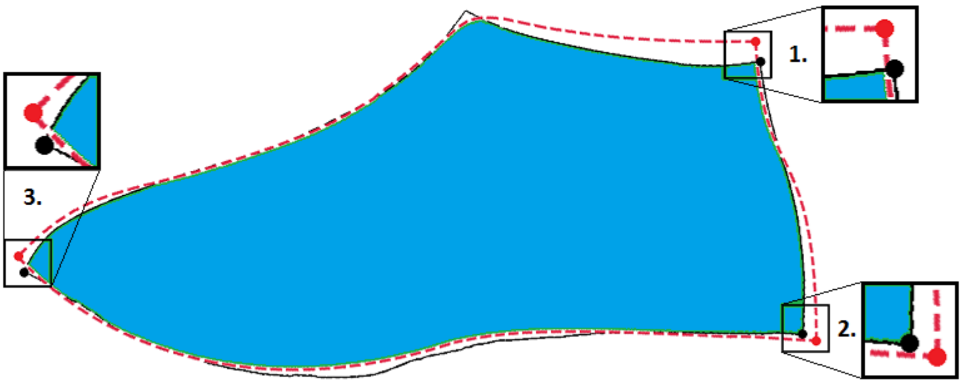


Figure 9: Comparison of molds of shoetrees

4.2 Distortion measures of 3D digital lasts' parameterization

In this section experiments for calculating triangular parameterization and angular D_{angle} and areal D_{area} distortion were performed.

$D_{angle} \in [2; \infty)$ shows the general conformal deformation including local authalic deformation of each triangle, $D_{area} \in [2; \infty)$ shows the general authalic deformation including local authalic deformation of each triangle. These measurements were calculated for all 20

Table 2: Distortion measures using ARAP, ABF++ and LSCM parameterization

Method			ARAP		ABF++		LSCM	
Shoetrees			D_{angle}	D_{area}	D_{angle}	D_{area}	D_{angle}	D_{area}
1st pair	L.	L.	2.00228	2.00197	67.48437	44.53729	73.65813	47.06853
		R.	2.00305	2.00254	193.84691	196.95579	178.14576	41.52362
	R.	L.	2.00250	2.00190	100.65913	44.91062	131.56172	53.34125
		R.	2.00306	2.00265	343.52222	263.49688	74.76634	48.81712
2nd pair	L.	L.	2.00401	2.00332	99.83757	56.97364	101.89899	85.38631
		R.	2.00303	2.00251	111.79438	52.92488	100.31810	129.67130
	R.	L.	2.00250	2.00194	69.26967	37.25687	70.82371	35.46770
		R.	2.00235	2.00174	85.85601	50.33586	74.26528	40.58363
3rd pair	L.	L.	2.00550	2.00486	643.61567	617.98049	432.56382	242.59852
		R.	2.00275	2.00215	472.90667	426.43866	329.35130	333.72100
	R.	L.	2.00317	2.00256	424.32068	176.17047	308.97918	349.21699
		R.	2.00275	2.00220	413.77531	624.16429	706.68631	252.61707
4th pair	L.	L.	2.00180	2.00137	143.98943	58.73077	180.26600	142.80674
		R.	2.00322	2.00273	275.07174	64.31684	202.32383	65.46264
	R.	L.	2.00326	2.00278	194.83687	61.12951	192.95828	84.46660
		R.	2.00236	2.00192	256.06296	85.19470	217.53689	100.51860
5th pair	L.	L.	2.00348	2.00288	128.61280	59.85048	159.24375	52.87524
		R.	2.00305	2.00250	169.29093	58.27662	185.67630	53.01303
	R.	L.	2.00326	2.00258	239.51547	47.49110	313.47793	82.37948
		R.	2.00382	2.00312	135.61499	49.24557	121.14847	43.91769
Average:			2.00306	2.00251	228.49419	153.81907	207.78251	114.27262

lasts' segments which were parameterized using ARAP, ABF++ and LSCM methods (table 2). The following results were received:

1. Using ARAP parameterization the measurements equal

$$D_{angle} = 2.00306,\ \ D_{area} = 2.00251.$$

2. Using ABF++ parameterization the measurements equal

$$D_{angle} = 228.49419,\ \ D_{area} = 153.81907.$$

3. Using LSCM parameterization the measurements equal

$$D_{angle} = 207.782501,\ \ D_{area} = 114.27262.$$

According to mean deformation results in comparison with ABF++ and LSCM method results ARAP method gives the best minimized conformal and authalic distortions, therefore, each triangle rigidity is best approached.

High valued measurement results of conformal parameterization methods ABF++ and LSCM show that by maintaining triangle similarity area of triangles were deformed mostly. Therefore usage of these methods in individual footwear production can result in leather stretching and wrinkle in the most deformed area of parameterization.

5 General conclusions

1. New automatic individual 3D digital last segmentation technology was designed. Using this automatic segmentation technology with isometric ARAP algorithm the result of parameterization partially complies with manual segmentation results by 97.85 %.

2. The experiments show that isometric type surface ARAP parameterization in comparison with ABF++ and LSCM parameterizations best comply with flattened results of individual footwear production using wax foil.

3. The experiments show that application of isometric ARAP parameterization for obtaining 2D digital molds distortion of each locally parameterized triangle is minimized. The general angular and areal deformations equal $D_{angle} = 2.00306$, $D_{area} = 2.00251$. This parameterization is far better than conformal ABF++ and LSCM parameterization results.

4. A new method for quadrangular surface subdivision for obtaining C^2 Bezier patches was proposed. The new surface subdivision algorithm has several features:

   a) Vertices of the output quadrangular surface near the featured vertices are equally spread.

   b) Output quadrangular surface parameter n can be freely chosen from the set $\mathbb{N}$. On the other hand correspondent parameter n can obtain values $2^1, 2^2, 2^3, \ldots$ using a well known Catmull-Clark surface subdivision.

5. The proposed analytic solution of 2×2 singular value decomposition suits for regular, singular and zero 2×2 real matrices. The application of analytical solutions $\sigma_{1,1}$ and $\sigma_{2,2}$ obtained in the thesis enabled to calculate D_{angle} and D_{area} much faster while it is not necessary to calculate $\mathbf{U}$ and $\mathbf{V}$ matrices.

List of publications

The articles published in the peer-reviewed periodical publications:

1. **Sabaliauskas M.**, Marcinkevičius V. An investigation of ABF++, LSCM, and ARAP methods for parameterization of shoetrees. *Science – Future of Lithuania*, Vilnius, Technika, ISSN 2029-2341, 2015, 7(3): 296–299.

2. **Sabaliauskas M.** An algorithm for approximation of Bezier surfaces for special case of quadrangular grid. *Computional Science and Techniques*, Klaipėda, Klaipėda University, ISSN 2029-9966, 2015, 3(2): 454–563.

3. **Sabaliauskas M.**, Mockus J. On the Nash equilibrium in the inspector problem. *Lithuanian Mathematical Journal*, ISSN 0132-2818, 2014, 55: 79–84.

4. Mockus J., **Sabaliauskas M.** On the exact polynomial time algorithm for a special class of bimatrix game. *Lithuanian Mathematical Journal*, ISSN 0132-2818, 2013, 54: 85–90.

The articles published in the conference proceedings:

1. **Sabaliauskas M.**, Marcinkevičius V. Segmentation model for flattening of individual 3D lasts. *EMC 2016: 6th International symposium "Engineering Management and Competetiveness": proceedings*, Kotor, Montenegro, ISBN: 9788676722846, 2016, 325–331.

2. **Sabaliauskas M.** A robust SVD algorithm for ARAP method. *EMC 2015: 5th International symposium "Engineering Management and Competetiveness": proceedings*, Zrejanin, Serbia, University of Novi Sad, ISBN 9788-676722563, 2015, 333–336.

Abstracts in the conference proceedings:

1. **Sabaliauskas M.**, Marcinkevičius V. An investigation of surface deformation using singular value decomposition. *Data analysis methods for software systems: 8th International Workshop*: [abstracts book], Druskininkai, Lithuania, ISBN 9789986680611, 2016, p. 52.

2. **Sabaliauskas M.** Skaitmeninių organinės prigimties kurpalių modelių segmentavimo ir projektavimo į plokštumą tyrimas. *Fizinių ir technologijos mokslų tarpdalykiniai*

tyrimai: šeštoji jaunųjų mokslininkų konferencija: pranešimų santraukos, Vilnius, 2016 m. vasario 10 d. Vilnius, Lietuvos mokslų akademijos leidykla, 2016, 57–60.

3. **Sabaliauskas M.**, Marcinkevičius V. An Application of ARAP Method for Parameterization of Shoetrees. *Data analysis methods for software systems: 7th International Workshop*: [abstracts book], Druskininkai, Lithuania, ISBN 9789986680581, 2015, 45–46.

4. **Sabaliauskas M.**, Marcinkevičius V. Comparison of mods of shoetrees obtained by theoretical and experimental methods. *Data analysis methods for software systems: 6th International Workshop*: [abstracts book], Druskininkai, Lithuania, 2014 [Vilnius], ISBN 9789986680505, 2014, 44.

About the author

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Nestandartinės avalynės gamybos formų paviršių kompiuterinio modeliavimo technologija

Tyrimų sritis

Fizinių objektų perkėlimas į skaitmeninius formatus taikant 3D skenavimo technologijas tapo dideliu žingsniu kuriant naujas gamybos technologijas. Sparčiai augant individualizuotų gaminių rinkai, reikalingi efektyvūs vienetinės gamybos (prekių gaminimo pagal individualius užsakymus) automatizavimo sprendimai. Gamyboje brėžiniai ir skaitmeniniai modeliai sudaro apie 70 % techninių dokumentų, tačiau trūksta techninių sprendimų, leidžiančių efektyviai panaudoti skaitmeninius duomenis būtent individualios gamybos technologijose. Vienas tokių inovatyvių sprendimų – nestandartinių trimačių avalynės gamybos formų paviršių skaitmeninių modelių paviršių išklotinių sudarymo automatizavimas.

Šio darbo tyrimų sritis yra paviršių netiesinės projekcijos metodų analizė, jų taikymo ir gerinimo būdai.

Problemos aktualumas

Vienetinės ir nestandartinės avalynės gamybinių modelių (kurpalių) paviršių išklotinių sudarymas šiuo metu yra rankinis darbas, nes nė viena šiandieninėje rinkoje esanti modeliavimo sistema netenkina gamybos proceso išklotinės tikslumo reikalavimo. Nestandartinių trimačių organinės prigimties formų objektų skaitmeninių modelių paviršių išklotinių sudarymo automatizavimas leistų paspartinti masinės vienetinės gamybos proceso trimačių paviršių išklotinių sudarymą.

Jeigu būtų sukurtas algoritmas, kuris automatiškai iš skaitmeninio kurpalio modelio sudarytų skaitmeninius lekalus, tada nebereikėtų atlikti šių vienetinės avalynės gamybos procesų:

1. Kurpalių paviršių segmentavimo.
2. Šių segmentų išplokštinimo naudojant vaško foliją.
3. Gamybinių brėžinių sudarymo pagal gautų vaško folijos išklotinių kontūrus.
4. Gautų gamybinių brėžinių suskaitmeninimo.

Dėl šio technologinio sprendimo modelio paruošimo gamybai laikas sutrumpėtų apie 30 %. Centralizuotos gamybos atvejais arba užsakant paslaugas nutolusiuose gamybos centruose mažėtų gamybinių modelių (kurpalių) siuntimo išlaidos bei trumpėtų siuntimo laikas. Siuntimo išlaidos sudaro santykinai didelę viso gamybos laiko ir sąnaudų dalį, nes masinės vienetinės gamybos visi gamybiniai modeliai yra skirtingi. Sukurtas metodas leistų spręsti panašias problemas, kuriant analogiškas sistemas kitoms pramonėms šakoms, kurioms yra svarbus trimačių objektų išklotinių gavimas.

Kita svarbi problema – išklotinių sudarymo algoritmų greitaveika. Kiekvieną skaitmeninį modelį apibrėžia gretimumo matrica, kurios eilė sutampa su šio skaitmeninio modelio viršūnių skaičiumi. Sudarant trianguliariojo paviršiaus išklotinę sprendžiamas optimizavimo uždavinys, kurio kiekvienos iteracijos metu apdorojama atvirkštinė gretimumo matrica, taigi iteracijos sudėtingumas tampa kvadratinis. Tačiau įvertinus, kad gretimumo matrica yra retoji matrica, taikant Choleckio dekompoziciją ar kitas matricų faktorizacijas gaunamos retosios matricos. Taigi išklotinių sudarymo algoritmo iteracijos sudėtingumas gali būti gerinamas iki tiesinio.

Tyrimų objektas

Disertacijos tyrimų objektas yra 3D skaitmeninių modelių trikampių ir keturkampių tinklo paviršių apdorojimo ir formavimo algoritmai: paviršiaus segmentavimo, paviršiaus išklotinės sudarymo, trianguliacijos, paviršiaus taškų tolygiojo paskirstymo, parametrinio paviršiaus konstravimo, grafų teorijos algoritmai. Šiems algoritmams ištirti ir realizuoti disertacijoje nagrinėjami globalaus optimizavimo, matricų dekompozicijos ir skaičiuojamosios geometrijos uždaviniai.

Tyrimų tikslas ir uždaviniai

Darbo tikslas – remiantis ortopedijos ekspertų žiniomis sukurti automatizuotą metodą, skirtą vienetinių kurpalių skaitmeninių modelių paviršių segmentavimui ir sudarytų segmentų projektavimui į plokštumą.

Šiam tikslui pasiekti sprendžiami šie uždaviniai:

1. Ištirti 3D skaitmeninių modelių paviršių formavimo ir apdorojimo algoritmus.

2. Ištirti parametrinių paviršių sudarymo metodus, kuriuos būtų galima pritaikyti formuojant parametrinius paviršius iš taškų debesų (skaitmeninių modelių), gautų iš

nuskenuotų kurpalių.

3. Ištirti, kurie trikampių tinklo paviršių išklotinių sudarymo metodai labiausiai minimizuoja paviršių deformacijas.
4. Ištirti retųjų matricų apdorojimo metodus, kuriais greitinamas paviršių išklotinių sudarymas.
5. Įvertinti, kurio iš standartinių paviršių parametrizavimo metodų (autalinių, konforminių ar izometrinių) rezultatas labiausiai atitinka vienetinės avalynės gamyboje rankiniu būdu sudaromus lekalus.
6. Sukurti automatinį kurpalių skaitmeninių modelių paviršių segmentavimo metodą, tinkantį nuskenuotų kurpalių paviršių išklotinėms sudaryti.
7. Palyginti rankinio ir sukurto automatinio kurpalių skaitmeninių modelių paviršių segmentavimo metodų rezultatus.
8. Sukurti programinę įrangą, skirtą nestandartinių avalynės formų paviršių segmentavimui ir šių segmentų paviršių išklotinių sudarymui.

Tyrimų metodai

Disertacijoje suformuluoti uždaviniai sprendžiami analitiniais ir eksperimentiniais metodais. Analizuojant mokslinius ir eksperimentinius rezultatus skaitmeninių modelių paviršių segmentavimo ir išplokštinimo srityse naudojami informacijos paieškos, sisteminimo, analizės ir apibendrinimo metodai. Teoriniai tyrimo metodai taikomi tiriant algoritmų sudėtingumą ir spartinimą. Apibendrinimo metodu įvertinami statistikos metodais gautos išklotinės palyginimo su vienetinės avalynės gamyboje naudojamais lekalais rezultatai.

Mokslinis naujumas

Disertacijos rezultatai:

1. Darbe atlikti eksperimentiniai tyrimai, skirti įvertinti kurpalių lekalų, gautų skirtingais paviršių išklotinių sudarymo algoritmais, atitikimui su lekalais, gautais vienetinės avalynės gamybos metu.

2. Išvesti analitiniai 2×2 matricų skaidymo singuliariosiomis reikšmėmis sprendiniai panaudoti lokalioje ARAP algoritmo optimizavimo fazėje, o Choleckio dekompozicija pritaikyta atvirkštinei retajai Laplaso matricai apskaičiuoti globalioje ARAP algoritmo optimizavimo fazėje. Visa tai leido iš esmės paspartinti izometrinį paviršių išklotinių sudarymo ARAP algoritmą.

3. Sukurtas naujas automatinis kurpalių skaitmeninių modelių paviršių segmentavimo algoritmas, pagal kurį sudarytų segmentų išklotinės apytiksliai 98 % santykiniu panašumu sutampa su vienetinės avalynės gamyboje naudojamais lekalais.

4. Kurpaliams modeliuoti pasiūlytas naujas kvadrianguliariųjų konstrukcijų aproksimavimo Bezjė paviršiais, atitinkančiais parametrinį C^2 paviršių, algoritmas.

Praktinė darbo reikšmė

Tikslus ir spartus skaitmeninių modelių paviršių ir jų fragmentų išplokštinimas teiktų didelę naudą daugeliui pramonės sričių: avalynės, tekstilės, automobilių, baldų, medicinos gaminių gamybos pramonei. Spartus virtualiųjų modelių paviršių plokštinimas paspartintų vienetinių ir unikalių produktų gamybą bei suteiktų daugiau technologinių galimybių ir gamybos proceso efektyvumo ypač toms pramonės sritims, kurios dirba su 3D skenavimo technologijomis ir gamina masinius individualius (angl. *mass customization*) bei vienetinius produktus.

Šioje disertacijoje pasiūlytas kurpalių skaitmeninių modelių paviršių segmentavimo algoritmas kartu su izometriniu paviršių išklotinių sudarymo algoritmu gali būti naudojami vienetinės avalynės gamyboje sudarant paviršių 2D brėžinius (lekalus).

Disertacijoje atliktas darbas, kuriame sukurti praktiškai pritaikomi algoritmai, 6-ojoje LMA jaunųjų mokslininkų konferencijoje „Fizinių ir technologijos mokslų tarpdalykiniai tyrimai“ įvertintas aukščiausiu asociacijos *INFOBALT* apdovanojimu.

Ginamieji teiginiai

1. Sukurtu automatiniu kurpalių skaitmeninių modelių paviršių segmentavimo algoritmu, naudojant izometrinį paviršių išklotinių sudarymo ARAP algoritmą, apskaičiuojamos išklotinės labiau atitinka vienetinės avalynės gamyboje naudojamus lekalus nei išklotinės, apskaičiuojamos rankiniu būdu segmentuojant kurpalių paviršių ir naudojant konforminius ABF++ ir LSCM paviršių išklotinių sudarymo algoritmus.

2. Naudojant izometrinį kurpalių skaitmeninių modelių segmentų išklotinių sudarymo ARAP algoritmą bendra visų trikampių ploto ir panašumo deformacija yra minimizuojama, skirtingai nei naudojant konforminius ABF++ ir LSCM paviršių išklotinių sudarymo algoritmus.

3. Sukurtas ir realizuotas Bezjė paviršių aproksimavimo algoritmas kvadrianguliariesiems įvesties paviršiams pasižymi šiomis savybėmis:

    a) ypatingųjų taškų srityse apskaičiuojamos viršūnės išsidėsto tolygiai,

    b) galima laisvai pasirinkti parametrinio paviršiaus detalumo parametrą $n \in \mathbb{N}$.

4. Pasiūlyti analitiniai 2×2 realiosios matricos $\mathbf{A}$ singuliariosios dekompozicijos $\mathbf{U\Sigma V}$ sprendiniai tinka apskaičiuoti trikampių tinklo paviršiaus deformacijos įverčius lokalioje ARAP paviršių išklotinių sudarymo algoritmo optimizavimo fazėje.

Disertacijos struktūra

Disertaciją sudaro 5 skyriai, literatūros sąrašas ir priedai. Disertacijos skyriai: Įvadas, Skaitmeninių modelių taikymo ir apdorojimo metodų apžvalga, Nauji 3D skaitmeninių modelių apdorojimo metodai, Išklotinių sudarymo eksperimentiniai tyrimai, Bendrosios išvados. Papildomai disertacijoje pateiktas paveikslų, lentelių, algoritmų, simbolių ir ženklų ir santrumpų sąrašas. Visa disertacijos apimtis 130 puslapių, pateikti 36 paveikslai, 7 lentelės ir 7 algoritmai. Disertacijoje remtasi 111 literatūros šaltinių.

Rezultatai ir išvados

1. Pasiūlyta nauja technologija, kuri skirta kurpalių skaitmeninių modelių paviršių dalijimui (segmentavimui). Pagal šią technologiją automatiškai sudaromi kurpalių paviršių segmentai. Juos išplokštinus ARAP algoritmu gaunamas 97,85 % atitikimas su gamyboje naudojamais lekalais.

2. Eksperimentiškai nustatyta, kad izometrinio tipo paviršiaus išklotinių sudarymo ARAP algoritmo rezultatai, lyginant su ABF++ ir LSCM algoritmų gautais rezultatais, labiausiai atitinka sudaromas išklotines vienetinės avalynės gamyboje naudojant vaško foliją.

3. Atlikus eksperimentus paaiškėjo, kad, taikant izometrinį trikampių tinklo išplokštinimo ARAP algoritmą kurpalių segmentų išklotinėms sudaryti, kiekvieno trikampio panašumo ir ploto deformacija minimizuojama beveik iki minimalaus dvejeto: $D_{pan} = 2{,}003062$, $D_{plot} = 2{,}002511$. Tai keliasdešimt kartų geresnis rezultatas už rezultatus, gautus, taikant konforminius ABF++ ir LSCM algoritmus.

4. Darbe pasiūlytas kvadrianguliariųjų paviršių aproksimavimo Bezjė paviršiais algoritmas pasižymi šiomis savybėmis:

   a) Ypatingųjų taškų aplinkose išvesties skaitmeninio modelio viršūnės visada išdėstomos tolygiai.

   b) Tinklo parametrą n galima laisvai pasirinkti iš aibės $\mathbb{N}$, o gerai žinomo Catmullo ir Clarko paviršiaus dalijimo algoritmo parametras n gali įgyti reikšmes $2^1, 2^2, 2^3, \ldots$

5. Pasiūlyti analitiniai 2×2 matricų skaidymo singuliariosiomis reikšmėmis sprendiniai tinka sudaryti dekompozicijai $\mathbf{A} = \mathbf{U\Sigma V}$, kai matrica $\mathbf{A}$ yra neišsigimusioji, išsigimusioji arba nulinė. Taikant disertacijoje išvestus analitinius $\sigma_{1,1}$ ir $\sigma_{2,2}$ sprendinius, paviršiaus suminės deformacijos D_{pan} ir D_{plot} apskaičiuojamos daug greičiau, nes nebereikia apskaičiuoti matricų $\mathbf{U}$ ir $\mathbf{V}$.

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KOMPIUTERINIO MODELIAVIMO TECHNOLOGIJA

Daktaro disertacija

Technologijos mokslai, informatikos inžinerija (07 T)

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COMPUTERIZED MODELING TECHNOLOGY OF UNIQUE
FOOTWEAR SURFACE MANUFACTURING DESIGN

Doctoral Thesis

Technological Sciences, Informatics Engineering (07 T)

Editor Danguolė Straižytė