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Master Thesis

Enhancing Portfolio Efficiency: Funds Separation in Markowitz Optimization Models

Portfelio efektyvumo didinimas: lėšų atskyrimas Markowitzo optimizavimo modeliuos

Author: Juba Hakem

Supervisor: Assoc. Prof. Dr. Andrius Grigutis

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Abstract

Keywords: Mutual Fund Separation Theorem, Risk-Return Trade-off, Modern Portfolio Theory, Mean-Variance Optimization

This study investigates the application of the Markowitz Portfolio Optimization Theory, which revolutionized the field of modern finance by introducing a quantitative approach to portfolio construction. The theory emphasizes the importance of diversification to achieve the optimal portfolio trade-off between risk and return. Key concepts such as the efficient frontier, risk-return trade-off, and portfolio optimization are discussed in detail, providing a comprehensive understanding of how investors can minimize risk while maximizing returns. The report also delves into the Mutual Fund Separation Theorem, which simplifies portfolio construction by demonstrating that any efficient portfolio can be represented as a linear combination of other efficient portfolios. This theorem offers practical implications, such as reducing complexity and transaction costs for investors.

By integrating these principles, the study highlights how Markowitz's theory remains a cornerstone of portfolio management, enabling investors to construct efficient and resilient portfolios that align with their financial goals and risk tolerance.

Santrauka

Raktiniai žodžiai: Savitarpio fondų atskyrimo teorema, rizikos ir grąžos kompromisas, modernioji portfelio teorija, vidutinės dispersijos optimizavimas.

Šiame pranešime nagrinėjama Markowitz portfelio optimizavimo teorija, kuri iš esmės pakeitė moderniosios finansų srities požiūrį į portfelio formavimą, įvesdama kiekybinį metodą. Teorijoje pabrėžiama diversifikacijos svarba, siekiant optimalios rizikos ir grąžos pusiausvyros. Detaliai aptariami pagrindiniai konceptai, tokie kaip efektyvusis frontas, rizikos ir grąžos kompromisas bei portfelio optimizavimas, suteikiant išsamų supratimą, kaip investuotojai gali sumažinti riziką ir padidinti grąžą. Taip pat išsamiai analizuojama savitarpio fondų atskyrimo teorema, kuri supaprastina portfelio formavimą, parodydama, kad bet kuris efektyvus portfelis gali būti išreikštas kaip kitų efektyvių portfelių tiesinė kombinacija. Ši teorema turi praktinių pritaikymo galimybių, tokių kaip portfelio valdymo supaprastinimas ir sandorių sąnaudų mažinimas.

Apibendrinant, pranešimas pabrėžia, kad Markowitz teorija išlieka kertiniu portfelio valdymo akmeniu, leidžiančiu investuotojams sukurti efektyvius ir atsparius portfelius, kurie atitinka jų finansinius tikslus ir rizikos toleranciją.

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(Rf)	Risk-free rate
(β)	Beta
(Rm)	Market return
(Rm -Rf)	Risk premium
R_P	Expected Portfolio Return
R_M	Market Portfolio Return
I_{RF}	Risk-free Rate of Interest
σ_M	Market's Standard Deviation
σ_P	Standard Deviation of Portfolio

1. Introduction

Markowitz Portfolio Theory formulated in 1952 and is “designed to build an optimal investment portfolio for the given level of risk return for an investment or the lowest amount of risk for a given degree of expected returns”. has revolutionized the investment strategy by formulating portfolio investment technique. Before Markowitz it was impractical for investors to constructing portfolio and they were mainly concerned with their particular assets. Markowitz was able to contradict this line of thinking by presenting a way through which portfolio can yield high returns while at the same time minimizing on risks by diversification. His theory laid the foundation for **Modern Portfolio Theory (MPT)**, which remains a cornerstone of investment management today. In finance, most recognized is the Markowitz model, the concept of an efficient portfolio; helps to choose the best portfolio of given securities by considering the portfolio of all possible options of the securities. MPT or mean-variance analysis is an analytical concept used for formulating a diversified efficient portfolio of securities, such that the probability of gain of the total portfolio is maximized for the given level of risk. It can be considered as a formalization and an extension of diversification in investors, the notion asserting that no concentration in one type of financial security is safer than having a variety of such securities. Portfolio concept is the focal concept of the modern capitalist economy and its main idea is that the risk and reward of an asset cannot be evaluated in isolation of other assets. It should be noted that the variance of return (or its transformation: the standard deviation) is employed as measure of risk because doing so offers mathematical tractability when assets are accumulated into portfolios. (Markowitz, 1952) Often, the historical variance and covariance of returns is used as a proxy for the forward-looking versions of these quantities, but other, more sophisticated methods are available (Bauwens et al., 2006).

2. Literature Review:

This is by the advice we have heard to not put all eggs in one basket so that all eggs do not break if the basket falls. With diversification, the risk borne in an investment can be reduced because all money is not put into one investment instrument. The more assets (basket), the lower the risk (Bjork et al., 2011). For instance, in a portfolio, an investor can invest in various products so as to yield different results. Studies have therefore focused on achieving maximum returns and the minimum level of risk (Ardia and Boudt, 2013). Markowitz, in 1952, popularized methods for efficient portfolio selection. For a given portfolio p with a weight vector w, investors typically pursue two main objectives:

i. Maximizing the Expected Return: This is represented by the expected value μp of the portfolio returns.

ii. Minimizing Portfolio Risk: Risk is quantified by the variance σp^2 of the portfolio.

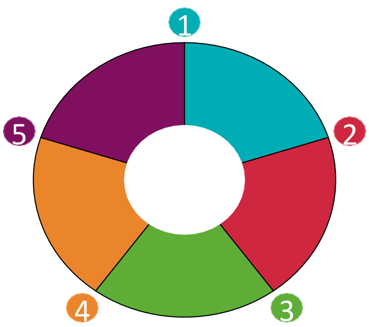
The Markowitz model holds that investors make two choices when it comes to constructing an investment portfolio, namely risk and return. This Markowitz diversification seeks to reduce the risk within the portfolio by adding such assets, and at the same time, does not eliminate any of the portfolio's return. As the correlation between assets in a portfolio reduces, the total portfolio risk (variance) reduces. This is because lower correlation enhances diversification because risk spreads out more comprehensively. It must be noted that despite inclusion of high-risk assets, overall risk of the portfolio can be reduced by proper diversification.

> *"Markowitz explains his approach to diversification as follows:*
>
> *Not only does [portfolio analysis] imply diversification, it implies the*
>
> *"right kind" of diversification for the "right reason." The adequacy of diversification is not thought by investors to depend on the number of*

different securities held. A portfolio with sixty different railway securities, for example, would not be as well diversified as the same size portfolio with some railroad, some public utility, mining, various sorts of manufacturing, etc. The reason is that it is generally more likely for firms within the same industry to do poorly at the same time than for firms in dissimilar industries. Similarly, in trying to make variance [of returns] small it is not enough to invest in many securities. It is necessary to avoid investing in securities with high covariances [or correlations] among themselves."

The asset portfolio optimization theory is one of the fundamental theories of quantitative investment, mainly used to construct optimal asset portfolios to achieve the purpose of maximizing investment returns or minimizing investment risks.

The efficient frontier curve in Markowitz's portfolio theory is one of his significant. Recommendations, expanding the investment options for the investors and some important implications for investment decisions. Li and Ng extended Markowitz's single-stage investment choice model to a multi-stage mean-variance model, taking into account the presence of risk-free assets in the portfolio. They obtain the efficient frontier of the mean-variance model when multi-stage and propose an optimal investment strategy in a dynamic investment environment Duan and Wan (2000) further developed this framework, presenting an optimal dynamic portfolio selection model using a multiperiod mean-variance formulation.



1 **The Concept of Diversification**

2 **Efficient Frontier**

3 **Risk and Return Tradeoff**

4 **Portfolio Optimization**

5 **Capital Asset Pricing Model (CAPM)**

2.1 The Concept of Diversification

The foundation of Markowitz's theory is thus based on the concept of diversification that assumes that the risks in the portfolio can be reduced by distributing the investments over different classes of securities. Diversification takes advantage of the fact that various investments do not always fluctuate in the same manner; their returns may well be negatively related. When these risks-return characteristics are combined, investors can be able to develop a portfolio that minimizes fluctuation and maximizes stability.

For example:

- **Equities**: The equity investments could have high returns, but it has a high-risk factor associated with it.
- **Bonds**: While bonds provide a fairly predictable income, they do not generate a high level of income.
- Both asset categories diversely when included in a portfolio, help to minimize the overall effect of market risks because the losses are likely to be offset by gains on the other assets the asset types within a portfolio reduces the overall impact of market fluctuations, as gains in one asset class can offset losses in another.

Markowitz then reduce this advantage into a numerical standard by examining expected return, variance (variation), and covariance (degree of relatedness) of return of the assets. He even started educating them about the risks for reward concept to enable them get the best out of their portfolios.

2.2 Efficient Frontier

The efficient frontier may be defined as a set of portfolios without superior ones. Portfolio A is called superior to B if A has a greater return B given that the risk of B is the same or larger. It represents the set of portfolios that offer the highest possible expected return for each level of risk. By plotting combinations of assets on a risk return graph, the efficient frontier creates a curve that helps investors visualize the most favorable trade-offs between risk and potential return.

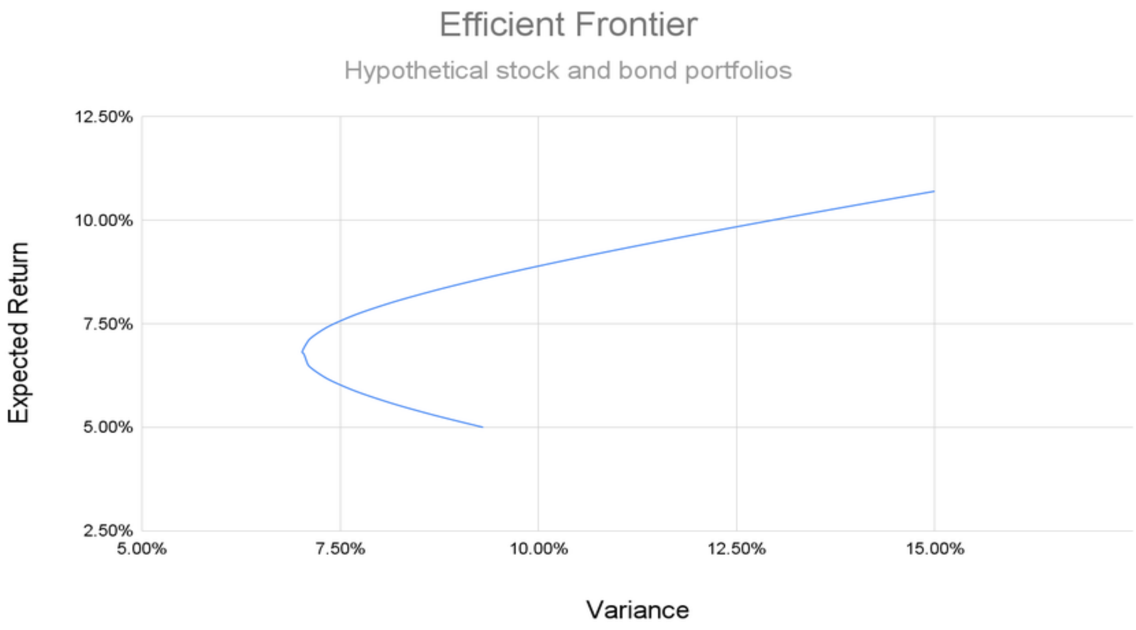


Figure 1 Efficient Frontier: Upper Branch of the Hyperbola

Portfolios that lie on this frontier are considered" efficient" because they maximize returns for their risk levels, while those below the frontier are" inefficient," offering lower returns for a given risk. The efficient frontier enables investors to identify the portfolio that best aligns with their risk tolerance and investment goals, guiding them toward asset allocations that can enhance returns without taking on unnecessary risk. This concept is crucial in portfolio optimization, as it encourages a strategic approach to diversification, balancing the benefits of higher returns with the need to manage and mitigate risk.

2.3 Trade-off between risk and return

In Markowitz's theory, the concept is drawn that return is a gain for bearing risk. Thus, the greater the expected return of an asset the greater the risk linked to that particular asset. But the good news is, by diversification, investors can avoid high risks and at the same reap good returns. This trade-off is depicted in the efficient frontier that shows the various possible portfolios that provide the greatest return given the amount of risk. Those portfolios that are situated below the line of efficient frontier are known as substandard, as they maximize the return at a given level of risk.

Figure 2 Risk and Return Trade Off Graph

Mathematically, the expected return of a portfolio is calculated as a weighted average of its constituent assets: for which location indicates the weight of the asset and its expected return.

The risk (variance or standard deviation) of a portfolio is determined using: where is located the correlation coefficient between assets and this formula highlights the importance of asset correlations: as we saw, the process of building a portfolio from assets with low or negative correlation coefficients minimizes total risk.

2.4 Markowitz Approach: Insights and Practical Applications

The Markowitz approach revolutionized how investors perceive risk and return by introducing a structured framework for decision-making:

Portfolio Optimization an important concept formulated by Markowitz's theory is portfolio optimization. Portfolio management on the other hand concentrates on identifying the right mix of investments within a portfolio that will minimize risk while offering maximum returns. Portfolio optimization aims at determining the mix of assets that enable the portfolio to have the maximum expected return per unit of risk or the minimum risk for a given level of expected return.

The process of portfolio optimization involves solving for the weights of assets that minimize the portfolio's variance (risk) while meeting a target return. This is often formulated as a **mean-variance optimization problem**, as introduced by Markowitz:

Subject to:

- *(the sum of portfolio weights must equal 1).*

i. **Portfolio Diversification:** Divestment redistributes risk over various securities and immunizes against big dips in value arising from poor performances in a given asset. This policy is crucial especially in risky markets where a company is likely to face a range of unfavorable conditions in the market.

   Example: While stocks might get, erode during such an environment of poor economic performance, it is possible to get good returns from government securities or gold.

ii. **Quantitative Analysis:** Markowitz was the first to bring mathematical approaches and measurability into portfolio design. Analyzing asset variance and covariance data allows an investor to decide on proper asset weighting to achieve a target rate of return with a given degree of risk.

iii. **Practical Implications**: Today investors, financial institutions, mutual funds and investment advisors apply Markowitz's principles to build portfolios to diversify risks depending on investors' risk-taking ability and investment objectives. It forms the basis of some of the most widely used today portfolio optimization technologies within the modern world of finance. Extensions like the **Capital Asset Pricing Model (CAPM)** and **Black-Litterman Model** address some of these limitations, further refining the optimization process.

3. Portfolio Optimization

Portfolio optimization is a central objective of Markowitz's theory. It focuses on determining the most efficient allocation of assets within a portfolio to achieve the optimal balance between risk and return. The goal of portfolio optimization is to identify the combination of assets that provides the highest expected return for a given level of risk or, equivalently, the lowest risk for a given level of expected return. Portfolio Selection, laid the groundwork for quantitative investment management by introducing the concept of mean-variance optimization (Markowitz, 1952).

Portfolio optimization has ideal implication in the real world where investors seek to meet their financial goals and objectives with due consideration of risk factors. Some of its practical uses include:

Customized Portfolios: To help minimize exposure to risk, improve returns, or both, investment managers employ optimization procedures to align portfolios with specific clients' characteristics such as risk appetite, time frame, and required returns.

Asset Allocation: Portfolio companies and their managers use optimization models to calculate the optimal proportions of equities and fixed income instruments in the portfolios they manage.

Risk Management: With correlations between assets, the investors can develop styles of investing portfolios, which will be safe from hazards that accompany market risks and economic downturns.

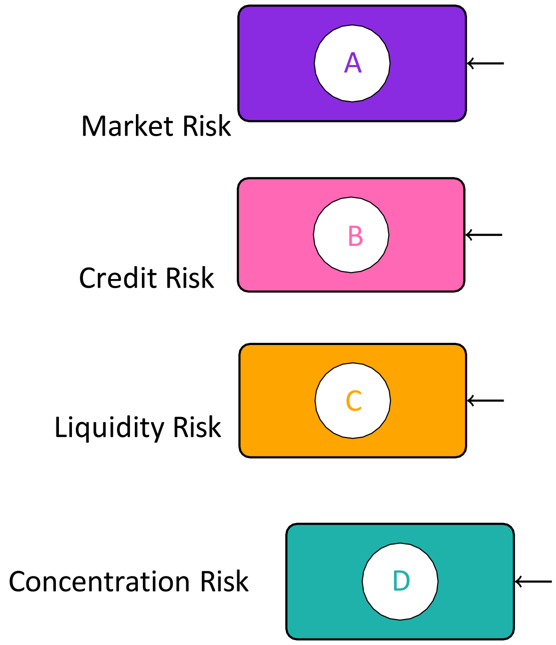
Performance Evaluation: Optimization frameworks specify conditions against which the efficiency of a portfolio in its utilization of risk for returns can be tested against.

3.1 Understanding the risk and the expected return

When it comes to investing, knowing how to balance risk and return is essential. It is what enables investors to make well-informed decisions and construct a portfolio that matches their expected returns and risk tolerance. Making sure you're not just safeguarding your money but also setting it up to develop in a way that suits you is the aim.

1. **The risk** refers to the volatility associated with the expected return on an investment, it is important to evaluate and understand the different types of risks. By analyzing these risks, investors can evaluate the probability of potential losses and take appropriate actions to minimize expo- sure and safeguard their investments.
2. **The Yield** represents the gains or the losses generated by an investment in a specific period and is usually measured in terms of percentage or monetary value.
3. **Diversification** is an investment strategy with potential risk diversification across asset classes. This has the effect of controlling portfolio losses from a large (underperformance) in any one investment. It is possible by not only investing in stocks, bonds and foreign exchange but also various other types of investment, thereby creating a greater level of stability within one port- folio.
4. **Risk assessment tools** are diverse and include standard deviation and value at risk (VaR). These tools give us on overlook on volatility and potential inconvenient of investments helping take the right decision.

3.2 Understanding Investment Risk



3.3 Calculating Expected Returns and Standard Deviation

The expected returns represent the average yield of a certain investment in a defined period of time

3.3.1 Expected return

The expected return of an asset is the weighted average of its potential returns, where the weights correspond to the probabilities of different outcomes.

Mathematically, if an asset has n possible outcomes with returns $R_1, R_2, \ldots, R_n$, and the cor- responding probabilities are $p_1, p_2, \ldots, p_n$, then the expected return $E(R)$ is given by:

$$E(R) = p_1R_1 + p_2R_2 + \ldots + p_nR_n$$

3.3.2 Expected portfolio return

For a portfolio composed of several assets, the overall expected return is a weighted sum of the expected returns of the individual assets.

suppose we have a portfolio with N assets, each with expected returns $E(R_1), E(R_2), \ldots, E(R_N)$.

The expected return of the portfolio is:

$$E(R_{\text{portfolio}}) = w_1 E(R_1) + w_2 E(R_2) + \ldots + w_N E(R_N)$$

Where w_i represents the weight (proportion) of the i-th asset in the portfolio.

$$E(R_{\text{portfolio}}) = 0{,}75 \cdot 10\% + 0{,}25 \cdot 5\% = 7{,}5\%$$

Example

1. Let's consider a portfolio of two assets: stocks with a 15% expected return and bonds with a 7% expected return. If an investor allocates 70% to stocks and 30% to bonds, the expected return of the portfolio is:

$$E(R_{\text{portfolio}}) = 0{,}7 \cdot 15\% + 0{,}3 \cdot 7\% = 12.6\%$$

and the standard deviation measures risk Standard deviation quantifies the variability or dispersion of an asset's returns.

3.3.3 Standard deviation

Standard deviation (σ) measures how far an asset's return deviate from their expected value

Mathematically, for an asset with returns $R_1, R_2, \ldots, R_n$, the standard deviation is calculated as follows:

Standard deviation of the portfolio

i. When combining assets in a portfolio the standard deviation not only depends on volatility but also on in their correlation.

ii. Diversification can reduce the global risk of a portfolio by combining assets with low or negative correlation

iii. diversification reduces the global risk of the portfolio by combining different assets with low or negative correlation.

iv. The standard deviation of the portfolio is given by:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i \, w_j \, \sigma_i \, \sigma_j \, \rho}$$

Where ρ_{ij} represents the coefficient of correlation between assets *i* et *j* and σ_i et σ_j are their respective standard deviations.

Let's suppose that we have a portfolio composed of stocks with standard deviation of 15% and bonds with 8% of standard deviation, If the coefficient of correlation between stocks and bonds is 0.3 and the portfolio weights are 60% stocks and 40% bonds, the standard deviation of the portfolio is:

$$\sigma_{portfolio} = \sqrt{0.6^2 * 0.15^2 + 0.4^2 * 0.08^2 + 2*(0.6*0.4*0.3*0.15*0.08)}$$

4. Finding the optimal Portfolio

In this section we will explain the concept of the efficient frontier that plays a crucial role in optimizing the compromise between the risk and the return in the context of Markowitz Portfolio Optimization. The Efficient Frontier represents a set of portfolios that offer the highest expected return for a given level of risk or the lowest risk for a given level of expected return.

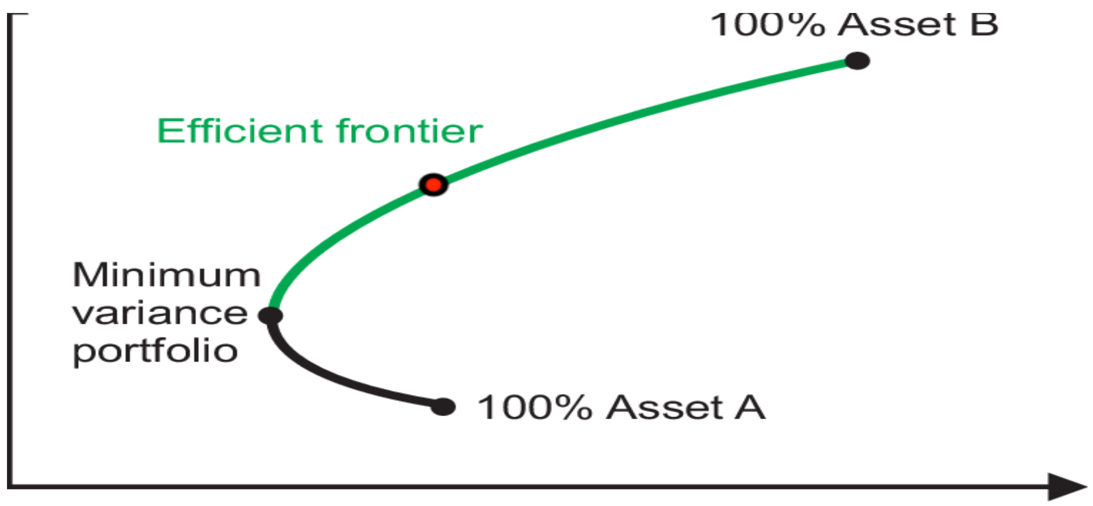


Fig. 3 Two Asset Efficient Frontier

4.1 Insights from various perspectives

- **Modern theory of portfolio:** The efficient frontier is the pillar of modern portfolio theory, developed by Harry Markowitz. This suggests that investors can maximize their return by diversifying their portfolio between different assets to reduce the risks.
- **Risk Return trade-off:** The efficient frontier illustrates the relationship between risk and re- turn. This shows that when an investor seeks higher returns, they must accept higher levels of risk.
- **Optimization of the Portfolio** The efficient frontier illustrates the relationship between

risk and return. That shows when an investor seeks higher returns, they must accept higher levels of risk.

4.2 Diversification of the Portfolio

Benefits of Diversification: The Efficient Frontier highlights the importance of the diversification in the reduction of the risk of the portfolio. By investing in different assets with low or negative correlation, investors can obtain a more effective portfolio that balances the risks and the returns.

- **Asset allocation:** The efficient Frontier guides investors in the optimal allocation of assets in their portfolio. It suggests that a well-diversified portfolio should include a mix of assets with varying levels of risk and return potential.

4.3 Construction of an effective Frontier

- **Estimations of risk and return:** to build an efficient Frontier investors have to estimate expected returns and risks on individual assets. Historical data, statistical models and expert opinions can be used for this purpose.

- **Portfolio Optimization Techniques:** Various mathematical techniques, such as mean variance optimization, can be used to identify portfolios lying on the efficient frontier. These techniques take into account the risk and return characteristics of assets to find the optimal allocation.

Mathematical Representation:

Let $w1,w2,...,wmw1,w2,...,wm$ be the weights of efficient portfolios with expected returns

$\mu0,1,\mu0,2,...,\mu0,m\mu0,1,\mu0,2,...,\mu0,m$.

The two-fund separation theorem shows that any portfolio $w=w1w1+w2w2+\ldots+wmwmw =w1w1+w2w2+\ldots+wmwm$ can achieve the desired expected return by adjusting the weights assigned to these portfolios.

[Mutual fund separation theorem].

Let $w1,w2, . . . ,wm$, $m \in N$, where $wi \in P$ for all

$i = 1, 2, . . . ,m$, be the efficient portfolios whose expected returns are $\mu0,1$, $\mu0,2$, . . . , $\mu 0,m$, respectively. Let $w \in R^m$ be such that

$$\begin{cases} w1 + w2 + \cdot \cdot \cdot + wm = 1, \\ w1\mu0,1 + w2\mu0,2 + \cdot \cdot \cdot + wm\mu0,m = \tilde{}\mu0 \_ \mu\sigma \min. \end{cases}$$

Then the portfolio $w1w1 + w2w2 + \cdots + wmwm$ is also efficient.

5. Results and Discussions

The portfolio analysis with R software was conducted using 10 stocks their names are (BHP - BHP Group Limited, VZ - Verizon Communications Inc, CVX - Chevron Corporation, PG - Procter & Gamble Co, CSL - Carlisle Companies Incorporated, HD - Home Depot, Inc, INTC - Intel Corporation. PFE - Pfizer Inc, CAT - Caterpillar Inc, KO Coca-Cola Company) from Yahoo Finance with data ranging between 2014 and 2023. Optimization was done using RStudio to plot the efficient frontier curve as per the mean-variance model for enhancing the portfolio. The Capital Allocation Line (CAL) was constructed from this line to identify the portfolio with the highest Sharpe ratio. By creating the CAL, the proportion of the stocks that should be included in the portfolio as well as the annual rate of return of the whole portfolio was determined. The stocks average return and their standard deviations (StdDev) are presented in below table:

Stock	Average Return %	StdDev %
BHP	0.959	9.349
VZ	0.326	5.011
CVX	0.899	7.909
PG	0.890	4.527
CSL	1.569	7.161
HD	1.663	6.273
INTC	1.163	8.068
PFE	0.501	6.289
CAT	1.558	8.519
KO	0.753	4.716

Table. 1

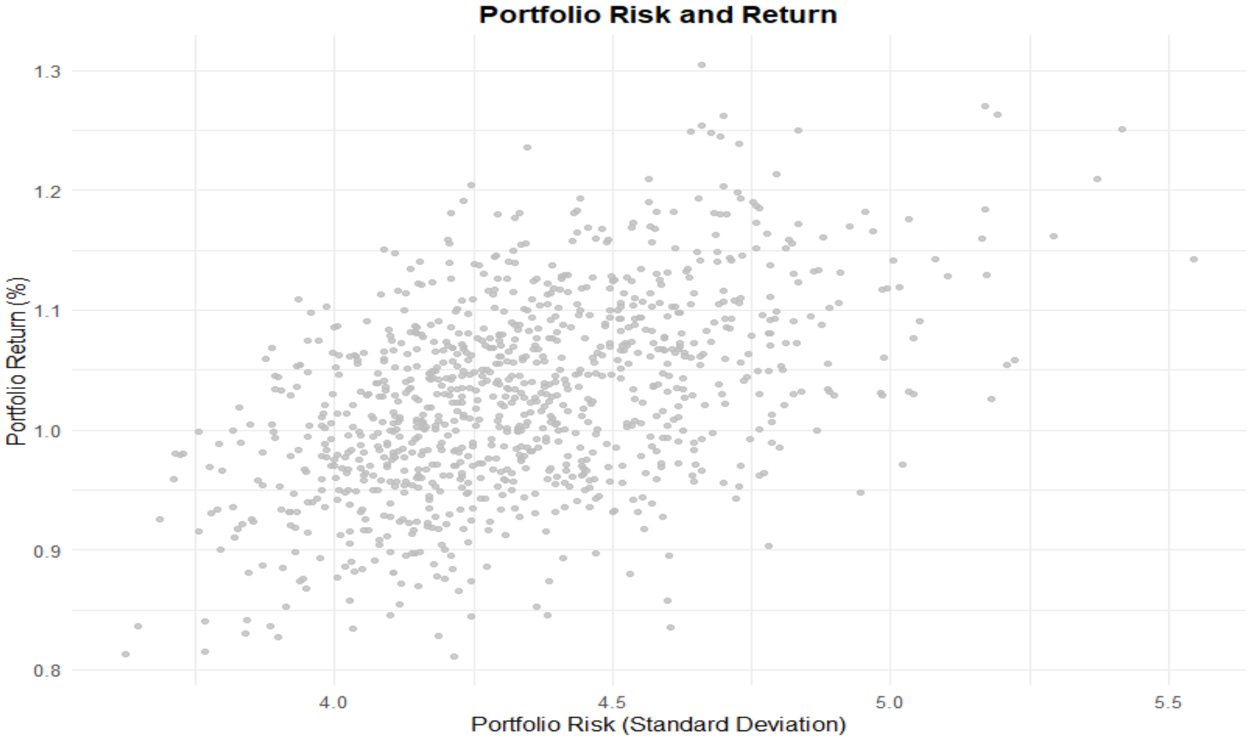


Fig 4 Two Fund Portfolio Risk and Return (Graph)

This bar graph demonstrates the standard deviation of returns for 10 different stocks over the period from 2014 to 2023. Standard deviation is a measure of risk, indicating how much the returns of a stock deviate from its average return. It highlights the positive correlation between risk and return, emphasizing that higher-risk portfolios are expected to yield higher returns.

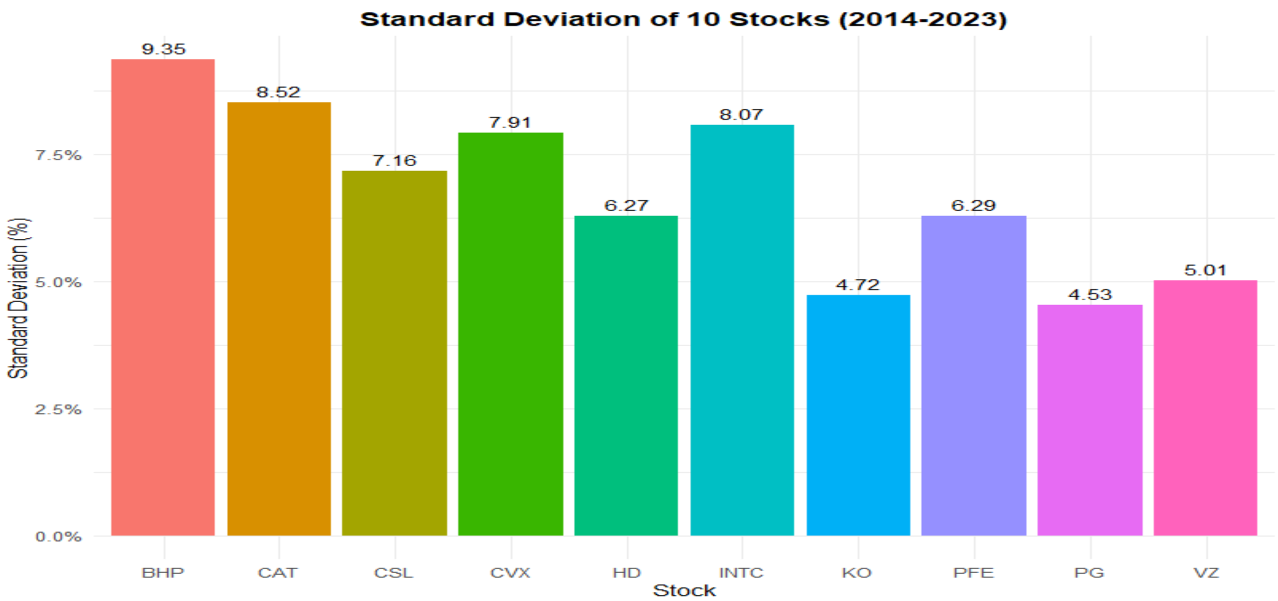


Fig.5 Standard Deviation of Stocks

This bar graph shows the standard deviation of 10 different stocks over the period of 2014 to 2023. Standard deviation is used as a measure of risk, where it is a measure of how much a stock's return will differ from its average return. Each stock is associated with a level of risk, which is represented by the height of each bar. A more volatile (riskier) set of returns is represented by a higher bar. This information can help investors find out how risky different stocks are. For one example, if the standard deviation of "BHP" is greater than for "KO," that indicates that BHP is more volatile (i.e. riskier). - Portfolio Construction: Understanding these risk levels helps investors use the Two Fund Separation Theorem to combine a risk-free asset (Treasury bond) with a portfolio of these risky stocks to achieve the desired risk return profile.

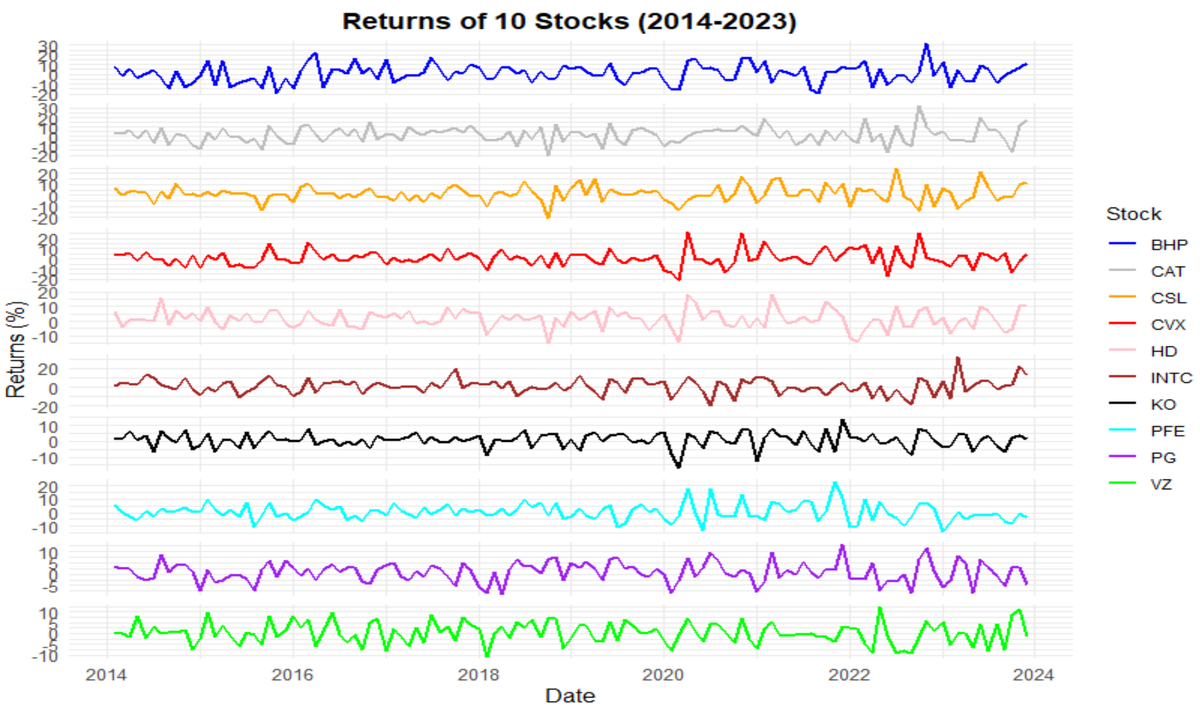


Fig. 6 Returns of Stocks

This line graph shows the same 10 stocks 'historical returns' as indicated by the title for the specified period. It shows the past trends how each stock has behaved in the past, of the stock in the trade. The graph enables the investors to be able to monitor fluctuations in the performance of the equities. For instance, the historical performance of an individual share can in fact be a good investment with an increasing trend of returns: The variations within each line suggest the volatility of revenue streams. High fluctuation stocks could be more actually risky than fluctuation little which shows that companies whose stocks have fluctuation lines are comparatively safer than those with smooth lines.

5.1 Correlation and Diversification:

It is possible for investors to study the behavior of these financial returns by observing how these stocks work. Two such stocks may offer a potential of diversification in case one is included in a portfolio since they mostly mirror opposite movements.

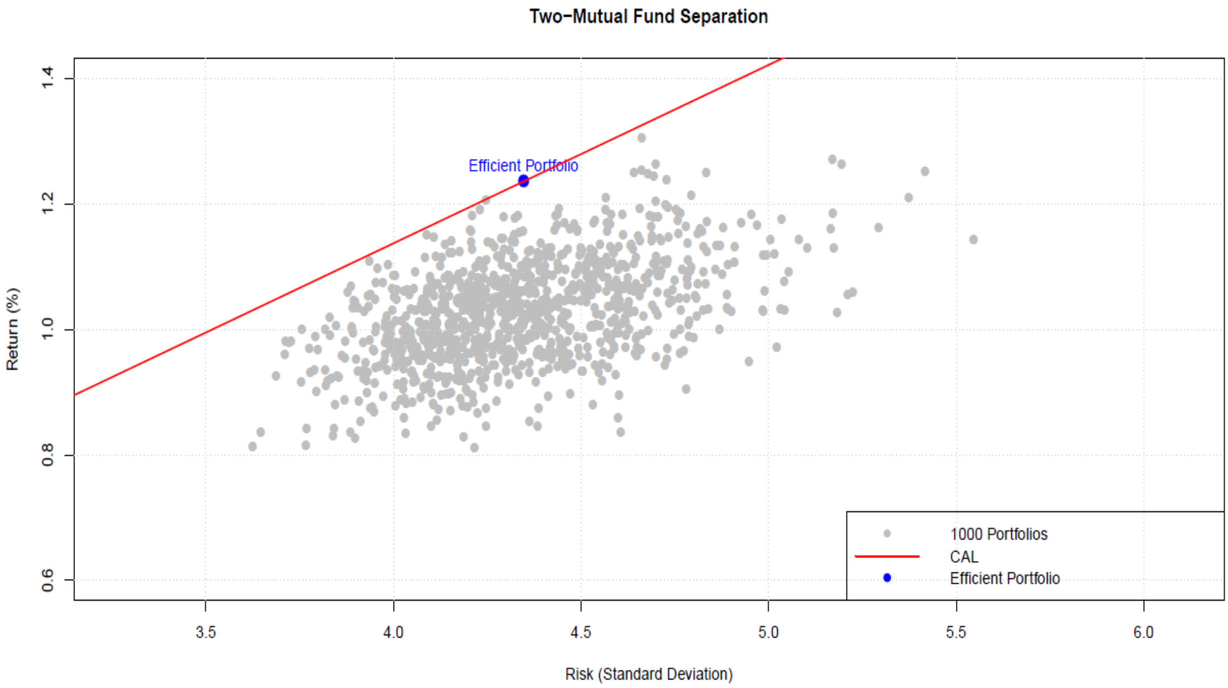


Fig 7 Two Fund Separation Graph

This graph illustrates the concept of mutual fund separation, showing the risk-return pro files of different portfolios. To optimize the portfolio R statistical tool on RStudio was used to apply optimization techniques and generate an efficient frontier curve according to the mean-variance model in Two Fund Separation Theorem. The Capital Allocation Line (CAL) was derived from this frontier to determine the portfolio with the largest Sharpe ratio. By establishing the CAL, the optimal portfolio ratio for each stock and the optimal annualized rate of return for the portfolio as a whole were calculated.

5.2 Efficient Frontier:

Indicates the portfolios that provide the best opportunity to earn more when there is an increase in risk. The blue point labelled "Efficient Portfolio" signifies a particularly optimal portfolio along the CAL. This portfolio offers the best possible expected return for its level of risk, thus illustrating the fundamental goal of Markowitz Portfolio Theory: to maximize returns while minimizing risk

5.3 Capital Allocation Line (CAL):

Shows the risk-reward opportunity for the investor when investing in an assortment of risky securities and a risk-free security. It demonstrates how investors can achieve optimal portfolios by selecting a mix of a risk-free asset and a diversified portfolio of risky assets. The grey dots represent a sample of 1,000 portfolios, showing various combinations of risk and return. The distribution of these portfolios around the CAL emphasizes the importance of diversification, as investors can achieve better risk-adjusted returns through careful asset allocation. Having created 1000 different portfolios, the sharpe ratio on tangent line was calculated at 0.284, and the risk and return values for the optimal portfolio are:

- **Risk (Standard Deviation):** 4.34%
- **Return:** 1.23%

This paper shows that the Markowitz model is useful in analyzing diversified investment portfolios for their optimality. Follow up of RStudio optimization techniques and the use of the utility functions provided adequate means for portfolio performance evaluation.

6. Capital Asset Pricing Model (CAPM) and its Role in Portfolio Optimization

6.1 Understanding the CAPM

Definition. The CAPM is a mathematical framework that quantifies the relationship between an asset's expected return and its risk relative to the overall market. It was developed by William F. Sharpe in the 1960s.

6.1.1 Hypothesis

- Investors are rational and risk averse.
- There is a risk-free asset (for example, government bonds).
- All investors have access to the same information.
- Markets are efficient.

6.1.2 Equation of CAPM:

$$\text{Expected return} = \text{Risk-free rate} + \beta \times (\text{Market return} - \text{Risk-free rate})$$

Where:

- β represents the asset's sensitivity to market movements (systematic risk).

6.1.3 Efficient Frontier Enhancement:

The Markowitz model defines an efficient portfolio line which presents the maximum return given the risk level. CAPM builds on this by the Capital Market Line (CML) which slopes and is tangent to the efficient frontier. They show that introducing risk-free asset into the equilibrium, the CML represents efficient portfolios.

6.1.4 Market Portfolio as Optimal Portfolio:

HOM lies in the imputation that all investors hold both riskless asset and market portfolio that consists of all risky securities. This is consistent with Markowitz diversification concept of reducing syntactical risks.

6.1.5 Risk Decomposition:

CAPM identifies two types of risks; the systematic risk and the unsystematic risk where the systematic risk is associated to the market risk. Compared to the Markowitz model, portfolio optimization in this case deals with the total variance and CAPM points out that systematic risk is the key component that defines expected returns.

6.1.6 Two-Fund Separation Theorem:

CAPM has a connection with the Two-Fund Separation Theorem often associated with Markowitz's model. This theorem asserts that there is a set of portfolios that is optimal for every investor and consists of the risk-free asset and the market portfolio irrespective of the risk tolerance of the investor.

Conclusion

Most people understand that expected returns and standard deviation are important factors in the building of well-balanced portfolios. And with an optimum trade-off between risk and return, an investor can hope to achieve his goals through uncertainty. Diversifications include the risks that are taken into consideration, as well as the major correlation. With the increasing complexities in business, daily operations and investment strategies, the position of Markowitz Portfolio Optimization will become even more important in the future. In a fast-growing global financial markets environment and structures, management of risks and diversification will always be essential in achieving good investment results. The ability to apply this theory across new technologies and different investment models reaffirms the role of the theory in contemporary finance. This adaptation reflects the real-world factors of risks such as market risks, investor psychology and therefore portfolio management is a dynamic process by nature because of this efficient frontier framework which balances risk and return the afterwards created a number of other models including the Capital asset pricing model and the Black-Litterman model. The inclusion of R in these concepts improves both theoretical based studies and practical investment approaches, making the conclusions helpful to both scholars and finance practitioners. Markowitz's groundwork revolutionized investment approaches and can also be considered as the basis for the contemporary theories on finance, including capital asset pricing and utility theories. In this paper, therefore, an emphasis is placed on presenting the arguments of portfolio selection in a stringent manner to benefit both mathematicians and finance practitioners in their understanding of Markowitz's work.

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Appendix

Rstudio Snippet

RStudio

File Edit Code View Plots Session Build Debug Profile Tools Help

Go to file/function Addins

Mutual fund separation code.r | portfolio Final code.r* | portfolio_data | avg_return_df | cov_matri

Source on Save Run Source

```r
library(ROI.plugin.quadprog)
library(doParallel)
library(dplyr)
# Step 1: Download Stock Data
symbols <- c("BHP", "VZ", "CVX", "PG", "CSL", "HD", "INTC", "PFE", "CAT", "KO")

install.packages("quantmod")
library(quantmod)
# Fetch Data
prices <- do.call(merge, lapply(symbols, function(x) {
  tryCatch({
    Ad(getSymbols(x, src = "yahoo", from = "2014-01-01", to = "2023-12-31", peri
  }, error = function(e) {
    message(paste("Error fetching data for:", x))
    return(NULL)
  })
}))

# Remove any NULL columns if any stock failed to download
prices <- prices[, colSums(is.na(prices)) < nrow(prices)]

colnames(prices) <- symbols

# Step 2: Calculate Monthly Returns
returns <- na.omit(ROC(prices, type = "discrete") * 100)

# Calculate Monthly Average Return
monthly_avg_return <- colMeans(returns, na.rm = TRUE)  # Calculate average retur
print(monthly_avg_return)

# Convert to data frame for ggplot
avg_return_df <- data.frame(Stock = names(monthly_avg_return), AverageReturn = m
```

209:1 (Top Level) R Scrip

Results through R program

Data	
adjusted_sharpe	num [1:1000, 1] 0.232 0.263 0.269 0.237 0.207 ...
avg_return_df	10 obs. of 2 variables
cov_matrix	num [1:10, 1:10] 87.41 11.41 35.35 6.72 21.42 ...
portfolio_data	1000 obs. of 2 variables
portfolio_returns	num [1:1000, 1] 0.962 1.061 1.262 0.971 0.967 ...
portplot	List of 11
prices	'xts' num [1:120, 1:10] 34.6 37.3 36.6 38.8 37.3...
returns	'xts' num [1:119, 1:10] 7.74 -1.64 5.91 -3.77 0....
returns_long	1190 obs. of 3 variables
returns_monthly	119 obs. of 11 variables
sharpe_ratios	num [1:1000, 1] 0.232 0.263 0.269 0.237 0.207 ...
std_dev_df	10 obs. of 2 variables
std_dev_plot	List of 11
weights	num [1:1000, 1:10] 0.068 0.119 0.106 0.136 0.198...
Values	
cal_returns	num [1:100] 0.0003 0.0162 0.0321 0.0481 0.064 ...
cal_risks	num [1:100] 0 0.056 0.112 0.168 0.224 ...
colors	chr [1:10] "blue" "green" "red" "purple" "orange" ...
efficient_idx	int [1:1000] 91 717 743 783 671 116 80 239 635 831...
efficient_returns	num [1:1000] 0.814 0.837 0.925 0.959 0.98 ...
efficient_risks	num [1:1000] 3.62 3.65 3.69 3.71 3.71 ...
mean_returns	Named num [1:10] 0.959 0.326 0.9 0.891 1.57 ...
monthly_avg_return	Named num [1:10] 0.959 0.326 0.9 0.891 1.57 ...
monthly_std_dev	Named num [1:10] 9.35 5.01 7.91 4.53 7.16 ...
num_assets	10L