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MASTER THESIS

IŠVESTINIŲ FINANSINIŲ PRIEMONIŲ VERTINIMAS: PAPRASTIEJI IR EGZOTINIAI PASIRINKIMO SANDORIAI	PRICING COMPLEXITIES IN DERIVATIVES: PLAIN VANILLA AND EXOTIC OPTIONS

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INTRODUCTION

Relevance of the topic. Fifty years ago, the willingness to curb financial risks had inspired financial engineers to release to the market in the standardized form, but on another hand, brand-spanking products – simple vanilla derivatives. As the innovation by investors and traders was accepted with goodwill, the different types and complexity derivative products have started to evolve uncontrollably and at a great agility. At the beginning of the 21st century, a famous American business magnate Warren Buffett (2002) has chosen a well-aimed label towards financial derivatives. It states that derivatives are "financial weapons of mass destruction, carrying dangers that, while now latent, are potential lethal". The development of such a careless and unregulated derivative market has led to financial downturns and huge material losses as well as a humiliated reputation and diminished trust in all the eyes of humankind. Even though more than a decade after one remarkable crash of financial markets has passed (i.e. 2007-2008), where derivative instruments played a contributory role by catalysing the financial losses and enormous spread around the markers. Notwithstanding, many legal adjustments and regulations (e.g. Dodd-Frank Act, MiFID II, etc.) around global markets were set in stone to prevent any similar crisis in the future. However, the Covid-19 pandemic, coupled with the global inflation crisis (2021–2023), the Russian invasion of Ukraine, and the subsequent energy crisis (2022–2023), along with escalating conflicts in Israel and ongoing geopolitical tensions, including the indirect confrontation between the United States and China, have made financial markets increasingly difficult to navigate. Despite these challenges, the U.S. market delivered remarkable S&P 500 returns in 2024, with an average gain of 23.31%, thus today users of numerous derivatives, especially option contracts with non-standard, so-called 'exotic' features, try to hedge against financial risks or acquire effortless profit with lack of consideration about the actual characteristics and valuation of the financial instrument itself. Such unintentional usage of options contracts, potential mispricing or misunderstandings in times of financial distress could cause unprecedented issues in the future. Ultimately, it all starts with the challenge of accurately pricing option contracts

The level of exploration of the topic. The exploration of option pricing models has significantly evolved since the inception of the Black-Scholes model in 1973 and continues to grow. Some authors such as Staelen and Hendy (2017), Wang and Yuan (2019), Rao and Manisha (2018), Mink and Weert (2023), Wu et al. (2023), D'Amico and Villani (2021) continue to focus on foundational option pricing models – Black-Scholes, Binomial and Monte-Carlo simulation – in order to evaluate plain vanilla and exotic option contracts in a simple and understandable manner. While another strand of contemporary research such as Kim et al. (2022), Ivascu (2021), Liu et al. (2019), Bekiros and Kouloumpou (2019), Ruf and Wang (2019), Jang et al. (2021), Shvimer and Zhu (2024), continue to expand on these foundational theories by incorporating heavier amount of data as well as Artificial Intelligence (AI) and advanced mathematical techniques, computational machine learning algorithms (e.g, regression trees, random forests, and neural networks) to enhance predictive accuracy and efficiency on the value of option contract. Nevertheless, a third pile or researchers pay attention to the non-standard parameters to be adhered into option valuation such as ESG - Cao et al. (2023), Ford et al. (2022), or macro-economic variables to better reflect market conditions – Dorion (2016), Lai (2017), Hasler and Jeanneret (2022), as well Covid-19 reproduction and economic policy uncertainty – Boswijk et al. (2024). This ongoing development is crucial for both theoretical advancement and practical application in financial markets.

The novelty of the Master thesis. In the theoretical section, the author will provide a concise review of various option pricing models, categorizing them into parametric and non-parametric groups, and will discuss their respective merits and drawbacks. For the valuation of an exotic option contract, specifically a chooser option, the author will employ a modified version of the Black-Scholes-Merton model. This unified approach will facilitate a comparative analysis of premium calculations between a European option and a chooser option, with a particular focus on the factors influencing the pricing of these contracts. Furthermore, the author will conduct a scenario analysis to juxtapose trading strategies with the exotic option, aiming to enhance understanding of its potential performance in an actual market environment.

The problem of the Master thesis. Among derivative instruments, the option contract offers the utmost and indefinite possibilities regarding the applicability to the needs of investors, nevertheless, these advantages complicate the valuation of different types of option contracts. This leads to the issue, because exotic options are far away from easy to evaluate and on how various kinds of this particular derivative segment respond to the changes in certain market conditions and the parameters affecting the value of the option.

The aim of the Master thesis. Evaluate distinctive parameters and their sensitivities, which alter the value of option contracts both plain vanilla and exotic options.

The objectives of the Master thesis:

1. Reveal the perception of derivatives and more specifically, option contracts, it's possible classification as well as identify the parameters that alter the value of this derivative contract and what valuation models are used.
2. Create a suitable valuation model, which is dedicated for computation of option price as well as assessment of sensitivities on the parameters of option price.

3. Using Black-Scholes-Merton model to compute the values of European and chooser options, assess their values based on effect from different parameters and observe their sensitivity using the Greeks.

The methods deployed by the Master thesis. Theoretical analysis of the thesis was performed using a systematic and comparative approach to scientific literature and articles. Practical analyses were conducted using the derivation of partial derivatives (the Greeks) and a modified Black-Scholes option pricing model, which could be suitable for evaluation of both plain vanilla and chooser options.

The description of the structure of the Master thesis. The thesis is arranged into three main parts. The first part covers scientific literature analysis on the derivatives and more specifically, the concept of option contracts, the different types and characteristics of both plain vanilla and exotic options, with a special focus on parameters affecting the value of option contract. The last sub-paragraph in the first part of the thesis presents research methods, models and variables used by scholars in previous studies to determine the impact on option contact's value as well as evaluation of risk sensitivities via partial derivatives (The Greeks). The second part discloses a more detailed description of the research model and presents important aspects based on theoretical knowledge gathered in the first part of the thesis and displays methods of how it would be evaluated. The third part introduces the empirical data analysis assessment of the option's value affecting factors and modelling of appropriate data along with the summary of the findings. Mathematical and statistical calculations were prepared using Excel.

The thesis contains 90 sources, 11 tables and 21 figures.

1. THEORETICAL IMPLICATION OF DERIVATIVE CONTRACT

This section of the thesis would contain a brief overview of derivative contracts and their characteristics, but more narrowed focus would be placed on option contracts, their specifics and differences among themselves as derivative contracts. there is a need to examine financial derivative contracts from the grassroots - their historical approach, main categories and characteristics with the factors affecting the value of option as well as potential possibilities of usage and consequentially leading drawbacks. The sensitivity valuation of factors and pricing of option contracts and the relevant models used to estimate it will be discussed in more depth as it is examined further in the thesis.

1.1. The primary definition of derivative contracts

The derivative is a financial instrument, whose value is dependent upon or derived from values of at least one underlying variable (e.g. asset, index, rate) as well it represents contracts under which the parties agree to transfer underlying assets on or before determined date in the future, according to prearranged price (Hull, 2021). While from the accounting and audit sense International Monetary Fund (1993), financial derivatives are described as not debt instruments, because in general, there is no requirement for principal payment to be repaid. In addition, financial derivatives themselves do not accrue income in the traditional sense as other financial instruments, such as dividends from stocks or interest from bonds (Gueant & Pu, 2015).

Before delving into all the financial derivative contracts, it would be a worthwhile to consider the size of its market – the value of exchange-traded derivatives (futures and options) has amounted to USD 134.40 billion, while outstanding over-the-counter derivatives reached USD 667 trillion[1] . As relevant to the comparison, the current world GDP in 2023 stands for USD 105,44 trillion[2]. Thus, the derivative market itself conducts a significant portion of the global financial landscape, and it is important to understand their working mechanisms behind.

Derivatives as financial contracts have an astonishing and colourful history. The allusions of derivative contracts track even in the early ages – the first example of financial derivatives could be found at Sumer civilization. The Sumerians used clay tokens stored in a clay vessel, and later clay writing tablets, to represent commodities, recording delivery date for goods being traded (CFA Institute, 2021). One of the biggest breakthroughs was reached at Eighteenth century in Japan. There was introduced the first secondary market for commodity derivatives with transferable rice vouchers, which were actively traded and could be settled for cash. Quite soon,

[1] Based on Bank for International Settlements - https://www.bis.org/publ/otc_hy2405.htm
[2] Based on World Bank national accounts data https://data.worldbank.org/indicator/NY.GDP.MKTP.CD

this approach has crossed the Ocean, and the Chicago Board of Trade (CBOT) was established in 1848 (Jarrow & Chatterjea, 2024). Due to the creation of this particular exchange, the trading of derivative contracts became more attractive as well as chaotic - thus leading to the introduction of standardized agreements and regulations in 1865. As it could be anticipated, this revolution has raised the efficiency of the market and reduced transaction costs. The next chapter in the story of derivatives has emerged with the ever-growing middle class and the invention of computing machines, especially with the expansion of personal computers, and telephones in ordinary daily life. Tarbert (2020) has named the period from the 1970s until nowadays as the modern age of derivative contracts. This technological development inspired creativity and experiments with various financial instruments as financial intermediaries have discovered that they could create derivatives of all forms and sell them to corporations, and institutions and manage their risks. The evolution of financial instruments with the timeline is presented in the APPENDIX 1.

Unavoidably, the thin line of safe usage of these newly created financial instruments was crossed. Grima and Eleftherios (2020) have identified, that although the initial intention was to protect against risk, from 2003 to 2007, derivatives became increasingly more complex and started to be used as speculative tools to leverage on more risk to maximize returns, while not always understanding how this instrument fully works, especially when the worst case scenarios materializes in the financial markets. Chance and Brooks (2021) assume the factors, which could lead to a sky-rocking growth of financial derivatives are as follows: increased volatility in asset prices in financial markets; increased integration of national financial markets with the international markets; and derivative market is characterized by relatively low transaction costs as they are designed to serve as a form of insurance and if the price of transaction costs will be relatively high, derivative instruments mostly would not exist.

However, the appetite increases while eating – commercial banks, and large corporations use various derivative contracts immoderately. The market of derivatives has grown tremendously until the hit of the crisis of 2008. Bartram et al. (2022) emphasize, that securitized products, like collateralized debt obligations (CDOs), and credit default swaps (CDS), were among the most popular products, which were leveraged in order to reap the highest possible gain. These particular financial instruments distinguish in the characteristics of difficult pricing. Due to these manipulations and the real estate bubble, we have been witnesses of a crisis, that crashed financial markets a decade ago. The famous phrase “too big to fail” acquired another meaning after the collapse of one biggest investment banks in the United States – Lehman Brothers. Meanwhile, derivatives have become the subject of several disputes in a broader academic community. Many researchers as Duran & Griffin (2021), and Su et al. (2022) have concluded, that derivatives were one of the causes of the recent crisis due to the lack of a healthy leverage rate and legal supervision.

Derivatives as one of the huge financial market segments are categorized along three main dimensions: **<u>The marketplace</u>**. There are generally two groups of derivatives – exchange-traded derivatives, where financial instruments are standardized and traded via specialized derivatives stock exchange or other exchanges, where contracts are standardized, more liquid and credit guaranteed; and over-the-counter (OTC) derivatives, which means that their terms are privately negotiated between two parties, these contracts could be customized but on the other hand, they are less transparent and less liquid. **<u>Underlying asset.</u>** Derivative financial contracts are so versatile, that they could be created using not commonly known financial instruments like equities, fixed income instruments, commodities, various market indices and currency exchange rates, but as per Boyle and McDougall (2019), even odd underlying could be used as basis: *weather*, such as average temperature, the amount of rainfall or natural disasters ("catastrophe bonds"); as well, various events like football games, movie box office receipts or internet data traffic. **<u>Type of relationship between the underlying asset and derivative underlying asset, and complexity.</u>** Boyle and McDougall (2019) divide financial derivatives according to their distinctive characteristics and nature: forward, future, option and swap. Interestingly, the Charter Financial Analyst (CFA) Institute (2021) suggests a bit different division of derivatives into two general categories forward commitments (i.e. forwards, futures, swaps) and contingent claims (i.e. options, callable bonds, etc.). However, the author will introduce distinctive features of the main four types of derivatives.

Following the description of Hull (2021) a **forward** contract is treated as a customized off-exchange contract, which permits participants to buy or sell an underlying asset at a predetermined forward price. Furthermore, forward contracts are bilateral contracts negotiated directly by the seller and buyer (CFA Institute, 2021). From a historical point of view, forward contracts are the oldest form of derivatives and take the biggest piece of the pie in the volumes of the derivatives market. Forward agreements serve primarily hedging purposes and could be written on various types of underlying like commodities (e.g. wheat, oil), financial variables (e.g. exchange rates, stock price) and other assets, even exotic as electricity or weather.

Futures is an exchange contract that enables participants to buy or sell an underlying asset at a predetermined forward price (Cuthbertson et al. (2019). As futures are traded in the exchange, all the terms of the contract are agreed upon and set at the beginning of the contract. Most futures contracts are physically settled, but for example, stock index futures are settled in cash. Moreover, futures exchanges offer contracts on a wide variety of underlying including agricultural commodities, metals, oil, equities and equity indices, interest rates, bonds and foreign currencies. Many researchers often combine futures and forwards under one category, due to their similarity in structure.

Going further, the third contract type in the basic derivative category - **option** – is distinguished in many characteristics. The option contract is an instrument, which grants the right, but not the obligation to buy or sell an underlying asset at a predetermined price (Hull, 2021). The main purpose for the usage of options is the ability to obtain downside protection to exposure even while retaining upside potential. As option contracts will be examined in the practical part, more characteristics will be provided in the next chapter.

Swaps are categorized as complex derivatives and as their complements forwards, are over-the-counter contracts. During a contract of swap, the two parties agree on multiple exchanges of cash flows over several dates in the future (Hull, 2021). Generally, there are different categories of swaps as interest-rate swaps, currency swaps, equity swaps, commodity swaps, and credit swaps. According to statistics provided by the Bank of International Settlements (2018), swaps are the largest component of the over-the-counter derivative market – they consist of around 60 per cent. This particular contract is very useful if swaps are used to transform the nature of exposure and hedge-financing costs and conduct profitable large-scale operations.

Moreover, derivative contracts could be split according to their complexity. There is main two groups: plain vanilla and exotic (Cuthbertson et al. (2019). Plain vanilla derivatives are the oldest ones, the most basic and standardized versions of a financial instrument, while the term exotic derivatives alter the components of traditional financial instruments but makes them more complex security.

Consequently, exotic derivatives could also become plain vanilla, in case of an increase in the general market demand or if the used underlying becomes more common. There is a real-life example with interest rate and currency swaps – these instruments were exotic back in the 1980s, but today they are standard financial tools (Boyle & McDougall, 2019).

Nejad (2022) states, that the dominant view prior to the crisis of 2007–2009 was that financial innovations are beneficial for the financial system. Thus, the emergency of such a variety of non-standard derivative contracts is accelerated by several main reasons:

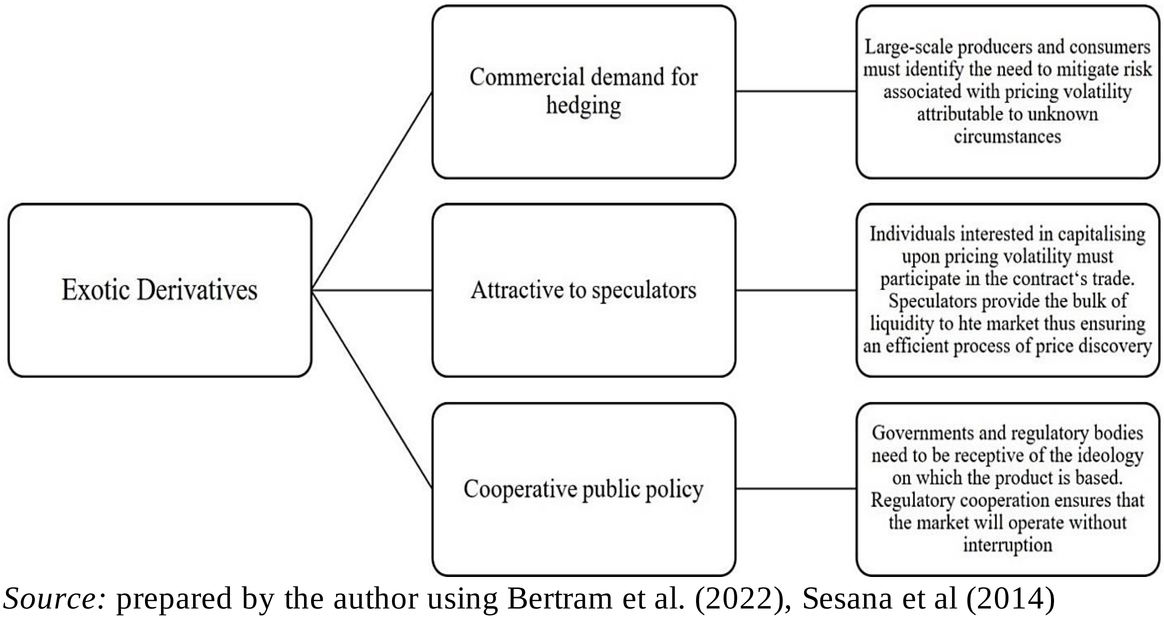
Requirements from customers. According to Cuthbertson et al. (2019), the ability to create a derivative on a new underlying or with different payoff features inspires financial engineers to meet individual customer needs and expectations. Exiting product innovations, as it could be expected, are firstly introduced at the over-the-counter market, because this type of market is more flexible in the means of standards and regulation as well as offer lower costs of initial investment, which even accelerates the production of innovation.

Development in technologies. Since the introduction of electronic trading in 1975, which continues to develop over the years today, traders can be satisfied with the result of the execution of trade in several seconds – there are no more physical papers, etc. However, according to Jayoela (2020), it is expected that blockchain technology in the upcoming years could be integrated into the trade reporting system for OTC derivatives as these derivatives remain in a grey zone.

Profit opportunities. The derivatives market suggests a great possibility to gain profit for all the participants in the market; however, standardised and over-the-counter markets have different pricing. Each transaction that occurs in the exchange is charged by transaction and clearing costs, while the over-the-counter market is wild-west. Thus, financial institutions and brave individuals could receive higher profits. However, as per Cuthbertson et al. (2019), the profitability of exchanges is significantly dependent on the trading volume and the ability to introduce new innovative derivative contracts, which could appeal to new customers and impress founders because it is really important to have a competitive advantage against opponents.

The experience of the crisis has led to – at least a partial reassessment of this view as, nowadays, product innovation in derivatives is often connected with credit events especially credit default swaps (CDS), which have partly caused the financial crisis. However, Bertram et al. (2022) highlighted that it should be noted that the crisis was raised not by derivative contracts themselves but due to the doubtful way of usage as derivatives have mainly satisfied the caprice of speculators for quick financial gain. If financial institutions use financial innovations to enhance risk measurement and risk control, Nejad states (2022) that these innovations could help protect the financial system from adverse shocks.

Figure 1. The classification of option contract



Source: prepared by the author using Bertram et al. (2022), Sesana et al (2014)

Professionals within the derivatives industry state, that there are several inputs crucial to the longevity and viability of a newly established product. These inputs are presented in the Figure 1 above. Corresponding to the Sesana et al. (2014), approximately all exotic derivatives (freight, inflation, weather, property, energy, etc.), categorized based on the underlying asset are tailored-made for risk management, except energy derivatives, which are traded as futures contracts and are used for speculation purposes. However, as per Shylow (2023) analysis, weather derivatives still lack usage and trading volumes in EU markets in comparison with the United States, but ongoing development in legislation for EU corporate sustainability standards potentially should lead to an increase in demand for such tools.

The use of derivatives is a contentious issue – at the same point, they are very attractive, but lots of financial specialists, traders and plain people look at derivatives suspiciously and with fear. Actually, derivative instruments have emerged with the primary purpose of risk management to avoid unexpected or negative outcomes (Hull, 2021).

Table 1. Complexities and challenges associated with derivatives

Factor	**Explanation**
Complexity in pricing and evaluation	According Hull (2021), the problematic situation relies on derivative evaluation itself and price. There are many determinants involved such as variety of different underlying assets, the specifics in the contract (type, maturity), market specifics and fluctuations, behaviour of investors and etc., which very much complicate the valuation of the contracts.
Risk management /High risk rate on the product itself	Even if all due diligence process of banks and companies are performed, derivatives themselves are considered a high risk associated instrument. For example, as per Bartram et al. (2022), there is a possibility of counter-party defaulting on its payment obligations and mostly these losses are unable to be compensated . In addition, absence of leverage is attractive to speculators, because it creates various unpredictable fluctuations in price. But, if there is a combination of leverage, volatility and illiquidity – this could lead to a turmoil and high-risk rates.
Increased need of regulation and transparency	After the global financial crisis in 2008, a lot of legislation as Dodd-Frank act, Directive of Basel III were issued. The main objectives of the below mentioned Dodd-Frank act are to minimize the systematic risk in the usage of derivatives and increase the transparency of the OTC derivatives market (CFA Institute, 2021). Moreover, the Act contains provisions as the Lincoln Rule and the Volcker Rule – they are created to discourage intermediaries from engaging into speculative trading. To the contrary, Le et al. (2018) argues, that imposed regulations after the crisis in the year of 2007 were too severe and to some extent destabilized the economy even more.
Incorrect usage could lead to higher figures of Bankruptcies	Derivatives have leveraged nature and often market players do not evaluate their position adequately, which eventually could lead to bankruptcies. For example, in 2008 Societe Generale Bank due to misuse of unauthorized future contracts had experienced a loss of 7.2 Billion USD (BIS, 2018). Or famous Enron scandal from dot-com bubble in 2001, where have been many different circumstances involved, but this particular bank was also using various complex financial derivatives (commodity swaps) and had some loopholes in accounting part – in total this scandal ended up with roughly 74 Billion USD loss for shareholders (Abdel, 2019).

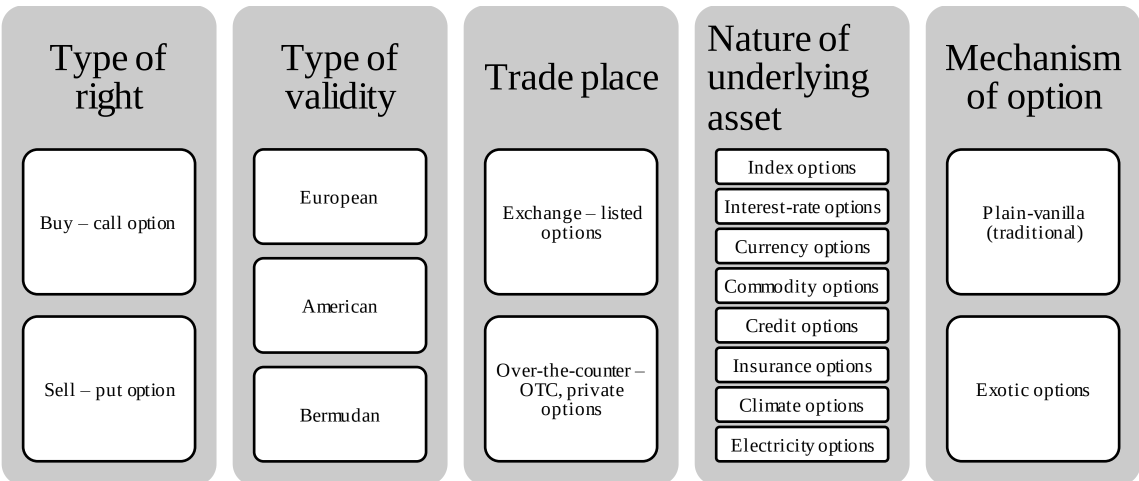
Source: summarized by the author

Despite all offering advantages, derivatives are highly controversial - Chance and Books (2021) elaborate that derivatives are often underestimated and receive negative sentiment because of the inability to use them properly or totally not understanding, how derivatives work in the market. It is crucial to keep in mind that the higher risk does not necessarily bring a higher return, thus before engaging in any activity with derivatives, there is a need to examine the risks, to which derivative contracts are involved to themselves, see Table 1 above. It could be summarized that derivatives possess great advantages in curbing turbulent risks in unpredictable financial markets, and offer a great variety of different forms of contracts, but on the other hand, as per Newall & Weiss-Cohen (2022), especially younger generation should not treat derivatives as a way to gamble in financial markets.

1.2. Plain vanilla versus exotic options and their characteristics

As it was mentioned before, the option contract is very different from obligation-bearing contracts such as futures and forwards. Following Cuthbertson et al. (2019), options have non-linear payoff profiles and there is a requirement that the buyer (or 'the holder') of the option should deposit a cash payment (i.e. premium) to a seller (i.e. writer) of this particular security at the beginning of the contract. Nevertheless, the development and offering in the options market evolve even further with a very complex structure. For Example, the Chicago Board Options Exchange (2024) is planning to introduce a product called 'VX Options', which corresponds to European style options on Cboe Volatility Index Futures, where VX futures are cash-settled futures on the Cboe Volatility Index (VIX® Index) listed for trading on CFE. As Escobar-Anel et al. (2024) displayed in selected GARCH model, 1 year-VIX is the worst to use, while 1 moth VIX – would be the most suitable choice for more flexible option pricing models. For easier observation of option contracts, firstly there is a need to evaluate common characteristics. Options could be classified according to several groupings as trade place, nature of the underlying asset, mechanism of option, bearing right and validity. Classification is presented in Figure 2.

Figure 2. Types of option contract



Source: prepared by the author using Boyle and McDougall *(2019),* Hull (2021)

There are two types of options according to the right: a call obtains the right to buy or a long position in the market, while a put offers the ability to sell or a short position in the market. Moreover, there are three validities/styles of the option, when a holder might exercise the option: only on maturity date (European), at any date before the maturity date (American) or Bermudan (mix of European and American). Consequently, options contracts as segments of derivatives could be traded on Exchange and over the counter due to their flexible features and contracts could be agreed on various types of underlying assets – from plain equity options to complicated weather options. Furthermore, to the contrary of futures and forwards, the holder of the option contract is not necessarily required to perform the agreement if it appears unprofitable for him. However, the agreement counterpart or writer has to fulfil the obligation at any conditions if the holder of the option has decided to exercise his right (Cuthbertson et al. (2019). Following the profit or loss occurrence from the seller and buyer's point of view (where maximum loss of the buyer is premium, while for writer the loss is way riskier due to obligations to execute the option), it can be stated, that the maximum loss for the buyer, in case the option becomes worthless, is the premium amount, while maximum profit depends on the price of the underlying. On the other side, the maximum loss of the seller depends on the profit, which is received by the buyer of the option, and the maximum profit is determined at the beginning of the contract – it is the received premium of the option.

This leads to the observation, that option contracts could be classified also regardless of their worth of exercising or (option contract term – its moneyness). There are three types of moneyness – in-the-money, at-the-money and out-the-money and as it could be expected, the different type of moneyness leads to diversified situations for the Call and Put options in the position of spot and strike price. It is also should be noted, that even if the option ends up in-the-money status, it does not mean that the holder of the option always experiences a profit – there is a need to consider the premium paid upfront.

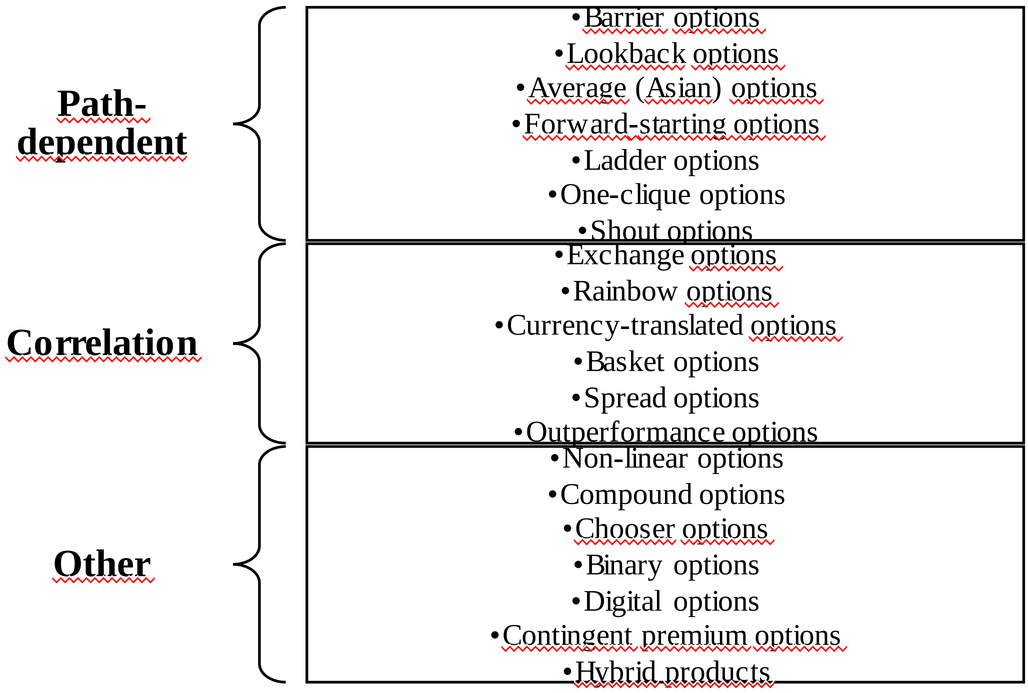
Where plain vanilla options lack flexibility due to their various limitations, there could emerge exotic options, which crack all common features of vanilla options. Generally, an exotic option is a more complex contract than a standard European and American call or put option on a stock, index, foreign currency, commodity or interest rate, and it is often constructed in a way to be tailored to meet specific investment, hedging or risk management needs, thus these contracts are used and traded less in the market (Hull, 2021). Also, as per Cuthbertson et al. (2019), with not frequent trading, exotic options are generally surrounded by less willing counterparties and it leads to less liquidity in the market, higher transaction costs with eventually harder evaluation of the price and actual worth/value of the contract. Another disadvantage of exotic options, corresponding to, is that the underlying market might become manipulative nature if immense amounts of exotic options are traded and approach maturity. However, as per Kim et al. (2022) technological development and expansion in financial modelling help to promote the attractiveness in using them in real-world financial markets, as mostly exotic options and their complexity attract more attention from an academic environment, by manipulating different properties of financial engineers could create a new ‘product’ to meet/hedge from specific market conditions. Actually, by selecting exotic options versus plain vanilla options, the trader could be surprised by tailor-made protection for an accessible price (Bekiros & Kouloumpou, 2019). For example, if a trader has tracked the declining volatility of his specific stocks in his portfolio and buys a barrier option as a solution due to cheaper price, compared with vanilla options. In addition, the writer of exotic options has a wider bid-offer spread and maintains a higher profit margin. Due to the flexibility of exotic options, there could be implemented different strategies from protecting a certain position to direct betting on the movement of the index.

The term ‘exotic’ option has come into force in the financial world since the 1980s. Primarily, the name ‘exotic option’ was applied only to Asian options, which have started to circulate in Tokyo (Poncet & Portrait, 2022). Perhaps, it was done due to some peculiarity of the country, as Japan has appeared quite exotic to the Western World since Ancient times. Nevertheless, according to Pirie (2021), American financial economist Marc Edward Rubinstein was among the first ones, who used the expression ‘exotic options’ in his research paper back in the year of 1990. Exotic options as their name implies involve unusual and difficult-to-value characteristics. Fisher Black (1973) has said - “with derivatives, you can have almost any payoff pattern you want. If you can draw it on paper, or describe it in words, someone can design a derivative that gives you that pay-off.”

Since the advent of exotic options, it is acceptable to call standard vanilla options first-generation, while ‘exotics’ refers to second-generation options. Second-generation options are also called special-purpose options or customer-tailored options, implying that each type of exotic option can somehow serve a special purpose, which standard options cannot do conveniently or cheaply (Jarrow & Chatterjea, 2024). Nevertheless, as mentioned in the introductory part of derivatives, the aftermath of the financial crisis back in 2008 has put a harder regulatory burden on the entire derivative market by trying to standardize it – accelerate transparency so that more trades happen in exchanges and via clearing houses as much as possible. This could be an explanation, of why exotic options came into existence and are used in significant varieties and volumes. Due to adversity, exotic options are usually traded over the counter by institutions and often many individual traders are attached to standard exchanges. As numerous distinctive types of exotic options are now operating mostly over the counter, it is quite hard for scientists to classify them into certain groups according to their characteristics and present a widely accepted and unified classification method. Currently, in scientific literature (Hull, 2021), Cuthbertson et al. (2019), Liu et al. (2024), the most important exotic options explored are as follows: Asian options, barrier options, compound options, chooser options, and lookback options.

According to Poncet and Portrait (2022), exotic options are divided into three groups (Figure 3): path-dependent, correlation and other options. The first group named path-dependent options has the payout, which depends on the price history of the underlying asset over all or part of the life of the option. Correlation options stand for options, whose payoffs are affected by more than one underlying asset These underlying assets could be either of the same or different asset classes. Of course, there are several types of options, which could not be classified as Path-dependent, or Correlation related and has left at the third group with the name 'other.'

Figure 3. Classification of exotic option



Source: prepared by the author using Poncet and Portrait (2022)

In addition, it has to be highlighted, that Nguyen and Wu (2013) in their paper have presented a new idea for the classification of existing exotic options by designing matrix with five traditional pillars, which could provide some insights even for not yet explored exotic instruments.

1) The option is unconditionally activated throughout the life period of the contract and cannot be cancelled before reaching maturity;

2) The maturity of the option can be neither reduced, nor extended;

3) The premium, paid by the buyer to the seller at the beginning of the life period of the option is obligatory and cannot be reimbursed;

4) All the variables of the option contract, namely the underlying asset price, the strike price, and the option price, are written in the same currency;

5) The option has only one underlying asset that is a basic risky asset, with a standard payoff, namely the positive or negative part of the difference between the spot price of the underlying asset and the pre-determined strike price.

When option does not meet at least one of the above-mentioned conditions, it could be categorized as non-traditional and depending on the number of not-fulfilled conditions, an 'exotic degree' could apply, and options could be grouped in certain categories. As there are so many different ways to categorize exotic options, the author has to briefly introduce only the most popular exotic options.

Compound or **Split-free options** give the owner the right, but not the obligation, to purchase another option at a specific price on or by a specific date. Usually, the underlying asset for a standard call or put option is an equity security, whereas for a compound option, it is another option (Bekiros & Kouloumpou, 2019). For the same reason, another tricky feature is that there are two strike prices and two exercise dates, which are applicable for any combination of calls and puts. This is the reason for the division of four compound option types with abbreviations: call on call - CoC; call on put - CoP; put on put - PoP; put on call – PoC. However, trading of compound options requires an advanced level and according to D'Amico and Villani (2021) it is more common to find compound options in currency, fixed-income markets or even in research and development (R&D) investments, as in these fields uncertainty exists regarding the option's risk protection capabilities.

Chooser Options are not path-dependant and give an opportunity for investors to choose whether the option is a put or call during a certain point of the life of the option. As per Martinkutė-Kaulienė (2012), chooser options usually have the same exercise price and expiration date regardless of what decision the holder ultimately makes. However, the holder has the right to exercise the option only on the expiration date, as after selection of put-or-call, the chooser option becomes a simple European option with a single expiration date, strike price and pay-off, which follows the same methodology as a plain vanilla call or put (Durica & Svabova, 2014). If a security is trading above its strike price at expiration, then the call option exercise is generally the most profitable, while if a security is trading below its strike price, if would be more beneficial to have put option. As per Hull (2021), for chooser or "As you liked it" options are two categorizations: simple and complex.

- *Simple* chooser is categorized, if the strike prices of both call and put are the same, just as their expirations;

- *Complex* chooser is an opposite version of simple chooser – call and put do not necessarily have to be of the same strike and maturity.

Nevertheless, even though the chooser option could offer more flexibility than the European option and potentially create more effective risk management, it is still complex to implement as it is mostly traded in over-the-counter markets.

Barrier options are quite similar to vanilla options but are exercised only when the underlying asset hits a selected price level. Barrier options are path-dependent and are typically classified as either knock-in or knock-out (Staelen & Hendy, 2017). *Knock-in* barrier option has no value until the underlying asset has reached a certain price. This type of barrier option is classified into sub-types as *up-and-in* and *down-and-in*. While *knock-out* option would expire worthless if the selected price of underlying will be assessed. These options are classified as *up-and-out* and *down-and-out*. Barrier options are always cheaper than European vanilla options because the payoff of barrier options is subject to additional constraints. Sometimes, a fixed cash rebate is paid out when the barrier condition is not satisfied. In addition, as per Staelen and Hendy (2017) finance engineers have introduced variations of barrier options as a rebate barrier option, a turbo warrant barrier option and a Parisian option.

Asian options are a type of financial derivative where the payoff is determined by the average price of the underlying asset over a specified period, rather than the price at maturity (Hull, 2021). This averaging feature helps mitigate the risk of price manipulation of the underlying asset at the time of settlement. It is totally opposite expression to standard options, as the holder of the Asian option could purchase or sell the underlying asset at the average price.

However, the expression 'average' should be specified in the option contract. Gao et al. (2020) agree, that the average could be calculated differently, for example, according to geometrical or arithmetic averages at discrete intervals, which are usually specified in the option contract. Asian options are usually used, when an individual is concerned about the average exchange rate, commodity price over a desired period of time or the underlying asset is highly volatile in the market. Meanwhile, Gan et al. (2020) employee the machine learning technique in their pricing and conclude, that the Asian contract itself is quite desirable as it could cost cheaper than American options, but not European ones, but there is a limited availability and liquidity of Asian options in the over-the-counter markets.

The lookback option according to Gao and Jia (2021) differs from other exotic options in the way that the holder has the advantage of tracking historical prices of the underlying asset and at the moment of exercise allowed to choose the most favourable price in respect to the time period of the option. This particular characteristic probably has influenced the other name of it – a *hindsight* option. There are two main types of lookback options: *fixed* lookback options offer the ability to select a strike price at the beginning of the contract, but at the time of exercise, the holder of this option could select the most beneficial price, which was achieved over the life of the contract. While *floating* lookback options have a bit different setup – the strike price is set automatically at the maturity of the option for most favourable terms, depending option type.

These particular options are available only in the over-the-counter market and they are quite expensive in comparison with vanilla options – the ability to select the most beneficial price costs against paid premium (Hull, 2021). However, lookback options eliminate the risks associated with the timing of market entry and highly reduce the possibility of worthless option.

The use of vanilla options on various stock markets has introduced a great opportunity for investors to hedge and speculate, as they are versatile and leveraged. Meanwhile, the extension of vanilla options, i.e. exotic options can create synthetic financial instruments, has lower the costs of trading strategies, and better consideration of the matter on taxes. has called exotic derivatives a promising risk management product. Following the idea, exotic options actually are very powerful and widely used tools for hedging financial risk. Nevertheless, due to the complexity of these contracts, the fundamental attributes of many exotic options remain poorly understood or are often misconstrued. While much of the research has concentrated on creating models for pricing exotic options, it is crucial to examine their distinctive features and comprehend them accurately. Moreover, accurately valuing exotic options is typically challenging.

However, as there is no ‘one-size fits all’ concept in the matter of exotic options, the competition in the exotic options market is not as strong as on plain options, because intermediaries, end users, regulators and others have to understand, how plain and exotic options work. Only after careful examination of the option’s parameters, which affect the value of option, real assumptions for disposing of them could be made. Overall, understanding of basic characteristics of plain option contracts could lead to the more extensive creation of complex contracts as exotic options, which could offer an even harder determination of value.

1.3. Factors impacting the price of option contract

In order for the valuation of options would be accurate and precise, there is a need to determine parameters, which make affect for value of an option. The scientists and market practitioners agree, as per Boyle and McDougall (2019), there are six main factors making a different impact for both call and put options.

1. The underlying price. For call options, as the price of the underlying asset increases above the strike price, the option's intrinsic value increases, thereby boosting the option's overall value. Conversely, put options become more valuable when the underlying asset's price decreases below the strike price, as their intrinsic value rises. Further in thesis discusses

the Greek letters as well play the role - The delta of the option quantifies how sensitive the option's price is to a unit change in the underlying asset's price. Additionally, other factors like gamma and Vega also play roles; gamma affects how delta changes as the underlying price moves, and Vega measures the sensitivity of the option's price to changes in the underlying asset's volatility. Therefore, fluctuations in the underlying price can significantly impact the price of the option, influenced by these sensitivity measures.

2. The strike price is specified in the option contract and does not change over time. Essentially, the strike price serves as the threshold that the underlying asset's price must cross for the option to have intrinsic value. For call options, the value increases as the underlying asset's price exceeds the strike price, making the option more valuable due to the greater likelihood of profitable exercise. Conversely, put options gain value when the underlying asset's price drops below the strike price, as this increases the potential payoff at exercise. Thus, the choice of strike price directly influences the risk and potential return of an option, shaping its market value and trading strategy implications.
3. Time to expiration is the time remaining for the option to expire. As the expiration date approaches, the value of the option typically decreases due to time decay. This decay accelerates as the option nears its expiration, reducing the time available for the underlying asset's price to move favourably. For options that are out of the money, this can lead to a rapid decline in value as the likelihood of achieving profitability diminishes. Conversely, more time until expiration generally means a higher premium for the option, as there is a greater chance that the underlying asset's price will move in a favourable direction, increasing the option's chance of profitability.
4. The parameter of the underlying asset’s volatility plays an important role in the price of an option. In simple terms, volatility is the uncertainty of returns. When volatility is high, the premium on the option increases because the greater uncertainty and potential for large price swings enhance the chances of the option ending in the money. Conversely, in low-volatility environments, option premiums tend to decrease as the expected range of price movement narrows, reducing the likelihood of the options expiring in the money. Bernales et al. (2020) while analysing using equity option contracts have identified that investors tend to follow the herd (e.g. if most people sell, others will start to do the same and vice versa), more specifically, as per Fang and Han (2025), when markets experience a high volatility risk (e.g. financial crisis in 2008 or following turmoil after Covid-19 pandemic).
5. The risk-free rate is the amount of return, guaranteed for an investor until the option expires in a no-risk scenario and it influences the discount rate used to present the value of the expected future cash flows from holding the options. In the situation where the risk-free

rate increases, the following call option would increase in value as a higher rate effectively decreases the present value of the expected payoff from exercising the option, while the put option would counteract the opposite action – the value of the put would decrease.

6. Distributed dividends of the underlying also have an effect on the price of the option. When dividends are paid, typically the price of underlying drops by the amount of any cash dividend on the ex-dividend date (Bekiros & Kouloumpou, 2019). For call options, this decrease in stock price can reduce their value, as the likelihood of the option being in the money (stock price above the strike price) diminishes. Conversely, put options may increase in value when dividends are announced, as the drop in the stock price can make it more likely that the put will be in the money (stock price below the strike price). Therefore, expected dividends are an important factor that options traders consider when evaluating the potential profitability of different options strategies.

The author would delve more into the sensitivities of these six main parameters being measured via 'The Greeks' in a further section, but it would be easy to access these pricing factors if they were straightforward and not influenced by other market factors.

Nevertheless, while trying to understand the full picture of what exactly affects option prices, nowadays researchers try to address other aspects as well because the economics are not simple. Of course, general market forces – *supply and demand* – as in any financial instrument naturally could shift prices towards one side or the other. It has been empirically evident that *short-selling stocks* and involving costs could significantly affect option prices (Atmaz & Basak, 2019). *Changes in regulation* or *economic indicators* as well as *important political events* such as presidential elections, and geopolitical events could create higher volatility in the market. Escobar-Anel et al. (2021) analyse the crisis periods and their impact on option prices. While He et al. (2024) in their own created model for European options are willing to adhere to potential liquidity risks and *fluctuations in economic cycles*. Moreover, *market liquidity* also plays a vital role – as more liquid options tend to have narrower bid-ask spreads.

Economic, Social and Government (ESG) factors could also have an influence on option pricing. For example, Cao et al. (2023) find that the effect of general ESG performance of the company is more prominent during the periods when the attention to ESG is higher and for firms that are more subject to ESG-related risks, thus investors to option contracts pay so-called ESG premium against jump risks, but not volatility risks. While Hu et al. (2024) try to incorporate ESG valuation into the recombining binomial trees model calculation and it provides differences

Lastly, *technological changes* or disruptions can also have a profound impact. For instance, the introduction of algorithmic trading and other high-tech trading methods has altered the landscape of options trading, potentially impacting the pricing structure by increasing market efficiency or altering the demand for certain types of options.

Even though derivatives, especially options, are used for different purposes, usually for the replacement or diversification of a portfolio, investors are buying certain options, which will help to expose risks. Typically, the price of an option contract is determined by the powers of the financial market – demand and supply, but there are various models introduced, which could try to get a fair value for the options and evaluate potential loss and gain (Li, 2024). However, there is a need to understand the repercussions of pricing and evaluate potential sensitivities of parameters affecting the price of the option – this matter is enclosed in the next chapter.

1.4. Sensitivity valuation in option contract pricing

In the world of finance, specifically in options trading, understanding how option price is affected and how sensitive it is to various factors, for example, change in the price of the underlying asset, change in volatility, etc., it is a crucial element for risk management, strategic planning as well as for potential income generation or speculation on any market player. This sensitivity or responsiveness is quantified through metrics known as "Option Greeks," or simply "Greeks." Each Greek measures the sensitivity of the option's price to a different variable, such as the underlying asset's price, time, volatility, and the risk-free rate of return (Hull, 2021).

Option Greeks are so named because they are denoted by Greek letters. As per Gao and Jia (2021), each Greek provides a different dimension of risk or sensitivity that can affect the value of an option either in isolation or in combination with others. There are a large number of these measures, including several second-order (e.g. gamma, Vanna) and third-order (e.g. ultima, zomma) partial derivatives, which are often called mixed partial derivatives, because they consider more than one variable. The variety of Greek letters is presented in Table 2.

Table 2. The main Greek letters and their possible extensions

Parameters	**Spot price (S)**	**Volatility (σ)**	**Time to maturity (T)**
Value (V)	Δ Delta	υ Vega	θ Theta
Delta (Δ)	τ Gamma	Vanna	Charm
Vega (υ)	Vanna	Vomma	Veta
Gamma (τ)	Speed	Zomma	Color
Vomma		Ultima	Totto

Source: Hull (2021)

In this paper, the author examines only the most popular and used in-practice, sensitivity measures as delta, gamma, vega, theta, and rho. The brief descriptions of the Greeks relevant in options trading are presented below, while their calculation formulas will be present in the methodology part.

Delta (Δ) is the measure for the sensitivity of the portfolio value to changes in the price of the underlying and it is in the range between -1 to 1 (Hull, 2021). This means that an increase or decrease in the underlying price will consequently be reflected in a proportional increase or decrease in the option value. It also has to be noted, that there is no ideal or uniform delta value to be reached, everything depends on the situation the investor or trade is willing to avoid. That's why traders usually refer to Delta as one of the major risk measures and as well as a *direction* (bullish/bearish/neutral) to follow and capture for a successful portfolio diversification or accurate hedge. Following the logic behind of Bernales et al. (2020), to keep the delta in a neutral position, where the delta would be close to 0, the portfolio should be offset together with preselected assets in the way that in the rise or fall of the asset's price, option contract would counteract the change, example if a hedge is intended for a long position in the underlying asset, the trader might purchase put options (i.e. negative delta) with potential ability to mitigate the loss in the underlying asset.

Positive delta (i.e. in the range 0 to 1) is applicable for call options and literally means a positive correlation of variables: that the option position will rise in value if the stock price rises and drop in value if the stock price falls. From a practical point of view, the delta being equal to 0.7 could be interpreted that an increase of 1 EUR in the underlying stock price would result in a EUR 0.70 increase in the value of the option contract. While *negative delta* (i.e. in the range of -1 to 0) is applied for put options and directly reflects on negative correlation: the value of the option will decrease, if the price of the underlying increases and vice versa. For example, if the delta is equal to -0.3, it means that a decrease of 1 Eur in stock price would lead to a 0.30 Eur increase in the value of the put option. Thus, the delta of an option depends largely on the price of the stock relative to the strike price. Therefore, when the stock price changes, the delta of the option changes (Zhang & Zhou, 2024). Likewise, risk managers were heeded about the changes in delta measure and introduced another index – gamma.

According to Cuthbertson et al. (2019), **Gamma (Γ)** has similarities towards delta, but it is a second-order partial derivative of the option value with respect to the change in the underlying asset value. Or by putting it in simple terms, Gamma indicates on how stable or quickly Delta of an option would change, which is important if investors/traders are trying to maintain a neutral delta setting, especially if Gamma is high. A positive gamma indicates that the delta of long calls will increase, approaching +1.00 as stock prices rise, and decrease, moving toward 0.00 as stock prices fall (Guo F. , 2024). It means that the delta of long puts will become more negative and move toward –1.00 when the stock price falls, and less negative and move toward 0.00 when the stock price rises. Gamma adjusts in quite predictable ways. Positive gamma is a relatively safe position for the options trader, as it will generate the delta, which could be a benefit from movement in the stock (Bernales et al. (2020). However, a position with a negative gamma could be even more dangerous as it ends with a negative delta measure.

The third important measure among risk parameters is **Vega (V).** Gao and Jia (2021) provide the description that Vega marks the sensitivity of the portfolio value to changes in the volatility of the underlying. In addition, this particular partial derivative is named *kappa* or *tau* in some academic literature (DeMarzo et al. (2016). Volatility is always valued positively (i.e. 0 to 1) and as variable itself refers to the potential stability of an underlying asset – if the volatility is quite high, there could be a great change in price applicable for both put and call, which enhances the potential for the option become profitable (Hull, 2021). Thus, higher volatility is associated with higher option premiums for both of the option types. For example, if Vega is equal to 0,15, the increase of 1% of volatility would lead to an increase of 0.15 Eur in the value of the option. Nevertheless, there are different types of volatility –historical, implied etc. However, the measurement of historical volatility (or statistical volatility) can be easily performed, while examination of current volatility (called implied volatility) is one of the great challenges for option pricing (Gueant & Pu, 2015).

The fourth Greek letter – **theta (Θ)** – gauges the sensitivity of the portfolio value to the passage of time (Hull, 2021). Besides, there is a need to keep in mind, that theta measure always is negative. According to Bernales et al. (2020), the cause of negative theta is easily explainable as options could be named as wasting assets, because the option premium consists of a time value, which continuously declines until the expiration of the option. A mathematical example could be as follows: if theta is equal to -0.20, it indicates, that a particular option loses 0.20 Eur of time value per day. Theta alongside rho is not a very compelling measure for investors and traders, based on the grounds that the flow of the time is unstoppable. Nevertheless, theta is greater for high volatility-bearing assets, because volatility increases the premium of the option by increasing the time value of the premium (Chance & Brooks, 2021).

The last measure and one of the lesser discussed by the Greeks are **rho (ρ).** Nevertheless, who plays a significant role and reflects on the sensitivity of the portfolio value to changes in the interest rate (Chance & Brooks, 2021). Even though this particular measure in stable markets is not volatile, there could be some specificities, for example, if the option contract is based on an underlying such as a bond or interest rate swap, the rho indicator should be adhered to more often as well as the general conditions in the market, because macroeconomic indicators as well influence the changes in interest rates as they are set by the Central Bank. Call options have a positive rho, i.e. if interest rates rise, the value of the option contract would increase, while inherently, the put option has a negative rho because raising interest rates would dampen the value of a put option. The mathematical expression could be as follows: if an option has a rho equal to

0.08, it indicates that if the risk rate would have an increase of 1%, the option value would be increasing by 0.08 EUR.

The Greeks of some exotic options, for example, barrier, and chooser options, could be computed using the same equations as for plain vanilla derivatives. Sadly, most exotics require a more precise look not only to their sensitivities, but as well numerical techniques due to their complex nature. Thus, further possible pricing models will be reviewed in the next chapter.

1.5. Models used in determining the price of option contracts

Ultimately, with the introduction of option contracts into financial markets, the valuation of these particular derivatives attracted a lot of attention from researchers as due to its complex nature, there is no straight way to predict the price with utmost certainty. Author decided to introduce option pricing models into a few folds according to their underlying data structure – *Parametric* and *Non-parametric*. Parametric models more rely on a finite set of parameters (e.g. volatility, interest rate, dividend yield, strike price and etc.), easier to calculate, while on the other hand, they are less flexible due to their data input. On the contrary, non-parametric models offer this desired flexibility without a predefined form for the data distribution and could provide a broader range of data outputs closer to reality; however, these non-parametric models are more complex, require much more data input as well as the availability of computational resources. A few exemplary models of each kind will be introduced in upcoming paragraphs.

<u>Parametric models</u>

Following the analysis of the literature, it can be found that grassroots towards option pricing theory could be traced to the model proposed by Louis Bachelier in 1900. **<u>Bachelier model</u>** is famously known as the earliest theoretical model involving stochastic processes, but this particular model possessed an important weakness and had an unrealistic expectation in financial markets – as there was a possibility of negative stock prices due to the assumption of normal distribution of stock price (Glaryrina & Melnikov, 2020), Choi et al. (2022).

Nevertheless, this pioneering approach of Bachelier laid out perfect fundamentals for further researchers, most notably Fisher Black and Myron Sholes, who proposed their own model - **<u>Black-Sholes (BS)</u>** in 1973 and overcame a persistent issue of the predecessor model. The duo of researchers assumed that the stock price rather would be following a geometric Brownian motion, which leads to a log-normal distribution of stock prices. Fischer Black and Myron Scholes could be stated as pioneers in the option pricing area. In 1973, the duo introduced one of the most extensive ideas to the theory of nowadays – the option pricing model, which was named after its founder's last names. The primary purpose of the Black-Scholes model is to determine the likelihood that an option will expire with intrinsic value.

In order this probability could be fulfilled, the model implements several assumptions (Black & Scholes, 1973):

1) There is an efficient market, which means the market movements cannot be predicted;
2) The options are European, which could be exercised only at the date of expiration;
3) The underlying price follows a Geometric Lognormal Diffusion process;
4) The risk-free rate and volatility of change in the price of the underlying asset are known and constant;
5) There are no taxes or transaction costs;
6) There are no cash flows on the underlying security (i.e. there are no distribution of dividends throughout the life of the option).

The key element of Black-Scholes model likewise as in Binomial model is risk-neutrality argument. The theoretical value of options obtains by five input variables: the strike price of an option, the current stock price, time until expiration, the risk-free rate and volatility In addition, it has to be noted, that the prices arrived at by using this model are only indicative. As well, according Hull (2021), it is important to note that the mathematical expression for Black-Scholes formula is divided into two parts:

- The first part, i.e. SN(d1), presents the expected benefits of purchasing the underlying asset – it shows the change in the call premium due to the change in the price of an underlying asset.
- The second part, i.e. N(d2)Ke-rt shows the current value of the exercise price that needs to be paid when the option is exercised.

The original Black-Scholes model (1973) has received many variations and extensions. For example, the *Black 76* model shares a common foundation with the paternal model, but it is designed to specifically price options on future contracts (Wang & Yuan, 2019). Another example is *Merton's Jump Diffusion model (1976)*, which extends the original B-S model by incorporating sudden and large movements in stock prices. Maurya et al. (2024) utilizes this method with a partial integro-differential equation to price both European and American options as they accurately capture volatility smiles and heavy tails. As this particular BS model with some additional alterations will be used in further calculations further description will follow in the second part of the thesis.

American economist Robert C. Merton (1976) was among the first theorists, which had adjusted the Black-Scholes model by eliminating assumption that underlying asset do not distribute dividends. Merton inferred that if the underlying asset is stock, it pays dividends persistently (Hull, 2021). Mink and Weert (2023) provocatively argues that Merton did not invent any formula; instead, he merely introduced a realistic element to an already existing formula to align it with the economics establishment, by eliminating the 'risk' parameter via 'dynamic hedging', but still not close the gap between theoretical values and market prices of option contracts.

The Black-Scholes-Merton method considers as standard model both in terms of approach and in terms of applicability (Rao & Manisha, 2018). Nevertheless, numerous studies indicate that the model often assigns too high a value to deep out-of-the-money calls and too low a value to deep in-the-money calls. Even though as the model is a simplification of reality, the assumption of model is a bit unrealistic due to these reasons:

- the Black-Scholes-Merton (BSM) model suspects that stock prices are distributed log-normally, while returns are normally dispersed. Nevertheless, as a real market practise has showed, the returns have much more of a tendency to exhibit outliers than would be the case if they were normally distributed.
- The BSM model states, that interest rates are constant and known and risk-free rate is used to replicate it in the computation formula. However, practically there is no such a thing as risk-free rate – according Binsbergen et al. (2022), the rate of high-quality Treasury bills is used as an alternative, because T-bills are the closest investment to being risk-free. On the other hand, even treasury rate is not constant and changes over time, especially during periods of high volatility.
- This particular model states, that volatility is perpetual. However, the volatility can be relatively stable for short periods, but it is hardly to believe, that it could remain consistent in the long periods and occurrence of various fluctuations and crashes in the market. In addition, financial markets do not operate continuously – they are closed every weekend and during public or national holidays.
- Geometric Brownian (GB) motion implies that stocks move in a manner such that investors cannot consistently predict the direction of the market or an individual stock. In simple terms, this called as a random walk. According to Parminder and Jasmeet (2020), a random walk means that at any given moment, the price of underlying stock can raise or fall down with the same probability.
- common market features as “volatility smile,” “leverage effect,” or underlying asset discontinuity is not directly incorporated.

Although the Black-Scholes model has introduced significant efficiencies, financial theorists like DeMarzo et al. (2016) argue that its implementation has inadvertently heightened market volatility in stocks and options. This increase is attributed to the surge in trading as investors frequently adjust their hedge positions.

In the contrary to all critics, the Black-Scholes model represents a major contribution to the efficiency of the options and stock markets, and it is still one of the most widely used financial tools among financial professionals in Wall Street and amateur investors. According Choi et al. (2022), besides providing a dependable way to price options, it helps investors understand how sensitive an option's price is to stock price movements. Thus, it aids investors in maximizing their portfolio efficiency by providing a method to calculate hedge ratios and implement portfolio insurance more effectively.

Even though Black-Scholes had an initial intention to be used a theoretical estimate of the price of plain vanilla European call options on non-dividend-bearing stocks, in the last decades it has been adapted to price other financial instruments. As well, the model itself was used as a foundational stone for event other model of pricing to be developed by expanding or breaking the laid original assumptions. models Regardless of various arguments that the Black-Scholes Merton model is mostly applicable to vanilla options, a lot of researches as Devreese et al. (2010), Staelen and Hendy (2017), Rao and Manisha (2018), Hu and Gan (2018), Wang and Yuan (2019) were conducted using this particular model for pricing exotic option as Asian, forward starting, double-barrier options and etc.

As practical part of the research will focus both on European call and put options as well as on chooser option, Black-Scholes-Merton model was adapted by Mark Rubinstein in 1991. According to Wu et al. (2023), the modifications of the Black-Scholes-Merton model were possible, as simple choosers have the same strike price and time to maturity for the call and the put. However, researchers more often try to examine and evaluate Asian, binary, and barrier options and only several authors presented the problem in the valuation of chooser option. Borkowski and Krawiec (2009) have tried to check the adaptability of chooser options to the European wheat market and have come to the conclusion, that chooser options may be useful for hedging the risk, but they are more expensive than standard vanilla options. While Martinkutė-Kaulienė (2012) has also analysed the chooser option and found out that the correlation between the value of the chooser option and strike price is not strong, but the influence of time until the choice of option's type is made is important and must be taken into account as it highly influences the price of chooser option. Moreover, Durica and Svabova (2014) have investigated the importance of certain partial derivatives as delta and gamma and their management using Tailor expansion for the big change in price of underlying asset.

A few years later, John Cox, Stephen Ross and Mark Rubinstein developed a contrary model to the Black-Scholes model, the so-called **Binomial Model** (1979). The main difference between these two models lies in the fact that the Binomial model assumes a discrete-time version of option pricing, while BS model takes the continuous-time framework.

Almost at the same time as the Binomial model, Phelim Boyle implements the idea of using **Monte Carlo Simulation** (1977) into option pricing as this particular method has been used in other science fields as physics. Even though Monte Carlo simulation sometimes could be understood as a non-parametric model due to its method of solving calculations through random sampling, but generally taking the real essence of the model – it is dependent on set of parameters such as volatility should be set as well as stochastic process (e.g. Geometric Brownian Motion) should be incorporated in order to calculate as accurate results as possible. Monte Carlo simulation due its parameters and considerable flexibility to adhere to multiple sources of uncertainty has the ability to manage complex pricing scenarios, where options have difficult pay-offs, or they are path-dependant and cannot be fully addressed by other analytical models. These distinctive features make Monte Carlo simulation in option pricing a power tool.

Lastly, it is worth to mention **Heston model** (1993) introduced by the namesake Steven Heston. This particular framework assesses both the price of underlying assets and volatility as stochastic processes, which means that they should be modelled as random components – this is quite opposite to the assumption of constant volatility in Black-Scholes model. Chavas et al. (2024) pay attention to volatility of the stock and how it affects the dynamics in pricing option in Black-Scholes model. By assuming that volatility is random process, it gives model more realistic market view with flexibility as well adherence to market phenomena – volatility smile - in realistic option pricing, especially for long maturity dates. APPENDIX 2 provides a summary of the five most popular models, including the types of option contracts used, starting variables needed in computation as well as advantages and disadvantages.

Other models. Nonetheless, there has been a significant amount of other parametric model variations or combinations of models, which are used to capture the value of options contracts such as *Finite Difference Methods (FDM)* could be used to solve the partial differential equations deriving from underlying parametric model, like in European option call pricing has showed Jeong et al. (2019), where FDM was blended with Monte Carlo simulation. Also, often *GARCH model* is often incorporated to enhance the better capture of volatility parameters, especially if there is a tendency of high volatility as Escobar-Anel et al. (2021) has implemented it together with the Heston model to analyse the crisis periods and its impact on options price. *Hull-White model (1990)* is much more widely used for interest rate derivatives as it models interest rates or volatility as stochastic processes. Authors such as Kim et al. (2024) used this particular model to price arithmetic Asian options, where pricing accuracy is better.

Non-parametric models

In addition to the advancements in parametric models, the rapid evolution of artificial intelligence and machine learning, particularly with neural networks, has significantly influenced the development of non-parametric approaches to option pricing, i.e. these type of models focuses on large sets of historical or observed data, as per Liu et al. (2024). For instance, Jang et al. (2021) introduced a model named DeepOption, symbolically integrating the principles of 'deep learning' with the complexities of financial derivatives to address option pricing and delta-hedging challenges. However, due to the inherent complexities in estimation and the computational intricacies of these algorithms, this thesis will not employ such methods for practical option valuation. Instead, it will focus on a theoretical review of these emerging techniques and their broader implications. *Dupire model,* as used in Labuschagne and Boetticher (2016) which belongs to the local volatility models, derives a volatility surface directly from observed option prices and do not assume constant volatility or any specific parametric distribution.

Kernel Smoothing method (2000s) in European and American option valuation helps to adapt to real market data and does not assume a specific distribution among its parameters. Heston et al (2023) mention that as key players it is advised to be the bandwidth and the kernel function type, which influence the smoothness and fidelity of the volatility surface estimation. In this way, it solves the discrepancies in the implied volatility estimation as it is coming from market prices of options and in this way, the market sentiment is represented more thoroughly, even though it requires more data resources and is computationally intensive. As per Heston et al (2023) blend Kernel smoothing with a single-factor diffusion model in OTM S&P 500 call and put options, could produce quite promising results in pricing versus market data.

Local Polynomial Regression (LPR) (2000s) is also quite similar to the method just mentioned, as the LPR technique has been used in both European and American options to estimate the local volatility surface more accurately by fitting polynomials to subsets of market data (Ait-Sahalia & Duarte, 2003). This method adapts to the local structure of the data by using polynomials of varying degrees, can oversee high-dimensional data and typically results in a smoother, more precise volatility surface compared to simpler models. Important parameters in LPR include the degree of the polynomial and the bandwidth, which dictate the complexity of the model and the degree of smoothing, respectively. As a downside, it is extremely expensive computationally and sensitive to bandwidth selection.

From the machine learning perspective, one of the subsets called *Artificial Neural Network (ANN)* has been heavily involved in option pricing in recent decades. As per Ruf and Wang (2019), these neural networks typically involve several layers of processing units that can learn nonlinear relationships within the data, making them particularly useful in capturing the intricacies of financial markets where linear models might fall short. There have been various developments for pricing European Options like Liu et al. (2019), Rombouts et al. (2020) created a flexible bivariate model has been constructed incorporating both U.S. stock and market index information, while Guo et al. (2024) use a dynamic ensemble framework with a time-varying parametric pricing model optimized using artificial intelligence algorithms and display that their model enhances accuracy in pricing, but as well maintains the stability of the model itself. For the American type of option contracts, Chen et al. (2021) introduce Laguerre neural network with three layers of neurons for solving the Black-Scholes model proposed equations. Also, the novel hybrid model introduced by Shimmer and Zhu (2024) shows greater precision in pricing both Call and Put options at all moneyness levels, outperforming traditional parametric and non-parametric option pricing models.

A second subset in machine learning are r*egression trees* and *Random forests*. Both terms are referred to in the literature as complementary parts. They are quite favoured approaches for incorporating multiway predictor interaction for all plain vanilla options as well as exotic ones. As well, this machine learning subset has clear decision rules and manages both numerical and categorical data. However, as per Vaswani et al. (2023), the pitfall lies in model tuning and pruning itself as it is very computationally demanding. In addition, these methods are prone to overfitting, plus it is hard to interpret the model itself due to its complexity. But it seems that it is rewarding due to accuracy in pricing - Ivascu (2021) has studied call options pricing, where the underlying asset was crude oil, and his constructed three decision-making methods: Random Forest, XGBoost and LightGMB have outperformed Black-Scholes and Corradu Su models based on accuracy and realistic option prices. (Han & Song, 2025). Brini and Lenz (2024) have selected a cryptocurrency as the main underlying asset and employed regression-tree methods in Machine learning, leading to conclusions that crypto options display more market inefficiencies than other regular underlying as equities, thus it leads in difficulties to measure option's price accurately.

This development in technology and models containing the ability to learn from themselves leads to evidential results that non-parametric models could outperform classical models (Jang et al (2021). However, the downside is the complexity of the models and the requirements of loads of data required for training of the model. Luo et al (2022) are able to prove that the parametric Heston model could outperform the machine learning model in certain conditions. With advances in Artificial intelligence-driven models competing around for the best possible replica of the market in order to determine the value of options contract, the landscape of financial analysis has transformed enormously, but nevertheless, so-called 'old-school' or sometimes even referred to as primitive models as Binomial or Black-Scholes remain relevant for several reasons. First of all, traditional parametric models have a grasping theoretical background

After conducting a comparative scientific literature analysis, the author gets acquainted with a broad description of derivatives, their designated purpose, a more thorough deep dive into options contracts and their peculiarities as well as still up-to-date existing challenge of difficulties in the valuation of option contracts, where not only traditional pricing methods are involved, but as well incorporated in advanced calculation algorithms. Nonetheless, history has shown that no model can account for every market aspect, as not all factors affecting the price of a financial security can be captured mathematically. According Ahmed et al. (2018), mathematical models can only attempt to capture some of the aspects of market behaviour. The convenience of using of one or another type of model will depend upon the valuation circumstances. In the upcoming part of the thesis, using gained theoretical knowledge, the author presents the methodology of how the analysis of the valuation of option contracts will be conducted.

2. METHODOLOGY FOR VALUATION OF OPTION CONTRACT

2.1. Purpose of the research, model and hypotheses

In this section of the thesis, the author presents the problem and purpose of the research, and its model and raises the following hypotheses.

The problem of the research. As it was already captured in the first part of the thesis - scientific literature review – the valuation of different types of option contracts is complex as it is dependent on the interaction of many factors such as the selected type of option contract, underlying asset's price fluctuation, selected strike price of option contract, time to maturity of the option contract as well as various direct and indirect market forces. The evaluation and comparison of pricing dynamics between plain vanilla options and exotic options have a variety of different models, with their own advantages and disadvantages. This particular study will focus on understanding the risk sensitivities of different types of option contracts as well as adapting the Black-Scholes-Merton model, traditionally used for plain vanilla options, to better fit the complex nature of exotic options, which often include features like path dependency and multiple exercise opportunities.

The aim of the research is to evaluate how alteration of different option parameters such as strike price, risk-free rate, volatility, dividend distribution, and time to maturity affect the value of plain vanilla option contact and exotic option (as examination items the author has selected the chooser option)

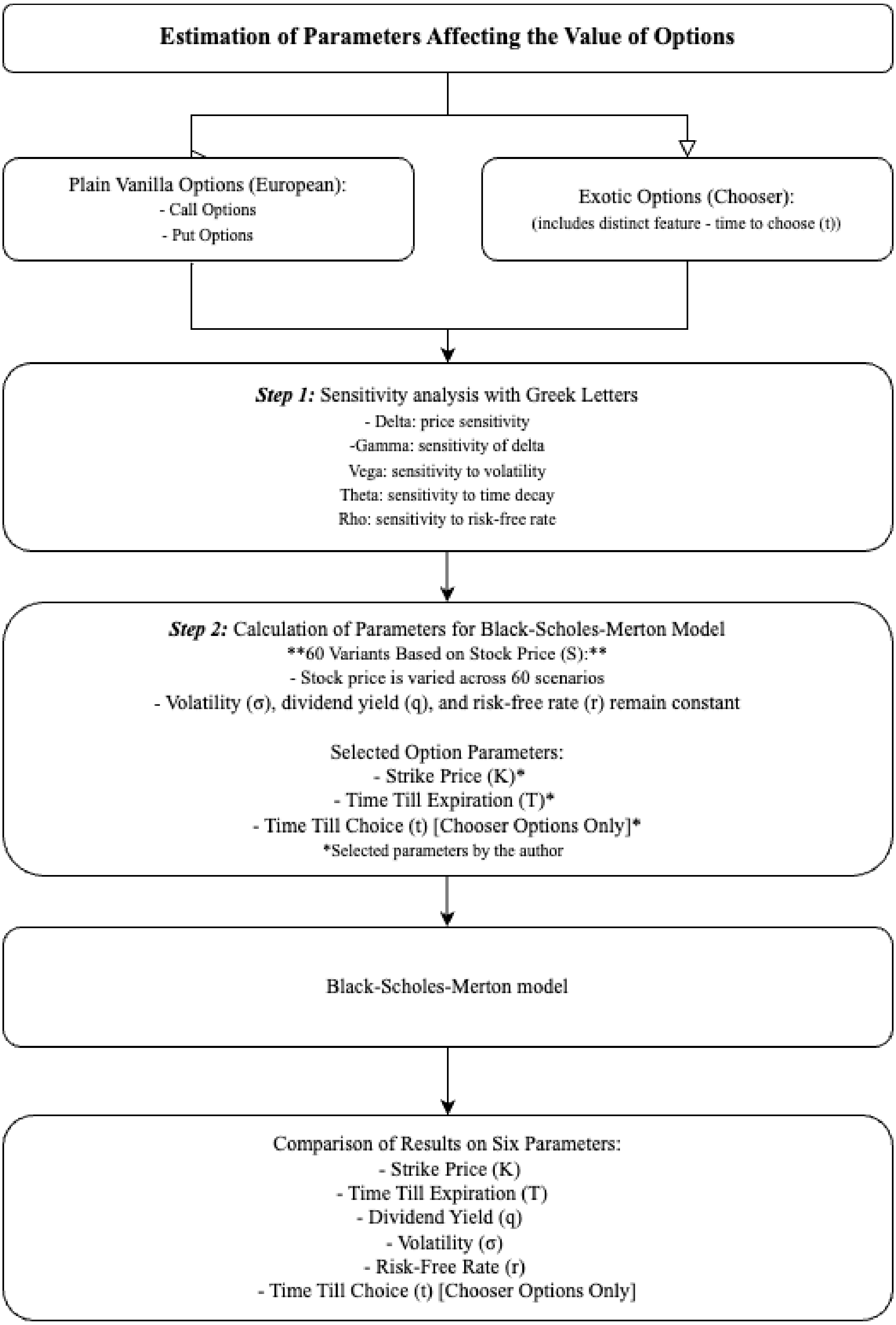
The author employs the Black-Scholes-Merton model for both options, as well the sensitivities to changes are assessed by previously described the Greeks. See Figure 4 for the detailed scheme of the proposed research methodology.

Based on collected theoretical knowledge, the author raises the following hypotheses to be assessed:

H1: Following Martinkutė-Kaulienė (2012) and Lian and Chen (2023), the chooser option has an embedded parameter of time-to-choose whether it should be exercised as call or put, the premium value will be higher than the European option.

H2: As per Lian and Chen (2023), the volatility and risk-free rate parameter has similar influence on the value of the chooser option and the European option

Figure 4. Proposed research methodology



Source: prepared by the author

The following paragraphs will introduce the selected research methodology with more detailed descriptions and computational formulas.

2.2. Sensitivity measure calculations

As it was introduced in the theoretical part, the main focus will be drawn on five Greek letters mentioned in the first-order partial derivatives, with the exception of Gamma, which is a second-order partial derivative. Each of the partial derivatives displays the relationship between the changes as follows (see Table 3):

Delta – change in the price of asset (f) versus change in the price of underlying (S)

Rho - change in the price of an asset (f) versus change in the risk-free rate (r)

Vega – change in the price of asset (f) versus change in volatility rate (σ)

Theta – change in the price of an asset (f) versus change in the time decay (t)

Gamma - change in delta (Δ) versus change in the price of underlying (S)

Table 3. Calculation formulas for partial derivatives

Delta	Rho	Vega	Theta	Gamma
$\Delta = \frac{\partial f}{\partial S}$	$\rho = \frac{\partial f}{\partial r}$	$v = \frac{\partial f}{\partial \sigma}$	$\theta = \frac{\partial f}{\partial t}$	$\tau = \frac{\partial^2 f}{\partial S^2} = \frac{\partial \Delta}{\partial S}$

Source: Hull (2021)

Interesting part is that the calculation of the Greeks could expressed as well using the variables from Black-Scholes model (see Table 4), which will be described further in the methodology.

Table 4. Expression of partial derivatives via Black-Scholes model

The Greeks	Call Option	Put Option
Delta	$e^{-qt}N(d_1)$	$e^{-qt}(N(d_1) - 1)$
Gamma	$\frac{e^{-qt}N'(d_1)}{s\sigma\sqrt{t}}$	$\frac{e^{-qt}N'(d_1)}{s\sigma\sqrt{t}}$
Rho	$KTe^{-rt}N(d_2)$	$-KTe^{-rt}N(-d_2)$
Vega	$e^{-qt}s\sqrt{t}N'(d_1)$	$e^{-qt}s\sqrt{t}N'(d_1)$
Theta	$-\frac{e^{-qt}s\sigma N'(d_1)}{2\sqrt{t}}$	$-\frac{e^{-qt}s\sigma N'(d_1)}{2\sqrt{t}}$

Source: Hull (2021)

Once the computation of the Greek letters is determined, we can move forward to actual option valuation model used in the research - Black-Scholes-Merton.

2.3. Research model – Black-Scholes-Merton

Firstly, scientific literature on the thesis topic was reviewed, revealing that option contract valuation typically involves the Black-Scholes, Binomial model, or Monte Carlo simulation, due to their popularity and ease of result assessment. For this thesis, the author chose a modified Black-Scholes (BS) model, incorporating the dividend yield feature, for its adaptability and clear parameters. This model was used to calculate both plain vanilla European put and call options, as well as the exotic - chooser - options. In the practical section, the plain vanilla option was evaluated alongside the chooser option, selected for its diversity and adaptability. To ensure consistency and comparability, the modified Black-Scholes model was applied to both options.

Model for European put & call options

The price of a European call option could be revealed by subtracting the second part from the first part. Mathematical expressions of standard Black-Scholes model are presented below in the equations *1-4*.

$$C = SN(d_1) - N(d_2)Ke^{-rt} \tag{1}$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)t}{s\sqrt{t}} \tag{2}$$

$$\mathrm{d}_2 = \mathrm{d}_1 - \mathrm{s}\sqrt{\mathrm{t}} \tag{3}$$

The price of European put option could be calculated as follows in equation *4*.

$$\mathrm{P} = \mathrm{N}(-\mathrm{d}_2)\mathrm{Ke}^{-\mathrm{rt}} - \mathrm{SN}(-\mathrm{d}_1) \tag{4}$$

Meanings in the formulas stands for: **C** – call premium; **S** – current stock price; **t** – time until expiration; **K** – option strike price; r – risk-free rate; **N** – normal distribution; **σ** - standard deviation; **e** – exponential (≈2,718281828…).

As discussed in theoretical review, Robert C. Merton has adjusted BS model, and it was appraised in the award of a Noble Memorial Prize in Economic Science in 1997. Particular equations (*5-6*) will be used throughout the practical analysis in the third part of the thesis.

$$C = Se^{-qt}N(d_1) - N(d_2)Ke^{-rt} \tag{5}$$

$$P = N(-d_2)Ke^{-rt} - Se^{-qt}N(-d_1) \tag{6}$$

In the equations *5-6*, the parameter '**q**' stands for annual dividend yield.

Model for chooser options

Thereupon, there is a time to introduce a second modification of the Black-Scholes model, which is customized for chooser options. The mathematical expression of the earlier introducted Rubinstein model is presented in equations *7-10* and will be used in the analysis.

$$\mathrm{P_{sc}} = \mathrm{Se}^{(\mathrm{b-r})\mathrm{T}}\mathrm{N(d)} - \mathrm{Ke}^{-\mathrm{rT}}\mathrm{N}\left(\mathrm{d} - \sigma\sqrt{\mathrm{T}}\right) - \mathrm{Se}^{(\mathrm{b-r})\mathrm{T}}\mathrm{N}(-\mathrm{y}) + \mathrm{Ke}^{-\mathrm{rT}}\mathrm{N}\left(-\mathrm{y} + \sigma\sqrt{\mathrm{t}}\right) \tag{7}$$

$$\mathrm{d} = \frac{\ln\left(\frac{\mathrm{S}}{\mathrm{K}}\right)+\left(\mathrm{b}+\frac{\sigma^2}{2}\right)\mathrm{T}}{\sigma\sqrt{\mathrm{T}}} \tag{8}$$

$$y = \frac{\ln\left(\frac{S}{K}\right)+bT+\frac{\sigma^2 t}{2}}{\sigma\sqrt{t}} \tag{9}$$

$$b = r - q \tag{10}$$

Meanings in the equations *7-10* stand for: **S** - the spot price of the asset; **b** - the cost of carry; **K** - the strike price; **σ** - the asset volatility; **T** - the time to maturity; $\mathbf{t_c}$ - the time to choose; N - the cumulative normal distribution.

With the necessary formulas for sensitivity measures and price valuation outlined, the following paragraph will address other components of the thesis.

2.4. Data collection methods and research instruments

As the model has been described (see *Figure 5*), it is time to collect the data, which would be used in calculations. To fulfil Black-Scholes model with dividends, author has aimed to select the company which has high capitalization in stock market as well there are some active trading in CBOE option market using this stock as underlying. Microsoft Inc. seems to be a perfect fit – as it has a long history of stable distribution of quarterly dividends and as per August 2024, Microsoft market capitalization took a whooping 3,11 trillion of dollars and of course, by crossing 3 trillion of dollars mark and currently being a second biggest company based on market capitalization, Microsoft continues to race with Nvidia and Apple against next milestone – 4 trillion. As well, Microsoft with its slogan/goal – "to empower every person and every organization on the planet to achieve more" - is an American multinational corporation with primary focus on technologies and developing software, offering applications, extra cloud storage and other solutions. Nevertheless, Microsoft Inc. stock belongs to the club called "Magnificent Seven" – this is the term coined by Michael Hartnett, investment strategist at Bank of America to group up and define high-performing, influential and leading the US market: Alphabet, Amazon, Apple, Meta, Microsoft, Nvidia and Tesla.

Also, during the observation period i.e. 250 trading days – from 1st of July 2023 until 30th of June 2024, there were no corporate actions (except quarterly dividends distributions), which could highly distort the price of the stock. According to Bloomberg Terminal data (2024), within the period 1987-2003, Microsoft Company has conducted nine stock splits, i.e. company divided its existing shares into multiple shares to boost the liquidity of shares in the way that stocks become cheaper and more attractive for investors, but imbalances the curve of stock price. Just to display the cumulative multiplier effect of these nine stock splits – for example, if you would have in your possession 1 MSFT stock back in 1987 and kept this stock up to date, after all the splits you would have 288 MSFT stocks.

In order to complete Black-Scholes-Merton model, which considers dividend payments, there is a need for a dividend yield. During selected observation period, there have been 4 dividend distribution with respective days as present using Bloomberg Terminal data (Table 5). Author has observed arithmetical average of dividend yield, which is equal to 0,71 % which will be used further in the analysis.

Table 5. Dividend yield for the last year payments

Ex-dividend date	Dividend amount	Dividend yield
16.08.2023	$0.68	0.830 %
15.11.2023	$0.75	0.736 %
14.02.2024	$0.75	0.691 %
15.05.2024	$0.75	0.681 %

Source: Bloomberg (2024)

As the cornerstone for the whole analysis is the historical data of Microsoft Company (the trading ticker is MSFT) stock price. The author has retrieved the prices of Microsoft stock from Bloomberg terminal (2024), where average price of Microsoft stock was 376,10 USD during selected period. Using this particular data set, there is a possibility to estimate other needed variable in order to be able model calculations. The volatility could be calculated as a standard deviation (σ^2) under the log returns of underlying asset's historical value. Arithmetical mean of the stock and standard deviation (σ^2) could be calculated using equations *11-12*.

$$\bar{\Delta} = \frac{\sum_{n=1}^{k} \Delta_{Ri}}{n} \tag{11}$$

$$\sigma = \sqrt{\frac{\sum(\Delta - \bar{\Delta})^2}{n-1}} \tag{12}$$

The meanings of equations stand for: **n** – numerical expression of observation periods, i.e. 251 days; $\boldsymbol{\sigma}$ – annual standard deviation of the changes in price of stock; Δ – logarithmic expression of the change of stock price per one day, i.e. $\ln(R_i / R_{i+1})$; $\bar{\Delta}$ – arithmetic average of stock price.

When one of the essential components – volatility – is revealed to be equal to 21,00%, there is a possibility to create a data set for underlying asset prices, which would be normally distributed to simulate various cases for assessment of option value. The data set of sixty randomly generated and normally distributed variables was generated using Excel and will be used as a basis for simulating different scenarios of the impact of parameters in option value. As the main focus of the research relies on the best possible way to display the effect of changes in the parameters estimating the value of option contract, the log-normal distribution technique would be more preferable due to more precise statistical inferences for data and more accurately describes the data where the spread of values is skewed (as in this particular case of stock prices)

Another component needed for establishing the model is a risk-free rate. Risk-free rate is pulled by the discount rate of 52 weeks termination of the treasury bills, issued in the United States on the 1st of July 2024. According to U.S. Department of the Treasury (2024), the ratio of treasury bills was 5,10 %.

The author selects the last components for the calculations manually: the strike price is equal to 390 USD and the duration of maturity is one year for both options. Additionally, as constant variable 'time-to-choose' for the chooser option is selected 6 months – the holder of chooser contract has a half of year till the decision, what option – call or put – he would love to have until the end of maturity. Summarizing all the modelling, Table 6 and Table 7 reveal main variables for calculations.

Table 6. Basic variables used for valuation of European option

Strike price (K)	The maturity of option (T)	Risk-free rate (r)	Volatility of stock price (σ)	Dividend yield (q)
390 USD	1 year	5,10 %	21,00 %	0,71%

Source: prepared by author

Table 7. Basic variables used for valuation of Chooser option

Strike price (K)	The maturity of option (T)	The time to choose (t)	Risk-free rate (r)	Volatility of stock price (σ)	Dividend yield (q)
390 USD	1 year	0,5 year	5,10 %	21,00 %	0,71%

Source: prepared by author

As all needed data is collected and concluding the second chapter - author has introduced the main sensitivities of parameters affecting the price of option, presented a classical option pricing model with and without the inclusion of dividends. After theoretical research, the author has selected the adapted Black-Scholes-Merton model, which encounters dividend payments into the estimation of the value of option. This model was selected due to its computational simplicity and widespread popularity among investors as well as a possibility to be adjusted in usage of exotic options as chooser options. All calculations are made via excel.

The whole methodology for the practical part will be conducted in the order as follows: the objective for investigation is to select two option contracts – plain European and chooser option. In the first part there will be examined the sensitivity using the Greek letters. In the second part of the practical part, there will be created a model-date set, which is suitable for further calculations using modified Black-Scholes model to check the impact of below mentioned parameters and their impact to the eventual value:

- The strike price (**K**);
- Time till expiration – maturity (**T**);
- Dividend yield (**q**)

- Volatility (**σ**)
- Risk-free rate (**r**)
- Time till choice of option contract (**t**) – applicable only for a chooser option contract

After all the results are compared, as the last item, there will be a study between the payout of trading strategy in plain vanilla options versus the chooser option. As the option premium is primarily composed of two elements: intrinsic value and time value, the actual payout and worth to exercise the option contract could be determined as follows. According to Cuthbertson et al. (2019), intrinsic value symbolizes the amount, which could be saved on the purchase of the underlying asset in comparison, to an investor who would have bought the same amount in the market.

$$Call\ option = current\ stock\ price - strike\ price \tag{13}$$

$$Put\ option = strike\ price - current\ stock\ price \tag{14}$$

The mathematical expressions for put and call options are presented in the equations (*13-14)*. In order to calculate the actual payout of the option contract, there is a need not to forget to multiply from 100, as per standard in the market, one option contract represents the claim to 100 shares.

3. ANALYSIS OF PARAMETERS AFFECTING THE VALUE OF OPTION CONTRACT

In the final chapter, the author analyses and compares selected plain vanilla and exotic options. European option was chosen due to its standard built and popularity in the derivative market. Meanwhile, from the pool of various exotic options, which have come up over the years, the chooser option was taken by the reason of similarity to traditional option, but its ability for diversion - the buyer of the chooser option after the choice time comes, could select the more favourable type of option – a call or a put.

Foremost, the author analyses the risk factor sensitivities for these particular options via the partial derivative method – the Greeks – in order to better understand the effect of factors such as time decay, volatility, the change in underlying asset's price. Afterwards, in the second part of the analysis the author contrives a model using historical data of existing company and realistic circumstances of the market in order to determine the value of selected option contracts using modified Black-Scholes-Merton model. A similar approach was taken by Jalan et al. (2021), while trying to evaluate the option price on bitcoin as well as risk inherent (sensitivities). Consequently, the impact of changes in several parameters, which affect the value of the option, are considered and appraised. Finally, the author evaluates the potential profit and loss of both option contracts.

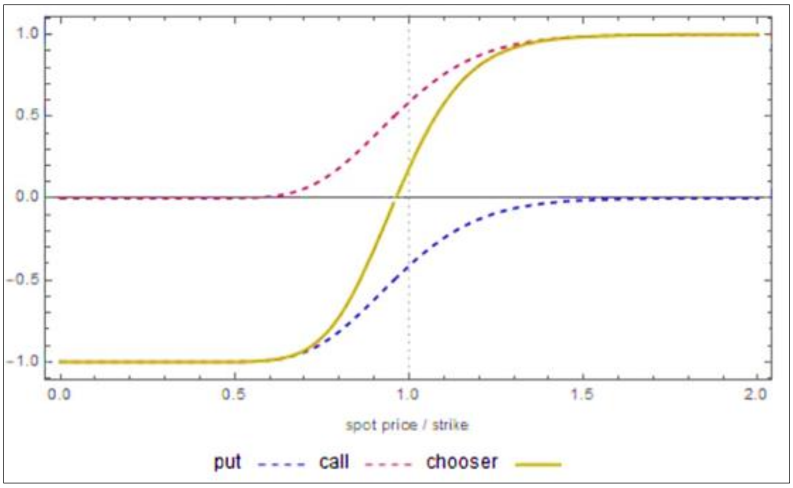
3.1. Comparison using the Greek letters

The comparison of European and chooser option will start with assessing standard calculations of risk sensitivity measures. The Greek letters as measure of risk sensitivity are common indicators and heavily used by the traders and portfolio managers on daily basis in order to oversee undesirable tendencies and suppress them on time. As described in the theoretical part, there are variety of different Greek letters, but the author opted out to make a comparison only for five most significant and popular ones: delta, rho, vega, gamma and theta.

For easier evaluation and comparison of the risk sensitivities for European and chooser option, author has selected visual presentation program 'Wolfram" as it offers ability to display and automatically generates graphical interpretation of risk sensitivities for chooser and both types of European options (Figures 6-10). Static key parameters used for comparison on all three options will be used throughout all the research and are as follows: time till maturity – 1 year, stock volatility is based on historical volatility of selected company Microsoft Corp., and it is equal to 21%, risk-free rate is based on US treasury bills as per 3 August 2024 and it is equal to 5,1%. The specific parameter of chooser option – time to choose - is equal to 6 months.

At the first glance, while comparing all five selected Greeks, it could be declared, that a chooser option is exposed to and possesses higher variety for sensitivity due to its embedded feature of choice (call or put), which adds more complexity in their valuation and calculation of its Greeks, in comparison with call and put vanilla options.

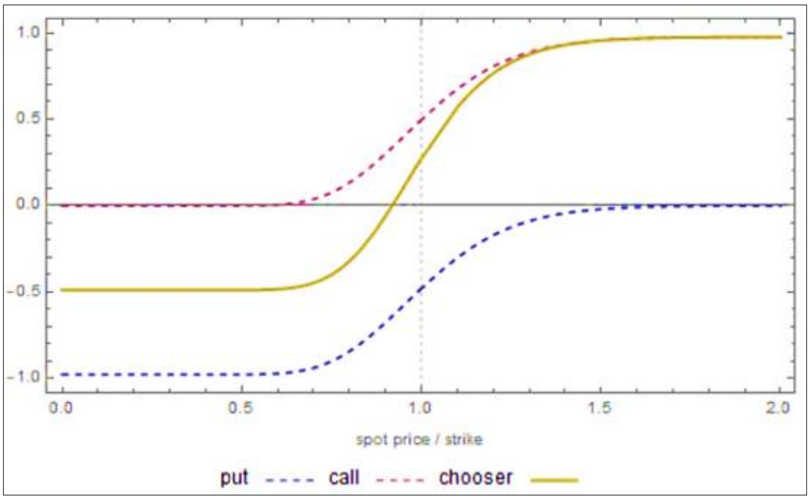
Figure 5. The estimation of delta



Source: prepared by author using Wolfram

First and practically the most important indicator for option traders – **delta** (Figure 5) for both types of European options disposes via parallel and expected manner (i.e. positive for a call option and negative for a put option), while a chooser option has a mixture of European option – when the stock price is lower than strike price, delta index follows the same path as European put option. Although if the strike price is overstepped by stock price, delta index quickly shifts and nearly equalizes with European call option. This type of arrangement shows that chooser option much better deals with diversification of risk, as for example, when spot price is equal to strike price, the sensitivity is equal practically for 0, while both European options remain as high as 0,5 for a call and -0,5 for a put.

Figure 6. The estimation of rho

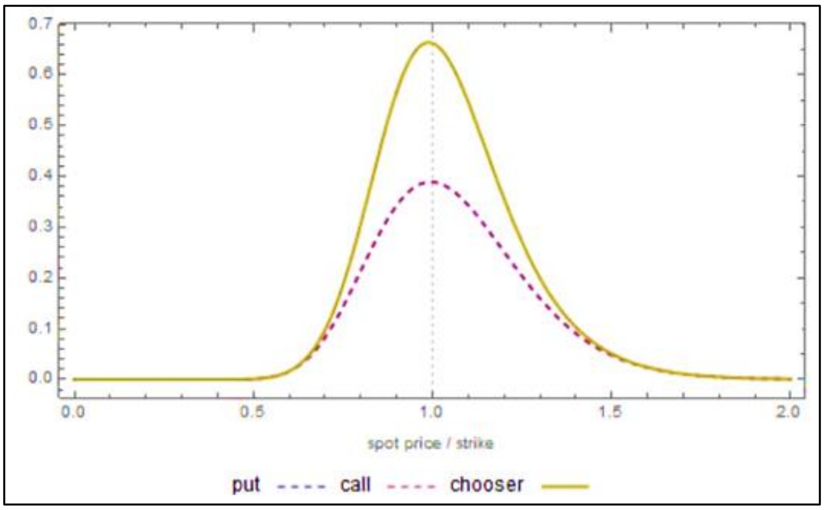


Source: prepared by author using Wolfram

Furthermore, other sensitivity indicator – **rho** (Figure 6) looks very similar in the meaning of arrangement to previously described delta. The effect of risk-free rate changes is practically identical for European options, whilst a chooser option arranges perfectly in the middle between European put and call options until stock price is below 20% than agreed strike price. With further increase in stock price, especially if stock price goes above 20% than strike price, it could be observed that rho quickly adapts and shift towards the path of European call.

The third variable of examination – **vega** (Figure 7) show the responsiveness to changes in the area of volatility. It can be seen, that both European options replicates to the volatility in the same manner – symmetrical parabolic form, in the range where the spot price is lower by 30% than strike price and higher by 30% than strike price. In this scenario, the highest point of volatility effect is reached, when stock price is equal to spot price. While chooser option in terms of vega reacts in the same range as European options, but the sensitivity level is two time higher while in comparison with European options.

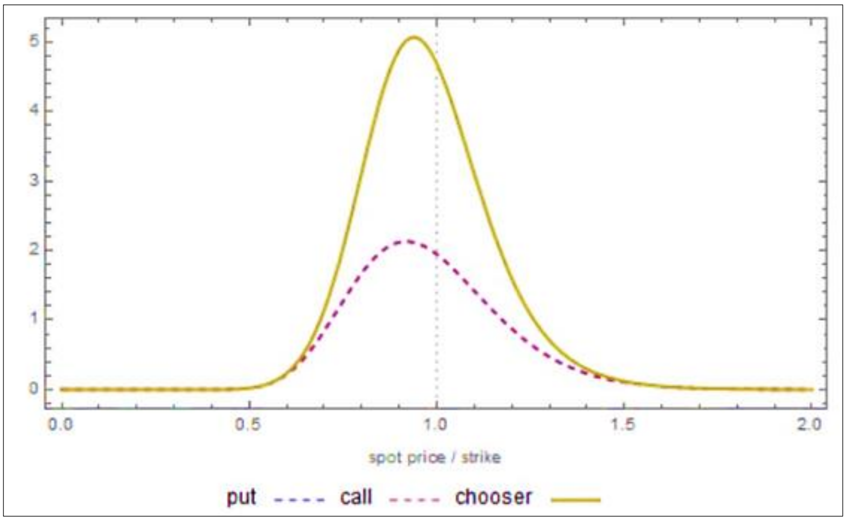
Figure 7. The estimation of vega



Source: prepared by author using Wolfram

Fourth indicator – **gamma** (Figure 8) as the second-generation Greek letter shows the reaction to changes of another variable – delta and looks very similar to previously discussed vega. Due to its specifics 'being the Greek letter on the Greek letter', the highest effect of gamma is seen on both types of options, when stock price is around 10% less than agreed strike price, i.e. delta has increased sensitivity at this point and due to this pronounced effect it automatically causes reaction in value of Gamma. Nevertheless, both European call and put falls under the same manner and reacts moderately in comparison to chooser option, which is much more sensitive in terms of changes in delta. This heightened sensitivity peaks as the investor is about to make their choice, reflecting the increased impact of price movements on the option's value.

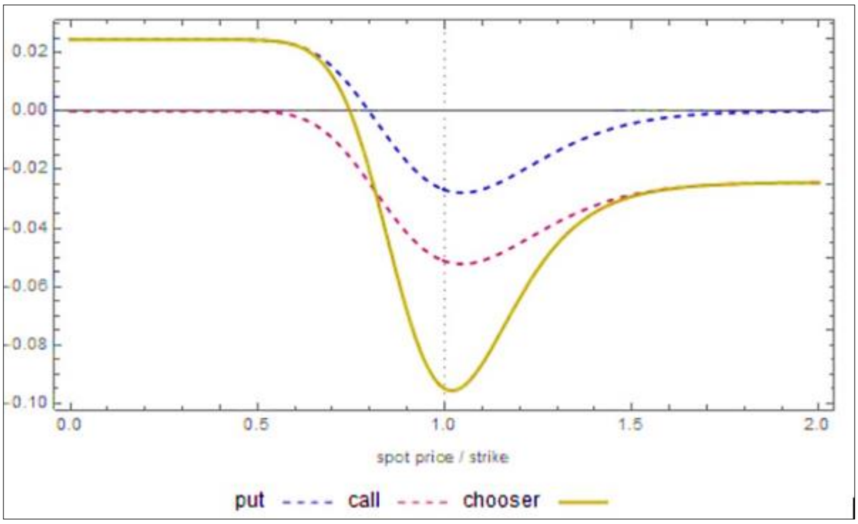
Figure 8. The estimation of gamma



Source: prepared by author using Wolfram

Last variable – **theta** (Figure 9) displays a reaction to the changes of the time decay for both types of European option and a chooser. Unavoidably, time in option contract cannot be increased, i.e. the value of theta is always negative due to predetermined exercise date of option. Nevertheless, in this situation the flexibility of chooser option comes into play: if stock price is 30% and lower than strike price, theta of chooser option would be equal to the theta of European put and becomes slightly positive.

Figure 9. The estimation of theta



Source: prepared by author using Wolfram

This effect could be expected as the intrinsic value of the put option (which is the difference between the strike price and the stock price when the option is in-the-money) may increase as the time decay causes the likelihood of the stock price recovering above the strike price to diminish. Essentially, as expiration nears, the certainty of the option finishing in-the-money increases, enhancing its value despite the usual erosive effect of time decay on option premiums.

This results in a positive theta, reflecting the increasing value of the put option as time passes. However, if stock price is close to strike price, the theta of chooser option is extremely negative by displaying the eroding effect of time in comparison to both European options. In this case, the holder is losing the "option to choose" as time expires, which is a significant component of the chooser option's initial extra value compared to standard European options, where the type of option (call or put) is fixed from the start. Nevertheless, the specific pronounced effect of chooser option is no longer visible and follows the same path of the call option, once stock price goes up by 40% in comparison to strike price.

All things considered, after the comparison of the Greeks for European call and put options versus a chooser option, several key differences emerge primarily due to the inherent flexibility of chooser options. European options, with their fixed nature (either call or put), display more predictable and stable Greek behaviours. In contrast, chooser options exhibit more complex behaviours due to the additional decision-making layer regarding the type of option to select at a future point. For instance, the delta of a chooser option may fluctuate more significantly as it approaches the decision point, reflecting the changing probabilities of selecting a call or put. Similarly, theta in chooser options tends to be more negative as the option value includes the premium of flexibility, which decays as the decision point nears. Gamma and vega in chooser options are also typically higher, capturing the heightened sensitivity to underlying price movements and volatility, given the dual possibilities of ending as a call or a put. Rho, reflecting interest rate sensitivity, can also differ notably depending on the prevailing economic conditions and how they might influence the choice between a call and a put. Overall, chooser options, with their added layer of decision and flexibility, tend to exhibit more volatile and sensitive Greek measurements compared to their European counterparts.

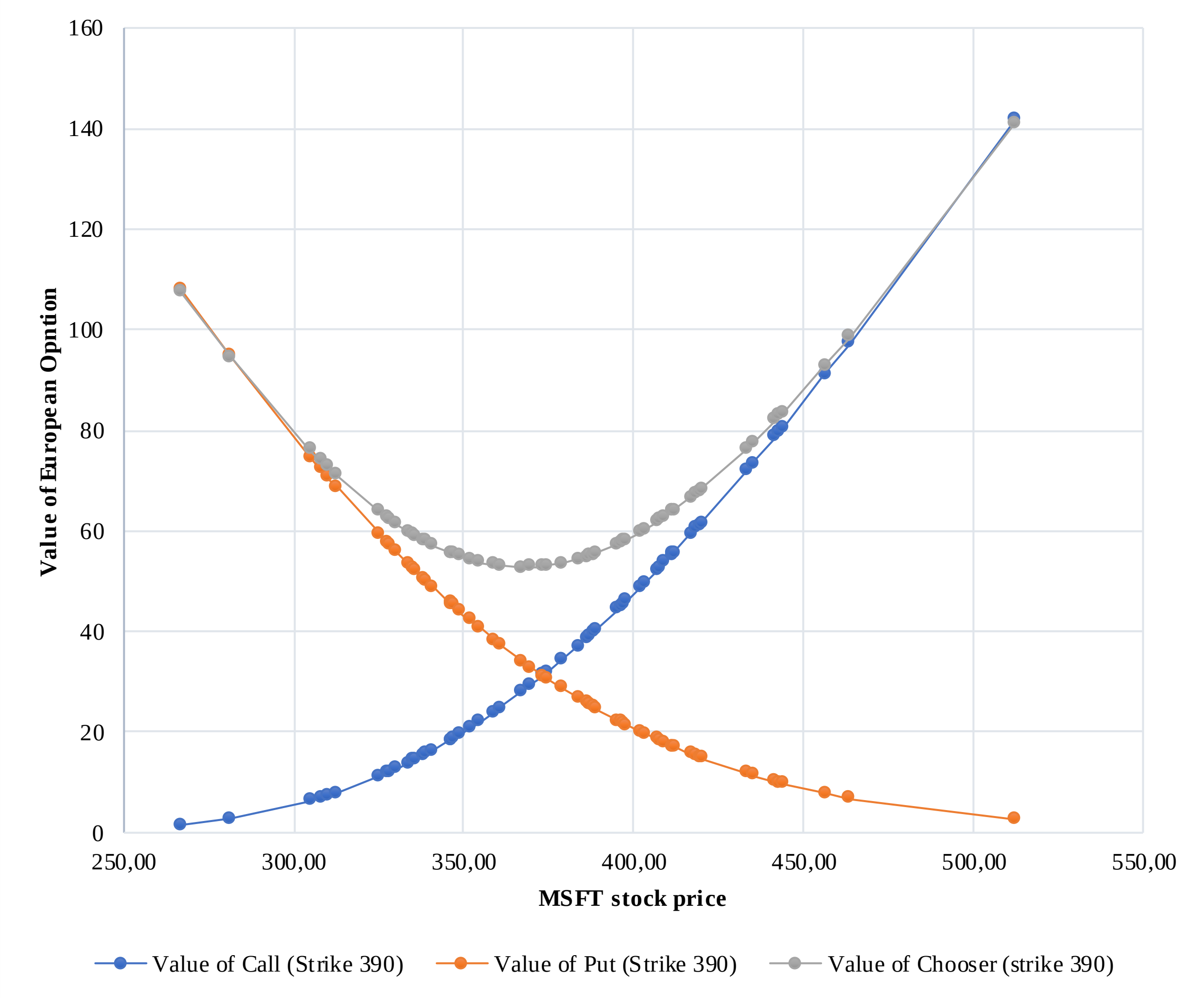
3.2. The estimation of value based on the Black-Scholes-Merton model

In order author could fulfil the Black-Scholes-Merton model, all needed variables and comparison parameters have been described in the paragraph 2.4 of the thesis.

General comparison of values between European put and call + chooser

As it can be seen from the graph in the Figure 10, the chooser option, which allows the holder to decide at the six-month mark (time to choose = 0.5 years) whether it functions as a call or a put, shows a unique pricing curve. Initially, the chooser option's value is higher compared to the individual European options due to its added flexibility. As the decision point approaches, the value of the chooser option converges towards that of the more valuable option between the call and the put, depending on the underlying asset's price movements.

Figure 10. The comparison of European and Chooser options, when K=390



Source: prepared by the author

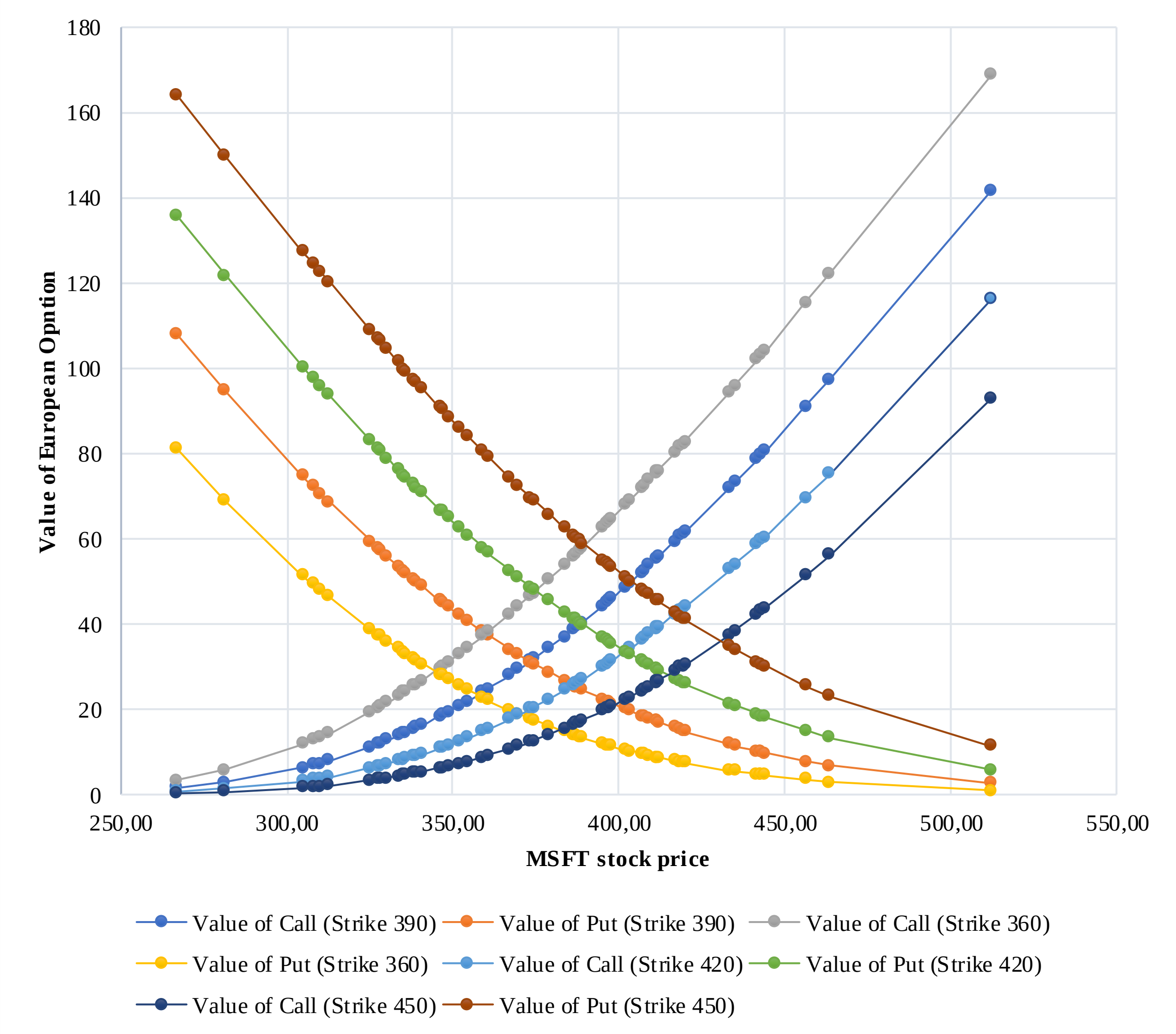
The European call and put options, on the other hand, have their values solely dependent on the asset's price relative to the strike price as the maturity approaches; the breakeven point for premium of both European options would be if at maturity the price of stock would be equal to USD 373,43, while premium would be USD 31,13. If chooser is executed as a call option, the value increases as the asset price rises above the strike price, i.e. option becomes in-the-money, while vice versa is applicable for the put option gains value when the asset price falls below the strike price. This graph effectively illustrates how the flexibility of the chooser option can offer a balance between the potential upside of price increases and the protective downside of price decreases; however, the underlying asset's price plays an important role in assessing the value of the option contract, especially it is visible, that the value of chooser option tends to keep higher.

<u>Examining parameters of European call and put options</u>

It is worth delving into other parameters as the strike price, time till expiration, dividend yield, volatility and risk-free rate of each plain vanilla and exotic option contract, to get a better understanding of how their changes affect the value of the option contract when all other variables in the model remain constant.

Starting the assessment the relationship between the value of the European option and the *strike price* (see the graph in Figure 11), where the x-axis represents the value of the European option, ranging from 164 USD to 0,27 USD, and the y-axis highlights stock price in USD.

Figure 11. The relationship between the value of European option and strike price

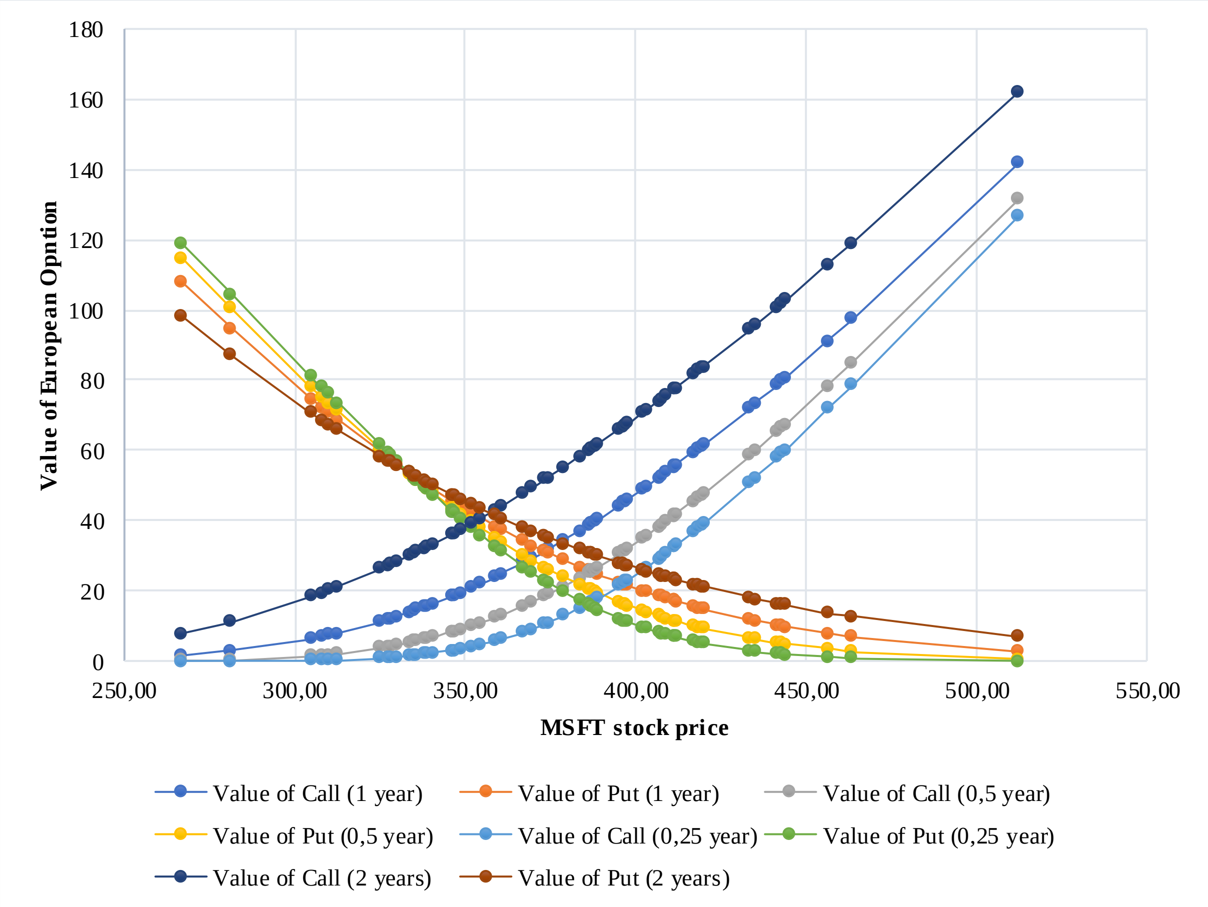


Source: prepared by the author

All of the parameters are kept unchanged and assessed through changes in 4 equivalents of the strike price - 360 USD, 390 USD, 420 USD, and 450 USD, there is a possibility to observe specific trends. For European call options, the value tends to decrease as the strike price increases, i.e. a direct inverse relationship between the strike price and the value of call options, e.g. stock price is 400 USD, while call option with strike price 360 USD is worth 67 USD, while call option with strike price 450 USD would be worth less – 22 USD. This decline occurs because a higher strike price means the underlying asset must surpass a greater value at expiration for the option to finish in the money, which statistically becomes less likely, thus reducing the option's value. In contrast, the value of European put options generally increases with an increasing strike price, i.e. a direct positive relationship between the strike price and the value of put options. This trend is because a higher strike price enhances the put option's profitability potential, as it allows the holder to sell the underlying asset at a higher price, which is advantageous if the actual market price falls significantly below this level at expiration. As well, interestingly, in all cases with the strike price, the intersection points of the European call and put of the same strike value would occur around the stock price being 5% lower than strike price. This type of nuanced behaviour is expected to as there should be a balance found in the intrinsic values and premiums paid of these options.

The second observation will be the relationship between the value of the European option and the *time till maturity* (see the graph in Figure 12), where the x-axis represents the value of the European option, ranging from 0,22 USD to 162 USD, and the y-axis displays the stock price in USD.

Figure 12. The relationship between the value of European option and maturity time

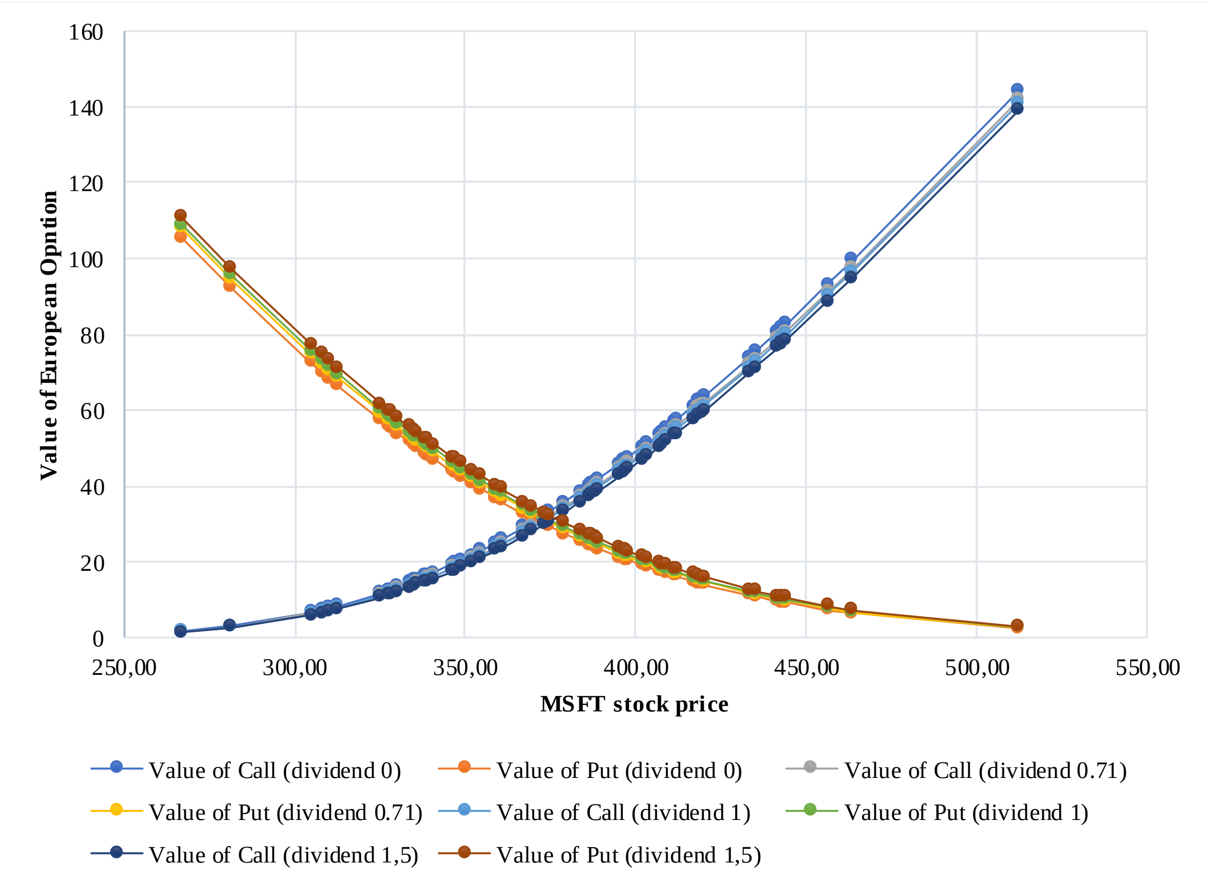


Source: prepared by the author

All of the parameters are kept unchanged and assessed through changes in 4 equivalents of time till expiration - 2 years, 1 year, 0,5 years (6 months) and 0,25 years (3 months). From the first glance, it is a visible trend for calls and puts, that if the stock price reaches around strike price and above and the longer time horizon to maturity results in a higher premium to be paid upfront, e.g. stock price is 400 USD, while call option with maturity in 4 months is worth 25 USD, while call option with maturity in 2 years would be worth more – 70 USD. This increase is because a longer duration until expiration provides more time for the underlying asset to move favourably relative to the option's strike price—increasing in value for call options and decreasing for put options. Additionally, a longer time amplifies the uncertainty and volatility in the asset's price, which enhances the probability that the option will be in-the-money at expiration. Consequently, this added time value results in a higher premium for both types of options, reflecting the greater potential for profitability and risk mitigation over an extended period. However, interesting details could be observed European put option, when the stock price is roughly below 15% and more of the agreed strike price – there we can see that a short-term (4-month) put option would be worth more than a long-term (2 years), because time decay (theta) is more sensitive to short-term options and causing value of these options to increase.

The third observation will be the relationship between the value of European option and the *dividend yield* (see the graph in the Figure 13), where y-axis represents the value of European option, ranging from 1,41 USD to 144,15 USD, and the x-axis displays stock price in USD.

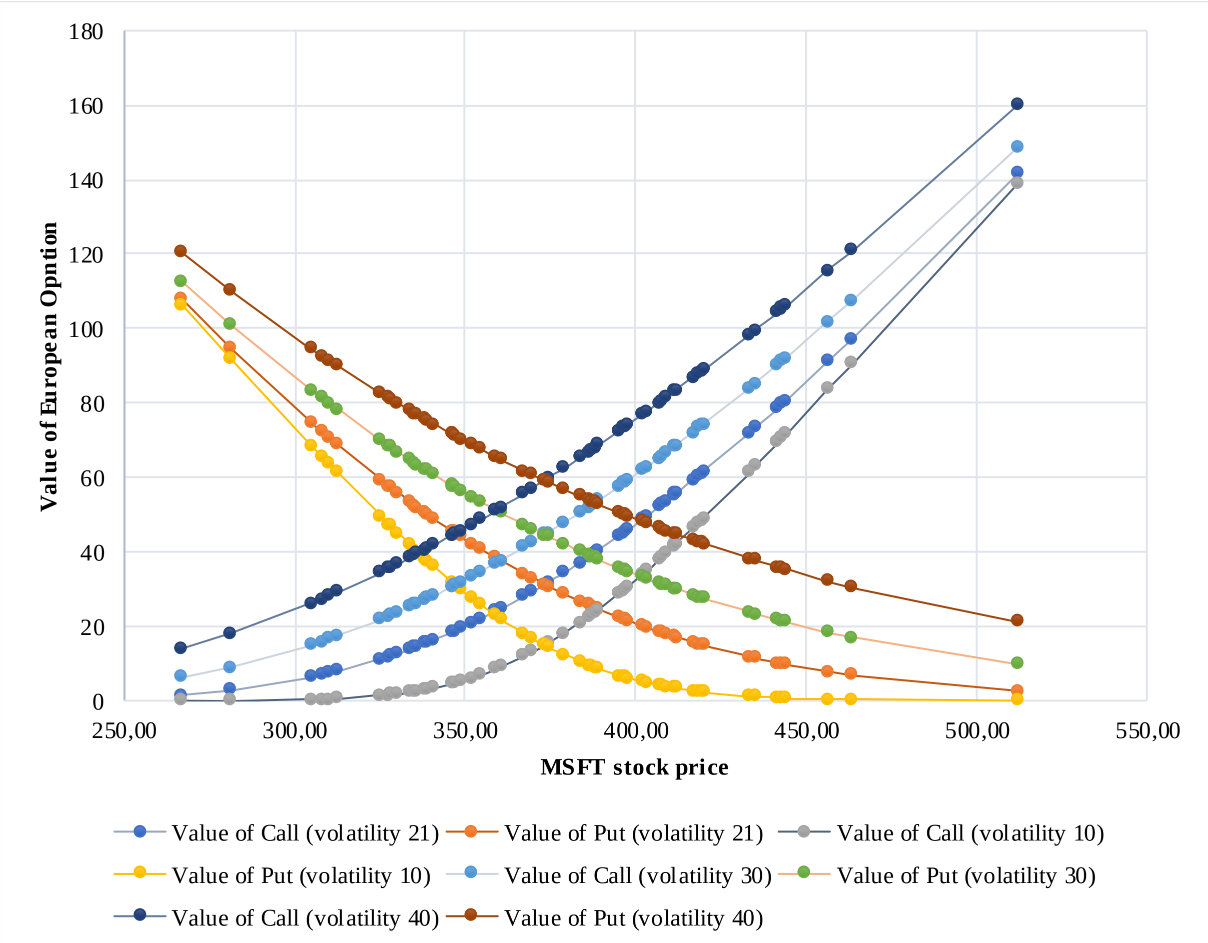
Figure 13. The relationship between the value of European options and dividend yield



Source: prepared by the author

All of the parameters are kept unchanged and assessed through changes in 4 equivalents of dividend yield - 0 %, 0,71 %, 1,00 % and 1,50 %. Typically, an increase in dividend yield typically leads to a decrease in the price of a call option (as the underlying stock price is expected to be lower due to higher dividend yield) and an increase in the price of a put option (higher dividends increase the likelihood that the option will be in-the-money), as it is visible in the graph above. This effect arises because dividends reduce the expected future price of the stock, as they represent a payout of company profits that could otherwise contribute to stock price growth. However, in this particular case, the dividend yield brackets are pretty low (0%-1,5%), thus when all the parameters are the same, the influence for both call and put options in terms of value is relatively low and practically insignificant, except two situations: put option value experiences different valuation, where stock price is below agreed strike price and vice versa situation goes for a call option value, if stock price reaches above strike price, call option with 0% dividend yield having a higher value than call option with 1,5%.

Figure 14. The relationship between the value of European option and volatility

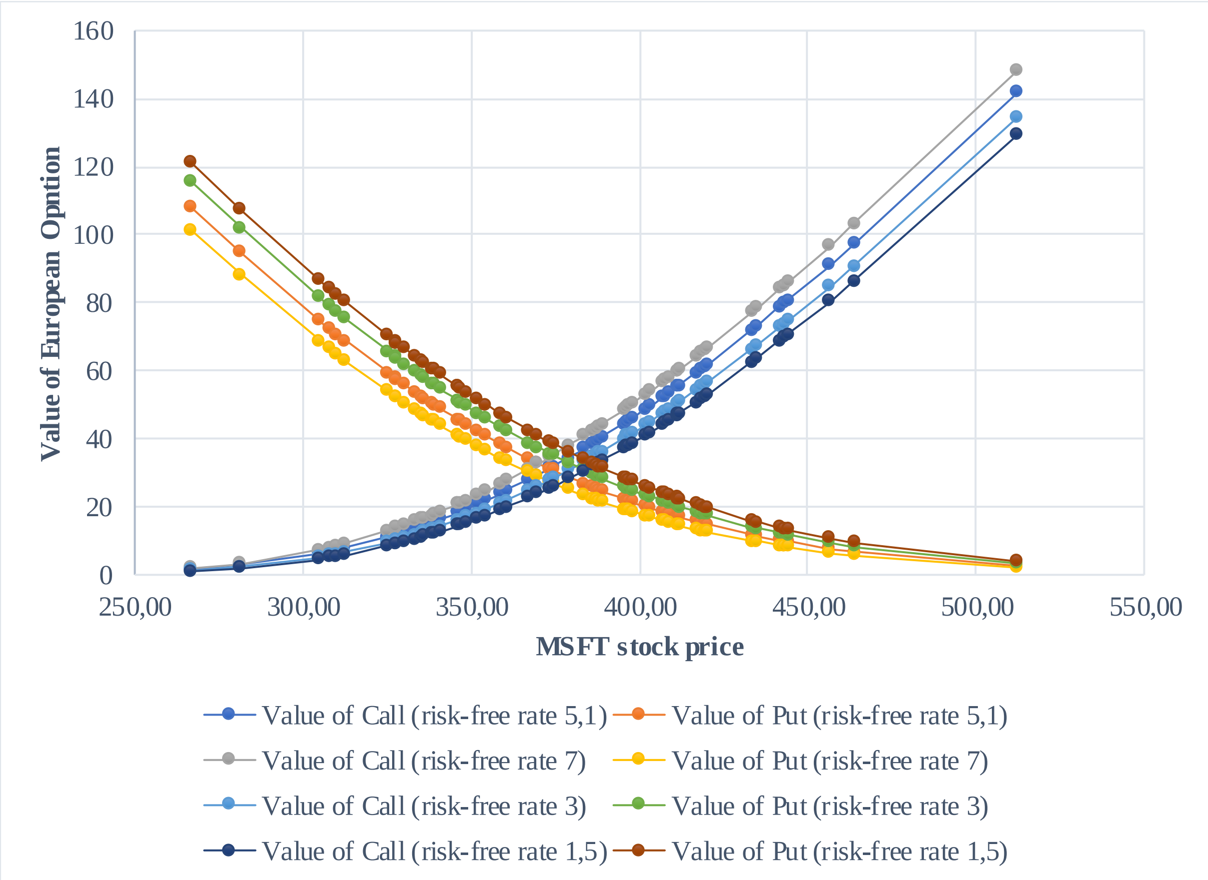


Source: prepared by the author

The fourth observation will be the relationship between the value of the European option and the *volatility* (see the graph in Figure 14), where the y-axis represents the value of the

European option, ranging from 0,01 USD to 160,17 USD, and the x-axis highlights the stock price in USD. All of the parameters are kept unchanged and assessed through changes in 4 equivalents of volatility - 10 %, 21 %, 30 % and 40 %. As demonstrated by the graph, increased volatility is associated with a higher valuation for both call and put options. This effect is most pronounced near the selected option strike price (k=390), where distinct behaviours can be observed depending on volatility levels. For instance, with a low volatility of 10%, the value line of the put option exhibits a more pronounced curvature across the range of stock prices. Conversely, at a higher volatility of 40%, the value-line for the put option tends to be flatter, indicating less sensitivity to changes in the stock price around the strike price. This graphical representation highlights the significant impact of volatility on option pricing dynamics, particularly in the vicinity of the strike price. To conclude, higher volatility enhances the potential reward for both call and put options, as calls benefit from upward price surges while puts benefit from downward movements, making the options more valuable as speculative tools or as hedging instruments.

Figure 15. The relationship between the value of the European option and a risk-free rate



Source: prepared by the author

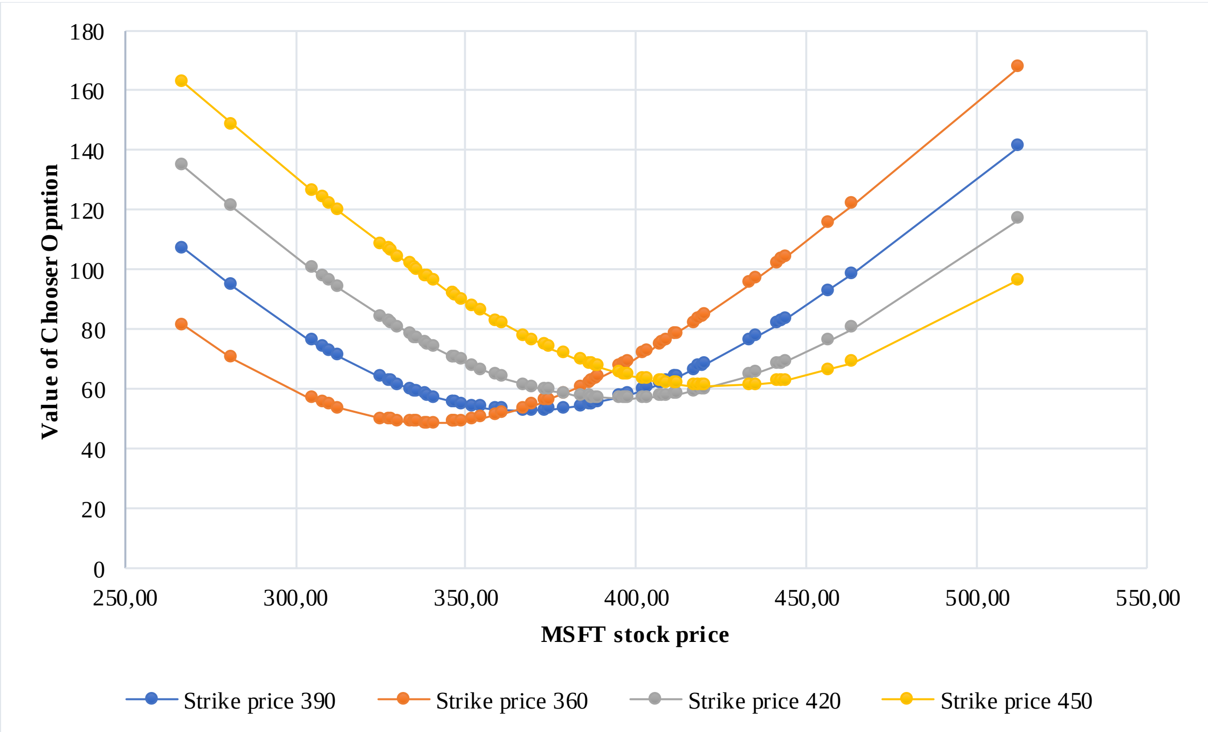
The fifth observation will be the relationship between the value of the European option and the *risk-free rate* (see the graph in Figure 15), where the y-axis represents the value of the European option, ranging from 1,03 USD to 148,23 USD, and the x-axis showcases stock price in

USD. All of the parameters are kept unchanged and assessed through changes in 4 equivalents of risk-free rate - 1,5 %, 3 %, 5,1 % and 7 %. From the graph above, it could be observed for a call option, that an increase in the risk-free rate leads to an increase in the value of a European call option, i.e. the impact of the risk-free rate at 7% for call option value is higher, than 1,5%, nevertheless, a more significant effect is more visible when the stock price is above the strike price.

On the contrary, using the Black-Scholes model the author has selected to evaluate the parameters of the chooser option as per the below list. They are similar to European put/call options, with the only separating feature - time till choice of option contract, which allows the holder to decide whether it functions as a call or a put option at a later date

Starting the assessment the relationship between the value of the chooser option and the *strike price* (see the graph in Figure 16), where the x-axis represents the value of the chooser option, ranging from 167,68 USD to 52,72 USD, and the y-axis showcases stock price in USD. All of the parameters are kept unchanged and assessed through changes in 4 equivalents of the strike price - 360 USD, 390 USD, 420 USD, and 450 USD.

Figure 16. The relationship between the value of chooser option and strike price

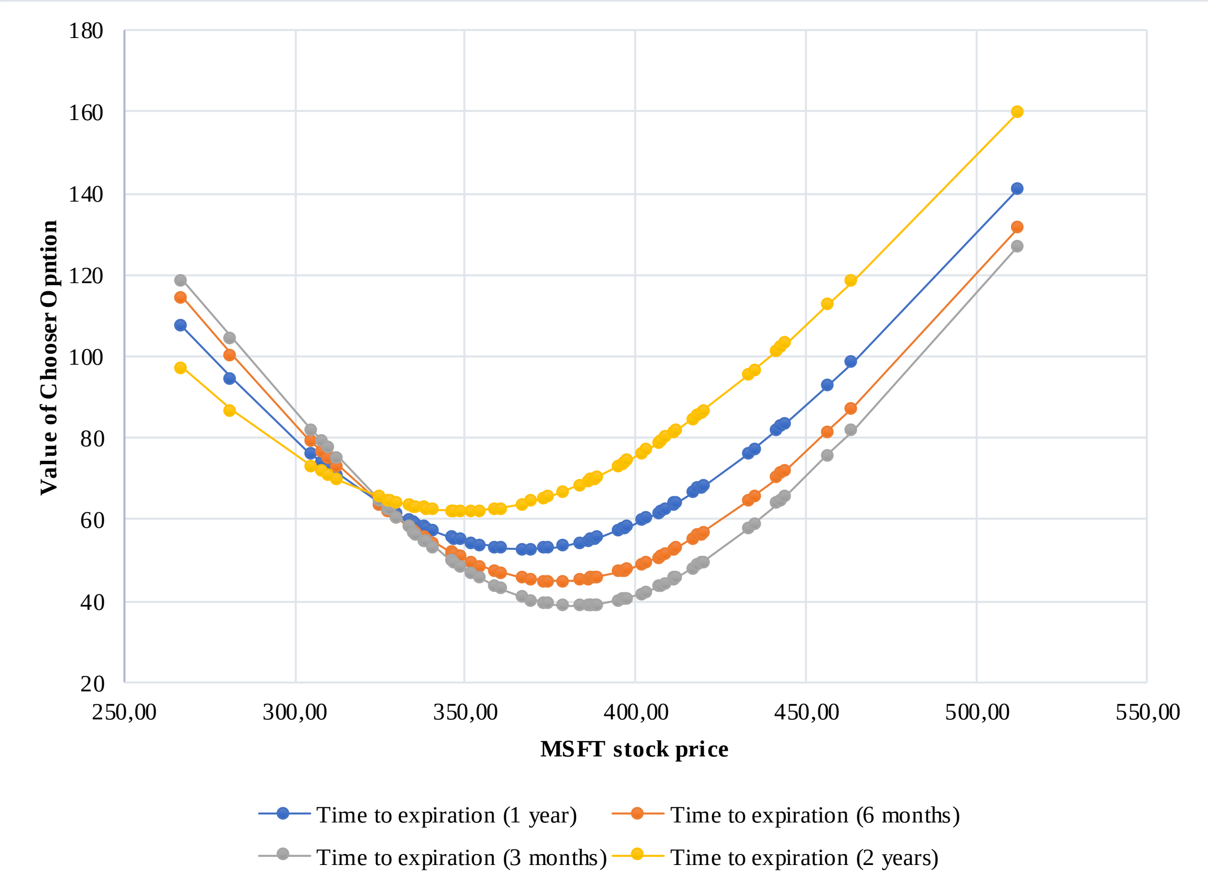


Source: prepared by the author

As previously the author was accessing the impact of a European call and put options, can definitely be observed the mix-and-match effect on the valuation of the chooser option and an assessment could be made from the put component and call component. In terms of value changes caused by strike price, the lowest value of the respective chooser option occurs roughly when the stock price is around 6-7% below the strike price. This point is a break-even point, as below this lowest point, the value path follows the path of European call (i.e. call component), while above it resembles the European put valuation (i.e. put component). As a chooser option has its embedded feature to make a choice of the type of option in the latter stage, it is reflected in its value by being more expensive than a separate call or put options.

The second observation will be the relationship between the value of the European option and the *time till maturity* (see the graph in Figure 17), where the x-axis represents the value of the chooser option, ranging from 40,26 USD to 159,86 USD, and the y-axis showcases stock price in USD. All of the parameters are kept unchanged and assessed through changes in 4 equivalents of time till expiration - 2 years, 1 year, 0,5 years (6 months) and 0,25 years (3 months).

Figure 17. The relationship between the value of chooser option and time to maturity

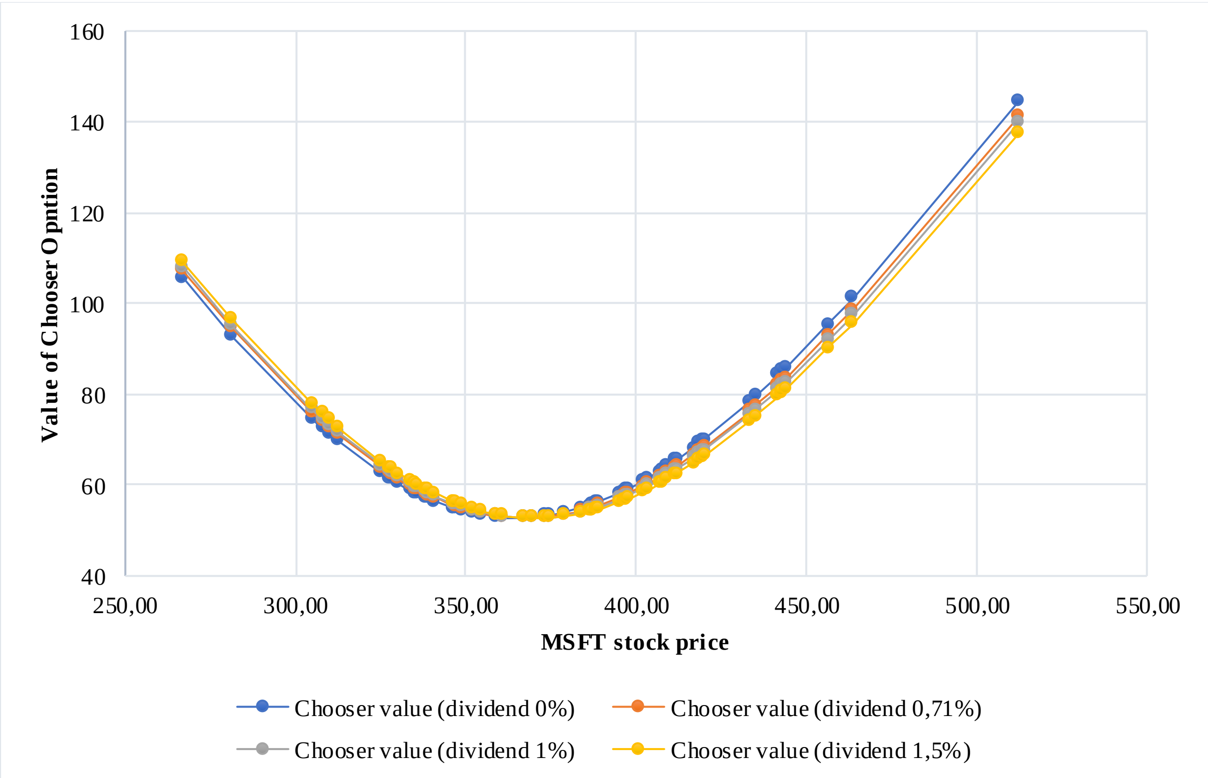


Source: prepared by the author

Interestingly, the break-even point for all option contracts with a strike price of USD 390 occurs, if the stock price reaches USD 327, i.e. it is below by 16% of the acquired right to buy/sell contact (as the reference (Figure 14), the same European put options had a similar crossing point at a bit higher level – USD 333). There is a visible trend, that the longer time horizon to maturity has a higher premium to be paid upfront when the stock price is around strike price and above (as in European calls), while if the value were less of stock price, shorter duration chooser options would cost more than a longer duration one.

The following observation will be the relationship between the value of the Chooser option and the *dividend yield* (see the graph in Figure 18), where the y-axis represents the value of the chooser option, ranging from 52,72 USD to 144,48; and the x-axis displays the stock price in USD. All of the parameters are kept unchanged and assessed through changes in 4 equivalents of dividend yield - 0 %, 0,71 %, 1,00 % and 1,50 %. An increase in dividend yield typically reduces the price of the underlying asset, which in turn influences the chooser option. For the call option component (in this particular example all the values below 333 USD in stock price), a higher dividend yield results in a lower asset price, making it less likely that the call will be exercised profitably, thereby decreasing its value. Conversely, for the put option component in the chooser option (i.e. when the stock price is above 333 USD), an increase in dividend yield enhances its value because the lower asset price increases the likelihood that the put will be in the money. Thus, similarly to European puts and calls, the effect of changes in the dividend field for a chooser option is limited.

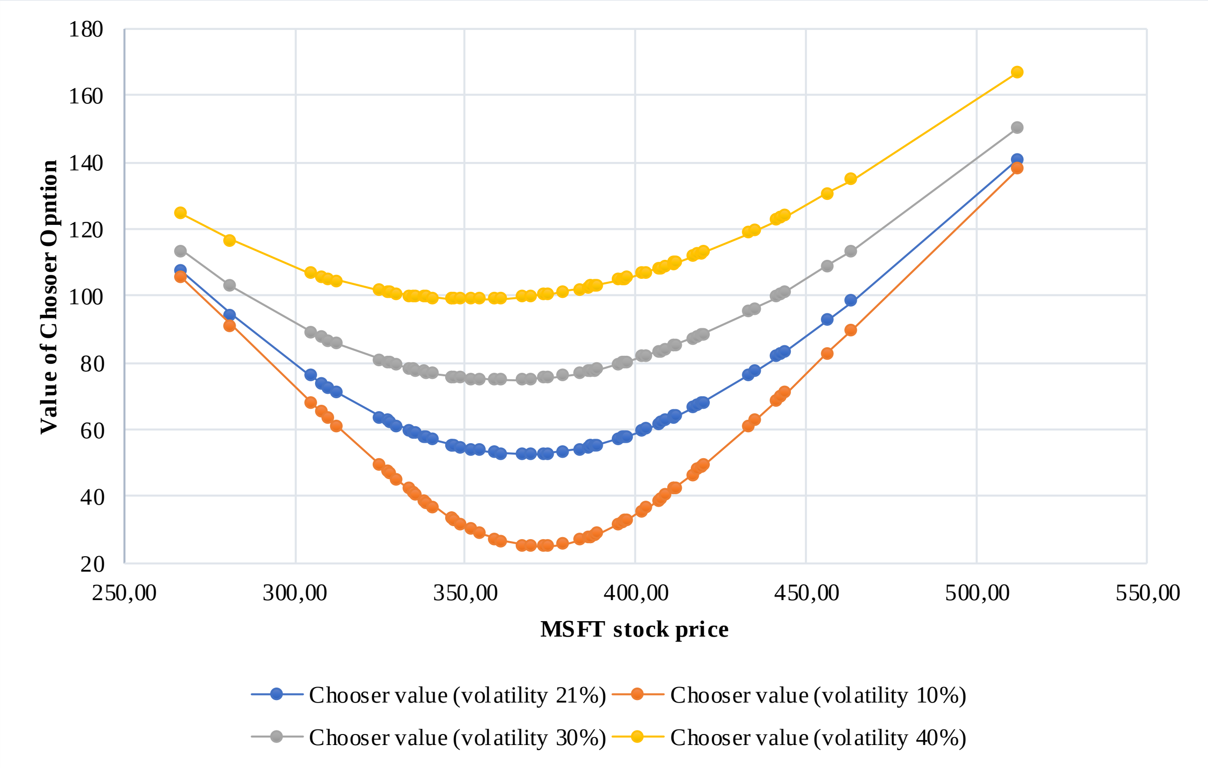
Figure 18. The relationship between the value of the chooser option and dividend yield



Source: prepared by the author

The fourth observation will be the relationship between the value of the chooser option and the *volatility* (see the graph in Figure 19), where the y-axis represents the value of chooser option, ranging from 25,05 USD to 167,08 USD, and the x-axis displays a stock price in USD. All of the parameters are kept unchanged and assessed through changes in 4 equivalents of volatility - 10 %, 21 %, 30 % and 40 %.

Figure 19. The relationship between the value of the chooser option and volatility



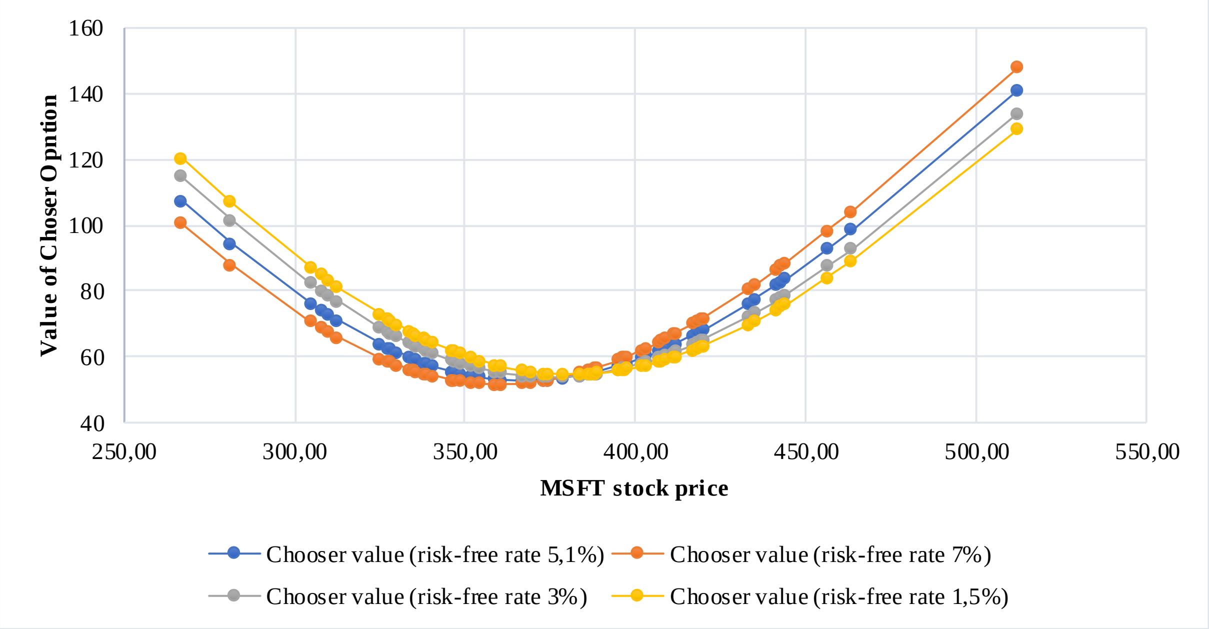
Source: prepared by the author

As it can be observed, in the graph, the chooser with a highest volatility rate (40%) is way more costly than the chooser at a volatility rate of 10%. This is because a higher volatility implies a greater uncertainty in the future price of the underlying asset, increasing the probability that the option will end up in-the-money, whether it is exercised as a call or a put. This enhanced potential for profitability makes the chooser option more valuable because it provides the flexibility to select the type of option (call or put) that could best capitalize on significant price movements, reflecting the increased chance of a favourable strike at maturity. As well, it should be noted that that in the range just a bit below expected strike price (below 15%), the valuation of chooser option has the most variation in between (e.g. at stock price 346 USD, identical chooser options except volatility would be worth from 33,42 USD (at 10% volatility) to 99,18 USD (at 40% volatility). So, a holder of chooser contract should be cautious about the change in volatility rate if the stock price is a bit below a strike price.

The fifth observation will be the relationship between the value of the chooser option and the *risk-free rate* (see the graph in Figure 20), where the y-axis represents the value of the chooser option, ranging from 1,03 USD to 148,23 USD, and the x-axis showcases the stock price in USD.

All of the parameters are kept unchanged and assessed through changes in 4 equivalents of risk-free rate - 1,5 %, 3 %, 5,1 % and 7 %.

Figure 20. The relationship between the value of the chooser option and the risk-free rate

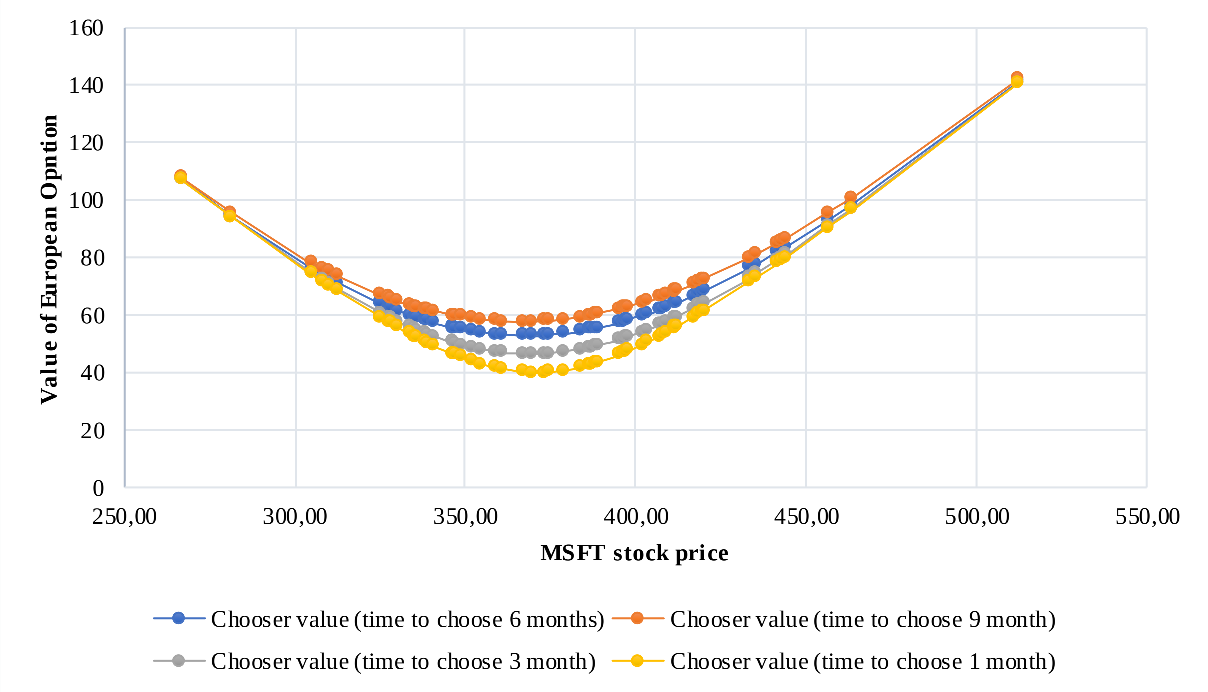


Source: prepared by the author

An increase in the risk-free rate typically enhances the value of the call component of the chooser option (this can be seen when the stock price goes above the strike price and a chooser option at 7% risk-free rate is price higher than the same chooser at 1,5% has a lower value). It is because the present value of the strike price, which is discounted using the risk-free rate, becomes cheaper, making it more attractive to exercise the call for a potential profit. Conversely, the put component of the chooser option generally decreases in value with a higher risk-free rate since the cost of paying the strike price in the future increases. However, since the chooser option provides the flexibility to select either the call or the put at a future date, the overall effect on the chooser option’s value will depend on the relative magnitude of these opposing effects. The net change in value can vary, but typically, the increase in the value of the call component due to higher risk-free rates might slightly outweigh the decrease in the put component, potentially leading to a modest overall increase in the value of the chooser option.

Lastly, the unique parameter for a chooser option – *time-to-choose* - the ability to delay the option type choice until the certain pre-agreed day in the future. It is a great strategic advantage point to select the most beneficial option type. See the graph in Figure 21, where the y-axis represents the value of the chooser option, ranging from 39,9 USD to 142,02 USD, and the x-axis showcases stock price in USD. All the parameters are constant, while the Time till choice of the option contract are 4 equivalents – 9 months, 6 months, 3 months and 1 month.

Figure 21. The relationship between the value of chooser option and time-to-chose



Source: prepared by the author

As it is visible from the graph, this extended observation period to make a decision of the type of contract can be particularly valuable but will cost a higher premium – a chooser with only a 1-month window to choose and 11 months of a call/a put would generally cost less if stock price is in a range of USD 350 and USD 400, i.e. in close proximity of strike price of USD 390. On the other hand, exactly the same chooser option with 9-month window to make a choice whether it is a put or a call and it offers increased potential for optimizing the financial outcome based on the most current market data would be around 20-25% more expensive in the same range (USD 350-400). However, if we suppose that the underlying stock price is very extremely far from the opted strike price, e.g. either less than USD 300 or above USD 450 - in that case, this parameter of choice becomes relatively insignificant and the premium in any case would cost similarly.

As in discussed graphs (*Figures 10-21*) revealed how different parameters impact the value of European put and call chooser options as well, the short summary of these effects as a partial conclusion is present in Table 8 below.

Table 8. Comparison of effect parameters between European and chooser option

Feature	European call/put	Chooser
Strike price	The effect: **medium** Call: if strike price increases, the value of option decreases Put: if strike prices increase, the value of option increases as well	The effect: **medium** Call: if strike price increases, the value of option decreases Put: if strike price increases, the value of option increases as well; nevertheless, value is higher as European option most of the time
Time till expiration	The effect: **high** Call and Put: if maturity is longer, the value of option increases	The effect: **high** Call and Put: if maturity is longer, the value of option increases
Dividend yield	The effect: **minor** Call: if dividend yield increases, the value of option decreases Put: if dividend yield increase, the value of option increases as well	The effect: **minor** Call: if dividend yield increases, the value of option decreases Put: if dividend yield increase, the value of option increases as well
Volatility	The effect: **high** Call and Put: if underlying stock is more volatile, the value of option increases	The effect: **high** Call and Put: if underlying stock is more volatile, the value of option increases
Risk-free rate	The effect: **minor** Call: if risk-free rate increases, the value of option increases Put: if risk-free rate prices increase, the value of option decreases	The effect: **minor** Call: if risk-free rate increases, the value of option increases Put: if risk-free rate prices increase, the value of option decreases
Time-to-choose	Not applicable	The effect: **medium** Call and Put: if time-to-choose increases, the value of option increases, as the exercise day is nearer and more time to evaluate market conditions

Source: prepared by the author

The comparison between European and chooser options reveals that both share similar sensitivities to strike price, time till expiration, dividend yield, volatility, and risk-free rate. However, chooser options generally have higher values than European options for the same strike price and feature a medium effect from the time-to-choose component, which increases option value as it allows more time to evaluate market conditions.

3.3. Chooser option versus plain vanilla trading strategy

The comparison of how the values of different plain vanilla and exotic options are affected by different parameters is enlightening, however, it is worth exploring the payout scenarios if there would be different trading strategies employed for European options versus a chooser option, which gives its flexibility in deciding whether the option should be a call or a put by the time of execution.

The author has selected to simulate common and popular option trading strategies, which could be compared with a chooser option and as well offer different market outlooks and risk tolerance levels. The simulation will not account for any brokerage fees, just pure premiums.

The straddle strategy involves two European options – put and call, and they are bought to have the same strike price.

Table 9. Variables involved in simulation with Straddle strategy

Options	Current underlying asset price	Strike price	Time-to-expiration (years)	Premium
European Call	$379,00	$390,00	1	$34,41
European Put	$379,00	$390,00	1	$28,66
Chooser	$379,00	$390,00	1	$53,53

Source: prepared by author

The author proposes the test scenario involving three different market situations, where time-to-choose in chooser option is equal to 6 months and the stock price is 1) USD 320,00 2) USD 400,00 3) USD 480,00.

Table 10. Computational results in pay-out of straddle strategy

Stock price	*Straddle*				
	Call	Status	Put	Status	Profit/loss
$320,00	-$34,41	OTM	$41,34	ITM	$692,73
$400,00	-$34,41	OTM	-$28,66	OTM	-$6 307,27
$480,00	$55,59	ITM	-$28,66	OTM	$2 692,73

Source: prepared by author

Table 11. Computational results in pay-out of chooser option

Stock price	*Chooser as a call*			*Chooser as a put*		
	Call	Status	Profit/loss	Put	Status	Profit/loss
$320,00	-$53,53	OTM	-$5 353,35	$16,47	ITM	$1 646,65
$400,00	-$53,53	OTM	-$5 353,35	-$53,53	OTM	-$5 353,35
$480,00	$36,47	ITM	$3 646,65	-$53,53	OTM	-$5 353,35

Source: prepared by author

As it can be visible from the results in the simulation (see Tables 10-11), a straddle strategy and a chooser option are beneficial as well as profitable, if the underlying asset is volatile. A chooser option provides a better edge in comparison to the premium paid amount (i.e. chooser's premium is equal to USD 53,53, while purchasing both European call and put equals to USD 63,07). As well, it could be visible, that a return from a chooser option, if determined correctly to exercise as put or call in the end, will be much higher than from a straddle strategy.

CONCLUSIONS AND PROPOSALS

The rapid development in technologies during the last couple of decades change the financial landscape by offering possibilities to create new variations and sub-types of financial instruments, to ensure better hedging possibilities and risk management, or even speculation. It could be defined, that the attractiveness of financial derivatives, especially on option contracts refer to their flexibility to adapt in various even unexpected situations or limit the loss till the paid premium to the writer of particular option.. With reference to scientific literature, it was revealed, that substantial impact for the value of option contract is made by following parameters: underlying price, strike price, time to expiration, volatility of underlying asset, risks-free rate and dividend yield (if underlying asset distributes dividends). Itself, their sensitivities could be assessed via The Greeks, which are common indices used by traders and portfolio managers for accurate identification of risk factors, which could be suppressed if the tendency of undesirable results is noticed on time. However, sensitivities might not be enough, the pricing of options involving the mentioned 6 parameters as key components, but macro-economic variables, ESG impact, and general market forces – demand and supply - as well should be incorporated into the valuation of option contracts. Moreover, there are loads of different type of option contracts created in theory, which might not be so used in practice due to its hard assessment in value, in particular, due to constantly changing market conditions. Pragmatic and pioneering option valuation methods (Black-Scholes, Binomial) are even further expanding with increased pressure of heavily data-driven computational models powered by AI, but these machine learning algorithms (random forests, regression trees) require emersed knowledge base from the user as well as computational energy, thus leading to as clear and concise understanding as possible.

The second part, covering the methodology in this research, demonstrates a sophisticated approach to the analysis of option pricing sensitivities using Greek letters, effectively capturing the nuanced impacts of varying parameters. The adaptation of the Black-Scholes-Merton model to incorporate a dividend yield parameter enhances its applicability across both plain vanilla and chooser options. This modification, coupled with the integration of a log-normal distribution for stock prices and inclusion of other real-market variables, provides a robust framework for comparing these two distinct types of option contracts across six key parameters: strike price, time till expiration, dividend yield, volatility, risk-free rate, and the unique time-to-choose parameter pertinent to chooser options. Furthermore, the comparative analysis of trading strategies between European and chooser options offers insightful perspectives on potential payouts, highlighting the practical implications of theoretical models in real market scenarios. This comprehensive methodological approach not only deepens the understanding of option pricing dynamics but also contributes valuable insights into strategic decision-making in financial markets.

Finally, in the third part, using historical data of existing companies and realistic circumstances of the market, the author has built the model for further examinations via the Greek letters and the Black-Scholes-Merton model. As comparative study objects are selected the pair of plain vanilla and exotic derivatives – European and chooser options. Starting the analysis of the Greek letters, it has shown, that chooser options are more sensitive to volatility and time decay than European options, while chooser options tend to easier adjust to delta and risk-free rate. After the conducted calculations via the Black-Scholes-Merton model, it can be briefly stated, that both European options are cheaper than simple chooser options, but the reaction to the change in volatility is proportionate. Moreover, the selected strike price actually modifies the value of both options, while time parameters - time for exercise and time-to-choose (applicable only for the chooser option as a distinctive feature) are utterly affected only when the price of the underlying asset is near the predetermined strike estimate. On the contrary, the changes in the dividend yield and risk-free rate do not have a considerable influence. At last, the chooser option shows better resistance to risks, and it has lower costs in comparison with the options trading strategy straddle when an investor buys two identical option contracts in opposite directions. Nevertheless, whether exotic options or diverse trading strategies are beneficial depends on the specific financial context, the strategic objectives of the investor, and their capacity to manage the associated risks and complexities. Each approach has its merits and can be more effective in different scenarios.

The exploration of option pricing continues to be a vibrant area of research, integrating insights from market behaviour, mathematical theories, and computational advancements to better understand and predict pricing dynamics in various market conditions. The risks associated with option contracts going forward into the future are multi-dimensional, reflecting the evolving landscape of financial markets. Volatility risk remains a significant challenge, as markets face heightened uncertainty due to factors like geopolitical tensions, economic crises and pandemics. Liquidity risk, particularly in less-traded or exotic options, can exacerbate price slippage, complicating trading strategies. Thus, besides a chooser option, further research could focus further on accessing the potential of further exotic options (i.e. barrier, Asian, compound), as research areas are quite limited due to over-counter trading and tailor-made parameters in exotic options. As well, as further development into reflecting the influence of other parameters such as ESG, inflation, liquidity of underlying assets or more specifically for exotic options - counterparty risk, to both European and Exotic options.

The future of option contract valuation, especially for exotic options, will be marked by greater sophistication and adaptability in pricing models, fuelled by advancements in AI, machine learning, quantum computing, and big data. But everyone should be mindful of both opportunities and risks in this kind of advancement. While AI can process vast data sets and enhance pricing accuracy, it is vulnerable to overfitting, lack of transparency, and reliance on historical data, which can result in poor predictions during market shocks. Additionally, AI's "black box" nature complicates understanding and mitigating model failures, potentially amplifying systemic risks. Counterparty risk and systemic risk also remain concerns, particularly in the OTC options market, where the failure of a major institution could trigger a broader financial collapse. The effectiveness of hedging strategies may be compromised by incorrect assumptions or sudden market shifts, further complicating risk management. Finally, behavioural and market sentiment risks, influenced by investor psychology and herd behaviour, can lead to mispricing and erratic price movements. As these risks become more interconnected, financial professionals will need to adapt continuously, integrating advanced modelling techniques, including AI, while remaining cognizant of their limitations in the face of unpredictable market dynamics. Ultimately, the evolution of option pricing will depend on the continued development of new technologies, regulatory scrutiny, improved risk management practices, and the ability of market participants to adapt to rapidly changing market conditions, but by better understanding the dynamics behind option pricing, financial professionals can make more informed decisions, helping to prevent the unintentional misuse of derivatives that could lead to unforeseen financial crises in the future.

LIST OF REFERENCES AND SOURCES

Abdel, A. (2019). How Enron Used Accounting for Prepaid Commodity Swaps to Delay Bankruptcy for One Decade: The Shadowy Relationships With Big Banks. *Journal of Accounting, Auditing & Finance, 34*(2), 309-328. doi:10.1177/0148558X17724249

Ahmed, D., Judge, A., & Mahmud, S. (2018). Does derivatives use reduce the cost of equity? *International Review of Financial Analysis, 60*, 1-16. Retrieved from https://doi.org/10.1016/j.irfa.2018.09.004

Ait-Sahalia, Y., & Duarte, J. (2003). Nonparametric option pricing under shape restrictions. *Journal of Econometrics, 116*(1-2), 9-47. doi:10.1016/S0304-4076(03)00102-7

Atmaz, A., & Basak, S. (2019). Option prices and costly short-selling. *Journal of Financial Economics, 134*(1), 1-28. doi:10.1016/j.jfineco.2019.04.004

Bartram, S. M., Conrad, J., Lee, J., & Subrahmanyam, M. G. (2022). Credit Default Swaps around the World. *The Review of Financial Studies, 35*, 2464-2524. doi:10.1093/rfs/hhab092

Bekiros, S., & Kouloumpou, D. (2019). On the pricing of exotic options: A new closed-form valuation approach. *Chaos, Solitons & Fractals, 122*, 153-162. Retrieved from https://doi.org/10.1016/j.chaos.2019.03.012

Bernales, A., Verousis, T., & Voukelatos, N. (2020). Do investors follow the herd in option markets? *Journal of Banking & Finance, 119*, 113-127. doi:10.1016/j.jbankfin.2016.02.002

Binsbergen, J., Diamond, W., & Grotteria, M. (2022). Risk-free interest rates. *Journal of Financial Economics, 143*(1), 1-29. doi:10.1016/j.jfineco.2021.06.012

BIS. (2018). *Statistics of OTC derivatives*. The Bank of International Settlements. Retrieved from https://stats.bis.org/statx/toc/DER.html

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy, 81*(3), 637-654. Retrieved from https://www.journals.uchicago.edu/doi/pdfplus/10.1086/260062

Bloomberg. (2024). *Dividend yield for Microsoft Corp. 01/07/2023 to 30/06/2024*. Retrieved from Bloomberg database

Boyle, P., & McDougall, J. (2019). *Trading and Pricing Financial Derivatives: a guide to futures, options and swaps.* Scotts Valley: CreateSpace independent publishing platform.

Borkowski, B., & Krawiec, M. (2009). Possibilities of investing on European Wheat market with the use of chooser options. *First International Conference Quantitative Methods in Economics, Sessions B* (p. 13 p.). Warsaw: Science24. Retrieved from http://science24.com/paper/17945

Boswijk, H. P., Laeven, R. J., & Vladimirov, E. (2024). Estimating option pricing models using a characteristic function-based linear state space representation. *Journal of Econometrics, 244*(1), 105-133. doi:10.1016/j.jeconom.2024.105864

Brini, A., & Lenz, J. (2024). Pricing cryptocurrency options with machine learning regression for handling market volatility. *Economic Modelling, 136*, 106-134. doi:10.1016/j.econmod.2024.106752

Buffet, W. E. (2002). *Annual report of Berkshire Hathaway Inc.* Omaha: Berkshire Hathaway Inc. Retrieved from http://www.berkshirehathaway.com/2002ar/2002ar.pdf

Cao, J., Goyal, A., Zhan, X., & Zhang, W. (2023). Unlocking ESG Premium from Options. *Swiss Finance Institute Research Paper No. 21-39*. doi:10.2139/ssrn.3878123

CFA Institute. (2021). *CFA Institute Investment Series: Derivatives*. New York: John Wiley & Sons.

Chance, D. M., & Brooks, R. (2021). *Introduction to Derivatives and Risk managment* (Vol. 10). Boston: Cengage Learning.

Chavas, J.-P., Li, J., & Wang, L. (2024). Option pricing revisited: The role of price volatility and dynamics. *Journal of Commodity Markets, 33*, 100-134. doi:10.1016/j.jcomm.2023.100381

Chen, Y., Yu, H., Meng, X., Xie, X., Hou, M., & Chevallier, J. (2021). Numerical solving of the generalized Black-Scholes differential equation using Laguerre neural network. *Digital Signal Processing, 112*. doi:10.1016/j.dsp.2021.103003

Chicago Board Options Exchange. (2024, August 12). *Options On Cboe Volatility Index Futures*. Retrieved August 19, 2024, from Chicago Board Options Exchange: https://www.cboe.com/VIX-OOF-pipeline-hub/

Choi, J., Kwak, M., Tee, C. W., & Wang, Y. (2022). A Black–Scholes user's guide to the Bachelier model. *The Journal of Futures Markets, 42*(5), 959-980. doi:10.1002/fut.22315

Cuthbertson, K., Nitzsche, D., & O'Sullivan, N. (2019). *Derivatives: Theory and Practice*. New York: Wiley.

D'Amico, G., & Villani, G. (2021). Valuation of R&D compound option using Markov chain approach. *Annals of Finance, 17*, 379-404. doi:10.1007/s10436-021-00389-1

DeMarzo, P. M., Kremer, I., & Mansour, Y. (2016). Robust option pricing: Hannan and Blackwell meet Black and Scholes. *Journal of Economic Theory, 163*, 410-434. Retrieved from https://doi.org/10.1016/j.jet.2016.01.009

Devreese, J. P., Lemmens, D., & Tempere, J. (2010). Path integral approach to Asian options in the Black-Scholes model. *Physica A; Statistical Mechanics and its Applications, 389*(4), 780-788. Retrieved from https://doi.org/10.1016/j.physa.2009.10.020

Dorion, C. (2016). Option Valuation with Macro-Finance Variables. *Journal of Financial Quantitative Analysis, 51*(4), 11-33. doi:10.1017/S0022109016000442

Duran, R. E., & Griffin, P. (2021). Smart contracts: will Fintech be the catalyst for the next global financial crisis? *Journal of Financial Regulation and Compliance, 29*(1), 104-122. doi:10.1108/JFRC-09-2018-0122

Durica, M., & Svabova, L. (2014). Delta and Gamma for Chooser Options. *17-th Applications of mathematics and statistics in economics*, (pp. 75-84). Poland. Retrieved from https://doi.org/10.15611/AMSE.2014.17.08

Escobar-Anel, M., Rastegari, J., & Stentoft, L. (2021). Option pricing with conditional GARCH models. *European Journal of Operational Research, 289*(1), 350-363. doi:10.1016/j.ejor.2020.07.002

Escobar-Anel, M., Stentoft, L., & Ye, X. (2024). Not all VIXs are (Informationally) equal: Evidence from affine GARCH option pricing models. *Finance Research Letters, 69*(A), 11-14. doi:10.1016/j.frl.2024.106053

Fang, Z., & Han, J.-Y. (2025). Realized GARCH Model in Volatility Forecasting and Option Pricing. *Computational Economics*. doi:10.1007/s10614-024-10826-8

Ford, J. M., Gehricke, S., & Zhang, J. (2022). Option traders are concerned about climate risks: ESG ratings and short-term sentiment. *Journal of Behavioral and Experimental Finance, 35*. doi:10.1016/j.jbef.2022.100687

Gan, L., Wang, H., & Yang, Z. (2020). Machine learning solutions to challenges in finance: An application to the pricing of financial products. *Technological Forecasting and Social Change, 153*, 119-144. doi:10.1016/j.techfore.2020.119928

Gao, Y., & Jia, L. (2021). Pricing formulas of barrier-lookback option in uncertain financial markets. *Chaos, Solitons & Fractals, 147*. doi:10.1016/j.chaos.2021.110986

Gao, R., Wu, W., Lang, C., & Lang, L. (2020). Geometric Asian barrier option pricing formulas of uncertain stock model. *Chaos, Solitons & Fractals, 140*, 110-121. doi:10.1016/j.chaos.2020.110178

Glaryrina, A., & Melnikov, A. (2020). Bachelier model with stopping time and its insurance application. *Insurance: Mathematics and Economics, 93*, 156-167. doi:10.1016/j.insmatheco.2020.04.012

Grima, S., & Thalassinos, E. (2020). Financial Derivatives: A Blessing or a Curse? *Financial Derivatives Use: A Literature Review*, 23-63. doi:10.1108/978-1-78973-245-020201003

Gueant, O., & Pu, J. (2015). Option pricing and hedging with execution costs and market impact. *27*(3), 803-831. doi:10.1111/mafi.12102

Guo, F. (2024). Pricing Vulnerable Options With Variance Gamma Systematic and Idiosyncratic Factors by Laplace Transform Inversion. *Journal of Futures Markets, 45*(1), 47-76. doi:10.1002/fut.22554

Guo, J., Wang, Y., & Kang, W. (2024). Multi-perspective option price forecasting combining parametric and non-parametric pricing models with a new dynamic ensemble framework. *Technological Forecasting and Social Change, 204*. doi:10.1016/j.techfore.2024.123429

Han, Q., & Song, X. (2025). Quantum walk option pricing model based on binary tree. *Physica A: Statistical Mechanics and its Applications, 658*, 130-144. doi:10.1016/j.physa.2024.130205

Hasler, M., & Jeanneret, A. (2022). A Macrofinance Model for Option Prices: A Story of Rare Economic Events. *Management Science, 69*(9), 33-48. doi:10.1287/mnsc.2022.4587

He, X.-J., Pasricha, P., & Lin, S. (2024). Analytically pricing European options in dynamic markets: Incorporating liquidity variations and economic cycles. *Economic Modelling, 139*. doi:10.1016/j.econmod.2024.106839

Heston, S., Jacobs, K., & Hyung , J. (2023). The Pricing Kernel in Options. *Finance and Economics Discussion Series*. doi:10.17016/FEDS.2023.053.

Hu, Y., Lindquist, W., & Rachev, S. (2024). ESG-valued discrete option pricing in complete markets. *Journal of Sustainable Finance & Investments*, 1-35. doi:10.1080/20430795.2024.2330518

Hu, J., & Gan, S. (2018). High order method for Black-Scholes PDE. *Computers & Mathematics with Applications, 75*(7), 2259-2270. Retrieved from https://doi.org/10.1016/j.camwa.2017.12.002

Hull. (2021). *Options, Futures, and Other Derivatives* (Vol. 11). London: Pearson.

IMF. (1993). *International Monetary Fund*. Retrieved August 5, 2024, from Financial derivatives. A supplement of the Balance of Payments Manual: https://www.imf.org/external/pubs/ft/fd/2000/finder.pdf

Institute, C. (2021). *CFA Institute Investment Series: Derivatives.* New York: John Wiley & Sons.

Ivascu, C.-F. (2021). Option pricing using Machine Learning. *Expert Systems with Applications, 163*. doi:10.1016/j.eswa.2020.113799

Jayeola, O. (2020). Inefficiencies in trade reporting for over-the-counter derivatives: Is blockchain the solution? *Capital Markets Law Journal, 15*(1), 48-69. doi:10.1093/cmlj/kmz028

Jalan, A., Matkovskyy, R., & Aziz, S. (2021). The Bitcoin options market: A first look at pricing and risk. *Applied Economics, 53*(17), 2026-2041. doi:10.1080/00036846.2020.1854671

Jang, J. H., Yoon, J., Kim, J., Gu, J., & Kim, H. Y. (2021). DeepOption: A novel option pricing framework based on deep learning with fused distilled data from multiple parametric methods. *Information Fusion, 70*, 43-59. doi:10.1016/j.inffus.2020.12.010

Jarrow, R. A., & Chatterjea, A. (2024). *An Introduction to Derivative securities, Financial Markets and Risk Management* (3 ed.). Singapore: World Scientific Publishing Company.

Jeong, D., Yoo, M., & Yoo, C. (2019). A Hybrid Monte Carlo and Finite Difference Method for Option Pricing. *Computational Economics, 53*, 111-124. doi:10.1007/s10614-017-9730-4

Kim, B., Kim, J., Yoon, H., & Lee, J. (2024). Pricing of discretely sampled arithmetic Asian options, under the Hull–White interest rate model. *The North American Journal of Economics and Finance, 74*, 22-45. doi:10.1016/j.najef.2024.102239

Kim, H.-G., Kwon, S.-J., & Kim, J.-H. (2022). Pricing path-dependent exotic options with flow-based generative networks. *Applied Soft Computing, 124*, 109-133. doi:10.1016/j.asoc.2022.109049

Labuschagne, C. C., & Boetticher, S. T. (2016). Dupire's formulas in the Piterbarg option pricing model. *The North American Journal of Economics and Finance, 38*, 148-162. doi:10.1016/j.najef.2016.09.002

Lai, Y.-W. (2017). Macroeconomic factors and index option returns. *International Review of Economics & Finance, 48*, 452-477. doi:10.1016/j.iref.2016.11.002

Le, V. P., Meenagh, D., & Minford, P. (2018). Financial stability: to regulate or not? A public choice inquiry. *Journal of International Financial Markets, Institutions and Money, 57*, 127-140. doi:10.1016/j.intfin.2018.07.004

Li, K. (2024). Option trading and equity price efficiency. *Journal of Corporate Finance, 88*, 102-120. doi:10.1016/j.jcorpfin.2024.102630

Lian, Y.-M., & Chen, J.-H. (2023). Valuation of chooser options with state-dependent risks. *Finance Research Letters, 52*. doi:10.1016/j.frl.2022.103527

Liu, S., Oosterlee, C. W., & Sander, B. M. (2019). Pricing Options and Computing Implied Volatilities using Neural Networks. *Modern Numerical Techniques and Machine-Learning in Pricing and Risk Management, 7*(1). doi:10.3390/risks7010016

Liu, Z., Li, D., Qian, L., & Yao, J. (2024). On the pricing of vulnerable Parisian options. *Finance Research Letters, 68*. doi:10.1016/j.frl.2024.105995

Luo, Q., Jia, Z., Li, H., & Wu, Y. (2022). Analysis of parametric and non-parametric option pricing models. *Heliyon - Business and Economics, 8*(11), 113-129. doi:10.1016/j.heliyon.2022.e11388

Martinkutė-Kaulienė, R. (2012). Exotic options: a chooser option and its pricing. *Business,Management and Education, 10*(2), 289-301. Retrieved from https://doi.org/10.3846/bme.2012.20

Maurya, V., Singh, A., Yadav, V. S., & Rajpoot, M. K. (2024). Efficient pricing of options in jump–diffusion models: Novel implicit–explicit methods for numerical valuation. *Mathematics and Computers in Simulation, 217*, 202-225. doi:10.1016/j.matcom.2023.10.025

Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics, 3*(1-2), 125-144. doi:10.1016/0304-405X(76)90022-2

Mink, M., & Weert, F. (2023). Black-Scholes-Merton Option Pricing Revisited: Did we Find a Fatal Flaw? *The review of FInancial studies*, 18. doi:10.2139/ssrn.4461094

Nejad, M. G. (2022). Research on financial innovations: an interdisciplinary review. *International Journal of Bank Marketing, 40*(3), 578-612. doi:10.1108/IJBM-07-2021-0305

Newall, P. W., & Weiss-Cohen, L. (2022). The Gamblification of Investing: How a New Generation of Investors Is Being Born to Lose. *International Economics, 19*(9), 67-89. doi:10.3390/ijerph19095391

Nguyen, T., & Wu, J. (2013). A New Classification of Exotic Options. *Bankers, Markets and Investors, No. 112, May - June 2011*, 1-9. Retrieved July 26, 2024, from https://ssrn.com/abstract=1735805

Parminder, B., & Jasmeet, K. (2020). Recent developments and applicability of the black and scholes model in option pricing: A literature review. *Journal of Finance and Accounting, 7*(2), 158-183. doi:10.17492/jpi.mudra.v7i2.722034

Poncet, P., & Portrait, R. (2022). *Exotic Options . In: Capital Market Finance.* Springer, Cham: Springer Texts in Business and Economics. doi:10.1007/978-3-030-84600-8_14

Rao, S. S., & Manisha. (2018). Numerical solution of generalized Black-Scholes model. *Applied Mathematics and Computation, 321*, 401-421. Retrieved from https://doi.org/10.1016/j.amc.2017.10.004

Rombouts, J. V., Stentoft, L., & Violante, F. (2020). Pricing individual stock options using both stock and market index information. *Journal of Banking & Finance, 111*. doi:10.1016/j.jbankfin.2019.105727

Ruf, J., & Wang, W. (2019). Neural Networks for Option Pricing and Hedging: A Literature Review. *Journal of Computational Finance, 32*. doi:10.2139/ssrn.3486363

Sesana, D., Marazzina, D., & Fusai, G. (2014). Pricing exotic derivatives exploiting structure. *European Journal of Operational Research, 236*(1), 369-381. doi:10.1016/j.ejor.2013.12.009

Shylow, M. (2023). Weather derivatives: general overview, legal, tax, and application issues. *Journal of Management and Financial Sciences, 49*, 53-72. doi:10.33119/JMFS.2023.49.3

Shvimer, Y., & Zhu, S.-P. (2024). Pricing options with a new hybrid neural network model. *Expert Systems with Applications, 251*. doi:10.1016/j.eswa.2024.123979

Staelen, R. D., & Hendy, A. (2017). Numerically pricing double barrier options in a time-fractional Black-Scholes model. *Computers&Mathematics with Applications, 74*(6), 1166-1175. Retrieved from https://doi.org/10.1016/j.camwa.2017.06.005

Su, K., Zhang, M., & Liu, C. (2022). Financial derivatives, analyst forecasts, and stock price synchronicity: Evidence from an emerging market. *Journal of International Financial Markets, Institutions and Money, 81*. doi:10.1016/j.intfin.2022.101671

Tarbert, H. (2020). Self-regulation in the derivatives markets: stability through collaboration. *Northwestern Journal of Internal Law & Business, 41*(2). Retrieved August 3, 2024, from https://scholarlycommons.law.northwestern.edu/njilb/vol41/iss2/2

The United StateS Government. (2024). *U.S. Department of the Treasury*. Retrieved August 10, 2024, from https://home.treasury.gov/policy-issues/financing-the-government/interest-rate-statistics

Vaswani, P., Mundakkad, P., & Jayaprakasam, K. (2023). Financial Option Pricing Using Random Forest and Artificial Neural Network: A Novel Approach. *Proceedings of International Joint Conference on Advances in Computational Intelligence*, 419-433. doi:10.1007/978-981-99-1435-7_36

Wang, S., & Yuan, F. (2019). The asymptotic behavior of the solutions of the Black-Scholes equation as volatilty tends to zero. *Computers & Mathematics with Applications*, 10 p. Retrieved from https://doi.org/10.1016/j.camwa.2019.03.028

Wu, J., Wang, Y., Zhu, M., Zheng, H., & Li, L. (2023). Exotic option pricing model of the Black–Scholes formula: a proactive investment strategy. *Sec. Mathematical Physics, 11*. doi:10.3389/fphy.2023.1201383

Zhang, J., & Zhou, K. (2024). Pricing options on the maximum or the minimum of several assets with default risk. *The North American Journal of Economics and Finance, 75*(A), 17-34. doi:10.1016/j.najef.2024.102272

IŠVESTINIŲ FINANSINIŲ PRIEMONIŲ VERTINIMAS: PAPRASTIEJI IR EGZOTINIAI PASIRINKIMO SANDORIAI

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Darbo vadovas prof. dr. A. Šapkauskienė, Vilnius, 2025

SANTRAUKA

68 puslapiai, 21 paveikslai, 11 lentelių, 90 šaltinių.

Baigiamajame magistro darbe nagrinėjamas opcionų vertės nustatymo sudėtingumas, daugiausia dėmesio skiriant įvertinti pačių paprastųjų finansinių išvestinių priemonių - europinių pirkimo/pardavimo – taip pat, egzotinių pasirinkimų sandorius, parametrų pokyčių įtaką jų vertei.

Magistro darbas suskirstytas į tris pagrindines dalis: Literatūros analizę, empirinį tyrimo modelį ir jo rezultatus, išvadas ir rekomendacijas.

Literatūros apžvalgoje nagrinėjama išvestinių finansinių priemonių kilmė, išryškinami jų kompleksiškumas, dėmesį sutelkiant į pasirinkimo sandorius bei jų jautrumą parametrų pokyčiams, pvz., perkamo turto vykdymo kaina, sandorio galiojimo laikas, bazinio turto rinkos kaina, bazinio turto dividendų pelningumas, bazinio turto kainos kintamumas, nerizikinga palūkanų norma. Taip pat pateikiamos istorinės ir teorinės pasirinkimo sandorių vertinimo retrospektyva - nuo Black-Scholes pasirinkimo sandorių kainodaros modelio iki pažangių ir sudėtingų DI taikymo metodų. Pažymėtina, kad makroekonominių kintamųjų ir ESG veiksnių integravimas kartu su pastarųjų pasaulinių įvykių poveikiu rinkos dinamikai palaipsniui nagrinėjamas ir įtraukiamas į pasirinkimo sandorių sutarčių vertinimą.

Atlikus literatūros analizę, metodologinėje dalyje pristatomas pasirinkimo sandorių kainos jautrumui nustatyti skirtas graikiškųjų raidžių metodas. Be to, paprastųjų europietiškų ir egzotinių pasirinkimo sandorių vertinimui yra pritaikomas modifikuotas Black-Scholes-Merton modelis, kuris lygina pagrindinius parametrus, tokius kaip antai bazinio turto kainos kintamumas, sandorio galiojimo laikas ir t.t. Taip pat, palyginimui naudojama opcionų prekybos strategijos. Empirinė duomenų analizė atliekama naudojant „Excel“, pateikiant įžvalgas apie praktinį teorinių modelių taikymą.

Išvados rodo, kad nors egzotiniai pasirinkimo sandorių suteikia didesnį atsparumą rizikai ir ekonomiškumą, palyginti su europietiškų pasirinkimo sandorių galimybėmis ir tam tikromis prekybos strategijomis, pasirinkimo sandorių vertei rinkos sąlygos diktuoja. Baigiamajame darbe pabrėžiama įvairių veiksnių, įskaitant dirbtinio intelekto (DI) ir skaičiavimo modelius, integravimo į pasirinkimo sandorių vertės nustatymą svarba, siekiant naršyti būsimus rinkos neapibrėžtumus ir optimizuoti pasirinkimo kainodarą, kartu veiksmingai valdant susijusią riziką.

PRICING COMPLEXITIES IN DERIVATIVES: PLAIN VANILLA AND EXOTIC OPTIONS

Ineta PRIALGAUSKAITĖ

Master Thesis

Finance and Banking

Faculty of Economics and Business Administration, Vilnius University

Supervisor prof. dr. A. Šapkauskienė, Vilnius, 2025

SUMMARY

68 pages, 21 figures, 11 tables, 90 references.

This master thesis explores the complexities of option pricing, with the main focus on both plain vanilla and exotic options, to the evaluate the effect of change in parameters essential in order to determine option contract value.

The Master thesis is structured into three primary sections: literature analysis, research model and its results, with conclusions and recommendations.

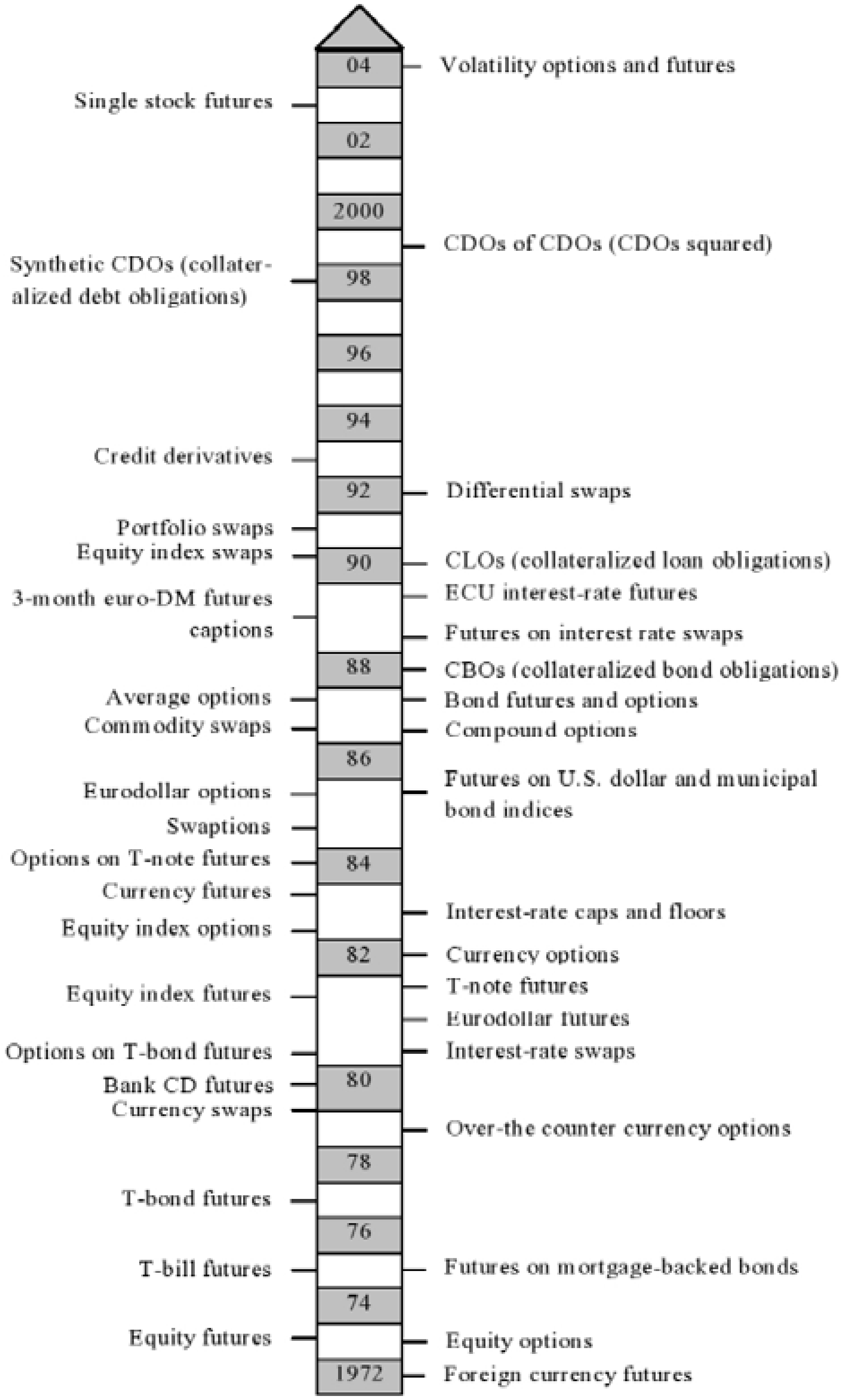
The literature review explores the background of derivatives, highlighting their complexities and focusing on option contracts and their sensitivity to changes in parameters such as strike price, time to maturity, spot price, dividend yield, volatility, and risk-free rate. Additionally, it reviews and compares the historical and theoretical development of option pricing models, from the Black-Scholes model to advanced AI-driven techniques. It is noteworthy that the integration of macroeconomic variables and ESG factors, along with the impact of recent global events on market dynamics, is gradually being examined and incorporated into the valuation of option contracts.

Following the literature analysis, the research uses a scenario-based approach to compare trading strategies, employing the Greeks to evaluate sensitivity to market changes. It also utilises a modified Black-Scholes-Merton model to assess the pricing of European and chooser options, focusing on key parameters like volatility, strike price, and time to expiration. Empirical data analysis is performed using Excel, offering insights into the practical application of theoretical models.

The findings indicate that while chooser options provide better risk resistance and cost efficiency compared to European options and certain trading strategies, their pricing is significantly affected by market conditions and specific parameters. The thesis concludes by highlighting the importance of integrating diverse factors, including artificial intelligence (AI) and computational models, in option pricing to navigate future market uncertainties and optimise option pricing while effectively managing associated risks.

ANNEXES

Annex 1. The evolution of derivative financial instruments



Source: Nguyen & Wu (2013)

Annex 2. Summary of parametric models used in option pricing

Model	Typical Option contracts used	Variables needed	Advantages of Model	Disadvantages of Model
Bachelier Model (1900)	Options on commodities and interest rates	Stock price (S) Strike price (K) Time to expiration (T) Risk-free rate (r) Volatility (σ)	-Models the option price as a function of the asset price in absolute terms -Suitable for assets with low prices	-Less commonly used -Assumes normal distribution of stock prices
Black-Scholes Model (1973)	European call & put options	Stock price (S); Strike price (K); Time to expiration (T) Risk-free rate (r) Volatility (σ)	-Simple and easy to understand -Widely used and recognized -Provides a closed-form solution	-Assumes constant volatility -Not suitable for options on stocks with dividends -Assumes log-normal distribution of stock prices
Binomial Model (1979)	American & European options Exotic options with complex features	Stock price (S) Strike price (K) Time to expiration (T) Risk-free rate (r) Volatility (σ) Number of time steps (n)	-Can manage varying volatility & dividends -Flexible, can be used for American options	-Computationally intensive for a large number of time steps -Less accurate with fewer steps
Monte Carlo Simulation (1977)	European & American options Exotic options, including path-dependent and path-independent types	Stock price (S) Strike price (K) Time to expiration (T) Risk-free rate (r) Volatility (σ) Dividend yield (q)	-Highly flexible, can be adapted for various complex options -Can manage path-dependent features	-Computationally expensive -Results can vary with number of simulations
Heston Model (1993)	European options, particularly those where volatility exhibits noticeable patterns	Stock price (S) Strike price (K) Time to expiration (T) Risk-free rate (r) Initial variance (v0) Mean reversion rate (kappa) Long-run variance (theta) Volatility of volatility (sigma) Correlation between stock and variance (rho)	-Accounts for stochastic volatility -Can capture the skewness and kurtosis of market returns	-More complex parameters to estimate; -Computationally demanding

Source: prepared by the author

Annex 3. Results of parameter analysis option pricing using BSM

1.1. The relationship between the value of European put & call option and strike price

Underlying price (S)	Value of Call, when strike = $390	Value of Put, when strike = $390	Value of Call, when strike = $360	Value of Put, when strike = $360	Value of Call, when strike = $420	Value of Put, when strike = $420	Value of Call, when strike = $450	Value of Put, when strike = $450
$266,90	1,5451	107,8949	3,4451	81,0834	0,6643	135,7257	0,2764	164,0492
$281,42	2,8040	94,6374	5,8154	68,9372	1,2935	121,8383	0,5759	149,8322
$304,88	6,3031	74,6692	11,7802	51,4348	3,2202	100,2978	1,5834	127,3724
$307,87	6,9052	72,2882	12,7494	49,4210	3,5705	97,6650	1,7764	124,5824
$309,95	7,3499	70,6455	13,4574	48,0415	3,8321	95,8392	1,9221	122,6407
$312,49	7,9163	68,6775	14,3501	46,3998	4,1687	93,6413	2,1113	120,2955
$325,12	11,1926	59,3239	19,3473	38,7671	6,1799	83,0228	3,2794	108,8337
$327,38	11,8632	57,7291	20,3408	37,4951	6,6038	81,1811	3,5326	106,8214
$327,93	12,0303	57,3452	20,5868	37,1903	6,7099	80,7363	3,5963	106,3343
$330,18	12,7276	55,7951	21,6086	35,9647	7,1554	78,9345	3,8654	104,3559
$333,52	13,8108	53,5416	23,1789	34,1983	7,8549	76,2973	4,2923	101,4461
$335,26	14,4004	52,3856	24,0258	33,2995	8,2394	74,9361	4,5291	99,9373
$335,96	14,6403	51,9286	24,3687	32,9455	8,3965	74,3963	4,6263	99,3376
$338,11	15,3954	50,5360	25,4426	31,8718	8,8937	72,7458	4,9357	97,4993
$338,87	15,6694	50,0474	25,8302	31,4967	9,0750	72,1645	5,0491	96,8501
$340,74	16,3531	48,8639	26,7930	30,5923	9,5298	70,7521	5,3349	95,2686
$346,10	18,4196	45,5668	29,6659	28,1017	10,9225	66,7812	6,2215	90,7917
$346,20	18,4605	45,5055	29,7222	28,0557	10,9503	66,7067	6,2394	90,7073
$346,68	18,6496	45,2235	29,9823	27,8448	11,0790	66,3644	6,3222	90,3191
$348,62	19,4428	44,0712	31,0694	26,9863	11,6215	64,9614	6,6727	88,7241
$351,73	20,7511	42,2732	32,8471	25,6578	12,5241	62,7577	7,2609	86,2060
$354,48	21,9531	40,7231	34,4648	24,5234	13,3615	60,8431	7,8121	84,0051
$358,93	23,9810	38,3023	37,1629	22,7727	14,7914	57,8242	8,7646	80,5089
$360,52	24,7323	37,4611	38,1532	22,1706	15,3263	56,7666	9,1244	79,2762
$367,18	28,0169	34,0887	42,4288	19,7891	17,6959	52,4792	10,7399	74,2347
$369,78	29,3630	32,8333	44,1576	18,9164	18,6808	50,8626	11,4211	72,3143
$373,43	31,3101	31,1304	46,6361	17,7449	20,1189	48,6507	12,4252	69,6685
$374,28	31,7752	30,7419	47,2245	17,4797	20,4647	48,1429	12,6683	69,0580
$379,00	34,4136	28,6592	50,5371	16,0712	22,4422	45,3993	14,0701	65,7386
$383,58	37,0781	26,7439	53,8418	14,7961	24,4656	42,8429	15,5239	62,6127
$386,16	38,6201	25,7120	55,7371	14,1175	25,6481	41,4515	16,3822	60,8970
$386,98	39,1165	25,3908	56,3447	13,9075	26,0305	41,0163	16,6610	60,3583
$387,00	39,1343	25,3794	56,3665	13,9001	26,0443	41,0009	16,6711	60,3391
$388,03	39,7601	24,9820	57,1307	13,6411	26,5276	40,4610	17,0245	59,6693
$389,08	40,4079	24,5791	57,9196	13,3794	27,0292	39,9120	17,3923	58,9865
$395,33	44,3676	22,2869	62,7012	11,9089	30,1252	36,7559	19,6851	55,0273
$396,45	45,0976	21,8939	63,5752	11,6600	30,7012	36,2090	20,1159	54,3352
$397,06	45,4954	21,6833	64,0507	11,5270	31,0159	35,9152	20,3517	53,9626
$397,80	45,9789	21,4307	64,6276	11,3679	31,3989	35,5622	20,6393	53,5141
$401,83	48,6688	20,0893	67,8204	10,5295	33,5422	33,6742	22,2585	51,1020

1.1. The relationship between the value of European put & call option and strike price (cont.)

Underlying price (S)	Value of Call, when strike = $390	Value of Put, when strike = $390	Value of Call, when strike = $360	Value of Put, when strike = $360	Value of Call, when strike = $420	Value of Put, when strike = $420	Value of Call, when strike = $450	Value of Put, when strike = $450
$403,31	49,6776	19,6125	69,0108	10,2343	34,3514	32,9977	22,8741	50,2320
$406,78	52,0698	18,5350	71,8190	9,5727	36,2813	31,4580	24,3515	48,2397
$407,57	52,6176	18,2982	72,4592	9,4284	36,7254	31,1176	24,6932	47,7968
$409,10	53,6937	17,8434	73,7140	9,1522	37,6002	30,4614	25,3683	46,9409
$411,28	55,2403	17,2127	75,5108	8,7717	38,8627	29,5466	26,3468	45,7422
$411,63	55,4927	17,1123	75,8032	8,7114	39,0692	29,4004	26,5074	45,5501
$411,90	55,6849	17,0363	76,0258	8,6658	39,2267	29,2896	26,6299	45,4043
$416,88	59,3028	15,6754	80,1951	7,8563	42,2073	27,2915	28,9634	42,7590
$418,63	60,5987	15,2186	81,6792	7,5876	43,2827	26,6141	29,8118	41,8547
$419,43	61,1946	15,0136	82,3601	7,4676	43,7785	26,3090	30,2041	41,4461
$419,99	61,6138	14,8713	82,8384	7,3845	44,1277	26,0968	30,4808	41,1614
$433,49	72,0188	11,7793	94,5702	5,6192	52,9206	21,3926	37,5580	34,7415
$435,11	73,3055	11,4490	96,0035	5,4355	54,0235	20,8785	38,4596	34,0261
$441,73	78,6578	10,1783	101,9293	4,7383	58,6454	18,8775	42,2696	31,2131
$442,95	79,6556	9,9586	103,0278	4,6194	59,5129	18,5275	42,9901	30,7162
$443,03	79,7253	9,9435	103,1045	4,6112	59,5736	18,5033	43,0406	30,6818
$444,10	80,6042	9,7544	104,0706	4,5093	60,3394	18,2011	43,6781	30,2513
$456,54	91,0616	7,7752	115,4629	3,4650	69,5505	14,9755	51,4434	25,5799
$463,72	97,2769	6,8018	122,1546	2,9680	75,1064	13,3428	56,2075	23,1554
$512,34	141,7204	2,6304	168,7997	0,9981	116,2115	5,8330	92,9367	11,2696

Source: prepared by the author

1.2. The relationship between the value of European put & call option and time to expiration

Underlying price (S)	Value of Call, when duration = 1 year	Value of Put, when duration = 1 year	Value of Call, when duration = 0,5 year	Value of Put, when duration = 0,5 year	Value of Call, when duration = 0,25 year	Value of Put, when duration = 0,25 year	Value of Call, when duration = 2 years	Value of Put, when duration = 2 years
$266,90	1,5451	107,8949	0,1262	114,7594	0,0019	118,8455	7,8591	98,1777
$281,42	2,8040	94,6374	0,3605	100,4772	0,0131	104,3402	11,1839	86,9861
$304,88	6,3031	74,6692	1,4469	78,0963	0,1566	81,0165	18,2895	70,6244
$307,87	6,9052	72,2882	1,6864	75,3529	0,2048	78,0816	19,3522	68,7040
$309,95	7,3499	70,6455	1,8720	73,4510	0,2456	76,0350	20,1175	67,3820
$312,49	7,9163	68,6775	2,1185	71,1630	0,3045	73,5594	21,0708	65,8008
$325,12	11,1926	59,3239	3,7442	60,1589	0,8072	61,4323	26,2142	58,3143
$327,38	11,8632	57,7291	4,1148	58,2640	0,9464	59,3061	27,2058	57,0404
$327,93	12,0303	57,3452	4,2089	57,8072	0,9831	58,7918	27,4500	56,7338
$330,18	12,7276	55,7951	4,6094	55,9604	1,1449	56,7063	28,4593	55,4957
$333,52	13,8108	53,5416	5,2550	53,2693	1,4244	53,6491	29,9953	53,6950
$335,26	14,4004	52,3856	5,6179	51,8865	1,5910	52,0700	30,8168	52,7708
$335,96	14,6403	51,9286	5,7677	51,3394	1,6617	51,4438	31,1481	52,4052
$338,11	15,3954	50,5360	6,2474	49,6714	1,8954	49,5298	32,1814	51,2908
$338,87	15,6694	50,0474	6,4244	49,0858	1,9844	48,8561	32,5527	50,8995
$340,74	16,3531	48,8639	6,8729	47,6671	2,2160	47,2205	33,4716	49,9512
$346,10	18,4196	45,5668	8,2833	43,7139	2,9994	42,6404	36,1872	47,3032
$346,20	18,4605	45,5055	8,3120	43,6403	3,0162	42,5549	36,2401	47,2538
$346,68	18,6496	45,2235	8,4450	43,3023	3,0943	42,1620	36,4841	47,0268
$348,62	19,4428	44,0712	9,0101	41,9219	3,4332	40,5554	37,5011	46,0983
$351,73	20,7511	42,2732	9,9649	39,7704	4,0312	38,0471	39,1550	44,6460
$354,48	21,9531	40,7231	10,8658	37,9193	4,6228	35,8866	40,6510	43,3899
$358,93	23,9810	38,3023	12,4336	35,0383	5,7096	32,5247	43,1294	41,4195
$360,52	24,7323	37,4611	13,0287	34,0409	6,1397	31,3623	44,0344	40,7320
$367,18	28,0169	34,0887	15,7122	30,0674	8,1855	26,7511	47,9174	37,9581
$369,78	29,3630	32,8333	16,8474	28,6011	9,0977	25,0618	49,4780	36,9171
$373,43	31,3101	31,1304	18,5222	26,6259	10,4880	22,8021	51,7074	35,4965
$374,28	31,7752	30,7419	18,9277	26,1778	10,8320	22,2925	52,2354	35,1709
$379,00	34,4136	28,6592	21,2644	23,7934	12,8650	19,6044	55,2006	33,4150
$383,58	37,0781	26,7439	23,6820	21,6312	15,0489	17,2085	58,1481	31,7827
$386,16	38,6201	25,7120	25,1050	20,4803	16,3676	15,9533	59,8347	30,8954
$386,98	39,1165	25,3908	25,5666	20,1243	16,8001	15,5682	60,3748	30,6180
$387,00	39,1343	25,3794	25,5832	20,1117	16,8157	15,5546	60,3942	30,6081
$388,03	39,7601	24,9820	26,1675	19,6728	17,3665	15,0822	61,0733	30,2640
$389,08	40,4079	24,5791	26,7748	19,2295	17,9427	14,6077	61,7740	29,9141
$395,33	44,3676	22,2869	30,5417	16,7443	21,5896	12,0026	66,0149	27,9030
$396,45	45,0976	21,8939	31,2454	16,3251	22,2834	11,5735	66,7893	27,5544
$397,06	45,4954	21,6833	31,6300	16,1013	22,6641	11,3458	67,2104	27,3671
$397,80	45,9789	21,4307	32,0985	15,8338	23,1292	11,0748	67,7213	27,1420
$401,83	48,6688	20,0893	34,7248	14,4288	25,7617	9,6761	70,5479	25,9373

1.2. The relationship between the value of European put & call option and time to expiration (cont.)

Underlying price (S)	Value of Call, when duration = 1 year	Value of Put, when duration = 1 year	Value of Call, when duration = 0,5 year	Value of Put, when duration = 0,5 year	Value of Call, when duration = 0,25 year	Value of Put, when duration = 0,25 year	Value of Call, when duration = 2 years	Value of Put, when duration = 2 years
$403,31	49,6776	19,6125	35,7179	13,9362	26,7674	9,1961	71,6015	25,5052
$406,78	52,0698	18,5350	38,0888	12,8374	29,1878	8,1468	74,0865	24,5205
$407,57	52,6176	18,2982	38,6347	12,5987	29,7486	7,9230	74,6531	24,3025
$409,10	53,6937	17,8434	39,7101	12,1432	30,8569	7,5003	75,7636	23,8821
$411,28	55,2403	17,2127	41,2623	11,5181	32,4640	6,9302	77,3539	23,2951
$411,63	55,4927	17,1123	41,5162	11,4193	32,7277	6,8412	77,6127	23,2012
$411,90	55,6849	17,0363	41,7098	11,3446	32,9289	6,7741	77,8098	23,1300
$416,88	59,3028	15,6754	45,3730	10,0290	36,7555	5,6220	81,5021	21,8435
$418,63	60,5987	15,2186	46,6934	9,5967	38,1428	5,2565	82,8172	21,4060
$419,43	61,1946	15,0136	47,3019	9,4043	38,7834	5,0962	83,4208	21,2086
$419,99	61,6138	14,8713	47,7303	9,2713	39,2348	4,9862	83,8449	21,0713
$433,49	72,0188	11,7793	58,4705	6,5144	50,6237	2,8780	94,2700	17,9994
$435,11	73,3055	11,4490	59,8095	6,2364	52,0488	2,6861	95,5474	17,6598
$441,73	78,6578	10,1783	65,3971	5,2010	57,9964	2,0107	100,8392	16,3285
$442,95	79,6556	9,9586	66,4413	5,0278	59,1073	1,9042	101,8221	16,0940
$443,03	79,7253	9,9435	66,5143	5,0159	59,1850	1,8969	101,8908	16,0777
$444,10	80,6042	9,7544	67,4348	4,8684	60,1639	1,8079	102,7557	15,8747
$456,54	91,0616	7,7752	78,4126	3,4095	71,8044	1,0117	112,9945	13,6768
$463,72	97,2769	6,8018	84,9475	2,7558	78,6926	0,7113	119,0439	12,5377
$512,34	141,7204	2,6304	131,3853	0,5786	126,6452	0,0489	161,9906	6,8694

Source: prepared by the author

1.3. The relationship between the value of European put & call option and dividend yield

Underlying price (S)	Value of Call, when dividend = 0%	Value of Put, when dividend = 0%	Value of Call, when dividend = 0,71%	Value of Put, when dividend = 0,71%	Value of Call, when dividend = 1%	Value of Put, when dividend = 1%	Value of Call, when dividend = 1,5 %	Value of Put, when dividend = 1,5 %
$266,90	1,6665	105,3757	1,5451	107,8949	1,4977	108,9315	1,4188	110,7290
$281,42	3,0046	92,1972	2,8040	94,6374	2,7253	95,6427	2,5939	97,3876
$304,88	6,6900	72,4154	6,3031	74,6692	6,1501	75,6002	5,8932	77,2196
$307,87	7,3209	70,0632	6,9052	72,2882	6,7407	73,2077	6,4641	74,8075
$309,95	7,7863	68,4413	7,3499	70,6455	7,1770	71,5567	6,8864	73,1424
$312,49	8,3787	66,4992	7,9163	68,6775	7,7331	69,5783	7,4248	71,1463
$325,12	11,7946	57,2853	11,1926	59,3239	10,9532	60,1685	10,5492	61,6409
$327,38	12,4920	55,7172	11,8632	57,7291	11,6131	58,5629	11,1907	60,0169
$327,93	12,6656	55,3398	12,0303	57,3452	11,7774	58,1764	11,3505	59,6258
$330,18	13,3899	53,8168	12,7276	55,7951	12,4638	56,6154	12,0183	58,0462
$333,52	14,5140	51,6042	13,8108	53,5416	13,5305	54,3454	13,0567	55,7480
$335,26	15,1254	50,4699	14,4004	52,3856	14,1114	53,1806	13,6226	54,5681
$335,96	15,3739	50,0216	14,6403	51,9286	14,3477	52,7200	13,8529	54,1015
$338,11	16,1562	48,6561	15,3954	50,5360	15,0919	51,3165	14,5782	52,6792
$338,87	16,4399	48,1772	15,6694	50,0474	15,3619	50,8239	14,8416	52,1799
$340,74	17,1475	47,0176	16,3531	48,8639	16,0359	49,6307	15,4990	50,9701
$346,10	19,2837	43,7902	18,4196	45,5668	18,0742	46,3054	17,4889	47,5964
$346,20	19,3259	43,7301	18,4605	45,5055	18,1146	46,2435	17,5283	47,5336
$346,68	19,5211	43,4543	18,6496	45,2235	18,3011	45,9590	17,7105	47,2448
$348,62	20,3400	42,3277	19,4428	44,0712	19,0839	44,7963	18,4755	46,0642
$351,73	21,6895	40,5709	20,7511	42,2732	20,3755	42,9816	19,7382	44,2207
$354,48	22,9282	39,0576	21,9531	40,7231	21,5625	41,4165	20,8996	42,6300
$358,93	25,0158	36,6964	23,9810	38,3023	23,5661	38,9714	22,8613	40,1429
$360,52	25,7886	35,8767	24,7323	37,4611	24,3086	38,1215	23,5888	39,2780
$367,18	29,1630	32,5941	28,0169	34,0887	27,5566	34,7124	26,7736	35,8058
$369,78	30,5441	31,3737	29,3630	32,8333	28,8884	33,4427	28,0807	34,5114
$373,43	32,5402	29,7198	31,3101	31,1304	30,8154	31,7197	29,9731	32,7538
$374,28	33,0167	29,3427	31,7752	30,7419	31,2759	31,3266	30,4255	32,3525
$379,00	35,7179	27,3228	34,4136	28,6592	33,8885	29,2181	32,9936	30,1995
$383,58	38,4426	25,4677	37,0781	26,7439	36,5283	27,2781	35,5906	28,2167
$386,16	40,0181	24,4693	38,6201	25,7120	38,0565	26,2324	37,0950	27,1472
$386,98	40,5251	24,1587	39,1165	25,3908	38,5485	25,9068	37,5795	26,8142
$387,00	40,5432	24,1477	39,1343	25,3794	38,5662	25,8953	37,5969	26,8023
$388,03	41,1823	23,7635	39,7601	24,9820	39,1866	25,4925	38,2079	26,3901
$389,08	41,8435	23,3741	40,4079	24,5791	39,8288	25,0840	38,8404	25,9721
$395,33	45,8823	21,1609	44,3676	22,2869	43,7560	22,7592	42,7111	23,5907
$396,45	46,6262	20,7819	45,0976	21,8939	44,4802	22,3605	43,4253	23,1820
$397,06	47,0316	20,5788	45,4954	21,6833	44,8749	22,1468	43,8147	22,9629
$397,80	47,5242	20,3353	45,9789	21,4307	45,3546	21,8904	44,2879	22,7001
$401,83	50,2632	19,0431	48,6688	20,0893	48,0242	20,5287	46,9222	21,3031

1.3. The relationship between the value of European put & call option and dividend yield (cont.)

Underlying price (S)	Value of Call, when dividend = 0%	Value of Put, when dividend = 0%	Value of Call, when dividend = 0,71%	Value of Put, when dividend = 0,71%	Value of Call, when dividend = 1%	Value of Put, when dividend = 1%	Value of Call, when dividend = 1,5 %	Value of Put, when dividend = 1,5 %
$403,31	51,2899	18,5841	49,6776	19,6125	49,0257	20,0446	47,9109	20,8062
$406,78	53,7232	17,5477	52,0698	18,5350	51,4009	18,9501	50,2566	19,6821
$407,57	54,2801	17,3201	52,6176	18,2982	51,9449	18,7095	50,7940	19,4350
$409,10	55,3740	16,8830	53,6937	17,8434	53,0137	18,2474	51,8500	18,9600
$411,28	56,9456	16,2773	55,2403	17,2127	54,5500	17,6064	53,3683	18,3010
$411,63	57,2019	16,1809	55,4927	17,1123	54,8006	17,5043	53,6161	18,1961
$411,90	57,3971	16,1079	55,6849	17,0363	54,9916	17,4270	53,8048	18,1166
$416,88	61,0705	14,8024	59,3028	15,6754	58,5865	16,0432	57,3597	16,6927
$418,63	62,3854	14,3646	60,5987	15,2186	59,8745	15,5785	58,6340	16,2142
$419,43	62,9899	14,1683	61,1946	15,0136	60,4669	15,3699	59,2201	15,9994
$419,99	63,4151	14,0320	61,6138	14,8713	60,8836	15,2251	59,6324	15,8503
$433,49	73,9565	11,0764	72,0188	11,7793	71,2319	12,0764	69,8816	12,6025
$435,11	75,2585	10,7613	73,3055	11,4490	72,5122	11,7397	71,1508	12,2546
$441,73	80,6710	9,5508	78,6578	10,1783	77,8394	10,4439	76,4340	10,9148
$442,95	81,6794	9,3418	79,6556	9,9586	78,8327	10,2198	77,4195	10,6829
$443,03	81,7499	9,3274	79,7253	9,9435	78,9021	10,2043	77,4884	10,6669
$444,10	82,6381	9,1476	80,6042	9,7544	79,7772	10,0114	78,3567	10,4673
$456,54	93,1956	7,2685	91,0616	7,7752	90,1927	7,9903	88,6986	8,3725
$463,72	99,4627	6,3470	97,2769	6,8018	96,3862	6,9951	94,8538	7,3391
$512,34	144,1557	2,4249	141,7204	2,6304	140,7245	2,7185	139,0061	2,8764

Source: prepared by the author

1.4. The relationship between the value of European put & call option and volatility

Underlying price (S)	Value of Call, when volatility = 21%	Value of Put, when volatility = 21%	Value of Call, when volatility = 10%	Value of Put, when volatility = 10%	Value of Call, when volatility = 30%	Value of Put, when volatility = 30%	Value of Call, when volatility = 40%	Value of Put, when volatility = 40%
$266,90	1,5451	107,8949	0,0033	106,3531	6,2243	112,5742	14,0063	120,3561
$281,42	2,8040	94,6374	0,0227	91,8561	8,9831	100,8165	18,1278	109,9611
$304,88	6,3031	74,6692	0,2689	68,6349	15,0026	83,3687	26,1804	94,5465
$307,87	6,9052	72,2882	0,3500	65,7330	15,9145	81,2975	27,3275	92,7105
$309,95	7,3499	70,6455	0,4184	63,7140	16,5728	79,8685	28,1467	91,4423
$312,49	7,9163	68,6775	0,5163	61,2774	17,3947	78,1558	29,1595	89,9206
$325,12	11,1926	59,3239	1,3318	49,4631	21,8610	69,9923	34,5017	82,6330
$327,38	11,8632	57,7291	1,5528	47,4187	22,7278	68,5937	35,5115	81,3773
$327,93	12,0303	57,3452	1,6107	46,9257	22,9417	68,2566	35,7594	81,0743
$330,18	12,7276	55,7951	1,8650	44,9326	23,8261	66,8937	36,7801	79,8477
$333,52	13,8108	53,5416	2,2997	42,0305	25,1756	64,9065	38,3236	78,0545
$335,26	14,4004	52,3856	2,5563	40,5415	25,8989	63,8841	39,1444	77,1296
$335,96	14,6403	51,9286	2,6647	39,9531	26,1909	63,4792	39,4746	76,7629
$338,11	15,3954	50,5360	3,0211	38,1617	27,1028	62,2434	40,5012	75,6418
$338,87	15,6694	50,0474	3,1559	37,5339	27,4309	61,8089	40,8690	75,2470
$340,74	16,3531	48,8639	3,5053	36,0160	28,2437	60,7544	41,7768	74,2876
$346,10	18,4196	45,5668	4,6694	31,8166	30,6530	57,8002	44,4409	71,5881
$346,20	18,4605	45,5055	4,6940	31,7390	30,7000	57,7450	44,4925	71,5375
$346,68	18,6496	45,2235	4,8087	31,3826	30,9171	57,4910	44,7306	71,3045
$348,62	19,4428	44,0712	5,3037	29,9322	31,8225	56,4509	45,7207	70,3491
$351,73	20,7511	42,2732	6,1681	27,6902	33,2980	54,8201	47,3240	68,8461
$354,48	21,9531	40,7231	7,0126	25,7827	34,6356	53,4057	48,7672	67,5373
$358,93	23,9810	38,3023	8,5403	22,8616	36,8579	51,1792	51,1452	65,4665
$360,52	24,7323	37,4611	9,1372	21,8661	37,6713	50,4001	52,0097	64,7386
$367,18	28,0169	34,0887	11,9253	17,9971	41,1721	47,2439	55,6988	61,7706
$369,78	29,3630	32,8333	13,1445	16,6148	42,5838	46,0541	57,1729	60,6432
$373,43	31,3101	31,1304	14,9779	14,7982	44,6051	44,4255	59,2714	59,0917
$374,28	31,7752	30,7419	15,4272	14,3939	45,0847	44,0514	59,7672	58,7339
$379,00	34,4136	28,6592	18,0513	12,2969	47,7827	42,0282	62,5436	56,7892
$383,58	37,0781	26,7439	20,8162	10,4820	50,4732	40,1390	65,2914	54,9572
$386,16	38,6201	25,7120	22,4617	9,5536	52,0164	39,1082	66,8588	53,9507
$386,98	39,1165	25,3908	22,9977	9,2720	52,5112	38,7855	67,3601	53,6344
$387,00	39,1343	25,3794	23,0170	9,2621	52,5289	38,7740	67,3781	53,6232
$388,03	39,7601	24,9820	23,6970	8,9189	53,1513	38,3733	68,0079	53,2298
$389,08	40,4079	24,5791	24,4054	8,5767	53,7941	37,9654	68,6573	52,8285
$395,33	44,3676	22,2869	28,8257	6,7449	57,6931	35,6123	72,5770	50,4962
$396,45	45,0976	21,8939	29,6550	6,4514	58,4066	35,2030	73,2909	50,0873
$397,06	45,4954	21,6833	30,1087	6,2966	58,7949	34,9828	73,6790	49,8669
$397,80	45,9789	21,4307	30,6615	6,1133	59,2662	34,7181	74,1497	49,6015
$401,83	48,6688	20,0893	33,7635	5,1840	61,8773	33,2979	76,7498	48,1704

1.4. The relationship between the value of European put & call option and volatility (cont.)

Underlying price (S)	Value of Call, when volatility = 21%	Value of Put, when volatility = 21%	Value of Call, when volatility = 10%	Value of Put, when volatility = 10%	Value of Call, when volatility = 30%	Value of Put, when volatility = 30%	Value of Call, when volatility = 40%	Value of Put, when volatility = 40%
$403,31	49,6776	19,6125	34,9371	4,8720	62,8522	32,7871	77,7176	47,6525
$406,78	52,0698	18,5350	37,7375	4,2027	65,1549	31,6201	79,9973	46,4625
$407,57	52,6176	18,2982	38,3818	4,0625	65,6806	31,3612	80,5166	46,1972
$409,10	53,6937	17,8434	39,6503	3,7999	66,7116	30,8613	81,5338	45,6835
$411,28	55,2403	17,2127	41,4787	3,4511	68,1897	30,1621	82,9894	44,9618
$411,63	55,4927	17,1123	41,7775	3,3972	68,4305	30,0502	83,2262	44,8459
$411,90	55,6849	17,0363	42,0053	3,3567	68,6138	29,9652	83,4065	44,7579
$416,88	59,3028	15,6754	46,3030	2,6756	72,0538	28,4264	86,7807	43,1534
$418,63	60,5987	15,2186	47,8458	2,4657	73,2813	27,9012	87,9813	42,6012
$419,43	61,1946	15,0136	48,5556	2,3747	73,8451	27,6641	88,5320	42,3510
$419,99	61,6138	14,8713	49,0550	2,3125	74,2413	27,4989	88,9189	42,1765
$433,49	72,0188	11,7793	61,4275	1,1880	84,0188	23,7793	98,4145	38,1750
$435,11	73,3055	11,4490	62,9494	1,0929	85,2215	23,3650	99,5765	37,7200
$441,73	78,6578	10,1783	69,2498	0,7704	90,2135	21,7340	104,3880	35,9086
$442,95	79,6556	9,9586	70,4184	0,7214	91,1425	21,4455	105,2815	35,5846
$443,03	79,7253	9,9435	70,4999	0,7181	91,2074	21,4255	105,3439	35,5621
$444,10	80,6042	9,7544	71,5275	0,6777	92,0253	21,1755	106,1301	35,2803
$456,54	91,0616	7,7752	83,6235	0,3370	101,7373	18,4509	115,4361	32,1496
$463,72	97,2769	6,8018	90,6959	0,2209	107,4999	17,0248	120,9367	30,4616
$512,34	141,7204	2,6304	139,0992	0,0091	148,8361	9,7461	160,1712	21,0811

Source: prepared by the author

1.5. The relationship between the value of European put & call option and risk-free rate

Underlying price (S)	Value of Call, when risk-free rate = 5,1%	Value of Put, when risk-free rate = 5,1%	Value of Call, when risk-free rate = 7%	Value of Put, when risk-free rate = 7%	Value of Call, when risk-free rate = 3%	Value of Put, when risk-free rate = 3%	Value of Call, when risk-free rate = 1,5%	Value of Put, when risk-free rate = 1,5%
$266,90	1,5451	107,8949	1,8881	101,2131	1,2288	115,4997	1,0382	121,0699
$281,42	2,8040	94,6374	3,3669	88,1754	2,2742	102,0286	1,9490	107,4641
$304,88	6,3031	74,6692	7,3790	68,7203	5,2595	81,5467	4,6010	86,6489
$307,87	6,9052	72,2882	8,0599	66,4181	5,7811	79,0852	5,0696	84,1344
$309,95	7,3499	70,6455	8,5614	64,8323	6,1676	77,3843	5,4175	82,3949
$312,49	7,9163	68,6775	9,1987	62,9351	6,6612	75,3435	5,8629	80,3058
$325,12	11,1926	59,3239	12,8551	53,9616	9,5424	65,5949	8,4791	70,2922
$327,38	11,8632	57,7291	13,5982	52,4393	10,1370	63,9240	9,0222	68,5698
$327,93	12,0303	57,3452	13,7830	52,0732	10,2853	63,5214	9,1577	68,1545
$330,18	12,7276	55,7951	14,5536	50,5964	10,9054	61,8940	9,7253	66,4747
$333,52	13,8108	53,5416	15,7475	48,4536	11,8714	59,5234	10,6115	64,0242
$335,26	14,4004	52,3856	16,3960	47,3564	12,3987	58,3050	11,0962	62,7631
$335,96	14,6403	51,9286	16,6594	46,9229	12,6134	57,8229	11,2937	62,2638
$338,11	15,3954	50,5360	17,4878	45,6036	13,2905	56,3523	11,9172	60,7396
$338,87	15,6694	50,0474	17,7879	45,1411	13,5365	55,8356	12,1440	60,2038
$340,74	16,3531	48,8639	18,5361	44,0221	14,1513	54,5832	12,7113	58,9039
$346,10	18,4196	45,5668	20,7904	40,9128	16,0161	51,0844	14,4367	55,2657
$346,20	18,4605	45,5055	20,8349	40,8550	16,0531	51,0192	14,4710	55,1977
$346,68	18,6496	45,2235	21,0405	40,5897	16,2242	50,7192	14,6296	54,8853
$348,62	19,4428	44,0712	21,9028	39,5064	16,9429	49,4924	15,2966	53,6068
$351,73	20,7511	42,2732	23,3218	37,8192	18,1311	47,5743	16,4011	51,6050
$354,48	21,9531	40,7231	24,6224	36,3677	19,2257	45,9169	17,4206	49,8725
$358,93	23,9810	38,3023	26,8105	34,1070	21,0785	43,3209	19,1505	47,1536
$360,52	24,7323	37,4611	27,6193	33,3233	21,7668	42,4167	19,7944	46,2050
$367,18	28,0169	34,0887	31,1439	30,1909	24,7866	38,7795	22,6268	42,3804
$369,78	29,3630	32,8333	32,5835	29,0290	26,0290	37,4204	23,7954	40,9474
$373,43	31,3101	31,1304	34,6612	27,4567	27,8306	35,5720	25,4931	38,9952
$374,28	31,7752	30,7419	35,1567	27,0986	28,2617	35,1496	25,9000	38,5484
$379,00	34,4136	28,6592	37,9623	25,1831	30,7128	32,8795	28,2166	36,1439
$383,58	37,0781	26,7439	40,7869	23,4279	33,1969	30,7838	30,5705	33,9181
$386,16	38,6201	25,7120	42,4177	22,4848	34,6383	29,6513	31,9391	32,7127
$386,98	39,1165	25,3908	42,9422	22,1917	35,1029	29,2983	32,3806	32,3367
$387,00	39,1343	25,3794	42,9610	22,1813	35,1196	29,2858	32,3964	32,3233
$388,03	39,7601	24,9820	43,6218	21,8189	35,7057	28,8487	32,9537	31,8574
$389,08	40,4079	24,5791	44,3053	21,4518	36,3128	28,4052	33,5313	31,3843
$395,33	44,3676	22,2869	48,4744	19,3688	40,0336	25,8739	37,0777	28,6787
$396,45	45,0976	21,8939	49,2413	19,0128	40,7212	25,4386	37,7343	28,2124
$397,06	45,4954	21,6833	49,6591	18,8221	41,0962	25,2052	38,0926	27,9622
$397,80	45,9789	21,4307	50,1666	18,5936	41,5521	24,9250	38,5282	27,6618
$401,83	48,6688	20,0893	52,9862	17,3819	44,0925	23,4342	40,9588	26,0611

1.5. The relationship between the value of European put & call option and risk-free rate (cont.)

Underlying price (S)	Value of Call, when risk-free rate = 5,1%	Value of Put, when risk-free rate = 5,1%	Value of Call, when risk-free rate = 7%	Value of Put, when risk-free rate = 7%	Value of Call, when risk-free rate = 3%	Value of Put, when risk-free rate = 3%	Value of Call, when risk-free rate = 1,5%	Value of Put, when risk-free rate = 1,5%
$403,31	49,6776	19,6125	54,0421	16,9522	45,0470	22,9031	41,8732	25,4899
$406,78	52,0698	18,5350	56,5425	15,9830	47,3139	21,7002	44,0473	24,1943
$407,57	52,6176	18,2982	57,1145	15,7703	47,8337	21,4354	44,5463	23,9087
$409,10	53,6937	17,8434	58,2374	15,3622	48,8554	20,9262	45,5278	23,3593
$411,28	55,2403	17,2127	59,8496	14,7972	50,3256	20,2191	46,9411	22,5953
$411,63	55,4927	17,1123	60,1125	14,7073	50,5656	20,1064	47,1720	22,4734
$411,90	55,6849	17,0363	60,3127	14,6393	50,7485	20,0210	47,3479	22,3811
$416,88	59,3028	15,6754	64,0762	13,4241	54,1960	18,4897	50,6683	20,7227
$418,63	60,5987	15,2186	65,4221	13,0172	55,4332	17,9742	51,8615	20,1632
$419,43	61,1946	15,0136	66,0406	12,8348	56,0025	17,7427	52,4109	19,9117
$419,99	61,6138	14,8713	66,4755	12,7083	56,4031	17,5818	52,7976	19,7370
$433,49	72,0188	11,7793	77,2376	9,9733	66,3848	14,0665	62,4596	15,9019
$435,11	73,3055	11,4490	78,5642	9,6829	67,6238	13,6884	63,6623	15,4875
$441,73	78,6578	10,1783	84,0735	8,5693	72,7875	12,2292	68,6821	13,8844
$442,95	79,6556	9,9586	85,0991	8,3773	73,7519	11,9761	69,6208	13,6056
$443,03	79,7253	9,9435	85,1707	8,3641	73,8193	11,9586	69,6864	13,5864
$444,10	80,6042	9,7544	86,0737	8,1991	74,6692	11,7406	70,5141	13,3461
$456,54	91,0616	7,7752	96,7915	6,4802	84,8103	9,4450	80,4110	10,8063
$463,72	97,2769	6,8018	103,1412	5,6413	90,8605	8,3066	86,3327	9,5394
$512,34	141,7204	2,6304	148,2266	2,1118	134,4971	3,3282	129,3272	3,9189

Source: prepared by the author

2.1. The relationship between the value of Chooser option and strike price

Underlying price (S)	Value of Chooser, when strike = $390	Value of Chooser, when strike = $360	Value of Chooser, when strike = $420	Value of Chooser, when strike = $450
$266,90	107,3230	81,2874	134,8072	162,8966
$281,42	94,4890	70,2693	121,1069	148,7996
$304,88	76,1192	56,5741	100,2198	126,6550
$307,87	74,0605	55,2733	97,7174	123,9176
$309,95	72,6729	54,4338	96,0032	122,0280
$312,49	71,0302	53,4829	93,9412	119,7379
$325,12	63,7781	50,0078	84,2641	108,6811
$327,38	62,6532	49,6112	82,6454	106,7693
$327,93	62,3879	49,5251	82,2571	106,3075
$330,18	61,3370	49,2159	80,6925	104,4326
$333,52	59,8818	48,8837	78,4426	101,6944
$335,26	59,1742	48,7706	77,3062	100,2903
$335,96	58,8995	48,7367	76,8561	99,7299
$338,11	58,0916	48,6738	75,4996	98,0252
$338,87	57,8190	48,6665	75,0296	97,4287
$340,74	57,1777	48,6814	73,8947	95,9750
$346,10	55,5736	48,9820	70,8179	91,9242
$346,20	55,5470	48,9912	70,7631	91,8503
$346,68	55,4211	49,0372	70,5011	91,4966
$348,62	54,9410	49,2537	69,4649	90,0825
$351,73	54,2687	49,7017	67,8807	87,8682
$354,48	53,7747	50,2001	66,5608	85,9666
$358,93	53,1760	51,2048	64,5907	83,0074
$360,52	53,0222	51,6219	63,9375	81,9868
$367,18	52,7217	53,6898	61,5003	77,9337
$369,78	52,7540	54,6329	60,6823	76,4529
$373,43	52,9395	56,0799	59,6629	74,4746
$374,28	53,0060	56,4369	59,4473	74,0312
$379,00	53,5335	58,5527	58,4014	71,6909
$383,58	54,2972	60,8126	57,6331	69,6233
$386,16	54,8342	62,1706	57,3078	68,5493
$386,98	55,0207	62,6147	57,2206	68,2219
$387,00	55,0253	62,6256	57,2186	68,2140
$388,03	55,2705	63,1918	57,1204	67,8126
$389,08	55,5326	63,7783	57,0330	67,4145
$395,33	57,3421	67,4572	56,7776	65,2795
$396,45	57,7103	68,1490	56,7795	64,9399
$397,06	57,9164	68,5297	56,7866	64,7606
$397,80	58,1715	68,9953	56,8010	64,5482
$401,83	59,6588	71,6006	56,9886	63,4942

2.1. The relationship between the value of Chooser option and strike price (cont.)

Underlying price (S)	Value of Chooser, when strike = $390	Value of Chooser, when strike = $360	Value of Chooser, when strike = $420	Value of Chooser, when strike = $450
$403,31	60,2455	72,5859	57,1037	63,1507
$406,78	61,7041	74,9530	57,4697	62,4380
$407,57	62,0521	75,5027	57,5717	62,2939
$409,10	62,7425	76,5784	57,7888	62,0341
$411,28	63,7629	78,1355	58,1422	61,7077
$411,63	63,9307	78,3881	58,2037	61,6602
$411,90	64,0608	78,5835	58,2521	61,6244
$416,88	66,5749	82,2599	59,2838	61,1062
$418,63	67,5079	83,5835	59,7082	60,9880
$419,43	67,9426	84,1937	59,9128	60,9450
$419,99	68,2500	84,6228	60,0599	60,9191
$433,49	76,3717	95,3993	64,5424	61,3105
$435,11	77,4315	96,7427	65,1958	61,4868
$441,73	81,9315	102,3288	68,1064	62,4876
$442,95	82,7891	103,3740	68,6835	62,7204
$443,03	82,8456	103,4427	68,7218	62,7362
$444,10	83,6051	104,3635	69,2388	62,9535
$456,54	92,8706	115,3025	75,9039	66,2905
$463,72	98,5420	121,7847	80,2549	68,8643
$512,34	141,0980	167,6778	116,8217	96,4082

Source: prepared by the author

2.2. The relationship between the value of Chooser option and time to expiration

Underlying price (S)	Value of Chooser, when duration = 1 year	Value of Chooser, when duration = 0,5 year	Value of Chooser, when duration = 0,25 year	Value of Chooser, when duration = 2 years
$266,90	107,3230	114,4780	118,7354	97,2159
$281,42	94,4890	100,4766	104,4465	86,8179
$304,88	76,1192	79,2641	82,0995	73,1623
$307,87	74,0605	76,7633	79,3715	71,7617
$309,95	72,6729	75,0591	77,4969	70,8381
$312,49	71,0302	73,0204	75,2362	69,7681
$325,12	63,7781	63,6752	64,5662	65,4380
$327,38	62,6532	62,1600	62,7783	64,8442
$327,93	62,3879	61,7992	62,3497	64,7082
$330,18	61,3370	60,3563	60,6238	64,1872
$333,52	59,8818	58,3160	58,1487	63,5186
$335,26	59,1742	57,3029	56,9029	63,2202
$335,96	58,8995	56,9053	56,4105	63,1099
$338,11	58,0916	55,7200	54,9313	62,8059
$338,87	57,8190	55,3143	54,4205	62,7110
$340,74	57,1777	54,3460	53,1920	62,5057
$346,10	55,5736	51,8138	49,9041	62,1388
$346,20	55,5470	51,7701	49,8462	62,1351
$346,68	55,4211	51,5620	49,5701	62,1188
$348,62	54,9410	50,7519	48,4854	62,0796
$351,73	54,2687	49,5581	46,8538	62,1059
$354,48	53,7747	48,6120	45,5256	62,2200
$358,93	53,1760	47,3031	43,6140	62,5833
$360,52	53,0222	46,9029	43,0048	62,7661
$367,18	52,7217	45,6171	40,8915	63,8281
$369,78	52,7540	45,2874	40,2638	64,3699
$373,43	52,9395	44,9881	39,5737	65,2476
$374,28	53,0060	44,9458	39,4453	65,4713
$379,00	53,5335	44,8980	38,9556	66,8426
$383,58	54,2972	45,1524	38,8424	68,3764
$386,16	54,8342	45,4241	38,9347	69,3252
$386,98	55,0207	45,5296	38,9874	69,6393
$387,00	55,0253	45,5323	38,9888	69,6470
$388,03	55,2705	45,6782	39,0712	70,0500
$389,08	55,5326	45,8416	39,1734	70,4704
$395,33	57,3421	47,1184	40,1534	73,1651
$396,45	57,7103	47,4011	40,3951	73,6818
$397,06	57,9164	47,5618	40,5350	73,9673
$397,80	58,1715	47,7631	40,7125	74,3176
$401,83	59,6588	48,9798	41,8262	76,2989

2.2. The relationship between the value of Chooser option and time to expiration (cont.)

Underlying price (S)	Value of Chooser, when duration = 1 year	Value of Chooser, when duration = 0,5 year	Value of Chooser, when duration = 0,25 year	Value of Chooser, when duration = 2 years
$403,31	60,2455	49,4767	42,2962	77,0569
$406,78	61,7041	50,7441	43,5226	78,8957
$407,57	62,0521	51,0524	43,8257	79,3260
$409,10	62,7425	51,6696	44,4370	80,1716
$411,28	63,7629	52,5947	45,3627	81,4032
$411,63	63,9307	52,7481	45,5172	81,6038
$411,90	64,0608	52,8674	45,6374	81,7592
$416,88	66,5749	55,2073	48,0220	84,7057
$418,63	67,5079	56,0909	48,9320	85,7768
$419,43	67,9426	56,5050	49,3600	86,2723
$419,99	68,2500	56,7987	49,6640	86,6214
$433,49	76,3717	64,7554	57,9922	95,5416
$435,11	77,4315	65,8146	59,1079	96,6719
$441,73	81,9315	70,3488	63,8879	101,4096
$442,95	82,7891	71,2185	64,8049	102,3024
$443,03	82,8456	71,2759	64,8654	102,3611
$444,10	83,6051	72,0475	65,6788	103,1493
$456,54	92,8706	81,5355	75,6569	112,6161
$463,72	98,5420	87,3888	81,7818	118,3064
$512,34	141,0980	131,4968	127,1270	159,8630

Source: prepared by the author

2.3. The relationship between the value of Chooser option and dividend yield

Underlying price (S)	Value of Chooser, when dividend = 0%	Value of Chooser, when dividend = 0,71%	Value of Chooser, when dividend = 1%	Value of Chooser, when dividend = 1,5%
$266,90	105,5961	107,3230	108,0282	109,2437
$281,42	92,7894	94,4890	95,1854	96,3888
$304,88	74,6160	76,1192	76,7412	77,8236
$307,87	72,5982	74,0605	74,6665	75,7229
$309,95	71,2413	72,6729	73,2671	74,3074
$312,49	69,6386	71,0302	71,6088	72,6196
$325,12	62,6258	63,7781	64,2629	65,1171
$327,38	61,5507	62,6532	63,1184	63,9398
$327,93	61,2988	62,3879	62,8481	63,6613
$330,18	60,2988	61,3370	61,7767	62,5555
$333,52	58,9242	59,8818	60,2896	61,0146
$335,26	58,2603	59,1742	59,5647	60,2606
$335,96	58,0034	58,8995	59,2829	59,9669
$338,11	57,2513	58,0916	58,4527	59,0993
$338,87	56,9989	57,8190	58,1722	58,8053
$340,74	56,4078	57,1777	57,5109	58,1102
$346,10	54,9537	55,5736	55,8469	56,2450
$346,20	54,9300	55,5470	55,8192	56,3153
$346,68	54,8179	55,4211	55,6877	56,1745
$348,62	54,3944	54,9410	55,1850	55,6334
$351,73	53,8147	54,2687	54,4757	54,8611
$354,48	53,4044	53,7747	53,9481	54,2765
$358,93	52,9442	53,1760	53,2936	53,5272
$360,52	52,8407	53,0222	53,1196	53,3187
$367,18	52,7545	52,7217	52,7326	52,7841
$369,78	52,8716	52,7540	52,7306	52,7234
$373,43	53,1768	52,9395	52,8676	52,7774
$374,28	53,2713	53,0060	52,9228	52,8131
$379,00	53,9542	53,5335	53,3871	53,1692
$383,58	54,8684	54,2972	54,0896	53,7664
$386,16	55,4897	54,8342	54,5921	54,2097
$386,98	55,7030	55,0207	54,7677	54,3666
$387,00	55,7083	55,0253	54,7721	54,3705
$388,03	55,9869	55,2705	55,0036	54,5785
$389,08	56,2831	55,5326	55,2518	54,8027
$395,33	58,2931	57,3421	56,9793	56,3887
$396,45	58,6968	57,7103	57,3330	56,7174
$397,06	58,9221	57,9164	57,5312	56,9020
$397,80	59,2005	58,1715	57,7768	57,1311
$401,83	60,8132	59,6588	59,2126	58,4780

2.3. The relationship between the value of Chooser option and dividend yield (cont.)

Underlying price (S)	Value of Chooser, when dividend = 0%	Value of Chooser, when dividend = 0,71%	Value of Chooser, when dividend = 1%	Value of Chooser, when dividend = 1,5%
$403,31	61,4453	60,2455	59,7807	59,0139
$406,78	63,0090	61,7041	61,1961	60,3545
$407,57	63,3806	62,0521	61,5344	60,6759
$409,10	64,1165	62,7425	62,2061	61,3152
$411,28	65,2009	63,7629	63,2001	62,2636
$411,63	65,3789	63,9307	63,3637	62,4199
$411,90	65,5169	64,0608	63,4906	62,5412
$416,88	68,1732	66,5749	65,9461	64,8950
$418,63	69,1549	67,5079	66,8589	65,7728
$419,43	69,6117	67,9426	67,2846	66,1826
$419,99	69,9345	68,2500	67,5856	66,4726
$433,49	78,4067	76,3717	75,5624	74,1970
$435,11	79,5058	77,4315	76,6058	75,2120
$441,73	84,1609	81,9315	81,0415	79,5356
$442,95	85,0460	82,7891	81,8876	80,3618
$443,03	85,1044	82,8456	81,9434	80,4162
$444,10	85,8878	83,6051	82,6930	81,1485
$456,54	95,4134	92,8706	91,8504	90,1172
$463,72	101,2211	98,5420	97,4652	95,6329
$512,34	144,4894	141,0980	139,7251	137,3753

Source: prepared by the author

2.4. The relationship between the value of Chooser option and volatility

Underlying price (S)	**Value of Chooser, when volatility = 21%**	**Value of Chooser, when volatility = 10%**	**Value of Chooser, when volatility = 30%**	**Value of Chooser, when volatility = 40%**
$266,90	107,3230	105,6002	113,3865	124,5904
$281,42	94,4890	91,2024	102,9959	116,7385
$304,88	76,1192	68,1672	89,2180	106,9928
$307,87	74,0605	65,2878	87,7647	106,0233
$309,95	72,6729	63,2988	86,7968	105,3859
$312,49	71,0302	60,8884	85,6637	104,6488
$325,12	63,7781	49,3470	80,8461	101,6607
$327,38	62,6532	47,3913	80,1309	101,2450
$327,93	62,3879	46,9220	79,9636	101,1493
$330,18	61,3370	45,0309	79,3075	100,7801
$333,52	59,8818	42,3189	78,4170	100,2977
$335,26	59,1742	40,9564	77,9927	100,0463
$335,96	58,8995	40,4188	77,8297	99,9948
$338,11	58,0916	38,8082	77,3566	99,7625
$338,87	57,8190	38,2542	77,1993	99,6881
$340,74	57,1777	36,9272	76,8346	99,5221
$346,10	55,5736	33,4364	75,9641	99,1808
$346,20	55,5470	33,3760	75,9503	99,1764
$346,68	55,4211	33,0889	75,8854	99,1558
$348,62	54,9410	31,9730	75,6444	99,0888
$351,73	54,2687	30,3406	75,3288	99,0350
$354,48	53,7747	29,0677	75,1220	99,0421
$358,93	53,1760	27,3708	74,9306	99,1614
$360,52	53,0222	26,8785	74,9048	99,2360
$367,18	52,7217	25,5000	75,0385	99,7291
$369,78	52,7540	25,2688	75,1954	99,9998
$373,43	52,9395	25,2380	75,5134	100,4528
$374,28	53,0060	25,2799	75,6037	100,5704
$379,00	53,5335	25,8446	76,2155	101,3057
$383,58	54,2972	26,9150	76,9852	102,1504
$386,16	54,8342	27,7338	77,4937	102,6822
$386,98	55,0207	28,0253	77,6664	102,8596
$387,00	55,0253	28,0326	77,6707	102,8640
$388,03	55,2705	28,4200	77,8954	103,0926
$389,08	55,5326	28,8384	78,1331	103,3320
$395,33	57,3421	31,7899	79,7249	104,8905
$396,45	57,7103	32,3970	80,0417	105,1936
$397,06	57,9164	32,7370	80,2182	105,3616
$397,80	58,1715	33,1580	80,4360	105,5682
$401,83	59,6588	35,6088	82,0233	106,7474

2.4. The relationship between the value of Chooser option and volatility (cont.)

Underlying price (S)	Value of Chooser, when volatility = 21%	Value of Chooser, when volatility = 10%	Value of Chooser, when volatility = 30%	Value of Chooser, when volatility = 40%
$403,31	60,2455	36,5718	82,1844	107,2030
$406,78	61,7041	38,9499	83,3975	108,3179
$407,57	62,0521	39,5134	83,6855	108,5808
$409,10	62,7425	40,6265	84,2556	109,0994
$411,28	63,7629	42,2594	85,0955	109,8595
$411,63	63,9307	42,5265	85,2334	109,9839
$411,90	64,0608	42,7334	85,3403	110,0803
$416,88	66,5749	46,6792	87,3988	111,9250
$418,63	67,5079	48,1189	88,1604	112,6031
$419,43	67,9426	48,7853	88,5151	112,9182
$419,99	68,2500	49,2547	88,7658	113,1407
$433,49	76,3717	61,1687	95,3838	118,9592
$435,11	77,4315	62,6589	96,2487	119,7143
$441,73	81,9315	68,8428	99,9302	122,9202
$442,95	82,7891	69,9965	100,6337	123,5317
$443,03	82,8456	70,0723	100,6801	123,5721
$444,10	83,6051	71,0874	101,3037	124,1139
$456,54	92,8706	83,0546	108,9603	130,7563
$463,72	98,5420	90,0586	113,6959	134,8632
$512,34	141,0980	138,1157	150,5145	167,0871

Source: prepared by the author

2.5. The relationship between the value of Chooser option and risk-free rate

Underlying price (S)	Value of Chooser, when risk-free rate = 5,1%	Value of Chooser, when risk-free rate = 7%	Value of Chooser, when risk-free rate = 3%	Value of Chooser, when risk-free rate = 1,5%
$266,90	107,3230	100,5315	114,8098	120,3073
$281,42	94,4890	88,0363	101,6673	106,9779
$304,88	76,1192	70,6072	82,4293	87,1995
$307,87	74,0605	68,7080	80,2209	84,8962
$309,95	72,6729	67,4369	78,7237	83,3293
$312,49	71,0302	65,9425	76,9410	81,1231
$325,12	63,7781	59,5227	68,8924	72,8933
$327,38	62,6532	58,5625	67,6080	71,5043
$327,93	62,3879	58,3378	67,3030	71,1733
$330,18	61,3370	57,4563	66,0872	69,8489
$333,52	59,8818	56,2600	64,3785	67,9724
$335,26	59,1742	55,6907	63,5349	67,0384
$335,96	58,8995	55,4722	63,2047	66,6713
$338,11	58,0916	54,8391	62,2239	65,5751
$338,87	57,8190	54,6291	61,8893	65,1990
$340,74	57,1777	54,1432	61,0935	64,2995
$346,10	55,5736	52,9958	59,0324	61,9298
$346,20	55,5470	52,9778	58,9971	61,8886
$346,68	55,4211	52,8935	58,8293	61,6923
$348,62	54,9410	52,5828	58,1784	60,9251
$351,73	54,2687	52,1851	57,2278	59,7843
$354,48	53,7747	51,9366	56,4835	58,8680
$358,93	53,1760	51,7391	55,4729	57,5726
$360,52	53,0222	51,7296	55,1703	57,1665
$367,18	52,7217	52,0350	54,2394	55,7941
$369,78	52,7540	52,3036	54,0238	55,4034
$373,43	52,9395	52,8194	53,8607	54,9928
$374,28	53,0060	52,9625	53,8460	54,9203
$379,00	53,5335	53,9120	53,9240	54,6763
$383,58	54,2972	55,0785	54,2550	54,6951
$386,16	54,8342	55,8387	54,5506	54,8154
$386,98	55,0207	56,0956	54,6608	54,8701
$387,00	55,0253	56,1019	54,6636	54,8715
$388,03	55,2705	56,4350	54,8133	54,9516
$389,08	55,5326	56,7863	54,9785	55,0459
$395,33	57,3421	59,1141	56,2203	55,8706
$396,45	57,7103	59,5730	56,4888	56,0652
$397,06	57,9164	59,8281	56,6408	56,1772
$397,80	58,1715	60,1424	56,8306	56,3185
$401,83	59,6588	61,9459	57,9672	57,1942

2.5. The relationship between the value of Chooser option and risk-free rate (cont.)

Underlying price (S)	Value of Chooser, when risk-free rate = 5,1%	Value of Chooser, when risk-free rate = 7%	Value of Chooser, when risk-free rate = 3%	Value of Chooser, when risk-free rate = 1,5%
$403,31	60,2455	62,6461	58,4275	57,5600
$406,78	61,7041	64,3651	59,5950	58,5091
$407,57	62,0521	64,7712	59,8778	58,7428
$409,10	62,7425	65,5728	60,4430	59,2137
$411,28	63,7629	66,7490	61,2879	59,9258
$411,63	63,9307	66,9415	61,4278	60,0446
$411,90	64,0608	67,0906	61,5366	60,1371
$416,88	66,5749	69,9453	63,6643	61,9709
$418,63	67,5079	70,9936	64,4658	62,6717
$419,43	67,9426	71,4804	64,8412	63,0017
$419,99	68,2500	71,8239	65,1074	63,2362
$433,49	76,3717	80,7451	72,3068	69,7212
$435,11	77,4315	81,8917	73,2653	70,6005
$441,73	81,9315	86,7271	77,3728	74,3990
$442,95	82,7891	87,6431	78,1617	75,1335
$443,03	82,8456	87,7034	78,2138	75,1821
$444,10	83,6051	88,5133	78,9141	75,8354
$456,54	92,8706	98,3077	87,5528	83,9734
$463,72	98,5420	104,2404	92,9121	89,0804
$512,34	141,0980	147,9215	134,1107	129,1547

Source: prepared by the author

2.6. The relationship between the value of Chooser option and time-to-choose

Underlying price (S)	Value of Chooser, when time to choose = 0,5 year	Value of Chooser, when time to choose = 0,75 year	Value of Chooser, when time to choose = 0,25 year	Value of Chooser, when time to choose = 0,0888 year
$266,90	107,3230	107,8565	107,1374	107,1311
$281,42	94,4890	95,4788	94,0013	93,9643
$304,88	76,1192	78,2101	74,5056	74,1458
$307,87	74,0605	76,3153	72,2363	71,7780
$309,95	72,6729	75,0437	70,6939	70,1548
$312,49	71,0302	73,5439	68,8533	68,2002
$325,12	63,7781	67,0057	60,5035	58,9848
$327,38	62,6532	66,0052	59,1696	57,4372
$327,93	62,3879	65,7698	58,8531	57,0660
$330,18	61,3370	64,8402	57,5927	55,5704
$333,52	59,8818	63,5598	55,8273	53,4244
$335,26	59,1742	62,9405	54,9596	52,3448
$335,96	58,8995	62,7007	54,6209	51,9184
$338,11	58,0916	61,9977	53,6188	50,6393
$338,87	57,8190	61,7614	53,2785	50,1986
$340,74	57,1777	61,2071	52,4731	49,1416
$346,10	55,5736	59,8346	50,4235	46,3496
$346,20	55,5470	59,8120	50,3890	46,3012
$346,68	55,4211	59,7054	50,2254	46,0708
$348,62	54,9410	59,3007	49,5975	45,1743
$351,73	54,2687	58,7407	48,7038	43,8609
$354,48	53,7747	58,3365	48,0317	42,8370
$358,93	53,1760	57,8634	47,1840	41,4793
$360,52	53,0222	57,7485	46,9539	41,0898
$367,18	52,7217	57,5747	46,4113	40,0449
$369,78	52,7540	57,6406	46,3843	39,9038
$373,43	52,9395	57,8581	46,5206	39,9635
$374,28	53,0060	57,9295	46,5814	40,0206
$379,00	53,5335	58,4672	47,1152	40,6293
$383,58	54,2972	59,2139	47,9452	41,6763
$386,16	54,8342	59,7299	48,5442	42,4526
$386,98	55,0207	59,9081	48,7539	42,7260
$387,00	55,0253	59,9126	48,7592	42,7328
$388,03	55,2705	60,1462	49,0359	43,0943
$389,08	55,5326	60,3954	49,3329	43,4826
$395,33	57,3421	62,1027	51,4001	46,1776
$396,45	57,7103	62,4484	51,8229	46,7245
$397,06	57,9164	62,6416	52,0597	47,0299
$397,80	58,1715	62,8808	52,3529	47,4072
$401,83	59,6588	64,2721	54,0635	49,5874

2.6. The relationship between the value of Chooser option and time-to-choose (cont.)

Underlying price (S)	**Value of Chooser, when time to choose = 0,5 year**	**Value of Chooser, when time to choose = 0,75 year**	**Value of Chooser, when time to choose = 0,25 year**	**Value of Chooser, when time to choose = 0,0888 year**
$403,31	60,2455	64,8200	54,7383	50,4374
$406,78	61,7041	66,1809	56,4141	52,5228
$407,57	62,0521	66,5053	56,8135	53,0145
$409,10	62,7425	67,1489	57,6049	53,9832
$411,28	63,7629	68,0999	58,7729	55,3989
$411,63	63,9307	68,2563	58,9647	55,6299
$411,90	64,0608	68,3776	59,1135	55,8087
$416,88	66,5749	70,7208	61,9779	59,2048
$418,63	67,5079	71,5907	63,0364	60,4383
$419,43	67,9426	71,9962	63,5288	61,0085
$419,99	68,2500	72,2829	63,8766	61,4099
$433,49	76,3717	79,8770	72,9639	71,5637
$435,11	77,4315	80,8709	74,1353	72,8344
$441,73	81,9315	85,1007	79,0752	78,1222
$442,95	82,7891	85,9085	80,0105	79,1122
$443,03	82,8456	85,9618	80,0721	79,1773
$444,10	83,6051	86,6777	80,8987	80,0495
$456,54	92,8706	95,4462	90,8740	90,4238
$463,72	98,5420	100,8453	96,8916	96,5858
$512,34	141,0980	142,0182	140,7347	140,7183

Source: prepared by the author