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DETERMINING THE GAUGE DEPENDENCE OF THE LOOP-CORRECTED LIGHT NEUTRINO PROPAGATOR

Master of Science Thesis

Study Programme – THEORETICAL PHYSICS AND ASTROPHYSICS

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1 Introduction

The Standard Model of particle physics predicts that neutrinos are massless particles. However, it is now a well known fact that neutrinos must have nonzero masses [1], albeit very small ones: currently established limit of the sum of neutrino masses is $\sum_i m_{\nu_i} < 0.12$ eV (95% C.L.) [2]. For comparison, the mass of the next lightest particle of the Standard Model, the electron, is ~511 keV, 6 orders of magnitude higher. The question of the origin of this huge mass difference is currently on the frontier of research of theoretical particle physics, and it is one of the most promising paths to the physics beyond the Standard Model.

The most ubiquitous explanation is the so-called *seesaw mechanism*. It has several versions; one of the simplest extends the Standard Model (SM) with an additional right-handed neutrino field and a second Higgs doublet, allowing to generate three massive neutrino fields — one very heavy (possibly of the order of 10^{14} GeV), two very light (order of 1 eV) — and one massless [3]. This model (let's further call it *the Grimus-Neufeld (GN) model*) seems promising, since it is economical (the SM is extended only slightly) and reproduces the mass hierarchy of the neutrino fields that is compatible with observations.

In [4], among other things, it is shown that in the GN model one of the light neutrino fields acquires mass through *one-loop corrections to the neutrino propagator*. It means that the expressions of one-loop corrections (and, consequently, the parameters of the GN model) can be connected with experimentally measurable quantities (squared mass differences of the neutrinos). This could allow making concrete numerical predictions and testing the validity of the GN model (if, for example, the second Higgs doublet is experimentally discovered at some point in the future). However, first we must be sure that the theory is consistent.

One of the most obvious consistency checks of a gauge theory is its *gauge invariance*: None of the physically observable predictions of the theory should depend on the value of an arbitrary gauge parameter. In [4] the authors concisely show that the one-loop corrections they obtain are gauge invariant. The aim of this work is to check this result using a different approach.

This master's thesis continues and finishes the work of my last three semester's papers. The first one [5] was an introduction to the quantum field theory and, more specifically, the concept of propagators. Second semester's work [6] focused on understanding concepts of spinors and gauge invariance, and presented our approach to deriving one-loop corrections of the neutrino propagator in a general model, not tied to any specific particle fields. Third semester's paper [7] connected the Lagrangian of this general model with the Standard Model extended by one right-handed neutrino field. And, finally, **this master's thesis aims to** (i) make the particle content of our model the same as in the GN model by introducing the second Higgs doublet, and (ii) check, whether the one-loop corrections to the neutrino propagator calculated in our formalism are gauge-invariant.

A brief overview of the structure of this thesis might make it more easily understandable.

In section 2 I provide a summary of the contents of the Standard Model, then introduce the second Higgs doublet and shortly discuss the origin of a gauge parameter in the theory. Section 3 is devoted to the seesaw mechanism: Along with the explanation of the mechanism itself it includes derivations of several relations that are needed in later calculations. In section 4 the main part of this work begins. First, I derive a connection between the coefficients of the generic model we are using (first introduced in my second semester's paper [6]) and the Standard Model extended by a right-handed neutrino field and a second Higgs doublet (the GN model). Then, in section 5, I use this result in order to rewrite the generic expressions of one-loop corrections to the neutrino propagator (derived in [6]) in terms of the particle fields of the GN model, and simplify them as much as possible. Finally, section 6 presents the investigation of the gauge dependence of one-loop corrections, and section 7 sums up the results.

2 Overview of the Standard Model with the Second Higgs Doublet

2.1 The Standard Model

The Standard Model (SM for short) has a symmetry group of $SU(3)_C \times SU(2)_L \times U(1)_Y$. The subscript C stands for color, L means that $SU(2)_L$ acts only on the left-handed fields, and Y stands for hypercharge and is intended to distinguish $U(1)_Y$ from the $U(1)$ for electromagnetism. The $SU(3)_C$ part leads to quantum chromodynamics, the theory of strong interactions, whereas the $SU(2)_L \times U(1)_Y$ subgroup describes the electroweak interactions.

The SM consists of fields of spins 0, $\frac{1}{2}$ and 1:

- The only spin-0 field of the SM is the Higgs field Φ. It is a complex scalar field, and it is also a doublet under the group $SU(2)_L \times U(1)_Y$. It couples left- and right-handed fermions together, and, via the spontaneous symmetry breaking mechanism, generates masses of the gauge bosons and fermions. A convenient definition of the Higgs doublet is

$$\Phi = \begin{pmatrix} \varphi^+ \\ \frac{1}{\sqrt{2}}\left(v + H + \mathrm{i}\varphi_Z\right) \end{pmatrix}. \tag{2.1}$$

  Here v is the vacuum expectation value of the Higgs doublet, H is the Higgs field, and φ^+ and φ_Z are, accordingly, the charged and neutral Goldstone bosons. The Goldstone bosons are unphysical, since they can be 'gauged away' by a gauge transformation of the $SU(2)_L$ group, leaving only the physical Higgs field:

$$\Phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\left(v + H\right) \end{pmatrix}. \tag{2.2}$$

  This gauge is called *the unitary gauge.*

- The spin-$\frac{1}{2}$ fields (fermions) of the SM are:

  - Quarks

    There are 3 quark doublets under the group $SU(2)_L \times U(1)_Y$,

$$Q^\alpha_{Li} = \begin{pmatrix} u^\alpha_{Li} \\ d^\alpha_{Li} \end{pmatrix}, \qquad i = 1, 2, 3, \tag{2.3}$$

    and 6 singlets:

$$u^\alpha_{Ri}, d^\alpha_{Ri}, \qquad i = 1, 2, 3. \tag{2.4}$$

Quarks also come in triplets under the group $SU(3)_C$, and that is what the index α in the above expressions denotes: $\alpha = 1, 2, 3$.

– Leptons

Similarly to the quark case, in the $SU(2)_L \times U(1)_Y$ group there are 3 lepton doublets,

$$L_{nL} = \begin{pmatrix} \nu_{nL} \\ e_{nL} \end{pmatrix}, \qquad n = e, \mu, \tau, \tag{2.5}$$

but only 3 singlets:

$$e_{nR}, \qquad n = e, \mu, \tau. \tag{2.6}$$

There are no right-handed neutrinos present in the Standard Model.

- The spin-1 particles are the gauge bosons that mediate the fundamental interactions of the SM:

  – Gluons,

$$G^A_\mu, \qquad A = 1, \ldots, 8, \tag{2.7}$$

appear in the $SU(3)_C$ group and mediate the strong interactions.

  – Three W bosons and B boson,

$$W^I_\mu, B_\mu, \qquad I = 1, 2, 3, \tag{2.8}$$

exist under $SU(2)_L \times U(1)_Y$ group and participate in the electroweak interactions. Through the spontaneous symmetry breaking, they combine with the Higgs field and produce the massless photon field

$$A_\mu, \tag{2.9}$$

and three massive gauge bosons,

$$W^+_\mu, W^-_\mu, Z_\mu, \tag{2.10}$$

with masses

$$m_W = \frac{gv}{2}, \qquad m_Z = \frac{v}{2}\sqrt{g^2 + g'^2} = \frac{m_W}{\cos\theta_W} \tag{2.11}$$

(g and g' are the gauge coupling constants of the $SU(2)_L$ group, v is the vacuum expectation value of the Higgs field, and *the weak mixing angle* θ_W is defined through $\cos\theta_W = g/\sqrt{g^2 + g'^2}$).

More extensive overview of the Standard Model can be found in, for example, [8] and [9].

2.2 The Second Higgs Doublet

In [3] the authors show that the addition of a second Higgs doublet to the SM extended with a right-handed neutrino field allows to generate mass for the second neutrino at one-loop level. Thus in our model we are also using two Higgs doublets:

$$\Phi_1 = \begin{pmatrix} \varphi^+ \\ \frac{1}{\sqrt{2}}\big(v + H + \mathrm{i}\varphi_Z\big) \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\big(H_R + \mathrm{i}H_I\big) \end{pmatrix}. \tag{2.12}$$

H^+ here is a charged scalar field, and H, H_R and H_I — neutral scalar fields. Eq. (2.12) is written in the so-called *Higgs basis*, in which only one of the two doublets has a non-zero vacuum expectation value. The second Higgs doublet does not produce any new gauge interaction terms in the Lagrangian, but appears in additional Yukawa terms (discussed in section 4.1).

Switching to the mass-basis of the two Higgs doublets, we would find [10] that the fields φ^+, φ_Z and H^+ are already in their mass eigenstates (although the masses of φ^+ and φ_Z vanish, as should be expected for Goldstone bosons), but the remaining three scalar fields (H, H_R and H_I) mix together to produce three physical massive Higgs fields h_1, h_2 and h_3:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} H \\ H_R \\ H_I \end{pmatrix}, \tag{2.13}$$

where R is an orthogonal transformation matrix that can be parametrized as

$$R = \begin{pmatrix} c_{13}c_{12} & -c_{23}s_{12} - c_{12}s_{13}s_{23} & -c_{12}c_{23}s_{13} + s_{12}s_{23} \\ c_{13}s_{12} & c_{12}c_{23} - s_{12}s_{13}s_{23} & -c_{23}s_{12}s_{13} - c_{12}s_{23} \\ s_{13} & c_{13}s_{23} & c_{13}c_{23} \end{pmatrix}, \tag{2.14}$$

with definitions $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. If we require CP conservation, the third neutral Higgs scalar H_I decouples from the other two, i.e., $\theta_{13} = \theta_{23} = 0$, and R significantly simplifies:

$$R = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.15}$$

Consequently, H_I is already in the mass state as well, so we are going to keep using this name for this field instead of h_3. And, since the R matrix now depends on only one mixing angle θ_{12}, let's rename it to α and recast h_1 and h_2 in the following form:

$$h_1 = H\cos\alpha - H_R \sin\alpha, \tag{2.16}$$

$$h_2 = H\sin\alpha + H_R \cos\alpha. \tag{2.17}$$

2.3 't Hooft's Gauges

The unitary gauge (2.2) for the Higgs field is useful when we are interested only in the physical particles' content of the theory, but for some other purposes (for instance, calculations of the loop corrections) leaving the Goldstone bosons φ^+ and φ_Z in the theory proves to be more convenient. However, in this case we are faced with a significant difficulty: the gauge boson propagator becomes undefined. Generally speaking, the reason for this is the fact that gauge theories introduce redundant degrees of freedom, which need to be removed by the procedure called *the gauge fixing* (extensively explained in, for example, [11]).

A clever way to do that was suggested by t' Hooft. He proposed using a gauge condition

$$\partial_\mu A^\mu = m\xi\chi, \tag{2.18}$$

for each gauge field A^μ and Goldstone field χ. m here is the mass of the gauge boson and ξ is an arbitrary gauge parameter. After imposing this gauge-fixing condition, the gauge boson propagator can be calculated, but it becomes dependent on ξ:

$$\mathrm{i}\left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2 - \xi m^2}\right]\frac{1}{k^2 - m^2}. \tag{2.19}$$

Luckily, ξ drops out when calculating values of physical observables, as it should: Quantities that can be measured experimentally should not depend on any arbitrary parameter like ξ.

In the case of the Standard Model, this procedure amounts to adding the following gauge-fixing term to the Lagrangian:

$$-\frac{1}{2\xi}\left\{\sum_{i=+,-}\left(\partial^\mu W^i_\mu + \xi m_W \varphi^i\right)^2 + \left(\partial^\mu Z_\mu + \xi m_Z \varphi_Z\right)^2 + \left(\partial^\mu A_\mu\right)^2\right\} \tag{2.20}$$

(φ^- is another Goldstone field that is defined as the complex conjugate of φ^+). As a consequence, the Goldstone fields $\varphi^{+/-}$ and φ_Z acquire masses that depend on ξ:

$$m^2_{\varphi+} = \xi m^2_W, \qquad m^2_{\varphi z} = \xi m^2_Z. \tag{2.21}$$

3 The Seesaw Mechanism

3.1 Dirac and Majorana Mass Terms

Quantum field theory allows two types of Lagrangian mass terms for a field ψ: *Dirac*,

$$\mathcal{L}^{D}_{\text{mass}} = -m_D \overline{\psi}_R \psi_L, \tag{3.1}$$

and *Majorana*,

$$\mathcal{L}^{L/R}_{\text{mass}} = \frac{1}{2} m_M \psi^T_{L/R} C^\dagger \psi_{L/R}. \tag{3.2}$$

(C is the charge conjugation matrix. Charge conjugation and Majorana particles are thoroughly discussed in [12].) In case of neutrinos, the SM assumes existence of only the left-handed fields ν_{nL} ($n = e, \mu, \tau$), thus only the Majorana mass term $\mathcal{L}^{L}_{\text{mass}}$ could exist. But it is not invariant under the $SU(2)_L \times U(1)_Y$ symmetry of the SM, therefore is forbidden. As a consequence, neutrinos are massless in the SM.

In order to generate masses of the neutrinos, it is enough to extend the Standard Model with only one right-handed neutrino field ν_R. Then the Dirac mass term becomes available and, since ν_R is a singlet of $SU(3)_C \times SU(2)_L \times U(1)_Y$, the Majorana mass term $\mathcal{L}^{R}_{\text{mass}}$ is also allowed. Actually, the Dirac mass term would suffice to explain the masses of the neutrinos (they would be generated in the same way as the masses of the charged leptons, via the Higgs mechanism), but the peculiar smallness of these masses still would remain a mystery. This is were the Majorana mass term comes in handy — it is an essential part of the *seesaw* mechanism that provides a sensible answer to the question, why the masses of the neutrinos are so much smaller than the masses of other particles of the Standard Model, but still are not equal to zero?

Let's start the presentation of the seesaw mechanism by writing down both Dirac and Majorana mass terms that are allowed by the SM, extended with one right-handed neutrino field ν_R:

$$\mathcal{L}^{D+M}_{\text{mass}} = \sum_{n=e,\mu,\tau} -m_{Dn} \bar{\nu}_R \nu_{nL} + \frac{1}{2} m_R \nu_R^T C^\dagger \nu_R + h.c. \tag{3.3}$$

($h.c.$ means the hermitian conjugate of the preceding terms). m_{Dn} are complex numbers, but m_R can be made real by redefining ν_R to consume the complex phase part of m_R. In the following I am going to assume that this has been done and $m_R \in \mathbb{R}$.

If we define the column matrix of left-handed chiral neutrino fields

$$N_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_R^C \end{pmatrix} = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ C\overline{\nu_R}^T \end{pmatrix}, \tag{3.4}$$

we can write the Dirac-Majorana mass term (3.3) in a simplified form:

$$\mathcal{L}_{\text{mass}}^{D+M} = \frac{1}{2} N_L^T C^\dagger M N_L + h.c., \tag{3.5}$$

with the symmetric mass matrix

$$M = \begin{pmatrix} 0 & 0 & 0 & m_{De} \\ 0 & 0 & 0 & m_{D\mu} \\ 0 & 0 & 0 & m_{D\tau} \\ m_{De} & m_{D\mu} & m_{D\tau} & m_R \end{pmatrix}. \tag{3.6}$$

Since the mass matrix M is not diagonal, it is clear that the chiral fields ν_{Ln} and ν_R do not have definite masses. We can find the fields of the massive neutrinos by using a unitary transformation of the chiral fields

$$n_L = U N_L = \begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} & U_{1R} \\ U_{2e} & U_{2\mu} & U_{2\tau} & U_{2R} \\ U_{3e} & U_{3\mu} & U_{3\tau} & U_{3R} \\ U_{4e} & U_{4\mu} & U_{4\tau} & U_{4R} \end{pmatrix} \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_R^C \end{pmatrix} \tag{3.7}$$

where

$$n_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \\ \nu_{4L} \end{pmatrix} \tag{3.8}$$

is the column matrix of chiral left-handed massive neutrino fields. The unitary matrix U must be such that

$$(U^{-1})^T M U^{-1} = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{pmatrix}, \tag{3.9}$$

with real $m_k \geq 0$. This can always be done, as is shown, for example, in [13].

With the transformation in eq. (3.7), the Dirac-Majorana mass term can be written as

$$\mathcal{L}_{\text{mass}}^{D+M} = \frac{1}{2} \sum_{k=1}^{4} m_k \nu_{kL}^T C^\dagger \nu_{kL} + h.c. = -\frac{1}{2} \sum_{k=1}^{4} m_k \bar{\nu}_k \nu_k, \tag{3.10}$$

which looks exactly like a Majorana mass term for the massive neutrino field

$$\nu_k \equiv \nu_{kL} + \nu_{kL}^C = \nu_{kL} + C \bar{\nu}_{kL}^T. \tag{3.11}$$

ν_k satisfies the Majorana condition

$$\nu_k = \nu_k^C, \tag{3.12}$$

therefore a Dirac-Majorana mass term (3.3) implies that massive neutrinos are Majorana particles.

Using definitions of U and ν_k, we can also find expressions connecting ν_R and ν_{nL} with ν_k. First of all, reversing eq. (3.7), we see that

$$\nu_{nL} = U^*_{mn} n_{Lm}, \qquad n = e, \mu, \tau, \qquad m = 1, \ldots, 4, \tag{3.13}$$

$$\nu_R^C = U^*_{m4} n_{Lm}, \qquad m = 1, \ldots, 4. \tag{3.14}$$

Here and in the rest of this work, summation over any two repeated indices is implied everywhere, unless stated otherwise. From the definition and properties of charge conjugation, it follows that

$$\nu_R = U_{m4} n_{Lm}^C. \tag{3.15}$$

Then, using the properties of the left- and right-handed projection matrices P_L and P_R (also discussed in [12]), we can write the following relations:

$$n_{Lm} = P_L \nu_m, \tag{3.16}$$

$$n_{Lm}^C = P_R \nu_m, \tag{3.17}$$

and finally get:

$$\nu_{nL} = U^*_{mn} P_L \nu_m, \tag{3.18}$$

$$\nu_R = U_{m4} P_R \nu_m, \tag{3.19}$$

and

$$\bar{\nu}_{nL} = \bar{\nu}_m U_{mn} P_R, \tag{3.20}$$

$$\bar{\nu}_R = \bar{\nu}_m U^*_{m4} P_L. \tag{3.21}$$

3.2 Diagonalization of Neutrino Mass Matrix

Since the matrix U, defined in eq. (3.7), is unitary, the diagonalization condition of neutrino mass matrix (3.9) can be rewritten in the following way:

$$(U^{-1})^T M U^{-1} = (U^\dagger)^T M U^\dagger = U^* M U^\dagger = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{pmatrix} \equiv \hat{m}, \tag{3.22}$$

and, using the fact that diagonalized mass matrix $\hat{m}$ is real,

$$U M^* U^T = \hat{m}. \tag{3.23}$$

Rank of the mass matrix M is 2, therefore only two of the masses m_j can be nonzero. Let's hold that the first two masses are the vanishing ones, i.e.,

$$m_1 = m_2 = 0. \tag{3.24}$$

Then, again using unitarity of U and the definition of M (eq. (3.6)), we can rewrite the seesaw condition (3.23) as

$$U M^* = \hat{m} U^* = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ m_3 U^*_{3e} & m_3 U^*_{3\mu} & m_3 U^*_{3\tau} & m_3 U^*_{3R} \\ m_4 U^*_{4e} & m_4 U^*_{4\mu} & m_4 U^*_{4\tau} & m_4 U^*_{4R} \end{pmatrix}. \tag{3.25}$$

Comparing the left-hand side of the previous equation,

$$\begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} & U_{1R} \\ U_{2e} & U_{2\mu} & U_{2\tau} & U_{2R} \\ U_{3e} & U_{3\mu} & U_{3\tau} & U_{3R} \\ U_{4e} & U_{4\mu} & U_{4\tau} & U_{4R} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & m^*_{De} \\ 0 & 0 & 0 & m^*_{D\mu} \\ 0 & 0 & 0 & m^*_{D\tau} \\ m^*_{De} & m^*_{D\mu} & m^*_{D\tau} & m_R \end{pmatrix} \tag{3.26}$$

(remember that m_R is real), with its right-hand side, we get

$$U_{1R} m^*_{De} = U_{1R} m^*_{D\mu} = U_{1R} m^*_{D\tau} = U_{2R} m^*_{De} = U_{2R} m^*_{D\mu} = U_{2R} m^*_{D\tau} = 0, \tag{3.27}$$

giving $U_{1R} = U_{2R} = 0$, and

$$U_{1e} m^*_{De} + U_{1\mu} m^*_{D\mu} + U_{1\tau} m^*_{D\tau} = U_{2e} m^*_{De} + U_{2\mu} m^*_{D\mu} + U_{2\tau} m^*_{D\tau} = 0, \tag{3.28}$$

telling us that the three vectors $(U_{1e}, U_{1\mu}, U_{1\tau}, 0)$, $(U_{2e}, U_{2\mu}, U_{2\tau}, 0)$ and $(m_{De}, m_{D\mu}, m_{D\tau}, m_R)$ are orthogonal (the orthogonality between U_{1j} and U_{2j} follows from the unitarity of U).

From the unitarity of U and the fact, that $U_{1R} = U_{2R} = 0$, follows the relation

$$|U_{3R}|^2 + |U_{4R}|^2 = 1, \tag{3.29}$$

which allows to parametrize U_{3R} and U_{4R} as

$$U_{3R} = \mathrm{e}^{\mathrm{i}\alpha_3} \sin\theta \equiv \eta_3 s \qquad \text{and} \qquad U_{4R} = \mathrm{e}^{\mathrm{i}\alpha_4} \cos\theta \equiv \eta_4 c. \tag{3.30}$$

Now eq. (3.25) takes the following form:

$$\begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} & 0 \\ U_{2e} & U_{2\mu} & U_{2\tau} & 0 \\ U_{3e} & U_{3\mu} & U_{3\tau} & \eta_3 s \\ U_{4e} & U_{4\mu} & U_{4\tau} & \eta_4 c \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & m_{De}^* \\ 0 & 0 & 0 & m_{D\mu}^* \\ 0 & 0 & 0 & m_{D\tau}^* \\ m_{De}^* & m_{D\mu}^* & m_{D\tau}^* & m_R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ m_3 U_{3e}^* & m_3 U_{3\mu}^* & m_3 U_{3\tau}^* & m_3 \eta_3^* s \\ m_4 U_{4e}^* & m_4 U_{4\mu}^* & m_4 U_{4\tau}^* & m_4 \eta_4^* c \end{pmatrix}. \tag{3.31}$$

From the first 3 columns we get the relations

$$m_3 U_{3n}^* = \eta_3 s m_{Dn}^*, \qquad n = 1, 2, 3, \tag{3.32}$$
$$m_4 U_{4n}^* = \eta_4 c m_{Dn}^*, \qquad n = 1, 2, 3, \tag{3.33}$$

and from the fourth one:

$$U_{3n} m_{Dn}^* + \eta_3 s m_R = m_3 \eta_3^* s, \tag{3.34}$$
$$U_{4n} m_{Dn}^* + \eta_4 c m_R = m_4 \eta_4^* c, \tag{3.35}$$

or, using eqs. (3.32) and (3.33):

$$\eta_3^* s \frac{m_D^2}{m_3} + \eta_3 s m_R = \eta_3^* s m_3, \tag{3.36}$$
$$\eta_4^* c \frac{m_D^2}{m_4} + \eta_4 c m_R = \eta_4^* c m_4, \tag{3.37}$$

where we have introduced

$$m_D^2 \equiv |m_{De}|^2 + |m_{D\mu}|^2 + |m_{D\tau}|^2. \tag{3.38}$$

Now if we assume the following mass scale hierarchy,

$$m_3 \ll m_4 \sim m_R \tag{3.39}$$

(a valid assumption, since the fourth neutrino would have already been detected experimentally if its mass was smaller than the mass of the Z boson), then equations (3.36) and (3.37) can be solved only if $\eta_3^2 = -1$ and $\eta_4^2 = +1$, and give as positive solutions

$$m_{3,4} = \frac{1}{2}\Big(\mp m_R + \sqrt{m_R^2 + 4m_D^2}\Big), \tag{3.40}$$

which produce the usual seesaw relations:

$$m_3 m_4 = m_D^2 \qquad \text{and} \qquad m_4 - m_3 = m_R. \tag{3.41}$$

Looking at these equations, the origin of the name of the seesaw mechanism becomes clear: the larger is m_4, the smaller m_3, and vice versa.

The seesaw mechanism provides a natural explanation of the smallnes of the neutrino masses with respect to the masses of charged leptons. The Dirac mass m_D, which can be generated through the Higgs mechanism of the SM, is expected to be of the order of charged lepton masses, or at least not much larger than the electroweak scale, which is of the order of 10^2 GeV. The reason for this is that a Dirac mass term can arise only as a consequence of symmetry breaking, as for the other particles in the SM. Hence, m_D is proportional to the symmetry-breaking scale. On the other hand, since a Majorana mass term is a singlet of the SM symmetries, it is not affected by them. It is plausible that the Majorana mass m_R is generated by new physics beyond the SM and its order of magnitude corresponds to the breaking scales of the symmetries of some high-energy theory, which may be at the grand unification scale of the order of 10^{14}-10^{16} GeV. Hence the seesaw expression (3.41) may give the mass of one of the light neutrinos m_3 of the order of $m_D^2/m_R \sim 10^{-10}$-10^{-12} GeV, which is consistent with experimental results [1].

Now, using unitarity of U and equations (3.32) and (3.33), we can get the following relations:

$$1 = |U_{3e}|^2 + |U_{3\mu}|^2 + |U_{3\tau}|^2 + s^2 = s^2 \frac{m_D^2}{m_3^2} + s^2 = s^2 \left(\frac{m_4}{m_3} + 1 \right) = s^2 \frac{m_3 + m_4}{m_3}, \tag{3.42}$$

$$1 = |U_{4e}|^2 + |U_{4\mu}|^2 + |U_{4\tau}|^2 + c^2 = c^2 \frac{m_D^2}{m_4^2} + c^2 = c^2 \left(\frac{m_3}{m_4} + 1 \right) = c^2 \frac{m_3 + m_4}{m_4}, \tag{3.43}$$

which give us the solutions

$$s^2 \equiv \sin^2\theta = \frac{m_3}{m_3 + m_4} \qquad \text{and} \qquad c^2 \equiv \cos^2\theta = \frac{m_4}{m_3 + m_4}, \tag{3.44}$$

and allow us to express the 3rd and 4th rows of U matrix as

$$U_{3n} = \mathrm{i}c \frac{s}{c} \frac{m_{Dn}}{m_3} = \mathrm{i}c \frac{\sqrt{m_3}}{\sqrt{m_4}} \frac{m_{Dn}}{m_3} = \mathrm{i}c \frac{m_{Dn}}{\sqrt{m_3 m_4}} = \mathrm{i}c \frac{m_{Dn}}{m_D}, \tag{3.45}$$

$$U_{4n} = s\frac{c}{s}\frac{m_{Dn}}{m_4} = s\frac{\sqrt{m_4}}{\sqrt{m_3}}\frac{m_{Dn}}{m_4} = s\frac{m_{Dn}}{\sqrt{m_3 m_4}} = s\frac{m_{Dn}}{m_D}. \tag{3.46}$$

This allows us to write the neutrino mixing matrix U in the following form:

$$U = \begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} & 0 \\ U_{2e} & U_{2\mu} & U_{2\tau} & 0 \\ \mathrm{i}c\frac{m_{De}}{m_D} & \mathrm{i}c\frac{m_{D\mu}}{m_D} & \mathrm{i}c\frac{m_{D\tau}}{m_D} & -\mathrm{i}s \\ s\frac{m_{De}}{m_D} & s\frac{m_{D\mu}}{m_D} & s\frac{m_{D\tau}}{m_D} & c \end{pmatrix} \approx \begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} & 0 \\ U_{2e} & U_{2\mu} & U_{2\tau} & 0 \\ \mathrm{i}\frac{m_{De}}{m_D} & \mathrm{i}\frac{m_{D\mu}}{m_D} & \mathrm{i}\frac{m_{D\tau}}{m_D} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{3.47}$$

where we have used the limit $m_3/m_4 \to 0$, in which $s \to 0$ and $c \to 1$. In this approximation the upper-left 3×3 block that decoupled from the rest of the matrix corresponds to the so-called PMNS (Pontecorvo-Maki–Nakagawa–Sakata) neutrino mixing matrix V, that is defined as

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \tag{3.48}$$

Comparing this definition with eq. (3.7), we can see that

$$U_{mn} = V^*_{nm}, \qquad \text{for } n, m = 1, 2, 3, \tag{3.49}$$

so

$$U_{3n} = \mathrm{i}\frac{m_{Dn}}{m_D} = V^*_{n3} \tag{3.50}$$

and

$$\frac{m_{Dn}}{m_D} = -\mathrm{i}V^*_{n3}. \tag{3.51}$$

Consequently, U matrix can be expressed in terms of the elements of V matrix:

$$U = \begin{pmatrix} V^*_{e1} & V^*_{\mu 1} & V^*_{\tau 1} & 0 \\ V^*_{e2} & V^*_{\mu 2} & V^*_{\tau 2} & 0 \\ cV^*_{e3} & cV^*_{\mu 3} & cV^*_{\tau 3} & -\mathrm{i}s \\ -\mathrm{i}sV^*_{e3} & -\mathrm{i}sV^*_{\mu 3} & -\mathrm{i}sV^*_{\tau 3} & c \end{pmatrix}. \tag{3.52}$$

4 The Coefficients of the Formal Lagrangian with the Second Higgs Doublet

A formal Lagrangian $\mathcal{L}$ of our model was introduced in [6]. This Lagrangian describes interactions relevant for calculations of one-loop corrections to neutrino propagator, in a model that contains an arbitrary number of Majorana fermions ψ, Dirac fermions χ, complex scalar fields S, real scalar fields R, complex vector fields W, and real vector fields V. In [6] I have also calculated neutrino one-loop corrections, and in [7] have derived a connection between the coefficients of the formal Lagrangian and the parameters of the Standard Model extended with one right-handed neutrino field. Here I am going to recalculate this connection, since our model now additionaly includes a second Higgs doublet.

First of all, let's repeat the expression of the formal Lagrangian, introduced in [6]:

$$\begin{aligned}\mathcal{L} = &-S_n\overline{\psi}_\alpha\Big(y^R_{\alpha an}P_R + y^L_{\alpha an}P_L\Big)\chi_a - S^*_n\overline{\chi}_a\Big(y^{R*}_{\alpha an}P_L + y^{L*}_{\alpha an}P_R\Big)\psi_\alpha \\ &- W^j_\mu\overline{\psi}_\alpha\gamma^\mu\Big(g^R_{\alpha aj}P_R + g^L_{\alpha aj}P_L\Big)\chi_a - W^{j*}_\mu\overline{\chi}_a\gamma^\mu\Big(g^{R*}_{\alpha aj}P_R + g^{L*}_{\alpha aj}P_L\Big)\psi_\alpha \\ &- R_n\overline{\psi}_\alpha\Big((y_n)_{\alpha\beta}P_R + (y^\dagger_n)_{\alpha\beta}P_L\Big)\psi_\beta - V^j_\mu\overline{\psi}_\alpha\gamma^\mu\Big((g^R_j)_{\alpha\beta}P_R + (g^L_j)_{\alpha\beta}P_L\Big)\psi_\beta. \end{aligned} \tag{4.1}$$

Here indices α and β distinguish different Majorana fermions, a — different Dirac fermions; n goes over different scalar fields (both complex and real), j — over vector fields (also complex and real); μ denotes spacetime components of vector fields; summation over all repeated indices is implied. The coefficients of the formal Lagrangian that I've mentioned previosly are $y^{R/L}_{\alpha an}$, $g^{R/L}_{\alpha aj}$, $(y_n)_{\alpha\beta}$ and $(g^{R/L}_j)_{\alpha\beta}$. They show the strengths of interactions between the fields of the Lagrangian.

The parts of the Lagrangian of our model that are relevant for this calculation are two of the Yukawa interaction terms:

$$\mathcal{L}_{\text{Yuk}} = -\overline{L}_{nL}Y^{(a)}_{Enm}\Phi_a e_{mR} - \overline{L}_{nL}Y^{(a)}_{Nn}\widetilde{\Phi}_a\nu_R + h.c., \tag{4.2}$$

and the gauge-kinetic term

$$\mathcal{L}_G = \mathrm{i}\overline{L}_{nL}\gamma^\mu D_\mu L_{nL}. \tag{4.3}$$

Let's investigate each of them in turn.

4.1 The Yukawa Term

First of all, let's discuss the meaning of each part of the Yukawa interaction term

$$\mathcal{L}_{\text{Yuk}} = -\overline{L}_{nL}Y^{(a)}_{Enm}\Phi_a e_{mR} - \overline{L}_{nL}Y^{(a)}_{Nn}\widetilde{\Phi}_a\nu_R + h.c. \tag{4.4}$$

L_{nL} are the left-handed lepton doublets

$$L_{nL} = \begin{pmatrix} \nu_{nL} \\ e_{nL} \end{pmatrix}, \qquad n = e, \mu, \tau; \tag{4.5}$$

e_{mR} are the right-handed parts of charged lepton fields; Φ_a is the ath Higgs doublet, and $\widetilde{\Phi}_a$ are the so-called *adjoint Higgs doublets*, defined as

$$\widetilde{\Phi}_a \equiv \mathrm{i}\sigma_2 \Phi^* \tag{4.6}$$

(σ_2 is the second Pauli matrix). In our two-Higgs-doublet model,

$$\widetilde{\Phi}_1 = \begin{pmatrix} \frac{1}{\sqrt{2}}\big(v + H - \mathrm{i}\varphi_Z\big) \\ -\varphi^- \end{pmatrix}, \qquad \widetilde{\Phi}_2 = \begin{pmatrix} \frac{1}{\sqrt{2}}\big(H_R - \mathrm{i}H_I\big) \\ -H^- \end{pmatrix}. \tag{4.7}$$

$Y_E^{(a)}$ and $Y_N^{(a)}$ are matrices of Yukawa coupling coefficients: $Y_E^{(1)}$ generates masses of charged lepton fields and $Y_N^{(1)}$ provides the Dirac masses of neutrino fields, so they are defined as

$$Y_{Enm}^{(1)} \equiv \frac{\sqrt{2}}{v} m_n \delta_{mn}, \qquad Y_{Nn}^{(1)} \equiv \frac{\sqrt{2}}{v} m_{Dn}^* \tag{4.8}$$

($Y_N^{(1)}$ is defined with the complex conjugate of m_{Dn} in order to later produce the same Dirac mass term, $-m_{Dn}\bar{\nu}_R \nu_{nL}$, as was defined in section 3.1). n, as before, goes over the three lepton generations: $n = e, \mu, \tau$, so that $Y_E^{(1)}$ is a diagonal 3×3 matrix, whereas $Y_N^{(1)}$ is a 3×1 column matrix.

$Y_E^{(2)}$ and $Y_N^{(2)}$ couple with the second Higgs doublet that has no vacuum expectation value, and therefore they do not participate in the Higgs mechanism of mass generation. We define them simply as

$$Y_{Enm}^{(2)} \equiv \sqrt{2} y_{Enm}^{(2)}, \qquad Y_{Nn}^{(2)} \equiv \sqrt{2} y_{Nn}^{(2)}, \tag{4.9}$$

where the $\sqrt{2}$ factor has been extracted for later convenience. The dimensions of these matrices are the same as of $Y_E^{(1)}$ and $Y_N^{(1)}$, but $Y_E^{(2)}$, differently from $Y_E^{(1)}$, is non-diagonal.

Expanding eq. (4.4), we get:

$$\begin{aligned}
\mathcal{L}_{\text{Yuk}} = &- \begin{pmatrix} \bar{\nu}_{nL} & \bar{e}_{nL} \end{pmatrix} \frac{\sqrt{2}}{v} m_n \delta_{mn} \begin{pmatrix} \varphi^+ \\ \frac{1}{\sqrt{2}}\big(v + H + \mathrm{i}\varphi_Z\big) \end{pmatrix} e_{mR} \\
&- \begin{pmatrix} \bar{\nu}_{nL} & \bar{e}_{nL} \end{pmatrix} \sqrt{2} y_{Enm}^{(2)} \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\big(H_R + \mathrm{i}H_I\big) \end{pmatrix} e_{mR} \\
&- \begin{pmatrix} \bar{\nu}_{nL} & \bar{e}_{nL} \end{pmatrix} \frac{\sqrt{2}}{v} m_{Dn}^* \begin{pmatrix} \frac{1}{\sqrt{2}}\big(v + H - \mathrm{i}\varphi_Z\big) \\ -\varphi^- \end{pmatrix} \nu_R
\end{aligned}$$

$$
\begin{aligned}
&- \begin{pmatrix} \overline{\nu}_{nL} & \overline{e}_{nL} \end{pmatrix} \sqrt{2} y^{(2)}_{Nn} \begin{pmatrix} \frac{1}{\sqrt{2}}\big(H_R - \mathrm{i}H_I\big) \\ -H^- \end{pmatrix} \nu_R + h.c. \\
= &- \frac{\sqrt{2}}{v} m_n \overline{\nu}_{nL} \varphi^+ e_{nR} - m_n \overline{e}_{nL} e_{nR} - \frac{m_n}{v} \overline{e}_{nL} H e_{nR} - \mathrm{i}\frac{m_n}{v} \overline{e}_{nL} \varphi_Z e_{nR} \\
&- \sqrt{2} y^{(2)}_{Enm} \overline{\nu}_{nL} H^+ e_{mR} - y^{(2)}_{Enm} \overline{e}_{nL} H_R e_{mR} - \mathrm{i} y^{(2)}_{Enm} \overline{e}_{nL} H_I e_{mR} \\
&- m^*_{Dn} \overline{\nu}_{nL} \nu_R - \frac{m^*_{Dn}}{v} \overline{\nu}_{nL} H \nu_R + \mathrm{i}\frac{m^*_{Dn}}{v} \overline{\nu}_{nL} \varphi_Z \nu_R + \frac{\sqrt{2}}{v} m^*_{Dn} \overline{e}_{nL} \varphi^- \nu_R \\
&- y^{(2)}_{Nn} \overline{\nu}_{nL} H_R \nu_R + \mathrm{i} y^{(2)}_{Nn} \overline{\nu}_{nL} H_I \nu_R + \sqrt{2} y^{(2)}_{Nn} \overline{e}_{nL} H^- \nu_R + h.c.
\end{aligned} \tag{4.10}
$$

Leaving only the terms that contain neutrino fields and making use of expressions connecting Higgs fields H and H_R with their mass eigenstates h_1 and h_2 (derived from eqs. (2.16) and (2.17)), namely

$$
\begin{aligned}
H &= h_1 \cos\alpha + h_2 \sin\alpha, \\
H_R &= -h_1 \sin\alpha + h_2 \cos\alpha,
\end{aligned} \tag{4.11}
$$

we get:

$$
\begin{aligned}
\mathcal{L}_{\text{Yuk}} = &- \frac{\sqrt{2}}{v} m_n \overline{\nu}_{nL} \varphi^+ e_{nR} - \sqrt{2} y^{(2)}_{Enm} \overline{\nu}_{nL} H^+ e_{mR} - m^*_{Dn} \overline{\nu}_{nL} \nu_R \\
&- \frac{m^*_{Dn}}{v} \overline{\nu}_{nL} \big(h_1 \cos\alpha + h_2 \sin\alpha\big) \nu_R + \mathrm{i}\frac{m^*_{Dn}}{v} \overline{\nu}_{nL} \varphi_Z \nu_R + \frac{\sqrt{2}}{v} m^*_{Dn} \overline{e}_{nL} \varphi^- \nu_R \\
&- y^{(2)}_{Nn} \overline{\nu}_{nL} \big(h_2 \cos\alpha - h_1 \sin\alpha\big) \nu_R + \mathrm{i} y^{(2)}_{Nn} \overline{\nu}_{nL} H_I \nu_R + \sqrt{2} y^{(2)}_{Nn} \overline{e}_{nL} H^- \nu_R + h.c. \\
= &- \frac{\sqrt{2}}{v} m_n \overline{\nu}_{nL} \varphi^+ e_{nR} - \sqrt{2} y^{(2)}_{Enm} \overline{\nu}_{nL} H^+ e_{mR} - m^*_{Dn} \overline{\nu}_{nL} \nu_R \\
&- \overline{\nu}_{nL} \left(\frac{m^*_{Dn}}{v} \cos\alpha - y^{(2)}_{Nn} \sin\alpha \right) h_1 \nu_R + \mathrm{i}\frac{m^*_{Dn}}{v} \overline{\nu}_{nL} \varphi_Z \nu_R + \frac{\sqrt{2}}{v} m^*_{Dn} \overline{e}_{nL} \varphi^- \nu_R \\
&- \overline{\nu}_{nL} \left(y^{(2)}_{Nn} \cos\alpha + \frac{m^*_{Dn}}{v} \sin\alpha \right) h_2 \nu_R + \mathrm{i} y^{(2)}_{Nn} \overline{\nu}_{nL} H_I \nu_R + \sqrt{2} y^{(2)}_{Nn} \overline{e}_{nL} H^- \nu_R + h.c.
\end{aligned} \tag{4.12}
$$

To replace left and right-handed field components with the full fields, we use eqs. (3.18)-(3.21) and $e_{nR/L} = P_{R/L} e_n$, $\overline{e}_{nR/L} = \overline{e}_n P_{L/R}$. Also remembering that $P^2_{R/L} = P_{R/L}$, we get the following expression for $\mathcal{L}_{\text{Yuk}}$:

$$
\begin{aligned}
\mathcal{L}_{\text{Yuk}} = &- \frac{\sqrt{2}}{v} m_n \overline{\nu}_k U_{kn} P_R \varphi^+ e_n - \sqrt{2} y^{(2)}_{Enm} \overline{\nu}_k U_{kn} P_R H^+ e_m - m^*_{Dn} \overline{\nu}_k U_{kn} P_R U_{l4} \nu_l \\
&- \overline{\nu}_k U_{kn} P_R \left(\frac{m^*_{Dn}}{v} \cos\alpha - y^{(2)}_{Nn} \sin\alpha \right) h_1 U_{l4} \nu_l + \mathrm{i}\frac{m^*_{Dn}}{v} \overline{\nu}_k U_{kn} P_R \varphi_Z U_{l4} \nu_l
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\sqrt{2}}{v}m^*_{Dn}\bar{e}_nP_R\varphi^-U_{l4}\nu_l-\bar{\nu}_kU_{kn}P_R\left(y^{(2)}_{Nn}\cos\alpha+\frac{m^*_{Dn}}{v}\sin\alpha\right)h_2U_{l4}\nu_l\\
&+\mathrm{i}y^{(2)}_{Nn}\bar{\nu}_kU_{kn}P_RH_IU_{l4}\nu_l+\sqrt{2}y^{(2)}_{Nn}\bar{e}_nP_RH^-U_{l4}\nu_l\\
&-\frac{\sqrt{2}}{v}m_n\bar{e}_n\varphi^-P_LU^*_{kn}\nu_k-\sqrt{2}y^{(2)*}_{Enm}\bar{e}_mH^-P_LU^*_{kn}\nu_k-m_{Dn}\bar{\nu}_lU^*_{l4}P_LU^*_{kn}\nu_k\\
&-\bar{\nu}_lU^*_{l4}h_1\left(\frac{m_{Dn}}{v}\cos\alpha-y^{(2)*}_{Nn}\sin\alpha\right)P_LU^*_{kn}\nu_k-\mathrm{i}\frac{m_{Dn}}{v}\bar{\nu}_lU^*_{l4}\varphi_ZP_LU^*_{kn}\nu_k\\
&+\frac{\sqrt{2}}{v}m_{Dn}\bar{\nu}_lU^*_{l4}\varphi^+P_Le_n-\bar{\nu}_lU^*_{l4}h_2\left(y^{(2)*}_{Nn}\cos\alpha+\frac{m_{Dn}}{v}\sin\alpha\right)P_LU^*_{kn}\nu_k\\
&-\mathrm{i}y^{(2)*}_{Nn}\bar{\nu}_lU^*_{l4}H_IP_LU^*_{kn}\nu_k+\sqrt{2}y^{(2)*}_{Nn}\bar{\nu}_lU^*_{l4}H^+P_Le_n. \qquad (4.13)
\end{aligned}
$$

Comparing these Lagrangian terms with the formal Lagrangian (4.1), we can find the expressions for several of the formal Lagrangian's coefficients:

1. $-\frac{\sqrt{2}}{v}m_n\bar{\nu}_kU_{kn}P_R\varphi^+e_n$ corresponds to $-S_1\overline{\psi}_\alpha y^R_{\alpha a1}P_R\chi_a$, where $S_1=\varphi^+$. Therefore,

$$y^R_{\alpha a1}=\frac{\sqrt{2}}{v}m_nU_{\alpha n}. \qquad (4.14)$$

2. $-\sqrt{2}y^{(2)}_{Enm}\bar{\nu}_kU_{kn}P_RH^+e_m$ corresponds to $-S_2\overline{\psi}_\alpha y^R_{\alpha a2}P_R\chi_a$, where $S_2=H^+$:

$$y^R_{\alpha a2}=\sqrt{2}y^{(2)}_{Ena}U_{\alpha n}. \qquad (4.15)$$

3. $\frac{\sqrt{2}}{v}m^*_{Dn}\bar{e}_nP_R\varphi^-U_{l4}\nu_l$ corresponds to $-S^*_1\overline{\chi}_ay^{L*}_{\alpha a1}P_R\psi_\alpha$:

$$y^L_{\alpha a1}=-\frac{\sqrt{2}}{v}m_{Da}U^*_{\alpha 4}. \qquad (4.16)$$

4. $\sqrt{2}y^{(2)}_{Nn}\bar{e}_nP_RH^-U_{l4}\nu_l$ corresponds to $-S^*_2\overline{\chi}_ay^{L*}_{\alpha a2}P_R\psi_\alpha$:

$$y^L_{\alpha a2}=-\sqrt{2}y^{(2)*}_{Na}U^*_{\alpha 4}. \qquad (4.17)$$

As for the terms that couple two neutrino fields, $-R_n\overline{\psi}_\alpha\Big((y_n)_{\alpha\beta}P_R+(y^\dagger_n)_{\alpha\beta}P_L\Big)\psi_\beta$ and $-V^j_\mu\overline{\psi}_\alpha\gamma^\mu\Big((g^R_j)_{\alpha\beta}P_R+(g^L_j)_{\alpha\beta}P_L\Big)\psi_\beta$, in [6] it was shown that coefficient matrices y_n and $g^{R/L}_j$ are symmetric. Since each term of the Lagrangian is a number, not a matrix, we can implement this symmetry condition in the according terms of our model's Lagrangian by replacing each term $\mathcal{L}$ by $\frac{1}{2}(\mathcal{L}+\mathcal{L}^T)$. Then, using the fact that for Majorana fermions the following relations are valid [12]:

$$\Psi^T=\overline{\Psi}C, \qquad \overline{\Psi}^T=-C^{-1}\Psi, \qquad (4.18)$$

and

$$(\overline{\Psi}_1\Psi_2)^T = -\Psi_2^T\overline{\Psi}_1^T, \tag{4.19}$$

we get:

1. When $\mathcal{L} = -\bar{\nu}_k U_{kn} P_R\Big(\frac{m^*_{Dn}}{v}\cos\alpha - y^{(2)}_{Nn}\sin\alpha\Big)h_1 U_{l4}\nu_l$,

$$\begin{aligned}\mathcal{L}^T &= -\bar{\nu}_l C U_{l4} h_1\left(\frac{m^*_{Dn}}{v}\cos\alpha - y^{(2)}_{Nn}\sin\alpha\right)C^{-1}P_R C U_{kn} C^{-1}\nu_k \\ &= -\bar{\nu}_l U_{l4} h_1\left(\frac{m^*_{Dn}}{v}\cos\alpha - y^{(2)}_{Nn}\sin\alpha\right)P_R U_{kn}\nu_k,\end{aligned} \tag{4.20}$$

and

$$\begin{aligned}\frac{1}{2}\big(\mathcal{L}+\mathcal{L}^T\big) &= \frac{1}{2}\Bigg[-\bar{\nu}_k U_{kn} P_R\left(\frac{m^*_{Dn}}{v}\cos\alpha - y^{(2)}_{Nn}\sin\alpha\right)h_1 U_{l4}\nu_l \\ &\quad -\bar{\nu}_l U_{l4} h_1\left(\frac{m^*_{Dn}}{v}\cos\alpha - y^{(2)}_{Nn}\sin\alpha\right)P_R U_{kn}\nu_k\Bigg] \\ &= -\frac{1}{2}\bar{\nu}_k\big(U_{kn}U_{l4}+U_{k4}U_{ln}\big)\left(\frac{m^*_{Dn}}{v}\cos\alpha - y^{(2)}_{Nn}\sin\alpha\right)P_R h_1\nu_l.\end{aligned} \tag{4.21}$$

This term corresponds to $-R_1\overline{\psi}_\alpha(y_1)_{\alpha\beta}P_R\psi_\beta$, where $R_1 = h_1$. Therefore,

$$(y_1)_{\alpha\beta} = \frac{1}{2}\big(U_{\alpha n}U_{\beta 4}+U_{\alpha 4}U_{\beta n}\big)\left(\frac{m^*_{Dn}}{v}\cos\alpha - y^{(2)}_{Nn}\sin\alpha\right). \tag{4.22}$$

2. When $\mathcal{L} = \mathrm{i}\frac{m^*_{Dn}}{v}\bar{\nu}_k U_{kn} P_R \varphi_Z U_{l4}\nu_l$,

$$\mathcal{L}^T = \mathrm{i}\frac{m^*_{Dn}}{v}\bar{\nu}_l C U_{l4}\varphi_Z C^{-1}P_R C U_{kn} C^{-1}\nu_k = \mathrm{i}\frac{m^*_{Dn}}{v}\bar{\nu}_l U_{l4}\varphi_Z P_R U_{kn}\nu_k, \tag{4.23}$$

$$\begin{aligned}\frac{1}{2}\big(\mathcal{L}+\mathcal{L}^T\big) &= \frac{\mathrm{i}}{2}\frac{m^*_{Dn}}{v}\Big(\bar{\nu}_k U_{kn} P_R\varphi_Z U_{l4}\nu_l + \bar{\nu}_l U_{l4}\varphi_Z P_R U_{kn}\nu_k\Big) \\ &= \frac{\mathrm{i}}{2}\frac{m^*_{Dn}}{v}\bar{\nu}_k\Big(U_{kn}U_{l4}+U_{k4}U_{ln}\Big)P_R\varphi_Z\nu_l.\end{aligned} \tag{4.24}$$

This term corresponds to $-R_2\overline{\psi}_\alpha(y_2)_{\alpha\beta}P_R\psi_\beta$, where $R_2 = \varphi_Z$. Therefore,

$$(y_2)_{\alpha\beta} = -\frac{\mathrm{i}}{2}\frac{m^*_{Dn}}{v}\Big(U_{\alpha n}U_{\beta 4}+U_{\alpha 4}U_{\beta n}\Big). \tag{4.25}$$

3. When $\mathcal{L} = -\bar{\nu}_k U_{kn} P_R \Big(y^{(2)}_{Nn} \cos\alpha + \frac{m^*_{Dn}}{v} \sin\alpha \Big) h_2 U_{l4} \nu_l$,

$$
\begin{aligned}
\mathcal{L}^T &= -\bar{\nu}_l C U_{l4} h_2 \left(y^{(2)}_{Nn} \cos\alpha + \frac{m^*_{Dn}}{v} \sin\alpha \right) C^{-1} P_R C U_{kn} C^{-1} \nu_k \\
&= -\bar{\nu}_l U_{l4} h_2 \left(y^{(2)}_{Nn} \cos\alpha + \frac{m^*_{Dn}}{v} \sin\alpha \right) P_R U_{kn} \nu_k ,
\end{aligned} \tag{4.26}
$$

$$
\begin{aligned}
\frac{1}{2}\Big(\mathcal{L} + \mathcal{L}^T\Big) &= \frac{1}{2}\Bigg[-\bar{\nu}_k U_{kn} P_R \left(y^{(2)}_{Nn} \cos\alpha + \frac{m^*_{Dn}}{v} \sin\alpha \right) h_2 U_{l4} \nu_l \\
&\quad - \bar{\nu}_l U_{l4} h_2 \left(y^{(2)}_{Nn} \cos\alpha + \frac{m^*_{Dn}}{v} \sin\alpha \right) P_R U_{kn} \nu_k \Bigg] \\
&= -\frac{1}{2}\bar{\nu}_k \Big(U_{kn} U_{l4} + U_{k4} U_{ln} \Big) \left(y^{(2)}_{Nn} \cos\alpha + \frac{m^*_{Dn}}{v} \sin\alpha \right) P_R h_2 \nu_l .
\end{aligned} \tag{4.27}
$$

This term corresponds to $-R_3 \overline{\psi}_\alpha (y_3)_{\alpha\beta} P_R \psi_\beta$, where $R_3 = h_2$. Therefore,

$$
(y_3)_{\alpha\beta} = \frac{1}{2}\Big(U_{\alpha n} U_{\beta 4} + U_{\alpha 4} U_{\beta n} \Big) \left(y^{(2)}_{Nn} \cos\alpha + \frac{m^*_{Dn}}{v} \sin\alpha \right). \tag{4.28}
$$

4. When $\mathcal{L} = \mathrm{i} y^{(2)}_{Nn} \bar{\nu}_k U_{kn} P_R H_I U_{l4} \nu_l$,

$$
\mathcal{L}^T = \mathrm{i} y^{(2)}_{Nn} \bar{\nu}_l C U_{l4} H_I C^{-1} P_R C U_{kn} C^{-1} \nu_k = \mathrm{i} y^{(2)}_{Nn} \bar{\nu}_l U_{l4} H_I P_R U_{kn} \nu_k , \tag{4.29}
$$

$$
\begin{aligned}
\frac{1}{2}\Big(\mathcal{L} + \mathcal{L}^T\Big) &= \frac{\mathrm{i}}{2} y^{(2)}_{Nn} \Big(\bar{\nu}_k U_{kn} P_R H_I U_{l4} \nu_l + \bar{\nu}_l U_{l4} H_I P_R U_{kn} \nu_k \Big) \\
&= \frac{\mathrm{i}}{2} y^{(2)}_{Nn} \bar{\nu}_k \Big(U_{kn} U_{l4} + U_{k4} U_{ln} \Big) P_R H_I \nu_l .
\end{aligned} \tag{4.30}
$$

This term corresponds to $-R_4 \overline{\psi}_\alpha (y_4)_{\alpha\beta} P_R \psi_\beta$, where $R_4 = H_I$. Therefore,

$$
(y_4)_{\alpha\beta} = -\frac{\mathrm{i}}{2} y^{(2)}_{Nn} \Big(U_{\alpha n} U_{\beta 4} + U_{\alpha 4} U_{\beta n} \Big). \tag{4.31}
$$

The Lagrangian terms in the last 4 lines of eq. (4.13) (which are the hermitian conjugate of the first 4 lines) would give no new information, so we do not take them into consideration.

4.2 The Gauge-Kinetic Term

In the term

$$\mathcal{L}_G = \mathrm{i}\overline{L}_{nL}\gamma^\mu D_\mu L_{nL}, \tag{4.32}$$

L_{nL}, as before, is the nth lepton doublet, and D_μ is the μth component of the covariant derivative, which has the following form for left-handed lepton doublets:

$$D_\mu = \partial_\mu + \mathrm{i}\eta\frac{g}{\sqrt{2}}\left(\sigma^+ W_\mu^+ + \sigma^- W_\mu^-\right) + \mathrm{i}\eta_e e Q A_\mu + \mathrm{i}\eta\frac{g}{\cos\theta_W}\left(\frac{\sigma_3}{2} - Q\sin^2\theta_W\right)\eta_Z Z_\mu, \tag{4.33}$$

where

$$\sigma^+ \equiv \frac{\sigma_1 \pm \mathrm{i}\sigma_2}{\sqrt{2}}. \tag{4.34}$$

σ_i are the Pauli matrices; Q is the electric charge; θ_W is the weak mixing angle; e is defined by $\eta_e e = \eta g \sin\theta_W$; g is the coupling constant of the $SU(2)_L$ group; and $\eta, \eta_e, \eta_Z = \pm 1$ are inserted to generalize all different conventions on signs that are prevalent in the field of elementary particle physics (more on them in [14]).

Expanding eq. (4.32), we have:

$$\begin{aligned}
\mathcal{L}_G &= \mathrm{i}\begin{pmatrix}\bar{\nu}_{nL} & \bar{e}_{nL}\end{pmatrix}\gamma^\mu\left[\begin{pmatrix}\partial_\mu & 0\\ 0 & \partial_\mu\end{pmatrix} + \mathrm{i}\eta\frac{g}{\sqrt{2}}\begin{pmatrix}0 & \sqrt{2}W_\mu^+\\ \sqrt{2}W_\mu^- & 0\end{pmatrix} + \begin{pmatrix}0 & 0\\ 0 & \mathrm{i}\eta_e e Q_e A_\mu\end{pmatrix}\right.\\
&\quad\left. + \mathrm{i}\eta\frac{g}{\cos\theta_W}\begin{pmatrix}\frac{1}{2} & 0\\ 0 & -\frac{1}{2} - Q_e\sin^2\theta_W\end{pmatrix}\eta_Z Z_\mu\right]\begin{pmatrix}\nu_{nL}\\ e_{nL}\end{pmatrix}\\
&= \mathrm{i}\begin{pmatrix}\bar{\nu}_{nL} & \bar{e}_{nL}\end{pmatrix}\gamma^\mu\begin{pmatrix}\partial_\mu + \mathrm{i}\eta\frac{g}{2\cos\theta_W}\eta_Z Z_\mu & \mathrm{i}\eta g W_\mu^+\\ \mathrm{i}\eta g W_\mu^- & \partial_\mu + \mathrm{i}\eta_e e Q_e A_\mu - \mathrm{i}\eta\frac{g}{\cos\theta_W}\left(\frac{1}{2} + Q_e\sin^2\theta_W\right)\eta_Z Z_\mu\end{pmatrix}\\
&\quad\times\begin{pmatrix}\nu_{nL}\\ e_{nL}\end{pmatrix}\\
&= \mathrm{i}\bar{\nu}_{nL}\gamma^\mu\left(\partial_\mu + \mathrm{i}\eta\frac{g}{2\cos\theta_W}\eta_Z Z_\mu\right)\nu_{nL} - \bar{\nu}_{nL}\gamma^\mu\eta g W_\mu^+ e_{nL} - \bar{e}_{nL}\gamma^\mu\eta g W_\mu^-\nu_{nL}\\
&\quad + \mathrm{i}\bar{e}_{nL}\gamma^\mu\left(\partial_\mu + \mathrm{i}\eta_e e Q_e A_\mu - \mathrm{i}\eta\frac{g}{\cos\theta_W}\left(\frac{1}{2} + Q_e\sin^2\theta_W\right)\eta_Z Z_\mu\right)e_{nL}\\
&= \mathrm{i}\bar{\nu}_k U_{kn}\gamma^\mu\left(\partial_\mu + \mathrm{i}\eta\frac{g}{2\cos\theta_W}\eta_Z Z_\mu\right)P_L U_{ln}^*\nu_l - \bar{\nu}_k U_{kn}\gamma^\mu P_L \eta g W_\mu^+ e_n - \bar{e}_n\gamma^\mu P_L\eta g W_\mu^- U_{ln}^*\nu_l\\
&\quad + \mathrm{i}\bar{e}_n\gamma^\mu\left(\partial_\mu + \mathrm{i}\eta_e e Q_e A_\mu - \mathrm{i}\eta\frac{g}{\cos\theta_W}\left(\frac{1}{2} + Q_e\sin^2\theta_W\right)\eta_Z Z_\mu\right)P_L e_n.
\end{aligned} \tag{4.35}$$

Neglecting all terms that do not concern us, we are left with

$$\mathcal{L}_G = -\bar{\nu}_k U_{kn}\gamma^\mu \eta \frac{g}{2\cos\theta_W}\eta_Z Z_\mu P_L U^*_{ln}\nu_l - \bar{\nu}_k U_{kn}\gamma^\mu P_L \eta g W^+_\mu e_n - \bar{e}_n \gamma^\mu P_L \eta g W^-_\mu U^*_{ln}\nu_l. \quad (4.36)$$

Let's consider each of them in turn.

1. The first term will be used to determine the symmetric coefficient matrix $g_j^{R/L}$, so we are going to symmetrize it in the same way as we did for several terms of the Yukawa Lagrangian.

   Denoting $\mathcal{L} = -\bar{\nu}_k U_{kn}\gamma^\mu \eta \frac{g}{2\cos\theta_W}\eta_Z Z_\mu P_L U^*_{ln}\nu_l$, we have

$$\mathcal{L}^T = \bar{\nu}_l C U^*_{ln} C^{-1} P_L C Z_\mu \eta_Z \frac{g}{2\cos\theta_W}\eta C^{-1}\gamma^\mu C U_{kn} C^{-1}\nu_k = \bar{\nu}_l U^*_{ln} Z_\mu \eta_Z \frac{g}{2\cos\theta_W}\eta\gamma^\mu P_R U_{kn}\nu_k, \quad (4.37)$$

$$\begin{aligned}\frac{1}{2}\Big(\mathcal{L}+\mathcal{L}^T\Big) &= \frac{1}{2}\Bigg(-\bar{\nu}_k U_{kn}\gamma^\mu \eta \frac{g}{2\cos\theta_W}\eta_Z Z_\mu P_L U^*_{ln}\nu_l + \bar{\nu}_l U^*_{ln} Z_\mu \eta_Z \frac{g}{2\cos\theta_W}\eta\gamma^\mu P_R U_{kn}\nu_k\Bigg)\\ &= -\frac{\eta\eta_Z g}{4\cos\theta_W} Z_\mu \bar{\nu}_k\gamma^\mu\Big(U_{kn}U^*_{ln}P_L - U^*_{kn}U_{ln}P_R\Big)\nu_l. \end{aligned} \quad (4.38)$$

   This term corresponds to $-V^1_\mu \overline{\psi}_\alpha \gamma^\mu \Big((g_1^R)_{\alpha\beta}P_R + (g_1^L)_{\alpha\beta}P_L\Big)\psi_\beta$, where $V^1_\mu = Z_\mu$. Therefore:

$$(g_1)^L_{\alpha\beta} = \frac{\eta\eta_Z g}{4\cos\theta_W}U_{\alpha n}U^*_{\beta n}, \qquad (g_1)^R_{\alpha\beta} = -\frac{\eta\eta_Z g}{4\cos\theta_W}U^*_{\alpha n}U_{\beta n}. \quad (4.39)$$

2. $-\bar{\nu}_k U_{kn}\gamma^\mu P_L \eta g W^+_\mu e_n$ corresponds to $-W^1_\mu \overline{\psi}_\alpha \gamma^\mu g^L_{\alpha a 1} P_L \chi_a$, where $W^1_\mu = W^+_\mu$. Therefore,

$$g^L_{\alpha a1} = \eta g U_{\alpha a}. \quad (4.40)$$

3. $-\bar{e}_n \gamma^\mu P_L \eta g W^-_\mu U^*_{ln}\nu_l$ corresponds to $-W^{j*}_\mu \overline{\chi}_a \gamma^\mu g^{L*}_{\alpha a j} P_L \psi_\alpha$ and gives the same result for $g^L_{\alpha a 1}$.

4.3 Summary of the Coefficients' Calculation Results

Gathering together all results, we have the following fields and coefficients in our model:

1. Two complex scalar fields, $S_1 = \varphi^+$ and $S_2 = H^+$:

$$y^L_{\alpha\beta j} = -Y^{(j)*}_{N\beta}U^*_{\alpha 4}, \qquad y^R_{\alpha\beta j} = Y^{(j)}_{En\beta}U_{\alpha n}, \quad (4.41)$$

$$\alpha = 1,\ldots,4 \qquad \beta, n = 1,2,3 \qquad j = 1,2. \quad (4.42)$$

2. One complex vector field $W^1_\mu = W^+_\mu$:

$$g^L_{\alpha\beta j} = \eta g U_{\alpha\beta}, \qquad g^R_{\alpha\beta j} = 0, \tag{4.43}$$

$$\alpha = 1, \ldots, 4 \qquad \beta = 1, 2, 3 \qquad j = 3. \tag{4.44}$$

3. Four real scalar fields, $R_1 = h_1$, $R_2 = \varphi_Z$, $R_3 = h_2$, $R_4 = H_I$:

$$(y_j)_{\alpha\beta} = \frac{1}{2\sqrt{2}}(U_{\alpha n}U_{\beta 4} + U_{\alpha 4}U_{\beta n})F^j_n, \tag{4.45}$$

$$\begin{aligned} F^4_n &= Y^{(1)}_{Nn}\cos\alpha - Y^{(2)}_{Nn}\sin\alpha, & F^5_n &= -\mathrm{i}Y^{(1)}_{Nn}, \\ F^6_n &= Y^{(1)}_{Nn}\sin\alpha + Y^{(2)}_{Nn}\cos\alpha, & F^7_n &= -\mathrm{i}Y^{(2)}_{Nn}, \end{aligned}$$

$$\alpha, \beta = 1, \ldots, 4 \qquad n = 1, 2, 3 \qquad j = 4, \ldots, 7. \tag{4.46}$$

4. One real vector field $V^1_\mu = Z_\mu$:

$$(g^L_j)_{\alpha\beta} = \frac{\eta\eta_Z g}{4\cos\theta_W}U_{\alpha n}U^*_{\beta n}, \qquad (g^R_j)_{\alpha\beta} = -\frac{\eta\eta_Z g}{4\cos\theta_W}U^*_{\alpha n}U_{\beta n}, \tag{4.47}$$

$$\alpha, \beta = 1, \ldots, 4 \qquad n = 1, 2, 3 \qquad j = 8. \tag{4.48}$$

5 Simplifying One-Loop Corrections

In this section the matrix $\mathcal{M}$ of one-loop corrections to the neutrino propagator (defined in [6]) is calculated. Each matrix element $\mathcal{M}_{\beta\alpha}$ consists of 4 terms:

$$\mathcal{M}_{\beta\alpha} = \mathcal{M}^{SD}_{\beta\alpha} + \mathcal{M}^{VD}_{\beta\alpha} + \mathcal{M}^{SM}_{\beta\alpha} + \mathcal{M}^{VM}_{\beta\alpha}, \tag{5.1}$$

where the SD and VD terms arise from the loops involving a Dirac fermion and, accordingly, a scalar or a vector boson, and the SM and VM terms — from the loops with a Majorana particle and, again, a scalar or a vector boson. In the following, each of the terms $\mathcal{M}^{VD}_{\beta\alpha}$ and $\mathcal{M}^{VM}_{\beta\alpha}$ is split into a longitudinal and a transverse part:

$$\mathcal{M}^{VD}_{\beta\alpha} = \mathcal{M}^{VT}_{\beta\alpha} + \mathcal{M}^{VL}_{\beta\alpha}, \tag{5.2}$$
$$\mathcal{M}^{VM}_{\beta\alpha} = \mathcal{M}^{TM}_{\beta\alpha} + \mathcal{M}^{LM}_{\beta\alpha}. \tag{5.3}$$

The expressions for each element of the $\mathcal{M}$ matrix in terms of our model have been found in [6]. Let's repeat them here:

$$\begin{aligned}\mathcal{M}^{SD}_{\beta\alpha}(p) = {} & \frac{1}{(4\pi)^2}\frac{1}{2p^0 2p'^0}\bar{u}^{(s_\beta)}_{(p)}\Big\{\not{p}\big(B_{0,ja} + B_{1,ja}\big)\Big[\big(y^{L*}_{\alpha aj}y^{L}_{\beta aj} + y^{R}_{\alpha aj}y^{R*}_{\beta aj}\big)P_R \\ & + \big(y^{R*}_{\alpha aj}y^{R}_{\beta aj} + y^{L}_{\alpha aj}y^{L*}_{\beta aj}\big)P_L\Big] + m_{1,a}B_{0,ja}\Big[\big(y^{L*}_{\alpha aj}y^{R}_{\beta aj} + y^{R}_{\alpha aj}y^{L*}_{\beta aj}\big)P_R \\ & + \big(y^{R*}_{\alpha aj}y^{L}_{\beta aj} + y^{L}_{\alpha aj}y^{R*}_{\beta aj}\big)P_L\Big]\Big\}u^{(s_\alpha)}_{(p')},\end{aligned} \tag{5.4}$$

$$\alpha,\beta = 1,\dots,4 \qquad a = 1,2,3 \qquad j = 1,2;$$

$$\begin{aligned}\mathcal{M}^{VT}_{\beta\alpha}(p) = {} & \frac{1}{(4\pi)^2}\frac{1}{2p^0 2p'^0}\bar{u}^{(s_\beta)}_{(p)}\Big\{(D-2)\not{p}(B_{0,ja} + B_{1,ja})\Big[(g^{R*}_{\alpha aj}g^{R}_{\beta aj} + g^{L}_{\alpha aj}g^{L*}_{\beta aj})P_R \\ & + (g^{R}_{\alpha aj}g^{R*}_{\beta aj} + g^{L*}_{\alpha aj}g^{L}_{\beta aj})P_L\Big] - DB_{0,ja}m_{1,a}\Big[(g^{R*}_{\alpha aj}g^{L}_{\beta aj} + g^{L}_{\alpha aj}g^{R*}_{\beta aj})P_R \\ & + (g^{R}_{\alpha aj}g^{L*}_{\beta aj} + g^{L*}_{\alpha aj}g^{R}_{\beta aj})P_L\Big]\Big\}u^{(s_\alpha)}_{(p')},\end{aligned} \tag{5.5}$$

$$\alpha,\beta = 1,\dots,4 \qquad a = 1,2,3 \qquad j = 3;$$

$$\begin{aligned}\mathcal{M}^{VL}_{\beta\alpha}(p) = {} & \frac{1}{(4\pi)^2}\frac{1}{2p^0 2p'^0}\bar{u}^{(s_\beta)}_{(p)}\Big\{\not{p}G_{ja}\Big[(g^{R*}_{\alpha aj}g^{R}_{\beta aj} + g^{L}_{\alpha aj}g^{L*}_{\beta aj})P_R + (g^{R}_{\alpha aj}g^{R*}_{\beta aj} + g^{L*}_{\alpha aj}g^{L}_{\beta aj})P_L\Big] \\ & + m_{1,a}(B_{0,ja} - \xi B^{\xi}_{0,ja})\Big[(g^{R*}_{\alpha aj}g^{L}_{\beta aj} + g^{L}_{\alpha aj}g^{R*}_{\beta aj})P_R + (g^{R}_{\alpha aj}g^{L*}_{\beta aj} + g^{L*}_{\alpha aj}g^{R}_{\beta aj})P_L\Big]\Big\} \\ & \times u^{(s_\alpha)}_{(p')},\end{aligned} \tag{5.6}$$

$$\alpha,\beta = 1,\dots,4 \qquad a = 1,2,3 \qquad j = 3;$$

$$
\begin{aligned}
\mathcal{M}^{SM}_{\beta\alpha}(p) = \frac{1}{(4\pi)^2}\frac{1}{p^0p'^0}\overline{u}^{(s_\beta)}_{(p)}\Bigg\{&\Big[(y_j)_{\beta\alpha}P_R + (y_j^\dagger)_{\beta\alpha}P_L\Big]\sum_\gamma \frac{2m_{1,\gamma}}{m^2_{0,j}}\,\mathrm{Re}[(y_j)_{\gamma\gamma}]A_0(m^2_{1,\gamma})\\
&+ \not{p}(B_{0,j\gamma}+B_{1,j\gamma})\Big[(y_j^\dagger)_{\beta\gamma}(y_j)_{\gamma\alpha}P_R + (y_j)_{\beta\gamma}(y_j^\dagger)_{\gamma\alpha}P_L\Big]\\
&+ m_{1,\gamma}B_{0,j\gamma}\Big[(y_j)_{\beta\gamma}(y_j)_{\gamma\alpha}P_R + (y_j^\dagger)_{\beta\gamma}(y_j^\dagger)_{\gamma\alpha}P_L\Big]\Bigg\}u^{(s_\alpha)}_{(p')},\\
&\alpha,\beta,\gamma = 1,\dots,4 \qquad j = 4,\dots,7;
\end{aligned}
\tag{5.7}
$$

$$
\begin{aligned}
\mathcal{M}^{TM}_{\beta\alpha}(p) = \frac{1}{(4\pi)^2}\frac{1}{p^0p'^0}\overline{u}^{(s_\beta)}_{(p)}\Bigg\{&(D-2)(B_{0,j\gamma}+B_{1,j\gamma})\not{p}\Big[(g_j)_{\beta\gamma}(g_j)_{\gamma\alpha}P_R - (g_j^*)_{\beta\gamma}(g_j^*)_{\gamma\alpha}P_L\Big]\\
&+ DB_{0,j\gamma}m_{1,\gamma}\Big[(g_j^*)_{\beta\gamma}(g_j)_{\gamma\alpha}P_R + (g_j)_{\beta\gamma}(g_j^*)_{\gamma\alpha}P_L\Big]\Bigg\}u^{(s_\alpha)}_{(p')},\\
&\alpha,\beta,\gamma = 1,\dots,4 \qquad j = 8;
\end{aligned}
\tag{5.8}
$$

$$
\begin{aligned}
\mathcal{M}^{LM}_{\beta\alpha}(p) = \frac{1}{(4\pi)^2}\frac{1}{p^0p'^0}\overline{u}^{(s_\beta)}_{(p)}\Bigg\{&\not{p}G_{j\gamma}\Big[(g_j)_{\beta\gamma}(g_j)_{\gamma\alpha}P_R - (g_j^*)_{\beta\gamma}(g_j^*)_{\gamma\alpha}P_L\Big]\\
&- m_{1,\gamma}(B_{0,j\gamma}-\xi B^\xi_{0,j\gamma})\Big[(g_j^*)_{\beta\gamma}(g_j)_{\gamma\alpha}P_R + (g_j)_{\beta\gamma}(g_j^*)_{\gamma\alpha}P_L\Big]\Bigg\}u^{(s_\alpha)}_{(p')},\\
&\alpha,\beta,\gamma = 1,\dots,4 \qquad j = 8.
\end{aligned}
\tag{5.9}
$$

$B_{0,ja}$ and $B_{1,ja}$ are the so-called *one-loop tensor integrals*:

$$
B_{0,ja} \equiv B_0(p^2, m^2_{0,j}, m^2_{1,a}) \equiv \frac{1}{\mathrm{i}\pi^2}\int \mathrm{d}^4k \frac{1}{(k^2-m^2_{0,a})\big[(k+p)^2 - m^2_{1,a}\big]}, \tag{5.10}
$$

$$
p^\mu B_{1,ja} \equiv p^\mu B_1(p^2, m^2_{0,j}, m^2_{1,a}) \equiv \frac{1}{\mathrm{i}\pi^2}\int \mathrm{d}^4k \frac{k^\mu}{(k^2-m^2_{0,a})\big[(k+p)^2 - m^2_{1,a}\big]}, \tag{5.11}
$$

whereas $B^\xi_{x,ja}$ and G_{ja} are defined as

$$
B^\xi_{x,ja} \equiv B_{x,ja}\Big(p^2, \xi m^2_{0,j}, m^2_{1,a}\Big), \qquad G_{ja} \equiv -\frac{p^2 - m^2_{1,a}}{m^2_{0,j}}\Big(B_{1,ja} - B^\xi_{1,ja}\Big) - \Big(B_{0,ja} - \xi B^\xi_{0,ja}\Big). \tag{5.12}
$$

More information about the properties of one-loop tensor integrals can be found in [15].

5.1 Inserting the Coefficients

Inserting the expressions of the coefficients from section 4, we get:

$$
\mathcal{M}^{SD}_{\beta\alpha}(p) = \frac{1}{(4\pi)^2}\frac{1}{2p^02p'^0}\overline{u}^{(s_\beta)}_{(p)}\Bigg\{\not{p}\big(B_{0,ja}+B_{1,ja}\big)\Big[\big(|Y^{(j)}_{Na}|^2U_{\alpha4}U^*_{\beta4} + Y^{(j)}_{Ena}U_{\alpha n}Y^{(j)*}_{Eka}U^*_{\beta k}\big)P_R
$$

$$+\big(Y^{(j)*}_{Ena}U^*_{\alpha n}Y^{(j)}_{Eka}U_{\beta k}+|Y^{(j)}_{Na}|^2U^*_{\alpha 4}U_{\beta 4}\big)P_L\Big]-m_{1,a}B_{0,ja}\Big[\big(U_{\alpha 4}U_{\beta n}$$

$$+U_{\alpha n}U_{\beta 4}\big)Y^{(j)}_{Na}Y^{(j)}_{Ena}P_R+\big(U^*_{\alpha 4}U^*_{\beta n}+U^*_{\alpha n}U^*_{\beta 4}\big)Y^{(j)*}_{Na}Y^{(j)*}_{Ena}P_L\Big]\Bigg\}u^{(s_\alpha)}_{(p')}, \tag{5.13}$$

$$\mathcal{M}^{VT}_{\beta\alpha}(p)=\frac{1}{(4\pi)^2}\frac{g^2}{2p^0 2p'^0}\bar{u}^{(s_\beta)}_{(p)}\not{p}\big(D-2\big)\big(B_{0,ja}+B_{1,ja}\big)\Big[U_{\alpha a}U^*_{\beta a}P_R+U^*_{\alpha a}U_{\beta a}P_L\Big]u^{(s_\alpha)}_{(p')}, \tag{5.14}$$

$$\mathcal{M}^{VL}_{\beta\alpha}(p)=\frac{1}{(4\pi)^2}\frac{g^2}{2p^0 2p'^0}\bar{u}^{(s_\beta)}_{(p)}\not{p}G_{ja}\Big[U_{\alpha a}U^*_{\beta a}P_R+U^*_{\alpha a}U_{\beta a}P_L\Big]u^{(s_\alpha)}_{(p')}, \tag{5.15}$$

$$\mathcal{M}^{SM}_{\beta\alpha}(p)=\frac{1}{(4\pi)^2}\frac{1}{p^0p'^0}\bar{u}^{(s_\beta)}_{(p)}\Bigg\{\frac{1}{2}\Big[\big(U_{\beta n}U_{\alpha 4}+U_{\beta 4}U_{\alpha n}\big)F^{(j)}_nP_R+\big(U^*_{\alpha n}U^*_{\beta 4}+U^*_{\alpha 4}U^*_{\beta n}\big)F^{(j)*}_nP_L\Big]$$

$$\times\sum_\gamma\frac{m_{1,\gamma}}{m^2_{0,j}}\operatorname{Re}[U_{\gamma k}U_{\gamma 4}F^{(j)}_k]A_0(m^2_{1,\gamma})$$

$$+\frac{1}{8}\not{p}(B_{0,j\gamma}+B_{1,j\gamma})\Big[\big(U^*_{\gamma n}U^*_{\beta 4}+U^*_{\gamma 4}U^*_{\beta n}\big)F^{(j)*}_n\big(U_{\gamma k}U_{\alpha 4}+U_{\gamma 4}U_{\alpha k}\big)F^{(j)}_kP_R$$

$$+\big(U_{\beta n}U_{\gamma 4}+U_{\beta 4}U_{\gamma n}\big)F^{(j)}_n\big(U^*_{\alpha k}U^*_{\gamma 4}+U^*_{\alpha 4}U^*_{\gamma k}\big)F^{(j)*}_kP_L\Big]$$

$$+\frac{1}{8}m_{1,\gamma}B_{0,j\gamma}\Big[\big(U_{\beta n}U_{\gamma 4}+U_{\beta 4}U_{\gamma n}\big)F^{(j)}_n\big(U_{\gamma k}U_{\alpha 4}+U_{\gamma 4}U_{\alpha k}\big)F^{(j)}_kP_R$$

$$+\big(U^*_{\gamma n}U^*_{\beta 4}+U^*_{\gamma 4}U^*_{\beta n}\big)F^{(j)*}_n\big(U^*_{\alpha k}U^*_{\gamma 4}+U^*_{\alpha 4}U^*_{\gamma k}\big)F^{(j)*}_kP_L\Big]\Bigg\}u^{(s_\alpha)}_{(p')}, \tag{5.16}$$

$$\mathcal{M}^{TM}_{\beta\alpha}(p)=\frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\bar{u}^{(s_\beta)}_{(p)}\Bigg\{(D-2)(B_{0,j\gamma}+B_{1,j\gamma})\not{p}\Big[U^*_{\beta n}U_{\gamma n}U^*_{\gamma k}U_{\alpha k}P_R$$

$$-U_{\beta n}U^*_{\gamma n}U_{\gamma k}U^*_{\alpha k}P_L\Big]+DB_{0,j\gamma}m_{1,\gamma}\Big[U_{\beta n}U^*_{\gamma n}U^*_{\gamma k}U_{\alpha k}P_R+U^*_{\beta n}U_{\gamma n}U_{\gamma k}U^*_{\alpha k}P_L\Big]\Bigg\}u^{(s_\alpha)}_{(p')}, \tag{5.17}$$

$$\mathcal{M}^{LM}_{\beta\alpha}(p)=\frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\bar{u}^{(s_\beta)}_{(p)}\Bigg\{\not{p}G_{j\gamma}\Big[U^*_{\beta n}U_{\gamma n}U^*_{\gamma k}U_{\alpha k}P_R-U_{\beta n}U^*_{\gamma n}U_{\gamma k}U^*_{\alpha k}P_L\Big]$$

$$-m_{1,\gamma}(B_{0,j\gamma}-\xi B^\xi_{0,j\gamma})\Big[U_{\beta n}U^*_{\gamma n}U^*_{\gamma k}U_{\alpha k}P_R+U^*_{\beta n}U_{\gamma n}U_{\gamma k}U^*_{\alpha k}P_L\Big]\Bigg\}u^{(s_\alpha)}_{(p')}. \tag{5.18}$$

$$\tag{5.19}$$

5.2 Relations Used in Further Calculations

Relations that are going to be useful in the following calculations:

1. The orthogonality of U and V matrices:

$$U_{nk}U^*_{mk}=U_{kn}U^*_{km}=\delta_{nm},\qquad k,n,m=1,\dots,4; \tag{5.20}$$

$$V_{nk}V^*_{mk}=V_{kn}V^*_{km}=\delta_{nm},\qquad k,n,m=1,\dots,3; \tag{5.21}$$

2. Rewriting eq. (3.51) in a slightly different way, we can express m_{Dn} in terms of V_{n3}:

$$m_{Dn} = -\mathrm{i} m_D V^*_{n3}, \qquad n = e, \mu, \tau; \tag{5.22}$$

3. From the last equation it is clear that m_{Dn} is orthogonal to the first two columns of V matrix:

$$V_{nm} m_{Dn} = -\mathrm{i} m_D \delta_{3m}, \qquad n = e, \mu, \tau; \tag{5.23}$$

4. We can easily formulate two useful relations between $Y_N^{(2)}$ and the elements of the PMNS matrix. First, we know that one of the neutrino mass states should stay massless even with one-loop corrections. This means that the Yukawa coupling of the second Higgs doublet to this state should vanish. If we pick the massless state to be the first neutrino ν_1, we get the following condition:

$$U_{1e} Y^{(2)}_{Ne} + U_{1\mu} Y^{(2)}_{N\mu} + U_{1\tau} Y^{(2)}_{N\tau} = 0, \tag{5.24}$$

or

$$V^*_{n1} Y^{(2)}_{Nn} = 0, \qquad n = e, \mu, \tau. \tag{5.25}$$

So, $Y_N^{(2)}$ is orthogonal to V_{n1}, and, since the V matrix is unitary, can be expressed as a linear combination of the two other orthogonal vectors, V_{n2} and V_{n3}:

$$Y^{(2)}_{Nn} \equiv \sqrt{2}\big(d V_{n2} + d' V_{n3}\big), \tag{5.26}$$

where we can choose the phase of $Y^{(2)}_{Nn}$ in such a way as to cancel the imaginary part of d: $0 < d \in \mathbb{R}$ and $d' \in \mathbb{C}$.

5. Using (5.23) and (5.25), we can derive:

$$V^*_{n1} F^4_n = V^*_{n1}\big(Y^{(1)}_{Nn} \cos\alpha - Y^{(2)}_{Nn} \sin\alpha\big) = V^*_{n1} \frac{\sqrt{2}}{v} m^*_{Dn} \cos\alpha = 0, \tag{5.27}$$

$$V^*_{n1} F^5_n = -\mathrm{i} V^*_{n1} Y^{(1)}_{Nn} = -\mathrm{i} V^*_{n1} \frac{\sqrt{2}}{v} m^*_{Dn} = 0, \tag{5.28}$$

$$V^*_{n1} F^6_n = V^*_{n1}\big(Y^{(1)}_{Nn} \sin\alpha + Y^{(2)}_{Nn} \cos\alpha\big) = V^*_{n1} \frac{\sqrt{2}}{v} m^*_{Dn} \sin\alpha = 0, \tag{5.29}$$

$$V^*_{n1} F^7_n = -\mathrm{i} V^*_{n1} Y^{(2)}_{Nn} = 0, \tag{5.30}$$

therefore,

$$V^*_{n1} F^j_n = U_{1n} F^j_n = 0, \qquad \text{for } j = 4, \ldots, 7; \quad n = e, \mu, \tau; \tag{5.31}$$

6. From (5.26), the definition of $Y_N^{(2)}$ (eq. (4.9)), and unitarity of V it follows that

$$V^*_{nm} y^{(2)}_{Nn} = d\delta_{2m} + d'\delta_{3m}. \tag{5.32}$$

7. Analogously to (5.27)-(5.30), the following relations hold (again, for $n = e, \mu, \tau$):

$$\begin{aligned}
V^*_{n2}F^4_n &= -\sqrt{2}d\sin\alpha, & V^*_{n3}F^4_n &= \sqrt{2}\Big(\mathrm{i}\frac{m_D}{v}\cos\alpha - d'\sin\alpha\Big), && (5.33)\\
V^*_{n2}F^5_n &= 0, & V^*_{n3}F^5_n &= \frac{\sqrt{2}}{v}m_D, && (5.34)\\
V^*_{n2}F^6_n &= \sqrt{2}d\cos\alpha, & V^*_{n3}F^6_n &= \sqrt{2}\Big(\mathrm{i}\frac{m_D}{v}\sin\alpha + d'\cos\alpha\Big), && (5.35)\\
V^*_{n2}F^7_n &= -\sqrt{2}\mathrm{i}d, & V^*_{n3}F^7_n &= -\sqrt{2}\mathrm{i}d'. && (5.36)\\
&&&&& (5.37)
\end{aligned}$$

5.3 Simplification of the Relevant Matrix Elements $\mathcal{M}_{\beta\alpha}$

We are going to be interested only in a part of the $\mathcal{M}$ matrix. First of all, we will neglect the 4th row and column, since they involve the heavy, undetected neutrino — we are investigating only corrections to the light neutrinos, and they are contained in the upper-left 3×3 block of $\mathcal{M}$. What is more, $\mathcal{M}$ is symmetric, so we can skip the calculations of $\mathcal{M}_{21}$, $\mathcal{M}_{31}$, and $\mathcal{M}_{32}$.

One last thing worth mentioning before focusing on the calculations is the fact that the masses of the first and the second neutrinos without loop corrections (*tree-level masses*), $m_{\nu 1}$ and $m_{\nu 2}$, are equal to 0 — this fact will be very useful in the rest of this section.

Now let's calculate each of the remaining matrix elements in turn.

5.3.1 $\beta = 1, \alpha = 1$

When $\beta = 1, \alpha = 1$, a big part of $\mathcal{M}^{SD}_{11}$ vanishes due to U_{14} elements, and we are left with

$$\mathcal{M}^{SD}_{11}(p) = \frac{1}{(4\pi)^2}\frac{1}{2p^0 2p'^0}\bar{u}^{(s_1)}_{(p)}\not{p}\big(B_{0,ja}+B_{1,ja}\big)\Big[Y^{(j)}_{Ena}U_{1n}Y^{(j)*}_{Eka}U^*_{1k}P_R + Y^{(j)*}_{Ena}U^*_{1n}Y^{(j)}_{Eka}U_{1k}P_L\Big]u^{(s_1)}_{(p')}, \tag{5.38}$$

but since spinors $\bar{u}^{(s)}_{(p)}$ obey the Dirac equation

$$\bar{u}^{(s_\alpha)}_{(p)}\not{p} = m_\alpha \bar{u}^{(s_\alpha)}_{(p)}, \tag{5.39}$$

and m_α in this case is the mass of the first neutrino $m_{\nu 1}$ that is equal to zero, the rest of $\mathcal{M}^{SD}_{11}$ vanishes as well:

$$\mathcal{M}^{SD}_{11}(p) = 0. \tag{5.40}$$

$\mathcal{M}_{11}^{VT}$ and $\mathcal{M}_{11}^{VL}$ are proportional to $\bar{u}_{(p)}^{(s_1)} \not{p}$, thus vanish for the same reason:

$$\mathcal{M}_{11}^{VT}(p) = 0, \qquad \mathcal{M}_{11}^{VL}(p) = 0. \tag{5.41}$$

$$\begin{aligned}\mathcal{M}_{11}^{SM}(p) &= \frac{1}{(4\pi)^2}\frac{1}{p^0 p'^0}\bar{u}_{(p)}^{(s_1)}\left\{\frac{1}{8}\not{p}(B_{0,j\gamma}+B_{1,j\gamma})\Big[U_{\gamma 4}^* U_{1n}^* F_n^{(j)*} U_{\gamma 4} U_{1k} F_k^{(j)} P_R\right.\\ &\quad + U_{1n}U_{\gamma 4}F_n^{(j)} U_{1k}^* U_{\gamma 4}^* F_k^{(j)*} P_L\Big] + \frac{1}{8} m_{1,\gamma} B_{0,j\gamma}\Big[U_{1n}U_{\gamma 4}F_n^{(j)} U_{\gamma 4} U_{1k} F_k^{(j)} P_R\\ &\quad \left. + U_{\gamma 4}^* U_{1n}^* F_n^{(j)*} U_{1k}^* U_{\gamma 4}^* F_k^{(j)*} P_L\Big]\right\} u_{(p')}^{(s_1)}\\ &= 0,\end{aligned} \tag{5.42}$$

due to eq. (5.31);

$$\begin{aligned}\mathcal{M}_{11}^{TM}(p) &= \frac{1}{(4\pi)^2}\frac{g^2}{16p^0 p'^0 \cos^2\theta_W}\bar{u}_{(p)}^{(s_1)}\left\{(D-2)(B_{0,j\gamma}+B_{1,j\gamma})\not{p}\Big[U_{1n}^* U_{\gamma n} U_{\gamma k}^* U_{1k} P_R\right.\\ &\quad \left. - U_{1n}U_{\gamma n}^* U_{\gamma k} U_{1k}^* P_L\Big] + D B_{0,j\gamma} m_{1,\gamma}\Big[U_{1n}U_{\gamma n}^* U_{\gamma k}^* U_{1k} P_R + U_{1n}^* U_{\gamma n} U_{\gamma k} U_{1k}^* P_L\Big]\right\} u_{(p')}^{(s_1)}\\ &= 0,\end{aligned} \tag{5.43}$$

since the $\not{p}$ part vanishes like before, as do the terms multiplied by $DB_{0,j1}m_{\nu 1}$ and $DB_{0,j2}m_{\nu 2}$, and what is left are the terms containing $U_{1n}U_{3n}^*$, $U_{1n}U_{4n}^*$, $U_{1n}^*U_{3n}$, and $U_{1n}^*U_{4n}$, which vanish due to the unitarity of U;

$$\begin{aligned}\mathcal{M}_{11}^{LM}(p) &= \frac{1}{(4\pi)^2}\frac{g^2}{16p^0 p'^0 \cos^2\theta_W}\bar{u}_{(p)}^{(s_1)}\left\{\not{p}G_{j\gamma}\Big[U_{1n}^* U_{\gamma n} U_{\gamma k}^* U_{1k} P_R - U_{1n}U_{\gamma n}^* U_{\gamma k} U_{1k}^* P_L\Big]\right.\\ &\quad \left. - m_{1,\gamma}(B_{0,j\gamma} - \xi B_{0,j\gamma}^{\xi})\Big[U_{1n}U_{\gamma n}^* U_{\gamma k}^* U_{1k} P_R + U_{1n}^* U_{\gamma n} U_{\gamma k} U_{1k}^* P_L\Big]\right\} u_{(p')}^{(s_1)}\\ &= 0,\end{aligned} \tag{5.44}$$

because of the same reasons as in the case of $\mathcal{M}_{11}^{TM}$.

5.3.2 $\beta = 1, \alpha = 2$

$$\begin{aligned}
\mathcal{M}_{12}^{SD}(p) &= \frac{1}{(4\pi)^2}\frac{1}{2p^0 2p'^0}\overline{u}_{(p)}^{(s_1)}\not{p}\big(B_{0,ja}+B_{1,ja}\big)\Big[Y_{Ena}^{(j)}U_{2n}Y_{Eka}^{(j)*}U_{1k}^{*}P_R + Y_{Ena}^{(j)*}U_{2n}^{*}Y_{Eka}^{(j)}U_{1k}P_L\Big]u_{(p')}^{(s_2)} \\
&= \frac{1}{(4\pi)^2}\frac{1}{2p^0 2p'^0}\overline{u}_{(p)}^{(s_1)}\Bigg\{\not{p}\big(B_{0,\varphi+a}+B_{1,\varphi+a}\big)\Big[Y_{Ena}^{(1)}V_{n2}^{*}Y_{Eka}^{(1)*}V_{k1}P_R + Y_{Ena}^{(1)*}V_{n2}Y_{Eka}^{(1)}V_{k1}^{*}P_L\Big] \\
&\quad + \not{p}\big(B_{0,H+a}+B_{1,H+a}\big)\Big[Y_{Ena}^{(2)}V_{n2}^{*}Y_{Eka}^{(2)*}V_{k1}P_R + Y_{Ena}^{(2)*}V_{n2}Y_{Eka}^{(2)}V_{k1}^{*}P_L\Big]\Bigg\}u_{(p')}^{(s_2)} \\
&= \frac{1}{(4\pi)^2}\frac{1}{2v^2 p^0 p'^0}\overline{u}_{(p)}^{(s_1)}\not{p}\Bigg\{m_a^2\big(B_{0,\varphi+a}+B_{1,\varphi+a}\big)\Big[V_{a2}^{*}V_{a1}P_R + V_{a2}V_{a1}^{*}P_L\Big] \\
&\quad + v^2\big(B_{0,H+a}+B_{1,H+a}\big)\Big[y_{Ena}^{(2)}V_{n2}^{*}y_{Eka}^{(2)*}V_{k1}P_R + y_{Ena}^{(2)*}V_{n2}y_{Eka}^{(2)}V_{k1}^{*}P_L\Big]\Bigg\}u_{(p')}^{(s_2)},
\end{aligned} \tag{5.45}$$

$$\begin{aligned}
\mathcal{M}_{12}^{VT}(p) &= \frac{1}{(4\pi)^2}\frac{g^2}{2p^0 2p'^0}\overline{u}_{(p)}^{(s_1)}\not{p}\big(D-2\big)\big(B_{0,Wa}+B_{1,Wa}\big)\Big[U_{2a}U_{1a}^{*}P_R + U_{2a}^{*}U_{1a}P_L\Big]u_{(p')}^{(s_2)} \\
&= \frac{1}{(4\pi)^2}\frac{g^2}{2p^0 2p'^0}\overline{u}_{(p)}^{(s_1)}\not{p}\big(D-2\big)\big(B_{0,Wa}+B_{1,Wa}\big)\Big[V_{a2}^{*}V_{a1}P_R + V_{a2}V_{a1}^{*}P_L\Big]u_{(p')}^{(s_2)},
\end{aligned} \tag{5.46}$$

$$\begin{aligned}
\mathcal{M}_{12}^{VL}(p) &= \frac{1}{(4\pi)^2}\frac{g^2}{2p^0 2p'^0}\overline{u}_{(p)}^{(s_1)}\not{p}G_{Wa}\Big[U_{2a}U_{1a}^{*}P_R + U_{2a}^{*}U_{1a}P_L\Big]u_{(p')}^{(s_2)} \\
&= \frac{1}{(4\pi)^2}\frac{g^2}{2p^0 2p'^0}\overline{u}_{(p)}^{(s_1)}\not{p}G_{Wa}\Big[V_{a2}^{*}V_{a1}P_R + V_{a2}V_{a1}^{*}P_L\Big]u_{(p')}^{(s_2)},
\end{aligned} \tag{5.47}$$

$$\begin{aligned}
\mathcal{M}_{12}^{SM}(p) &= \frac{1}{(4\pi)^2}\frac{1}{p^0 p'^0}\overline{u}_{(p)}^{(s_1)}\Bigg\{\frac{1}{8}\not{p}(B_{0,j\gamma}+B_{1,j\gamma})\Big[U_{\gamma 4}^{*}U_{1n}^{*}F_n^{(j)*}U_{\gamma 4}U_{2k}F_k^{(j)}P_R \\
&\quad + U_{1n}U_{\gamma 4}F_n^{(j)}U_{2k}^{*}U_{\gamma 4}^{*}F_k^{(j)*}P_L\Big] + \frac{1}{8}m_{1,\gamma}B_{0,j\gamma}\Big[U_{1n}U_{\gamma 4}F_n^{(j)}U_{\gamma 4}U_{2k}F_k^{(j)}P_R \\
&\quad + U_{\gamma 4}^{*}U_{1n}^{*}F_n^{(j)*}U_{2k}^{*}U_{\gamma 4}^{*}F_k^{(j)*}P_L\Big]\Bigg\}u_{(p')}^{(s_2)} \\
&= 0,
\end{aligned} \tag{5.48}$$

due to $U_{1n}F_n^j$ terms;

$$\mathcal{M}_{12}^{TM}(p) = \frac{1}{(4\pi)^2}\frac{g^2}{16p^0 p'^0\cos^2\theta_W}\overline{u}_{(p)}^{(s_1)}\Bigg\{(D-2)(B_{0,j\gamma}+B_{1,j\gamma})\not{p}\Big[U_{1n}^{*}U_{\gamma n}U_{\gamma k}^{*}U_{2k}P_R$$

$$
\begin{aligned}
&- U_{1n}U^*_{\gamma n}U_{\gamma k}U^*_{2k}P_L\Big] + DB_{0,j\gamma}m_{1,\gamma}\Big[U_{1n}U^*_{\gamma n}U^*_{\gamma k}U_{2k}P_R + U^*_{1n}U_{\gamma n}U_{\gamma k}U^*_{2k}P_L\Big]\Bigg\}u^{(s_2)}_{(p')} \\
= &\frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\overline{u}^{(s_1)}_{(p)}\Bigg\{(D-2)(B_{0,j\nu 1}+B_{1,j\nu 1})\not{p}\Big[U^*_{1k}U_{2k}P_R - U_{1k}U^*_{2k}P_L\Big] \\
&+ (D-2)(B_{0,j\nu 2}+B_{1,j\nu 2})\not{p}\Big[U^*_{1n}U_{2n}P_R - U_{1n}U^*_{2n}P_L\Big] \\
&+ (D-2)(B_{0,j\nu 3}+B_{1,j\nu 3})\not{p}\Big[U^*_{1n}U_{3n}U^*_{3k}U_{2k}P_R - U_{1n}U^*_{3n}U_{3k}U^*_{2k}P_L\Big] \\
&+ (D-2)(B_{0,j\nu 4}+B_{1,j\nu 4})\not{p}\Big[U^*_{1n}U_{4n}U^*_{4k}U_{2k}P_R - U_{1n}U^*_{4n}U_{4k}U^*_{2k}P_L\Big] \\
&+ DB_{0,j\nu 3}m_{\nu 3}\Big[U_{1n}U^*_{3n}U^*_{3k}U_{2k}P_R + U^*_{1n}U_{3n}U_{3k}U^*_{2k}P_L\Big] \\
&+ DB_{0,j\nu 4}m_{\nu 4}\Big[U_{1n}U^*_{4n}U^*_{4k}U_{2k}P_R + U^*_{1n}U_{4n}U_{4k}U^*_{2k}P_L\Big]\Bigg\}u^{(s_2)}_{(p')} \\
= &\,0, \qquad (5.49)
\end{aligned}
$$

since in each term there is a factor of $U^*_{1n}U_{mn}$ or $U_{1n}U^*_{mn}$, with $m \neq 1$, that is equal to 0;

$$
\begin{aligned}
\mathcal{M}^{LM}_{12}(p) = &\frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\overline{u}^{(s_1)}_{(p)}\Bigg\{\not{p}G_{j\gamma}\Big[U^*_{1n}U_{\gamma n}U^*_{\gamma k}U_{2k}P_R - U_{1n}U^*_{\gamma n}U_{\gamma k}U^*_{2k}P_L\Big] \\
&- m_{1,\gamma}(B_{0,j\gamma} - \xi B^{\xi}_{0,j\gamma})\Big[U_{1n}U^*_{\gamma n}U^*_{\gamma k}U_{2k}P_R + U^*_{1n}U_{\gamma n}U_{\gamma k}U^*_{2k}P_L\Big]\Bigg\}u^{(s_2)}_{(p')} \\
= &\,0, \qquad (5.50)
\end{aligned}
$$

in the same way as $\mathcal{M}^{TM}_{12}$.

5.3.3 $\beta = 1, \alpha = 3$

$$
\begin{aligned}
\mathcal{M}^{SD}_{13}(p) = &\frac{1}{(4\pi)^2}\frac{1}{2p^0 2p'^0}\overline{u}^{(s_1)}_{(p)}\Bigg\{\not{p}\big(B_{0,ja}+B_{1,ja}\big)\Big[Y^{(j)}_{Ena}U_{3n}Y^{(j)*}_{Eka}U^*_{1k}P_R + Y^{(j)*}_{Ena}U^*_{3n}Y^{(j)}_{Eka}U_{1k}P_L\Big] \\
&- \mathrm{i}sm_{1,a}B_{0,ja}\Big[-U_{1n}Y^{(j)}_{Na}Y^{(j)}_{Ena}P_R + U^*_{1n}Y^{(j)*}_{Na}Y^{(j)*}_{Ena}P_L\Big]\Bigg\}u^{(s_3)}_{(p')} \\
= &\frac{1}{(4\pi)^2}\frac{1}{2p^0 2p'^0}\overline{u}^{(s_1)}_{(p)}\Bigg\{\not{p}c\big(B_{0,\varphi+a}+B_{1,\varphi+a}\big)\Big[Y^{(1)}_{Ena}V^*_{n3}Y^{(1)*}_{Eka}V_{k1}P_R + Y^{(1)*}_{Ena}V_{n3}Y^{(1)}_{Eka}V^*_{k1}P_L\Big] \\
&- \mathrm{i}sm_{1,a}B_{0,\varphi+a}\Big[-V^*_{n1}Y^{(1)}_{Na}Y^{(1)}_{Ena}P_R + V_{n1}Y^{(1)*}_{Na}Y^{(1)*}_{Ena}P_L\Big] \\
&+ \not{p}c\big(B_{0,H+a}+B_{1,H+a}\big)\Big[Y^{(2)}_{Ena}V^*_{n3}Y^{(2)*}_{Eka}V_{k1}P_R + Y^{(2)*}_{Ena}V_{n3}Y^{(2)}_{Eka}V^*_{k1}P_L\Big]
\end{aligned}
$$

$$
\begin{aligned}
&- \mathrm{i}sm_{1,a}B_{0,H+a}\Big[- V_{n1}^* Y_{Na}^{(2)} Y_{Ena}^{(2)} P_R + V_{n1} Y_{Na}^{(2)*} Y_{Ena}^{(2)*} P_L\Big]\Bigg\} u_{(p')}^{(s_3)} \\
&= \frac{1}{(4\pi)^2}\frac{1}{2v^2 p^0 p'^0}\bar{u}_{(p)}^{(s_1)}\Bigg\{ \not{p} c m_a^2 \big(B_{0,\varphi+a} + B_{1,\varphi+a}\big)\Big[V_{a3}^* V_{a1} P_R + V_{a3} V_{a1}^* P_L\Big] \\
&- \mathrm{i}sm_a^2 B_{0,\varphi+a}\Big[- V_{a1}^* m_{Da}^* P_R + V_{a1} m_{Da} P_L\Big] \\
&+ \not{p} c v^2 \big(B_{0,H+a} + B_{1,H+a}\big)\Big[y_{Ena}^{(2)} V_{n3}^* y_{Eka}^{(2)*} V_{k1} P_R + y_{Ena}^{(2)*} V_{n3} y_{Eka}^{(2)} V_{k1}^* P_L\Big] \\
&- \mathrm{i}sm_a v^2 B_{0,H+a}\Big[- V_{n1}^* y_{Na}^{(2)} y_{Ena}^{(2)} P_R + V_{n1} y_{Na}^{(2)*} y_{Ena}^{(2)*} P_L\Big]\Bigg\} u_{(p')}^{(s_3)} \\
&= \frac{1}{(4\pi)^2}\frac{1}{2v^2 p^0 p'^0}\bar{u}_{(p)}^{(s_1)}\Bigg\{ \not{p} c m_a^2 \big(B_{0,\varphi+a} + B_{1,\varphi+a}\big)\Big[V_{a3}^* V_{a1} P_R + V_{a3} V_{a1}^* P_L\Big] \\
&- s m_a^2 m_D B_{0,\varphi+a}\Big[V_{a1}^* V_{a3} P_R + V_{a1} V_{a3}^* P_L\Big] \\
&+ \not{p} c v^2 \big(B_{0,H+a} + B_{1,H+a}\big)\Big[y_{Ena}^{(2)} V_{n3}^* y_{Eka}^{(2)*} V_{k1} P_R + y_{Ena}^{(2)*} V_{n3} y_{Eka}^{(2)} V_{k1}^* P_L\Big] \\
&- \mathrm{i}sm_a v^2 B_{0,H+a}\Big[- V_{n1}^* y_{Na}^{(2)} y_{Ena}^{(2)} P_R + V_{n1} y_{Na}^{(2)*} y_{Ena}^{(2)*} P_L\Big]\Bigg\} u_{(p')}^{(s_3)}, \qquad (5.51)
\end{aligned}
$$

where in the last line we have used relation (5.22);

$$
\begin{aligned}
\mathcal{M}_{13}^{VT}(p) &= \frac{1}{(4\pi)^2}\frac{g^2}{2p^0 2p'^0}\bar{u}_{(p)}^{(s_1)}\not{p}\big(D-2\big)\big(B_{0,Wa} + B_{1,Wa}\big)\Big[U_{3a} U_{1a}^* P_R + U_{3a}^* U_{1a} P_L\Big] u_{(p')}^{(s_3)} \\
&= \frac{1}{(4\pi)^2}\frac{g^2 c}{2p^0 2p'^0}\bar{u}_{(p)}^{(s_1)}\not{p}\big(D-2\big)\big(B_{0,Wa} + B_{1,Wa}\big)\Big[V_{a3}^* V_{a1} P_R + V_{a3} V_{a1}^* P_L\Big] u_{(p')}^{(s_3)}, \qquad (5.52)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{13}^{VL}(p) &= \frac{1}{(4\pi)^2}\frac{g^2}{2p^0 2p'^0}\bar{u}_{(p)}^{(s_1)}\not{p} G_{Wa}\Big[U_{3a} U_{1a}^* P_R + U_{3a}^* U_{1a} P_L\Big] u_{(p')}^{(s_3)} \\
&= \frac{1}{(4\pi)^2}\frac{g^2 c}{2p^0 2p'^0}\bar{u}_{(p)}^{(s_1)}\not{p} G_{Wa}\Big[V_{a3}^* V_{a1} P_R + V_{a3} V_{a1}^* P_L\Big] u_{(p')}^{(s_3)}, \qquad (5.53)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_{13}^{SM}(p) = \frac{1}{(4\pi)^2}\frac{1}{p^0 p'^0}\bar{u}_{(p)}^{(s_1)}\Bigg\{ &\frac{\mathrm{i}s}{2}\Big[- U_{1n} F_n^{(j)} P_R + U_{1n}^* F_n^{(j)*} P_L\Big] \times \sum_\gamma \frac{m_{1,\gamma}}{m_{0,j}^2}\,\mathrm{Re}[U_{\gamma k} U_{\gamma 4} F_k^{(j)}] A_0(m_{1,\gamma}^2) \\
&+ \frac{1}{8}\not{p}(B_{0,j\gamma} + B_{1,j\gamma})\Big[U_{\gamma 4}^* U_{1n}^* F_n^{(j)*}\big(- \mathrm{i}sU_{\gamma k} + U_{\gamma 4} U_{3k}\big) F_k^{(j)} P_R \\
&\qquad + U_{1n} U_{\gamma 4} F_n^{(j)}\big(U_{3k}^* U_{\gamma 4}^* + \mathrm{i}sU_{\gamma k}^*\big) F_k^{(j)*} P_L\Big] \\
&+ \frac{1}{8} m_{1,\gamma} B_{0,j\gamma}\Big[U_{1n} U_{\gamma 4} F_n^{(j)}\big(- \mathrm{i}sU_{\gamma k} + U_{\gamma 4} U_{3k}\big) F_k^{(j)} P_R \\
&\qquad + U_{\gamma 4}^* U_{1n}^* F_n^{(j)*}\big(U_{3k}^* U_{\gamma 4}^* + \mathrm{i}sU_{\gamma k}^*\big) F_k^{(j)*} P_L\Big]\Bigg\} u_{(p')}^{(s_3)}
\end{aligned}
$$

$$= 0, \tag{5.54}$$

since each term contains a $U_{1n}F_n^j$ factor that, according to (5.31), is equal to 0;

$$\begin{aligned}\mathcal{M}_{13}^{TM}(p) = & \frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\bar{u}_{(p)}^{(s_1)}\Big\{(D-2)(B_{0,j\gamma}+B_{1,j\gamma})\not{p}\Big[U_{1n}^*U_{\gamma n}U_{\gamma k}^*U_{3k}P_R \\ & - U_{1n}U_{\gamma n}^*U_{\gamma k}U_{3k}^*P_L\Big] + DB_{0,j\gamma}m_{1,\gamma}\Big[U_{1n}U_{\gamma n}^*U_{\gamma k}^*U_{3k}P_R + U_{1n}^*U_{\gamma n}U_{\gamma k}U_{3k}^*P_L\Big]\Big\}u_{(p')}^{(s_3)} \\ = & \, 0,\end{aligned} \tag{5.55}$$

$$\begin{aligned}\mathcal{M}_{13}^{LM}(p) = & \frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\bar{u}_{(p)}^{(s_1)}\Big\{\not{p}G_{j\gamma}\Big[U_{1n}^*U_{\gamma n}U_{\gamma k}^*U_{3k}P_R - U_{1n}U_{\gamma n}^*U_{\gamma k}U_{3k}^*P_L\Big] \\ & - m_{1,\gamma}(B_{0,j\gamma}-\xi B_{0,j\gamma}^{\xi})\Big[U_{1n}U_{\gamma n}^*U_{\gamma k}^*U_{3k}P_R + U_{1n}^*U_{\gamma n}U_{\gamma k}U_{3k}^*P_L\Big]\Big\}u_{(p')}^{(s_3)} \\ = & \, 0,\end{aligned} \tag{5.56}$$

arguing analogously to the case $\beta = 1, \alpha = 2$.

5.3.4 $\beta = 2, \alpha = 2$

$$\begin{aligned}\mathcal{M}_{22}^{SD}(p) = & \frac{1}{(4\pi)^2}\frac{1}{2p^02p'^0}\bar{u}_{(p)}^{(s_2)}\not{p}\big(B_{0,ja}+B_{1,ja}\big)\Big[Y_{Ena}^{(j)}U_{2n}Y_{Eka}^{(j)*}U_{2k}^*P_R + Y_{Ena}^{(j)*}U_{2n}^*Y_{Eka}^{(j)}U_{2k}P_L\Big]u_{(p')}^{(s_2)} \\ = & \, 0,\end{aligned} \tag{5.57}$$

since $\bar{u}_{(p)}^{(s_2)}\not{p}$ can be replaced with $\bar{u}_{(p)}^{(s_2)}m_{\nu 2}$, and the tree-level mass of the second neutrino, $m_{\nu 2}$, is equal to 0. The same reasoning applies to $\mathcal{M}_{22}^{VT}$ and $\mathcal{M}_{22}^{VL}$:

$$\mathcal{M}_{22}^{VT}(p) = \frac{1}{(4\pi)^2}\frac{g^2}{2p^02p'^0}\bar{u}_{(p)}^{(s_2)}\not{p}\big(D-2\big)\big(B_{0,Wa}+B_{1,Wa}\big)U_{2a}U_{2a}^*u_{(p')}^{(s_2)} = 0, \tag{5.58}$$

$$\mathcal{M}_{22}^{VL}(p) = \frac{1}{(4\pi)^2}\frac{g^2}{2p^02p'^0}\bar{u}_{(p)}^{(s_2)}\not{p}G_{Wa}U_{2a}U_{2a}^*u_{(p')}^{(s_2)} = 0. \tag{5.59}$$

As for $\mathcal{M}_{22}^{SM}, \mathcal{M}_{22}^{TM}$, and $\mathcal{M}_{22}^{LM}$, they consist not only of the $\not{p}$ parts, so not everything disappears:

$$\begin{aligned}
\mathcal{M}_{22}^{SM}(p) &= \frac{1}{(4\pi)^2}\frac{1}{p^0p'^0}\bar{u}_{(p)}^{(s_2)}\Bigg\{\frac{1}{8}\not{p}(B_{0,j\gamma}+B_{1,j\gamma})\Big[U_{\gamma4}^*U_{2n}^*F_n^{(j)*}U_{\gamma4}U_{2k}F_k^{(j)}P_R \\
&\quad + U_{2n}U_{\gamma4}F_n^{(j)}U_{2k}^*U_{\gamma4}^*F_k^{(j)*}P_L\Big] + \frac{1}{8}m_{1,\gamma}B_{0,j\gamma}\Big[U_{2n}U_{\gamma4}F_n^{(j)}U_{\gamma4}U_{2k}F_k^{(j)}P_R \\
&\quad + U_{\gamma4}^*U_{2n}^*F_n^{(j)*}U_{2k}^*U_{\gamma4}^*F_k^{(j)*}P_L\Big]\Bigg\}u_{(p')}^{(s_2)} \\
&= \frac{1}{(4\pi)^2}\frac{1}{p^0p'^0}\bar{u}_{(p)}^{(s_2)}\Bigg\{-\frac{1}{8}m_{\nu3}B_{0,j\nu3}s^2\Big[U_{2n}F_n^{(j)}U_{2k}F_k^{(j)}P_R+U_{2n}^*F_n^{(j)*}U_{2k}^*F_k^{(j)*}P_L\Big] \\
&\quad + \frac{1}{8}m_{\nu4}B_{0,j\nu4}c^2\Big[U_{2n}F_n^{(j)}U_{2k}F_k^{(j)}P_R+U_{2n}^*F_n^{(j)*}U_{2k}^*F_k^{(j)*}P_L\Big]\Bigg\}u_{(p')}^{(s_2)} \\
&= \frac{1}{(4\pi)^2}\frac{1}{p^0p'^0}\bar{u}_{(p)}^{(s_2)}\frac{1}{8}\Big[\big(U_{2n}F_n^{(j)}\big)^2P_R+\big(U_{2n}^*F_n^{(j)*}\big)^2P_L\Big]\Big(m_{\nu4}B_{0,j\nu4}c^2-m_{\nu3}B_{0,j\nu3}s^2\Big)u_{(p')}^{(s_2)} \\
&= \frac{1}{(4\pi)^2}\frac{d^2}{4p^0p'^0}\bar{u}_{(p)}^{(s_2)}\Bigg\{\sin^2\alpha\Big(m_{\nu4}B_{0,h1\nu4}c^2-m_{\nu3}B_{0,h1\nu3}s^2\Big) \\
&\quad + \cos^2\alpha\Big(m_{\nu4}B_{0,h2\nu4}c^2-m_{\nu3}B_{0,h2\nu3}s^2\Big)-\Big(m_{\nu4}B_{0,HI\nu4}c^2-m_{\nu3}B_{0,HI\nu3}s^2\Big)\Bigg\}u_{(p')}^{(s_2)},
\end{aligned} \tag{5.60}$$

$$\begin{aligned}
\mathcal{M}_{22}^{TM}(p) &= \frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\bar{u}_{(p)}^{(s_2)}\Bigg\{(D-2)(B_{0,Z\gamma}+B_{1,Z\gamma})\not{p}\Big[U_{2n}^*U_{\gamma n}U_{\gamma k}^*U_{2k}P_R \\
&\quad - U_{2n}U_{\gamma n}^*U_{\gamma k}U_{2k}^*P_L\Big]+DB_{0,Z\gamma}m_{1,\gamma}\Big[U_{2n}U_{\gamma n}^*U_{\gamma k}^*U_{2k}P_R+U_{2n}^*U_{\gamma n}U_{\gamma k}U_{2k}^*P_L\Big]\Bigg\}u_{(p')}^{(s_2)} \\
&= 0,
\end{aligned} \tag{5.61}$$

since in the summation over γ all terms vanish either due to $m_{\nu1}$, $m_{\nu2}$ factors in front or due to the sums $U_{2n}U_{3n}^*, U_{2n}U_{4n}^*$;

$$\begin{aligned}
\mathcal{M}_{22}^{LM}(p) &= \frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\bar{u}_{(p)}^{(s_2)}\Bigg\{\not{p}G_{Z\gamma}\Big[U_{2n}^*U_{\gamma n}U_{\gamma k}^*U_{2k}P_R-U_{2n}U_{\gamma n}^*U_{\gamma k}U_{2k}^*P_L\Big] \\
&\quad - m_{1,\gamma}(B_{0,Z\gamma}-\xi B_{0,Z\gamma}^{\xi})\Big[U_{2n}U_{\gamma n}^*U_{\gamma k}^*U_{2k}P_R+U_{2n}^*U_{\gamma n}U_{\gamma k}U_{2k}^*P_L\Big]\Bigg\}u_{(p')}^{(s_2)} \\
&= 0,
\end{aligned} \tag{5.62}$$

analogously to $\mathcal{M}_{22}^{TM}$.

5.3.5 $\beta = 2, \alpha = 3$

$$\begin{aligned}
\mathcal{M}_{23}^{SD}(p) &= \frac{1}{(4\pi)^2}\frac{1}{2p^0 2p'^0}\bar{u}_{(p)}^{(s_2)}\bigg\{\not{p}\big(B_{0,ja}+B_{1,ja}\big)\Big[Y_{Ena}^{(j)}U_{3n}Y_{Eka}^{(j)*}U_{2k}^{*}P_R + Y_{Ena}^{(j)*}U_{3n}^{*}Y_{Eka}^{(j)}U_{2k}P_L\Big] \\
&\quad + \mathrm{i}sm_{1,a}B_{0,ja}\Big[U_{2n}Y_{Na}^{(j)}Y_{Ena}^{(j)}P_R - U_{2n}^{*}Y_{Na}^{(j)*}Y_{Ena}^{(j)*}P_L\Big]\bigg\}u_{(p')}^{(s_3)} \\
&= \frac{1}{(4\pi)^2}\frac{1}{2p^0 2p'^0}\bar{u}_{(p)}^{(s_2)}\bigg\{\not{p}\big(B_{0,\varphi+a}+B_{1,\varphi+a}\big)\Big[Y_{Ena}^{(1)}U_{3n}Y_{Eka}^{(1)*}U_{2k}^{*}P_R + Y_{Ena}^{(1)*}U_{3n}^{*}Y_{Eka}^{(1)}U_{2k}P_L\Big] \\
&\quad + \mathrm{i}sm_{1,a}B_{0,\varphi+a}\Big[U_{2n}Y_{Na}^{(1)}Y_{Ena}^{(1)}P_R - U_{2n}^{*}Y_{Na}^{(1)*}Y_{Ena}^{(1)*}P_L\Big] \\
&\quad + \not{p}\big(B_{0,H+a}+B_{1,H+a}\big)\Big[Y_{Ena}^{(2)}U_{3n}Y_{Eka}^{(2)*}U_{2k}^{*}P_R + Y_{Ena}^{(2)*}U_{3n}^{*}Y_{Eka}^{(2)}U_{2k}P_L\Big] \\
&\quad + \mathrm{i}sm_{1,a}B_{0,H+a}\Big[U_{2n}Y_{Na}^{(2)}Y_{Ena}^{(2)}P_R - U_{2n}^{*}Y_{Na}^{(2)*}Y_{Ena}^{(2)*}P_L\Big]\bigg\}u_{(p')}^{(s_3)} \\
&= \frac{1}{(4\pi)^2}\frac{1}{2v^2 p^0 p'^0}\bar{u}_{(p)}^{(s_2)}\bigg\{\not{p}cm_a^2\big(B_{0,\varphi+a}+B_{1,\varphi+a}\big)\Big[V_{a3}^{*}V_{a2}P_R + V_{a3}V_{a2}^{*}P_L\Big] \\
&\quad + \mathrm{i}sm_a^2 B_{0,\varphi+a}\Big[V_{a2}^{*}m_{Da}^{*}P_R - V_{a2}m_{Da}P_L\Big] \\
&\quad + \not{p}cv^2\big(B_{0,H+a}+B_{1,H+a}\big)\Big[y_{Ena}^{(2)}V_{n3}^{*}y_{Eka}^{(2)*}V_{k2}P_R + y_{Ena}^{(2)*}V_{n3}y_{Eka}^{(2)}V_{k2}^{*}P_L\Big] \\
&\quad + \mathrm{i}sv^2 m_a B_{0,H+a}\Big[V_{n2}^{*}y_{Na}^{(2)}y_{Ena}^{(2)}P_R - V_{n2}y_{Na}^{(2)*}y_{Ena}^{(2)*}P_L\Big]\bigg\}u_{(p')}^{(s_3)} \\
&= \frac{1}{(4\pi)^2}\frac{1}{2v^2 p^0 p'^0}\bar{u}_{(p)}^{(s_2)}\bigg\{\not{p}cm_a^2\big(B_{0,\varphi+a}+B_{1,\varphi+a}\big)\Big[V_{a3}^{*}V_{a2}P_R + V_{a3}V_{a2}^{*}P_L\Big] \\
&\quad - sm_a^2 m_D B_{0,\varphi+a}\Big[V_{a2}^{*}V_{a3}P_R + V_{a2}V_{a3}^{*}P_L\Big] \\
&\quad + \not{p}cv^2\big(B_{0,H+a}+B_{1,H+a}\big)\Big[y_{Ena}^{(2)}V_{n3}^{*}y_{Eka}^{(2)*}V_{k2}P_R + y_{Ena}^{(2)*}V_{n3}y_{Eka}^{(2)}V_{k2}^{*}P_L\Big] \\
&\quad + \mathrm{i}sv^2 m_a B_{0,H+a}\Big[V_{n2}^{*}y_{Na}^{(2)}y_{Ena}^{(2)}P_R - V_{n2}y_{Na}^{(2)*}y_{Ena}^{(2)*}P_L\Big]\bigg\}u_{(p')}^{(s_3)},
\end{aligned} \tag{5.63}$$

$$\begin{aligned}
\mathcal{M}_{23}^{VT}(p) &= \frac{1}{(4\pi)^2}\frac{g^2}{2p^0 2p'^0}\bar{u}_{(p)}^{(s_2)}\not{p}\big(D-2\big)\big(B_{0,ja}+B_{1,ja}\big)\Big[U_{3a}U_{2a}^{*}P_R + U_{3a}^{*}U_{2a}P_L\Big]u_{(p')}^{(s_3)} \\
&= \frac{1}{(4\pi)^2}\frac{g^2 c}{2p^0 2p'^0}\bar{u}_{(p)}^{(s_2)}\not{p}\big(D-2\big)\big(B_{0,Wa}+B_{1,Wa}\big)\Big[V_{a3}^{*}V_{a2}P_R + V_{a3}V_{a2}^{*}P_L\Big]u_{(p')}^{(s_3)},
\end{aligned} \tag{5.64}$$

$$\mathcal{M}_{23}^{VL}(p) = \frac{1}{(4\pi)^2}\frac{g^2}{2p^0 2p'^0}\bar{u}_{(p)}^{(s_2)}\not{p}G_{ja}\Big[U_{3a}U_{2a}^{*}P_R + U_{3a}^{*}U_{2a}P_L\Big]u_{(p')}^{(s_3)}$$

$$= \frac{1}{(4\pi)^2}\frac{g^2 c}{2p^0 2p'^0}\bar{u}^{(s_2)}_{(p)}\not{p}G_{Wa}\Big[V^*_{a3}V_{a2}P_R + V_{a3}V^*_{a2}P_L\Big]u^{(s_3)}_{(p')}, \tag{5.65}$$

$$\begin{aligned}\mathcal{M}^{SM}_{23}(p) = \frac{1}{(4\pi)^2}\frac{1}{p^0p'^0}\bar{u}^{(s_2)}_{(p)}\Bigg\{&\frac{\mathrm{i}s}{2}\Big[-U_{2n}F^{(j)}_n P_R + U^*_{2n}F^{(j)*}_n P_L\Big]\sum_\gamma \frac{m_{1,\gamma}}{m^2_{0,j}}\,\mathrm{Re}[U_{\gamma k}U_{\gamma 4}F^{(j)}_k]A_0(m^2_{1,\gamma})\\
&+\frac{1}{8}\not{p}(B_{0,j\gamma}+B_{1,j\gamma})\Big[U^*_{\gamma 4}U^*_{2n}F^{(j)*}_n\big(-\mathrm{i}sU_{\gamma k}+U_{\gamma 4}U_{3k}\big)F^{(j)}_k P_R\\
&\qquad + U_{2n}U_{\gamma 4}F^{(j)}_n\big(U^*_{3k}U^*_{\gamma 4}+\mathrm{i}sU^*_{\gamma k}\big)F^{(j)*}_k P_L\Big]\\
&+\frac{1}{8}m_{1,\gamma}B_{0,j\gamma}\Big[U_{2n}U_{\gamma 4}F^{(j)}_n\big(-\mathrm{i}sU_{\gamma k}+U_{\gamma 4}U_{3k}\big)F^{(j)}_k P_R\\
&\qquad + U^*_{\gamma 4}U^*_{2n}F^{(j)*}_n\big(U^*_{3k}U^*_{\gamma 4}+\mathrm{i}sU^*_{\gamma k}\big)F^{(j)*}_k P_L\Big]\Bigg\}u^{(s_3)}_{(p')},\end{aligned} \tag{5.66}$$

$$\begin{aligned}\mathcal{M}^{TM}_{23}(p) &= \frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\bar{u}^{(s_2)}_{(p)}\Bigg\{(D-2)(B_{0,j\gamma}+B_{1,j\gamma})\not{p}\Big[U^*_{2n}U_{\gamma n}U^*_{\gamma k}U_{3k}P_R\\
&\quad - U_{2n}U^*_{\gamma n}U_{\gamma k}U^*_{3k}P_L\Big] + DB_{0,j\gamma}m_{1,\gamma}\Big[U_{2n}U^*_{\gamma n}U^*_{\gamma k}U_{3k}P_R + U^*_{2n}U_{\gamma n}U_{\gamma k}U^*_{3k}P_L\Big]\Bigg\}u^{(s_3)}_{(p')}\\
&= 0,\end{aligned} \tag{5.67}$$

since with every value of $\gamma = 1, \ldots, 4$, each term of $\mathcal{M}^{TM}_{23}$ contains a factor of the type $U_{2m}U^*_{nm}, n \neq m$;

$$\begin{aligned}\mathcal{M}^{LM}_{23}(p) &= \frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\bar{u}^{(s_2)}_{(p)}\Bigg\{\not{p}G_{j\gamma}\Big[U^*_{2n}U_{\gamma n}U^*_{\gamma k}U_{3k}P_R - U_{2n}U^*_{\gamma n}U_{\gamma k}U^*_{3k}P_L\Big]\\
&\quad - m_{1,\gamma}(B_{0,j\gamma}-\xi B^{\xi}_{0,j\gamma})\Big[U_{2n}U^*_{\gamma n}U^*_{\gamma k}U_{3k}P_R + U^*_{2n}U_{\gamma n}U_{\gamma k}U^*_{3k}P_L\Big]\Bigg\}u^{(s_3)}_{(p')}\\
&= 0,\end{aligned} \tag{5.68}$$

following the same reasoning as in the previous case.

5.3.6 $\beta = 3, \alpha = 3$

$$\begin{aligned}\mathcal{M}^{SD}_{33}(p) = \frac{1}{(4\pi)^2}\frac{1}{2p^0 2p'^0}\bar{u}^{(s_3)}_{(p)}\Bigg\{&\not{p}\big(B_{0,ja}+B_{1,ja}\big)\Big[\big(s^2|Y^{(j)}_{Na}|^2 + Y^{(j)}_{Ena}U_{3n}Y^{(j)*}_{Eka}U^*_{3k}\big)P_R\\
&+\big(Y^{(j)*}_{Ena}U^*_{3n}Y^{(j)}_{Eka}U_{3k} + s^2|Y^{(j)}_{Na}|^2\big)P_L\Big]\end{aligned}$$

$$+\,2\mathrm{i}sm_{1,a}B_{0,ja}\Big[U_{3n}Y^{(j)}_{Na}Y^{(j)}_{Ena}P_R-U^*_{3n}Y^{(j)*}_{Na}Y^{(j)*}_{Ena}P_L\Big]\Bigg\}u^{(s_3)}_{(p')}$$

$$=\frac{1}{(4\pi)^2}\frac{1}{2p^0 2p'^0}\overline{u}^{(s_3)}_{(p)}\Bigg\{m_{\nu 3}\big(B_{0,\varphi+a}+B_{1,\varphi+a}\big)\Big[s^2|Y^{(1)}_{Na}|^2+Y^{(1)}_{Ena}U_{3n}Y^{(1)*}_{Eka}U^*_{3k}P_R$$

$$+\,Y^{(1)*}_{Ena}U^*_{3n}Y^{(1)}_{Eka}U_{3k}P_L\Big]+2\mathrm{i}sm_{1,a}B_{0,\varphi+a}\Big[U_{3n}Y^{(1)}_{Na}Y^{(1)}_{Ena}P_R-U^*_{3n}Y^{(1)*}_{Na}Y^{(1)*}_{Ena}P_L\Big]$$

$$+\,m_{\nu 3}\big(B_{0,H+a}+B_{1,H+a}\big)\Big[s^2|Y^{(2)}_{Na}|^2+Y^{(2)}_{Ena}U_{3n}Y^{(2)*}_{Eka}U^*_{3k}P_R+Y^{(2)*}_{Ena}U^*_{3n}Y^{(2)}_{Eka}U_{3k}P_L\Big]$$

$$+\,2\mathrm{i}sm_{1,a}B_{0,H+a}\Big[U_{3n}Y^{(2)}_{Na}Y^{(2)}_{Ena}P_R-U^*_{3n}Y^{(2)*}_{Na}Y^{(2)*}_{Ena}P_L\Big]\Bigg\}u^{(s_3)}_{(p')}$$

$$=\frac{1}{(4\pi)^2}\frac{1}{2v^2p^0p'^0}\overline{u}^{(s_3)}_{(p)}\Bigg\{m_{\nu 3}\big(B_{0,\varphi+a}+B_{1,\varphi+a}\big)\Big[s^2|m_{Da}|^2+m_a^2c^2|V_{a3}|^2\Big]$$

$$+\,2\mathrm{i}scm_a^2B_{0,\varphi+a}\Big[V^*_{a3}m^*_{Da}P_R-V_{a3}m_{Da}P_L\Big]$$

$$+\,m_{\nu 3}v^2\big(B_{0,H+a}+B_{1,H+a}\big)\Big[s^2|y^{(2)}_{Na}|^2+c^2y^{(2)}_{Ena}V^*_{n3}y^{(2)*}_{Eka}V_{k3}P_R+c^2y^{(2)*}_{Ena}V_{n3}y^{(2)}_{Eka}V^*_{k3}P_L\Big]$$

$$+\,2\mathrm{i}scm_av^2B_{0,H+a}\Big[V^*_{n3}y^{(2)}_{Na}y^{(2)}_{Ena}P_R-V_{n3}y^{(2)*}_{Na}y^{(2)*}_{Ena}P_L\Big]\Bigg\}u^{(s_3)}_{(p')}$$

$$=\frac{1}{(4\pi)^2}\frac{1}{2v^2p^0p'^0}\overline{u}^{(s_3)}_{(p)}\Bigg\{m_{\nu 3}|V_{a3}|^2\big(B_{0,\varphi+a}+B_{1,\varphi+a}\big)\Big[s^2m_D^2+c^2m_a^2\Big]$$

$$-\,2scm_a^2m_D|V_{a3}|^2B_{0,\varphi+a}$$

$$+\,m_{\nu 3}v^2\big(B_{0,H+a}+B_{1,H+a}\big)\Big[s^2|y^{(2)}_{Na}|^2+c^2y^{(2)}_{Ena}V^*_{n3}y^{(2)*}_{Eka}V_{k3}P_R+c^2y^{(2)*}_{Ena}V_{n3}y^{(2)}_{Eka}V^*_{k3}P_L\Big]$$

$$+\,2\mathrm{i}scm_av^2B_{0,H+a}\Big[V^*_{n3}y^{(2)}_{Na}y^{(2)}_{Ena}P_R-V_{n3}y^{(2)*}_{Na}y^{(2)*}_{Ena}P_L\Big]\Bigg\}u^{(s_3)}_{(p')},\qquad(5.69)$$

$$\begin{aligned}\mathcal{M}^{VT}_{33}(p)&=\frac{1}{(4\pi)^2}\frac{g^2}{2p^0 2p'^0}\overline{u}^{(s_3)}_{(p)}\not{p}\big(D-2\big)\big(B_{0,ja}+B_{1,ja}\big)|U_{3a}|^2u^{(s_3)}_{(p')}\\&=\frac{1}{(4\pi)^2}\frac{g^2c^2}{2p^0 2p'^0}\overline{u}^{(s_3)}_{(p)}m_{\nu 3}\big(D-2\big)\big(B_{0,Wa}+B_{1,Wa}\big)|V_{a3}|^2u^{(s_3)}_{(p')},\end{aligned}\qquad(5.70)$$

$$\begin{aligned}\mathcal{M}^{VL}_{33}(p)&=\frac{1}{(4\pi)^2}\frac{g^2}{2p^0 2p'^0}\overline{u}^{(s_3)}_{(p)}\not{p}G_{Wa}|U_{3a}|^2u^{(s_3)}_{(p')}\\&=\frac{1}{(4\pi)^2}\frac{g^2c^2}{2p^0 2p'^0}\overline{u}^{(s_3)}_{(p)}m_{\nu 3}G_{Wa}|V_{a3}|^2u^{(s_3)}_{(p')}\end{aligned}\qquad(5.71)$$

$$\mathcal{M}^{SM}_{33}(p)=\frac{1}{(4\pi)^2}\frac{1}{p^0p'^0}\overline{u}^{(s_3)}_{(p)}\Bigg\{\mathrm{i}s\Big[-U_{3n}F^{(j)}_nP_R+U^*_{3n}F^{(j)*}_nP_L\Big]\sum_\gamma\frac{m_{1,\gamma}}{m^2_{0,j}}\,\mathrm{Re}[U_{\gamma k}U_{\gamma 4}F^{(j)}_k]A_0(m^2_{1,\gamma})$$

$$
\begin{aligned}
&+\frac{1}{8}\not{p}(B_{0,j\gamma}+B_{1,j\gamma})\Big[\big(\mathrm{i}sU^*_{\gamma n}+U^*_{\gamma 4}U^*_{3n}\big)F^{(j)*}_n\big(-\mathrm{i}sU_{\gamma k}+U_{\gamma 4}U_{3k}\big)F^{(j)}_k P_R \\
&\qquad+\big(U_{3n}U_{\gamma 4}-\mathrm{i}sU_{\gamma n}\big)F^{(j)}_n\big(U^*_{3k}U^*_{\gamma 4}+\mathrm{i}sU^*_{\gamma k}\big)F^{(j)*}_k P_L\Big] \\
&+\frac{1}{8}m_{1,\gamma}B_{0,j\gamma}\Big[\big(U_{3n}U_{\gamma 4}-\mathrm{i}sU_{\gamma n}\big)F^{(j)}_n\big(-\mathrm{i}sU_{\gamma k}+U_{\gamma 4}U_{3k}\big)F^{(j)}_k P_R \\
&\qquad+\big(\mathrm{i}sU^*_{\gamma n}+U^*_{\gamma 4}U^*_{3n}\big)F^{(j)*}_n\big(U^*_{3k}U^*_{\gamma 4}+\mathrm{i}sU^*_{\gamma k}\big)F^{(j)*}_k P_L\Big]\Bigg\}u^{(s_3)}_{(p')},
\end{aligned}
\tag{5.72}
$$

$$
\begin{aligned}
\mathcal{M}^{TM}_{33}(p)&=\frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\bar{u}^{(s_3)}_{(p)}\Bigg\{(D-2)(B_{0,j\gamma}+B_{1,j\gamma})\not{p}\Big[U^*_{3n}U_{\gamma n}U^*_{\gamma k}U_{3k}P_R \\
&\quad-U_{3n}U^*_{\gamma n}U_{\gamma k}U^*_{3k}P_L\Big]+DB_{0,j\gamma}m_{1,\gamma}\Big[U_{3n}U^*_{\gamma n}U^*_{\gamma k}U_{3k}P_R+U^*_{3n}U_{\gamma n}U_{\gamma k}U^*_{3k}P_L\Big]\Bigg\}u^{(s_3)}_{(p')} \\
&=\frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\bar{u}^{(s_3)}_{(p)}\Bigg\{m_{\nu 3}c^4(D-2)(B_{0,Z\nu 3}+B_{1,Z\nu 3})\Big[P_R-P_L\Big] \\
&\quad+m_{\nu 4}s^2c^2(D-2)(B_{0,Z\nu 4}+B_{1,Z\nu 4})\Big[P_R-P_L\Big] \\
&\quad+DB_{0,Z\nu 3}m_{\nu 3}c^4-DB_{0,Z\nu 4}m_{\nu 4}s^2c^2\Bigg\}u^{(s_3)}_{(p')} \\
&=\frac{1}{(4\pi)^2}\frac{g^2c^2}{16p^0p'^0\cos^2\theta_W}\bar{u}^{(s_3)}_{(p)}\Bigg\{(D-2)\Big[m_{\nu 3}c^2(B_{0,Z\nu 3}+B_{1,Z\nu 3})+m_{\nu 4}s^2(B_{0,Z\nu 4}+B_{1,Z\nu 4})\Big] \\
&\quad\times\Big[P_R-P_L\Big]+D\Big(m_{\nu 3}c^2B_{0,Z\nu 3}-m_{\nu 4}s^2B_{0,Z\nu 4}\Big)\Bigg\}u^{(s_3)}_{(p')},
\end{aligned}
\tag{5.73}
$$

$$
\begin{aligned}
\mathcal{M}^{LM}_{33}(p)&=\frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\bar{u}^{(s_3)}_{(p)}\Bigg\{\not{p}G_{Z\gamma}\Big[U^*_{3n}U_{\gamma n}U^*_{\gamma k}U_{3k}P_R-U_{3n}U^*_{\gamma n}U_{\gamma k}U^*_{3k}P_L\Big] \\
&\quad-m_{1,\gamma}(B_{0,Z\gamma}-\xi B^{\xi}_{0,Z\gamma})\Big[U_{3n}U^*_{\gamma n}U^*_{\gamma k}U_{3k}P_R+U^*_{3n}U_{\gamma n}U_{\gamma k}U^*_{3k}P_L\Big]\Bigg\}u^{(s_3)}_{(p')} \\
&=\frac{1}{(4\pi)^2}\frac{g^2}{16p^0p'^0\cos^2\theta_W}\bar{u}^{(s_3)}_{(p)}\Bigg\{m_{\nu 3}G_{Z\nu 3}c^4\Big[P_R-P_L\Big]-m_{\nu 3}(B_{0,Z\nu 3}-\xi B^{\xi}_{0,Z\nu 3})c^4 \\
&\quad+m_{\nu 3}G_{Z\nu 4}s^2c^2\Big[P_R-P_L\Big]+m_{\nu 4}(B_{0,Z\nu 4}-\xi B^{\xi}_{0,Z\nu 4})s^2c^2\Bigg\}u^{(s_3)}_{(p')} \\
&=\frac{1}{(4\pi)^2}\frac{g^2c^2}{16p^0p'^0\cos^2\theta_W}\bar{u}^{(s_3)}_{(p)}\Bigg\{m_{\nu 3}\Big(G_{Z\nu 3}c^2+G_{Z\nu 4}s^2\Big)\Big[P_R-P_L\Big] \\
&\quad-m_{\nu 3}(B_{0,Z\nu 3}-\xi B^{\xi}_{0,Z\nu 3})c^2+m_{\nu 4}(B_{0,Z\nu 4}-\xi B^{\xi}_{0,Z\nu 4})s^2\Bigg\}u^{(s_3)}_{(p')}.
\end{aligned}
\tag{5.74}
$$

6 Investigating the Gauge Dependence of One-Loop Corrections

In the expressions obtained in the last section, the gauge dependence appears in three ways:

- explicitly, through the factor ξ;
- in the $B^{\xi}_{0,ja}$ and $B^{\xi}_{1,ja}$ integrals;
- through the masses of the Goldstone bosons φ^{+} and φ_Z (eq. (2.21)).

Therefore, the gauge dependence may be present only in $\mathcal{M}^{SD}_{\beta\alpha}$, $\mathcal{M}^{VL}_{\beta\alpha}$, $\mathcal{M}^{SM}_{\beta\alpha}$, and $\mathcal{M}^{LM}_{\beta\alpha}$ terms. In this section we are going to investigate them more closely.

6.1 B_0 and B_1 Integrals

The definitions of B_0 and B_1 integrals were introduced in eqs. (5.10) and (5.11). Since we are interested in the processes with a very small total 4-momentum p (it is equal to the mass of the incoming/outgoing neutrino), the expressions of B_0 and B_1 can be simplified by expanding them in power series of p^2 around the point $p^2 = 0$ and neglecting the terms $\mathcal{O}(p^2)$.

From [15] we know that B_0 can be written as:

$$B_0(p^2, m_0^2, m_1^2) = \Delta - \int_0^1 \mathrm{d}x \ln \frac{p^2 x^2 - x(p^2 - m_0^2 + m_1^2) + m_1^2}{\mu^2}, \tag{6.1}$$

where Δ is the divergent part of the integral, and μ is a mass scale parameter. After expanding the integrand in power series and leaving only the first two terms, we get:

$$\begin{aligned} B_0(p^2, m_0^2, m_1^2) &\approx \Delta - \int_0^1 \mathrm{d}x \ln \frac{m_1^2 + x\big(m_0^2 - m_1^2\big)}{\mu^2} - p^2 \int_0^1 \mathrm{d}x \frac{x(x-1)}{m_1^2 + x\big(m_0^2 - m_1^2\big)} \\ &= \Delta - \frac{m_0^2 \ln m_0^2 - m_1^2 \ln m_1^2}{m_0^2 - m_1^2} - p^2 \left(\frac{m_0^2 m_1^2}{\big(m_0^2 - m_1^2\big)^3} \ln \frac{m_0^2}{m_1^2} - \frac{m_0^2 + m_1^2}{2\big(m_0^2 - m_1^2\big)^2} \right). \end{aligned} \tag{6.2}$$

Whereas B_1 can be expressed in terms of B_0 (more details in [16]):

$$B_1(p^2, m_0^2, m_1^2) = \frac{1}{2p^2} \Big[A_0\big(m_0^2\big) - A_0\big(m_1^2\big) - \big(p^2 - m_1^2 + m_0^2\big) B_0(p^2, m_0^2, m_1^2) \Big], \tag{6.3}$$

where

$$A_0(m^2) \equiv \frac{1}{\mathrm{i}\pi^2} \int \mathrm{d}^4 q \frac{1}{q^2 - m^2} = m^2 \Big(\Delta - \ln \frac{m^2}{\mu^2} + 1 \Big). \tag{6.4}$$

Also, from the definition (5.12) of $B^{\xi}_{x,ja}$ and the expressions of masses of the Goldstone bosons φ^+ and φ_Z (eq. (2.21)), it follows that:

$$B_{x,\varphi+a} = B^{\xi}_{x,Wa}, \qquad B_{x,\varphi za} = B^{\xi}_{x,Za}. \tag{6.5}$$

Now we are ready to investigate the gauge dependence of the calculated matrix elements $\mathcal{M}_{\beta\alpha}$.

6.2 The Gauge Dependent Parts of the $\mathcal{M}$ Matrix

6.2.1 $\beta = 1, \alpha = 1$

Since all parts of $\mathcal{M}_{11}$ vanish, there is, of course, no gauge dependence left.

6.2.2 $\beta = 1, \alpha = 2$

$$\mathcal{M}^{SD\xi}_{12}(p) = \frac{1}{(4\pi)^2}\frac{1}{2v^2p^0p'^0}\bar{u}^{(s_1)}_{(p)}\not{p}m_a^2\big(B_{0,\varphi+a} + B_{1,\varphi+a}\big)\Big[V^*_{a2}V_{a1}P_R + V_{a2}V^*_{a1}P_L\Big]u^{(s_2)}_{(p')}, \tag{6.6}$$

$$\mathcal{M}^{VL\xi}_{12}(p) = \frac{1}{(4\pi)^2}\frac{g^2}{2p^02p'^0}\bar{u}^{(s_1)}_{(p)}\not{p}G_{Wa}\Big[V^*_{a2}V_{a1}P_R + V_{a2}V^*_{a1}P_L\Big]u^{(s_2)}_{(p')}. \tag{6.7}$$

6.2.3 $\beta = 1, \alpha = 3$

$$\begin{aligned}\mathcal{M}^{SD\xi}_{13}(p) = \frac{1}{(4\pi)^2}\frac{m_a^2}{2v^2p^0p'^0}\bar{u}^{(s_1)}_{(p)}\Big\{&c\not{p}\big(B_{0,\varphi+a} + B_{1,\varphi+a}\big)\Big[V^*_{a3}V_{a1}P_R + V_{a3}V^*_{a1}P_L\Big]\\ &- sm_D B_{0,\varphi+a}\Big[V^*_{a1}V_{a3}P_R + V_{a1}V^*_{a3}P_L\Big]\Big\}u^{(s_3)}_{(p')},\end{aligned} \tag{6.8}$$

$$\mathcal{M}^{VL}_{13}(p) = \frac{1}{(4\pi)^2}\frac{g^2c}{2p^02p'^0}\bar{u}^{(s_1)}_{(p)}\not{p}G_{Wa}\Big[V^*_{a3}V_{a1}P_R + V_{a3}V^*_{a1}P_L\Big]u^{(s_3)}_{(p')}. \tag{6.9}$$

6.2.4 $\beta = 2, \alpha = 2$

The only nonzero part of $\mathcal{M}_{22}$ is $\mathcal{M}^{SM}_{22}$, but the gauge dependent terms in it (the ones that contain couplings with the Goldstone field φ^+) vanish, thus $\mathcal{M}_{22}$ is gauge independent.

6.2.5 $\beta = 2, \alpha = 3$

The ξ dependence of $\mathcal{M}_{23}$ is contained in:

$$\mathcal{M}_{23}^{SD\xi}(p) = \frac{1}{(4\pi)^2}\frac{m_a^2}{2v^2p^0p'^0}\overline{u}_{(p)}^{(s_2)}\Big\{c\not{p}\big(B_{0,\varphi+a}+B_{1,\varphi+a}\big)\Big[V_{a3}^*V_{a2}P_R+V_{a3}V_{a2}^*P_L\Big]$$
$$-sm_DB_{0,\varphi+a}\Big[V_{a2}^*V_{a3}P_R+V_{a2}V_{a3}^*P_L\Big]\Big\}u_{(p')}^{(s_3)} \tag{6.10}$$

and

$$\mathcal{M}_{23}^{VL\xi}(p) = \frac{1}{(4\pi)^2}\frac{g^2c}{2p^02p'^0}\overline{u}_{(p)}^{(s_2)}\not{p}G_{Wa}\Big[V_{a3}^*V_{a2}P_R+V_{a3}V_{a2}^*P_L\Big]u_{(p')}^{(s_3)}. \tag{6.11}$$

6.2.6 $\beta = 3, \alpha = 3$

$$\mathcal{M}_{33}^{SD\xi}(p) = \frac{1}{(4\pi)^2}\frac{|V_{a3}|^2}{2v^2p^0p'^0}\overline{u}_{(p)}^{(s_3)}\Big\{m_{\nu3}\big(B_{0,\varphi+a}+B_{1,\varphi+a}\big)\Big[s^2m_D^2+c^2m_a^2\Big]-2scm_Dm_a^2B_{0,\varphi+a}\Big\}u_{(p')}^{(s_3)}, \tag{6.12}$$

$$\mathcal{M}_{33}^{VL\xi}(p) = \frac{1}{(4\pi)^2}\frac{g^2c^2}{2p^02p'^0}\overline{u}_{(p)}^{(s_3)}m_{\nu3}G_{Wa}|V_{a3}|^2u_{(p')}^{(s_3)}, \tag{6.13}$$

$$\mathcal{M}_{33}^{SM\xi}(p) = \frac{1}{(4\pi)^2}\frac{1}{p^0p'^0}\overline{u}_{(p)}^{(s_3)}\Big\{\mathrm{i}sc\Big[-V_{n3}^*F_n^{(5)}P_R+V_{n3}F_n^{(5)*}P_L\Big]\sum_\gamma\frac{m_{1,\gamma}}{m_{\varphi z}^2}\mathrm{Re}[U_{\gamma k}U_{\gamma4}F_k^{(5)}]A_0(m_{1,\gamma}^2)$$
$$+\frac{1}{8}\not{p}(B_{0,\varphi z\gamma}+B_{1,\varphi z\gamma})\Big[\big(\mathrm{i}sU_{\gamma n}^*+U_{\gamma4}^*U_{3n}^*\big)F_n^{(5)*}\big(-\mathrm{i}sU_{\gamma k}+U_{\gamma4}U_{3k}\big)F_k^{(5)}P_R$$
$$+\big(U_{3n}U_{\gamma4}-\mathrm{i}sU_{\gamma n}\big)F_n^{(5)}\big(U_{3k}^*U_{\gamma4}^*+\mathrm{i}sU_{\gamma k}^*\big)F_k^{(5)*}P_L\Big]$$
$$+\frac{1}{8}m_{1,\gamma}B_{0,\varphi z\gamma}\Big[\big(U_{3n}U_{\gamma4}-\mathrm{i}sU_{\gamma n}\big)F_n^{(5)}\big(-\mathrm{i}sU_{\gamma k}+U_{\gamma4}U_{3k}\big)F_k^{(5)}P_R$$
$$+\big(\mathrm{i}sU_{\gamma n}^*+U_{\gamma4}^*U_{3n}^*\big)F_n^{(5)*}\big(U_{3k}^*U_{\gamma4}^*+\mathrm{i}sU_{\gamma k}^*\big)F_k^{(5)*}P_L\Big]\Big\}u_{(p')}^{(s_3)}$$
$$=\frac{1}{(4\pi)^2}\frac{1}{p^0p'^0}\overline{u}_{(p)}^{(s_3)}\Big\{\frac{s^2c^2m_D^2}{v^2}m_{\nu3}(B_{0,\varphi z\nu3}+B_{1,\varphi z\nu3})+\frac{m_D^2}{4v^2}m_{\nu3}\big(c^2-s^2\big)^2(B_{0,\varphi z\nu4}+B_{1,\varphi z\nu4})$$
$$-\frac{s^2c^2m_D^2}{v^2}m_{\nu3}B_{0,\varphi z\nu3}+\frac{m_D^2}{4v^2}m_{\nu4}\big(c^2-s^2\big)^2B_{0,\varphi z\nu4}\Big\}u_{(p')}^{(s_3)}$$

$$
\begin{aligned}
&= \frac{1}{(4\pi)^2}\frac{m_D^2}{v^2p^0p'^0}\overline{u}^{(s_3)}_{(p)}\Bigg\{s^2c^2m_{\nu3}B_{1,\varphi z\nu3} + \frac{\left(c^2-s^2\right)^2}{4}\Big[\left(m_{\nu3}+m_{\nu4}\right)B_{0,\varphi z\nu4} + m_{\nu3}B_{1,\varphi z\nu4}\Big]\Bigg\} \\
&\quad\times u^{(s_3)}_{(p')} \\
&= \frac{1}{(4\pi)^2}\frac{1}{4v^2p^0p'^0}\frac{m_{\nu3}m_{\nu4}}{(m_{\nu3}+m_{\nu4})^2}\overline{u}^{(s_3)}_{(p)}\Bigg\{4m_{\nu3}^2m_{\nu4}B_{1,\varphi z\nu3} + \left(m_{\nu4}-m_{\nu3}\right)^2 \\
&\quad\times\Big[\left(m_{\nu3}+m_{\nu4}\right)B_{0,\varphi z\nu4} + m_{\nu3}B_{1,\varphi z\nu4}\Big]\Bigg\}u^{(s_3)}_{(p')}.
\end{aligned} \tag{6.14}
$$

$$
\begin{aligned}
\mathcal{M}^{LM\xi}_{33}(p) &= \frac{1}{(4\pi)^2}\frac{g^2c^2}{16p^0p'^0\cos^2\theta_W}\overline{u}^{(s_3)}_{(p)}\Bigg\{m_{\nu3}\Big(G_{Z\nu3}c^2+G_{Z\nu4}s^2\Big)\Big[P_R-P_L\Big] \\
&\quad - m_{\nu3}(B_{0,Z\nu3}-\xi B^{\xi}_{0,Z\nu3})c^2 + m_{\nu4}(B_{0,Z\nu4}-\xi B^{\xi}_{0,Z\nu4})s^2\Bigg\}u^{(s_3)}_{(p')} \\
&= \frac{1}{(4\pi)^2}\frac{g^2c^2}{16p^0p'^0\cos^2\theta_W}\overline{u}^{(s_3)}_{(p)}\Bigg\{m_{\nu3}\Big(G_{Z\nu3}c^2+G_{Z\nu4}s^2\Big)\Big[P_R-P_L\Big] \\
&\quad + \frac{m_{\nu3}m_{\nu4}}{m_{\nu3}+m_{\nu4}}\Big(B_{0,Z\nu4}-\xi B^{\xi}_{0,Z\nu4}-B_{0,Z\nu3}+\xi B^{\xi}_{0,Z\nu3}\Big)\Bigg\}u^{(s_3)}_{(p')} \\
&= \frac{1}{(4\pi)^2}\frac{m_Z^2}{4v^2p^0p'^0}\frac{m_{\nu3}m_{\nu4}}{m_{\nu3}+m_{\nu4}}\overline{u}^{(s_3)}_{(p)}\Bigg\{\Big(G_{Z\nu3}+\frac{m_{\nu3}}{m_{\nu4}}G_{Z\nu4}\Big)\Big[P_R-P_L\Big] \\
&\quad + B_{0,Z\nu4}-\xi B^{\xi}_{0,Z\nu4}-B_{0,Z\nu3}+\xi B^{\xi}_{0,Z\nu3}\Bigg\}u^{(s_3)}_{(p')}.
\end{aligned} \tag{6.15}
$$

6.3 Discussing the Gauge Dependence of $\mathcal{M}$

As was shown in the previous section, there are quite a lot of ξ dependent parts in the $\mathcal{M}$ matrix. Luckily, ξ dependence in the off-diagonal matrix elements need not concern us, since these matrix elements directly contribute not to the neutrino mass corrections, but to the wavefunction, and thus can be cancelled by suitable counterterms during the neutrino wavefunction renormalization procedure.

This leaves us with only the diagonal gauge dependent terms $\mathcal{M}^{\xi}_{11}$, $\mathcal{M}^{\xi}_{22}$, and $\mathcal{M}^{\xi}_{33}$. The first two of them vanish and thus are gauge independent. What is more, the full mass correction of the first neutrino $\mathcal{M}_{11}$ is 0, i.e., it remains massless, whereas mass corrections of the second neutrino, $\mathcal{M}_{22}$, are nonzero and finite, since the infinities contained in the $B_{x,ja}$ integrals vanish. The last fact can be shown by replacing $B_{0,ja}$ in the only nonvanishing term of M_{22}, M^{SM}_{22}, by its leading part, the infinity Δ:

$$
\begin{aligned}
\mathcal{M}_{22}^{SM} &= \frac{1}{(4\pi)^2}\frac{d^2}{4p^0p'^0}\bar{u}_{(p)}^{(s_2)}\Bigg\{\sin^2\alpha\Big(m_{\nu 4}B_{0,h1\nu 4}c^2 - m_{\nu 3}B_{0,h1\nu 3}s^2\Big) \\
&\quad + \cos^2\alpha\Big(m_{\nu 4}B_{0,h2\nu 4}c^2 - m_{\nu 3}B_{0,h2\nu 3}s^2\Big) - \Big(m_{\nu 4}B_{0,HI\nu 4}c^2 - m_{\nu 3}B_{0,HI\nu 3}s^2\Big)\Bigg\}u_{(p')}^{(s_2)} \\
&\approx \frac{1}{(4\pi)^2}\frac{d^2}{4p^0p'^0}\bar{u}_{(p)}^{(s_2)}\Bigg\{\sin^2\alpha\Big(m_{\nu 4}c^2\Delta - m_{\nu 3}s^2\Delta\Big) \\
&\quad + \cos^2\alpha\Big(m_{\nu 4}c^2\Delta - m_{\nu 3}s^2\Delta\Big) - \Big(m_{\nu 4}c^2\Delta - m_{\nu 3}s^2\Delta\Big)\Bigg\}u_{(p')}^{(s_2)} \\
&= \frac{1}{(4\pi)^2}\frac{d^2}{4p^0p'^0}\bar{u}_{(p)}^{(s_2)}\Bigg\{m_{\nu 4}c^2\Delta\Big(\sin^2\alpha + \cos^2\alpha\Big) \\
&\quad - m_{\nu 3}s^2\Delta\Big(\sin^2\alpha + \cos^2\alpha\Big) - \Big(m_{\nu 4}c^2\Delta - m_{\nu 3}s^2\Delta\Big)\Bigg\}u_{(p')}^{(s_2)} \\
&= 0.
\end{aligned}
\tag{6.16}
$$

This result (zero mass corrections for the first neutrino, and nonzero, finite corrections for the second one) is consistent with the GN model.

As for $\mathcal{M}_{33}$, we are left with the following gauge dependent parts:

$$
\begin{aligned}
\mathcal{M}_{33}^{\xi W} &= \frac{1}{(4\pi)^2}\frac{|V_{a3}|^2}{2v^2p^0p'^0}\bar{u}_{(p)}^{(s_3)}\Bigg\{m_{\nu 3}\big(B_{0,\varphi+a} + B_{1,\varphi+a}\big)\Big[s^2m_D^2 + c^2m_a^2\Big] - 2scm_Dm_a^2B_{0,\varphi+a}\Bigg\}u_{(p')}^{(s_3)} \\
&\quad + \frac{1}{(4\pi)^2}\frac{g^2c^2}{2p^02p'^0}\bar{u}_{(p)}^{(s_3)}m_{\nu 3}G_{Wa}|V_{a3}|^2u_{(p')}^{(s_3)} \\
&= \frac{1}{(4\pi)^2}\frac{|V_{a3}|^2}{2v^2p^0p'^0}\frac{m_{\nu 3}m_{\nu 4}}{m_{\nu 3}+m_{\nu 4}}\bar{u}_{(p)}^{(s_3)}\Bigg\{\big(B_{0,\varphi+a} + B_{1,\varphi+a}\big)\Big[m_{\nu 3}^2 + m_a^2\Big] - 2m_a^2B_{0,\varphi+a}\Bigg\}u_{(p')}^{(s_3)} \\
&\quad + \frac{1}{(4\pi)^2}\frac{|V_{a3}|^2}{2v^2p^0p'^0}\frac{m_{\nu 3}m_{\nu 4}}{m_{\nu 3}+m_{\nu 4}}\bar{u}_{(p)}^{(s_3)}2m_W^2G_{Wa}u_{(p')}^{(s_3)} \\
&= \frac{1}{(4\pi)^2}\frac{|V_{a3}|^2}{2v^2p^0p'^0}\frac{m_{\nu 3}m_{\nu 4}}{m_{\nu 3}+m_{\nu 4}}\bar{u}_{(p)}^{(s_3)}\Bigg\{\big(B_{0,\varphi+a} + B_{1,\varphi+a}\big)\Big[m_{\nu 3}^2 + m_a^2\Big] - 2m_a^2B_{0,\varphi+a} \\
&\quad + 2m_W^2G_{Wa}\Bigg\}u_{(p')}^{(s_3)},
\end{aligned}
\tag{6.17}
$$

and

$$
\mathcal{M}_{33}^{\xi Z} = \frac{1}{(4\pi)^2}\frac{1}{4v^2p^0p'^0}\frac{m_{\nu 3}m_{\nu 4}}{(m_{\nu 3}+m_{\nu 4})^2}\bar{u}_{(p)}^{(s_3)}\Bigg\{4m_{\nu 3}^2m_{\nu 4}B_{1,\varphi z\nu 3} + \big(m_{\nu 4} - m_{\nu 3}\big)^2
$$

$$
\begin{aligned}
&\times \Big[\big(m_{\nu 3}+m_{\nu 4}\big)B_{0,\varphi z\nu 4}+m_{\nu 3}B_{1,\varphi z\nu 4}\Big]\Bigg\}u^{(s_3)}_{(p')}+\frac{1}{(4\pi)^2}\frac{m_Z^2}{4v^2p^0p'^0}\frac{m_{\nu 3}m_{\nu 4}}{m_{\nu 3}+m_{\nu 4}}\bar{u}^{(s_3)}_{(p)}\\
&\times\Bigg\{\Big(G_{Z\nu 3}+\frac{m_{\nu 3}}{m_{\nu 4}}G_{Z\nu 4}\Big)\Big[P_R-P_L\Big]+B_{0,Z\nu 4}-\xi B^{\xi}_{0,Z\nu 4}-B_{0,Z\nu 3}+\xi B^{\xi}_{0,Z\nu 3}\Bigg\}u^{(s_3)}_{(p')}\\
=&\frac{1}{(4\pi)^2}\frac{1}{4v^2p^0p'^0}\frac{m_{\nu 3}m_{\nu 4}}{(m_{\nu 3}+m_{\nu 4})^2}\bar{u}^{(s_3)}_{(p)}\Bigg\{4m_{\nu 3}^2m_{\nu 4}B_{1,\varphi z\nu 3}+\big(m_{\nu 4}-m_{\nu 3}\big)^2\\
&\times\Big[\big(m_{\nu 3}+m_{\nu 4}\big)B_{0,\varphi z\nu 4}+m_{\nu 3}B_{1,\varphi z\nu 4}\Big]+m_Z^2(m_{\nu 3}+m_{\nu 4})\Big(G_{Z\nu 3}+\frac{m_{\nu 3}}{m_{\nu 4}}G_{Z\nu 4}\Big)\\
&\times\Big[P_R-P_L\Big]+m_Z^2(m_{\nu 3}+m_{\nu 4})\Big(B_{0,Z\nu 4}-\xi B^{\xi}_{0,Z\nu 4}-B_{0,Z\nu 3}+\xi B^{\xi}_{0,Z\nu 3}\Big)\Bigg\}u^{(s_3)}_{(p')}. \qquad (6.18)
\end{aligned}
$$

$\mathcal{M}^{\xi W}_{33}$ comes from the interactions with charged particles W^+ and φ^+, and therefore, as argued in [4], is a subdominant correction and can be neglected. But $\mathcal{M}^{\xi Z}_{33}$, that comes from the interactions with neutral particles Z and φ_Z, cannot be discarded in the same way. Unfortunately, after inserting into (6.18) the definitions of the B integrals, it can be seen that the ξ dependence does not vanish, even after neglecting the part proportional to $P_R - P_L$ that contributes not to the mass corrections but to the corrections of the wavefunction.

There are many possible sources of this inconsistency with the results of the [4] paper. We think that the most probable one is the fact that the third neutrino mass is nonzero already at the tree-level, thus it is additionaly subject to the corrections due to the counterterms of its wavefunction renormalization. Including wavefunction renormalization into our model would have required a significant amount of additional work from the very beginning, thus it was decided not to do that at the time the work began. It may be that the problem lies exactly in this decision. Nevertheless, this is just a motivated guess — determining the real source of the inconsistency will require a great deal of further investigation.

7 Results and Conclusions

In this master's thesis:

- a connection between our generic model (introduced in [6]) and the Standard Model extended by a right-handed neutrino field and a second Higgs doublet (called the Grimus-Neufeld model in this work) has been made;

- one-loop mass corrections of the neutrino fields have been investigated. The following things have been determined:

  1. the mass corrections $\mathcal{M}_{11}$ of the first neutrino are equal to zero, i.e., the first neutrino remains massless in this model;

  2. the mass corrections $\mathcal{M}_{22}$ are nonzero and finite.

  These results are consistent with the Grimus-Neufeld model of [3].

- the gauge dependence of the mass corrections has been investigated:

  1. $\mathcal{M}_{11}$ is gauge independent, since it vanishes;

  2. $\mathcal{M}_{22}$ is also gauge independent;

  3. the gauge dependence of $\mathcal{M}_{33}$ does not vanish.

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Dmitrij Chomčik

LENGVŲJŲ NEUTRINŲ SKLIDIMO INTEGRALO PRIKLAUSOMYBĖS NUO KALIBRAVIMO PARAMETRO NUSTATYMAS VIENOS KILPOS ARTINYJE

Santrauka

Standartiniame dalelių fizikos modelyje neutrinai yra bemasės dalelės. Tačiau šiuo metu jau yra eksperimentiškai nustatyta, kad neutrinai privalo turėti nors ir labai mažas, bet nenulines mases [1]: dabartinė neutrinų masių sumos riba yra nustatyta ties 0.12 eV (95% patikimumas) [2]. Palyginimui, sekančios pagal lengvumą Standartinio modelio dalelės — elektrono — masė yra lygi ~511 keV, taigi 6 eilėmis didesnė. Šio didžiulio masių skirtumo priežastis yra aktualus šiuolaikinės dalelių fizikos klausimas.

Vienas iš plačiausiai paplitusių paaiškinimų yra vadinamasis “sūpuoklės“ mechanizmas (angl. *seesaw*), turintis keletą versijų. Vienoje iš jų Standartinis modelis (SM) yra praplečiamas dar vienu neutrinų lauku ir antruoju Higso dubletu [3]. Tokiame modelyje (pavadinkime jį Grimus-Neufeld (GN) modeliu) du neutrinai natūraliai įgauna labai mažas mases (1 eV eilės), vienas — labai didelę (galimai ~10^{14} GeV eilės), o dar vienas neutrinas lieka bemasiu. Kadangi šie rezultatai neprieštarauja eksperimentiniams stebėjimams, o pats GN modelis praplečia Standartinį modelį labai nežymiai, jis gali būti laikomas viena iš daugiausiai žadančių “sūpuoklės“ mechanizmo atmainų.

[4] šaltinyje yra parodoma, kad GN modelyje vienas iš neutrinų įgauna masę per *vienos kilpos pataisas*. Tai reiškia, kad vienos kilpos pataisų išraiškos (ir, atitinkamai, GN modelio parametrai) gali būti susietos su eksperimentiškai išmatuojamais dydžiais (neutrinų masių kvadratų skirtumais), kas potencialiai gali leisti ateityje patikrinti šio modelio spėjimus jam būdinguose procesuose (pavyzdžiui, su antruoju Higso dubletu, jeigu jis kada nors bus aptiktas). Tačiau visų pirma mes turime įsitikinti, kad modelis nėra prieštaringas.

Vienas iš akivaizdžiausių kalibravimo teorijos prieštaringumo patikrinimų yra jos *kalibravimo invariantiškumas*. Modelio prognozės bet kokiam eksperimentiškai išmatuojamam dydžiui neturi priklausyti nuo laisvai parenkamos kalibravimo parametro vertės. [4] darbe autoriai parodo, kad jų gaunamos vienos kilpos pataisų išraiškos yra invariantiškos kalibravimo atžvilgiu. Šio darbo tikslas buvo patikrinti šį rezultatą naudojant kitokį priėjimą.

Šis magistro baigiamasis darbas užbaigia tai, kas buvo pradėta ir vystoma trijuose mano ankstesniuose kursiniuose darbuose. Pirmasis iš jų [5] yra įžanga į kvantinę lauko teoriją ir, konkrečiau, Feinmano sklidimo integralo sąvoką. Antrajame [6] buvo išnagrinėti spinorai ir kalibravimo invariantiškumas, bei suskaičiuotos neutrinų vienos kilpos pataisos mūsų formalizme, nepririštame prie jokių konkrečių dalelių laukų. Trečiasis kursinis darbas [7] susiejo šį formalizmą su Standartiniu modeliu, praplėstu vienu papildomu neutrinų lauku. Ir, galiausiai,

šio magistrinio darbo tikslas buvo (i) praplėsti mūsų modelį antruoju Higso dubletu, kad jo ir GN modelio aprašomi laukai sutaptų, bei (ii) patikrinti, ar neutrinų vienos kilpos pataisos, suskaičiuotos mūsų formalizme, nepriklauso nuo kalibravimo parametro.

Šie tikslai buvo įvykdyti: 2 skyriuje antrasis Higso dubletas yra pristatomas, o 4 skyriuje — susiejamas su mūsų modeliu; 6 skyriuje yra tiriama vienos kilpos pataisų priklausomybė nuo kalibravimo parametro. Nustatyta, kad pirmojo ir antrojo neutrinų masių pataisos nuo jo nepriklauso, kaip ir buvo tikėtasi remiantis [4] darbu. Tačiau trečiojo neutrino masių pataisose kalibravimo parametras išliko. 6 skyriaus pabaigoje yra trumpai aptariamos galimos tokio rezultato priežastys.