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Summary of Doctoral Dissertation

Scientific Problem and Research Object

The research object of the dissertation is a set of strongly additive functions such that $f_x(p) \in \{0, 1\}$ for all primes p and all $x \geqslant 2$. It follows from the strong additivity that for every positive integer n

$$f_x(n) = \sum_{p|n} f_x(p) = \sum_{\substack{p|n \\ f_x(p)=1}} 1 = \sum_{p|n}^{f} 1.$$

Scientific problem is studying of the weak convergence of the distributions for additive functions on shifted primes, for the sum of additive functions with shifted arguments and for the sum of additive functions on shifted primes to the discrete uniform law $\mathcal{U}(u, L)$.

Aims and Problems

The aim of the dissertation is to consider the cases of a weak convergence of distributions of a set of strongly additive functions to the discrete uniform law $\mathcal{U}(u, L)$, more precisely, to solve the following problems:

- to obtain the sufficient and necessary conditions for a weak convergence of distributions of a set of strongly additive functions the arguments of which run through shifted primes to the discrete uniform law;
- to obtain the sufficient and necessary conditions for a weak convergence of distributions of a sum of sets of strongly additive functions on shifted arguments to the discrete uniform law;
- to obtain the sufficient and necessary conditions for a weak convergence of distributions of a sum of sets of strongly additive functions on shifted primes to the discrete uniform law.

Actuality

The idea to consider the limit behaviour of a sum of additive functions with shifted arguments is not new. Almost all classical results and their historical context can be found in the books of P.D.T.A. Elliott [3], [4] and J. Kubilius [14] . The first results of the limit theorems of additive functions were obtained by P. Erdős, A. Wintner [9] (1939) and P. Erdős, M. Kac [8] (1940). The first result of the limit behaviour of the sum of additive functions belongs to W.J. LeVeque [15] (1949). More general results later were established by J. Kubilius [14], G. Halász [10], I. Kátai [13], A. Hildebrand [11], P.D.T.A. Elliott [3]–[6] and by some others (see [21], [23], [27], [28], [29], [32]). In these works, the different classes of additive functions were considered. The cases when values of additive functions could be taken on different arithmetic progressions, on shifted primes, as well as the number of additive functions as summands could slowly increase together with x were examined.

The cases when the values of additive functions were taken on shifted primes were considered by M.B. Barban, A.I. Vinogradov, B.V. Levin [1], I. Kátai [12] and by others (see [4], [20], [22], [30], [31]).

All these results on weak convergence were given by using different methods. Among them, elementary methods, sieve methods, the method of characteristic functions, the method of factorial moments, and the Kubilius model of probability spaces were used.

In the articles [16]–[23] the case of the Poisson distribution as a limit law was considered by J. Šiaulys and G. Stepanauskas. The Bernoulli, geometrical, binomial, discrete uniform distributions as limit laws were investigated in [24] - [26].

The discrete uniform law as the limit distribution for additive functions on shifted primes, for the sum of additive functions with shifted arguments and for the sum of additive functions on shifted primes has been never considered.

The obtained results describe the limit behaviour of the sets of additive functions and can be applied in mathematical research that require knowledge of the asymptotic behaviour of the additive functions.

Methods

In the proofs of the obtained results we used the method of factorial moments and the method of characteristic functions.

Novelty

All results presented in the dissertation are new. The results have theoretical aspects. Discrete uniform law as the limit distribution for additive functions on shifted primes, for the sum of additive functions with shifted arguments and for the sum of additive functions on shifted primes were proved first time.

Structure of Dissertation

The dissertation is written in Lithuanian. It consists of introduction, four sections, conclusions and the list of references. The introduction provides aim and problems of the dissertation, research object, methods, approbation and publications of the dissertation results. The first chapter describes classical results and historical context of additive functions on shifted arguments. Other three chapters provides dissertation research results and mathematical proofs. The size of the work is 67 pages.

Review and Main Results

In the first part of the work the sufficient and necessary conditions for a weak convergence of distributions of a set of strongly additive functions on shifted primes

$$\nu_x(p \leqslant x, f_x(p+1) < u) = \frac{1}{\pi(x)} \sum_{\substack{p \leqslant x \\ f_x(p+1)<u}} 1 \tag{1}$$

to the discrete uniform law

$$\mathcal{U}(u, L) := \sum_{\substack{k=0,1,\dots,L-1 \\ k<u}} \frac{1}{L}\,, \tag{2}$$

where the parameter $L \in \mathbb{N},\ L \geqslant 2$, were obtained.

Theorem 1 *Let f_x, $x \geqslant 2$, be a set of strongly additive functions. Assume that $f_x(p) \in \{0,1\}$ for all prime numbers p and*

$$\lim_{x\to\infty} \log x \sum_{x^\gamma < p \leqslant x}^{f} \frac{1}{p} = 0 \tag{H}$$

for all $\gamma \in (0,1)$*. The distributions* $\nu_x(f_x(p+1) < u)$ *converge weakly to the limit discrete uniform law* $\mathcal{U}(u,L)$*, as* $x \to \infty$*, if and only if* $L = 2$ *and*

$$f_x(3) = 1, \quad \lim_{x\to\infty} \sum_{\substack{p\leqslant x\\ p\neq 3}}{}^{f} \frac{1}{p} = 0. \tag{3}$$

The proof of this theorem is based on the following lemmas on the concentration of additive functions on shifted primes and on the limit behaviour of factorial moments of the distributions $\nu_x(f_x(p+1) < u)$.

Lemma 2 ([7]) *For every real valued additive function* h*, every real number* b *and every integer* a

$$\nu_x(\ h(p+a) = b) \ll \left(4 + \sum_{\substack{p\leq x\\ h(p)\neq 0}} \frac{1}{p}\right)^{-1/2}.$$

Lemma 3 *Let* f_x*,* $x \geqslant 2$*, be a set of strongly additive functions such that* $f_x(p) \in \{0,1\}$ *for all primes* p*. If distributions* (1) *converge weakly to some distribution function* $F(u)$ *with a jump at the point* $u = 0$*, as* $x \to \infty$*, then the quantities*

$$\beta(l,x) := \frac{1}{\pi(x)} \sum_{p\leqslant x} f_x(p+1)(f_x(p+1)-1)\dots(f_x(p+1)-l+1), \quad l = 1,2,\dots,$$

have finite limits

$$\lim_{x\to\infty} \beta(l,x) = g_l, \tag{4}$$

where g_l *is the* l^{th} *factorial moment of the limit law.*

Lemma 4 ([20], Lemma 2) *If a set of strongly additive functions* f_x *satisfies the conditions of Theorem* 1 *and*

$$\sum_{p\leqslant x}{}^{f} \frac{1}{p} \ll 1, \tag{5}$$

then

$$\beta(l,x) = \sum_{\substack{p_1,p_2,\dots,p_l\leqslant x\\ p_i\neq p_j,\, i\neq j}}{}^{f} \frac{1}{(p_1-1)(p_2-1)\dots(p_l-1)} + \varepsilon_l(x), \ l = 1,2,\dots$$

According to this statement and equality (4) in the case of convergence of $\nu_x(f_x(p+1) < u)$, we have that

$$\lim_{x\to\infty} \sum_{\substack{p_1,p_2,\dots,p_l\leqslant x\\ p_i\neq p_j,\, i\neq j}}{}^{f} \frac{1}{(p_1-1)(p_2-1)\dots(p_l-1)} = g_l. \tag{6}$$

for each $l \in \{1,2,\dots\}$.

Lemma 5 *Let f_x, $x \geqslant 2$, be a set of strongly additive functions such that $f_x(p) \in \{0,1\}$ for all primes p and condition (H) hold. If distributions* (1) *converge weakly to the distribution F_ξ of the random variable ξ with a finite support $\{0,1,\dots,L-1\}$, $L \geqslant 2$, then there exists some constant $D \geqslant 2$ such that*

$$\limsup_{x\to\infty} \#\{p \leqslant D : f_x(p) = 1\} \leqslant L-1, \tag{7}$$

$$\lim_{x\to\infty} \sideset{}{^f}\sum_{D<p\leqslant x^{1/L}} \frac{1}{p} = 0, \tag{8}$$

$$\lim_{x\to\infty} \sideset{}{^f}\sum_{\substack{p_1,\dots,p_l\leqslant D\\ p_i\neq p_j,\, i\neq j}} \frac{1}{(p_1-1)(p_2-1)\dots(p_l-1)} = g_l, \quad l = 1,2,\dots,L-1. \tag{9}$$

Moreover, the characteristic function of the limit distribution F_ξ is equal to

$$1 + \sum_{l=1}^{L-1} \frac{g_l}{l!}(\mathrm{e}^{it}-1)^l.$$

In the second part of the work the sufficient and necessary conditions for a weak convergence of distributions of a sum of sets of strongly additive arithmetic functions with shifted arguments

$$\nu_x(n \leqslant x, f_x(n) + g_x(n+1) < u) = \frac{1}{[x]} \sum_{\substack{n\leqslant x\\ f_x(n)+g_x(n+1)<u}} 1 \tag{10}$$

to the discrete uniform law $\mathcal{U}(u,L)$, where $L \in \mathbb{N},\ L \geqslant 2$, were obtained.

Theorem 6 *Let f_x and g_x, $x \geqslant 2$, be two sets of strongly additive functions such that $f_x(p), g_x(p) \in \{0,1\}$ for all primes p. Let, in addition,*

$$\lim_{x\to\infty} \frac{1}{\log x} \sum_{p\leqslant x} \frac{(f_x(p)+g_x(p))\log p}{p} = 0. \tag{11}$$

Distributions (10) *converge weakly to the discrete uniform law $\mathcal{U}(u,L)$, as $x \to \infty$, if and only if $L = 2$ and*

$$f_x(2) + g_x(2) = 1, \tag{12}$$

$$\lim_{x\to\infty} \sum_{2<p\leqslant x} \frac{f_x(p)+g_x(p)}{p} = 0. \tag{13}$$

The proof of this theorem is based on the following assertions on the concentration of a sum of additive functions and on the limit behaviour of factorial moments of the distributions $\nu_x(f_x(n)+g_x(n+1)<u)$.

Lemma 7 ([21]) *Let f_x and g_x, $x \geqslant 2$, be two sets of strongly additive functions such that $f_x(p), g_x(p) \in \{0,1\}$ for all primes p. If distributions (10) converge weakly to some distribution function, as $x \to \infty$, then the quantities*

$$\phi(x,l) := \frac{1}{x}\sum_{n\leqslant x}\prod_{k=0}^{l-1}(f_x(n)+g_x(n+1)-k), \quad l=1,2,\ldots,$$

have the finite limits

$$\lim_{x\to\infty}\phi(x,l) =: \phi_l, \tag{14}$$

and ϕ_l is the l^{th} factorial moment of the limit law. On the contrary, if (14) holds for every fixed positive integer l and the series

$$\sum_{l=1}^{\infty}\frac{2^l\phi_l}{l!}$$

converges, then distribution functions (10) converge weakly to some distribution with the characteristic function

$$1+\sum_{l=1}^{\infty}\frac{\phi_l}{l!}\left(e^{it}-1\right)^l. \tag{15}$$

Lemma 8 ([10]) *Let h be an arbitrary real valued additive function. Then*

$$\sum_{\substack{n\leqslant x\\ h(n)=a}} 1 \ll x\left(\sum_{\substack{p\leqslant x\\ h(p)\neq 0}}\frac{1}{p}\right)^{-1/2}$$

holds uniformly for all real numbers a and $x \geqslant 2$.

Lemma 9 *Let f_x and g_x, $x \geqslant 2$, be two sets of strongly additive functions such that $f_x(p), g_x(p) \in \{0,1\}$ for all primes p and let condition (11) be satisfied. If distributions (1) converge weakly to some limit law, then*

$$\lim_{x\to\infty}\sum_{k=0}^{l}\binom{l}{k}\sum_{\substack{p_1,\ldots,p_k,p_{k+1},\ldots,p_l\leqslant x\\ p_i\neq p_j, i\neq j}}\frac{f(p_1)\ldots f(p_k)g(p_{k+1})\ldots g(p_l)}{p_1\ldots p_k p_{k+1}\ldots p_l} = \phi_l \tag{16}$$

for all fixed positive integers l. Moreover, the limit distribution has characteristic function (15).

Lemma 10 *Let f_x and g_x, $x \geqslant 2$, be two sets of strongly additive functions such that $f_x(p), g_x(p) \in \{0,1\}$ for all primes p and condition* (11) *hold. If distributions* (1) *converge weakly to the distribution F_ξ with the finite support $\{0,1,2,\dots,L-1\}$, $L \geqslant 2$, then there exists some constant $D \geqslant 2$ such that*

$$\limsup_{x\to\infty} \#\{p \leqslant D : f_x(p) + g_x(p) \neq 0\} \leqslant L-1, \tag{17}$$

$$\lim_{x\to\infty} \sum_{D<p\leqslant x} \frac{f_x(p)+g_x(p)}{p} = 0, \tag{18}$$

$$\lim_{x\to\infty} \sum_{k=0}^{l} \binom{l}{k} \sum_{\substack{p_1,\dots,p_k,p_{k+1},\dots,p_l\leqslant D\\ p_i\neq p_j, i\neq j}} \frac{f(p_1)\dots f(p_k)g(p_{k+1})\dots g(p_l)}{p_1\dots p_k p_{k+1}\dots p_l} = \phi_l, \tag{19}$$

$l = 1,2,...,L-1$. *Moreover, the characteristic function of the limit distribution F_ξ has form* (15) *with $\phi_l = 0$ for $l \geqslant L$.*

In the third part of the work the sufficient and necessary conditions for a weak convergence of distributions of a sum of sets of strongly additive arithmetic functions with shifted primes

$$\nu_x(p \leqslant x, f_x(p+1) + g_x(p+2) < u) = \frac{1}{\pi(x)} \sum_{\substack{p\leqslant x\\ f_x(p+1)+g_x(p+2)<u}} 1 \tag{20}$$

to the discrete uniform law $\mathcal{U}(u,L)$, where $L \in \mathbb{N}, L \geqslant 2$, were obtained.

Theorem 11 *Let f_x and g_x, $x \geqslant 2$, be two sets of strongly additive functions. Assume that $f_x(p), g_x(p) \in \{0,1\}$ for all prime numbers p and*

$$\lim_{x\to\infty} \log x \sum_{x^\gamma<p\leqslant x} \frac{f_x(p)+g_x(p)}{p} = 0 \tag{21}$$

for every $\gamma \in (0,1)$. The distributions

$$\nu_x(f_x(p+1) + g_x(p+2) < u)$$

converge weakly to the discrete uniform law $\mathcal{U}(u,L)$ as $x \to \infty$, if and only if $L = 2$,

$$\lim_{x\to\infty} \sum_{\substack{p\leqslant x\\ p\neq 3,5}} \frac{f_x(p)+g_x(p)}{p} = 0, \tag{22}$$

and, for every sufficiently large x, either

$$f_x(3) + g_x(3) = 1 \text{ and } f_x(5) + g_x(5) = 0 \tag{23}$$

or

$$f_x(3) + g_x(3) = 0 \text{ and } f_x(5) + g_x(5) = 2. \tag{24}$$

The proof of this theorem is based on the following lemmas.

Lemma 12 ([2]) *Let K be a positive real number. Then there exists a further real number L such that uniformly for all $x \geq 2$*

$$\sum_{d \leq x^{1/2}(\log x)^{-L}} \max_{(d,v)=1} \left| \pi(x,d,v) - \frac{\operatorname{li} x}{\phi(d)} \right| \ll \frac{x}{(\log x)^K}.$$

Lemma 13 ([22]) *Let f_x and g_x, $x \geq 2$, be two sets of strongly additive functions such that $f_x(p)$, $g_x(p) \in \{0,1\}$ for all primes p and*

$$\sum_{p \leqslant x} \frac{f_x(p) + g_x(p)}{p} \leqslant c. \tag{25}$$

For positive integers l let

$$\beta(l,x) := \frac{1}{\pi(x)} \sum_{p \leq x} (f_x(p+1) + g_x(p+2)) \\ \times (f_x(p+1) + g_x(p+2) - 1) \dots (f_x(p+1) + g_x(p+2) - l + 1).$$

Then

$$\beta(l,x) \ll_{l,c} 1.$$

Lemma 14 *Let f_x and g_x, $x \geqslant 2$, be two sets of strongly additive functions such that $f_x(p), g_x(p) \in \{0,1\}$ for all primes p and let inequality (25) be satisfied. If distributions (20) converge weakly to some distribution function $F(u)$ as $x \to \infty$, then*

$$\lim_{x \to \infty} \beta(l,x) = \beta_l, \quad l = 1, 2, \dots, \tag{26}$$

where β_l is the l^{th} factorial moment of the limit law.

Lemma 15 *Let sets of strongly additive functions f_x and g_x satisfy the conditions of Theorem* 1 *and let inequality* (25) *hold. If distributions* (20) *converge weakly to some distribution function $F(u)$ as $x \to \infty$, then*

$$\beta_l = \lim_{x\to\infty} \sum_{k=0}^{l} \binom{l}{k} \sum_{\substack{p_1,p_2,\ldots,p_l\le x\\ p_i\neq p_j, i\neq j}} \frac{f_x(p_1)\ldots f_x(p_k)g_x(p_{k+1})\ldots g_x(p_l)}{\phi(p_1p_2\ldots p_l)}. \tag{27}$$

Lemma 16 *Let f_x and g_x, $x \geqslant 2$, be two sets of strongly additive functions such that $f_x(p) \in \{0,1\}$ for all primes p and let condition* (21) *hold. If distributions* (20) *converge weakly to the distribution F_ξ of the random variable ξ with a finite support $\{0,1,\ldots,L-1\}$, $L \geqslant 2$, then there exists some constant $D \geqslant 2$ such that*

$$\limsup_{x\to\infty} \#\{p \leqslant D : f_x(p) + g_x(p) \neq 0\} \leqslant L-1, \tag{28}$$

$$\lim_{x\to\infty} \sum_{D<p\leqslant x} \frac{f_x(p)+g_x(p)}{p} = 0, \tag{29}$$

$$\lim_{x\to\infty} \sum_{k=0}^{l} \binom{l}{k} \sum_{\substack{p_1,\ldots,p_k,p_{k+1},\ldots,p_l\leqslant D\\ p_i\neq p_j, i\neq j}} \frac{f(p_1)\ldots f(p_k)g(p_{k+1})\ldots g(p_l)}{\phi(p_1p_2\ldots p_l)} = \beta_l, \tag{30}$$

$l = 1,2,\ldots,L-1$. *Moreover, the characteristic function of the limit distribution F_ξ is equal to*

$$1 + \sum_{l=1}^{L-1} \frac{\beta_l}{l!}(\mathrm{e}^{it}-1)^l.$$

Conclusions

In the dissertation, the following results are established:

- the sufficient and necessary conditions for a weak convergence of distributions of a set of strongly additive functions on shifted primes to the discrete uniform law are obtained;

- the sufficient and necessary conditions for a weak convergence of distributions of a sum of sets of strongly additive functions on shifted arguments to the discrete uniform law are obtained;

- the sufficient and necessary conditions for a weak convergence of distributions of a sum of sets of strongly additive functions on shifted primes to the discrete uniform law are obtained.

Approbation

The results of the dissertation were presented at the Sixth International Conference "Analitic and Probabilistic Methods in Number Theory"(Palanga, Lithuania, 2016) and at the Conferences of Lithuanian Mathematical Society (2014, 2015, 2016).

Principal Publications

1. G. Stepanauskas, L. Žvinytė, Discrete uniform limit law for additive functions on shifted primes, *Nonlinear Anal. Model. Control*, **21**(4):437-447, 2016.

2. G. Stepanauskas, L. Žvinytė, Discrete uniform distribution for a sum of additive functions, *Lith. Math. J*, **57**(3):391-400, 2017.

3. J. Šiaulys, G. Stepanauskas, L. Žvinytė, Discrete uniform limit law for a sum of additive functions on shifted primes, *Lith. Math. J.*, 2017, in print.

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Reziumė

Šiame darbe nagrinėtas stipriai adityviųjų funkcijų sekų f_x skirstinių silpnas konvergavimas į ribinį diskretų tolygųjį dėsnį. Čia $f_x(p)$ įgyja reikšmę 0 arba 1 bet kuriam pirminiam skaičiui p ir visiems $x \geqslant 2$.

Darbą sudaro įvadas, keturi skyriai, išvados, naudotos literatūros sąrašas.

Įvade pateikiamas darbo tikslas ir uždaviniai, nagrinėtos problemos aktualumas, suformuluoti pagrindiniai darbo rezultatai, publikacijų disertacijos tema sąrašas. Pirmame skyriuje aptariami stipriai adityviųjų funkcijų su paslinktais argumentais ribinio elgesio klasikiniai rezultatai ir jų istorinis kontekstas. Kituose trijuose skyriuose suformuluotos ir įrodytos stipriai adityviųjų funkcijų sekų su paslinktais pirminiais, sumų su paslinktais argumentais bei sumų su paslinktais pirminiais skirstinių silpnojo konvergavimo į diskretų tolygųjį skirstinį būtinos ir pakankamos sąlygos.

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