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Letter

Faddeev-type calculation of nonelastic breakup in deuteron-nucleus scattering

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ABSTRACT

The nonelastic breakup (NEB), one of channels in (d, p) inclusive reactions, is studied using the Faddeev-type scattering theory. The NEB differential cross section is obtained in terms of the imaginary part of the neutron-nucleus optical potential sandwiched between the Alt-Grassberger-Sandhas three-body transition operators. The momentum-space calculations including the Coulomb force are extended to higher charge numbers. Well converged numerical results are obtained for the energy distribution of the NEB cross section, being roughly consistent with previous works. The spin-dependent interaction terms do not play a significant role. The optical potential nonlocality effect shows up at higher proton energies, but is comparable to local potential uncertainties.

1. Introduction

The collisions of two nuclei are highly complicated many-body problems, whose complexity led to the development of simplified approaches with rather few effective degrees of freedom. The simplest of them is the introduction of optical potentials that by an appropriate fit of parameters can be quite successful in reproducing the elastic scattering observables and total reaction cross section for two colliding nuclei. The picture becomes more complicated if at least one of the involved nuclei is weakly bound, such as the deuteron (d) or halo nucleus. The need to account for the breakup possibility leads to an effective three-body problem at least. For the deuteron induced reactions the active degrees of freedom are the proton (p), neutron (n) and the involved nucleus A, that is most often treated as an inert particle, whereas its compositeness is effectively accounted for via nucleon-nucleus optical potentials. Within such a model space the elastic scattering, deuteron breakup and stripping (nucleon transfer) reactions have been calculated using a number of approximate [1,2] and rigorous three-body methods [3,4]. All these calculations refer to a class of processes where the nucleus A remains in its ground state. The inclusion of few lowest excited states and the respective reaction channels have been achieved by formulation of the problem in an extended Hilbert space [5–8].

However, attempts have also been made to calculate the reactions beyond the explicitly considered model space, i.e., when following the deuteron breakup $d + A \to p + X$ only one of the nucleons (proton) is detected, while another nucleon and the nucleus A can be in any state X, including disintegration of A. It is called the inclusive (d, p) breakup with contributions from several different reaction mechanisms: the elastic breakup (EB), nonelastic breakup (NEB), and preequilibrium and compound nucleus (PE + CN). The latter typically is modeled using the statistical Hauser-Feshbach theory [9] and is beyond the scope of the present work. The EB is the standard three-body breakup with A remaining in its ground state, while NEB includes the breakup with simultaneous excitation of A and inelastic $n + A$ processes. Both in EB and NEB three particles $(A + p + n)$ are involved explicitly, thus, the three-body calculations at least are needed.

Under the assumption that the detected particle acts as a spectator, i.e., predominantly scatters from A elastically, several approaches by Udagawa and Tamura [10], Kasano and Ichimura [11], Hussein and McVoy [12], and Ichimura, Austern and Vincent [2,13] have been proposed to calculate the NEB semi-inclusive differential cross section for the detected particle. All of them explicitly include only the ground state of the nucleus A and rely on the closure method to sum up implicitly all final states of the subsystem X. Although a formal derivation of the scattering wave functions has been performed also using rigorous Faddeev theory [14], most practical calculations, e.g., by Jia et al. [9], Moro and Lei [15,16], Potel et al. [17], Liu et al. [18], Torabi and Carlson [19] so far used three-body scattering wave functions based on the distorted-wave Born-approximation (DWBA) or Glauber model. Recently also the continuum-discretized coupled-channel method has been used [20,21]. In particular, Ref. [16] pointed out that “the solution of the Faddeev equations is too complicated for many practical applications and, even if this solution is available, its implementation for NEB will be very challenging.”

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Experiments performed for the inclusive deuteron breakup, e.g., [22,23], aimed to determine the yield of various particles (p, n, α, ...). The collisions of deuterons with nuclei, possible both in direct and inverse kinematics, also enable an indirect study of nucleons capture by nuclei; see Ref. [17] for a recent overview. Theoretical and experimental studies of the inclusive (d,p) processes revealed that at low proton energies the PE + CN mechanism totally dominates the differential cross section with small contributions from EB and NEB. This kinematic condition implies also low relative proton-nucleus energy, corresponding to long interaction time in the classical picture, and is incompatible with the proton spectator assumption. In contrast, at medium to highest proton energies the PE + CN mechanism yields very small cross section whereas the NEB dominates, with a moderate contribution of EB as well. The theoretical predictions [9,15–19] are successful in reproducing the experimental data qualitatively, while at the quantitative level some discrepancies remain, also between different theoretical approaches for NEB. This calls for new independent methods that could possibly resolve the existing disagreements.

Given previous successful practical applications of the Faddeev-type theory to elastic, inelastic, transfer, and breakup reactions in three-body systems [7,24], its extension to NEB appears to be relevant and timely. In contrast to Ref. [14] that considered wave functions, in this work the Alt, Grassberger, and Sandhas (AGS) equations [4] for three-body transition operators will be employed; the calculations proceed in the momentum space. This framework offers one more advantage, namely, a convenient inclusion of nonlocal optical potentials that have a significant impact in one-nucleon transfer reactions [25,26] and deuteron inelastic scattering [27]. The coordinate space approaches can include the nonlocality via integro-differential equations as well, but this has not been performed yet in the context of NEB. Thus, after establishing the calculational scheme for NEB, the optical potential nonlocality effect will be evaluated as well.

Another key feature of momentum-space calculations [25–27] is the inclusion of the Coulomb force via the method of screening and renormalization [28–31]. It works very well for light nuclei but becomes tedious for high charges or at very low energies, such that practical applications are limited so far to ^{58}Ni [24]. An extension to higher charge numbers is desirable, especially in the view of NEB where majority of the data refer to heavier nuclei.

Section 2 contains the AGS three-body scattering formalism. It shows how the extended version simulates the processes beyond the standard three-body space and includes NEB. The Appendix A presents an alternative derivation of the same result using a many-body approach. Section 3 contains the results, while Section 4 collects conclusions.

2. Three-body transition operators and NEB

The AGS equations [4] represent the integral equation formulation of the three-body Faddeev scattering theory [3] for transition operators $U_{\gamma\beta}$ that are directly related to scattering amplitudes; Greek subscripts label the initial and final spectator particle (or the remaining pair in the odd-man-out notation), whereas $\gamma = 0$ stands for three free particles. In a number of previous works AGS equations have been used for the description of reactions with three inert particles [24,25] while later extensions include also the dynamical excitations of the core nucleus [7,26,27]. Notably, both versions can be cast into the same form of equations, i.e.,

$$U_{\gamma\beta} = (1-\delta_{\gamma\beta})\, G_0^{-1} + \sum_{\alpha=1}^{3} (1-\delta_{\alpha\gamma})\, t_\alpha\, G_0 U_{\alpha\beta} \tag{1}$$

where G_0 is the free resolvent, and the pairwise interaction potentials v_α enter via the respective two-particle transition operators

$$t_\alpha = v_\alpha + v_\alpha G_0\, t_\alpha. \tag{2}$$

If excitations of the nucleus A are allowed, all operators in Eq. (1) become multicomponent operators acting in an extended Hilbert space with multiple sectors [7] corresponding to different internal states of the nucleus A. The basis states $|\mathbf{p}_\alpha \mathbf{q}_\alpha a\rangle \equiv |\mathbf{p}_\alpha\rangle \otimes |\mathbf{q}_\alpha\rangle \otimes |a\rangle$ for the relative motion are characterized by Jacobi momenta for the pair ($\mathbf{p}_\alpha$) and spectator ($\mathbf{q}_\alpha$), plus the label a for the internal state of the nucleus A with the excitation energy E_a, i.e., $E_a = 0$ for the ground state. While previous studies with explicit core excitation included only few lowest bound states of A, formally also the continuum states corresponding to the breakup of A can be discretized and included into the set $\{a\}$ as pseudostates. With a finite but sufficiently large number of such states (pseudostates) one may expect to account accurately for the A continuum. In contrast, no discretization is used for the $A+p+n$ three-body continuum, described by two continuous variables, the Jacobi momenta $\mathbf{p}_\alpha$ and $\mathbf{q}_\alpha$. The explicit multicomponent equations can be obtained from Eq. (1) inserting the completeness relation

$$1 = \sum_a \int d^3\mathbf{p}_\alpha d^3\mathbf{q}_\alpha\, |\mathbf{p}_\alpha \mathbf{q}_\alpha a\rangle\langle \mathbf{p}_\alpha \mathbf{q}_\alpha a|. \tag{3}$$

The neutron-proton interaction and the multicomponent free resolvent are diagonal with respect to the different Hilbert sectors, e.g.,

$$\langle a'|G_0|a\rangle = \delta_{a'a}\,(E + i0 - H_0 - E_a)^{-1}. \tag{4}$$

Here E is the available system energy in the center-of-mass (c.m.) frame and H_0 the kinetic energy operator for the relative motion of the three clusters $A+p+n$, with eigenvalues $p_\alpha^2/2\mu_\alpha + q_\alpha^2/2M_\alpha$, where μ_α and M_α are the pair and spectator reduced masses. In contrast, the real nucleon-nucleus potentials v_α couple the different Hilbert sectors, with nonvanishing $\langle a'|v_\alpha|a\rangle$ for any combination of a' and a. As a consequence, the nucleon-nucleus transition operators in Eq. (2) and three-body transition operators in Eq. (1) couple the different Hilbert sectors. Though the above equations are of the three-body type, the many-body character of the problem resides in the multicomponent nucleon-nucleus potentials $\langle a'|v_\alpha|a\rangle$, whose microscopic calculation would require the solution of the many-body problem. The solution of the multicomponent three-body Eq. (1) may be also highly challenging, if large number of states a is included. However, both difficulties can be avoided in the calculations of NEB cross section as will be shown in the following. The breakup operator is a special case of Eq. (1) with $\gamma = 0$, i.e.,

$$U_{0\beta} = G_0^{-1} + \sum_{\alpha=1}^{3} t_\alpha\, G_0 U_{\alpha\beta} \tag{5a}$$

$$= \delta_{\beta\alpha} G_0^{-1} + (1 + t_\alpha\, G_0) U_{\alpha\beta}, \tag{5b}$$

whereas the last equation is valid for any $\alpha = 1, 2$, or 3. For the calculation of physical observables the operator (5) has to be sandwiched between the initial and final channel states. The reaction is initiated by the collision of particle 1 and the bound pair of particles (23) with energy $\epsilon_1 < 0$, the initial relative momentum being $\mathbf{q}_1^i$ and the available energy $E = (q_1^i)^2/2M_1 + \epsilon_1$. Thus, the initial channel state is given by the direct product of the bound-state wave function $|\phi_1\rangle$ and plane wave $|\mathbf{q}_1^i\rangle$, A being in its ground state. The final breakup state could be expressed in any of the Jacobi configurations, but for the detected particle α the most convenient choice is $|\mathbf{p}_\alpha \mathbf{q}_\alpha a\rangle$. The corresponding semi-inclusive differential cross section is obtained from the fully exclusive one [32] via summation and integration over all states of the undetected particles, i.e.,

$$\frac{d^3\sigma_b}{d^3\mathbf{q}_\alpha} = (2\pi)^4 \frac{M_1}{f_s\, q_1^i} \sum_a \int d^3\mathbf{p}_\alpha\, \delta\!\left(E - \frac{p_\alpha^2}{2\mu_\alpha} - \frac{q_\alpha^2}{2M_\alpha} - E_a\right) \times |\langle \mathbf{p}_\alpha \mathbf{q}_\alpha a|U_{01}|\phi_1 \mathbf{q}_1^i\rangle|^2. \tag{6}$$

The summation is performed also over all initial and final spin states, but for the notational brevity it is not explicitly indicated; instead, the presence of $1/f_s$ in equations will indicate the performed spin summation. The initial-state spin factor $f_s = (2s_1+1)(2S_1+1)$ takes care of the spin averaging, with s_1 (S_1) being the spin of the initial-state spectator (pair). Note that due to the energy conservation $q_\alpha^2 \le 2M_\alpha E$. Applying the Eq. (5b) and the general relation $\langle \mathbf{p}_\alpha a|(1 + t_\alpha\, G_0) = \langle \psi^-(\mathbf{p}_\alpha) a|$

between the two-particle transition matrix and scattering wave function one gets

$$\langle \mathbf{p}_\alpha \mathbf{q}_\alpha a | U_{01} | \phi_1 \mathbf{q}_1^i \rangle = \langle \psi^-(\mathbf{p}_\alpha) \mathbf{q}_\alpha a | U_{\alpha 1} | \phi_1 \mathbf{q}_1^i \rangle. \tag{7}$$

The first term in Eq. (5b) does not contribute assuming that the detected particle α is not free in the initial state, i.e., $\alpha \neq 1$. Further manipulations are similar to those of previous works [2,10–13], i.e., the δ-function in Eq. (6) is rewritten as the imaginary part of the energy denominator according to $\delta(x) = -(1/\pi)\mathrm{Im}(x+i0)^{-1}$, resulting in

$$\frac{d^3\sigma_b}{d^3\mathbf{q}_\alpha} = -(2\pi)^4 \frac{M_1}{\pi f_s q_1^i} \mathrm{Im}\Bigg[\sum_a \int \langle \phi_1 \mathbf{q}_1^i | U_{\alpha 1}^\dagger | \psi^-(\mathbf{p}_\alpha)\mathbf{q}_\alpha a\rangle \times \frac{d^3\mathbf{p}_\alpha}{E + i0 - \frac{p_\alpha^2}{2\mu_\alpha} - \frac{q_\alpha^2}{2M_\alpha} - E_a} \langle \psi^-(\mathbf{p}_\alpha)\mathbf{q}_\alpha a | U_{\alpha 1} | \phi_1 \mathbf{q}_1^i \rangle \Bigg]. \tag{8}$$

The differential cross section for nucleon transfer reactions, where in the final state the undetected nucleon is captured by the nucleus A to one of the bound states $|\phi_\alpha^j\rangle$ with energy ϵ_α^j, is given as [26]

$$\frac{d^3\sigma_t}{d^3\mathbf{q}_\alpha} = (2\pi)^4 \frac{M_1}{f_s q_1^i} \sum_j \delta\left(E - \frac{q_\alpha^2}{2M_\alpha} - \epsilon_\alpha^j \right) |\langle \phi_\alpha^j \mathbf{q}_\alpha | U_{\alpha 1} | \phi_1 \mathbf{q}_1^i \rangle|^2. \tag{9}$$

Note, the summation is over the bound state label j, since $|\phi_\alpha^j\rangle$ may contain several components with different a [26]. After rewritting the δ-function as in Eq. (8) it becomes

$$\frac{d^3\sigma_t}{d^3\mathbf{q}_\alpha} = -(2\pi)^4 \frac{M_1}{\pi f_s q_1^i} \mathrm{Im}\Big[\sum_j \langle \phi_1 \mathbf{q}_1^i | U_{\alpha 1}^\dagger | \phi_\alpha^j \mathbf{q}_\alpha \rangle \times \frac{1}{E + i0 - \frac{q_\alpha^2}{2M_\alpha} - \epsilon_\alpha^j} \langle \phi_\alpha^j \mathbf{q}_\alpha | U_{\alpha 1} | \phi_1 \mathbf{q}_1^i \rangle \Big]. \tag{10}$$

The channel resolvent embedded into the three-body space

$$G_\alpha = \sum_a \int \frac{|\psi^-(\mathbf{p}_\alpha)\mathbf{q}_\alpha a\rangle\, d^3\mathbf{p}_\alpha \,\langle \psi^-(\mathbf{p}_\alpha)\mathbf{q}_\alpha a|}{E + i0 - \frac{p_\alpha^2}{2\mu_\alpha} - \frac{q_\alpha^2}{2M_\alpha} - E_a} + \sum_j \frac{|\phi_\alpha^j \mathbf{q}_\alpha\rangle \langle \phi_\alpha^j \mathbf{q}_\alpha|}{E + i0 - \frac{q_\alpha^2}{2M_\alpha} - \epsilon_\alpha^j} \tag{11}$$

has continuum and bound-state contributions that can be easily identified in Eqs. (8) and (10), respectively. The full cross section for the detected particle α, being the sum of (8) and (10), becomes

$$\frac{d^3\sigma}{d^3\mathbf{q}_\alpha} = -(2\pi)^4 \frac{M_1}{\pi f_s q_1^i} \mathrm{Im}\big[\langle \phi_1 \mathbf{q}_1^i | U_{\alpha 1}^\dagger G_\alpha U_{\alpha 1} | \phi_1 \mathbf{q}_1^i \rangle_q \big]. \tag{12}$$

The subscript q for the matrix element in Eq. (12) indicates that the intermediate variable $\mathbf{q}_\alpha$ is not integrated out, i.e., it is a matrix element with respect to the two-body subspace of the undetected pair only. Kinematically the nucleon transfer and breakup reactions remain separated, as they correspond to q_α values above and below $2M_\alpha E$, respectively.

A simple interpretation of Eq. (12) is that the reaction proceeds via transfer of the particle $\beta \neq \alpha$ from the bound pair (23) into the continuum of the subsystem (1β), with subsequent rescattering in that subsystem, whereas the particle α acts as a spectator. Thus, under the α spectator assumption one may expect a reasonable description of the cross section also when using a simplified model space. In the standard three-body approach to deuteron-nucleus reactions only the ground state of the nucleus A is included explicitly, thus, all involved operators are projected onto the ground state of A, while other states are approximately accounted for via optical potentials v_α. The label a becomes redundant and will be omitted, implying that states $|\mathbf{p}_\alpha \mathbf{q}_\alpha\rangle$ refer to this simplified model space to be considered in the following. The reaction is then described by the AGS Eq. (1) with the single-component version of the free resolvent (4), while the channel resolvent becomes

$$G_\alpha = (E + i0 - H_0 - v_\alpha)^{-1} \tag{13a}$$

$$= G_0 + G_0 t_\alpha G_0. \tag{13b}$$

In particular,

$$\mathrm{Im}G_\alpha \equiv (1/2i)\{G_\alpha^\dagger[(G_\alpha^\dagger)^{-1} - G_\alpha^{-1}]G_\alpha\} \tag{14a}$$

$$= -\pi(1 + G_0^\dagger t_\alpha^\dagger)\delta(E - H_0)(1 + t_\alpha G_0) + G_\alpha w_\alpha G_\alpha, \tag{14b}$$

where $w_\alpha = \mathrm{Im} v_\alpha$ is the imaginary part of the optical potential acting within the undetected pair. The relation (14b) in a slightly different but equivalent form has been obtained in previous works, e.g., [11], and shown to be essential in disentangling EB and NEB. In particular, the first term of Eq. (14b) together with (5b) leads to the cross section (6) contribution

$$\frac{d^3\sigma_{\mathrm{EB}}}{d^3\mathbf{q}_\alpha} = (2\pi)^4 \frac{M_1}{f_s q_1^i} \int d^3\mathbf{p}_\alpha\, \delta\left(E - \frac{p_\alpha^2}{2\mu_\alpha} - \frac{q_\alpha^2}{2M_\alpha} \right) \times |\langle \mathbf{p}_\alpha \mathbf{q}_\alpha | U_{01} | \phi_1 \mathbf{q}_1^i \rangle|^2, \tag{15}$$

which is the standard breakup cross section with the nucleus A remaining in its ground state, i.e., the elastic breakup. The second term of Eq. (14b) together with (13b) and (5b) leads to NEB cross section

$$\frac{d^3\sigma_{\mathrm{NEB}}}{d^3\mathbf{q}_\alpha} = -(2\pi)^4 \frac{M_1}{\pi f_s q_1^i} \langle \phi_1 \mathbf{q}_1^i | U_{01}^\dagger G_0^\dagger w_\alpha G_0 U_{01} | \phi_1 \mathbf{q}_1^i \rangle_q \tag{16a}$$

$$= -(2\pi)^4 \frac{M_1}{\pi f_s q_1^i} \int [d^3\mathbf{p}'_\alpha d^3\mathbf{p}_\alpha w_\alpha(\mathbf{p}'_\alpha, \mathbf{p}_\alpha) \times \langle \mathbf{p}'_\alpha \mathbf{q}_\alpha | G_0 U_{01} | \phi_1 \mathbf{q}_1^i \rangle^* \langle \mathbf{p}_\alpha \mathbf{q}_\alpha | G_0 U_{01} | \phi_1 \mathbf{q}_1^i \rangle]. \tag{16b}$$

The obtained NEB differential cross section (16), consistently with previous works [2,10–13], is given by the imaginary part of the optical potential for the undetected pair. The difference lies in the calculation of scattering wave functions, that in the present work are given in terms of three-body transition operators. While EB requires only the on-shell elements of the breakup operator (5), the NEB involves integration of its half-shell matrix elements.

The AGS equations are solved numerically in the momentum-space partial-wave representation. The orbital angular momenta up to 3, 12 and 22 are included for the neutron-proton, neutron-nucleus, and proton-nucleus pairs, respectively, with total angular momentum up to 35. See Refs. [24–27,32] for further details. The Coulomb interaction is included via the screening and renormalization method [28–31]. In fact, the renormalization factors cancel in the expressions for the cross section. Nevertheless, one has to ensure that the screening radius is large enough and the results become practically independent of it; this will be demonstrated in the next section.

3. Results

The primary goals of the present work are to establish the calculation of NEB in the framework of rigorous three-body AGS equations and to evaluate the optical potential nonlocality effect. The comparison with the experimental data would involve the preequilibrium and compound nucleus contributions that are beyond the reach of the present calculations and therefore is left to future studies. The most convenient physical observable for this investigation is the distribution of the energy $E_\alpha = q_\alpha^2/2m_\alpha$ of the detected particle in the three-body center-of-mass (c.m.) frame,

$$\frac{d\sigma_{\mathrm{NEB}}}{dE_\alpha} = m_\alpha q_\alpha \int d^2\hat{\mathbf{q}}_\alpha \frac{d^3\sigma_{\mathrm{NEB}}}{d^3\mathbf{q}_\alpha}. \tag{17}$$

This observable can be calculated directly in partial waves, performing the angular integration within Eq. (16a). In the considered case of deuteron-nucleus scattering the detected particle is assumed to be the proton, though the whole formalism could be applied equally well also to (d,n) inclusive reactions.

A realistic neutron-proton interaction with spin-orbit and tensor terms such as CD Bonn [33] can be included into solution of the AGS equations in the partial-wave representation. However, it was found that the differential cross section (17) is insensitive to fine details of the potential, especially of its noncentral part, therefore a simple Gaussian potential from Ref. [2] reproducing the deuteron binding energy and low-energy $^3S_1$ phase shift was adopted; it has been used in many reaction calculations. Likewise, the optical nucleon-nucleus potentials are taken without noncentral parts, which allows the omission of spin degrees of freedom and leads to a significant reduction in the number of partial waves and calculation time. Few examples with the full spin-dependent interaction will be shown as well. Several parametrizations of the nucleon-nucleus optical potential are used, including those by Koning and Delaroche (KD) [34], Weppner et al. [35], Watson et al. [36], Chapel Hill (CH89) group [37], and Giannini and Ricco (GR) [38]. The latter has nonlocal and its equivalent local versions, to be used to estimate the nonlocality effect. Except for the nonlocal one [38], the other potential contain energy-dependent parameters. In the present study they are taken at half the deuteron energy, a standard choice in the calculations of elastic scattering and breakup. It is perhaps too naive for NEB where one could consider also explicit energy dependence for w_α in Eq. (16), but should be fully sufficient for the purpose of the present work, i.e., establishing the calculational scheme and evaluating the nonlocality effect.

First, the convergence of NEB cross section with respect to the Coulomb screening radius is demonstrated in Fig. 1. The screening function $e^{-(r/R)^n}$ is taken over from Ref. [31] with $n = 8$, though the results are quite insensitive to n between 4 and 10. Two cases are considered, deuteron scattering from ^{12}C and ^{90}Zr nuclei at 50 and 80 MeV beam

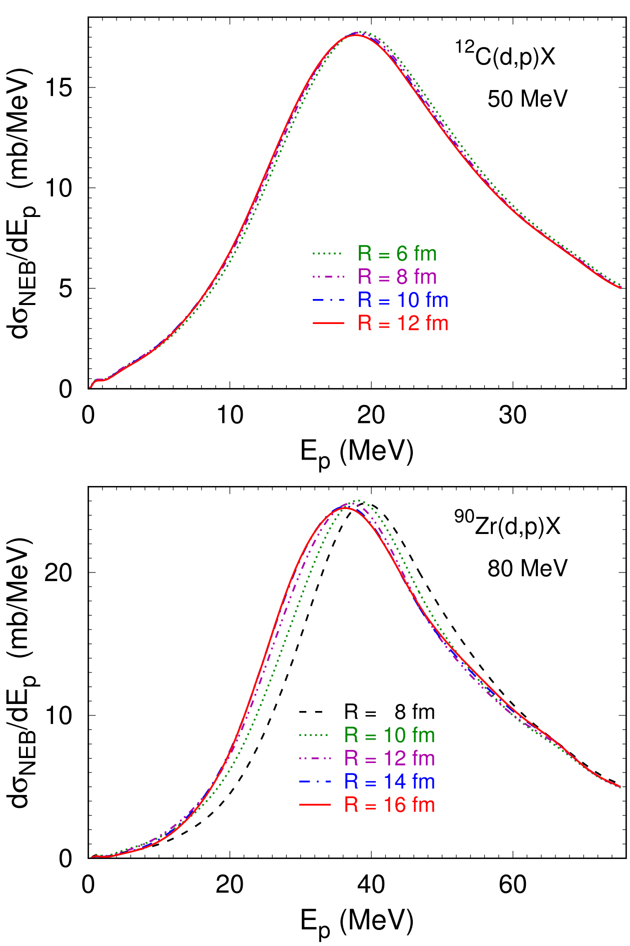

Fig. 1. The convergence of the semi-inclusive NEB cross section with the Coulomb screening radius R. 50 MeV (80 MeV) deuteron scattering from ^{12}C (^{90}Zr) nucleus is shown in the top (bottom) panel. KD optical potential is used.

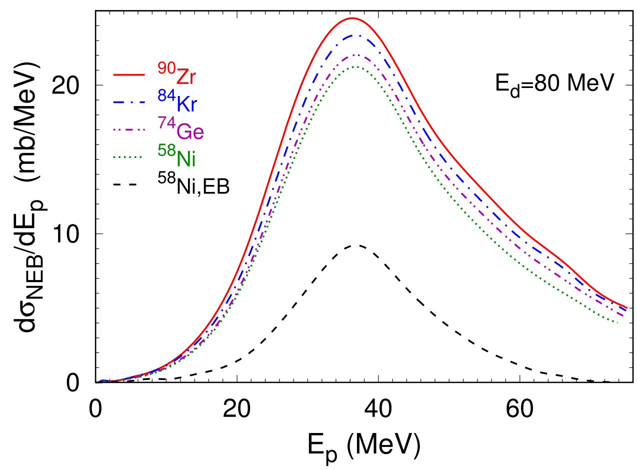

Fig. 2. The semi-inclusive NEB cross section for 80 MeV deuteron scattering from ^{58}Ni, ^{74}Ge, ^{84}Kr, and ^{90}Zr nuclei as a function of the proton energy in c.m. frame. KD optical potential is used. In the case of ^{58}Ni also the EB cross section is displayed by dashed curve.

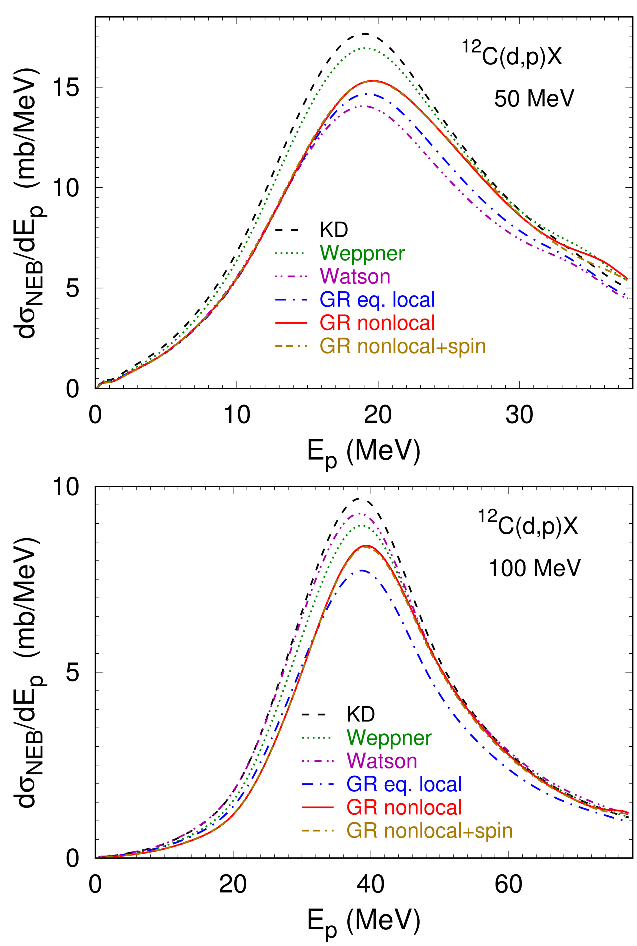

Fig. 3. The semi-inclusive NEB cross section for deuteron-^{12}C scattering at 50 and 100 MeV beam energy as a function of the proton energy in c.m. frame. Results with several neutron-proton and nucleon-nucleus optical potential parametrizations are compared.

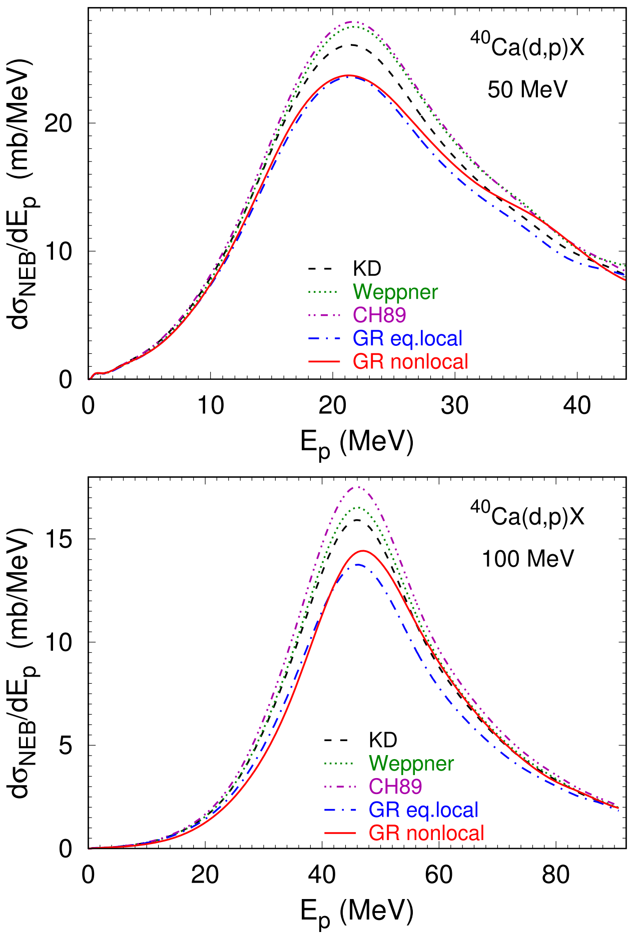

Fig. 4. The semi-inclusive NEB cross section for deuteron-^{40}Ca scattering at 50 and 100 MeV beam energy as a function of the proton energy in c.m. frame. Results with several optical potential parametrizations are compared.

energy, respectively. While for $Z = 6$ target the convergence is very fast, even $R = 6$ fm is quite sufficient, the convergence becomes considerably slower for $Z = 40$, where R around 15 fm at least is needed. Nevertheless, achieving well converged results for ^{90}Zr with $Z = 40$ is an important step forward compared to previous Faddeev-type calculations with the screening and renormalization approach, limited to ^{58}Ni with $Z = 28$.

Results for 80 MeV deuteron scattering from most abundant isotopes of $Z = 28$, 32, 36, and 40 nuclei, i.e., ^{58}Ni, ^{74}Ge, ^{84}Kr, and ^{90}Zr are presented in Fig. 2. The differential NEB cross section has an asymmetric bell shape and shows increase with Z, which is not strictly linear, probably indicating the dependence on the neutron-proton asymmetry $(A - 2Z)/A$ as well. In the case of ^{58}Ni also the corresponding EB cross section is shown, which is considerably lower than the NEB one. The EB peak around half of the deuteron beam energy corresponds to the quasifree neutron scattering off the nucleus A. As the half-shell breakup operator appears also in Eq. (16), the NEB cross section peaks in the same energy region. These ^{58}Ni results, transformed to the lab frame, allow a comparison with previous works, e.g., [15,19]. Their energy distributions also have an asymmetric bell shape (with few wiggles in Ref. [15]), but the Faddeev-type approach predicts larger cross section: at the peak the results of Refs. [15] and [19] are lower by 10 and 35 %, respectively. At the same time, as shown in Fig. 3 of Ref. [15], the PE + CN contribution is sizable and even becomes dominant at lower proton energies.

The optical potential nonlocality effect is investigated in Figs. 3 and 4. The GR potentials [38] were developed for symmetric $N = Z$ nuclei, and further improved for ^{12}C, ^{16}O and ^{40}Ca [25] by an additional readjustment of parameters to fit the experimental data. The present study explores deuteron scattering from ^{12}C and ^{40}Ca nuclei at 50 and 100 MeV beam energy. In addition to the GR nonlocal optical potential and its roughly equivalent local potential, results were obtained also with other commonly used local potentials, i.e., KD [34], Weppner [35], Watson [36], and CH89 [37]. The nonlocality effect, taken as the difference between local and nonlocal GR potentials, visibly increases the NEB cross section at the peak and beyond. On the other hand, the sensitivity to the model of the local potential appears even more sizable. Noteworthy, at higher E_p values the nonlocal GR potential results agree quite well with those of most local potentials except the GR one. Furthermore, the shape of the NEB cross section depends on the beam energy as well, the high-energy tail becoming lower with increasing beam energy.

Fig. 3 also studies the importance of spin-orbit force in the optical potential and the realistic neutron-proton potential CD Bonn [33]. Both are included in the results labeled “GR nonlocal + spin”. Quite surprisingly, the effect for NEB cross section turns out to be almost negligible. Thus, even a very simple and unrealistic np potential might be sufficient for NEB calculations.

4. Conclusions

The nonelastic breakup, comprising a class of channels in (d, p) inclusive reactions, was studied using the Faddeev-type three-body scattering theory. The key steps in deriving the NEB cross section, i.e., the spectator assumption for the proton, the closure over final states and the subsequent projection onto the ground state of the nucleus A, are the same as in previous works. Consistently, the NEB cross section is driven by the imaginary part of the optical potential for the undetected pair. The difference lies in the description of scattering states which in the present study is given in terms of the AGS transition operators, calculated in the momentum-space partial-wave representation.

Well converged numerical results were obtained for the energy distribution of the NEB cross section. They have asymmetric bell shape as found in previous works but are 10 to 35% higher. Benchmark calculations are left for future studies. They are very important given the existing differences between various NEB approaches [9,17–19].

The effect of the optical potential nonlocality was also investigated. It is possibly more pronounced at higher proton energies, but remains within uncertainties due to different local optical potential parametrizations. The NEB cross section appears to be insensitive to the spin-dependent interaction terms.

Finally, performing calculations with nuclei of higher charge number, presently up to $Z = 40$, is an important step towards application of the Faddeev-type momentum-space equations also to other reactions involving medium mass and heavy nuclei, e.g., the deuteron stripping (d, p). Preliminary studies [39] indicate that a combination with machine learning methods could be a powerful tool for a further extension, whereas the results of direct calculations like here could serve as the training data.

Data availability

Data will be made available on request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Many-body approach to NEB

I consider the collision of the bound pair $(\alpha\beta)$ with a bound A-body system in the c.m. frame, having the $A(d,p)X$ reaction in mind. This is an $(A+2)$-body problem, the Hamiltonian H_0+V contains $(A+1)$ kinetic energy contributions H_0 from all relative motions plus the sum of all $(A+2)(A+1)/2$ pairwise potentials, eventually supplemented by three-body and higher forces, all being real. Let E be the available energy, $|\Psi^+(\mathbf{q}_1^i)\rangle$ the full many-body scattering wave function with the initial relative deuteron-nucleus momentum $\mathbf{q}_1^i$, and $\mathbf{q}_\alpha$ the final relative momentum between the detected particle α and the remaining subsystem X which can be in any state.

If X forms a bound nucleus with wave function $|\phi_X^j\rangle$ and energy ϵ_X^j, where j labels the different bound states, the reaction amplitude can be given as $\langle \phi_X^j \mathbf{q}_\alpha | V_{\alpha X} | \Psi^+(\mathbf{q}_1^i)\rangle$. Here $V_{\alpha X}$ is the interaction that is external to the final $\alpha + X$ channel, i.e., the sum of all potentials (two-body, three-body, etc.) acting between the particle α and the subsystem X. The above amplitude also defines the transfer component of the transition operator

$$U_{\alpha X, dA} |\phi_d \phi_A \mathbf{q}_1^i\rangle \equiv V_{\alpha X} |\Psi^+(\mathbf{q}_1^i)\rangle, \tag{A.1}$$

acting on the initial channel state, given as product of bound-state wave functions of deuteron and nucleus A and plane wave for their relative motion.

If X is broken into n clusters X_j with internal energies ϵ_j, $j=1,\dots,n$, the reaction amplitude can be given as $\langle \psi^-(\{X_j\},\{\mathbf{p}_k\})\,\mathbf{q}_\alpha | V_{\alpha X} | \Psi^+(\mathbf{q}_1^i)\rangle$. Here $\langle \psi^-(\{X_j\},\{\mathbf{p}_k\})|$ is the full wave function of the subsystem X corresponding to the set of outgoing clusters $\{X_j\}$ with reduced masses μ_k and asymptotic relative momenta $\mathbf{p}_k$, $k=1,\dots,n-1$. Note that μ_k and $\mathbf{p}_k$ depend on the clustering $\{X_j\}$, but this dependence is suppressed for brevity. The energy of such a state is $E(\{X_j\},\{\mathbf{p}_k\}) = \sum_k p_k^2/2\mu_k + \sum_j \epsilon_j$. The differential cross section for the detected particle α is the sum over all possible channels,

$$\begin{aligned}\frac{d^3\sigma}{d^3\mathbf{q}_\alpha} = (2\pi)^4 \frac{M_1}{f_s q_1^i} \Big[&\sum_j \delta\Big(E - \frac{q_\alpha^2}{2M_\alpha} - \epsilon_X^j\Big) |\langle \phi_X^j \mathbf{q}_\alpha | V_{\alpha X} | \Psi^+(\mathbf{q}_1^i)\rangle|^2 \\ &+ \sum_{\{X_j\}} \int \Big(\prod_k d^3\mathbf{p}_k\Big) \delta\Big(E - \frac{q_\alpha^2}{2M_\alpha} - E(\{X_j\},\{\mathbf{p}_k\})\Big) \\ &\times |\langle \psi^-(\{X_j\},\{\mathbf{p}_k\})\,\mathbf{q}_\alpha | V_{\alpha X} | \Psi^+(\mathbf{q}_1^i)\rangle|^2 \Big],\end{aligned} \tag{A.2}$$

i.e., the second sum runs over all possible clusterings.

Rewriting the δ-functions as in Section 2, one can isolate in Eq. (A.2) the resolvent for the subsystem X, i.e.,

$$\begin{aligned}G_X = &\sum_j \frac{|\phi_X^j \mathbf{q}_\alpha\rangle \langle \phi_X^j \mathbf{q}_\alpha|}{E + i0 - \frac{q_\alpha^2}{2M_\alpha} - \epsilon_X^j} \\ &+ \sum_{\{X_j\}} \int \Big(\prod_k d^3\mathbf{p}_k\Big) \frac{|\psi^-(\{X_j\},\{\mathbf{p}_k\})\,\mathbf{q}_\alpha\rangle\langle \psi^-(\{X_j\},\{\mathbf{p}_k\})\,\mathbf{q}_\alpha|}{E + i0 - \frac{q_\alpha^2}{2M_\alpha} - E(\{X_j\},\{\mathbf{p}_k\})}.\end{aligned} \tag{A.3}$$

Together with Eq. (A.1) the differential cross section (A.2) becomes

$$\frac{d^3\sigma}{d^3\mathbf{q}_\alpha} = -(2\pi)^4 \frac{M_1}{\pi f_s q_1^i} \mathrm{Im}\big[\langle \phi_d \phi_A \mathbf{q}_1^i | U_{\alpha X, dA}^\dagger G_X U_{\alpha X, dA} | \phi_d \phi_A \mathbf{q}_1^i\rangle_q\big]. \tag{A.4}$$

where, as in Section 2, the subscript q indicates that the matrix element in Eq. (A.4) is taken with respect to X variables only.

Finally, the three-body reduction, i.e., the projection onto the ground state of the nucleus A implies the replacement $G_X \to G_\alpha$ and $U_{\alpha X, dA}|\phi_d \phi_A \mathbf{q}_1^i\rangle \to U_{\alpha 1}|\phi_1 \mathbf{q}_1^i\rangle$, leading exactly to the three-body result as in Eq. (12).

The above derivation is quite similar to those given in Refs. [2,13,14], but does not assume the infinitely heavy nucleus A, and instead of the distorted proton wave introduces the transition operator.

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