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Evolution of Fuzzy Sets in Digital Transformation Era

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Abstract. Nowadays, it is agreed that fuzzy sets are suitable for capturing and representing the concept of vagueness and uncertainty, and various fuzzy reasoning systems are being developed based on them. Researchers have proposed fuzzy set extensions to improve the performance and accuracy of these systems. The research questions arise regarding how fuzzy sets have evolved and what the main trends in their evolution are. To address these questions, our research presents a chronological and bibliometric analysis of fuzzy sets based on papers extracted from the Web of Science database. The main findings and contributions have been identified, systematized and visualized in a fuzzy set keyword map of 65 fuzzy set extensions. These extensions are primarily used for decision-making, reasoning, and prediction, particularly in the context of digital transformation, by integrating digital technologies into all areas of business, transforming operations and enhancing value delivery to customers. As organisations increasingly adopt digital technologies, the need for robust frameworks to manage uncertainty becomes critical. The main trends indicating the directions of fuzzy sets development, an overview of the variety and popularity of fuzzy sets over the years, and the impact of countries engaged in fuzzy set research are also identified and reported. The results support researchers and practitioners working on fuzzy sets and their applications by providing valuable insights into the fuzzy set topic, its existing extensions, and, more generally, to any field of investigation where fuzzy sets are relevant, particularly in the realm of digital transformation.

Key words: bibliometrics, fuzzy sets, uncertainty, fuzzy set extension, digital transformation, artificial intelligence.

1. Introduction

According to Abiodun *et al.* (2023), the concept of digitalization,[1] defined as the transformation of something into digital form, was first mentioned in 1954. Additionally, the invention of the Internet can be considered one of the foundations of digital transformation. Since then, efforts have been made to digitalize various activities, such as education, business, medicine, etc., to reduce the costs of searching, replicating, transporting, tracking, and verification (Walter *et al.*, 2022), as well as to shrink every industry and nearly

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[1] https://www.merriam-webster.com/dictionary/digitalization

every citizen in the world into a virtual environment in which its members interact and share online (García-Galera *et al.*, 2018).

Recently, digital transformation has been particularly associated with digitalizing complex and non-linear human tasks in the real world, involving the incorporation of artificial intelligence techniques and fuzzy inference mechanisms to simulate complex and vague human thinking. In this context, it becomes important to systematize and understand how fuzzy sets have evolved over time and how they have contributed to digital transformation.

In 1965, Zadeh (1965) proposed fuzzy set theory to express uncertain concepts more realistically in real-case applications. Since then, fuzzy sets have been widely used by incorporating them into existing techniques for approximate reasoning, like pattern classification and bidirectional approximate reasoning (Ganie *et al.*, 2024b), approximate deductive reasoning for natural language semantics processing (Rubio-Manzano and Pereira-Fariña, 2019), fuzzy approximations for data mining of Machine Learning Databases from the UCI Repository (Huang *et al.*, 2017), fuzzy control systems (Patel and Shah, 2021), and for enhanced ship engine vibration reduction with the help of Adaptive Neuro-Fuzzy Control System (ANFIS) (Sharma *et al.*, 2025), pattern recognition using different distance measures in Athira *et al.* (2019) and fuzzy sets with two new measures of correlation coefficients and weighted correlation coefficients based on the newly defined information energy measure (Guleria and Bajaj, 2021b), multi-criterial decision making for hospital recommender which considers the patient preferences in Bani-Doumi *et al.* (2024), bone graft selection problem (Xie *et al.*, 2024), similarity measurement between grey and fuzzy theory (Khuman, 2021), for medical diagnosis (Abdel-Basset *et al.*, 2021), for enhanced decision-making for the assessment of the internet skills (Sarfraz and Azeem, 2024), object recognition (Pisharady *et al.*, 2014), classification and feature selection using SWARM algorithms with improved initialization (Maini *et al.*, 2019), facial features extraction (Li *et al.*, 2017), for classification of medical datasets (Vommi and Battula, 2024), or remote-sensing image classification (Meher *et al.*, 2024), group decision making (Garg and Kumar, 2019; Lai *et al.*, 2022), which are summarized in the survey (Çil and Cebi, 2025), mobile robots (Jhang *et al.*, 2019) or mobile robots in virtual environments (Pérez-Juárez *et al.*, 2025), etc.

Uncertainty comes in various forms and is independent of fuzzy set theory or any specific approach used to handle it. It is unavoidable when dealing with real-world cases. At the empirical level, uncertainty arises in almost all measurements due to measurement errors or resolution limits of measuring tools. At the cognitive level, it originates from the vagueness and ambiguity of natural language. At the social level, uncertainty is often produced and maintained by people for various purposes, such as security, privacy, etc. Generally, regardless of the causes of uncertainty, it represents an information deficiency resulting from information inaccuracy, fragmentation, incompleteness, and vagueness (cf. Klır and Wierman, 1998).

According to the nature of uncertainty (Klır and Wierman, 1998), there are three types of uncertainty: fuzziness (or vagueness), which arises from the imprecise boundaries of fuzzy sets (i.e. linguistic imprecision); non-specificity, which comes from the inaccurate size (or cardinality) of the relevant alternative set (i.e. information-based imprecision);

and disagreement, which arises from conflicting sets of alternatives. Depending on the described nature of the uncertainty, various extensions of fuzzy sets have been proposed for better modelling to obtain realistic, reliable, and high-quality results (Işık, 2023b). However, fuzzy set extensions are chosen for their methods without considering the nature of uncertainty (Işık, 2023b, 2023a). This occurs because, in many real-world problems, it is impossible to explicitly define the nature of uncertainty as it is multifaceted. Authors simply refer to it as "fuzzy".

A traditional fuzzy set is a set of elements whose membership degrees are given in interval [0; 1]. However, this membership degree should be defined by applying a particular approach or a human expert (Miliauskaite and Kalibatiene, 2020). Fuzzy set extensions are defined to model uncertainties of different natures (Işık, 2023a). For example, Atanassov (1999) proposed intuitionistic fuzzy sets to model set elements with a lack of information about membership degree. Intuitionistic fuzzy sets generalize fuzzy sets by providing each element with degrees of membership and non-membership. Smarandache (2005) introduced neutrosophic sets to generalize intuitionistic fuzzy sets by providing independent membership degree, non-membership degree, and indeterminacy degree for inconsistent data modelling.

Pythagorean fuzzy sets, proposed by Yager (2013), are intuitionistic fuzzy sets extended theoretically using the Pythagorean theorem, where inconsistency occurs under a threshold, i.e. the sum of the squares of the membership degree and non-membership degree. Yager (2017) proposed q-rung orthopair fuzzy sets, which generalize Pythagorean fuzzy sets by limiting the qth power of the membership degree and non-membership degree by 1. This allows us to handle a higher level of inconsistency. Senapati and Yager (2020) defined Fermatean fuzzy sets by replacing q with 3. Picture fuzzy sets, proposed by Cuong (2014), Cuong and Kreinovich (2013), generalize intuitionistic fuzzy sets by including the refusal element into the set. Mahmood *et al.* (2019) defined spherical fuzzy sets as a generalization of picture fuzzy sets and Pythagorean fuzzy sets. Spherical fuzzy sets address an interesting scenario, where Pythagorean fuzzy sets and picture fuzzy sets failed, by including the neutral degree. In certain cases, the sum of the positive, neutral, and negative membership degrees in picture fuzzy sets exceeds 1, which leads us to the spherical fuzzy set (Ashraf *et al.*, 2019). In fuzzy Mandelbrot sets (Ince and Ersoy, 2022), a membership function describes the degrees of belonging of points to the Mandelbrot set under iteration, even if the orbit of the point is not limited. The introduction of fuzzy Mandelbrot sets provides a more detailed view by using higher maximum iteration numbers (Ince and Ersoy, 2023) as computers cannot handle infinity.

The research question arises: "*how have fuzzy sets evolved over time, and what are the main trends in their evolution*?" To answer this question, the paper presents a chronological and bibliometric analysis of fuzzy sets. In the literature, various reviews and surveys on fuzzy sets can be found. However, they are limited to specific domains (i.e. decision-making, He and Sun, 2018; Guleria and Bajaj, 2021a), fuzzy set extension types (i.e. spherical fuzzy sets, Gündoğdu and Kahraman, 2019; Donyatalab *et al.*, 2020, type-2 fuzzy sets, De *et al.*, 2022), time periods, or descriptions (i.e. Guleria and Bajaj, 2021a; De *et al.*, 2022). The authors of Kumar and Sharma (2019) present a systematic literature review on fuzzy-logic-based text summarization with a period limitation from 2003 to 2017. In Kahraman *et al.* (2016), the authors qualitatively reviewed fuzzy sets and presented their classification until 2010.

Our research aims to fill this gap by not limiting itself to a specific application domain, fuzzy set extension types, or time periods. Instead, it takes a broader approach to fuzzy set extensions. Moreover, systematic literature reviews are usually limited in terms of the number of analysed publications, i.e. 40–300 (Donthu *et al.*, 2021; Snyder, 2019). Since the authors of this paper aim to cover a larger number of publications without restrictions on time period or application area, bibliometric analysis is applicable to answer the defined research question. Thus, our study aims to present the chronological and bibliometric analysis of fuzzy sets and their various extensions, providing insights into the fuzzy set topic and its existing extensions in the realm of digital transformation.

Why do we study fuzzy set extensions? As organizations increasingly adopt digital technologies, it becomes more crucial to have robust frameworks in place to manage uncertainty. This paper is a chronological and bibliometric analysis of fuzzy set extensions, aiming to gather, analyse, and report the trends and prospects of fuzzy set evolution from its beginning to the present. It highlights the directions of fuzzy sets development, provides an overview of the variety and popularity of fuzzy sets over the years, and examines the impact of countries working on the fuzzy set topic.

This research enhances the intellectual structure of the fuzzy set topic by presenting a fuzzy set map of the subject of interest and identifying the main topics in the research area. The paper seeks to support researchers and practitioners working on fuzzy sets and their application by providing the intellectual structure of the fuzzy set topic, demonstrating existing fuzzy set extensions, and contributing to any field of investigation where fuzzy sets are relevant. This paper also presents the countries contributing to fuzzy sets research. The main contribution and the novelty of our research are the following:

1. 65 fuzzy set extensions are identified, systematized chronologically, and visualized in a keyword map.
2. The main trends in the evolution of fuzzy sets are discovered and reported.
3. The trends are identified and reported in relation to countries represented by scientists involved in fuzzy set research.

The rest of this paper is organized as follows. Section 2 reviews related works on fuzzy sets. Section 3 presents the research methods. Section 4 reports the results of the chronological and bibliometric analysis of fuzzy sets. Section 5 discusses our findings. Finally, Section 6 concludes this paper.

2. Related Works

Here, the authors of this paper present the analysis of the related literature reviews, surveys, and bibliometric analysis works on fuzzy sets found by applying the methodology described in Section 3. The papers relevant to this analysis are summarized in Table 1, which consists of columns as follows: 1) Reference (Ref.) of the paper; 2) Period covering

Table 1
The related literature analysis on fuzzy sets (fs).

No.	Ref.	Period	DL	Paper type	Domain	Fuzzy set extensions
1.	(Kahraman *et al.*, 2016)	1965–2015	Scopus	Literature review (narrative)	Not limited	Ordinary fs; interval-valued fs; type-n fs; intuitionistic fs; fuzzy multisets; nonstationary fs; hesitant fs
2.	(Işık, 2023b)	2012–2022	NA[a]	Regular paper	All types of engineering problems	Intuitionistic fs; type-2 fs; interval fs; z-number; neutrosophic fs; hesitant fs; l-fuzzy set; pythagorean fs; bipolar fs; picture fs; orthopair fs; spherical fs; m-polar fs; type-n fs; non-stationary fs; fermatean fs
3.	(Bustince *et al.*, 2016)	ND[b]	ND	Regular paper	Not limited	Atanassov intuitionistic fs; bipolar-valued fs of Lee; bipolar-valued fs of Zhang; complex fs; fuzzy rough sets; fuzzy soft sets; grey sets; hesitant fs; interval type-2 fs; interval-valued fs; m-polar-valued fs; neutrosophic fs; pythagorean fs; set-valued fs; shadow sets; type-2 fs; type-n fs; typical hesitant fs; vague sets
4.	(He and Sun, 2018)	FS in 1965–2017; fuzzy decision-making methods 2015–2018	ND	Survey	Different fs; fuzzy decision-making methods	Type-1 fs; general type-2 fs; interval-valued type-2 fs; intuitionistic fs; interval-valued intuitionistic fs; fuzzy multisets; hesitant fs; probabilistic hesitant fs
5.	(Sotoudeh-Anvari, 2020)	2010–2020	WoS	Critical review	Fuzzy arithmetic in the field of logical operator “OR” in decision problems	Ordinary fs; interval-valued intuitionistic fs; intuitionistic fs; neutrosophic soft sets; hesitant fs; fuzzy soft sets; single-valued neutrosophic hesitant fs; soft rough sets; pythagorean fs; hesitant interval-valued fs; type-2 fs; complex fs; interval type-2 fs
6.	(Gundogdu and Kahraman, 2019)	NA	NA	Regular paper	Generalized 3D spherical fs in WASPAS	Ordinary fs; type-2 fs; interval-valued fs; intuitionistic fs; fuzzy multisets; neutrosophic fs; nonstationary fs; hesitant fs; q-rung orthopair fs; spherical fs
7.	(Gündoğdu and Kahraman, 2019)	NA	NA	Regular paper	Spherical fs in TOPSIS	Ordinary fs; type-2 fs; interval-valued fs; intuitionistic fs; fuzzy multisets; neutrosophic fs; nonstationary fs; hesitant fs
8.	(Donyatalab *et al.*, 2020)	NA	NA	Regular paper	Spherical fuzzy linear assignment method in group decision-making	Taken from (Gundogdu and Kahraman, 2019)
9.	(Guleria and Bajaj, 2021a)	NA	NA	Regular paper	Decision-making problems	Intuitionistic fs; type-2 fs; picture fs; spherical fs; t-spherical fs; eigen fs; soft sets and matrices; pythagorean fuzzy soft sets; complex fuzzy soft sets
10.	(Laengle *et al.*, 2021)	1978–2016	WoS	Bibliometric overview	“Fuzzy Sets and Systems” journal	Not defined
11.	(De *et al.*, 2022)	ND	ND	Literature survey	Type-2 fs	General type-2 fs; interval type-2 fs; type-2 intuitionistic fs; type-2 hesitant fs; type-2 fuzzy rough set
12.	(Boltürk and Kahraman, 2022)	NA	NA	Regular paper	Investment analysis	Ordinary fs; type-2 fs; intuitionistic fs; interval-valued fs; nonstationary fs; neutrosophic fs; fuzzy multisets; pythagorean fs; picture fs; q-rung orthopair fs; fermatean fs; spherical fs; circular intuitionistic fs
13.	(Büyüközkan *et al.*, 2024)	left open	Scopus, WoS	Literature review	Fermatean fuzzy sets (ffs)	Interval-valued ffs; hesistant ffs; 2-tuple ffs; trapezoidal ffs; triangular ffs; ff liguistic sets; ff soft sets; feratean cubic fs; 3,4-quasirung fs; rough ff; ff N-soft sets; interval-valued hesistant ffs; dempster-shafer theory-based ffs
14.	(Valdez *et al.*, 2025)	2002-2025	Scopus, WoS	Bibliometric overview	Adoption of intelligent methods, involving Type-3 fuzzy logic systems	Type-3 fs; interval type-3 fs; general type-3 fs

[a] NA – not applicable. [b] ND – not determined.

the analysed papers; 3) Digital Library (DL), from which papers for the analyses were taken; 4) Paper type, which can be a review or a regular paper; 5) Domain, in which a survey, review, or other analysis is performed; 6) Fuzzy set extensions, found or analysed by the authors.

As can be seen from Table 1, there is a lack of comprehensive literature reviews and surveys on fuzzy set extensions. Authors tend to analyse a particular feature of fuzzy set extensions, like limiting an application domain to decision-making in He and Sun (2018) and Guleria and Bajaj (2021a), a particular multi-criteria decision making (MCDM) method as WASPAS in Gundogdu and Kahraman (2019) or TOPSIS in Gündoğdu and Kahraman (2019), analysing only fuzzy arithmetic in the field of OR in decision problems (Sotoudeh-Anvari, 2020), emphasizing spherical fuzzy sets in Gündoğdu and Kahraman (2019) and Donyatalab *et al.* (2020), type-2 fuzzy sets in De *et al.* (2022), etc. In the analysed reviews and surveys (Table 1), their authors do not present the complete description of the applied methodology for the review or the survey, such as do not present a search string and research questions in He and Sun (2018), Gündoğdu and Kahraman (2019), De *et al.* (2022). The authors of Laengle *et al.* (2021) concentrate solely on one journal, "Fuzzy Sets and Systems". Moreover, among the found reviews and surveys, there is no complete bibliometric chronological review of fuzzy set extensions. Consequently, in this paper, we aim to cover this gap and contribute by increasing the knowledge of the fuzzy set extensions by providing a fuzzy set map and grasping the main topics in the fuzzy set field.

3. Research Methodology

This analysis consists of two parts. First, we searched the literature for a chronological and bibliometric analysis of fuzzy sets. Second, we searched for related works on fuzzy sets that provide historical, definitional, operational, and applied overviews of fuzzy sets in general rather than focusing on specific fuzzy sets.

The analysis was systematically organized following the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA 2020) guidelines (Page *et al.*, 2021) and presented in Fig. 1. Note, the present study is a bibliometric analysis, but not a systematic literature review. They have some differences in their final goal, the research questions, search process, search strategy requirements, quality evaluation, and results (Petersen *et al.*, 2015; Kitchenham *et al.*, 2011; Zhang and Budgen, 2012).

The chronological and bibliometric analysis presented in this paper has been conducted following the guidelines set out in Donthu *et al.* (2021). The analysis procedure is presented in Table 2 and described in the forthcoming sub-sections.

3.1. *Research Questions*

This chronological and bibliometric analysis aims to explore the chronological relationships among fuzzy sets (Table 2, Step 1). Based on this aim, the following main research questions (MRQ) and sub-questions (RQ) are defined:

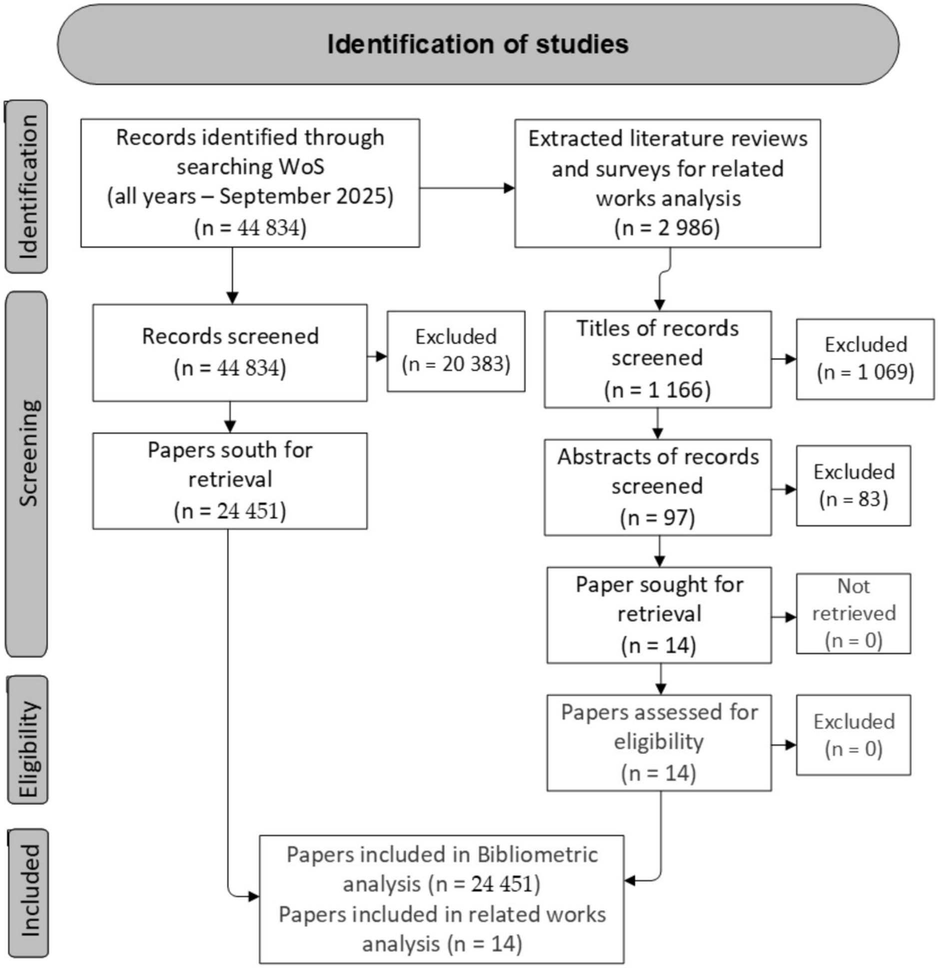

Fig. 1. The schema of the current bibliometric analysis (adopted from Yepes-Nuñez *et al.*, 2021).

Table 2
The bibliometric analysis procedure.

Step	Definition
1. Aim and scope	Aim: to explore the chronological relationships among fuzzy sets by applying a bibliometric analysis. Scope suitability for the use of bibliometric analysis: Yes (i.e. the number of papers >1000).
2. Techniques	Co-word analysis (to identify notable keywords and future research directions). Co-authorship analysis (to identify the relationship among countries and their intellectual collaboration).
3. Data	Search terms exemplify the scope of the analysis: Yes Coverage of the database adequate: Yes Data free of errors (i.e. duplicates, erroneous entries, etc.): Yes Final dataset fulfils the requirements of the bibliometric analysis techniques: Yes
4. Findings	Answering the defined questions, Bibliometric summary, and Results validity.

Table 3
Search features in WoS (CS – Computer Science).

Digital library	Search string	Document type	Language	Category	Search result
WoS	"fuzzy set*"	article OR proceeding paper OR review article	English	CS	24.451
WoS	"fuzzy set*" AND ("review*" OR "surve*")	article OR proceeding paper OR review article	English	CS	1.166

MRQ1: How have fuzzy sets evolved over time?
- **RQ1:** When are papers on fuzzy sets published?
- **RQ2:** What extensions of fuzzy sets are found in scientific papers?

MRQ2: What are the main trends in the evolution of fuzzy sets?
- **RQ3:** What are the main trends and future directions discovered in the analysed topic?
- **RQ4:** What are the noticeable trends for the countries involved in the study of fuzzy sets?
- **RQ5:** What is the connection between fuzzy sets and artificial intelligence?
- **RQ6:** Which methods are used with fuzzy sets? The search protocol developed by the first author and reviewed by the second author to eliminate subjectivity is presented as follows.

3.2. *Conducting the Search*

Since this research focuses on fuzzy sets without specific limitations, the main search term is "fuzzy set". According to PICOC (Population, Intervention, Comparison, Outcomes, Context) (Kitchenham and Charters, 2007), the search is defined as follows:

Population – research papers on fuzzy sets. *Intervention* – currently, only a partial analysis of the evolution of fuzzy sets exists. Therefore, there is a need for a more comprehensive study of fuzzy sets evolution. *Comparison* – not applicable. *Outcomes* – the measurable outcome is not considered in the analysed papers. *Context* – academic context, scientific papers on fuzzy sets.

The chosen keyword was transformed into the search string and processed on the Web of Science (WoS) digital library. Table 3 presents the search limitations. The categories were limited to Computer Science to focus on this research area and to restrict the final quantity of papers, since initially 44.834 papers were found, which would be difficult to process manually. Moreover, after excluding other categories, the number of scientific papers remains big enough and suitable for the bibliometric analysis.

This study was conducted in May 2023, supplemented with the newest publications in January 2025 and additionally supplemented in September 2025. There were no restrictions by year in the search. The document type has been limited to articles, proceedings papers and reviews. The WoS digital library has been chosen for this chronological and bibliometric analysis based on the experience published in Kalibatiene and Miliauskaitė

(2021). In addition, the authors of Donthu *et al.* (2021) recommend deciding on one suitable digital library to avoid the impact of the consolidation of different formats of references and possible human errors, which ensures the construct validity of the analysis.

3.3. *Research Paper Selection and Quality Assessment*

The research paper selection was conducted in two branches as follows (Fig. 1):

1. Selecting all found papers for the chronological and bibliometric analysis. In this branch, the research paper selection (inclusion/exclusion) was not performed, since we are interested in the global chronological view of the fuzzy set evolution. Also, WoS contains non-duplicating high-quality refereed papers (Kalibatiene and Miliauskaitė, 2021), which helps us to ensure study selection validity (Ampatzoglou *et al.*, 2019). Nevertheless, the quantitative approach is more common for mapping studies, like in this paper, and can also complement systematic reviews (Petersen *et al.*, 2008).
2. Selecting literature reviews and surveys for related works analysis. In this branch, after downloading search results from WoS, an initial set of all found research papers consisted of 1 166 references (see Table 3, row 2). The review of titles and keywords of these papers was performed by both authors to ensure internal validity (Ampatzoglou *et al.*, 2019) according to the inclusion (IC) and exclusion (EC) criteria as follows:
   - IC1: Include papers that are literature reviews, surveys or bibliometric analysis on fuzzy sets.
   - EC1: Exclude papers that analyse generic topics, such as fuzzy mathematical operations, fuzzy concepts, similarity measures, etc., without analysis of fuzzy sets evolution.
   - EC2: Exclude papers that contain relevant research keywords, but fuzzy sets and their extensions are not reviewed.
   - EC3: Exclude papers that contain a review of only one extension of fuzzy sets, or compare two fuzzy set extensions, since here we do not investigate properties of fuzzy sets.
   - EC4: Exclude papers that contain applications of fuzzy sets.

After excluding non-relevant papers for the related works analysis, a set of 95 relevant papers was obtained for further reading of their abstracts. The same IC and EC were applied when reading abstracts. Finally, 12 research papers were selected for full reading of their text. An initial set of papers, a set of excluded papers and a set of included papers are presented in GitHub (https://github.com/Jolantux13/A-Chronological-and-Bibliometric-Analysis-Fuzzy-Sets (accessed on February 2025)). The results of the related work analysis are presented in Table 1. Moreover, such well-defined qualitative IC and EC allow us to ensure the validity of the results (Alves *et al.*, 2010).

3.4. *Data Extraction and Keyword Map Development*

To answer the defined research questions and develop a keyword map on fuzzy sets, a bibliographic data mapping and visualization tool, VOSviewer (https://www.vosviewer.com/

(accessed on February 2025)) was employed. This tool implements the chosen techniques and allows us to perform co-word and co-citation analyses (Table 2).

A co-word analysis based on the developed keyword map helps enrich understanding of the thematic clusters and predict future research trends (Donthu *et al.*, 2021). In this research, notable keywords and future research directions (RQ2, RQ3) are identified. The words in the co-word analysis were derived from "All keywords". The co-word analysis assumes that words appearing together frequently have a thematic relationship with each other. Donthu *et al.* (2021) have identified downsides of co-word analysis, such as multiple contexts, a high level of abstraction, etc. To mitigate the potential downsides of co-word analysis and perform data cleaning (Van Eck and Waltman, 2019), the thesaurus (https://github.com/Jolantux13/A-Chronological-and-Bibliometric-Analysis-Fuzzy-Sets (accessed on February 2025)) was created by the authors of this paper according to the following rules:

1. Merging different spellings of words that essentially represent the same concept, like "complex fuzzy sets" and "complex fuzzy set", "correlation-coefficients" and "correlation coefficient", etc.;
2. Merging keywords with their abbreviations, like "multi-criteria decision making" and "MCDM", "fuzzy-set qualitative comparative analysis" and "fsQCA", etc.;
3. Merging synonyms, like "fuzzy logic" and "fuzzy set logic";
4. Excluding general keywords, like method, algorithm, etc., since they provide very general information, and the specificity of the resulting map increases when they are excluded.

The developed thesaurus includes 569 items for word merging or exclusion. Its inclusion into data extraction allows us to ensure the validity of the results (Ampatzoglou *et al.*, 2019). Finally, VOSviewer identified 40.371 keywords, 374 of which are found at least 20 times in the chosen 23 712 scientific papers.

A co-authorship analysis, which is used to examine intellectual collaboration among scholars (Donthu *et al.*, 2021; Acedo *et al.*, 2006; Cisneros *et al.*, 2018), was applied to analyse the relationship among countries (RQ4). The tools used for the current analysis are shown in Table 4.

Table 4
Tools for data processing and visualization.

Software/tools	Data analysis
VOSviewer	It is used for the Keyword Map Analysis (RQ2, RQ3) (Fig. 3, Fig. 4), Countries/Regions Participating in the Study Analysis (RQ4) (Fig. 8), artificial intelligence and fuzzy sets (RQ5) (Fig. 11), MCDM and fuzzy sets (RQ6) (Table 6)
Microsoft Excel	It is used for the evolution of papers in Fig. 2 (RQ1), visualization of co-occurrence matrixes of fuzzy set extensions (Fig. 5, Fig. 6, and Fig. 7). The co-occurrence of countries (Fig. 9). The detailed chronological analysis of countries participation in the fuzzy set research (Fig. 10).
Microsoft Visio	The flow diagram of the current analysis (Fig. 1).

3.5. *Validity Evaluation*

As proposed by the guidelines of Petersen *et al.* (2015), Wohlin *et al.* (2012), this analysis covers internal, construct, external and conclusion validity. Ampatzoglou *et al.* (2019) presented a more extended list of validity threats for the secondary studies. Some of them are described while presenting the methods of this analysis. Here we discuss only the four main threats to validity.

Construct validity refers to the operational measures for the concept being studied, i.e. fuzzy set extensions. Since we are interested in analysing all possible extensions of fuzzy sets, a general term "fuzzy set" was chosen for the search string without any specific limitation of the studied concept. Our research questions and search string may not be able to completely cover all papers on fuzzy set extensions. However, according to Donthu *et al.* (2021), we have obtained a sufficient set of papers (Table 3) for the analysis, and can provide valuable findings for researchers and practitioners with its overview.

Internal validity refers to whether an experimental condition affects the results. The individual researcher's bias influences the selection of literature reviews and surveys for the related works analysis (the second branch in Fig. 1, Section 3). To mitigate the bias of researchers, the authors of this paper clearly defined the search strategy, carefully examined titles and abstracts of primarily found papers and assessed the obtained results together. Also, the authors searched only one online digital library for the bibliometric analysis, as recommended in Donthu *et al.* (2021).

External validity refers to the generalizability of the results. Though we have limited the search results to Computer Science to reduce the final number of papers (the initial set contained 42 640 papers that would be manually difficult to process), it (∼58%) still remains sufficient and suitable for the bibliometric analysis and valid for other research areas. Other research areas actively involved in the development and application of fuzzy set extensions are Engineering (∼29%) and Mathematics (∼13%). However, their inclusion in the analysed set of papers only slightly influences the final results by adding only a few new fuzzy set extensions.

Conclusion validity deals with the degree to which conclusions are reasonable within the data collected, i.e. interpretive validity. In this paper, a potential threat to conclusion validity is the interpretation of the extracted data presented by keyword maps. To guarantee the conclusion's validity to a reasonable extent, we have reviewed other related bibliometric surveys on fuzzy set extensions. The conclusions drawn do not contradict the previous studies.

To ensure the replicability of this research, the authors have explicitly defined steps performed in the chronological and bibliometric analysis, developed a thesaurus, and provided evidence about those findings in GitHub.

4. Results

4.1. *Chronological Analysis (RQ1)*

Figure 2 shows the chronological distribution of the scientific papers published on the topic of fuzzy sets in the period from 1990 to September 2025 (RQ1).

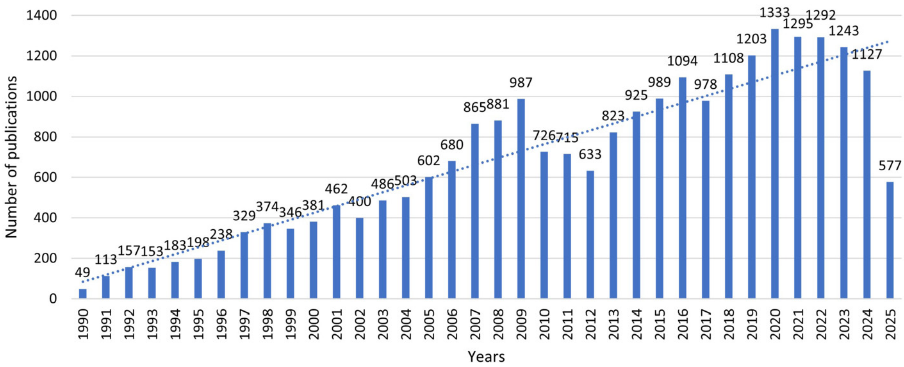

Fig. 2. Chronological distribution of the published papers on fuzzy sets (RQ1).

Although the trendline in the number of publications is increasing every year (in Fig. 2, dotted line), the growth is wavy. In the last 14 years (2012–2025), the number of publications (14 620, i.e. 60%) exceeded the total number of publications from 1990 to 2011 (i.e. 22-year interval) (9 828, i.e. 40%). This indicates the increasing interest of researchers in fuzzy sets over the last period. Also, two visible waves, with peaks in 2009 (987 publications) and 2020 (1 333 publications), are observed in Fig. 2.

4.2. *Keyword Map Analysis (RQ2 and RQ3)*

A keyword map of fuzzy sets was developed and visualized with VOSviewer (Fig. 3). It should be analysed from two perspectives: 1) year perspective, and 2) occurrence perspective. From a year perspective, VOSviewer coloured and visualized keywords according to Average Publication Year (APY) of the paper, in which a keyword occurs (Van Eck and Waltman, 2019). In the Fig. 3 legend, the APY interval from the oldest to the latest is coloured from blue to yellow. Also, larger bubbles represent more frequently occurring keywords. Smaller bubbles represent keywords found less frequently in the analysed articles. Based on the colour and size of the bubbles, we can see whether the keyword is relevant in today's context. For example, small and yellow bubbles represent newly appearing keywords in the analysed articles, while small and blue bubbles show keywords that are disappearing and have been analysed for a long time and little. Note that this explanation should be used to decode all the keyword maps presented in this paper (see Figs. 3–7).

From the occurrence perspective, VOSviewer identified, sized and visualized keywords according to their density of occurrence. Keywords with the biggest bubbles have the largest occurrence.

Currently, the newest fuzzy set extensions (RQ2) found in the keyword map of fuzzy sets (Fig. 3) are as follows: Fermatean fuzzy set (32, APY = 2021.32), T-spherical fuzzy set (25, APY = 2021), probabilistic hesitant fuzzy set (21, APY = 2020.7), spherical fuzzy set (71, APY = 2020.51) and q-rung orthopair fuzzy set (91, APY = 2020.30). Their occurrence comparing to the whole number of the analysed papers (23 712) is rather small,

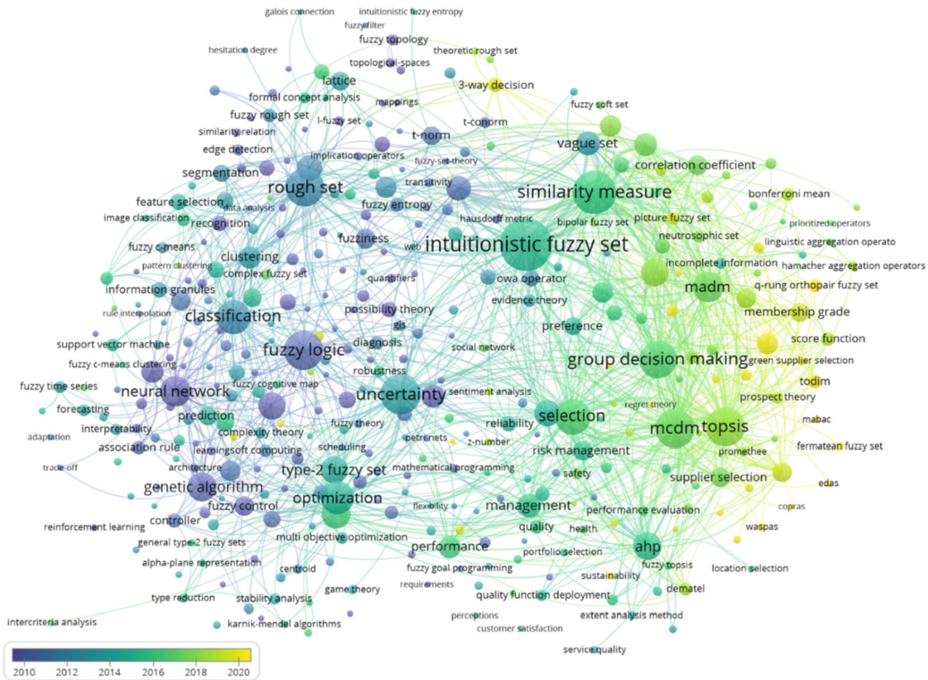

Fig. 3. Keyword map on fuzzy sets according to years.

however, observable, which shows the future direction of the analysed topic (RQ3). Also, among the newest keywords there are multi-criteria decision making (MCDM) methods, like WASPAS, EDAS, COPRAS and Best-Worst Method (BWM), which shows a future direction in the application of fuzzy sets (RQ3).

From this initial keyword map of fuzzy sets, we have extracted 65 extensions of fuzzy sets (RQ2), which are visualized by VOSviewer in Fig. 4 and summarized in Table 5. From this analysis, we can see that five most occurring fuzzy sets are as follows: intuitionistic fuzzy set (1 513, APY = 2015.22), interval type-2 fuzzy set (501, APY = 2016.20), type-2 fuzzy set (463, APY = 2014.22), hesitant fuzzy set (431, APY = 2017.97), and interval-valued fuzzy set (290, APY = 2012.18). These sets are presented by the biggest bubbles. Their APYs are older (i.e. green coloured), since they are the main and oldest extensions of fuzzy sets used in the research papers until now.

It is also worth paying attention to the medium and smaller bubbles in blue. These bubbles represent fuzzy set extensions, which APY tend to be oldest, and they are less mentioned in current papers, like convex fuzzy set (30, APY = 2001.97), eigen fuzzy set (8, APY = 2005.5), balanced fuzzy set (6, APY = 2005.5), l-fuzzy set (53, APY = 2006.74), three-dimensional fuzzy set (6, APY = 2008.83), and normal fuzzy set (5, APY = 2010.2), etc. They have been less used in current papers or have evolved into other fuzzy set extensions.

Table 5 presents all found fuzzy set extensions and the most occurring keywords with them. Keywords, like multi-criteria decision making (MCDM), multi-attribute decision making (MADM), and group decision making, are found with the majority of fuzzy set extensions. Therefore, they are generalized into a "decision-making" keyword for the sake of simplicity. The whole version of Table 5 with relevant references is presented in GitHub (https://github.com/Jolantux13/A-Chronological-and-Bibliometric-Analysis-Fuzzy-Sets (accessed on February 2025)).

In Table 5, a "decision making" (dm) keyword was found within 41 fuzzy set extensions. Additionally, 19 particular MCDM methods are observed 81 times (i.e. several MCDM methods can be repeated within the same fuzzy set extension). A keyword "select" variations, like "supplier selection", "facility location selection", etc., are found within 14 fuzzy set extensions. Similarly, a keyword "diagnos" variations, such as "medical diagnosis" (found 8 times), "fault diagnosis" (found 1 time), "diagnosis" (found 1 time), are observed within 10 fuzzy set extensions. The most popular keywords found within fuzzy set extensions are as follows: decision making (41), topsis (31), correlation coefficient (18), vikor (15), todim (11), medical diagnosis (8) and supplier selection (8).

Also, according to the fuzzy set extensions occurrence (see Fig. 3), the most commonly occurring fuzzy set extensions, like type-2 fuzzy set, Pythagorean fuzzy set, intuitionistic fuzzy set, etc., have large communities of keywords (Table 5). These rich keyword communities show the popularity of these fuzzy set extensions and their wide applicability. Contrary, less occurred fuzzy set extensions, such as Fermatean fuzzy set, Pythagorean fuzzy soft sets, etc., have fewer related keywords. These poor or middle keywords communities indicate the lower applicability of these fuzzy set extensions.

We can summarize that MCDM tasks remain the most important and significant real-life problems with uncertain features of input data due to the complex information collection process and influence of different interests of stakeholders (Beg *et al.*, 2022). So, to present the information and handle with the imprecision of data, different fuzzy set extensions and their theories have recently been developed.

Based on Fig. 4, the newest fuzzy set extensions (RQ2) with APY > 2020 are presented in Table 6. Their occurrences comparing to the whole number of the analysed papers (21 379) are rather small, which shows a future direction and new extensions of fuzzy sets (RQ3). Moreover, the newest APYs of those fuzzy set extensions show a greater interest in them. Among fuzzy set extensions that have engaged greater interest from scientists in the analysed papers, we observe the following sets: q-rung orthopair fuzzy set, spherical fuzzy set and Fermatean fuzzy set.

Links in the keyword map show relationships between the keywords. These links can be characterized by the distance and the strength, showing the number of papers in which two keywords occur together. Thicker lines and shorter distances represent stronger relationships between the keywords.

Based on the developed keyword map of fuzzy sets (Fig. 4), it was determined that among all found extensions of fuzzy sets, there are only 8.60% relationships and their link strength falls in the interval [1; 51]. In addition, 44.02% of existing links have a strength equal to 1,47.85% have strengths ranging in the interval (1; 10], and only 8.13% have strengths ranging in the interval (10; 51]. Therefore, for better visualization, three co-occurrence matrices of fuzzy set extensions are developed as follows: 1) with link

Table 5
The found fuzzy set extensions (RQ2, RQ3)[a].

No.	FS[b] extensions	Keywords
1	Atanassov intuitionistic fs	dm, terminological difficulties
2	Axiomatic fs	semantic interpretation, eigenfaces
3	Balanced fs	fuzzy neural network, learning
4	Bipolar fs	graph representation, relational analysis method, terminological difficulties
5	Complex fs	granular computing, graph representation, machine learning, particle swarm optimization
6	Complex intuitionistic fs	dmb
7	Complex pythagorean fs	dm
8	Complex q-rung orthopair fs	topsis
9	Convex fs	relational database
10	Dynamic fs	dm, image segmentation, medical diagnosis, moving object detection
11	Dual hesitant fs	dm, topsis, correlation coefficient
12	Dual hesitant fuzzy soft set	dm, correlation coefficient, expert system, medical diagnosis
13	Eigen fs	image analysis, convex combination
14	Fermatean fs	dm, topsis
15	Fuzzy multiset	information fusion, medical diagnosis, terminological difficulties
16	Fuzzy rough set	information system, machine learning, object recognition, particle swarm optimization, 3-way decision
17	Fuzzy soft set	normal parameter reduction
18	General type-2 fs	karnik-mendel algorithm, computing with words, controller
19	Hesitant fuzzy linguistic term set	topsis, vikor, dm
20	Hesitant fs	dm, medical diagnosis, pattern recognition, quality function, supplier selection, todim, topsis, vikor, 3-way decision, big data
21	Hesitant fuzzy soft set	topsis, dm
22	Interval neutrosophic hesitant fs	dm, topsis, vikor, correlation coefficient
23	Interval neutrosophic set	dm, correlation coefficient
24	Interval type-2 fs	karnik-mendel algorithm, dm, mobile robot, particle swarm optimization, perceptual computing, qualiflex, risk management, software development, stability analysis, supplier selection, topsis, vikor, best-worst method, c-means algorithm, controller, data envelopment analysis, dematel, dynamic system, edge detection, facility location selection, fmea, green supplier selection, construction
25	Interval-valued dual hesitant fs	topsis, correlation coefficient, dm
26	Interval-valued fs	terminological difficulties, topsis, dm
27	Interval-valued hesitant fs	dm, pattern recognition, topsis, vikor, correlation coefficient, green supplier selection
28	Interval-valued intuitionistic fs	dm, pattern recognition, programming, risk management, topsis, vikor, ahp, supplier selection
29	Interval-valued pythagorean fs	dm, topsis
30	Interval-valued q-rung orthopair fs	topsis, 3-way decision, aras, fmea, dm
31	Interval-valued spherical fs	topsis, dm
32	Intuitionistic fuzzy rough sets	minimization
33	Intuitionistic fs	expert system, fault diagnosis, fmea, fuzzy c-mean, fuzzy clustering, fuzzy neural network, fuzzy time series, anp, genetic algorithm, gra, image processing, information system, fuzzy c-means, mabac, magnetic resonance imaging, mathematical programming, medical diagnosis, multimoora, particle swarm optimization, pattern recognition, programming, promethee, quality, recommender system, renewable energy, risk management, 3-way decision, supplier selection, supply chain, todim, topological spaces, topsis, vendor selection, vikor, correlation coefficient, data envelopment analysis, data mining, decision support system, dematel, electre, dm

(*continued on next page*)

Table 5
(*continued*)

No.	FS[b] extensions	Keywords
34	Intuitionistic fuzzy soft set	gra, dm
35	l-fuzzy set	qualitative reasoning, topological spaces
36	Linguistic hesitant fs	quality, todim, topsis, vikor, best-worst method, correlation coefficient, dm
37	Linguistic intuitionistic fs	topsis, decision support model, dm, sentiment analysis, cognition
38	Linguistic pythagorean fs	topsis, gra, dm
39	m-polar fs	topsis, disease, fuzzy concept lattice, graph representation, dm
40	Multi-fuzzy set	diagnosis
41	Multi-valued neutrosophic set	qualiflex, todim, correlation coefficient, electre, dm
42	n-dimensional fs	cloud computing, fuzzy negations, information fusion
43	Neutrosophic set	supplier selection, topsis, vikor, dm, image segmentation
44	Neutrosophic soft set	todim, topsis, vikor, correlation coefficient, edas, dm
45	Normal fs	todim, correlation coefficient, dm
46	Paired fs	terminological difficulties
47	Picture fs	relational analysis method, todim, topsis, vikor
48	Pythagorean fs	renewable energy, risk management, service quality, supplier selection, sustainability, todim, topsis, vikor, waspas, codas, conflict analysis, copras, correlation coefficient, deep learning, dematel, edas, gra, green supplier selection, linmap, market volatility, moora, occupational-health, promethee
49	Pythagorean fuzzy soft sets	topsis, ahp, correlation coefficient, dm
50	Pythagorean hesitant fs	qualiflex, topsis, dm
51	Polygonal fs	rule interpolation, sparse fuzzy rule-based systems
52	Probabilistic fs	time series prediction, c-means algorithm, k-means clustering, controller
53	Probabilistic hesitant fs	todim, vikor, correlation coefficient, dm
54	q-rung orthopair fs	quality, todim, topsis, vikor, 3-way decision, correlation coefficient, mabac, dm, supplier selection
55	Random fs	c-means algorithm, machine learning
56	Rough fs	3-way decision, decision support system, gaussian kernel, granular computing, incremental learning, information system
57	Simplified neutrosophic set	correlation coefficient, dm, medical diagnosis
58	Single valued neutrosophic set	topsis, ahp, correlation coefficient, dm, power average
59	Spherical fs	todim, correlation coefficient, dm
60	Three-dimensional fs	distributed parameter system, stability analysis
61	Type-1 fs	computing with words, facility location selection, karnik-mendel algorithm, dm, perceptual computer, topsis
62	Type-2 fs	vikor, artificial intelligence, data envelopment analysis, data mining, edge detection, fuzzy ahp, fuzzy c-mean, fuzzy image processing, granular computing, hidden markov models, image processing, inference system, dm, medical diagnosis, mobile robot, ontology, particle swarm optimization, regression model, risk management, social network, supplier selection, terminological difficulties
63	Typical hesitant fs	correlation coefficient, dm
64	t-spherical fs	topsis, correlation coefficient, dm, medical diagnosis, pattern recognition
65	z-number	ahp, delphi, dm, mobile robot, qualiflex, sustainability, topsis

[a] Note that all terms in Table 5 are presented as they are extracted from WoS and processed by VOSviewer.
[b] Fuzzy set – fs, decision making – dm.

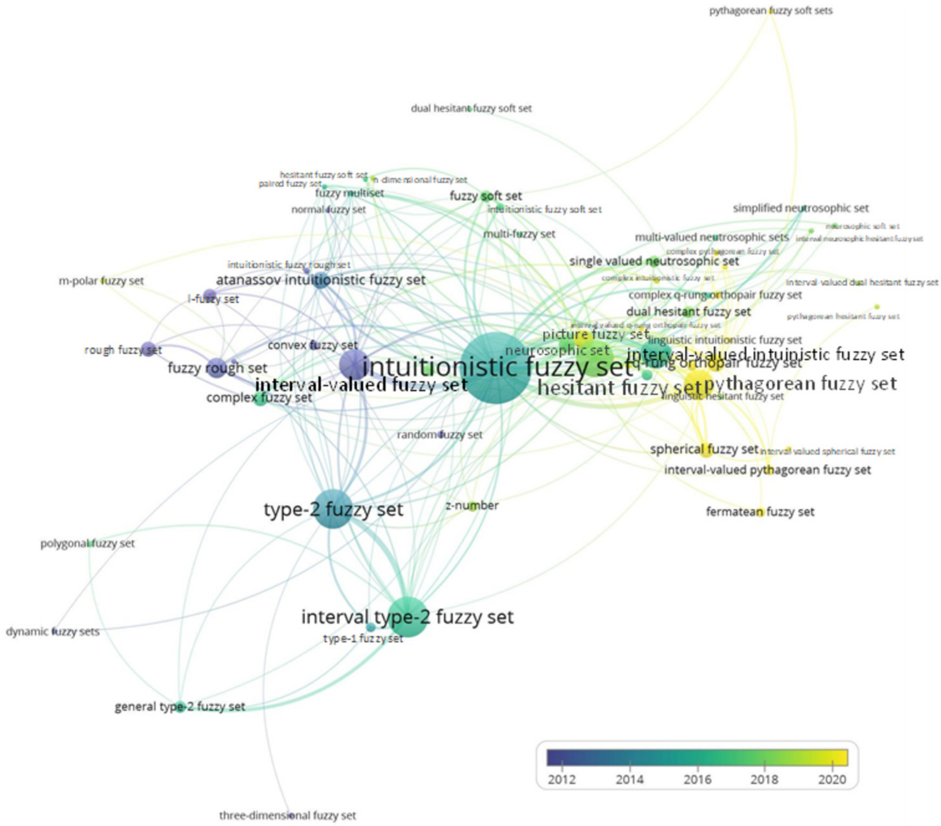

Fig. 4. Keyword map on fuzzy set extensions according to years (RQ2).

Table 6
The newest extensions of fuzzy sets found in the keyword map.

Keyword	Occurrence	APY
Fermatean fs	32	2021.32
Interval-valued spherical fs	8	2021.13
Complex pythagorean fs	9	2021
Complex q-rung orthopair fs	17	2021
Interval-valued q-rung orthopair fs	6	2021
t-spherical fs	25	2021
Probabilistic hesitant fs	29	2020.57
Spherical fs	71	2020.51
Pythagorean fuzzy soft set	5	2020.5
q-rung orthopair fs	96	2020.26
Linguistic pythagorean fs	5	2020.2

strengths 1 (Fig. 5); 2) having link strengths in the interval (1; 10] (Fig. 6); and 3) with link strengths in the range (10; 51] (Fig. 7). Note that these three matrices are symmetric. However, for better visualization, only one part of the symmetry is presented in Fig. 5,

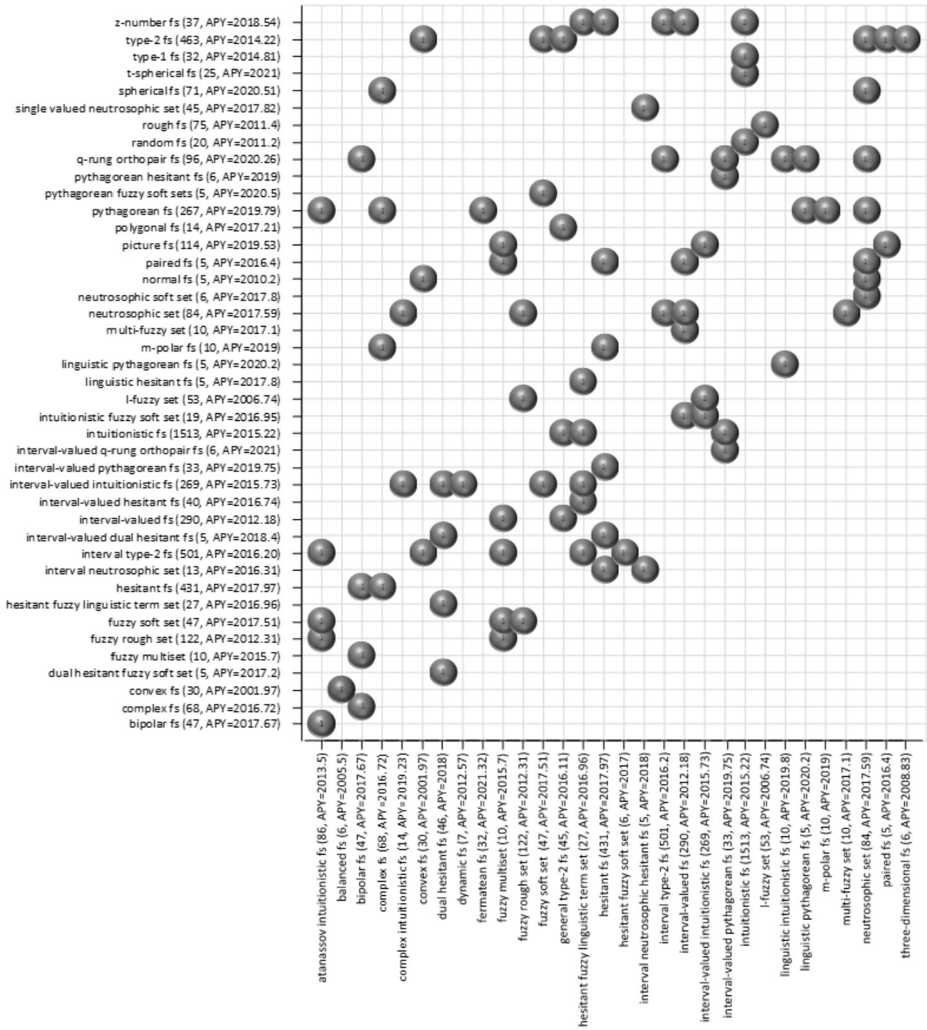

Fig. 5. The co-occurrence matrix of fuzzy set (fs) extensions with link strengths 1.

Fig. 6 and Fig. 7. In Fig. 5, the axes present fuzzy set extensions, and the bubbles – their links with strength 1. It is observed that such links exist between fuzzy set extensions with different occurrences and APY, which are shown in the brackets after the title of a fuzzy set extension. The fuzzy set extensions with the most links having a link strength of 1 are as follows: neutrosophic set (7), hesitant fuzzy set (6), type-2 fuzzy set (6), q-rung orthopair fuzzy set (6), Pythagorean fuzzy set (6). This finding shows the greater popularity and development of the found sets in the analysed papers.

In Fig. 6, the axes present fuzzy set extensions, the bubbles – links with strengths in the range (1; 10]. The bubbles are sized and coloured according to the link strength (Fig. 6).

It is observed that such links exist between fuzzy set extensions with different occurrences and APY, which are shown in the brackets after the title of a fuzzy set extension. As

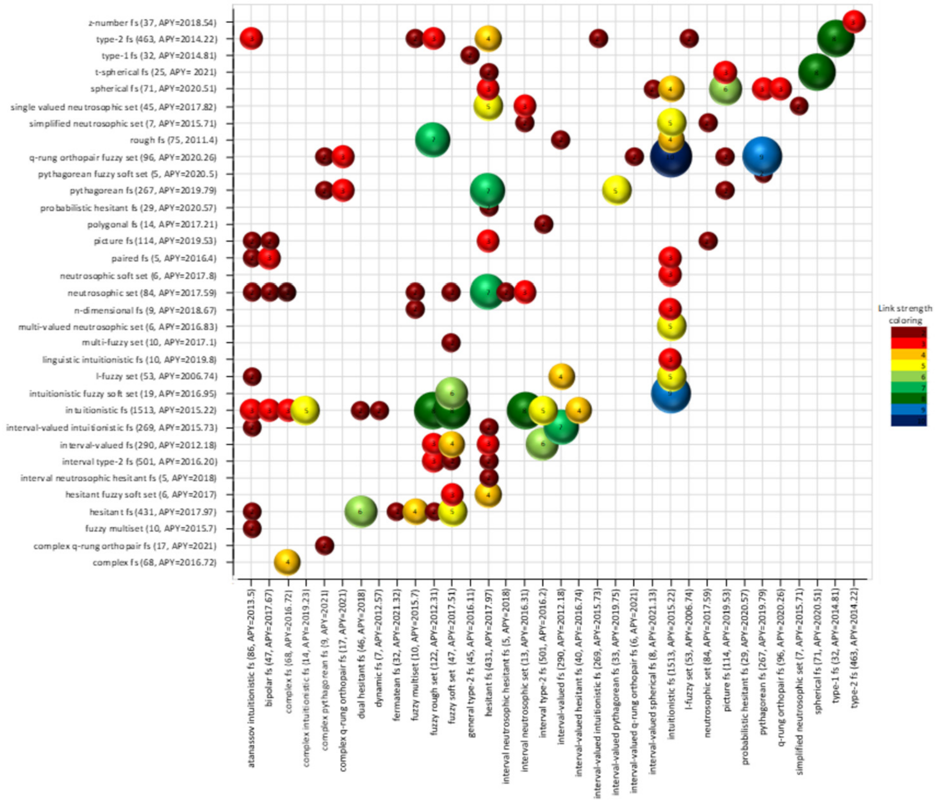

Fig. 6. The co-occurrence matrix of fuzzy set extensions with link strengths (1; 10].

can be seen, the majority (43%) of the bubbles represent a link strength equal to 2. The size distribution of the bubbles is as follows: link strength equal to 3–24%; link strength equal to 4–9%; link strength equal to 5–8%; link strength equal to 6 and 7–4%; link strength equal to 8–5%; link strength equal to 9–2%; and link strength equal to 10–1%. From Fig. 6, we found that the following fuzzy sets have the biggest link strength in the range (1; 10]: intuitionistic fuzzy set – q-rung orthopair fuzzy set (10), pythagorean fuzzy set – q-rung orthopair fuzzy set (9), intuitionistic fuzzy set – intuitionistic fuzzy soft set (9), intuitionistic fuzzy set – fuzzy rough set (8), intuitionistic fuzzy set – fuzzy soft set (8), intuitionistic fuzzy set – interval neutrosophic set (8), spherical fuzzy set – t-spherical fuzzy set (8), type-1 fuzzy set – type-2 fuzzy set (8). The found link strengths between fuzzy sets in most cases can be explained by the fact that some fuzzy sets are extensions of other fuzzy sets, like intuitionistic fuzzy set and intuitionistic fuzzy soft set, spherical fuzzy set and t-spherical fuzzy set, etc.

In Fig. 7, the axes present fuzzy set extensions, and the bubbles – links with strengths in the range (10; 51]. The bubbles are sized and coloured according to the link strength. It is observed that such links exist mainly between fuzzy set extensions with large occurrence and older APY. Exceptions are as follows: intuitionistic fuzzy rough set with small

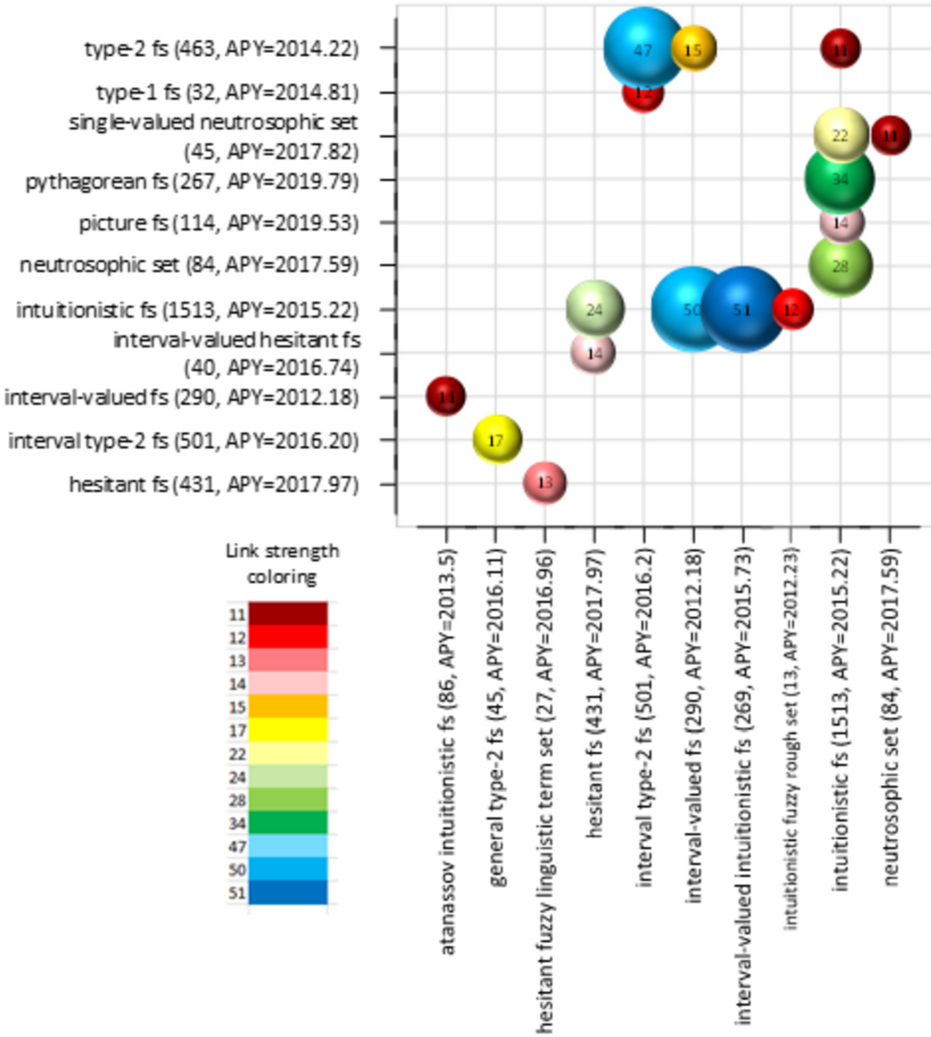

Fig. 7. The co-occurrence matrix of fuzzy set extensions with link strengths (10; 51].

occurrence, and picture fuzzy set and Pythagorean fuzzy set with newer APY. Therefore, their link strengths are weaker compared to other link strengths. Moreover, in Fig. 7 we can observe the same trend – the link between fuzzy sets in most cases is explained by the fact that some fuzzy sets are extensions of other fuzzy sets, such as type-2 fuzzy sets, interval type-2 fuzzy sets (Snyder, 2019).

4.3. *Countries/Regions Participating in the Study (RQ4)*

VOSviewer has created a country/region occurrence map from the analysed papers as presented in Fig. 8. VOSviewer has identified 124 countries, of which 93 countries have

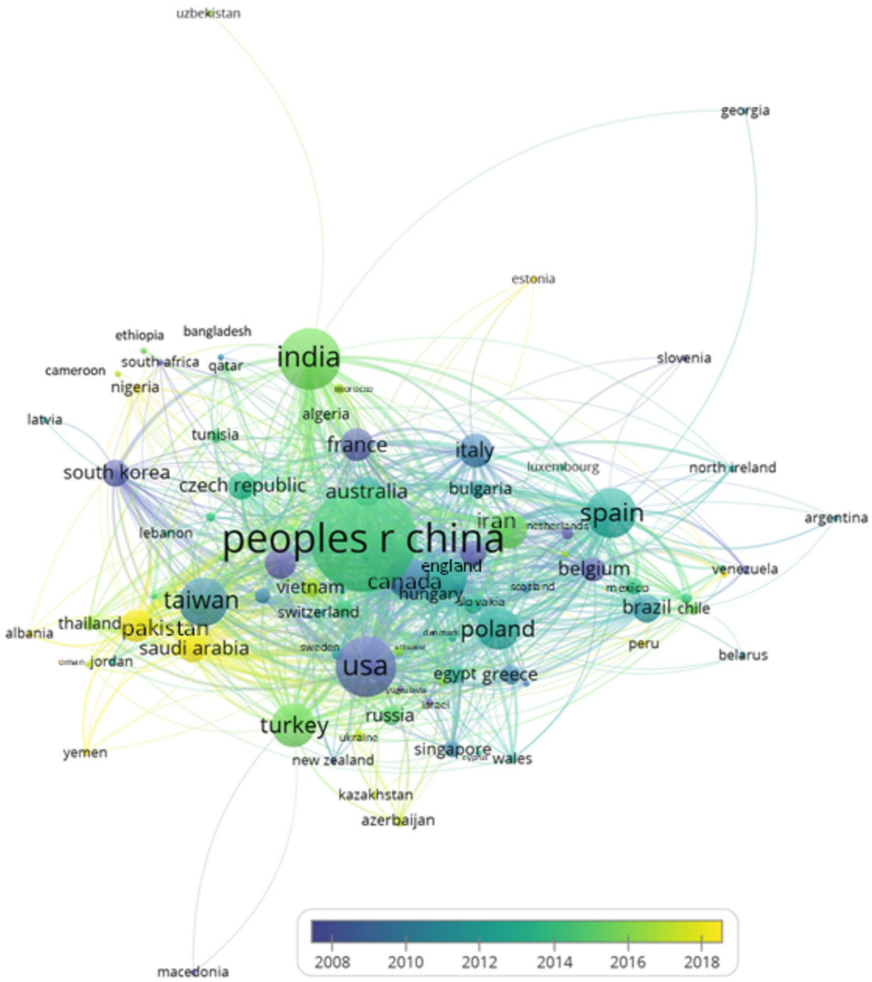

Fig. 8. The country map on the fuzzy set topic (note, countries are titled as they appear in WoS).

published at least 5 papers on the analysed topic. Note that WoS indicates not only countries but also regions. For the sake of simplicity, below we write "countries".

The five most popular countries found in the map are as follows: China (6 464, APY = 2014.3), India (2 281, APY = 2015.31), USA (2 125, APY = 2008.48), Taiwan (1 383, APY = 2010.23) and Spain (1 369, APY = 2012.27), where the number near the country indicates the number of papers published by the country and APY. The APY of those countries is older, since they have started publishing papers on fuzzy sets earlier. The newer countries-contributors to the fuzzy set topic are as follows: Yemen (18, APY = 2021.44), Iraq (70, APY = 2019.61), Palestine (7, APY = 2019.5), Pakistan (730, APY = 2019.02), Ecuador (9, APY = 2018.87), Nigeria (39, APY = 2018.5), Estonia (6, APY = 2018.5), Saudi Arabia (471, APY = 2018.37), and Peru (6, APY = 2018.2). As can be seen in Fig. 8, links represent relationships between countries. Thicker links and shorter

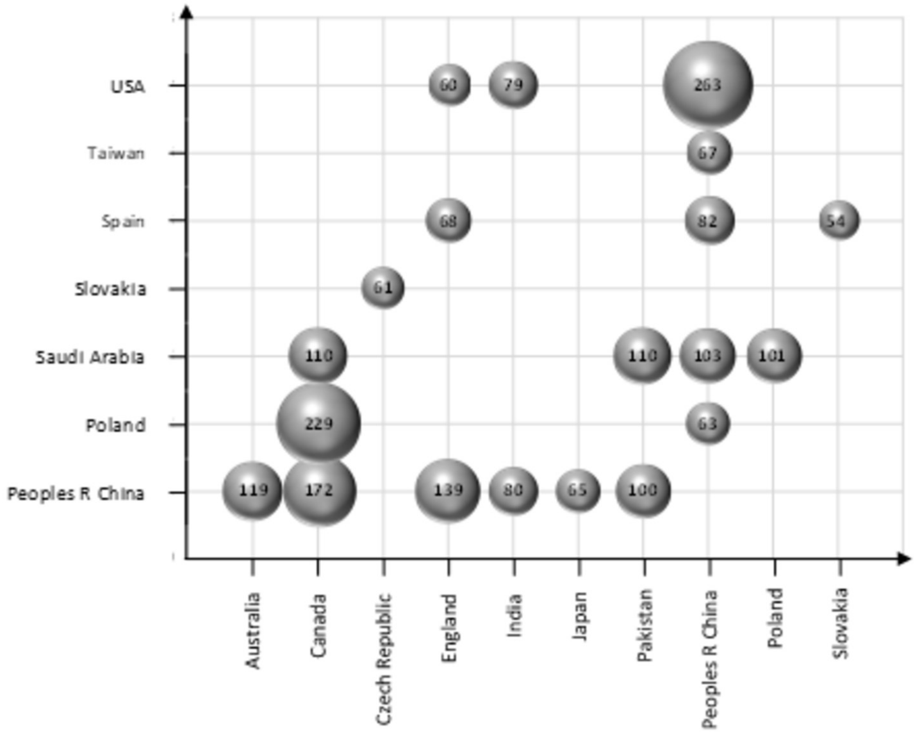

Fig. 9. The co-occurrence of countries with link strength in the interval (50; 263].

distances present stronger relationships. Based on the developed country map of fuzzy sets (Fig. 10), all link strengths range in the interval [1; 263] as follows: 38.04% – equal to 1, 46.82% – (1; 10], 13.12% – (10; 50], and 2.02% – (50; 263].

The co-occurrence of countries with link strengths in the interval (50; 263] is presented in Fig. 9. They can be understood as strong relationships between countries in fuzzy set research. It can be seen that the three strongest relationships are between the USA and China (263), Poland and Canada (229), and Canada and China (172). Links with strength (10; 50] can be treated as developing relationships, for example, between South Korea and Poland (40), Spain and Italy (31), Turkey and England (29), etc. Links with strength (1; 10] can be understood as weak, and it is not clear how they will develop. Also, from these relationships, we can see that the countries with the most links are as follows: USA (66), China (68), England (62), India (61), and Spain (60).

The detailed chronological analysis of 15 mostly publishing countries participating in the fuzzy set research is presented in Fig. 10 (RQ1, RQ4). According to this visualization, all 15 countries can be classified into three classes according to their involvement into the research on fuzzy sets as follows: 1) from the beginning of the period and having small increase of papers during the entire period (like England, USA, Japan); 2) from the beginning of the period and having significant increase of papers at a certain point of time (like China, India); 3) from a certain point of the period and having some increase of

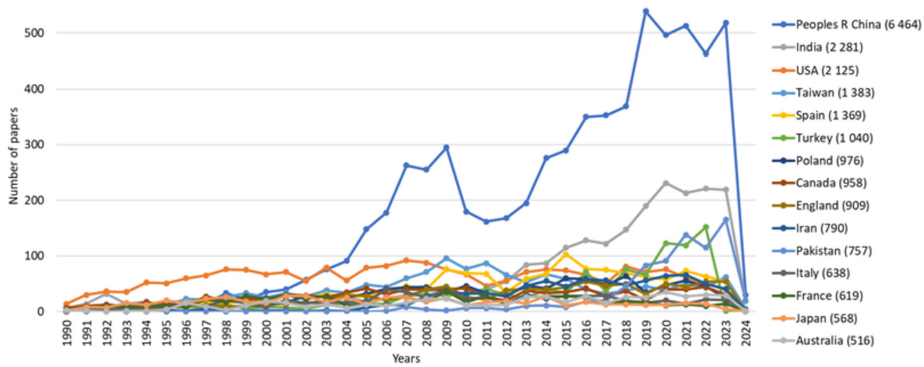

Fig. 10. The detailed chronological analysis of countries' participation in the fuzzy set research.

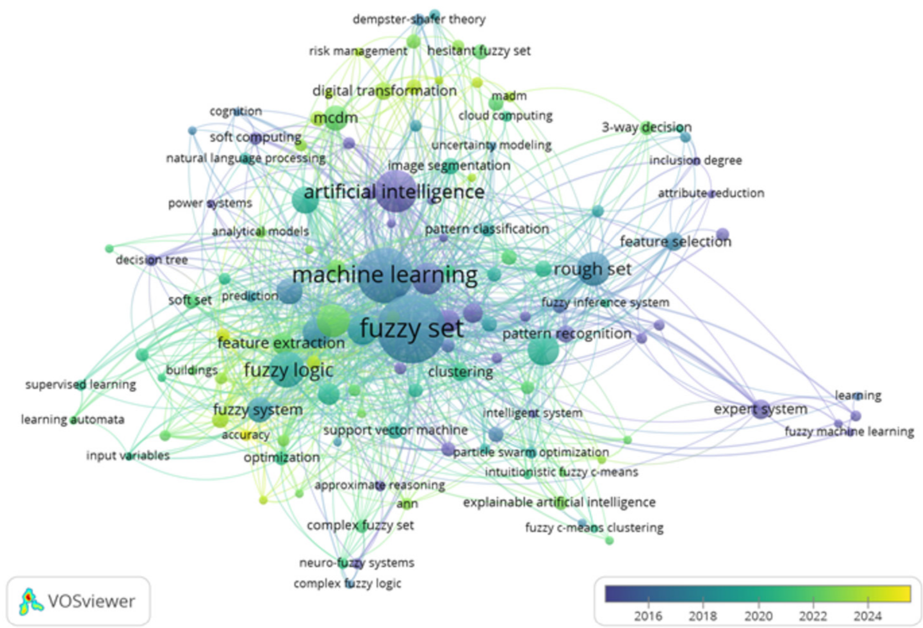

Fig. 11. Relationship between fuzzy sets and artificial intelligence (RQ5).

papers (like Pakistan, Spain). Fig. 10 shows that the intensive participation of China has a significant impact on the overall publication of papers on fuzzy sets (Fig. 2).

4.4. *Fuzzy Sets, Artificial Intelligence and Other Decision Making Methods (RQ5 and RQ6)*

To answer this research question, we have explored the found keywords from Fig. 3 and Table 4 from the artificial intelligence perspective. The obtained results are presented in Fig. 11. The developed Fig. 11 shows significant relationship between fuzzy sets and artificial intelligence (found in 60 papers of 712 analysed in this topic), machine learning (found in 109 papers of 712 analysed in this topic), neural networks (found in 28 papers of 712 analysed in this topic) and deep learning (found in 37 papers of 712 analysed in this topic). The relationship between fuzzy sets and artificial intelligence is long-standing and old since its (APY = ∼2013). However, currently this concept is converting into explainable artificial intelligence (API = ∼2023). The same is a long-standing and old relationship exists between fuzzy sets and machine learning (API = ∼2016) and neural networks (API = ∼2018), where the newest development is fuzzy neural networks (API = ∼2024). The relationship between fuzzy sets and deep learning is a new one and current (API = ∼2023). In this context, they all form the new concept of digital transformation (API = ∼2023). Alkan and Kahraman (2023b) present the concept of a "digital supply chain" by proposing an interval-valued Fermatean fuzzy analytic hierarchy process method for the digital transformation in the supply chain. Otay *et al.* (2023) introduced the Interval Valued Circular Intuitionistic Fuzzy Analytic Hierarchy Process method and applied it to evaluate digital transformation projects with a complex nature. In Yang *et al.* (2024), authors proposed to perform the evaluation of digital transformation effectiveness in retail enterprises through multi-attribute group decision making, employing a spherical fuzzy number EDAS and utilising the Hamming distance and logarithmic distance measures. Mishra *et al.* (2024) proposed to rank the alternatives by applying a picture fuzzy extension of the alternative ranking order model, accounting for a two-step normalization approach. The applicability of their approach was illustrated by assessing digital transformation in higher education institutions.

Also, from Fig. 11 we can observe that the conjunction of fuzzy sets, artificial intelligence, and decision making is highly related. These two clusters of artificial intelligence and decision making in the form of different MCDM methods can be viewed in Fig. 3, also. The most popular MCDM methods used with fuzzy sets are as follows in Table 7. We have found the main 17 MCDM methods, which are extended with a particular type of fuzzy set, to handle the uncertainty and imprecision inherent in many decision-making problems. As can be seen, the most used and extended with fuzzy sets are TOPSIS, AHP, VIKOR, TODIM and ELECTRE. Those methods have the biggest number of different extensions with fuzzy sets, while others have been less extended because of their narrower applicability. Moreover, some methods are used in conjunction to solve some decision-making problems, like COPRAS and EDAS (Ganie *et al.*, 2024a), MOLTIMOORA and BWM in Yang *et al.* (2020), Nemati (2024); SAW and ARAS in Gül (2021b); AHP and TOPSIS in Alkan and Kahraman (2023a), Swethaa and Felix (2023), etc.

Emerging information technologies, such as cloud computing, big data, and artificial intelligence, have changed MCDM methods by evolving them into data-driven MCDM (DDMCDM) methods, which apply different machine learning techniques to process a large amount of data when making decisions (Fu *et al.*, 2021). Combination of MCDM methods with machine learning techniques faces two main challenges (Fu *et al.*, 2021) as follows: 1) how to select the appropriate machine learning algorithm for a specific MCDM problem, and 2) how to build a bridge between the appropriate machine learning algorithm and the evidential reasoning approach.

Table 7
MCDM methods applied with fuzzy sets (RQ6).

No.	MCDM method	Found some fsa extensions for MCDM methods	No. of occurrences
1.	AHP	intuitionistic fs (Alkan and Kahraman, 2023a); intuitionistic dense fs (Swethaa and Felix, 2023); picture fs (Kahraman, 2024); type-2 fs (Kahraman *et al.*, 2014); 3d spherical fs (Monika and Sangwan, 2022); triangular fermatean fs (Fahmi, 2023)	519
2.	ARAS	fermatean fs (Gül, 2021b); linear diophantine fs (Aydoğdu *et al.*, 2024); spherical fs (Gül, 2021a);	34
3.	BWM	hesitant fs (Yang *et al.*, 2020); hesitant fs and probabilistic hesitant fs (Wang *et al.*, 2022); p, q, r-spherical fs (Rahim *et al.*, 2025); interval type-2 trapezoidal fs (Nemati, 2024)	88
4.	CoCoSo	hesitant fermatean fs (Nemati, 2024); circular spherical fs (Ahmad *et al.*, 2024); spherical fs (Ghoushchi *et al.*, 2022); intuitionistic fs (Manish and Kumar, 2025)	47
5.	CODAS	hesitant fs, intuitionistic fs, and spherical fs (Kahraman *et al.*, 2020); pythagorean fs (Khan *et al.*, 2023; Alkan and Kahraman, 2024); hesitant fs, intuitionistic fs, and spherical fs (Kahraman *et al.*, 2020)	62
6.	COPRAS	hesitant fs, intuitionistic fs, and spherical fs (Kahraman *et al.*, 2020); pythagorean fs (Thao and Smarandache, 2019); spherical fs (Ganie *et al.*, 2024a); intuitionistic fs (Zavadskas *et al.*, 2014)	65
7.	EDAS	spherical fs (Yang *et al.*, 2024); hesitant fs, intuitionistic fs, and spherical fs (Ganie *et al.*, 2024a)	100
8.	ELECTRE	circular spherical fs and disc spherical fs (Ashraf *et al.*, 2023); hesitant interval-valued fs (Wang *et al.*, 2019); hesitant fs, intuitionistic fs, and spherical fs (Kahraman *et al.*, 2020); interval-valued fs (Vahdani and Hadipour, 2011); q-rung picture fs (Pinar and Boran, 2022)	108
9.	MABAC	hesitant fs, intuitionistic fs, and spherical fs (Kahraman *et al.*, 2020); fermantean fs (Aydin, 2021)	69
10.	MULTI-MOORA	hesitant fs, intuitionistic fs, and spherical fs (Kahraman *et al.*, 2020); hesitant fs (Yang *et al.*, 2020); interval type-2 trapezoidal fs (Nemati, 2024)	83
11.	PROMETHEE	hesitant fs, intuitionistic fs, and spherical fs (Kahraman *et al.*, 2020); pythagorean fs (Molla *et al.*, 2021); interval type-2 fs (Chen, 2014)	88
12.	SAW	fermatean fs (Gül, 2021b); interval-valued fs (Chen, 2012); pythagorean fs (Gocer, 2022)	15
13.	SWARA	spherical fs (Ghoushchi *et al.*, 2022); fermatean fs (Aydoğan and Ozkir, 2024); spherical fs (Ghoushchi *et al.*, 2022)	58

(*continued on next page*)

Table 7
(*continued*)

No.	MCDM method	Found some fsa extensions for MCDM methods	No. of occurrences
14.	TOPSIS	interval-valued spherical fs (Kutlu Gündoğdu and Kahraman, 2019b); fermatean fs (Ganie, 2022); q-rung orthopair fs (Ünver and Olgun, 2023); t-spherical fs (Karamaz and Karaaslan, 2025); intuitionistic fs (Alkan and Kahraman, 2023a); intuitionistic dense fs (Swethaa and Felix, 2023); q-rung orthopair multi-fuzzy soft set (Vimala *et al.*, 2023); hesitant fs, intuitionistic fs, spherical fs (Kahraman *et al.*, 2020); spherical fs (Kutlu Gündoğdu and Kahraman, 2019a); triangular fermatean fs (Fahmi, 2023)	1.280
15.	TODIM	hesitant fs, intuitionistic fs, and spherical fs (Kahraman *et al.*, 2020); triangular interval type-2 fs (Tian *et al.*, 2023); hesitant fs (Song *et al.*, 2023)	217
16.	VIKOR	fermatean fs (Gül, 2021b); hesitant fs, intuitionistic fs, and spherical fs (Kahraman *et al.*, 2020); spherical fs (Kahraman *et al.*, 2020)	353
17.	WASPAS	hesitant fs, intuitionistic fs, and spherical fs (Kutlu Gündoğdu and Kahraman, 2019a); complex fs (Khan *et al.*, 2024); intuitionistic fs (Alrasheedi *et al.*, 2023; Deb *et al.*, 2023)	84

Also, authors Ma and Li (2024) stressed the challenges of the supplier quality evaluation because of a large amount of qualitative and quantitative data, multi-dimensional attributes, a large number of items and suppliers, etc. Even though a number of combinations of MCDM methods (Ma and Li, 2024; Zavadskas *et al.*, 2016) have been proposed, not all existing aggregation methods are efficient/effective for the supplier evaluation problem with high dimensions and big data (Ma and Li, 2024). Machine learning techniques may be applicable here to overcome the drawbacks of existing MCDM methods, especially when a large quantity of real business data is available (Ma and Li, 2024). Therefore, Ma and Li (2024) proposed to evaluate the alternatives according to the selected criteria by applying six MCDM methods, like TOPSIS, VIKOR, PROMETHEE with a usual type preference function, PROMETHEE with a Gaussian type preference function, MOORA with the ratio system, and MOORA with the reference approach, and then, construct an aggregated model, in which disagreements are integrated with the help of loss functions from machine learning, like L2 loss, L1 loss, Log-cosh loss, and M-estimator loss. Finally, authors make a comparison analysis to find the appropriate loss function and the final aggregated evaluation.

5. Discussion

The motivation behind this chronological and bibliometric analysis of fuzzy sets is to draw the attention of researchers by informing them about the development of fuzzy set extensions and inspire them to develop both theoretical and practical applications of this subject. In the literature, various fuzzy set extensions can be found. However, there is a lack of bibliometric analysis or systematic review on the existing fuzzy set extensions, how fuzzy sets have evolved over time, and what the main trends in their evolution are. Since they have evolved, been applied to solve a variety of tasks and continue to inspire new research and innovation in the field of artificial intelligence (Kahraman and Haktanır, 2024), our research presents a chronological and bibliometric analysis of fuzzy sets by identifying fuzzy set extensions, systematizing, and visualizing them chronologically in a keyword map.

The number of papers on the topic of fuzzy sets exhibits a fluctuating growth trend over the years (RQ1). The performed analysis shows noticeable peaks in 2009 and 2020. The detailed analysis of the chronological publication of papers by country shows that these peaks are directly related to the highly active publications from China and less active publications from India, Pakistan, and Turkey (Fig. 10) (RQ4). Other countries have a moderate impact on publishing on the fuzzy set topic.

Based on this research, the authors have identified 65 fuzzy set extensions (RQ2) (Table 5). According to the developed chronological keyword map of fuzzy sets (Fig. 3), the following trends are identified:

1. The newer fuzzy set extensions are the Fermatean fuzzy set, T-spherical fuzzy set, probabilistic hesitant fuzzy set, spherical fuzzy set and q-rung orthopair fuzzy set.
2. The most frequently occurring fuzzy set extensions are the intuitionistic fuzzy set, interval type-2 fuzzy set, type-2 fuzzy set, hesitant fuzzy set, and interval-valued fuzzy set.
3. The less frequently mentioned fuzzy set extensions in the newest papers are the convex fuzzy set, eigen fuzzy set, balanced fuzzy sets, l-fuzzy set, three-dimensional fuzzy set, and normal fuzzy set.

The currently identified collection of fuzzy set extensions is basic but not complete due to the limitations of the bibliometric analysis (i.e. it is based on quantitative keyword identification in the analysed papers, and sets with fewer occurrences are not detected). In this analysis, we used the occurrence limitation of not less than 5. Fewer occurrences generate a lot of informational noise, making the selection of meaningful keywords difficult and requiring significant human time and resources, and ultimately, the result may not meet expectations. It is more appropriate to use qualitative analysis methods, such as a systematic literature review, to search for such insignificant, newly emerging fuzzy set extensions. For this reason, our chronological and bibliometric analysis does not discover newly appearing fuzzy set extensions, such as the Mandelbrot fuzzy set (Ince and Ersoy, 2022), Fuzzy superior Mandelbrot set (Mahmood and Ali, 2022), and Polytopic fuzzy set (Beg *et al.*, 2022) (RQ2, RQ3). However, this limitation points to further research directions without diminishing the significance of the current study.

The main identified fuzzy sets are depicted in chronological sequence in Fig. 12. Based on the above-identified trends, it can be summarized that fuzzy sets continue to evolve, especially through their use and application with artificial intelligence techniques for solving

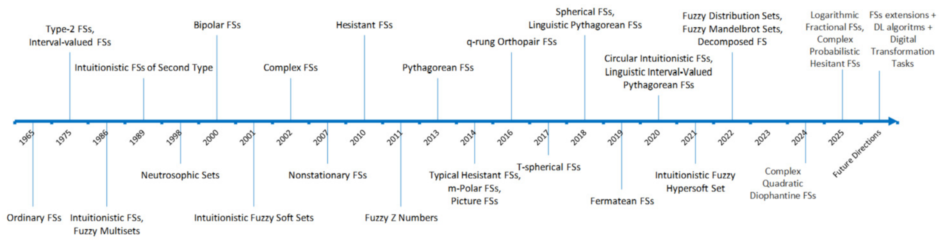

Fig. 12. Evolution of fuzzy sets.

digital transformation tasks (RQ5) (Fig. 11). Their applications have led to new research and innovation, including fuzzy decision-making, fuzzy clustering, and fuzzy inference. Moreover, new directions have emerged, such as explainable artificial intelligence, fuzzy neural networks, and fuzzy machine learning.

The keywords co-occurrence analysis shows that fuzzy set extensions are most commonly associated with the following keywords (RQ3, RQ6): multi-criteria decision making (MCDM), multi-attribute decision making (MADM), decision making, group decision making, selection, optimization, uncertainty, approximation, classification and TOPSIS. This indicates that decision-making is the most developed application area of fuzzy set extensions due to the imprecision, subjectivity, and uncertainty of the decision-making environment. From a general perspective, fuzzy set extensions are applied to selection, approximation, and optimization problems, which are also uncertain, vague, and imprecise. Among the newest keywords found in the fuzzy set map (Fig. 3) are MCDM methods, such as fuzzy WASPAS, fuzzy EDAS, fuzzy COPRAS and fuzzy Best-Worst Method (RQ3, RQ6).

The co-occurrence analysis of fuzzy set extensions and related keywords shows that the most frequent and middle APY fuzzy set extensions have a large community of keywords, while less frequent fuzzy set extensions have fewer related keywords (Table 5, RQ2, RQ3). Those keywords communities indicate the popularity, applicability and usability of the fuzzy set extensions. The bigger keywords communities show the bigger popularity, wider applicability and greater usability of some fuzzy set extensions. The smaller keywords communities show lower popularity, narrower applicability and fewer usability of those fuzzy set extensions. Such smaller or larger occurrences and keywords communities of fuzzy set extensions can be explained as follows: 1) the emergence of new fuzzy set extensions; 2) calculation complexity; 3) the formalization of new uncertain concepts. However, to determine the reasons for fuzzy set extensions co-occurrence with other keywords, a deeper analysis, like systematic literature analysis, should be performed in future work.

The analysis of country occurrences shows that the countries that contribute the most to the study of fuzzy sets are as follows: China, India, USA, Spain and Taiwan. The activity of those countries in the research can be explained by their size and economic development. The latest countries joining the research on fuzzy set extensions are as follows: Yemen, Iraq, Palestine, Pakistan, Ecuador, Nigeria, Estonia, Saudi Arabia and Peru. Among these recent countries, Pakistan and Saudi Arabia stand out in terms of publishing intensity.

The strongest relationships are found between the USA and China, Poland and Canada, and Canada and China. Also, it can be seen that the dominant countries in the fuzzy set extension topic have strong international collaboration. Thus, it can be concluded that such international participation among countries allows them to extend the analysed topic and increase international and intercontinental visibility.

In summary, the following general trends are found within this research. First, fuzzy sets are penetrating into other sets, extending them with uncertainty, like complex fuzzy sets, bipolar fuzzy sets, picture fuzzy sets, etc. Second, the spread of fuzzy sets to other sets also has either a reverse and synergy effect, i.e. the theory of fuzzy sets is supplemented and expanded with new concepts. Such new extensions allow us to apply newly developed approaches based on fuzzy set extensions for previously unsolved or partially solved problems by obtaining more accurate results, which is not always possible with classical sets. In addition, the evolution of fuzzy sets can be explained by the need to solve more complex decision problems, simulated human thinking and digitalization of the different complex and vague real-world nonlinear problems.

Consequently, the future research areas of fuzzy sets are concerned with their extension by other sets and their application, particularly in the context of digital transformation. We are moving towards the development of intelligent digital systems that increasingly reflect human thinking, behaviour, decision-making and reduce human errors. This evolution is confirmed by our developed fuzzy set keyword map, timeline and the insights discussed in this section. Moreover, there is a visible and increasing cooperation between countries and continents in developing such intelligent systems, which are essential for navigating the complexities of digital transformation.

Based on the found and described answers to the research questions, we have proposed an increased awareness of the fuzzy set extensions topic. This awareness inspires further research directions for other extensions of fuzzy sets and their application in decision-making, reasoning, diagnosis, prediction, etc., and overall digital transformation of all areas. As organizations increasingly rely on digital technologies, the integration of fuzzy sets can enhance the effectiveness of these applications. However, some in-depth research methods (i.e. systematic literature review or meta-analysis) and other bibliometric analysis methods (i.e. co-citation, co-authorship, clustering, etc.) are applicable here for future research to discover new fuzzy set extensions, their relationships with various application domains, and how fuzzy sets enhance the effectiveness of digital transformation initiatives. In addition, it would be useful to include other scientific databases in the current research.

5.1. *Practical Implications*

The presented bibliometric study on fuzzy sets can offer several practical implications for practitioners, especially in AI applications and decision-making systems. They are as follows:

1. Toward explainable AI, practitioners can extend their AI-based system design by adding a supplementary fuzzy logic component to better explain the obtained inference results.

2. Choice of a fuzzy set – the identified 65 fuzzy set extensions help practitioners prioritize them for their applications.
3. Enhancing collaboration and innovation – the presented bibliometric maps of country collaboration can help practitioners identify key experts for partnerships.
4. Innovation opportunities – the current study encourages innovation by applying novel or hybrid fuzzy models in AI systems where traditional models fall short.

6. Conclusions

This research presents a chronological and bibliometric analysis of the fuzzy set extensions topic. It also presents a developed fuzzy set keywords map, which provides a systematic and chronological view of the analysed topic and captures the field's main ideas.

The analysis of the chronological distribution of papers on fuzzy set extensions shows the fluctuating growth of the number of articles, with notable peaks observed in 2009 and 2020, which is directly related to the scientific impact of China, India, Pakistan, and Turkey. This shows that the analysed topic is relevant and widely investigated. Therefore, it can be concluded that research on intelligent systems has been strongly developing in these countries.

According to the developed fuzzy set keywords map, 65 fuzzy set extensions have been identified, but this number is not final and is expected to be supplemented with new fuzzy set extensions in the near future. According to the co-occurrence analysis, we have identified the research emphasis in decision making, reasoning, diagnosis, prediction, etc. The newest keywords found in the fuzzy set keyword map are also mainly related to MCDM methods, such as fuzzy WASPAS, fuzzy EDAS, fuzzy COP-RAS, and fuzzy Best-Worst Method.

Notably, the results obtained in this research are based on a quantitative analysis presented through keyword mapping. However, to gain more specific insights and identify the newest fuzzy set extensions, a systematic literature review incorporating both quantitative and qualitative research should be conducted in future work. In addition, as the number of fuzzy set extensions grows, it is necessary to conduct a systematic study to compare as many fuzzy set extensions as possible and develop recommendations or guidelines on which fuzzy set extensions are more suitable for solving various reasoning tasks in digital environments. Additionally, a method for comparing different fuzzy set extensions is necessary.

Furthermore, in future work, it would be useful to investigate the application of fuzzy sets in different areas by conducting a systematic literature review.

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