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ON THE RENORMALIZATION OF NEUTRINOS IN THE SEESAW EXTENSION OF THE TWO-HIGGS DOUBLET MODEL*

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We present the complex mass renormalization scheme for mixed Majorana fermions using the Weyl spinor notation. Showing the expressions for field and mass renormalization constants, we discuss the differences to the on-shell renormalization scheme. Working in a seesaw extended two-Higgs doublet model, we apply the complex mass scheme for neutrino masses and mixings.

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1. Introduction

The most commonly used renormalization scheme is the on-shell scheme (OS). However, it is shown that for unstable particles this scheme leads to gauge non-invariant definitions of masses [1]. In the seesaw mechanism (for a review, see [2]), the heaviest neutrino is by no means a stable particle. Original assumptions of the seesaw mechanism put the heaviest neutrino beyond the reach of any possible experiment. This partially justifies the use of OS, since we are looking only at the light neutrinos. However, for a more precise study of the model, the assumptions on the unmeasured parameters should be relaxed and this justification is lost.

The extension of the OS for unstable particles is the complex mass scheme (CMS) [3–5]. It is the analytical continuation of the propagator to the complex domain. In that way, the information about the decay width of the particle is included in the renormalized mass as the imaginary part of self energies. One can formally prove gauge invariance of the definition of mass at all loop levels [1] with the help of Nielsen identities [6].

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In Section 2, we present the main definitions used to define the renormalization scheme. In Section 3, we outline the derivation of mass and field renormalization constants for the Majorana fermions in the complex mass scheme and discuss the implications. Finally, in Section 4, we present the restrictions on the specific renormalization constants and one-loop Green's functions in the two-Higgs doublet model (2HDM) with one seesaw neutrino.

2. Definitions

We use the Weyl spinor notation in the chiral representation as in [7, 8]. Let us say we have left-handed Weyl spinors ν_{0i} with bare Majorana masses m_{0i}. We can always fix the phase of ν_{0i} so that the mass parameter m_{0i} is real. Then we can write the multiplicative renormalization constants as:

$$\begin{aligned} \nu_{0i} &= Z_{ij}^{\frac{1}{2}}\nu_j\,, \qquad \nu_{0i}^{\dagger} = Z_{ij}^{\frac{1}{2}\dagger}\nu_j^{\dagger}\,, \qquad m_{0i} = m_i Z_{m_i}\,, \\ Z_{m_i} &= 1+\delta_{m_i}\,, \qquad Z_{ij}^{\frac{1}{2}} = 1_{ij}+\tfrac{1}{2}\delta_{ij}\,. \end{aligned} \tag{1}$$

However, as we will see later, these multiplicative constants are not enough to absorb the imaginary parts coming from the loop functions for unstable particles. So we increase the degrees of freedom for the field renormalization part:

$$\nu_{0i} = Z_{ij}^{\frac{1}{2}}\nu_j\,, \qquad \nu_{0i}^{\dagger} = \bar{Z}_{ij}^{\frac{1}{2}}\bar{\nu}_j \Rightarrow \left(Z_{ij}^{\frac{1}{2}}\nu_j\right)^{\dagger} = \bar{Z}_{ij}^{\frac{1}{2}}\bar{\nu}_j\,. \tag{2}$$

This is equivalent to dropping the pseudohermicity requirement as suggested in [3].

The renormalized Green functions are

$$\langle\phi_1\ldots\phi_n\rangle_{1PI}^{[\mathrm{loop}]} = \frac{\delta^n\hat{\Gamma}^{[\mathrm{loop}]}}{\delta\phi_1\ldots\delta\phi_n} \equiv \hat{\Gamma}_{\phi_1\ldots\phi_n}^{[\mathrm{loop}]} \equiv \Gamma_{\phi_1\ldots\phi_n}^{[\mathrm{loop}]} + \delta\Gamma_{\phi_1\ldots\phi_n}^{[\mathrm{loop}]}\,, \tag{3}$$

where $\hat{\Gamma}$ is the renormalized effective action and $\delta\Gamma$ stands for counterterms. Then the tree-level Green's functions read as:

$$\hat{\Gamma}_{\nu_i\nu_i}^{[0]} = -m_i\,, \qquad \hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_i}^{[0]} = -m_i\,, \qquad \hat{\Gamma}_{\bar{\nu}_i\nu_i}^{[0]} = p\bar{\sigma}\,, \qquad \hat{\Gamma}_{\nu_i\bar{\nu}_i}^{[0]} = p\sigma\,, \tag{4}$$

where σ and $\bar{\sigma}$ connect spinors to four vectors as defined in [7, 8]. Due to Lorentz invariance, we can factor out the scalar parts of Green's functions as:

$$\begin{aligned} \hat{\Gamma}_{\nu_i\nu_i} &= m_i\hat{\Sigma}_{\nu_i\nu_i}\,, \qquad \hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_i} = m_i\hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i}\,, \\ \hat{\Gamma}_{\nu_i\bar{\nu}_j} &= p\sigma\hat{\Sigma}_{\nu_i\bar{\nu}_j}\,, \qquad \hat{\Gamma}_{\bar{\nu}_i\nu_j} = p\bar{\sigma}\hat{\Sigma}_{\bar{\nu}_i\nu_j}\,. \end{aligned} \tag{5}$$

Then we can express the counterterms using Eqs. (1)–(5) and write the loop functions as:

$$\begin{aligned} \hat{\Sigma}_{\nu_i\nu_i} &= -\delta_{m_i} - \delta_{ii} + \Sigma_{\nu_i\nu_i}\,, & \hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i} &= -\delta_{m_i} - \bar{\delta}_{ii} + \Sigma_{\bar{\nu}_i\bar{\nu}_i}\,, \quad (6)\\ \hat{\Sigma}_{\bar{\nu}_i\nu_j} &= \tfrac{1}{2}\left(\delta_{ij} + \bar{\delta}_{ji}\right) + \Sigma_{\bar{\nu}_i\nu_j}\,, & \hat{\Sigma}_{\nu_i\bar{\nu}_j} &= \tfrac{1}{2}\left(\bar{\delta}_{ij} + \delta_{ji}\right) + \Sigma_{\nu_i\bar{\nu}_j}\,. \quad (7)\end{aligned}$$

3. From on shell to complex mass shell

With these definitions, the resummed propagators are:

$$\langle\bar{\nu}_i\nu_i\rangle = i\bar{\sigma}p\left[p^2\left(1+\hat{\Sigma}_{\nu_i\bar{\nu}_i}\right) - m_i^2\left(1-\hat{\Sigma}_{\nu_i\nu_i} - \hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i} - \hat{\Sigma}_{\bar{\nu}_i\nu_i}\right)\right]^{-1}\,, \quad (8)$$

$$\langle\nu_i\nu_i\rangle = im_i\left[p^2\left(1+\hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i} + \hat{\Sigma}_{\nu_i\bar{\nu}_i} + \hat{\Sigma}_{\bar{\nu}_i\nu_i}\right) - m_i^2\left(1-\hat{\Sigma}_{\nu_i\nu_i}\right)\right]^{-1}\,, \quad (9)$$

together with analogous two propagators that can be obtained from Eq. (8) and Eq. (9) by changing $\nu_i \leftrightarrow \bar{\nu}_i$. Abbreviating $D_i \equiv p^2 - m_i^2$, the mixed two-point correlation functions ($i \neq j$) are:

$$\langle\nu_i\nu_j\rangle = -i(D_iD_j)^{-1}\left(m_im_j\hat{\Gamma}_{\nu_i\nu_j} + p^2\left[m_j\hat{\Sigma}_{\bar{\nu}_i\nu_j} + \hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_j} + m_i\hat{\Sigma}_{\nu_i\bar{\nu}_j}\right]\right)\,, \quad (10)$$

$$\langle\bar{\nu}_i\nu_j\rangle = -i\bar{\sigma}p(D_iD_j)^{-1}\left(m_j\hat{\Gamma}_{\nu_i\nu_j} + m_im_j\hat{\Sigma}_{\bar{\nu}_i\nu_j} + m_i\hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_j} + p^2\hat{\Sigma}_{\nu_i\bar{\nu}_j}\right)\,, \quad (11)$$

and $\nu_i \leftrightarrow \bar{\nu}_i$. The OS renormalization condition for a mass counterterm can be derived by requiring that the real part of the pole of the diagonal propagator coincides with the renormalized mass. The requirement that the mixed propagators vanish and that the residue of the diagonal propagator is equal to one gives the conditions for the wave function renormalization. Generalization from the OS to the CMS is obtained by just dropping the reality requirement and evaluating self energy functions at the exact complex pole of the propagator. Hence in the CMS, these conditions are:

$$\left(\hat{\Sigma}_{\nu_i\nu_i} + \hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i} + \hat{\Sigma}_{\nu_i\bar{\nu}_i} + \hat{\Sigma}_{\bar{\nu}_i\nu_i}\right)\Big|_{p^2=m_i^2} = 0\,, \quad (12)$$

$$\hat{\Sigma}_{\bar{\nu}_i\nu_i} = -m_i^2\frac{\partial}{\partial p^2}\left(\hat{\Sigma}_{\nu_i\nu_i} + \hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i} + \hat{\Sigma}_{\nu_i\bar{\nu}_i} + \hat{\Sigma}_{\bar{\nu}_i\nu_i}\right)\Big|_{p^2=m_i^2}\,, \quad (13)$$

$$\hat{\Sigma}_{\bar{\nu}_i\nu_i}\Big|_{p^2=m_i^2} = \hat{\Sigma}_{\bar{\nu}_i\nu_i}\Big|_{p^2=m_i^2} = -\hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i}\Big|_{p^2=m_i^2} = -\hat{\Sigma}_{\nu_i\nu_i}\Big|_{p^2=m_i^2}\,, \quad (14)$$

$$\left(\hat{\Gamma}_{\nu_i\nu_j} + m_j\hat{\Sigma}_{\nu_i\bar{\nu}_j}\right)\Big|_{p^2=m_i^2} = 0\,, \qquad \left(\hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_j} + m_j\hat{\Sigma}_{\bar{\nu}_i\nu_j}\right)\Big|_{p^2=m_i^2} = 0\,. \quad (15)$$

The condition of Eq. (12) comes from the requirement of the position of the pole for Eq. (8) and Eq. (9); the conditions of Eq. (13) and Eq. (14) come from the requirement that the residue of Eq. (8) and Eq. (9) is one and the conditions of Eq. (15) come from the requirement that the expressions in Eq. (10) and Eq. (11) vanish at $p^2 = m_i^2$ and $p^2 = m_j^2$. Inserting the expressions from Eq. (6) and Eq. (7) into these conditions leads to:

$$\delta_{m_i} = \frac{1}{2}\left(\Sigma_{\nu_i\nu_i} + \Sigma_{\bar{\nu}_i\bar{\nu}_i} + \Sigma_{\nu_i\bar{\nu}_i} + \Sigma_{\bar{\nu}_i\nu_i}\right)\Big|_{p^2=m_i^2}, \tag{16}$$

$$\frac{1}{2}\left(\bar{\delta}_{ii} + \delta_{ii}\right) = -\Sigma_{\bar{\nu}_i\nu_i} - m_i^2\frac{\partial}{\partial p^2}\left(\Sigma_{\nu_i\nu_i} + \Sigma_{\bar{\nu}_i\bar{\nu}_i} + \Sigma_{\nu_i\bar{\nu}_i} + \Sigma_{\bar{\nu}_i\nu_i}\right)\Big|_{p^2=m_i^2}, \tag{17}$$

$$\bar{\delta}_{ii} - \delta_{ii} = \left(\Sigma_{\nu_i\nu_i} - \Sigma_{\bar{\nu}_i\bar{\nu}_i}\right)\Big|_{p^2=m_i^2}, \tag{18}$$

$$\bar{\delta}_{ij} = \frac{2}{m_i^2 - m_j^2}\left(m_j\Gamma_{\nu_i\nu_j} + m_j^2\Sigma_{\nu_i\bar{\nu}_j} + m_i\Gamma_{\bar{\nu}_i\bar{\nu}_j} + m_i m_j\Sigma_{\bar{\nu}_i\nu_j}\right)\Big|_{p^2=m_j^2}, \tag{19}$$

$$\delta_{ij} = \frac{2}{m_i^2 - m_j^2}\left(m_i\Gamma_{\nu_i\nu_j} + m_i m_j\Sigma_{\nu_i\bar{\nu}_j} + m_j\Gamma_{\bar{\nu}_i\bar{\nu}_j} + m_j^2\Sigma_{\bar{\nu}_i\nu_j}\right)\Big|_{p^2=m_j^2}. \tag{20}$$

Equations (12)–(20) are consistent with the expressions in [3]. If we used the multiplicative constants only in the form of Eq. (1), without the field renormalization constants shown in Eq. (2), we could not absorb the imaginary parts from the loop functions. This can be easily seen from Eq. (17): using only constans from Eq. (1) would lead to an always real combination of constants $\delta_{ii}^{\dagger} + \delta_{ii}$ in the LHS of Eq. (17) instead of $\bar{\delta}_{ii} + \delta_{ii}$ which, in general, can be complex. On the other side, we see that the mass counterterm in Eq. (16), generalizes straightforwardly to the complex mass scheme by just dropping this reality condition. Actually, this would not be the case if we did not absorb the phase of the bare mass parameters in the Weyl spinors. Then we would have needed to introduce some new $\bar{m}_i$ and $\bar{\delta}_{m_i}$ in analogy to $\bar{\delta}_i$ and $\bar{\nu}_i$. However, there is no need for this additional complication, since we can always fix the phase of Majorana fermions.

Another interesting and somewhat odd feature of this scheme is that the renormalized field in the Lagrangian is not related to the corresponding antifield by Hermitian conjugation, whereas the bare fields are. The relation is altered by the wave function renormalization constants from Eq. (2):

$$\left(Z_{ij}^{\frac{1}{2}}\nu_j\right)^{\dagger} = \bar{Z}_{ij}^{\frac{1}{2}}\bar{\nu}_j \Rightarrow \nu_i^{\dagger} = \bar{\nu}_i + \tfrac{1}{2}\left(\bar{\delta}_{ij} - \delta_{ij}^{\dagger}\right)\bar{\nu}_j + O\left(\delta^2\right). \tag{21}$$

From Eq. (19) and Eq. (20), we can see that if ν_j is stable, we have $\bar{\delta}_{ij} = \delta_{ij}^{\dagger}$. This means that the relation of Eq. (21) reduces to $\nu_i^{\dagger} = \bar{\nu}_i$ if every ν_j is stable and we recover the usual on-shell conditions. However, if at least one particle entering Eq. (21) is unstable, we get $\bar{\nu}_i \neq \nu_i^{\dagger}$ for all particles that mix, even if the particle under consideration is stable. This is not inconsistent: all particles mix at the Lagrangian level. To see how this is consistent, we should look at Green's functions instead. Let us assume that the particle ν_i is stable, then at one-loop level:

$$\begin{aligned}\frac{\delta^2}{\delta\nu_i\delta\bar{\nu}_i}\hat{\Gamma} &= \int\limits_j \frac{\delta\nu_j^\dagger}{\delta\bar{\nu}_i}\frac{\delta^2}{\delta\nu_i\delta\nu_j^\dagger}\hat{\Gamma}\,,\\ \frac{\delta\nu_j^\dagger}{\delta\bar{\nu}_i} &= 1_{ji}+\frac{1}{2}\left(\bar{\delta}_{ji}-\delta_{ji}^\dagger\right)=1_{ij}\Rightarrow\hat{\Gamma}_{\nu_i\bar{\nu}_i}=\hat{\Gamma}_{\nu_i\nu_i^\dagger}\,.\end{aligned}\tag{22}$$

If ν_i is unstable, similar manipulations give:

$$\hat{\Gamma}^{[\geq 1]}_{\nu_i\bar{\nu}_i}=\left(1+\frac{1}{2}\left(\bar{\delta}_{ii}-\delta_{ii}^\dagger\right)\right)\hat{\Gamma}^{[0]}_{\nu_i\nu_i^\dagger}+\hat{\Gamma}^{[1]}_{\nu_i\nu_i^\dagger}\,.\tag{23}$$

Here, we also used the assumption that the basis is chosen in such a way that there are no mixed terms at tree level. As an example, let us assume that all the couplings that go into the expression for $\bar{\delta}_{ii}-\delta_{ii}^\dagger$ are real. Then $\bar{\delta}_{ii}=\delta_{ii}$ and we can rewrite Eq. (23) as:

$$\hat{\Gamma}^{[\geq 1]}_{\nu_i\bar{\nu}_i}=e^{i\,\mathrm{Im}\,\delta_{ii}}\hat{\Gamma}^{[0]}_{\nu_i\nu_i^\dagger}+\hat{\Gamma}^{[1]}_{\nu_i\nu_i^\dagger}\,.\tag{24}$$

We see that the instability of ν_i is seen as the additional phase in its two-point Green's function, while a Green's function of a stable particle stays the same.

4. Renormalization constants for 4 neutrinos in the 2HDM

The Yukawa sector for neutrinos in the 2HDM in the Higgs basis includes four neutrinos, two neutral scalars h', H', one neutral pseudoscalar A', a charged scalar $H^\pm$ and Goldstone bosons. In general, all neutral scalars mix, giving the mass eigenstates h, H, and A. The seesaw mixing is defined between the third and the fourth neutrino ($s^2=\frac{m_{03}}{m_{04}+m_{03}}$, $c^2=\frac{m_{04}}{m_{04}+m_{03}}$). The full Yukawa Lagrangian for this model can be found in [9]. The Yukawa part that includes only the neutral scalars can be written as:

$$\begin{aligned}\mathcal{L}_\nu &= -\frac{1}{2}m_{03}\,\nu_{03}\nu_{03}-\frac{1}{2}m_{04}\,\nu_{04}\nu_{04}\\ &-\frac{1}{\sqrt{2}}\left[y\left(h'+i\chi^0\right)-d'\,\left(H'+iA'\right)\right]\left(cs\nu_{03}\nu_{03}+i\left(c^2-s^2\right)\nu_{03}\nu_{04}+cs\nu_{04}\nu_{04}\right)\\ &-\frac{1}{\sqrt{2}}d\left(H'+iA'\right)\nu_{02}\left(-is\nu_{03}+c\nu_{04}\right)+\text{h.c.}\end{aligned}\tag{25}$$

y is given by the neutrino masses and the vacuum expectation value, hence the only free parameters in this part of the Lagrangian are

$$m_{03}, m_{04}, d \in \mathbb{R} \quad \text{and} \quad d' \in \mathbb{C}\,. \tag{26}$$

There are no bare masses for ν_2 and ν_1, hence no mass counterterms and no counterterms for their mixing:

$$\delta_{m_1} = \delta_{m_2} = \delta_{12} = \bar{\delta}_{12} = \delta_{21} = \bar{\delta}_{21} = 0\,. \tag{27}$$

ν_1, ν_2 and ν_3 are stable at one-loop level, so the counterterms are the same as we would have in the OS scheme:

$$\nu_j^\dagger = \bar{\nu}_j\,, \quad \delta_{m_3} \in \mathbb{R}\,, \quad \delta_{jj}^\dagger = \bar{\delta}_{jj}\,, \quad \delta_{ij}^\dagger = \bar{\delta}_{ij}\,, \quad i = 1,2,3,4\,; \quad j = 1,2,3\,. \tag{28}$$

For an unstable ν_4, we have:

$$\delta_{m_4}, \delta_{i4}, \bar{\delta}_{i4}, \delta_{44}, \bar{\delta}_{44} \in \mathbb{C}\,, \quad \nu_4^\dagger = \left(1 - \tfrac{1}{2}\bar{\delta}_{44} + \tfrac{1}{2}\delta_{44}^\dagger\right)\bar{\nu}_4\,, \quad i = 1,2,3\,. \tag{29}$$

Since we chose a basis in such a way that ν_1 does not interact with any neutral scalar, it stays massless after loop corrections as well. Since the counterterms of Eq. (27) are zero, there should be no mixing between ν_1 and ν_2 after a loop correction, so:

$$\Gamma_{\nu_1\nu_1} = \Gamma_{\nu_1\nu_2} = 0\,. \tag{30}$$

Note that δ_{13} and δ_{14} are not equal to zero and they are used to absorb the mixing coming from $\Gamma_{\nu_1\nu_3}$ and $\Gamma_{\nu_1\nu_4}$, which are not zero due to a loop with a charged fermion and a charged scalar.

Finally, there is no counterterm for the mass term for ν_2, which means that the one-loop mass term

$$m_2 = -\Gamma_{\nu_2\nu_2}(0) \tag{31}$$

is finite and gauge invariant.

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REFERENCES

[1] P. Gambino, P.A. Grassi, *Phys. Rev. D* **62**, 076002 (2000) [arXiv:hep-ph/9907254].

[2] G. Senjanović, *Riv. Nuovo Cim.* **34**, 1 (2011).

[3] D. Espriu, J. Manzano, P. Talavera, *Phys. Rev. D* **66**, 076002 (2002) [arXiv:hep-ph/9907254].

[4] A. Denner, S. Dittmaier, *Nucl. Phys. Proc. Suppl.* **160**, 22 (2006) [arXiv:hep-ph/0605312].

[5] B.A. Kniehl, A. Sirlin, *Phys. Rev. D* **77**, 116012 (2008) [arXiv:0801.0669 [hep-th]].

[6] N.K. Nielsen, *Nucl. Phys. B* **101**, 173 (1975).

[7] H.K. Dreiner, H.E. Haber, S.P. Martin, *Phys. Rep.* **494**, 1 (2010) [arXiv:0812.1594 [hep-ph]].

[8] V. Dūdėnas, T. Gajdosik, *Lith. J. Phys.* **56**, 149 (2016).

[9] W. Grimus, H. Neufeld, *Nucl. Phys. B* **325**, 18 (1989).