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THE ONE-LOOP IMPROVED LAGRANGIAN OF THE GRIMUS–NEUFELD MODEL*

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Adding a single gauge singlet fermion and a second Higgs doublet to the original Standard Model allows an explanation for the observed smallness of the neutrino masses using the seesaw mechanism. This model predicts two neutral fermions with vanishing mass. The one-loop contribution to the neutral fermion masses due to the second Higgs doublet lifts this degeneracy and allows to fit the model parameters to the observed neutrino mass differences. We determine the values of the additional Yukawa couplings by requiring the correct prediction of the mass differences and mixings in the neutrino sector. We also discuss the ambiguities of the model parameters.

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1. Lagrangian of the model

The Lagrangian of the Grimus–Neufeld model [1] adds to the Standard Model (SM) a single gauge singlet fermion N_{R} and a second Higgs doublet. The scalar sector is a general Two-Higgs-Doublet Model (2HDM), as described in [2]. Even though this description might be enough, we want to explicitly write the parts as, this way, we also define our parameters.

For the bare Lagrangian, we start with the gauge sector of the SM, which is given by the gauge group $\mathcal{G}_{\mathrm{SM}} = \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{SU}(3)_{\mathrm{color}}$. The gauge field strength tensors form the Yang–Mills Lagrangian

$$\mathcal{L}_{\mathrm{G}} = -\tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} - \tfrac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \tfrac{1}{4} G^b_{\mu\nu} G^{b\mu\nu} \,. \tag{1}$$

The fermion-gauge sector uses the covariant derivative with the fermions being in the fundamental representation of the gauge group $\mathcal{G}_{\mathrm{SM}}$

$$\mathcal{L}_{\text{G-F}} = \sum_{\psi} \bar{\psi}\, i\gamma^\mu D_\mu \, \psi \,. \tag{2}$$

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The Higgs potential $V(\phi_a)$ is written in the Higgs basis [2], where only the first doublet develops the vacuum expectation value (vev) v. The Higgs doublets are also in the fundamental representation of the gauge group, giving them the covariant derivative

$$D_\mu\phi_a = \partial_\mu\phi_a + \tfrac{i}{2}g_{\mathrm{U}(1)}B_\mu\phi_a + \tfrac{i}{2}g_{\mathrm{SU}(2)}W^j_\mu\sigma^j\phi_a\,, \tag{3}$$

and the Lagrangian

$$\mathcal{L}_{\text{G-H}} = (D^\mu\phi_a)^\dagger(D_\mu\phi_a) - V(\phi_a)\,. \tag{4}$$

In the fermion–Higgs sector, we have the Yukawa couplings, which are defined with the Higgs doublets in the Higgs basis

$$\mathcal{L}_{\text{F-H}} = -\bar{\ell}^0_{\mathrm{L}j}\,\phi_a\left(Y^{(a)}_E\right)_{jk}e^0_{\mathrm{R}k} - \bar{\ell}^0_{\mathrm{L}j}\,\tilde{\phi}_a\left(Y^{(a)}_N\right)_j N^0_{\mathrm{R}} + \text{h.c.} \tag{5}$$

The Yukawa couplings of the quarks belong to the model, but since we are only interested in the neutrinos, we omit them from our discussion. The last sector is the Majorana mass term for the singlet fermion N^0_{R}

$$\mathcal{L}_{\mathrm{Maj}} = \tfrac{1}{2}M_{\mathrm{R}}{N^0_{\mathrm{R}}}^\top\boldsymbol{C}^{-1}N^0_{\mathrm{R}} + \text{h.c.}\,, \tag{6}$$

where $\boldsymbol{C}$ is the "charge conjugation matrix" [3]. The kinetic term is already included in the fermion-gauge Lagrangian Eq. (2) with a simple covariant derivative $D_\mu N^0_{\mathrm{R}} = \partial_\mu N^0_{\mathrm{R}}$.

2. Tree-level mass eigenstates

Inserting the vacuum expectation value v into the Yukawa Lagrangian Eq. (5), we get the mass matrices for the charged fermions

$$M_e = \tfrac{v}{\sqrt{2}}\left(Y^{(1)}_E\right)\,, \qquad M_u = \tfrac{v}{\sqrt{2}}\left(Y^{(1)}_U\right)\,, \qquad M_d = \tfrac{v}{\sqrt{2}}\left(Y^{(1)}_D\right)\,. \tag{7}$$

Diagonalization of these mass matrices

$$U_{e\mathrm{L}}M_eU^\dagger_{e\mathrm{R}} = \mathrm{diag}\{m_e, m_\mu, m_\tau\} \tag{8}$$

defines the mass eigenstates of the charged fermions

$$e_{\mathrm{L}k} = (U_{e\mathrm{L}})_{kn}e^0_{\mathrm{L}n} \qquad \text{and} \qquad e_{\mathrm{R}k} = (U_{e\mathrm{R}})_{kn}e^0_{\mathrm{R}n}\,. \tag{9}$$

Applying the same rotation to the neutral leptons defines their flavor states

$$\{\nu_e,\nu_\mu,\nu_\tau\}_k = \nu_{\mathrm{L}k} = (U_{e\mathrm{L}})_{kn}\nu^0_{\mathrm{L}n}\,, \tag{10}$$

which are used for the definition of the neutrino mixing matrix. The Yukawa coupling with the neutral fermions gives the so-called Dirac mass term

$$M_{\mathrm{D}} = \tfrac{v}{\sqrt{2}}\left(Y_N^{(1)}\right) \tag{11}$$

that describes the tree-level mixing of the neutral leptons with the gauge singlet. In our case, it is a 3×1 matrix. Together with the Majorana mass term, it forms a symmetric mass matrix

$$M_\nu = \begin{pmatrix} M_{\mathrm{L}} & M_{\mathrm{D}}^\top \\ M_{\mathrm{D}} & M_{\mathrm{R}} \end{pmatrix}, \quad \text{with} \quad \begin{array}{rcl} M_{\mathrm{L}} & = & 0_{3\times 3}\,, \\ M_{\mathrm{D}} & = & \tfrac{v}{\sqrt{2}} Y_N^{(1)}\,, \end{array} \tag{12}$$

which is obviously rank 2 and hence gives only two non-zero masses. We can diagonalize M_ν by the unitary matrix $U_{(\nu)}$. Writing the diagonalization in a linear form (the "seesaw relation")

$$U_{(\nu)}{}^* M_\nu = \mathrm{diag}(m_1, m_2, m_3, m_4) U_{(\nu)} =: \hat{m} U_{(\nu)} \quad \text{with} \quad m_1 = m_2 = 0\,, \tag{13}$$

we can determine the general form of $U_{(\nu)}$ as

$$U_{(\nu)} = \begin{pmatrix} u_{1e} & u_{1\mu} & u_{1\tau} & 0 \\ u_{2e} & u_{2\mu} & u_{2\tau} & 0 \\ c u_{3e} & c u_{3\mu} & c u_{3\tau} & is \\ is u_{3e} & is u_{3\mu} & is u_{3\tau} & c \end{pmatrix}, \quad \text{where} \quad \begin{array}{rcl} c^2 & = & \frac{m_4}{m_4+m_3} \\ s^2 & = & \frac{m_3}{m_4+m_3} \end{array}. \tag{14}$$

The entries $u_{\alpha k}$ can be understood as the tree-level 3×3 neutrino mixing matrix. Ignoring the heavy state, $u_{\alpha k}$ tells us how the three light mass eigenstates mix to form the interaction eigenstates $\nu_{\mathrm{L}k}$.

3. Tree-level Yukawa couplings

Looking at the linear seesaw relation, Eq. (13), from the perspective of the involved Yukawa couplings, we can write the decoupling of the tree-level degenerate neutrinos $\zeta_{1,2}^M$ from the first Higgs doublet as

$$u_{1k}^*\left(Y_N^{(1)}\right)_k = u_{2k}^*\left(Y_N^{(1)}\right)_k = 0\,, \tag{15}$$

where $\sum_k$ is implied. Knowing that the radiative mass generation will lift the degeneracy, we distinguish the two neutrinos by their coupling to the second Higgs doublet. We require that ζ_1^M has no coupling to the second Higgs doublet, and we parametrize the coupling of ζ_2^M by the parameter d

$$u_{1k}^*\left(Y_N^{(2)}\right)_k = 0 \qquad \text{and} \qquad u_{2k}^*\left(Y_N^{(2)}\right)_k =: d \neq 0\,. \tag{16}$$

Since $u_{\alpha k}$ is a unitary 3×3 matrix, its rows act as a basis of 3 orthonormal vectors. We can use these vectors to parametrize the Yukawa couplings

$$\left(Y_N^{(1)}\right)_k = \tfrac{\sqrt{2} m_D}{v} u_{3k}\,, \qquad \left(Y_N^{(2)}\right)_k := d\, u_{2k} + d' u_{3k}\,. \tag{17}$$

4. The Grimus–Lavoura procedure

Grimus and Lavoura [4] made a series of approximations that simplified the needed one-loop corrections and described the essential feature of the model in a gauge invariant way. The first step was to consider only the "light" neutrino states $\zeta^M_{1,2,3}$ and to use only the interaction eigenstate basis. On this basis, the zero block of the neutrino mass matrix M_ν, Eq. (12), is explicit. Reducing the mass matrix to an effective 3×3 mass matrix $\mathcal{M}_\nu$ gives the tree-level matrix

$$\mathcal{M}_\nu^{\text{tree}} = -M_{\text{D}}^\top\, M_{\text{R}}^{-1}\, M_{\text{D}}\,, \tag{18}$$

and the one-loop matrix

$$\mathcal{M}_\nu^{\text{1-loop}} = \mathcal{M}_\nu^{\text{tree}} + \delta\mathcal{M}_\nu\,, \tag{19}$$

with

$$\begin{aligned}\delta\mathcal{M}_\nu \;=\; & \delta M_{\text{L}} - \delta M_{\text{D}}^\top\, M_{\text{R}}^{-1}\, M_{\text{D}} - M_{\text{D}}^\top\, M_{\text{R}}^{-1}\,\delta M_{\text{D}} \\ & + M_{\text{D}}^\top\, M_{\text{R}}^{-1}\,\delta M_{\text{R}}\, M_{\text{R}}^{-1}\, M_{\text{D}}\,,\end{aligned} \tag{20}$$

where the counter-terms come only from the parameters appearing in $\mathcal{M}_\nu^{\text{tree}}$. δM_{L} has no counter-terms, as it is absent at tree-level, but loops generate a finite contribution. Following the discussion in [4], we note that the corrections coming from δM_{D} are suppressed by the squares of normal Yukawa couplings compared to the contribution arising in δM_{L}. And since we do not aim at measuring the mass of the heavy neutrino, $m_4 \sim M_{\text{R}}$, we can redefine its bare mass to adjust the counterterm δM_{R} to any arbitrary value, hence its contribution is irrelevant.

The second step in the Grimus–Lavoura procedure consists of calculating the correction δM_{L}, where the loop integrals, shown in Fig. 1, are evaluated at vanishing external momentum. From the Lorentz structure of the loop corrections, we see that only the Z boson and the neutral Higgs bosons contribute to $(\delta M_{\text{L}})_{\alpha\beta}$. That allows to write the effective mass matrix as

$$\left(\mathcal{M}_\nu^{\text{1-loop}}\right)_{jk} = u_{2j}u_{2k}A + (u_{2j}u_{3k} + u_{3j}u_{2k})B + u_{3j}u_{3k}C\,, \tag{21}$$

which is only of rank 2, hence producing only a single vanishing neutrino mass $\tilde{m}_1 = 0$.

The coefficients are given by

$$A = d^2 f_1\,, \qquad B = d' d f_1 + id\tfrac{m_D}{v} f_2\,, \qquad C = {d'}^2 f_1 + 2id'\tfrac{m_D}{v} f_2 + \tfrac{m_D^2}{v^2} f_3\,, \tag{22}$$

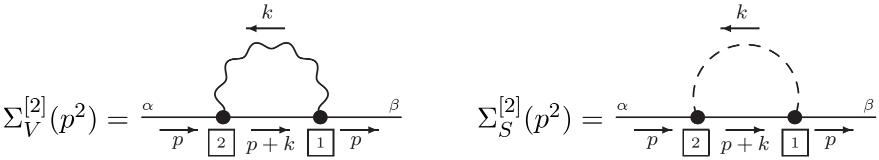

Fig. 1. Two Feynman diagrams contributing to the effective mass matrix $(\delta M_{\rm L})_{\alpha\beta}$ at one loop.

with the functions f_i depending only on the Higgs sector

$$f_1 = \frac{1}{32\pi^2}\left[\frac{s_{12}^2 m_h^2}{m_4}\ln\frac{m_4^2}{m_h^2} + \frac{c_{12}^2 m_H^2}{m_4}\ln\frac{m_4^2}{m_H^2} - \frac{m_A^2}{m_4}\ln\frac{m_4^2}{m_A^2}\right], \tag{23}$$

$$f_2 = \frac{\sqrt{2}s_{12}c_{12}}{32\pi^2}\left[\frac{m_h^2}{m_4}\ln\frac{m_4^2}{m_h^2} - \frac{m_H^2}{m_4}\ln\frac{m_4^2}{m_H^2}\right], \tag{24}$$

$$f_3 = \frac{1}{16\pi^2}\left[\frac{3m_Z^2}{m_4}\ln\frac{m_4^2}{m_Z^2} - \frac{c_{12}^2 m_h^2}{m_4}\ln\frac{m_4^2}{m_h^2} - \frac{s_{12}^2 m_H^2}{m_4}\ln\frac{m_4^2}{m_H^2}\right] - \frac{v^2}{m_4}. \tag{25}$$

5. Using neutrino masses as input

Since the mass of the lightest neutrino vanishes in this model, we can determine the "experimentally measured" light neutrino masses from the measured mass squared differences, that are determined in the neutrino oscillation experiments. We use the summary [5].

On the theory side, we get the masses $\tilde{m}_k$ from the reduced mass matrix, Eq. (21). Since we already know $\tilde{m}_1 = 0$, we can reduce the matrix to an effective size of 2×2

$$R^* \times \begin{pmatrix} A & B \\ B & C \end{pmatrix} = \begin{pmatrix} \tilde{m}_2 & 0 \\ 0 & \tilde{m}_3 \end{pmatrix} \times R\,, \tag{26}$$

where the mixing matrix

$$R = e^{i\frac{\alpha}{2}}\begin{pmatrix} \tilde{c} & \tilde{s} \\ -\tilde{s}^* & \tilde{c}^* \end{pmatrix}, \qquad \text{with} \qquad \begin{array}{l} \tilde{c} = e^{\frac{i}{2}(\gamma+\delta)}\cos\beta \\ \tilde{s} = e^{\frac{i}{2}(\gamma-\delta)}\sin\beta \end{array} \tag{27}$$

describes the mixing between neutrino fields ζ_2 and ζ_3. The angles and phases of R are fully determined

$$\tan^2\beta = \frac{|A|^2+|B|^2-\tilde{m}_2^2}{\tilde{m}_3^2-|A|^2-|B|^2}, \qquad e^{i\delta} = -\frac{(A^*B+B^*C)\tan\beta}{|A|^2+|B|^2-\tilde{m}_2^2} = \ldots, \tag{28}$$

$$e^{2i\alpha} = \frac{\tilde{m}_2\,\tilde{m}_3}{AC-B^2}, \qquad e^{i\gamma} = \frac{e^{i\alpha}}{\tilde{m}_3}(Ce^{-i\delta} - B\tan\beta) = \ldots, \tag{29}$$

where "..." indicate different possible analytic expressions.

But we can determine the masses $\tilde{m}_2$ and $\tilde{m}_3$ from A, B, and C alone by using the identities for the trace

$$\tilde{m}_2^2+\tilde{m}_3^2=\mathrm{Tr}\left[R\begin{pmatrix} A & B \\ B & C\end{pmatrix}^{\dagger}\begin{pmatrix} A & B \\ B & C\end{pmatrix}R^{\dagger}\right]=|A|^2+2|B|^2+|C|^2 \tag{30}$$

and the determinant

$$\tilde{m}_2\,\tilde{m}_3=\det[R^*]\det\begin{bmatrix} A & B \\ B & C\end{bmatrix}\det\left[R^{\dagger}\right]=e^{-2i\alpha}\left[AC-B^2\right]. \tag{31}$$

Due to the specific structure of Eq. (22), the determinant is very simple

$$\tilde{m}_2\,\tilde{m}_3=\left|AC-B^2\right|=d^2\tfrac{m_D^2}{v^2}\left|f_1f_3+f_2^2\right| \tag{32}$$

and allows the direct determination of d,

$$d^2=\frac{v^2}{m_D^2}\frac{\tilde{m}_2\,\tilde{m}_3}{\left|f_1f_3+f_2^2\right|}\,, \tag{33}$$

in terms of the measured masses

$$\tilde{m}_2=\sqrt{\Delta m_{\mathrm{sol}}^2}\qquad\text{and}\qquad\tilde{m}_3=\sqrt{\Delta m_{\mathrm{atm}}^2}\,. \tag{34}$$

Using these masses as input, Eq. (30) can be seen as a fourth-order equation for $|d'|$, where $\phi'=\arg[d']$

$$\begin{aligned} 0 &= -\tilde{m}_2^2-\tilde{m}_3^2+|A|^2+2|B|^2+|C|^2 \\ &= a_4\left|d'\right|^4+a_3\left|d'\right|^3+a_2\left|d'\right|^2+a_1\left|d'\right|+a_0\,, \end{aligned} \tag{35}$$

with

$$a_0 = d^4f_1^2+2d^2\tfrac{m_D^2}{v^2}f_2^2+\tfrac{m_D^4}{v^4}f_3^2-\tilde{m}_2^2-\tilde{m}_3^2\,, \tag{36}$$

$$a_1 = 4\sin\phi'\tfrac{m_D}{v}f_2\left[d^2f_1-\tfrac{m_D^2}{v^2}f_3\right], \tag{37}$$

$$a_2 = 2\left[d^2f_1^2+\left(2f_2^2+\cos 2\phi' f_1f_3\right)\tfrac{m_D^2}{v^2}\right], \tag{38}$$

$$a_3 = 4\sin\phi'\tfrac{m_D}{v}f_1f_2\,, \tag{39}$$

$$a_4 = f_1^2\,, \tag{40}$$

giving the dependence

$$\left|d'\right|=\left|d'\right|\left[v^2;m_h,m_H,m_A,s_{12};\tilde{m}_2,\tilde{m}_3,\tilde{m}_4;m_D^2;\phi'\right], \tag{41}$$

which is displayed graphically by the scatter plot in Fig. 2.

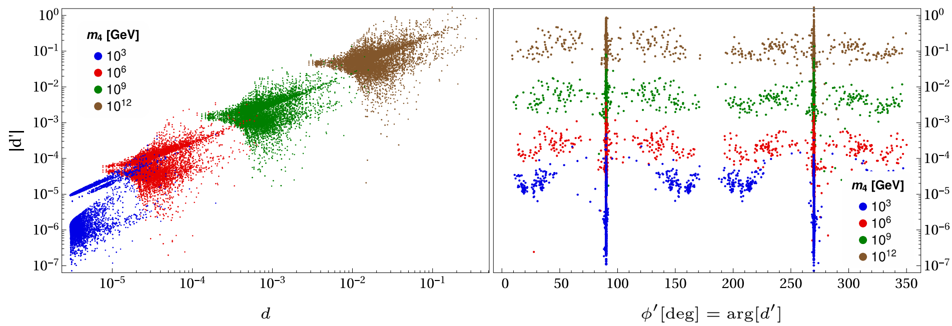

Fig. 2. (Color online) Scatter plot of the parameters d, $|d'|$, and ϕ'. $\tilde{m}_2 = 8.7\,\mathrm{meV}$ and $\tilde{m}_3 = 49.6\,\mathrm{meV}$ are calculated from Eq. (34) with the Δm^2 taken from [5]. For $\tilde{m}_4$, we assume the values $\{10^3, 10^6, 10^9, 10^{12}\}$ GeV and $m_D^2 = 0.81\,\tilde{m}_3\tilde{m}_4$. The parameters of the Higgs sector are taken from [6].

6. Using the PMNS matrix as input

The mixing matrix R, Eq. (27), transforms the two light neutrino states $\zeta_{2,3}$ into the mass eigenstates. Therefore, the mixing matrix $u_{\alpha k}$ from Eq. (14) is not the full PMNS matrix that is defined by the coupling of W^+ to the charged leptons and the neutrino mass eigenstates. We can relate $u_{\alpha k}$ with the PMNS matrix by

$$u_{1k} = U_{1k}^{\mathrm{PMNS}}\,, \qquad \begin{pmatrix} u_{2k} \\ u_{3k} \end{pmatrix} = R^\dagger \cdot \begin{pmatrix} U_{2k}^{\mathrm{PMNS}} \\ U_{3k}^{\mathrm{PMNS}} \end{pmatrix}, \tag{42}$$

giving the final definition of the Yukawa couplings in terms of the PMNS matrix elements

$$\left(Y_N^{(1)}\right)_k = \frac{\sqrt{2}m_D}{v} e^{-i\frac{\alpha}{2}} \left(\tilde{s}^* U_{2k}^{\mathrm{PMNS}} + \tilde{c} U_{3k}^{\mathrm{PMNS}}\right), \tag{43}$$

$$\left(Y_N^{(2)}\right)_k = e^{-i\frac{\alpha}{2}} \left[\left(\tilde{c}^* d + \tilde{s}^* d'\right) U_{2k}^{\mathrm{PMNS}} - \left(\tilde{s} d - \tilde{c} d'\right) U_{3k}^{\mathrm{PMNS}}\right]. \tag{44}$$

7. Summary

For the masses alone, the Grimus–Lavoura procedure [4] gives a very simple and yet also very accurate description of the physical neutrino sector. Our study, illustrated in Fig. 3, suggests that the Grimus–Lavoura procedure does not always give a perfect match to the physical neutrino sector. The subdominant contributions from the charged loops might contribute to the dependencies derived from the neutrino mixing, as discussed in [4] as well. A logical next step is the inclusion of the full renormalization of the model. The initial phase of the study is reported in [7].

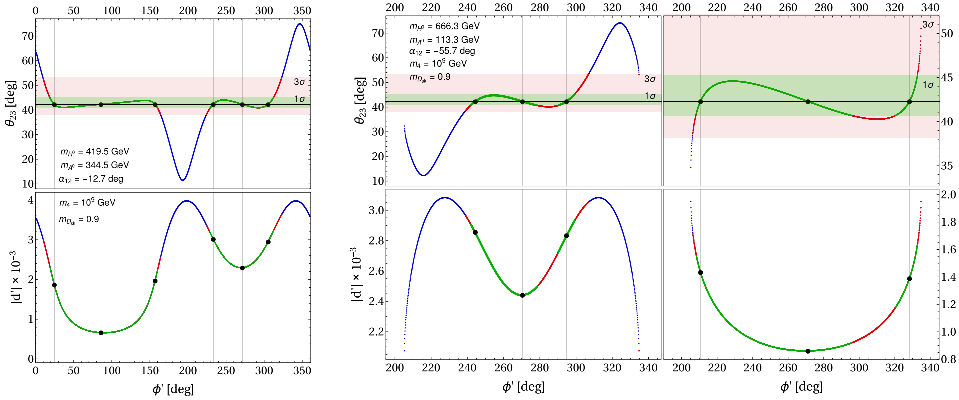

Fig. 3. (Color online) The values of the $|d'|$ and the PMNS mixing angle θ_{23} in dependence of the phase ϕ'. The left plot represents a single solution of the quartic equation for $|d'|$, Eq. (35), that is allowed for the whole range of phases. The right plots shows the case when Eq. (35) has more than one real positive solution, but not for the whole range of phases: $205^\circ \leq \phi' \leq 335^\circ$.

We have implemented checks, where our model gives a good self-consistent description of the physical neutrino sector. In these regions of the parameter space, the predictions of the Yukawa couplings, defined in Eqs. (43) and (44), should be reliable.

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