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Research Article

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Observational data and orbits of the asteroids discovered at the Molėtai Observatory in 2010–2012

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Abstract: This paper is devoted to the discovery of asteroids at the Molėtai Astronomical Observatory (MAO) in 2010-2012 together with the orbital analysis of two dynamically interesting Near Earth Objects (NEOs) discovered at the MAO, namely 2006 SF77 and 2010 BT3. We used the OrbFit software v.5.0 to compute orbits and to analyze orbital evolution of 2006 SF77 and 2010 BT3. We computed value of the Lyapunov time: 830 years for 2006 SF77 and 1650 year for 2010 BT3. We also searched for possible impacts of 2006 SF77 and 2010 BT3 with the Earth, Venus and Mars in the next 15000 years.

Keywords: minor planets, asteroids: search, astrometry, orbits

1 Introduction

This is our fourth paper in the series of papers devoted to the asteroids discovered at the Molėtai Observatory. The aim of our project is the determination of more precise orbits of asteroids and comets. Our main site of observations is the Molėtai Astronomical Observatory (MAO) of Vilnius University, located at the longitude 25.5633° E, latitude 55.3166° N and altitude 210 m. Its IAU code is 152. Most of the asteroids were discovered with the Maksutov (0.35/0.51 m f/3.5 + CCD) telescope in the morning sky about 10-30 days before their opposition time at elongations 140-170°. For observations of asteroids we used nitrogen cooled CCD camera VersArray with dimensions 26 × 26 mm. The scale of the system was 3.4" per pixel. Limiting magnitude of detection for asteroids in CCD frame is of about 20.5 R, using time of exposures 360 s. The sky survey has been done close to the ecliptic (mostly not more than 15° from the ecliptic line), taking three CCD images on the same field, with 15-20 min time spans between the exposures. For the measurements, the Astrometrica software (Raab 2003) was applied. The catalogs USNO-A2.0, USNO-B1.0 and UCAC-2 were used for the selection of reference stars.

During 2010-2012, about 4200 CCD images for astrometry of asteroids were made. The 14811 astrometric positions of 3450 asteroids, including 10 NEOs, were published in the Minor Planet Circulars (MPC) and Electronic Minor Planet Circulars (MPEC) (Cernis and Zdanavicius 2010, 2011; Černis *et al.* 2012).

In 2010-2012, during sky survey in the ecliptic regions and NEO asteroid follow-up observations, 99 new asteroids have been discovered.

Till now, 2017 May 10, the credits for discovery of 9 asteroids have been received, and one of them have been named. In the near future, we expect to get numbers and credits for another 12 asteroids: 2010 BO5, 2010 BL131, 2010 EN35, 2010 EJ66, 2010 EN106, 2010 RN122, 2010 TG65, 2010 TH187, 2011 EJ44, 2011 SR96, 2011 UZ401, 2011 UH332. The asteroids 2011 FO3, 2011 FR51 and 2013 HX46 have got their designation only in 2012-2013, however, these objects were first spotted at Molėtai observatory.

It is interesting that in years 2000-2012 the Molėtai Observatory discovered 357 asteroids that account for 0.07% of all 538425 asteroids discovered worldwide during this period. Moreover, in 2000-2012 we observed 41 NEOs. We studied asteroids discovered at the Molėtai Observatory in years: 2008-2009 (Černis *et al.* 2016b), 2005-2007 (Černis *et al.* 2016a) and 2000-2004 (Černis *el al.* 2014). We also studied two dynamically interesting asteroids discovered at the Molėtai Observatory: Amor Group Asteroid 2010 BT3 (Černis *et al.* 2012) and Aten Group Asteroid 2006 SF77 (Černis *et al.* 2008).

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Table 1. Statistics of asteroid discoveries and astrometric observations of the asteroids (both new and known) at the Molėtai Observatory in 2010–2012.

Year	Number of asteroid discoveries	Number of asteroid observations	Number of asteroids observed	References (MPC No.)
2010	69	9614	2275	68215,68671,69205,69727,70194 70654,71064,71531,71955,72441
2011	29	3915	890	73051,73670,74029,74385,74814 75146,75401,75596,75854,77168
2012	1	1282	285	77584,77926,78330,78789,79142 79474,79747,80457,81144,81618
Total	99	14811	3450	

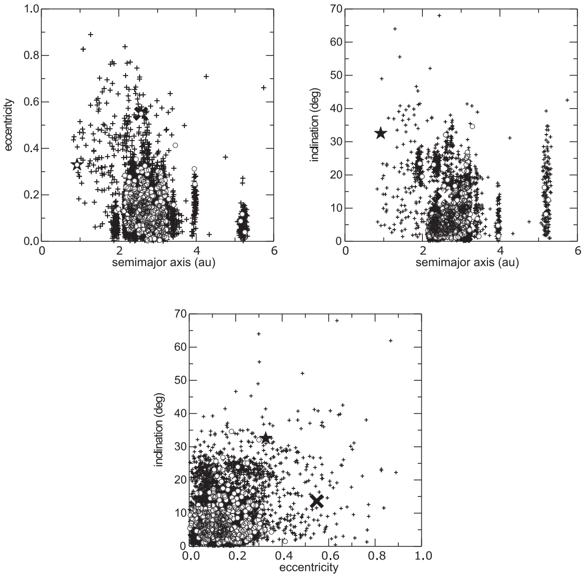

Fig. 1. 357 asteroids discovered at the MAO in 2000-2012 in the phase planes (a, e), (a, i), (e, i). They are presented 10000 first numbered asteroids (crosses) and 357 discovered asteroids (circles). Star denotes position of 2006 SF77 and great cross - 2010 BT3.

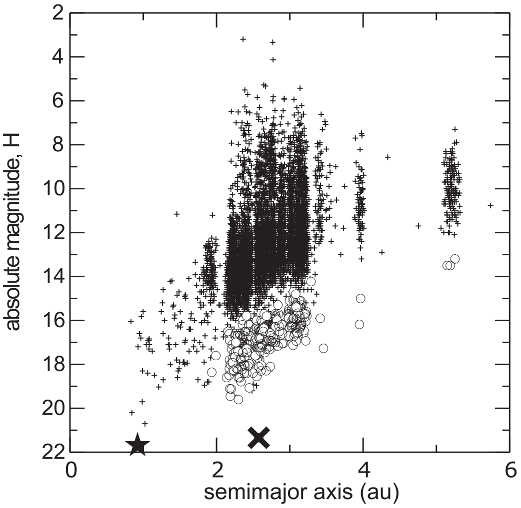

Fig. 2. 357 discovered asteroids at the MAO in 2000-2012 in the phase plane (*a, H*) (circles) placed together with the first 10000 numbered asteroids (crosses).

2 Discoveries of minor planets at MAO in 2010-2012

Table 1 presents statistics of 99 discovered asteroids and their astrometric observations made at the Molėtai Observatory during 2010–2012.

It, worth noticing that between 2000 and 2012, at the MAO, 357 asteroids were discovered from a total of 538425 discoveries around the world in this time, *i.e.* of about 0.07%. Figure 1 lists all 357 asteroids discovered at the MAO in years 2000-2012 plotted against only 10000 first numbered asteroids from all 496815 asteroids listed as of 2017 July 9 in the Minor Planet Center Archive Statistics (http://www.minorplanetcenter.net/iau/lists/ArchiveStatistics.html). They are presented in (a, e), (a, i) and (e, i) phase spaces where a denotes the semimajor axis, e - the eccentricity and i - the inclination of orbit of asteroids. Only semimajor axes of the first 10000 asteroids which are in the interval (0;6)au interval are plotted. Star denotes position of 2006 SF77 and great cross - 2010 BT3. It is visible that asteroids discovered at the MAO lie mainly in the center of the main belt of asteroids.

In Figure 2 it is presented, the 357 discovered asteroids at the MAO in the plane of the first 10000 numbered asteroids in (a, H), (a, i) and (e, i) phase spaces where a denotes the semimajor axis and H - the absolute magnitude of asteroids. Two asteroids in the uppermost part of the Figure 2 have the highest absolute magnitude, H: the Amor Group Asteroid 2010 BT3 (H=21.34 mag) and the Aten Group Asteroid 2006 SF77 (H=21.69 mag). On the other hand, the MAO discovered three asteroids with the lowest H - see Figure 2, *i.e.* three numbered asteroids: (352655) with H=13.2 mag, (264068) and (353194) both with H=13.5 mag. They are Jupiter Trojans.

Table 2. List of asteroids discovered at the Molėtai Observatory in 2010–2012.

No.	Date of discovery	Designation	Number	Name	Discoverers	Status
1	2010 Jan 23	2010 BT3			KC, JZ	*
2	2010 Jan 24	2010 BJ5	378881			Id
3	2010 Jan 24	2010 BN5	411159		KC, JZ	*
4	2010 Jan 24	2010 BO5			KC, JZ	+
5	2010 Jan 24	2010 BR5	452987			Id
6	2010 Jan 24	2010 BS5	473780			Id
7	2010 Jan 24	2010 BT5				Id
8	2010 Jan 24	2010 BU5				Id
9	2010 Jan 24	2010 BL131			KC, JZ	+
10	2010 Mar 10	2010 EP30	392317			Id
11	2010 Mar 10	2010 EQ30	284942		KC, JZ	*
12	2010 Mar 10	2010 ER30	338105			Id
13	2010 Mar 10	2010 ES30	439513			Id
14	2010 Mar 10	2010 ET30	392318			Id
15	2010 Mar 10	2010 EU30				Id
16	2010 Mar 10	2010 EN35			KC, JZ	+
17	2010 Mar 10	2010 EJ66			KC, JZ	+
18	2010 Mar 10	2010 EK66				Lost
19	2010 Mar 10	2010 EL66	389472		KC, JZ	*
20	2010 Mar 10	2010 EM66				Lost
21	2010 Mar 10	2010 EC70	350986			Id
22	2010 Mar 10	2010 EH74	453572			Id
23	2010 Mar 10	2010 EJ74				Id
24	2010 Mar 10	2010 EK74	296967		KC, JZ	*
25	2010 Mar 10	2010 EO74	350407			Id
26	2010 Mar 10	2010 EP74	398152		KC, JZ	*
27	2010 Mar 10	2010 EQ74	343475			Id
28	2010 Mar 10	2010 ER74				Id
29	2010 Mar 12	2010 ES74	296968	Ignatianum	KC, JZ	*
30	2010 Mar 10	2010 EK104	455047			Id
31	2010 Mar 10	2010 EG105	443662			Id
32	2010 Mar 10	2010 EH105	343498			Id
33	2010 Mar 10	2010 EH106	346985			Id
34	2010 Mar 10	2010 EJ106	382237			Id
35	2010 Mar 10	2010 EN106			KC, JZ	+
36	2010 Mar 11	2010 EO106	347456			Id
37	2010 Mar 11	2010 EX106	425469		KC, JZ	*
38	2010 Mar 10	2010 EW123	478696			Id
39	2010 Sep 8	2010 RC64	407346			Id
40	2010 Sep 8	2010 RF120	325813			Id
41	2010 Sep 8	2010 RG120				Id
42	2010 Sep 8	2010 RH120	392440		KC, JZ	*
43	2010 Sep 8	2010 RN122			KC, JZ	+
44	2010 Sep 8	2010 RO122				Lost
45	2010 Sep 8	2010 RX164	442075			Id
46	2010 Sep 8	2010 RY164	439422			Id
47	2010 Oct 1	2010 TL4				Lost

Continued on next page

Table 2. ... continued

No.	Date of discovery	Designation	Number	Name	Discoverers	Status
48	2010 Oct 1	2010 TS6				Id
49	2010 Oct 1	2010 TT6	264235			Id
50	2010 Oct 7	2010 TU6	365551			Id
51	2010 Oct 1	2010 TK25	328901			Id
52	2010 Oct 1	2010 TM25	362493			Id
53	2010 Oct 1	2010 TP25				Id
54	2010 Oct 5	2010 TB38	372806			Id
55	2010 Oct 6	2010 TO38	326145			Id
56	2010 Oct 5	2010 TX57				Lost
57	2010 Oct 7	2010 TD65	364451			Id
58	2010 Oct 7	2010 TE65				Id
59	2010 Oct 8	2010 TF65				Lost
60	2010 Oct 6	2010 TG65			KZ, JZ	+
61	2010 Oct 8	2010 TK81				Id
62	2010 Oct 8	2010 TT150	400211			Id
63	2010 Oct 8	2010 TM173	356416			Id
64	2010 Oct 1	2010 TN173	451307			Id
65	2010 Oct 1	2010 TX173				Id
66	2010 Oct 5	2010 TV174	408755			Id
67	2010 Oct 1	2010 TW174				Id
68	2010 Oct 1	2013 HX46			KZ, JZ	+
69	2010 Oct 1	2010 TH187			KZ, JZ	+
70	2011 Mar 9	2011 EH40	336823			Id
71	2011 Mar 9	2011 EJ40	429588			Id
72	2011 Mar 8	2011 EJ44			KZ, JZ	+
73	2011 Mar 8	2011 EK44	322272			Id
74	2011 Mar 8	2011 EL44				Id
75	2011 Mar 9	2011 EM44	384640			Id
76	2011 Mar 9	2011 EN44	376234			Id
77	2011 Mar 9	2011 EJ45	448734			Id
78	2011 Mar 9	2011 EP74				Id
79	2011 Mar 9	2011 EQ74				Id
80	2011 Mar 9	2011 EE77	458571			Id
81	2011 Mar 9	2011 EQ77	465995			Id
82	2011 Mar 9	2011 ER77	456182			Id
83	2011 Mar 9	2011 EE88	443685			Id
84	2011 Mar 9	2011 FO3			KZ, JZ	+
85	2011 Mar 9	2011 FR51			KZ, JZ	+
86	2011 Sep 24	2011 SR96			KZ, JZ	+
87	2011 Sep 24	2011 SL107				Id
88	2011 Sep 24	2011 SF218				Id
89	2011 Sep 26	2011 SY221	355133			Id
90	2011 Sep 24	2011 SN247				Lost
91	2011 Sep 30	2011 SP276				Lost
92	2011 Oct 26	2011 US294	362734			Id
93	2011 Oct 26	2011 UT294				Lost
94	2011 Oct 26	2011 UU294				Id

Continued on next page

Table 2. ... continued

No.	Date of discovery	Designation	Number	Name	Discoverers	Status
95	2011 Oct 26	2011 UQ306	407724			Id
96	2011 Oct 26	2011 UR306	406876			Id
97	2011 Oct 26	2011 UZ401			KZ, JZ	+
98	2011 Oct 26	2011 UH332			KZ, JZ	+
99	2012 Oct 18	2012 UG104				Id

Notes:

KC, JZ	Kazimieras Černis, Justas Zdanavičius
*	Credited for discoverers from MAO
Lost	The lost asteroid
Id	An independent discovery
+	Waiting for crediting MAO

Concluded

Table 3. Asteroids 2010 BT3 and 2006 SF77. Keplerian orbital elements with their uncertainties for the epoch JD2457800.5=2017 Feb. 16 TDB.

semimajor axis	eccentricity	inclination	longitude of ascending node	argument of perihelion	mean anomaly
au		deg	deg	deg	deg
		Asteroid 2010 BT3			
2.57551	0.549150	13.5867	308.603805	149.93718	261.60
0.00049	0.000080	0.0014	0.000060	0.00070	0.18
		Asteroid 2006 SF77			
0.92160	0.32894	32.440	1.2519	224.332	16.5
0.00015	0.00010	0.026	0.0013	0.027	1.1

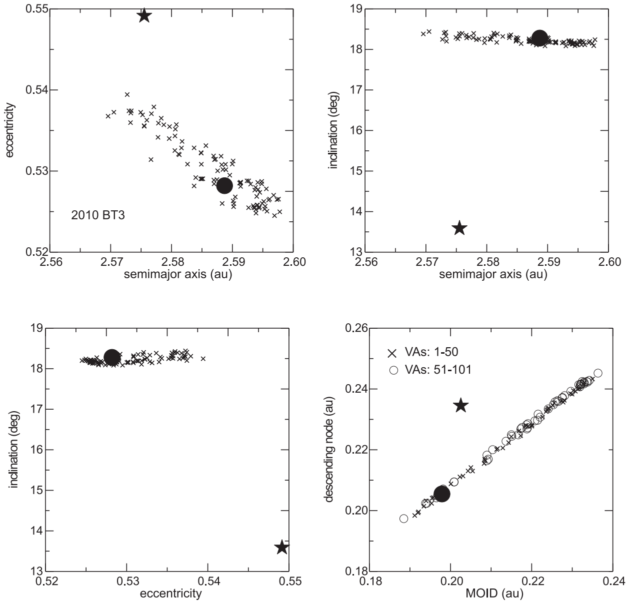

Fig. 3. 2010 BT3. Phase space of orbital elements (*a, e, i*) and (*MOID, descending node*) at the end of integration in 16999. Start position of nominal orbit of asteroid is denoted by star and final by dot.

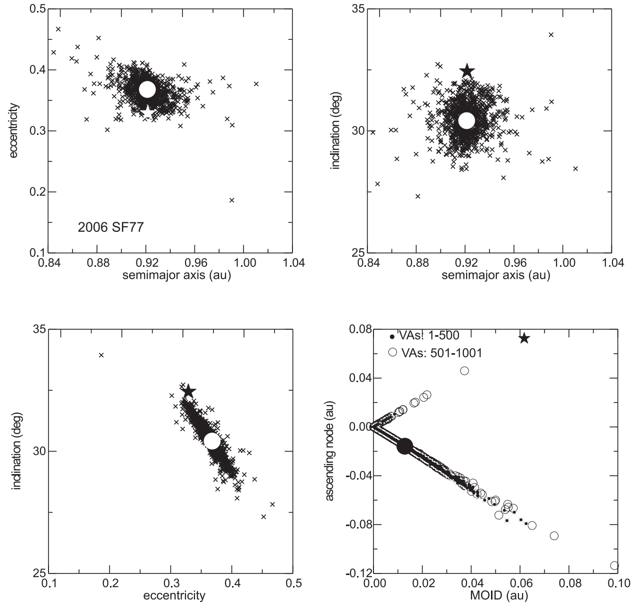

Fig. 4. 2006 SF77. Phase space of orbital elements (*a, e, i*) and (*MOID, ascending node*) at the end of integration in 16999. Start position of nominal orbit of asteroid is denoted by star and final by dot.

3 Orbits

All asteroids discovered at the MAO, have absolute magnitudes in the range of H=(13.2, 18.6) for numbered asteroids and H=(14.23, 19.6) for multiopposition asteroids. Moreover, two NEOs: 2006 SF77 has H=21.69 mag, and 2010 BT3 has H=21.34 mag. Table 2, lists all asteroids discovered at the Molėtai Observatory in 2010–2012. It was not possible to compute orbits of the following asteroids with short observational arc: 2010 EK66, 2010 EM66, 2010 RO122, 2010 TL4, 2010 TX57, 2010 TF65, 2011 SN247, 2011 SP276, 2011 UT294. They are listed in Table 2 as lost asteroids.

4 Amor-type asteroid 2010 BT3 and Aten group asteroid 2006 SF77

4.1 Orbits

We recomputed the orbits of two Near-Earth Asteroids: Amor Group Asteroid 2010 BT3 (Černis *et al.* 2012) and Aten Group Asteroid 2006 SF77 (Černis *et al.* 2008) with the new version of the OrbFit software 5.0.

Table 3 lists new computed orbits of the asteroids 2010 BT3 and 2006 SF77 by the authors where: a denotes semimajor axis, e - eccentricity, i - inclination, Ω - longitude of the ascending node, ω - argument of perihelion and M - mean anomaly.

The orbit of 2010 BT3 is computed from all 96 astrometric positions and is based on observations from 2010 Jan. 23.10902 UTC to 2010 March 05.34983 UTC.

The orbit of 2006 SF77 is computed from all 230 astrometric positions and is based on observations from 2006 Sept. 23.02142 UTC to 2006 Oct. 16.12686 UTC.

First, we computed orbit of both asteroids based on all observations using the OrbFit software: http://adams.dm.unipi.it/~orbmaint/orbfit/. 17 perturbing asteroids were used according to Farnocchia *et al.* (2013a,b)) and similarly to Włodarczyk (2015).

We used the new version of the OrbFit Software, namely OrbFit5.0 which has the new error model based upon Chesley *et al.* (2010) and also the debiasing and weighting scheme described in Farnocchia *et al.* (2015). Moreover, we used the DE431 version of JPL's planetary ephemerides.

For asteroid 2010 BT3, we computed: $RMSast$(normalized RMS(root mean squares) of the fit of orbit to observations)=0.6343", H=(21.343±0.501) mag, $MOID$ (Minimum Orbit Intersection Distance) = 0.2026 au, nodes: ascending node-Earth separation = 2.4138 au, descending node-Earth separation = 0.2346 au.

And for asteroid 2006 SF77 we got: $RMSast$=0.6907", H=(21.695±0.511) mag, MOID (Minimum Orbit Intersection Distance)=0.0618 au, nodes: ascending node-Earth separation=0.07267 au, descending node-Earth separation=-0.33110 au.

4.2 Long time orbital evolution

Next, we studied long time orbital evolution of both asteroids: Amor Group Asteroid 2010 BT3 and Aten Group Asteroid 2006 SF77. First, 101 and 1001 virtual asteroids (clones) were computed using the multiple solution method and the Line of Variation (LOV) (Milani *et al.* 2005a,b) for the 3σ uncertainty around the nominal orbit of asteroids 2010 BT3 and 2006 SF77, respectively. Hence, we got 50 and 500 clones on both sides of the LOV, respectively. Due to the high consumption of computer time to study orbital evolution of asteroid 2010 BT3 (many close approaches with planets) we did not expand the number of clones to 1001 at this stage of research. We did this in the next Section.

We propagated these clones 15000 years forwards using the OrbFit software v.5.0 and the JPL DE431. Results are presented in Figure 3 and in Figure 4.

Panel (a, e) in Figure 3 shows, for asteroid 2010 BT3, scattering of clones in semimajor axis: (2.570-2.598) au and concentrated around a = 2.585 au, and also scattering in eccentricity: (0.524-0.539) and centered around e = 0.531. In both cases centers of orbital evolution after 15 000 years integration differs from starting position of asteroid 2006 SF77 (a=2.576 au , e=0.549). Start position of nominal orbit of asteroid is denoted by star and final by dot.

Panel (a, i). The scatter in inclination, i is small. It lies in the limits (18.1 - 18.4)° and concentrated around the value i = 18.3° - over the i of starting asteroid value.

Summarizing, the average a and e do not change much, while the i increases by 26% , which means that the orbit deviates from the ecliptic plane and thus avoids collision with the planets.

Panel (*MOID, descending node*). Minimum Orbital Intersection Distance (MOID) with the Earth changes its value in the limits (0.188, 0.236) au and value of the descending node-Earth separation changes between (0.197, 0.245) au. First 50 clones lying on the left side of the LOV are marked by crosses, others clones by circles. It is difficult to separate both group of clones, mainly because of chaotic motion of clones to this date. On the other hand, there are not possible impacts over the next 15,000 years.

Figure 4 presents state of orbital elements (a, e, i) and (*MOID, ascending node*) for asteroid 2006 SF77 after integration of equations of motion of 1001 clones computed on both sides of LOV using 3-sigma uncertainty.

In panel (a, e) we can see that scattering of clones in semimajor axis: (0.845-1.010) au is concentrated around starting position of 2006 SF77 at a = 0.922 au. Scattering in eccentricity: 0.186-0.467 is centered around e = 0.353, *i.e.* significantly higher than asteroid's 2006 SF77 start position at e = 0.329. Start position of nominal orbit of asteroid is denoted by a star and the final position by dot.

In general, the average a after integration remains the same, *i.e.* near a of asteroid.

In panel (a, i) we see that scattering in the inclination, i, is within the limits (27.3 - 33.9)° and is concentrated around i = 30.45° - below the i of asteroid value.

Hence, while the average a does not change, while the value of e increases by 7 % and the value of i decreases by 6 %, which means that the orbit elongates and its orbital plane approaches the ecliptic, which can lead to a collision with the planets.

In next panel (*MOID, ascending node*) we can see that Minimum Orbital Intersection Distance with the Earth changes its value in the limits (0, 0.1) au while the value of the ascending node-Earth separation changes between (−0.114, 0.046) au. First 500 clones lying on the left side of the LOV are marked by black dots, others clones by circles. First group lies closer to the point (0, 0), *i.e.* closer to the place with possible impacts with the Earth. Generally, it is difficult to separate both group of clones, mainly because of chaotic motion of clones to this date. Hence, impacts with the Earth are possible but it is difficult to compute them. However, if the drift rate in eccentricity and inclination continued then, after of about 14 times the time span of our integration, *i.e.* after 14x15000=225000 years, most of clones can achieve ecliptic plane and possible impacts with the Earth can occur.

It is interesting that top panels of Figure 3 and Figure 4 are so different. Eccentricity and inclination in Figure 3 seems to be drifting, while in Figure 4 these parameters seems to be concentrated around a given value. Probably MMR 20:8J with MMR 3:1J (see Figure 7 and Section 4.4) and close approaches with Mars (see Figure 5 and Section 4.3) are responsible for this effect.

4.3 Close approaches with planets

Next we searched for approaches between asteroids and planets. In the case of the asteroid 2006 SF77, we used results of our computations from Section 4.2. For asteroid 2010 BT3, we additionally computed 1001 virtual asteroids (clones) using the same multiple solution method (Milani *et al.* 2005a,b) and for 3σ uncertainty as in the case of asteroid 2006 SF77. We propagated both swarm of clones 15000 years forwards using the OrbFit software v.5.0 and the JPL DE431 and searched for close approach with planets.

Figure 5 presents the results of our computations: left panel shows many close approach of clones with Mars and right panel with the Earth. We observed many close approach es with Mars. About every 3600 years, there are two pairs of deep close approach with Mars. Minimal distances are in:
4236 at 0.0052 au
7875 - 0.00267 au
11257 - 0.00208 au
14575 - 0.00127 au.
The last close approach is the deepest close approach with Mars.

After 550 years of each presented dates, we observed shallower approaches to Mars. One can see the proximity of clones to Mars - maybe in the future it will lead to a collision with Mars. But despite the over two weeks of calculations on the computer cluster, we have not computed any collisions.

In the right panel, we observed the appearance of the first approach of clones with the Earth at a distance of less than 0.2 au and only about a year 15318 at a distance of 0.199 au. The deepest approach will take place at 16992 at 0.194 au.

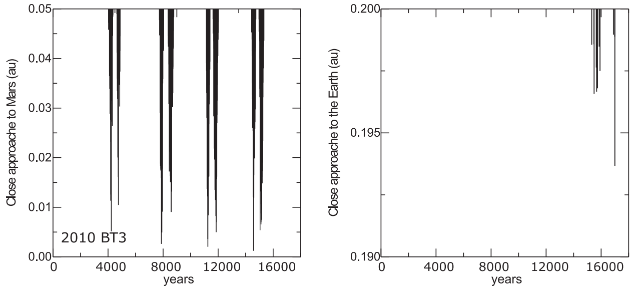

Fig. 5. 2010 BT3. Close approaches with Mars and the Earth. 1001 clones with 3 sigma between years 2000 - 17000.

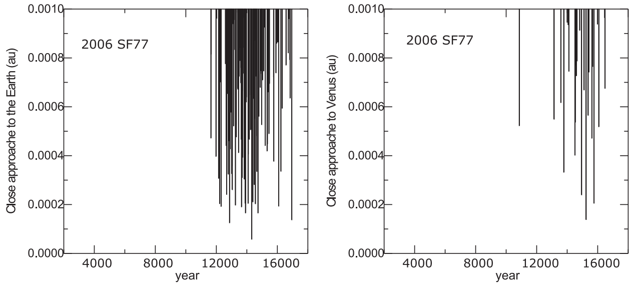

Fig. 6. 2006 SF77. Close approaches with the Earth and Venus. 1001 clones with 3 sigma between years 2000 - 17000.

As in the case of the search for collisions with Mars, despite the long-term computing, we have not found any collisions with the Earth in the next 15,000 years.

Figure 6 presents close approaches of clones of asteroid 2006 SF77 with the Earth and Venus. In both approaches, we see that they are significantly different from those for 2010 BT3. The picture is more irregular and chaotic. Another close approach is hard to predict.

Deep approaches with the Earth, greater than 0.001 au, appear after about year 12000. They last almost until the end of the integration period. Likewise they behave in close proximity with Venus. Although in the case of Venus they are rarer.

We were also looking for possible collisions with the Earth and Venus and we did not find them.

However, they may be probable with the Earth what may suggest Figure 4 in the panel with (*MOID, ascending node*).

4.4 Mean motion resonances

Next, we searched for mean motion resonances for the asteroids under study, for which time evolution of semimajor axis is presented in Figure 7 and Figure 8.

We followed a similar method of searching of mean motion resonance as in Sekhar et al. (2016) but we used the new code named atlas2bgeneral developed by Gallardo (2014) available at http://www.fisica.edu.uy/~gallardo/atlas/2bmmr.html. Gallardo takes typical orbital values of NEAs (e=0.46, i=15°) and calculates MMR with all planets from Mercury to Neptune with parameters $|p+q|$ <100 and order q <100.

We studied the order of resonance for the orbital elements at the moment when they have semimajor axes close to predicted mean motion resonance as shown in Figure 7 and Figure 8.

According to the notation of Gallardo for interior resonances, we have ($|p+q| > |p|$), degree p<0 and order q>=0. Hence, for the asteroid 2010 BT3, for mean motion resonance 20:7 with Jupiter (notation MMR 20:7J), we have p=7, q=13.

It is important to take into account that to identify a resonant motion, it is necessary that the oscillation period of the semimajor axis be the same as the oscillation or circulation period of the critical angle.

In Figure 7 for asteroid 2010 BT3 - bottom panel, we present a time evolution of the critical angle of resonance, σ which circulates around variables in time values between 0 and 360° in the years 2000 and 3000. It means that asteroid is close but not captured in exact resonance. We mark the position of the exact resonance 20:7J in semimajor axis/time diagram.

The asteroid 2010 BT3 has an oscillation in semimajor axis of about 77 years while the circulation period of the critical angle of the resonance 20:7J is about 210 years as we can see in Figure 7 bottom left panel. Then, this resonance is not the main cause of the evolution of semimajor axis. It was the strongest resonance (a=2.58312 au, strength 0.208 according to map of Gallardo) close to the semimajor axis of 2010 BT3. It is easy to verify that the cause of the oscillations in semimajor axis is the influence of the proximity (a=2.50045 au) of the very strong resonance 3:1J. The critical angle of 3:1J is circulating with the same frequency of the semimajor axis as is presented in right bottom panel of Figure 7. Observed MMR 20:7J is weak and and lies between strong resonances 3:1J, 1:4E and 11:4J as is shown in Gallardo plots of location and strength of the MMRs (Gallardo 2006).

In Figure 8 we presented time evolution and MMR of the next asteroid under study - 2006 SF77. Plotting the semimajor axis, there are visible several flat runs in many intervals of time. The asteroid 2006 SF77 shows a chaotic evolution in the semimajor axis from t=4000 year up to t=14000 year when it seems to be captured in a resonance up to t=16000. We shows a line with position of semimajor axis connected with MMR 9:8E (a=0.92448 au). The most relevant resonances between 0.920 au and 0.925 au seems

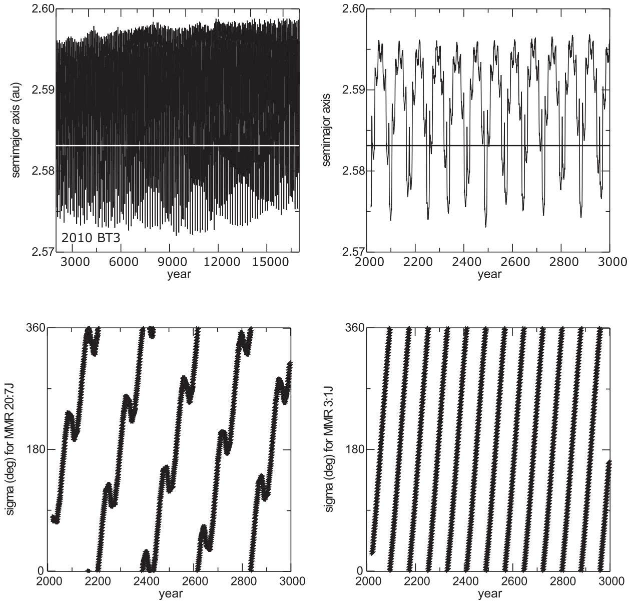

Fig. 7. 2010 BT3. Nominal orbit. Mean motion resonance 20:7 with Jupiter.

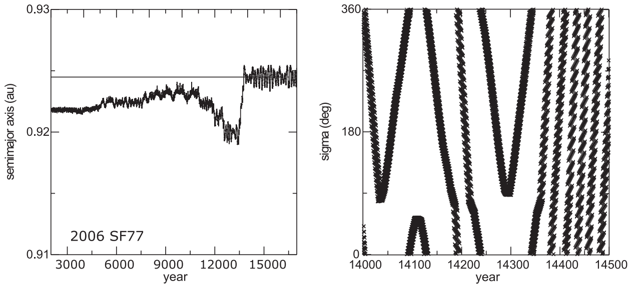

Fig. 8. 2006 SF77. Nominal orbit. Mean motion resonances 9:8 with the Earth.

to be 9:13V (a=0.92428 au) and 16:23V (a=0.92152 au) and probably resonance 26:23E at semimajor axis, a=0.92152 au (between year 2000 and 4000), and also 3:11Me (Me means Mercury) at a=0.92046 au (in years 13000-14000). The resonance with Mercury is very weak and can be discarded. All these MMRs lie between two strong MMR 2:3V and 1:1E. These neighboring resonances overlap each other and destabilize the motion of the asteroid, f.e. (Morbidelli 2002). Mean motion resonance 9:8E is the strongest and is presented in bottom right panel in Figure 8. The influence of the MMRs in the motion of the asteroid 2006 SF77 are visible in Figure 6 where chaotic changes in close approaches with the Earth and Venus after 12000 are visible. Similar effect in the motion of the asteroid 2010 BT3 is visible after year 12000 as is visible in Figure 5 right panel.

4.5 Lyapunov time

As was presented in Section 4.3, both asteroids are characterized by many deep close approaches with the Earth, Mars and Venus.

It can lead to chaotic behavior of the orbit of asteroid and the position of asteroid in space.

The measure of the chaotic nature of the orbit is called maximum Lyapunov characteristic exponent, y (Tancredi *et al.* 2000):

$$y = lim(1/t)ln(\frac{d}{d_0}) \tag{1}$$

where d is the separation between two nearby trajectories at time t, and d_0 is an initial separation. Moreover, $t \rightarrow \infty$ and $d_0 \rightarrow 0$. Lyapunov time is the inverse of y.

First, the starting orbital elements of both asteroids and planets for the same epoch were computed with the software Mercury (Chambers 1999). Next, we added these starting orbital elements as input files to the Swift software, mainly *swift_rmvs3_f* (Broz 2003) and integrated 101 clones of both asteroids separately for a period of 1 My and with orbital elements produced every 10 years.

For the first time, we used here our new method of finding the value of the Lyapunov time by taking three neighboring orbits from the integration (above) which differ in starting positions from the nominal orbit by 10^{-8}, 10^{-9} and 10^{-10} au, respectively. We found the mean value of the Lyapunov times, T_L=1650 years for asteroid 2010 BT3 and T_L=830 years for asteroid 2006 SF77. We compared our method of computing Lyapunov time on the example of the NEO asteroid (3200) Phaethon with the procedure in the OrbFit software. In both cases we got similar value of Lyapunov time of about 400 years. Hence, because of the chaotic nature of the asteroid motion, the long term evolution, greater than of about T_L=1650 years and 830 years, can be studied statistically using a swarm of clones not only the nominal orbit.

Moreover, in both integrations, the results of time evolution of the asteroid 2010 BT3 and 2006 SF77 depend on the starting elements of their orbits, the method of integration of the equations of motion, the Solar system model used, the method of selection and weighting of observational material and the error model of existing reference star catalogues.

Table 4. Ephemerides for geocentric observer of asteroids 2010 BT3 and 2006 SF77 discovered at the Molėtai Observatory.

Date	RA	DEC	Mag	Solar	Lunar	Sky plane error		*PA*
				elongations		Err1	Err2	
2010 BT3 – Amor Group Asteroid								
13 Dec 2059	10 45 12.380	+25 28 51.66	22.0	106.5	-150.1	8.974	0.000	149.3
23 Dec 2059	11 4 4.236	+18 27 19.44	21.9	109.9	-23.2	8.855	0.000	146.8
2 Jan 2060	11 16 4.675	+11 19 53.16	21.7	114.9	94.2	8.696	0.000	141.7
12 Jan 2060	11 21 11.899	+ 4 26 13.97	21.6	121.3	-129.7	8.634	0.000	134.7
22 Jan 2060	11 19 35.581	− 1 53 16.69	21.5	129.2	9.9	8.749	0.000	127.3
1 Feb 2060	11 11 46.169	− 7 15 56.14	21.4	137.9	120.9	8.979	0.000	121.4
11 Feb 2060	10 59 13.376	−11 20 31.42	21.4	146.7	-93.8	9.105	0.000	118.0
21 Feb 2060	10 44 39.425	−13 56 59.43	21.5	153.8	39.3	8.916	0.000	117.0
2 Mar 2060	10 31 1.348	−15 10 43.37	21.7	157.0	149.6	8.363	0.000	117.6
2006 SF77 – Aten Group Asteroid								
13 Sep 2021	12 44 0.949	−64 1 59.58	21.9	-69.6	55.6	2.883	0.000	-139.4
23 Sep 2021	17 18 28.060	−59 8 50.49	21.1	-84.8	111.6	2.918	0.000	106.9
3 Oct 2021	19 24 1.452	−31 50 36.59	21.0	-98.2	-143.1	4.094	0.000	47.3
13 Oct 2021	20 8 56.849	−11 52 27.29	21.5	-102.1	-18.3	3.583	0.000	-148.2

5 Ephemerides for the asteroids with orbits of low accuracy

Table 4 presents ephemerides for geocentric observer computed for two Near-Earth Asteroids: Amor Group Asteroid 2010 BT3 and Aten Group Asteroid 2006 SF77, when they will be the brightest. These ephemerides correspond to the next observational opprotunity. We used the OrbFit software version 5.0 and JPL DE431 planetary and lunar ephemerides with 17 additional perturbing asteroids according to (Farnocchia *et al.* 2013a,b). The ephemerides are computed for the value of the non-gravitational parameter $A2 = 0$.

In Table 4 the columns list, in succession, right ascensions (h, m, s) and declinations (deg, arcmin, arcsec), expected magnitudes, solar and lunar elongations (deg), the sky plane errors with long axis ($Err1$) and short axis ($Err2$) in deg, and the position angle (PA, deg). Great error in long axis ($Err1$) is connecting with properties of error propagation of the orbital elements. The errors in magnitudes are of the order of 0.5 mag. We need additional astrometric observations to extend their observational arcs and to improve the orbital elements.

6 Summary

During the years 2010–2012, we obtained at the Molėtai Observatory a total of 14 811 astrometric positions of 3450 asteroids. Of these asteroids, 99 are new discoveries that account for 0.08% of all 118434 asteroids discovered worldwide during 2010-2012. It is interesting that from 2000 to 2012 at the Molėtai Observatory were discovered 357 asteroids. We recomputed orbits of two dynamically interesting Near-Earth Asteroids: Amor Group Asteroid 2010 BT3 Aten Group Asteroid 2006 SF77 and studied their orbital evolution in the next 15000 years.

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