



Tetraneutron: Rigorous continuum calculation

A. Deltuva

Institute of Theoretical Physics and Astronomy, Vilnius University, Saulėtekio al. 3, LT-10257 Vilnius, Lithuania

ARTICLE INFO

Article history:

Received 10 April 2018

Received in revised form 9 May 2018

Accepted 13 May 2018

Available online 17 May 2018

Editor: J.-P. Blaizot

Keywords:

Four-body scattering

Transition operators

Resonance

Universality

ABSTRACT

The four-neutron system is studied using exact continuum equations for transition operators and solving them in the momentum-space framework. A resonant behavior is found for strongly enhanced interaction but not a the physical strength, indicating the absence of an observable tetraneutron resonance, in contrast to a number of earlier works. As the transition operators acquire large values at low energies, it is conjectured that this behavior may explain peaks in many-body reactions even without a resonance.

© 2018 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

1. Introduction

The four-neutron ($4n$) system is an exotic few-body system challenging experimental techniques as well as theoretical understanding of the nuclear force and methods for the description of the few-particle continuum. It has attracted a great interest in the last few years [1–5], but, nevertheless, remains highly controversial. An experimental observation of few events in the double charge-exchange reaction ${}^4\text{He}({}^8\text{He}, {}^8\text{Be})$, that were interpreted as a formation of a tetraneutron resonance with the energy $E_r = 0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst})$ MeV and width $\Gamma \leq 2.6$ MeV [1], still awaits a confirmation in the analysis of further experiments. The theoretical predictions for the tetraneutron are even more contradictory: They range from a narrow near-threshold resonance with $E_r \approx 0.8$ MeV and $\Gamma \approx 1.4$ MeV [2] or $E_r \approx 2.1$ MeV [3] to a broad resonance with $E_r \approx 7.3$ MeV and $\Gamma \approx 3.7$ MeV [4] while other authors [5,6] predict no observable tetraneutron resonance at all, i.e., negative E_r and very large Γ . Despite these differences, all above works concluded that tetraneutron properties are insensitive to the details of the neutron–neutron (nn) and three-neutron ($3n$) interaction models as long as they remain realistic. Thus, those very different predictions cannot be explained by differences in employed potentials but raise question on the reliability of at least some of the above calculations. Indeed, the $4n$ system resides in the continuum whose exact treatment is much more complicated as compared to bound states. However, among the above-mentioned works only the solution of the complex-scaled

Faddeev–Yakubovsky (FY) equations [5,6] treats the continuum rigorously; if no further simplifications are made unlike in Ref. [4] this applies also to the no-core Gamow shell model. In contrast, the harmonic oscillator representation of the continuum [2] and the bound-state quantum Monte Carlo with the extrapolation to the continuum [3] approaches are not natural methods for a rigorous description of the four-particle continuum. In fact, none of the approaches from Refs. [2–4] has been applied successfully to other four-nucleon scattering processes, in contrast to FY equations [7]. However, the only method that so far provided reliable results for *all* four-nucleon reactions above the complete breakup threshold, i.e., for elastic, charge-exchange, transfer, and breakup processes in nucleon–trineutron and deuteron–deuteron collisions, is the momentum-space transition operator method [8,9]. Furthermore, it provided the most accurate results in the field of the universal four-fermion [10] and four-boson [11] physics, including the properties of resonant (unstable) four-particle states. The method is an exact integral version of FY equations [12] proposed by Alt, Grassberger, and Sandhas (AGS) [13,14]. Its application to the $4n$ problem is highly desirable, since being a rigorous continuum method it should provide reliable conclusions regarding the tetraneutron resonance, much like in the case of the $3n$ system [15], where it clearly supported the earlier conclusion [5,16] on the $3n$ resonance unobservability in contrast to Ref. [3]. Another very important advantage of the transition operator method is its ability to determine not only the resonance position and width but also the nonresonant (background) contribution to scattering amplitudes, thereby making solid conclusions regarding the resonance observability in physical processes.

E-mail address: arnoldas.deltuva@tfai.vu.lt.

2. Theory

AGS equations for four-nucleon transition operators have been applied to the study of reactions initiated by all possible two-cluster collisions [8,9]. The situation is different in the $4n$ system that has no bound subsystems and the only possible reaction is the elastic scattering of four free particles. Starting from Ref. [17], the operator for this $4 \rightarrow 4$ process can be split into two-, three-, and four-particle components, i.e.,

$$T_{4 \rightarrow 4} = \sum_j t_j + \sum_{j\beta} t_j G_0 U_\beta^{ji} G_0 t_i + \sum_{ji\beta\alpha} T_{\beta\alpha}^{ji}. \quad (1)$$

Here Latin (sub)superscripts denote pairs while Greek subscripts denote two-cluster partitions (subsystems) that can be of $3+1$ or $2+2$ type. Furthermore, G_0 is the free resolvent at the available energy E , $t_j = v_j + v_j G_0 t_j$ are the pair transition operators with pair potentials v_j , and $U_\beta^{ji} = G_0^{-1} \delta_{ji} + \sum_k \delta_{jk} t_k G_0 U_\beta^{ki}$ are the subsystem transition operator where $\delta_{ji} = 1 - \delta_{ji}$. The four-particle transition operators obey the system of integral equations

$$T_{\beta\alpha}^{ji} = \sum_k t_j G_0 U_\beta^{jk} \delta_{\beta\alpha} G_0 t_k G_0 U_\alpha^{ki} G_0 t_i + \sum_{\gamma k} t_j G_0 U_\beta^{jk} G_0 \delta_{\beta\gamma} T_{\gamma\alpha}^{ki}. \quad (2)$$

Taking into account identity of neutrons the equations (2) can be symmetrized, reducing the number of $j\beta$ components from 18 to just two, one being of the $3+1$ type and another of the $2+2$ type; in the following they will be abbreviated by subscripts 1 and 2, respectively. For example, four-neutron operators $T_{\beta 2}$ are obtained from integral equations

$$T_{12} = t G_0 U_1 G_0 t G_0 U_2 G_0 t + t G_0 U_1 G_0 (T_{22} - P_{34} T_{12}), \quad (3a)$$

$$T_{22} = t G_0 U_2 G_0 (1 - P_{34}) T_{12}, \quad (3b)$$

where P_{34} is the permutation operator of particles 3 and 4, while t and U_β are symmetrized pair and subsystem operators, respectively [18]. Kernel of Eqs. (3) is built from the same operators (just in a different order) as in Refs. [8,9,18] for two-cluster reactions. Thus, the solution technique to a large extent can be taken over from Refs. [8,9,18]. It is performed in the momentum-space partial-wave representation [18], whereas kernel singularities arising from G_0 are treated by the complex-energy method with special integration weights [19]. As the four-cluster matrix elements exhibit stronger dependence on the imaginary part ε of the energy, smaller values $0.1 \text{ MeV} \leq \varepsilon \leq 1 \text{ MeV}$ as compared to Refs. [8,9,19] have to be used, which implies larger number (around 80) of grid points for the discretization of Jacobi momenta k_x, k_y , and k_z in the notation of Refs. [11,19].

A pure $4n$ scattering experiment is practically impossible, with presently available experiment techniques one may only indirectly observe $4n$ as a final subsystem in a more complicated reaction such as ${}^4\text{He}({}^8\text{He}, {}^8\text{Be})$. It is complicated many-body process that cannot be described rigorously by presently available methods, however, half-shell matrix elements of $T_{\beta\alpha}$ that determine the $4n$ wave function, together with some simplified reaction model, may provide estimation for the properties of the final $4n$ subsystem, e.g., its energy distribution. Therefore it is important to evaluate also half-shell matrix elements of $T_{\beta\alpha}$.

3. Results

In the following I consider the $4n$ state with total angular momentum and parity $\mathcal{J}^\Pi = 0^+$; namely in this state Refs. [2–4]

predict the $4n$ resonance. In order to make comparison with those works, I use chiral effective field theory (χ EFT) potential at next-to-leading order (NLO) [20], an improved version of the local NLO potential used in Ref. [3], and a low-momentum potential that should have similar behavior as those used in Refs. [2,4]. It is based on a realistic Argonne V18 potential [21] evolved using the similarity renormalization group (SRG) transformation [22] with the flow parameter $\lambda = 1.8 \text{ fm}^{-1}$. It is important that this is one of few models able to reproduce quite well not only the ${}^3\text{H}$ binding energy but also the cross section for n - ${}^3\text{H}$ scattering in the energy regime with pronounced four-nucleon ($3n$ + proton) resonances [23]. For this reason it can be considered as a well suited model for the $4n$ resonance study.

In order to follow the evolution of the $4n$ resonance, I also perform calculations enhancing the nn potential by a factor $f > 1$ in mn partial waves with the total angular momentum $j_x < 3$ except for the 1S_0 wave where the physical potential strength is kept, ensuring that there are no bound dineutrons. The calculations include nn partial waves with $j_x < 3$ and $3n$ partial waves with the total angular momentum $J_y < \frac{7}{2}$, while subsystem orbital angular momenta are $l_y, l_z < 5$. With these cutoffs the results appear to be well converged.

Since the $4n$ resonance corresponds to the pole of the transition operators $T_{\beta\alpha}$ in the complex energy plain at $E_r - i\Gamma/2$, these values are extracted from the energy dependence of calculated $T_{\beta\alpha}$ matrix elements much like in Ref. [15] for the $3n$ resonance. In general they are functions of six Jacobi momentum variables, but since all of them exhibit the same resonant behavior, they are shown for few initial and final on-shell and off-shell states only, abbreviated by $|k_a\rangle$ and $|k_a^{\text{off}}\rangle$, respectively. They are chosen with $l_x = l_y = l_z = 0$ and two vanishing Jacobi momenta $k_j = 0$ for $j \neq a$, the remaining one being $k_a = \sqrt{2\mu_a E}$ and $k_a^{\text{off}} = \sqrt{2\mu_a(E + \epsilon_{\text{off}})}$, respectively. For example, the state $|k_z\rangle$ in the $2+2$ configuration corresponds to two pairs of neutrons with vanishing relative nn momentum, that can be interpreted as a two (unbound) dineutron state. In the above relations μ_a is the associated reduced mass while ϵ_{off} measures how much off-shell the system is. A typical value in the shown results is $\epsilon_{\text{off}} = 2 \text{ MeV}$ that roughly corresponds to ${}^8\text{He}$ binding with respect to the ${}^4\text{He} + 4n$ threshold.

$4n$ transition operators $T_{\beta\alpha}$ in the $\mathcal{J}^\Pi = 0^+$ wave calculated using the physical nn potential, i.e., $f = 1$, show no indications of resonance, but for sufficiently large f a resonant behavior is clearly seen in all matrix elements; two examples for the SRG potential with $f = 1$ and $f = 5$ are presented in Fig. 1. The results indicate that nonresonant contributions are very important even at $f = 5$ with $E_r - i\Gamma/2 \approx (5.9 - 0.6i) \text{ MeV}$. The $\mathcal{J}^\Pi = 0^+$ resonance position and width extracted at different f values are displayed in Fig. 2. The $4n$ system becomes bound at $f = 5.29$. Thus, the tetra-neutron is lower in energy than the trineutron that in the SRG model becomes bound only at $f > 6$ [15]. The results for $f \geq 5.3$ obtained solving the standard bound state FY equations connect to $f \leq 5.3$ results indicating the consistency between the simpler bound state and much more complicated continuum calculations. Surely, the resonance trajectory depends on the particular enhancement scheme used and therefore is not identical with those in Refs. [5,6]. Nevertheless it exhibits a typical behavior [5,6,16,15]: decreasing the enhancement factor f the pole first moves to higher energy and away from the real axis until the turning point where E_r starts to decrease while Γ continues increasing rapidly. For $f < 4.3$ the pole of $T_{\beta\alpha}$ becomes too far from the real axis to be discernible from the nonresonant continuum, which results in increasing theoretical error bars estimated as in Ref. [15]. Thus, an unrealistically large enhancement of the higher-wave potential is needed to support an observable $4n$ resonance, which strongly suggests that at the physical interaction strength there is

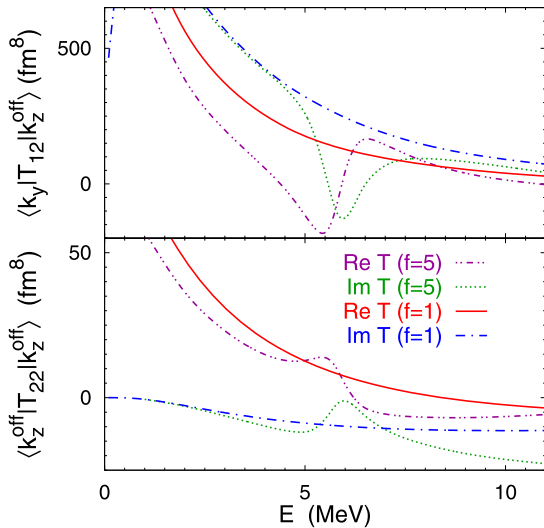


Fig. 1. (Color online) Energy dependence of real and imaginary parts of selected $\mathcal{T}^{\Pi=0^+}$ four-neutron transition matrix elements calculated using the SRG potential with higher wave enhancement factors $f = 1$ and 5.

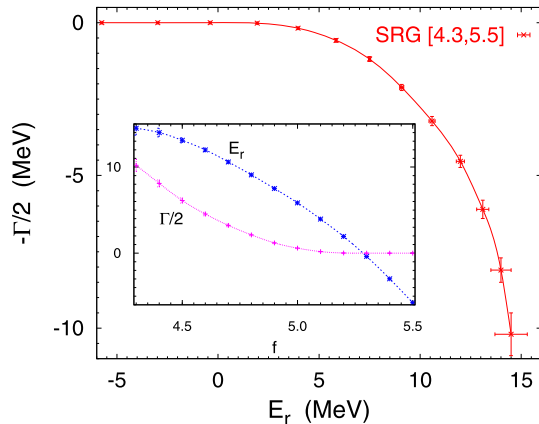


Fig. 2. (Color online) Four-neutron $\mathcal{T}^{\Pi=0^+}$ resonance trajectory obtained with the SRG potential varying the higher-wave enhancement factor f from 5.5 to 4.3 with the step of 0.1. The inset shows the individual dependence of E_r and Γ on f . Lines are for guiding the eye only.

no observable $4n$ resonance, in agreement with Refs. [5,6] and in contradiction with Refs. [2–4].

The absence of an observable $4n$ resonance with a physical nn interaction is shown in Fig. 3 over a broader energy range on the example of still another matrix elements of $T_{\beta\alpha}$. Also predictions with the NLO potential are presented. In fact, the results are almost independent of the force model, as observed also in previous works. Calculations using the CD Bonn potential [24], not shown in Fig. 3, provide an additional confirmation. Furthermore, the results appear to be insensitive to P - and higher-wave interaction: SRG calculations including only the 1S_0 nn force agree quite well with full SRG results. The dominance of the S -wave interaction may indicate a manifestation of the four-fermion universality where observables are governed by a large nn scattering length. This point of view also supports the absence of an observable $4n$ resonance since the universal four-fermion system is very far from being bound: a positive scattering length for two difermions indicates that their effective interaction is repulsive [10,25].

Despite that no observable $4n$ resonance is predicted, matrix elements of transition operators $T_{\beta\alpha}$ acquire large absolute values at low energies. This can be seen in both Figs. 1 and 3, and is con-

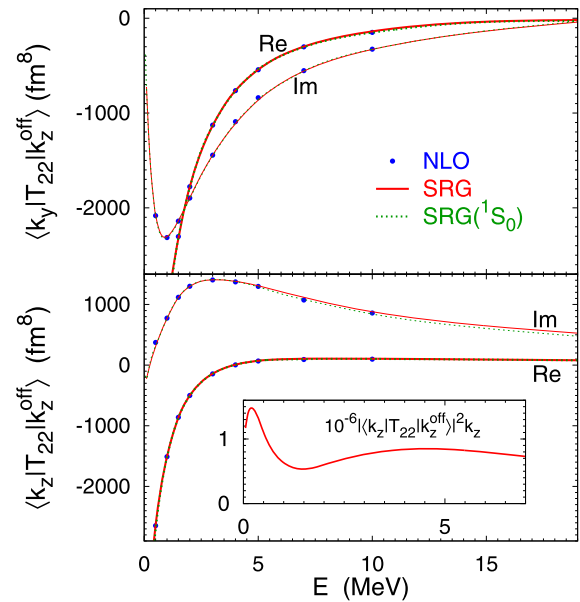


Fig. 3. (Color online) Energy dependence of selected $4n$ transition matrix elements obtained using the physical NLO (dots) and SRG (solid curves) potentials. For the latter also the results including only the 1S_0 nn interaction are given by dotted curves. The inset shows the squared matrix element multiplied with k_z arising from the phase-space factor.

firmed by further calculations not shown here. One may conjecture that this low-energy enhancement could manifest itself also in more complicated many-body reactions with the $4n$ subsystem in the final state such as ^4He (^8He , ^8Be) of Ref. [1]. The amplitude for such a reaction could be approximated by a many-body double charge-exchange matrix elements for the involved clusters (^8Be and $4n$) weighted with the corresponding initial and final-state wave functions [26]. It also depends on the double charge-exchange operator that is not well known; note that a choice made in Ref. [26] has not produced a pronounced peak without a resonance. Nevertheless, to illustrate the possibility of the low-energy enhancement, in the inset of Fig. 3 the squared matrix element of $T_{\beta\alpha}$ multiplied with k_z due to the phase-space factor is plotted; this product would be a factor in the integrand determining the cross section $d^6\sigma/d^3k_x d^3k_y$ for two (unbound) dineutrons. Indeed, this quantity exhibits a two-peak shape: a sharp and narrow one around 0.25 MeV and a broad one around 4.5 MeV. Note, that peaks are possible even in repulsive systems; a textbook example is given in Ref. [27].

Finally, it is important to understand the difference to Refs. [2–4] that predicted a tetra-neutron resonance. Among the approaches used in those works there is also one based on the nn force enhancement by a factor f and subsequent extrapolation of the obtained bound-state energy to the $f = 1$ limit in the continuum.¹ However, Refs. [2–4] apply the same factor f in all nn waves thereby generating a bound 1S_0 dineutron once f exceeds roughly 1.1. Thus, $4n$ states interpreted in Refs. [2–4] as bound tetra-neutrons are in fact above the two-dineutron threshold. Strictly speaking, no stable $4n$ bound state is possible above the two-dineutron threshold, only scattering states. Thus, a calculation of $4n$ bound states in the regime above the two-dineutron threshold and extrapolation of their energies is meaningless. A similar situation arises for the $4n$ system in an external trap where a tetra-neutron “bound” at the $4n$ threshold [3] is above the dineutron threshold.

¹ In Ref. [2] this was an auxiliary method used beside the harmonic-oscillator representation.

These simple considerations indicate serious shortcomings in the calculations of Refs. [2–4] and question the reliability of their results.

4. Conclusions

The $4n$ system was studied using one of the most reliable four-nucleon continuum methods. The integral equations for transition operators were solved in the momentum space leading to well-converged results. Strongly enhancing the nn force in higher partial waves the $4n$ model system in the $\mathcal{J}^{\pi} = 0^{+}$ state was made bound or resonant. In the latter case the resonant behavior was seen in all transition matrix elements, their energy dependence was used to extract the resonance position and width. However, reducing the enhancement factor the resonant behavior disappears well before the physical strength is reached. This indicates the absence of an observable $4n$ resonance, in agreement with Refs. [5,6] and in contradiction with Refs. [2–4], a possible reason being the neglect of the dineutron threshold in the latter works.

Even without an observable $4n$ resonance the transition operators exhibit pronounced low-energy peaks. It is conjectured that they may be seen also in more complicated reactions such as ${}^4\text{He}({}^8\text{He}, {}^8\text{Be})$ of Ref. [1] with the $4n$ subsystem in the final state. The present calculation of half-shell matrix elements of $4n$ transition operators is a first step toward understanding of those reactions.

Acknowledgements

The author acknowledges discussions with R. Lazauskas, the support by the Alexander von Humboldt Foundation under Grant No. LTU-1185721-HFST-E, and the hospitality of the Ruhr-Universität Bochum where a part of this work was performed.

References

- [1] K. Kismori, et al., Phys. Rev. Lett. 116 (2016) 052501.
- [2] A.M. Shirokov, G. Papadimitriou, A.I. Mazur, I.A. Mazur, R. Roth, J.P. Vary, Phys. Rev. Lett. 117 (2016) 182502.
- [3] S. Gandolfi, H.-W. Hammer, P. Klos, J.E. Lynn, A. Schwenk, Phys. Rev. Lett. 118 (2017) 232501.
- [4] K. Fosse, J. Rotureau, N. Michel, M. Płoszajczak, Phys. Rev. Lett. 119 (2017) 032501.
- [5] E. Hiyama, R. Lazauskas, J. Carbonell, M. Kamimura, Phys. Rev. C 93 (2016) 044004.
- [6] R. Lazauskas, J. Carbonell, Phys. Rev. C 72 (2005) 034003.
- [7] R. Lazauskas, Phys. Rev. C 91 (2015) 041001.
- [8] A. Deltuva, A.C. Fonseca, Phys. Rev. Lett. 113 (2014) 102502.
- [9] A. Deltuva, A.C. Fonseca, Phys. Lett. B 742 (2015) 285.
- [10] A. Deltuva, Phys. Rev. A 96 (2017) 022701.
- [11] A. Deltuva, Phys. Rev. A 85 (2012) 012708.
- [12] O.A. Yakubovsky, Sov. J. Nucl. Phys. 5 (1967) 937.
- [13] P. Grassberger, W. Sandhas, Nucl. Phys. B 2 (1967) 181.
- [14] E.O. Alt, P. Grassberger, W. Sandhas, JINR report E4-6688, 1972.
- [15] A. Deltuva, Phys. Rev. C 97 (2018) 034001.
- [16] R. Lazauskas, J. Carbonell, Phys. Rev. C 71 (2005) 044004.
- [17] A.C. Fonseca, in: Lecture Notes in Physics, vol. 273, Springer, Heidelberg, 1987, p. 161.
- [18] A. Deltuva, A.C. Fonseca, Phys. Rev. C 75 (2007) 014005.
- [19] A. Deltuva, A.C. Fonseca, Phys. Rev. C 86 (2012) 011001.
- [20] E. Epelbaum, H. Krebs, U.-G. Meißner, Phys. Rev. Lett. 115 (2015) 122301.
- [21] R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. C 51 (1995) 38.
- [22] S.K. Bogner, R.J. Furnstahl, A. Schwenk, R.J. Perry, Phys. Lett. B 649 (2007) 488.
- [23] A. Deltuva, A.C. Fonseca, S.K. Bogner, Phys. Rev. C 77 (2008) 024002.
- [24] R. Machleidt, Phys. Rev. C 63 (2001) 024001.
- [25] D.S. Petrov, C. Salomon, G.V. Shlyapnikov, Phys. Rev. Lett. 93 (2004) 090404.
- [26] R. Lazauskas, J. Carbonell, E. Hiyama, Prog. Theor. Exp. Phys. 2017 (2017) 073D03.
- [27] S. Flügge, Practical Quantum Mechanics I, Springer, Berlin, 1971, p. 44.