



From Eons to Epochs: multifractal Geological Time and a compound multifractal-Poisson model

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Geological time is punctuated by events that define biostrata and the Geological Time Scale's (GTS) hierarchy of eons, eras, periods, epochs, ages. Paleotemperatures and macroevolution rates, have already indicated that the range ≈ 1 Myr to (at least) several hundred Myrs is a scaling (hence hierarchical) "megaclimate" regime. We apply analysis techniques including Haar fluctuations, structure functions trace moment and extended self-similarity to the temporal density of the boundary events ($r(t)$) of two global and four zonal series. We show that $r(t)$ itself is a new paleoindicator and we determine the fundamental multifractal exponents characterizing the mean fluctuations, the intermittency and the degree of multifractality. The strong intermittency allows us to show that the (largest) megaclimate scale is at least ≈ 0.5 Gyr. We also analyze a Precambrian series going back 3.4 Gyrs directly confirming this limit and allowing us to quantitatively compare the Phanerozoic with the Proterozoic eons.

We find that the probability distribution of the intervals ("gaps") between boundaries and find that its tail is also scaling with an exponent $q_D \approx 3.3$ indicating huge variability with occasional very large gaps such that its third order statistical moment barely converges. The scaling in time implies that record incompleteness increases with its resolution (the "Resolution Sadler effect"), while scaling in probability space implies that incompleteness increases with sample length (the "Length Sadler effect").

The density description of event boundaries is only a useful characterization over time intervals long enough for there to be typically one or more events. In order to model the full range of scales (and low to high $r(t)$), we introduce a compound Poisson-multifractal model in which the multifractal process determines the probability of a Poisson event. The model well reproduces all the observed statistics.

Scaling changes our understanding of life and the planet and it is needed for unbiasing many statistical paleobiological and geological analyses, including unbiasing spectral analysis of the bulk of geodata that are derived from cores.

