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Aristotle's Numbers *as* Hylomorphic Compounds

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Abstract: One plausible approach to Aristotle's philosophy of arithmetic is that he held numbers to be hylomorphic compounds. While research has been conducted to show the benefits of this theory, particularly in solving the problem of numbers' unity, none has been extensively dedicated to justifying the initial hypothesis that numbers have hylomorphic structure. The absence of such an account is felt acutely due to Aristotle's repetitive negation throughout *Metaphysics* M–N that numbers are not and could not be either forms or hylomorphic compounds. This paper argues that despite these explicit statements, Aristotle does, in a way, consider numbers to be hylomorphic without resorting to Platonic hypostatization. His thinking is grounded in the distinction between numbers' ontological status in the natural world and their mental epistemological mode of being. In the natural world, numbers exist as quantitative properties of things, being numbers just potentially. However, in his understanding, the mathematician separates these properties and considers them as if they were individual entities possessing hylomorphic structure. This is how numbers mentally appear to be hylomorphic without actually being such. The paper further clarifies the epistemological procedure of abstraction which bridges the gap between numbers' dual mode of existence and explores various aspects of numerical form and matter to strengthen the initial hypothesis and show that Aristotle indeed had a coherent understanding of the ontology of number.

Keywords: aristotle; arithmetic; ontology of number

1 Introduction

Although Aristotle's views about the ontological status of numbers remain an enigmatic topic and scholarly literature is still relatively scarce, two approaches seem to have crystallised. The first group of scholars believes that individual numbers are species of the genus number, with specific cases (e.g., three dogs) being instances of

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those species. The second argues that numbers are hylomorphic compounds.[1] Neither position is unproblematic. The hylomorphic solution, however, holds particular appeal, as it seems to solve one of the most pressing puzzles about number – that of its unity. In *Metaphysics* H 3, Aristotle ponders about two possibilities:

> Further, number must be something in virtue of which it is a unity, though people cannot now say what it is that makes it so, if indeed it is. (For either it is not, but is like a heap, or it is, and then it should be explained what it is that makes it one out of many.)[2]

As Edward Halper famously puts it, 'Aristotle reserves the term "heap" for what has little or no ontological status', and that for him, 'to be something is *not* to be a heap'.[3] Quite a few scholars[4] have pointed out that the problem of unity is at the core of Aristotle's critique of his predecessors' accounts of number.[5] In his comment on the H 3 passage, David Bostock helpfully summarizes that Aristotle poses this question to his predecessors, especially the Platonists since it is essential in presenting a consistent theory of number. Therefore, Aristotle must have a theory that accounts for the unity of numbers to avoid the criticism his predecessors were subject to.[6]

A hylomorphic line of thought does just that: it is not difficult to imagine units[7] as matter, structured by a numerical form. Despite being an appealing solution, this approach was recently seriously criticized by Emily Katz. Her main objection is that 'Aristotle's insistence that his opponents account for the unity of number is always conditional upon another of their key commitments: the identification of numbers

1 The first approach, with its variations, is advocated by Mignucci (1987, pp. 193–201), Halper (1989, p. 261f.), and Katz (2022, p. 141f.). Among the supporters of the second theory are Gaukroger (1980, p. 188), Bostock (1994, p. 167f.), Mueller (1987, p. 251f.), Cleary (1995, p. 375) and Galluzzo (2018, pp. 296–8). Pappas' (2018) PhD thesis is a more interesting case: in the first part, it tends towards interpreting numbers as species, but when Pappas reaches the question of unity, he seems to think that Aristotle's answer must be hylomorphism. Finally, he concludes that how these two different views should be reconcile is unclear.

2 1044a3–5, trans. Bostock.

3 Halper (1989, p. 256).

4 Halper (*ibid.*), Mueller (1987, p. 252), Galluzzo (2018, p. 303f.).

5 Cf. Aristotle, *Metaphysics*, A 9, 992a1f.; K 2, 1060b10–12; *Λ* 10, 1075b34–6.

6 Bostock (1994, p. 269).

7 The Greeks thought of number (*arithmos*) as a plurality of units. That is why they regarded two to be the first number, as one (as a unit) was only the principle of counting and not a plurality (N 1, 1088a7f.). As a concept, then, number designated numerical types of sets of units that could be found in the natural world. It is important to distinguish this notion from the modern post-Renaissance one, when number began to be apprehended as an abstract concept of quantity. For the Greeks and Aristotle as well, number was always a number of something, of some kind of units (see I 1, 1053a30; M 9, 1085b22; N 5, 1092b19f.). Furthermore, Aristotle speaks about numbers in form-matter terms, naming the units matter and the number form (for instance, see M 8, 1084b3–29; N 5 1092b19).

and substances.'[8] This is an identification that, according to her, Aristotle himself does not hold.[9] In a sense, Katz is right. As she proceeds to show, every passage (including the already mentioned H 3) usually quoted in literature indeed has conditional context: *if* numbers are substances, *then* they must have unity. Moreover, Aristotle often explicitly states that numbers are not or could not be substances, i.e., either forms or compounds of form and matter. He denies this from the beginning of the *Metaphysics* (see A.9) and repeats it frequently throughout books M−N (e.g., M 2, 1077a33–b15; N 3, 1090a29; N 5, 1092b16–25).

In this paper, I argue that there is, after all, a way for numbers to be treated as hylomorphic entities that do not commit Aristotle to a Platonic tendency to hypostatize mathematical objects. My reasoning rests on the discernment of the nuanced way of Aristotle's thinking about mathematical objects' different modes of being. In the natural world around us, he sees things having mathematical properties. However, after perceiving a group of objects as being of a certain number, this grasped entity, which exists only in understanding and not in reality, appears as a hylomorphic being. How such a shift occurs is connected to Aristotle's notion of abstraction. I dedicate Sections 3–5 to discussing its nature and various aspects and issues, especially how we can understand abstracted numerical form and matter. Finally, in Section 6, I offer further clarifications on the nature of numerical form, the unity of numbers and what the proposed analysis tells us about Aristotle's concept of arithmetic. In this last section, I also discuss in what sense we should understand numbers as hylomorphic entities. For clarity, I have to state here at the beginning that I do not hold numbers to be hylomorphic in the same way as natural substances or equate the two. Nevertheless, I hold them to have a hylomorphic structure that can be analysed into form and matter, and the nature of which will hopefully be accounted for in due course.

While all of these topics are complex and deserve more individual detailed treatment than is offered here, I hope to sketch a preliminary map of answers to a number of interconnected problems. Scholars often treat various questions concerning the Aristotelian nature of numbers separately, and I believe that to be the reason why it is so difficult to find any definite answers to any of them. An attempt to tackle different aspects of Aristotle's theory of number, on the other hand, in a holistic manner, as a whole at once should indicate whether reasoning about each problem is correct not only by its internal coherence but by coherence with answers

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8 Katz (2021, p. 199).

9 Katz does agree, though, that an ontology that identifies substances with numbers for Aristotle *would* be required to answer the question of unity (2021, p. 200). Furthermore, she does not entirely dismiss the idea that numbers should have some unity, and so in her article, Katz argues for a peculiar kind of unity. For reasons why her argument conflicts with the pillars of Aristotelian metaphysics, see Section 6.

to other problems. This bigger picture should open a path for a more detailed inquiries into each philosophical puzzle in the future.

2 Aristotle's Dual Ontology of Number

In discussions concerning numbers' mode of being, the text frequently employed is *Metaphysics* book M. At the end of M 2, Aristotle suggests that mathematical objects either do not exist at all (not a viable alternative), or they exist in a certain way that is not unqualified, for the word 'exist' has many senses.[10] In M 3, he offers some clarification by proposing that the arithmetician or the geometrician studies mathematical objects by taking that which does not exist separately and positing it as such.[11] Upon closer analysis, this brief statement in M 3 seem to echo far more detailed account in the *Physics*. An especially illuminating passage is found in B 2:

> Now the mathematician too busies himself about these things, although not insofar as each of them is the limit of a natural body, nor does he get a theoretical grasp on the coincidents of natural bodies insofar as they are such. That is why he separates them. For they are separable in the understanding from movement, and so their being separated makes no difference, nor does any falsehood result from it. Those who speak about the Ideas do this too, though it escapes their notice. For they separate natural objects though these are less separable than mathematical ones. This would become clear if one were to try to state the definitions both of the objects themselves and their coincidents. For odd and even, straight and curved, and furthermore number, line, and figure will be without movement, whereas flesh, bone, and human will not, but rather all of them are said of things just as snub nose is and not as curved is.[12]

The general idea is relatively straightforward: mathematicians consider their objects as separable (*chōrista*) from change or matter. It is worth noting that these objects are not *actually* separate but can be separated in mind through mental activity (the nature of this process is discussed in Section 3). Thus, the subsequent question in determining numbers' mode of being seems obvious, yet never explicitly asked in the context of Aristotle's philosophy of arithmetic: what does it mean to be separate?

To quote Phil Corkum, 'The Greek *chōris* and its cognates, when unqualified, typically in Aristotle refers to the separation that he ascribes to primary substances. (When qualified, the term can refer to other notions, such as local, temporal, or definitional separation.)'[13] Although the theme of separation is complex and controversial and cannot be thoroughly tackled here, the most significant aspect has

10 1077b16f.

11 1078a22f.

12 193b32–4a8, trans. Reeve (2018).

13 Corkum (2016, p. 2).

already been addressed. Separation, first of all, is an ontological notion reserved for primary substances (i.e., hylomorphic compounds) that exhibit a capacity for independent existence. There is a good basis to believe that, in a way, Aristotelian forms are ontologically separate as well.[14] A conclusion would follow that Aristotle equates being separate with being a substance. In her recent article, Katz makes the same equation, claiming that the conditional 'if number is separate' stands for 'if number is substance'.[15] Yet Aristotle dedicates M 2 to demonstrating that mathematical objects cannot be separate (*kechōrismenoi*) from sensible ones without ontological consequences.[16] Moreover, in M 3 he stressed that their existence is qualified – not unqualified, like substances'. The difficulty is evident, and Katz's disproval that numbers for Aristotle could be independent substances seems sound.

Nevertheless, if the reasoning of this paper is correct so far, *Physics* B 2 and *Metaphysics* M 3 imply *considering* numbers as substances. The nuance, then, is indeed extremely subtle and can be easily overlooked: Aristotle differentiates between the actual existence of things in the ontological structure of the world and the epistemological potential of their mental existence. Hence, while numbers do not exist as independent beings, Aristotle suggests that they appear as such in understanding. While the exact metaphysical status of these numerical entites is yet to be determined, it is safe to assume that they should exhibit the form-matter structure of natural substances, as to them the mental existence of numbers is equated.

Even so, in addition to the complexities that arise from regarding numbers as hylomorphic compounds, the primary and fundamental issue is that 'separate' cannot be simply equated with 'hylomorphic' if forms can be separate as well. Furthermore, in *Physics* B 2, it is stated that numbers are considered separate from change. Since change in Aristotelian (meta)physics is enabled by matter, the question is then, is matter eliminated altogether, or does it still exist in a certain way? These questions should be elucidated by an inquiry into the process of acquiring numbers, to which we now turn.

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14 For more on this, see Katz (2017). To briefly summarize, the ascription of separateness to forms was thought to mean separate in thought or definition. Except that this makes the *Metaphysics* inconsistent: in various passages where the form is said to be separate, separation has substantial ontological implications (e.g., *Δ* 8, 1017b23–6; Z 1, 1028a33f.; H 1, 1042a28–31; see Katz (2017, pp. 42–52) for a detailed analysis). Therefore, it became necessary to accommodate an ontological meaning of separation to forms to save Aristotle's philosophy from being incoherent. One evident difficulty of such an approach is the lack of textual evidence, namely, Aristotle's direct discussion of the notion of separation. On the other hand, the differentiation of meanings is deduced within Aristotle's theoretical framework, making it probable and convincing.

15 Katz (2022, p. 205).

16 See 1076b11–7b11.

3 Notion of Abstraction Revisited

Vangelis Pappas accurately observes that it is important for Aristotle to highlight mathematics' close ties to the natural world; otherwise, they would not apply to it. Consequently, mathematical objects' existence should be accommodated within those ties.[17] In M 3, Aristotle stresses that sciences can apply their propositions to sensible objects because these objects have respective relevant properties: this also stands for mathematics. He proceeds to compare the applicability of mathematics with that of physics. Physicists consider their objects *as* moving and do not take into account their nature or other characteristics. It does not follow that there is some moving object separate from sensible substances or that they have a separate moving nature.[18] The same will apply to mathematics – there will be propositions, in arithmetic's case, that could be applied to objects *as* indivisible units.[19] One can conceptualize it as a selective focus, where scientists consider solely the property essential to their scientific field. Things get confusing later on. Aristotle further writes that if things that the mathematician considers are coincidentally sensible, it does not follow that mathematical sciences are about sensible objects.[20] Nor does it imply, though, that objects of mathematics exist separately from these. The already-mentioned positive answer is presented at the end of M 3 – the mathematician takes that which does not exist separately and considers it *as if* it were separate (i.e., as if it were a hylomorphic compound).[21] E.g., the arithmetician posits a man as one indivisible, and then studies what is incidental to him as such. Aristotle's train of thought then seems to be as follows: the arithmetician, observing a group of objects, posits them to be indivisible units and then separates their incidental *properties* in understanding as if they were matter and form. These quasi-hylomorphic entities, then, are the mathematical objects that the mathematician further inquires into. Once

17 Pappas (2018, p. 75).

18 1077b24–33.

19 The concept of being an indivisible unit can be puzzling. For instance, in M 3, Aristotle takes a man to be indivisible. Yet it can be argued that a man can be easily divided into parts. Following Pappas' (2018, p. 144) and Galluzzo's (2018, p. 206) interpretation, Aristotle's suggestion here is to consider beings indivisible *per se*, meaning that by dividing a man, one cannot get more men. However, Barnes (1985, p. 114) presents a complicated case that by dividing a cube, one can get more identical cubes. That is why Katz's (2021, p. 211 n. 72) argument seems the most valid. She argues that it is not necessary for a unit to be indivisible as a unit – it is more of a coincidence that a man cannot be divided into more men. Bearing in mind that a unit is always some kind of measure, Katz notes how Aristotle writes about defining this measure by taking a thing as one according to the senses (*pros tēn aisthēsin*) (N1, 1087b37–8a3). Therefore, while theoretically, a cube can be divided into more cubes, the senses perceive it as one cube, which will be the starting point of further counting.

20 1078a3–5.

21 1078a18–29.

again, they do not somehow exist *in* sensible substances. The only thing that belongs to natural substances are mathematical attributes that fall into the category of quantity, which is dependable on substances. However, the mathematician separates these quantitative properties in his mind, and this separation entails their transformation into independent mental entities. This is how mathematics preserves its ties to the natural world: if natural substances did not possess mathematical properties, there would not be anything to separate.

It the light of *Metaphysics* K 3, it can be quite soundly stated that such mathematical objects are obtained by a process of abstraction (*aphairesis*).[22] Although the nature of it has already been extensively discussed, not much consensus has been reached (except that this is indeed the way the mathematicals are obtained). That should not come as a surprise insofar as the questions are interconnected: it is difficult to clarify the method when it is not clear what results one should get by following it. However, in the framework of 'numbers as substances' hypothesis, we can look at the concept of abstraction in K 3 once again:

> But just as the mathematician produces his theoretical knowledge about things that result from abstraction, for he gets his theoretical grasp on them having first stripped away all the perceptible attributes (for example, weight and lightness, hardness and its contrary, and further, also heat and cold, and the other perceptible contrarieties), and leaves behind only the quantitative and the continuous (sometimes in one, sometimes in two, sometimes in three dimensions) and the attributes of things insofar as they are quantitative and continuous, and does not get a theoretical grasp on any other aspect of them, but investigates the relative positions of some and what belongs to them, and the commensurabilities and incommensurabilities of others, and the ratios of others.[23]

Abstraction appears as a mental procedure where the mathematician not only focuses on what concerns him particularly in sensible objects, but also 'leaves' only those qualities, taking away everything sensible.[24] In this case, the mathematician leaves quantity, which is constituted by units and attributes that are intrinsic to it. To illustrate, let us say there are three people. According to Aristotle's reasoning, when the arithmetician takes away everything that constitutes them as people, he gets abstract units. And as the only thing left characteristic of their abstractedness is the numeric property of being three, one could say that the numeric property is what then *constitutes* the abstractedness. Therefore, after the group of objects have been

22 To name a few who are in favour of Aristotle's philosophy of mathematics being abstractionistic: Mueller (1970, p. 161), Gaukroger (1980, p. 188), Mignucci (1987, p. 181f.), Cleary (1995, pp. 312–18), Katz (2019, pp. 486–92; 2022, p. 137).

23 1061a29–b2, trans. Reeve (2016).

24 The K 3 passage makes it clear that this applies not only to arithmetic but to geometry as well. However, in this discussion, I am leaving the case of geometry aside, as it deserves separate attention within a broader analysis of Aristotle's philosophy of geometry.

reduced just to their quantity, the numerical property appears to act as a form that holds the units (i.e., matter) together.[25] That is how, by grasping the elements that are left after the abstraction, the mathematician performs their mental separation. Moreover, these elements are inevitably thought of in form-matter categories. Stephen Gaukroger (1980) aptly names them 'noetic mathematical objects' which have form as well as matter, with both components perceived as abstracted and separate in understanding from their sensible counterparts. But before moving on to the elucidation of the nature of noetic matter, a few clarifications should be offered about numerical form.

4 The Peculiarities of Numerical Forms

In M 3, Aristotle stresses that by considering as separate that which does not exist separately, mathematicians not only do not fall into error, but they also treat something that really exists, for things exist in two ways – in actuality or as matter (i.e., potentially).[26] One plausible interpretation is that mathematical entities exist only potentially in sensible things and are brought out to actuality by the mind. Mario Mignucci is right in saying that this explanation is rather obscure.[27] However, in the light of a hylomorphic interpretation of numbers, the sentence in M 3 is elucidated by reading *de Anima*. Aristotle writes that objects of thought – forms – exist in things that have matter only potentially and are actualized in understanding.[28] Now, Mignucci disagrees with connecting the M 3 and *de Anima* passages by arguing that this would imply numbers being substances, which Aristotle denies. According to him, the process of actualization 'concerns the condition of forms with respect to mind which thinks of them, but not their ontological status. To think of a dog does not imply that the form of the dog is contained only potentially in the dog and that it is brought to actuality by the thinker. The form of the dog is actually in the individual dog' – meaning that when applied to mathematical objects, it would make them

25 Cf. M 8, 1084b6f., where it is said that if number is separate, the units act as a material part, and the number is the form of the units.

26 1078a21–31.

27 Mignucci (1987, p. 183).

28 See III 4, 429b10–23; 430a5–9; III 8, 431b20–2a2. It is true that in his theory about understanding and knowing by reason throughout *de Anima* III.4–8, Aristotle does not dedicate separate attention to acquisition of mathematical objects. However, the mathematical examples being offered together with more customary ones (e.g., stones) suggests that mathematical objects are obtained the same way as physical entities are. For instance, at III 4, 429b11, Aristotle notes that a magnitude is not the same as the essence of magnitude.

forms that exist in sensible objects.[29] Again, this reasoning seems sensible and in accordance with Aristotle's thought that numbers cannot be independent beings. But the normal cognitive process of actualization in the case of mathematical objects is supplemented by the process of abstraction. Abstraction is an additional, special element in this epistemological equation that allows attributes to be treated as separate forms. Thus, there is no reason why numbers could not be actualized in the mind: existing as quantitative attributes, after abstraction they appear as forms. Abstraction should not be viewed as a psychological but a logical procedure – it allows to separate a predicate from its sensible matter and consider it independently.

However, doesn't that mean that other sciences should also consider relevant separate forms? Should not motion to a physicist and health to a doctor be abstracted attributes considered as forms? Aristotle answers this in the same *Physics* B 2 chapter, discussing the differences between mathematicians and natural scientists. Indeed, a natural scientist, too, concerns himself with forms. Nevertheless, nature is twofold – form *and* matter. Accordingly, it belongs to a natural scientist to know both, e.g., a doctor knows health, and also bile and phlegm, in which health resides. This twofoldness is echoed again in *Metaphysics* E 1:

> Among things defined, i.e. those which are *what* something is, some are like the snub, others like the concave, and the difference between these is that in the snub matter is implicit – for the snub is a concave *nose* – whereas concavity is independent of perceptible matter. So if every naturally existing thing is called [what it is] in the same way as the snub, as for instance nose, eye, face, flesh, bone, and animal as a whole, and leaf, root bark, and plant as a whole – for the formulae of none of them are independent of change but always include matter – the manner in which we need to investigate and define what a thing is in the case of naturally existing things is plain.[30]

The main idea is that physical things cannot be reasoned about without considering matter. That is why mathematical sciences are unique and universal: the relevant predicates can be completely separated from matter.[31] Physicists, on the other hand, cannot fully 'actualize' pure forms, although they perform abstraction to an extent to

29 Mignucci (1987, p. 183). Pappas (2018, p. 98) is in agreement.

30 1025b32–6a5, trans. Kirwan (1993).

31 It is worth noting how Aristotle speaks about 'separating' relevant qualities in other sciences as well. This strengthens the present argument that separation implies studying certain aspects *as if* they were autonomous beings in themselves. Which, of course, would force one to think about them as substances, as that is the category of primary beings. However, the difference between mathematical and natural sciences is that the latter cannot isolate relevant qualities from sensible matter completely, meaning that theorising and applying knowledge about the natural 'objects' becomes limited and contingent. E.g., a doctor, even possessing universal theoretical knowledge of health, has to take into consideration the condition of a patient and tailor his knowledge accordingly (*Metaph* Z 7, 1032b1–15). This does not apply to mathematics due to the capacity of the qualities studied by it to be completely isolated from sensible matter. That is why Aristotle repeats that no error can fall out of isolate their subject-matter. Moreover, as it will become clear from the next section, mathematical forms do not require specific matter to be applied to, meaning that anything can be counted and made into mental hylomorphic compounds. As Aristotle stresses, a unit is a measure, and thus, 'a number means a measured plurality and a plurality of measures'.[32] The measure can be perceived as an identification of the matter that the number form will be applied to, because choosing a measure is choosing what kind of units, i.e., matter, will be counted. This being said, let us proceed to the question of the nature of mathematical objects' matter.

5 The Nature and Necessity of Intelligible Matter

As it happens, nowhere does Aristotle speak about the complete elimination of matter, including the passages quoted above – mathematical objects appear to have matter just as other objects do. But, in the same K 3 and E 1, Aristotle tells us that mathematical matter is abstracted, which makes it a peculiar kind of matter. The standing consensus is to identify this abstracted matter with noetic or intelligible matter (*hylē noētē*). Although *Metaphysics* does not elucidate its nature much, it does look as though Aristotle considered the matter of mathematical objects to be of this type. He indicates this and even offers a kind of definition in Z 10 and Z 11:

> Some matter is perceptible, e.g. bronze, wood, and all changeable matter, while some is intelligible, namely that which is present in perceptible things but not *qua* perceptible. Such is the matter of the objects of mathematics.[33]

> Yet the objects of mathematics are not perceptible objects. … But in fact there is no difference, for some non-perceptible objects also have matter. Indeed everything has matter of some sort unless it is not a this but a what-being-is and a form itself in its own right. Accordingly the semicircles will not be parts of the universal circle, but they will be parts of particular circles, as was said before. For matter may be either perceptible or intelligible.[34]

These passages do state rather directly that objects of mathematics have intelligible matter, and that these are the type of objects that a mathematician considers. It is also briefly stated on the nature of intelligible matter that it is present in sensible things not *qua* sensible. This is an important claim, as by it, Aristotle emphasizes intelligible matter's connection to the sensible one – it is not a distinct, other-worldly matter of

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mathematical reasoning (*Phys* B 2, 193b34f.; *Metaph* M 3, 1078a20). As matter is the factor that makes things contingent and changeable, the elimination of it leads to results being invariable.

32 N 1, 1088a5f. Cf. I 1, 1052b32f.; I 6, 1057a3f.). For choosing a measure, see N 1, 1088a8–14.

33 1036a9–11, trans. Bostock.

34 1036b32–7a4, trans. Bostock.

sorts. As stated in the previous section, abstraction should be viewed as logical procedure that allows us, in the case of numbers, to separate only 'the attributes of things insofar as they are quantitative and continuous'. These quantitative attributes, as argued, can then be considered as forms. But in his notion of abstraction in K 3, Aristotle also applies this procedure to mathematical objects' matter: the mathematician in his mind strips away everything that is perceptible and leaves only pure quantity (or continuity), bounded by mathematical attributes. Thus, it is another step of the double method of abstraction as a logical procedure by which a mathematician can attain the notion of individual units *as if* they would be devoid of sensible properties, no matter that such abstract units do not exist in reality (hence the name 'mental', *noētē*, matter). Similarly in geometry, a circle can have (and will have, as it will be a particular object) specific matter, yet by the method of abstraction the mathematician will attain its idea as an abstract extension, bounded by a geometrical form.[35]

However, Aristotle does not fail to stress that in reasoning we do not make any use of the fact that mathematical objects have an underlying subject. In *Posterior Analytics* I 13, 79a8–10, he explicitly states that mathematical sciences are concerned with forms (i.e., abstracted properties) and do not reason about a particular substrate. Even if mathematical objects are said of a particular substrate, they are not studied in relation to it. One can ask, then, what is the role of matter, and what happens with it in mathematical reasoning?

In principle, matter allows us to perform various calculations. A form, although designating the thing's essence, giving it an identity and bringing it to actuality, without the matter is not yet the thing in question. That is, in the case of arithmetic, one can think about a mentally separated form as a concept that implies a certain number of units – but the form denoting three units is not yet a number three that comprises three units. The number comes into being only when the form becomes rooted in matter, i.e., the relevant number of units. Thus, matter is essential for

35 In this interpretation, I follow Gaukroger (1980, p. 188). Gaukroger gives a more extensive account, but the foundations of this notion of noetic matter were already laid out by Mueller (1970). Although there is some criticism of this approach, none is substantial enough to reject the theory altogether. Annas (1976, p. 30) even acknowledges that this interpretation of intelligible matter is appealing and makes Aristotle's philosophy coherent. Yet, according to her, Mueller relies too much upon later commentators of Aristotle in building his argument. This view of Annas seems undeserved. While it is true that Mueller appeals to later commentators to bolster his theory, his approach to them is cautious and critical, and a careful reading of the *Metaphysics* and the *Posterior Analytics* generates his initial idea. Pappas' (2018, p. 97) criticism bears more weight, as he is concerned with the problem of precision: geometrical objects should be ideal objects (e.g., a circle should be ideally circular), but that does not seem possible because sensible objects cannot be ideal embodiments. It may be an interesting point for further discussions, nonetheless, it is irrelevant to arithmetical objects (i.e., numbers). Moreover, no viable alternatives to the notion of noetic matter have been suggested.

existence. As argued, the number's measure can be considered as the identification of material units that will be counted: in a number of 10 horses the number form of 10 will comprise 10 units whose nature it is to be a horse. Hylomorphism, then, allows us to count perceptible magnitudes.

Furthermore, abstract matter enables to perform theoretical mathematical operations. In *Metaphysics Δ* 27, Aristotle states that a number cannot be mutilated (*kolobousthai*) because to be mutilated means to stay the same in substance just without some part (e.g., a man is still a man after losing his hand). Meanwhile, any change in the case of numbers implies a substantial change.[36] Mathematical operations such as subtracting or adding lead to a change in numbers' nature, meaning that, for instance, from adding 3 and 4, the resulting 7 is a whole new being. And in the case of change where one thing ceases to exist and another comes into being, there must be something that persists.[37] That is precisely the matter (in this case, intelligible) that acts as a substrate, which becomes constituted by a new form after the change.[38]

We can find additional support for this argument in *de Anima* I 4, 408b30–9a31, where Aristotle refutes the possibility that the soul is a number of some sort by arguing that souls have features which numbers lack. This feature is the soul's capability to be divided without forcing a change in species. It is characteristic of many plants and animals: when divided into parts, they continue to live and have the same soul. Meanwhile, 'if someone subtracts a number, or a unit, from a number, what remains is a different number.'[39]

Two concerns might be raised here. First, there appears to be a tension between Aristotle's claims that the mathematician is only concerned with forms and then the necessity of matter's involvement, albeit abstracted, in mathematical operations. I will get back to this question in the last section of the paper. For now, one can raise another: if numbers have matter, how do they differ from the objects of physics?[40]

The first thing to accentuate is that the role of matter in theoretical mathematical operations is strictly functional. Although it is a necessary condition for mathematical

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36 1024a18–24. Cf. H 3, 1043b36–8, trans. Bostock: 'And just as, if you add or substract anything – however small – from the things of which a number is composed, the result is no longer the same number but a different one'.

37 Aristotle, *Physics*, A 7, 190a13–1a22.

38 The same stands for operations with sensible numbers, providing the matter of them is the same – one cannot add 10 horses to four dogs. In such case, one has to search for a new measure, which here could be 'animal' (see *Metaph* N 1, 1088a8–14).

39 406a7f., trans. Shields. For a detailed treatment of this passage, see Shields commentary (2016, p. 147f.). Shields is in agreement that mathematical operations lead to a number differing in kind from the original one.

40 I would like to thank to the anonymous reviewer for pressing this issue.

objects to exist, it does not in any way affect the number's nature – while matter is existentially essential, its nature is incidental, and the mathematician does not take it into consideration, perceiving it only as an abstract quantity. Consequently, pure numbers as such are free of change, since matter does not affect the nature of the number.[41] This is not possible in natural sciences, where the object's matter must be taken into consideration (e.g., only a nose can be snub), and which makes sensible objects subject to change (see the quoted *Phys* B 2, *Metaph* E 1 passages). A more pressing point is that numbers, as argued, admit to one type of change – that of substantial change when a new number emerges after performing calculations. However, the capacity to change is not a sufficient condition for an object to be the subject-matter of natural sciences, as there are different ways *how* things change. Aristotle brings this to our attention in *Physics* B 1, claiming that entities existing by nature have a principle of change within themselves, while the changes of artefacts are brought about externally. Although mathematical objects are not artefacts, they can be paralleled to certain extent: first of all, they are not natural entities, and secondly, their existence is brought about by intellectual capacities.[42] While it is true that numbers have ties with the natural world around us, they exist only as mathematical properties – numbers as concepts and pure entities are attained and exist only in the mind. From *Phys Δ* 14, 223a22–9, we learn that the existence of numbers is dependent on the existence of the soul (more precisely, reason). I agree with Mignucci's careful analysis here that this passage does not commit Aristotle to saying that if the soul did not exist, then the numbers would not exist *at all*. Rather, he maintains that the possibility of numbers' existence is dependent on the soul for its realisation.[43] This is consistent with the claim put forward at the beginning of this paper, that Aristotle differentiates between the actual existence of things in the ontological structure of the world and the epistemological potential of their mental existence. In addition, the change of numbers in mathematical operations then can be seen as a purely speculative change that happens due to human reasoning, and thus falling outside of the natural sciences' domain.

41 In a sense, sensible numbers are free of change as well. Although the matter of the objects counted can change – for instance, two acorns grow into two oak trees – it does not look like the number changes from within. More likely, as numbers are mental entities, one number (that of acorns) ceases to exist, and another one takes its place (that of trees). Sensible numbers, as I see it, are also capable only of substantial change.

42 The efficient cause of an artefact lies within the artisan, since to create an object, the artisan has to attend to a relevant *logos*. Only then can he proceed to a material production of the artefact. Importantly, to be an efficient cause of an object appertains only to humans. Aristotle does not hold the productive activities of other animals as crafts precisely because they are not results of deliberation (*Phys* B 8, 199a20f.).

43 Mignucci (1987, pp. 184–6).

A more interesting case might be the so-called 'mixed' or 'subordinate' sciences of mathematics, which include astronomy, harmonics, optics and mechanics, which fall under the relevant theoretical branch of mathematics, e.g., harmonics falls under arithmetic. In *Physics*, Aristotle even calls these sciences (the first three) the more natural-science-like parts of mathematics.[44] However, they are not truly natural sciences, and Aristotle is cautious in his distinction. These branches of mathematics are still mathematical sciences, only their domain happens to be of the natural world.[45] In the framework of this paper, this means that subordinate sciences have sensible matter. How can they not be natural sciences, then?

The difference lies in the fact that in these kind of mathematical deliberations matter does not enter reasoning. While deliberations are about a particular substrate, the substrate is relevant only inasmuch as it exhibits mathematical properties, i.e., lines and numbers:

> The same account applies to harmonics and optics; neither studies its objects as seeing or as utterance, but as lines and numbers (these being proper attributes of the former); and mechanics likewise.[46]

Harmonics and optics, then, do not consider the faculties of seeing and hearing, nor the causes of sight or voice or any other matters of this sort. The only thing that is important to these sciences are how sight and voice exhibit mathematical properties, which can be abstracted just the same.[47] Now, the understanding of, let us say, health, requires to take into account the state of a human's bile and phlegm – it is not possible to state anything about a person's health without knowing the condition of his or her body. Hence, Aristotle offers us a rather clear distinction between natural and mathematical sciences. Whether it is sensible or intelligible matter, it should be perceived as a necessary condition for an object of mathematical interest to exist, yet as a condition that does not in any way affect mathematical consideration of said object.

One relevant obstacle Gaukroger rightly points out in the current argument is that all of Aristotle's examples regarding intelligible matter are about geometry and

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44 B 2, 194a7–9. Cf. *Metaph* M 2, 1077a1–8; *APo* I 13, 79a10–16. Mechanics is included in *Metaph* M 3, 1078a13–16; *APo* I 13, 78b35–9a3.

45 It is worthwile to stress this because some scholars have held them to be the more mathematical of the natural sciences, instead of the more natural of the mathematical sciences. For more on this point and arguments for astronomy, harmonics, optics, and mechanics being mathematical sciences, see Distelzweig (2013, pp. 6–9).

46 *Metaph* M 3, 1078a13–16, trans. Annas.

47 For a more detailed treatment of the topic, see Distelzweig's (2013) article, esp. p. 14f. This reasoning also stands for sensible numbers. While their matter is perceptible, it does not enter mathematical deliberation.

geometrical objects, and that it is difficult to understand how intelligible matter should be perceived in the case of numbers.[48] That may also be the reason why the literature dedicated to noetic matter primarily discusses it only in relation to geometry. However, Gaukroger develops an interpretation that Aristotle held conception of arithmetic as a metrical geometry, which appears to be highly possible considering the mathematical practices of the time. Not only geometry was more prominent than arithmetic, but there was no abstract notation for mathematical proofs neither digits for numbers. Thus, because the numbers are always numbers *of* something, mathematical operations for the Greeks were performed using geometrical entities, of which the most basic entities were points and line lengths.

I would like to draw attention to a crucial claim put forward by Gaukroger that this does not mean that either the Greeks or Aristotle *identified* noetic numbers with noetic lines.[49] Aristotle is very clear on the fact that quantity is of two kinds, plurality and (spatial) magnitude, these being respectively inquired by separate sciences of arithmetic and geometry.[50] Moreover, in *Phys* Z 1, Aristotle advances the idea that lines, which are continuous entities, cannot consist of indivisible points – lines are infinitely divisible. As numerical plurality is a plurality of discrete, non-continuous points, it is simply impossible to identify numbers with lines, which as entities are objects of geometry. Nevertheless, line lengths can be treated arithmetically by holding a line length to be an indivisible unit (cf. n. 19), and the whole of units of length to be a number, which is exactly what was done in Antiquity. Given arithmetic's geometrical notation, it would explain why Aristotle provides only geometrical examples of intelligible matter while stating that it is the matter of '*ta mathēmatika*', all kinds of mathematical objects. Hence, the arithmetically treated line lengths were possibly the way to think about the arithmetical number comprising abstract units (*monadikos*), of which Aristotle talks at *Metaph* M 8, 1083b16f. Plausibly, this 'arithmetical' number is the same mathematical number made up of undifferentiated units and called *ho monadikos* that was discussed throughout M.6–7.[51]

48 Gaukroger (1980, p. 187). Mueller's theory concerns only geometry as well. Nevertheless, in another article, he briefly remarks that in the case of numbers, one should probably understand noetic matter as an abstract plurality of units (1987, p. 251f.). According to Mueller, it is by no means certain that Aristotle held such a conception, yet there seems to be no plausible alternative conceptions to assign to him.

49 Gaukroger (1980, p. 192).

50 *Metaph Δ* 13, 1020a7ff.

51 Cf. M 6, 1080a22f.; 1081a19f.; M 7, 1082b5–7.

6 Clarifications on Number's Unity and Aristotle's Concept of Arithmetic

Finally, we can again turn to the question from which it all started – that of number's unity. As Gabrielle Galluzzo points out, this is fundamentally a question about number's identity.[52] It inquires why a number is a unique number, e.g., why a number of six units is an individual number six and not, let us say, an instance of a doubled three. The passage that leads to thinking that numbers have a single nature in their own right is *Metaphysics Δ* 14, where Aristotle discusses the notion of quality (*to poion*).[53] Given that quality means the differentia of substance, he writes that numbers are qualitative in this way too, and differ from each other not only by quantity: 'For the substance of each thing is what [it] is once, as for instance what six is, not two or three times, but once; for six is once six.'[54] The already mentioned example of the number six shows that while of course, it is true that 2 + 2 + 2 or 3 + 3 adds up to 6, what makes the number six a six is not the combination of such sums – a relevant quantity of units is only a part of number's identity.[55]

If we consider that numbers exist *as* hylomorphic, then this unifying and identity-establishing agent should naturally be identified with a specific numerical form. It should tie the separate parts together and establish a collective identity. Galluzzo (2018) explores this approach by discussing the *Metaphysics* H 3 passage, in which Aristotle tries to shed light on the issue of unity by drawing an analogy between substances and numbers. Galluzzo's extensive and persuasive reading of the passage and analysis of relevant connected issues leaves little to be desired. Nonetheless, I would like to touch additionally upon the question of numbers' hylomorphic status and the nature of their form.

So far, this paper has argued for a hylomorphic understanding of numbers. But numbers constitute a very marginal case of entities in the scope of beings among which, we know, only natural ones can be counted as true hylomorphic substances with certainty. Even the closest to natural ones – physical artefacts – are still subject to doubt among scholars, whether they can be treated as substances and whether we can even establish that artefacts have forms. One way to think about this issue is to hold that various entities exhibit forms in ways analoguous to natural substances,

52 Galluzzo (2018, p. 304).

53 Aristotle returns to this once again in M 8, 1083a1–17, where he argues that units do not qualitatively differ from one another, yet numbers made up of units do.

54 1020b8f.

55 Pappas (2018, p. 174) also points out that one should understand the symbol of congruence as a symbol of equivalence, meaning that the number of units is the same. The same number of units, however, does not imply the same numerical identity.

and thus hylomorphic model can be employed to analyse them just the same, although their ontological status may differ. This is also the view of Galluzzo, who argues that Aristotle's comparison of substances to numbers in H 3 invites us to consider a *vice versa* analogy and proceeds to the conclussion that numbers possess a structural or formal component.

Aristotle lays ground for such thinking in the preceding chapter H 2, where he explains how to search for the cause of perceptible composite beings which should be the form as actuality. In his inquiry, Aristotle identifies various kinds of essential *differentiae*, among which we find human-dependent objects: the cause of being of a threshold is the specific position of its wood or stones, and this position of a threshold's matter differentiates it from a lintel, and honey-water is defined as a mixture of honey and water. Importantly, *differentiae* can also be perceptible attributes, such as hardness and softness, density and rarity, excess and deficiency. In general, the objects that Aristotle names are heterogeneous (books, dinner, breakfast, wind, houses, harmonies, ice, etc.), which highlights the fact that *differentiae* are peculiar to the entity's specific kind. Aristotle goes on to stress that these are not merely qualities of an object's matter:

> From this it is clear that, since the substance of a thing is a cause of its being, it is among these that we must look to determine what is the cause of the being of each of these things. (In fact none of these *differentiae* just mentioned is a substance, not even when it is coupled with matter; nevertheless it is in each case what is analogous to substance. Just as in substances what is predicated of the matter is the actuality itself, so in other definitions it is what is closest to actuality.)[56]

It is clearly spelled out that *differentiae* are not truly forms. However, structurally and functionally they act analogously to the actuality of substance, thus also being the cause of an entity's unity. Now, it is evident that one cannot hold numbers to be hylomorphic substances in the true sense of the word – they simply do not exist as such in the natural world, and Aristotle is in definite denial of such a possibility. Regardless, his statements about numbers *as if* they were independent beings together with constant allusion to them in terms of form and matter encourage us to think that numbers should possess structural similarity to natural substances. As a result, although we cannot speak about numbers being hylomorphic substances literally, I favour Galluzzo's view that numbers should be considered as unified structured wholes due to their possession of numerical forms in an analogous way.

Even so, it is not fully clear how a numerical form should unify the units. While we can hold it true that it gives numerical identity (e.g., a unit in a number five is not an individual entity but rather one of five parts that should be considered together as

56 H 2, 1043a2–7, trans. Bostock.

one being), no one has yet proposed even a hypothesis on how exactly should units be tied together *structurally*. It appears to be almost a futile task given the discrete nature of units – by definition, they are points without position which do not come into any kind of contact.[57] None of the usual *differentiae*, which could establish some kind of continuity and thus unity, such as blending or composition, apply.[58] Having said that, in the corpus we do find a promising notion that numbers are successive. At K 12, Aristotle presents us with this notion's definition:

> Successive is what is after the starting-point (the order being determined by position or form or in some other way) and has nothing of the same kind (*genos*) between it and what it succeeds – for example, lines in the case of line, units in that of unit, or houses in that of house (but there is nothing to prevent something of some other kind from being in between). For what is successive is successive of something and is something posterior. For one does not succeed two, nor the first day of the month the second.[59]

Aristotle further writes that things that are continuous are by default successive, as they have parts that come one after another. And as they each have limits that touch and make contact, such entities form a continuity and become one. The contrary, however, does not necessarily hold: things that are successive might not be continuous. The units in a number is exactly such a case, because they have no point of continuous contact between them. Nevertheless, this would not appear very helpful if Aristotle did not clarify that the order of successiveness might be determined by position or form (or in some other unspecified way). We can, for instance, think about pages in a book which should be organised in a logical sequence for the book to make sense. Here, position of the order of the pages is relevant.

As the position is not relevant to numbers, I propose that successive order is dictated by the numerical form. If we think of a generation of a number, it does come about by counting and successively adding one unit to another. At M 6, 1080a20–3, Aristotle explicitly states that units are all directly successive and combinable with any other, as they do not differ among themselves. Hence, if we have number six, it will designate six units where one comes after another in an order, while the specific position of a unit will be irrelevant. A perceptible example could possibly make this concept more intelligible. Let us take the line lengths and say we have six of them. Now, it is true that one could take the third section and place it instead of the sixth, placing the latter in the place of the third. We would still have the same six line lengths as before, because position of the units have no significance. The important thing is, we would have them in an orderly sequence – it is inevitable that one line

57 *Δ* 6, 1016b20–9; M 8, 1085a23–32.

58 Cf. M 7, 1082a20–3.

59 1068b31–9a1. Almost the same definition is found at *Phys* E 3, 226b34–7a5.

will come after another, and we will have six of them, as this is the number that numerical form indicates.

We encounter one obstacle to this argument as we continue reading to the end of K 12. There Aristotle writes that 'in things in which there is no contact, there is no natural unity (*symphysis*).'[60] At first glance, one might expect that Aristotle here is talking strictly about natural substances. However, in *Metaphysics Δ* 4 and *Physics* E 3 lies explanation that Aristotle also has in mind continuous single entities that can come about in artificial ways, such as pinning, glueing, composing, or grafting. As units are discrete and thus seem to have no natural unity, we still have the problem that numerical successiveness may not be able to act as a unifying *differentia*. This is salvaged, as I see it, by *Phys* E 3, 227a17–20:

> It is also evident that the successive is primary. For things that make contact are necessarily successive, but not all things that are successive make contact. That is why succession is found in things, such as numbers, that are prior in account, whereas contact is not.[61]

Plausibly, Aristotle's thought here is that physical entities that exhibit unity by continuity are prior in nature and being. Consequently, their continuous unity is unity *par excellence* and should apply as a criterion for other sensible things which we wish to call unified wholes. Be that as it may, numbers, albeit having ties with the natural world, are mental entities that have priority in account, which results in them being more fundamental conceptually. Furthermore, the successiveness as a type of order found within them is also in the same way more fundamental than the order established by contact and continuity. This would mean that while being in a discrete sequence is certainly not enough for a being to be called a unity in the natural world, it appears to be enough to mentally unify units in a number. A numerical form, therefore, designates a number of units standing in a successive order, and this orderly relation formed between the units constitutes the unity of number.

Having laid out these preliminaries of how we can understand number's unity, I would like to address the only alternative understanding of number's unity that has been proposed so far and to provide my reasoning as to why that account is not convincing.

Gaukroger and Katz advocate an approach which claims that the unification of number is performed by measure.[62] As one selects a measure and, after counting, says that, for instance, there are 10 sheep, one unifies an aggregate of units by ascribing a common measure. In other words, then, units should be unified by

60 1069a10f.

61 Trans. Reeve (2018).

62 Gaukroger (1982, pp. 316–20), Katz (2021, pp. 212–14).

identifying what they are units of. Gaukroger emphasizes how Aristotle's account of number hinges on the fact that we can only count things of the same kind: 'This requirement clearly rules out the idea that unity could be conferred on a collection simply in virtue of our ascribing a number to it, since homogeneity is a precondition of counting, not its consequence.'[63]

This theory faces two problems. As already stressed, the question of unity concerns identity or nature. In hylomorphic compounds, the form not only structures material parts in a relevant way – it ties their identity to the whole so that neither parts could be understood without a reference to the whole, nor a whole could be comprehended without certain parts that make it up. How can, then, a common nature of the things counted account for their unitary interdependence? It seems that precisely the process of counting is what brings things together in the first place. The measure theory would reverse this relation as if somehow the shared nature of things could tie them in a *numerical* relation.[64] If so, then it should account for how the nature counted determines the numerical aspect of the plurality. For now, it seems that measure cannot explain how the things counted are a single *number*.

The hylomorphic understanding of numbers further unfolds this problem by highlighting its conflict with the framework of Aristotelian metaphysics as such. As argued previously, measure is a factor that individualizes units (i.e., matter) and allows us to have everyday numbers. To say that measure unifies units, then, would be the same as to reason that the human body's skin colour unifies the parts of the body. While, of course, a human being's skin cannot be without colour and numbers analogically have always to be numbers *of* something, it does not follow that an attribute could somehow establish unity. This capacity is strictly reserved to form. In a way, Aristotle himself criticises such an approach in *Metaph* M 8, 1084b2–32, where he protests against the inconsistent wish of the Platonists to make one both the material and the formal principle of number. The Platonists do share the notion that numbers are composed of units, therefore making unit a common material principle of numbers. Yet by saying that as a principle unit is prior to number and not specifying the mode of this priority, they appear to claim that unit acts as a formal unifying principle as well.[65] Clearly, then, Aristotle denies the possibility of attributing the capacity to unify to unit. If we remember that unit is always a measure, it

63 Gaukroger (1982, p. 316).

64 Halper (1989, p. 258f.) also notices this. Halper's own position on the issue of unity is peculiar. In his view, numbers are abstracted from single entity's parts, and such nature automatically renders a unitary number. For instance, we take a line, which is potentially made of points, and effect its mental division to sections. Then, we abstract numbers from sections while in actuality the line remains a single being, as the whole process is mental. However, this account fails to explain how the unity of the object should 'transfer' to the number which acquires its own separate mental existence.

65 For a full reconstruction of the argument see Annas (1976, pp. 181–4).

can be argued that by extension, Aristotle is also disagreeing with the possibility that unification is performed by measure.[66]

Finally, the hylomorphic theory accounts for one last difficulty I want to note. Concerns were raised that, compared to Plato, Aristotle's concept of arithmetic appears to be limited even in the context of his contemporaries' understanding, let alone present-day mathematics. If a number, as Aristotle repeatedly states, is ultimately the units counted, does he really have a theory of arithmetic, or is it just *logistikē*?[67] The art of calculation was held to be as a subsidiary of arithmetic, and the differences between the two were acknowledged. *Logistikē* was a functional art, its purpose being to consider mutual relations of counted collections of units and the operations performed on them (multiplication, division and so on). *Arithmētikē*, by contrast, as theoretical branch studied numbers' properties and kinds as they are in themselves, aside from countable things (i.e., counted or calculated units). One would find it curious if Aristotle indeed failed to distinguish between these two mathematical branches, especially as it was undoubtedly important to Plato. According to Julia Annas, precisely because of his commitment to a distinction between *logistikē* and *arithmētikē*, Plato felt a need to postulate number as separate, given independently of our activities, so it would explain how the act of counting is enabled in the first place. In other words, numbers for Plato can be known in their own right; numbers have a nature distinct from the interrelations they are in.[68]

The hylomorphic approach inclines me to think that Aristotle would agree with such a reasoning, as it shows that number for him is something more than the sum of units. A number has a principle that unifies the units and makes numbers what they are – a numerical form. This implies that numbers have essences that can be known and investigated. Indeed, in *Posterior Analytics* Aristotle even gives an example of the essence of three: it is a number, it is odd and it is prime in both senses that it is neither measured by number nor compouded from numbers (meaning that it does not have factors and is not a sum of two or more numbers).[69] Hylomorphism, thus, allows for Aristotle to posit such a notion of number that it could be the object of *logistikē* as well as *arithmētikē*.

66 N 1, 1088a5–8: 'One means a measure of some plurality, and number means a measured plurality and a plurality of measures'.

67 This concern is raised by Gaukroger (1980, p. 195) and Pappas (2018, p. 169f.).

68 Annas (1976, p. 7).

69 See *APo*. II.13, 96a36–b2.

7 Concluding Remarks

The paper has argued that a comprehensive understanding of Aristotle's philosophy of arithmetic necessitates recognizing the dual ontological status of numbers in his thinking. This approach bridges the natural world and numbers as entities, as they exist firstly as numerical properties of things. Aristotelian mathematics, then, asserts that mathematical truths are grounded in reality while avoiding the Platonic tendency to hypostatize predicates. Nevertheless, Aristotle assigns numbers a hylomorphic structure through abstraction, thus accounting for their unity, individual numerical natures, and their intelligibility as scientific entities accessed by reason. This hylomorphic interpretation aligns with Aristotle's broader metaphysical framework and distinguishes his approach to arithmetic from mere calculation, suggesting a theoretical understanding of numbers' properties and kinds. Future inquiries could extend this hylomorphic theory to geometrical objects, as Aristotle often theorizes about mathematical objects in general, not differentiating between whether these are arithmetical or geometrical. If hylomorphism holds for geometry as well, it would indicate Aristotle's consistent view of the ontological status of mathematical objects across different branches of mathematics.

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