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NIELSEN TAPATYBIŲ AUTOMATIZAVIMAS GRIMUS–NEUFELD MODELIUI

MAGISTRANTŪROS STŪDIJŲ BAIGIAMASIS DARBAS

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AUTOMATIZING THE NIELSEN IDENTITIES FOR THE GRIMUS–NEUFELD MODEL

MASTER'S FINAL THESIS

Theoretical physics and astrophysics study program

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Introduction

In Quantum Field Theory (QFT), the gauge symmetries play a fundamental role in ensuring consistency in interactions. However, in gauge theory, our description of gauge fields introduces redundant (unphysical) degrees of freedom. In order to remove redundant degrees of freedom, one gauge fixes the theory, which breaks the original gauge invariance. To address this, Becchi, Rouet, Stora (BRS) [1, 2] and Tyutin [3] showed that such gauge-fixed theory has a global symmetry – BRST symmetry, that allows the consistent treatment of the gauge-fixed system.

Gauge fixing introduces an additional parameter to parametrize our choice of gauge. While physical observables, such as pole masses, remain gauge independent, gauge fixing leads to intermediate quantities (Green's functions like self-energy) being dependent of the gauge parameter. Thus we want to know how these intermediate quantities depend on gauge. One of the tools to analyze the gauge dependence are Nielsen Identities.

BRST quantization (and so Nielsen Identities) finds applications in a wide range of theories and extensions – string theory, higher derivative quantum theories etc. A particularly interesting extension beyond-the-Standard-Model is the Grimus–Neufeld Model [4–6], in which we apply the BRST quantization. This model is one of the various theories proposed to explain the small masses of neutrinos. It employs the Seesaw mechanism, along with an additional Higgs doublet and a right-handed neutrino singlet.

The goal of this thesis is to implement Nielsen Identities of the neutrino self-energy in the Grimus–Neufeld model and automate them using computer algebra systems.

The concepts that are required to understand the BRST formalism and the Grimus–Neufeld Model are discussed to some degree. In sections 1 and 2 we discuss the Faddeev–Popov procedure [7], which indirectly leads to the symmetry mentioned above, as well as BRST formalism. Generalized Ward–Takahashi Identities [8, 9] for non-Abelian theory are introduced, which in turn leads to Nielsen Identities [10]. In section 3 we discuss the handedness of neutrinos in the Standard Model, neutrino mass generation, Seesaw mechanism; how, through spontaneous symmetry breaking, gauge bosons acquire mass as well as the Grimus–Neufeld model. In section 4 we derive BRST transformations for the Higgs and neutrino sectors in mass eigenstates and explicitly construct BRST Lagrangian, required for Nielsen Identities. Lastly, in sections 5 and 6 accordingly, we provide confirmation of consistency of the Nielsen Identity implementation in the Grimus–Neufeld Model, as well as work and insights towards automatizing via software.

1 BRST Symmetry

1.1 Over counting field problem and Faddeev–Popov ghost field

Consider a simple non–abelian gauge theory Lagrangian:

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2, \tag{1.1.1}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{1.1.2}$$

is the gauge boson field strength tensor, a denotes a gauge group index, $A_\mu = A^a_\mu T^a$ is the gauge boson, T^a denotes generator(s) of the gauge group, f^{abc} – structure constants of the gauge group algebra and coupling constant g; α is the gauge transformation parameter and $(A^\alpha)^a_\mu = A^a_\mu + \frac{1}{g}\partial_\mu \alpha^a + f^{abc} A^b_\mu \alpha^c$ – gauge field after finite gauge transformation. Then for the quantization (to derive Feynman rules), the path integral is of the form

$$\int \mathcal{D}A \exp\left[-i \int d^4x \frac{1}{4}(F^a_{\mu\nu})^2\right]. \tag{1.1.3}$$

The problem arises is that the integral $\mathcal{D}A$ is counting in physically equivalent paths in phase space of A field configurations (which correspond to local gauge transformations). We can remove these unnecessary paths by constraining all gauge paths with a global gauge-fixing condition:

$$G(A^a) = 0, \tag{1.1.4}$$

where it can take many forms, such as generalized Lorenz gauge ($\partial^\mu A^a_\mu = 0$), axial ($n^\mu A^a_\mu = 0$, for fixed four-vector n^μ) or non-linear covariant gauges (the condition may not necessarily lead to unique configuration either. There can be multiple gauge field configurations that satisfy the same condition, which is known as Gribov Ambiguity [11]). Following Faddeev's and Popov's method [12], we can impose this constraint with path integral identity

$$1 = \int D\alpha(x)\, \delta(G(A^a)) \det\left(\frac{\delta G(A^a)}{\delta \alpha}\right). \tag{1.1.5}$$

Then, once we insert the condition into eq. (1.1.3), one can find that integral over parameter α can be factored out from the eq. (1.1.3) as an overall normalization:

$$\int \mathcal{D}A e^{\mathrm{i}S[A]} = \left(\int \mathcal{D}\alpha\right) \int \mathcal{D}A\, e^{iS[A]} \delta(G(A^a)) \det\left(\frac{\delta G(A^a)}{\delta \alpha}\right) \tag{1.1.6}$$

and that

$$\det\left(\frac{\delta G(A^a)}{\delta \alpha}\right) = \det\left(\frac{\partial^\nu D_\nu}{g}\right), \tag{1.1.7}$$

where we picked $G(A^a) = \partial^\mu A^a_\mu$ and D_ν is the gauge covariant derivative. Because $D_\nu = D_\nu(A_\nu)$, we aim for this ”residual” term to be reflected in the new, re-canonicalized Lagrangian, which can be achieved by introducing Grassmann numbers. Faddev and Popov defined this determinant as a path integral for a new pair of anticommuting fields, $c(x)$ and $\bar{c}(x)$, called ghost fields:

$$\det\left(\frac{1}{g}\partial^\mu D_\mu\right) = \int \mathcal{D}[c]\mathcal{D}[\bar{c}] \exp\left[-\mathrm{i}\int d^4x\,(\partial^\nu \bar{c})(D_\nu c)\right]. \tag{1.1.8}$$

So, our re-canonicalized integral in eq. (1.1.3),

$$\left(\int \mathcal{D}\alpha\right)\int \mathcal{D}A\,\mathcal{D}c\,\mathcal{D}\bar{c}\, \exp\left[-\mathrm{i}\int d^4x\,\left(\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{c}(\partial^\nu D_\nu)c\right)\right]\delta(G(A^\alpha)) \tag{1.1.9}$$

and new Lagrangian is of the form, including the typical gauge-fixing term and matter terms,

$$\mathcal{L}_{\text{new}} = \mathcal{L} + \mathcal{L}_{\text{matter}} + \mathcal{L}_{GF} + \mathcal{L}_{FP} = \mathcal{L}_{\text{matter}} - \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2\xi}\left(\partial^\mu A^a_\mu\right)^2 - \bar{c}^a\partial^\nu D^{ad}_\nu c^d, \tag{1.1.10}$$

where ξ is a gauge parameter that arises from gauge-fixing procedure of gauge field (not a gauge transformation parameter like α), $\mathcal{L}_{\text{matter}} = \mathcal{L}_{\text{matter}}(\phi, D_\mu\phi)$ and ϕ — matter field.

One thing to keep in mind is that while ghost fields are not physical, i.e. they are anticommuting scalar fields that do not follow the spin-statistics relation of physical fields, but they can still appear as virtual particles in specific Feynman diagrams and their computations.

Also, after re-canonicalizing the functional integral above in Faddeev-Popov way, we no longer have invariance under local gauge transformation, after introducing $\mathcal{L}_{GF} + \mathcal{L}_{FP}$. What is important is that the new Lagrangian in eq. (1.1.10) can be invariant under new transformation, i.e. will have a new symmetry that it obeys.

1.2 BRST Formalism

As it was mentioned at the end of previous section, the invariance of the Lagrangian can be preserved by having a broader symmetry. Becchi, Rouet and Stora (and Tyutin later on) defined new global gauge transformation, called BRST (BRS previously) transformation, under which the Lagrangian stays invariant, using ghost fields as a tool. In upcoming sections we examine BRST transformations on fields, BRST charge operator (its nilpotency) as well as Slavnov–Taylor Identities.

1.2.1 BRST transformations

The BRST transformation is defined as a set of transformations acting on the fields, with the original gauge transformation parameter α expressed in terms of the ghost field and an infinitesimal Grassmann parameter, ϵ,

$$\alpha^a(x) = \epsilon c^a(x), \tag{1.2.1}$$

for matter fields ϕ_i, gauge field A_μ, ghost fields as well as auxiliary field B:

$$\delta A^a_\mu(x) = \epsilon D^{ab}_\mu c^b(x), \tag{1.2.2a}$$

$$\delta\phi(x) = g\epsilon c(x)^a T^a \phi(x), \tag{1.2.2b}$$
$$\delta c^a(x) = -\epsilon \frac{g}{2} f^{abc} c(x)^b c(x)^c, \tag{1.2.2c}$$
$$\delta \bar{c}^a(x) = \epsilon B^a(x), \tag{1.2.2d}$$
$$\delta B^a(x) = 0, \tag{1.2.2e}$$

where f^{abc} is the structure constant of the gauge group. $B(x)$ is an auxiliary field, i.e. field without physical dynamics, introduced for specific constraint conditions as well as to ensure the nilpotency of the BRST charge. Modifying the gauge-fixing Lagrangian term by auxiliary field $B(x)$, we have

$$\mathcal{L}_{\text{GF}} = -(\partial^\mu B)A_\mu + \frac{\xi}{2}B^2. \tag{1.2.3}$$

It can be shown that the Lagrangian remains invariant under BRST transformations. Matter and gauge field strength terms in eq. (1.1.10) are invariant due to their gauge transformation. Gauge-fixing term that involves auxiliary field B and gauge parameter is invariant due to $\delta B^a(x) = 0$. A bit more work is needed to show that the first gauge-fixing and ghost field terms are invariant under eq. (1.2.2): ghost field term with transformed $D_\nu^{ad} c^d$:

$$\begin{aligned} -\bar{c}^a \partial^\nu \delta\left(D_\nu^{ad} c^d\right) &= -\bar{c}^a \partial^\nu (D_\nu^{ad} \delta c^d + g f^{abc} \delta A_\nu^b c^c) \\ &= -\bar{c}^a \partial^\nu (-\frac{1}{2} g\epsilon \partial_\nu \left(f^{abd} c^b c^d\right) - \frac{1}{2} g^2 \epsilon f^{abd} f^{dle} A_\nu^b c^l c^e \\ &\quad + g\epsilon f^{abd} (\partial_\nu c^b) c^d + g^2 \epsilon f^{abd} f^{ble} A_\nu^l c^e c^d), \end{aligned} \tag{1.2.4}$$

and since $f^{ade} f^{bcd} + f^{bde} f^{cad} + f^{cde} f^{abd} = 0$ (Jacobi identity), the eq. (1.2.4) is zero. And lastly, we notice that transformation of $\bar{c}^a$ in the same ghost field term in eq. (1.1.10), $-\bar{c}^a \partial^\nu D_\nu^{ad} c^d$, cancels the first gauge-fixing term with transformed A_μ, i.e.

$$(\partial^\mu B)(\delta A_\mu) = -(\delta \bar{c}^a)\partial^\nu D_\nu^{ad} c^d, \tag{1.2.5}$$

hence making the Lagrangian invariant.

1.2.2 BRST charge

We know now, from previous chapter, that BRST transformations generates a continuous global symmetry for gauge-fixed theory. By Noether's theorem [13, 14], such symmetry will have its associated conserved BRST current and charge.

If we take Lagrangian in eq. (1.1.10) with gauge-fixing term in eq. (1.2.3) and apply Noether's theorem, we have

$$j_\mu^{\text{BRST}} = D^\nu c^a \frac{\partial \mathcal{L}}{\partial(\partial^\mu (A^\nu)^a)} + i(c^a g T^a \phi)_j \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_j)} - \frac{1}{2} g (f^{abd} c^b c^d) \frac{\partial \mathcal{L}}{\partial(\partial^\mu c_a)} + i B^a \frac{\partial \mathcal{L}}{\partial(\partial^\mu \bar{c}_a)}. \tag{1.2.6}$$

Since conserved charge is obtained by integrating the time component of the conserved BRST current over the spatial coordinates,

$$Q_{\text{BRST}} = \int d^3x j_0^{\text{BRST}}, \tag{1.2.7}$$

it can be derived that the charge is

$$Q_{\text{BRST}} = \int d^3x \left[B^a \partial_0 c^a + gB^a A_0 f^{abd} c^b c^d + \frac{g}{2} \partial_0 \bar{c}^a f^{ael} c^b c^d\right]. \tag{1.2.8}$$

The BRST charge is the generator of BRST symmetry. Acting on a field through the commutator (or anti-commutator, depending on the statistics), the charge generates BRST transformation of the field:

$$\mathrm{i}[\epsilon Q_{\text{BRST}}, \Sigma]_{\pm} = \delta\Sigma, \tag{1.2.9}$$

where $\pm$ indicates that the brackets can be either a commutator or an anti-commutator, depending on the grassmann parity of Σ, which can be a ghost, matter or gauge field. One notable feature of this generator is that it is nilpotent:

$$\begin{aligned}
\mathrm{i}\eta(Q_{\text{BRST}})^2 &= \left[\mathrm{i}\epsilon Q_{\text{BRST}}, \int \mathrm{d}^3x \left(B^a D_0 c^a - \partial_0 B^a c^a + \frac{\mathrm{i}g}{2} \partial_0 \bar{c}^a f^{abd} c^b c_d\right)\right] \\
&= \left\{\delta B^a = 0,\ \delta(D_\mu c^a) = 0,\ \delta(f^{abd} c^b c^d) = 0\right\} \\
&= \int \mathrm{d}^3x \left(-\partial_0 B^a \delta c^a + \frac{ig}{2} \partial_0 (\delta \bar{c}^a) f^{abd} c^b c^d\right) = 0.
\end{aligned} \tag{1.2.10}$$

The nilpotency of BRST charge in eq. (1.2.10) has implication on states: a state $|\phi\rangle$ is physical if

$$Q_{\text{BRST}} |\phi\rangle = 0. \tag{1.2.11}$$

That ensures physical states are BRST invariant and Q_{BRST} acting as projection operator on the states, isolating the physical subspace from the broader Hilbert space.

1.2.3 Slavnov-Taylor Identities

Slavnov–Taylor Identities (STI) [15, 16] – generalized Ward-Takahashi Identities (WTI) for non-abelian theory, are identities involving Green functions which are to ensure that physical observables are independent of the gauge we choose. The STI can be derived from the condition given in eq. (1.2.11):

$$\begin{aligned}
0 &= \langle 0|[\mathrm{i}\eta Q_{\text{BRST}}, T\{\Omega_1(x_1)\cdots\Omega_n(x_n)\}]|0\rangle \\
&= \sum_{k=1}^{n} \langle 0|T\{\Omega_1(x_1)\cdots\Omega_{k-1}(x_{k-1})\delta\Omega_k(x_k)\Omega_{k+1}(x_{k+1})\cdots\Omega_n(x_n)\}|0\rangle,
\end{aligned} \tag{1.2.12}$$

where $T\{\cdots\}$ is time ordering operator and Ω_k's are field operators.

Consider an example of deriving STI for the generating functional of one Particle-Irreducible

(1PI) vertex functions, with a functional source $\mathcal{J}$:

$$\begin{aligned}\mathcal{J}[J,K] = \int \mathrm{d}^4x \big(& J_\mu A^\mu + J_i\phi_i + \bar{J}_c^a c^a + J_{\bar{c}}^a \bar{c}^a \\ & + J_B^a B^a + K_\mu^a D^\mu c^a + \mathrm{i}K_i(c^a gT^a)_{ij}\phi_j - \frac{1}{2}K_c^a g f^{abe} c^b c^e \big),\end{aligned} \tag{1.2.13}$$

where J and K are the appropriate sources and whether J_i and K_i are commuting or anti-commuting depends on type of matter terms (fermion, ghost, boson). Again, due to the nilpotency of BRST charge (1.2.10),

$$\delta B^a = \delta(D_\mu c^a) = \delta(f^{abd}c^b c^d) = 0, \tag{1.2.14}$$

so that we get

$$\begin{aligned} 0 =& \langle 0|[\mathrm{i}\eta Q_{\mathrm{BRST}}, T\left\{\mathrm{e}^{i\mathcal{J}[J,K]}\right\}|0\rangle \\ =& \mathrm{i}\epsilon \int \mathrm{d}^4x \langle 0|T\big(J_\mu^a D^\mu c^a + \mathrm{i}J_i(c^a gT^a)_{ij}\phi_j \\ & + \frac{1}{2}\bar{J}_c^a g f^{abd} c^b c^d - \mathrm{i}J_{\bar{c}}^a B^a\big)\mathrm{e}^{i\mathcal{J}[J,K]}|0\rangle, \end{aligned} \tag{1.2.15}$$

and STI for simple field operators are obtained by differentiating with respect to sources. Since the generating functional of connected Green functions $W[J,K]$ is of the form

$$\mathrm{e}^{\mathrm{i}W[J,K]} = \langle 0|T\left\{\mathrm{e}^{\mathcal{J}[J,K]}\right\}|0\rangle,, \tag{1.2.16}$$

and is related to the generating functional Γ of 1PI vertices through the Legendre transformation,

$$\Gamma[\Phi, K] = W[J,K] - J_\Phi \Phi \tag{1.2.17}$$

(where Φ are fields and J_Φ are their associated sources), then the identity for the generating functional of 1PI is derived from eq. (1.2.12):

$$\int \mathrm{d}^4x \left(\frac{\delta\Gamma}{\delta A_\mu}\frac{\delta\Gamma}{\delta K^\mu} + \frac{\delta\Gamma}{\delta\phi_i}\frac{\delta\Gamma}{\delta K_i} + \frac{\delta\Gamma}{\delta c^a}\frac{\delta\Gamma}{\delta K_c^a} + \mathrm{i}\frac{\delta\Gamma}{\delta\bar{c}^a}B^a \right) = 0, \tag{1.2.18}$$

where integration over space-time is shown explicitly. STI of generating functionals of 1PI vertices are called Lee Identities and in such a form are independent of theory parameters.

2 Nielsen Identities

Another notable feature of STI is that we can derive identities, called Nielsen Identities (NI) [10], that relate the generating functional Γ to gauge parameter ξ, i.e. showing Γ gauge dependence. NI can be a useful tool in investing gauge-dependence of quantities (or to make sure that physical quantities do not depend on gauge), such as self-energies of particles, in the Standard Model (SM) and its extensions. In sections below, we provide derivations of NI in both QED and QCD by following the derivation of Lee Identities (1.2.18), as well as providing an example of gauge dependence of self–energies of the quarks.

2.1 Examples of Nielsen Identities in QED and QCD

Since Nielsen Identities are relating the generating functionals to gauge parameter ξ, we must also extend the BRST symmetry by variations of this parameter. In general, gauge-fixing eq. (1.2.3) with ghosts can be written in a form

$$\mathcal{L}_{\text{fix+ghost}} = \delta\left(\bar{c}^a\left(\frac{\xi}{2}B^a + G(A^a)\right)\right). \tag{2.1.1}$$

Having transformations of gauge parameter, i.e.

$$\delta\xi = \epsilon\chi, \tag{2.1.2a}$$

$$\delta\chi = \delta^2\xi = 0, \tag{2.1.2b}$$

where χ is the BRST source of gauge parameter ξ, $\mathcal{L}_{\text{fix+ghost}}$ in eq. (2.1.1) becomes

$$\begin{aligned}\mathcal{L}_{\text{fix+ghost}} &= \delta\left(\bar{c}^a\left(\frac{\xi}{2}B^a + G(A^a)\right)\right)\\ &= \frac{\xi}{2}(B^a)^2 - \bar{c}^a\frac{\chi}{2}B^a - \delta(\bar{c}^a G(A^a)).\end{aligned} \tag{2.1.3}$$

For QED, the second term becomes

$$\mathcal{L}_\chi = -\bar{c}\frac{\chi}{2}B = \bar{c}\frac{\chi}{2\xi}G(A) = -\frac{\chi}{2\xi}\bar{c}(\partial^\mu A_\mu). \tag{2.1.4}$$

Now, focusing on the QED case, the generating functional is

$$\begin{aligned}\mathcal{Z} &= \exp(W)\\ &= \int \mathcal{D}A\ \exp\left\{i\int d^4x\,\mathcal{L}_{\text{QED}} + J_\mu A^\mu + \bar{J}_\psi\psi + \bar{\psi}J_{\bar{\psi}} + BJ_B\right.\\ &\left.+\bar{\eta}_\psi\delta\psi + (\delta\bar{\psi})\eta_{\bar{\psi}}\right\},\end{aligned} \tag{2.1.5}$$

we also include the BRST–exact terms $\mathcal{L}_{BRST} = \sum_i \eta_i\delta\Phi_i$, that is, BRST transformations of fields coupled to their associated sources, as we did with functional source $\mathcal{J}$ in eq. (1.2.13). Then we can write generating functional Γ for proper vertex functions,

$$\begin{aligned}\Gamma(A^{\mu},\psi,\bar{\psi},B,c,\bar{c},\chi,\xi,\bar{\eta}_{\psi},\eta_{\bar{\psi}}) = W\left(J_{\mu},J_{\bar{\psi}},\bar{J}_{\psi},J_B,\eta_{\bar{\psi}},\bar{\eta}_{\psi},\chi,\xi\right) \\ -\int d^4x (J_{\mu}A^{\mu} + J_B B + \bar{J}_{\psi}\psi + \bar{\psi}J_{\bar{\psi}}).\end{aligned} \tag{2.1.6}$$

Since due to transformations (1.2.2) and (2.1.2), our Lagrangian remains invariant, the transformation of the generating functional Γ is of the form

$$\begin{aligned}\delta\Gamma \equiv 0 &= \delta A_{\mu}\frac{\delta\Gamma}{\delta A_{\mu}} + \delta\psi\frac{\delta\Gamma}{\delta\psi} + \delta\bar{\psi}\frac{\delta\Gamma}{\delta\bar{\psi}} + \delta\bar{c}\frac{\delta\Gamma}{\delta\bar{c}} + \delta\xi\frac{\partial\Gamma}{\partial\xi} \\ &= \partial_{\mu}c\frac{\delta\Gamma}{\delta A_{\mu}} + \frac{\delta\Gamma}{\delta\bar{\eta}_{\psi}}\frac{\delta\Gamma}{\delta\psi} + \frac{\delta\Gamma}{\delta\eta_{\bar{\psi}}}\frac{\delta\Gamma}{\delta\bar{\psi}} + B\frac{\delta\Gamma}{\delta\bar{c}} + \chi\frac{\partial\Gamma}{\partial\xi}.\end{aligned} \tag{2.1.7}$$

Taking a functional derivative with respect to the BRST source χ (then setting $\chi = 0$), we obtain Nielsen Identities for QED:

$$\begin{aligned}-\frac{\partial\Gamma}{\partial\xi} &= -\partial_{\mu}c\frac{\delta^2\Gamma}{\delta\chi\delta A_{\mu}} + \frac{\delta^2\Gamma}{\delta\chi\delta\bar{\eta}_{\psi}}\frac{\delta\Gamma}{\delta\psi} - \frac{\delta\Gamma}{\delta\bar{\eta}_{\psi}}\frac{\delta^2\Gamma}{\delta\psi\delta\chi} \\ &\quad + \frac{\delta^2\Gamma}{\delta\chi\delta\eta_{\bar{\psi}}}\frac{\delta\Gamma}{\delta\bar{\psi}} + \frac{\delta\Gamma}{\delta\eta_{\bar{\psi}}}\frac{\delta^2\Gamma}{\delta\chi\delta\bar{\psi}} + B\frac{\delta^2\Gamma}{\delta\chi\delta\bar{c}} = \\ &= B\frac{\delta^2\Gamma}{\delta\chi\delta\bar{c}} + \frac{\delta^2\Gamma}{\delta\chi\delta\bar{\eta}_{\psi}}\frac{\delta\Gamma}{\delta\psi} + \frac{\delta^2\Gamma}{\delta\chi\delta\eta_{\bar{\psi}}}\frac{\delta\Gamma}{\delta\bar{\psi}},\end{aligned} \tag{2.1.8}$$

where some terms vanish in the second line due to one-point functions and ghost number conservation.

By analogy to QED, we can also derive NI for the QCD theory. Due to the non-abelian nature of the theory, the ghost sector becomes non-trivial, giving ghost-gluon interactions at the loop level due to ghost field term in eq. (1.1.8). Because of this, we now include their source terms in the generating functional:

$$\begin{aligned}\mathcal{Z} = \int \mathcal{D}A \;\text{exp}\Big\{i\int d^4x\, \mathcal{L}_{\text{QCD}} + J_{\mu}A^{\mu} + \bar{J}_{\psi}\psi + \bar{\psi}J_{\bar{\psi}} + \bar{J}_c c + \bar{c}J_{\bar{c}} + BJ_B \\ + K_{\mu}\delta A^{\mu} + \bar{K}_c\delta c + \bar{\eta}_{\psi}\delta\psi + (\delta\bar{\psi})\eta_{\bar{\psi}}\Big\},\end{aligned} \tag{2.1.9}$$

and by following the same procedure as we did for QED, we have

$$-\frac{\partial\Gamma}{\partial\xi} = \frac{\delta^2\Gamma}{\delta\chi\delta K_{\mu}}\frac{\delta\Gamma}{\delta A_{\mu}} + \frac{\delta^2\Gamma}{\delta\chi\delta\bar{\eta}_{\psi}}\frac{\delta\Gamma}{\delta\psi} + \frac{\delta^2\Gamma}{\delta\chi\delta\eta_{\bar{\psi}}}\frac{\delta\Gamma}{\delta\bar{\psi}} + B\frac{\delta^2\Gamma}{\delta\chi\delta\bar{c}}. \tag{2.1.10}$$

We see that the identity in eq. (2.1.10) has an additional term that involves source K_{μ}, which couples gauge field A_{μ} and ghost field, as seen in eq. (2.1.9). So as we mentioned already, this makes the ghost field not decoupled anymore, leading to ghost field contributions on loop level.

Then, as an example, one can obtain NI for the fermion 2–point function — propagator by differentiating eq. (2.1.10) w.r.t. to quark fields $\psi(x)$, $\bar{\psi}(y)$ and abiding the quark conservation,

$$\begin{aligned}-\frac{\partial}{\partial\xi}\frac{\delta^2\Gamma}{\delta\psi(y)\delta\bar{\psi}(x)} &= \frac{\delta^2\Gamma}{\delta K_\mu\delta\chi}\frac{\delta^3\Gamma}{\delta\psi(y)\delta\bar{\psi}(x)\delta A^\mu} + \frac{\delta^4\Gamma}{\delta\psi(y)\delta\bar{\psi}(x)\delta K_\mu\delta\chi}\frac{\delta\Gamma}{\delta A^\mu} \\ &+ \frac{\delta^3\Gamma}{\delta\psi(y)\delta\bar{\eta}_\psi\delta\chi}\frac{\delta^2\Gamma}{\delta\bar{\psi}(x)\delta\psi} + \frac{\delta^2\Gamma}{\delta\psi(y)\delta\bar{\psi}}\frac{\delta^3\Gamma}{\delta\bar{\psi}(x)\delta\eta_{\bar{\psi}}\delta\chi} \\ &= \frac{\delta^3\Gamma}{\delta\psi(y)\delta\bar{\eta}_\psi\delta\chi}\frac{\delta^2\Gamma}{\delta\bar{\psi}(x)\delta\psi} + \frac{\delta^2\Gamma}{\delta\psi(y)\delta\bar{\psi}}\frac{\delta^3\Gamma}{\delta\bar{\psi}(x)\delta\eta_{\bar{\psi}}\delta\chi},\end{aligned} \tag{2.1.11}$$

where first and second terms vanish due to Lorentz invariance in zero momentum limit — terms involve insertions of operators with a free Lorentz index that are not contracted; in more compact notation,

$$-\partial_\xi\Gamma_{\psi\bar{\psi}} = \Gamma_{\psi\bar{\eta}_\psi\chi}\Gamma_{\bar{\psi}\psi} + \Gamma_{\psi\bar{\psi}}\Gamma_{\bar{\psi}\eta_{\bar{\psi}}\chi}. \tag{2.1.12}$$

Now, summing over quark generations, we have

$$-\partial_\xi\Gamma_{\psi_j\bar{\psi}_i} = \sum_{k=1,2,3}\left(\Gamma_{\psi_j\bar{\eta}_{\psi_k}\chi}\Gamma_{\bar{\psi}_i\psi_k} + \Gamma_{\psi_j\bar{\psi}_k}\Gamma_{\bar{\psi}_i\eta_{\bar{\psi}_k}\chi}\right), \tag{2.1.13}$$

where $k = 1, 2, 3$. If we expand proper vertex functions in terms of loops, we have

$$\Gamma_{\psi_j\bar{\eta}_{\psi_k}\chi} = \Gamma^{(1)}_{\psi_j\bar{\eta}_{\psi_k}\chi} + \Gamma^{(2)}_{\psi_j\bar{\eta}_{\psi_k}\chi} + \dots \tag{2.1.14}$$

and

$$\Gamma_{\bar{\psi}_i\psi_j} = \Gamma^{(0)}_{\bar{\psi}_i\psi_j} + \Gamma^{(1)}_{\bar{\psi}_i\psi_j} + \Gamma^{(2)}_{\bar{\psi}_i\psi_j} + \dots, \tag{2.1.15}$$

where 3–point vertex function $\Gamma_{\psi_j\bar{\eta}_{\psi_k}\chi}$ vanish at tree level and $\Gamma^{(1)}_{\psi_j\bar{\psi}_i}$ is self–energy of quarks. Since self–energy, at tree level, is an inverse propagator, that is $\Gamma^{(0)}_{\psi_i\bar{\psi}_j} = (\not{p} - m_{\psi_j})\delta_{ij}$, we can simplify the identity more,

$$-\partial_\xi\Gamma^{(1)}_{\psi_j\bar{\psi}_i} = \Gamma_{\psi_j\bar{\eta}_{\psi_i}\chi}(\not{p} - m_{\psi_i}) + (\not{p} - m_{\psi_j})\Gamma_{\bar{\psi}_i\eta_{\bar{\psi}_j}\chi}. \tag{2.1.16}$$

So we end up with a rather simple expression of a gauge dependence of self–energy of quarks. We only need to calculate 3–point functions $\Gamma^{(1)}_{\psi_j\bar{\eta}_\psi\chi}$ and $\Gamma^{(1)}_{\bar{\psi}\eta_{\bar{\psi}}\chi}$, which can be done easily using Lagrangians of the BRST source χ and BRST–exact terms for quarks [17].

Worth noting that if we're looking specifically at the fermionic sector, we have

$$-\partial_\xi\Gamma = \sum_I\left[\Gamma_{\chi\bar{\eta}_I}\Gamma_{\psi_I} + \Gamma_{\chi\eta_I}\Gamma_{\bar{\psi}_I} + (\psi_I \leftrightarrow \eta_I)\right], \tag{2.1.17}$$

where direction of funtional differentiation, due to anticommutation rules, is not shown explicitly. From eq. (2.1.17) we obtain actual NI, as we did above, for proper vertex functions by differentiation w.r.t to specific Standard Model's field(s).

3 Neutrinos and 2HDM

3.1 Handedness of a Dirac spinor and Majorana neutrinos

With the weak sector within the SM, we know that only left-handed neutrino and the right-handed antineutrino participate in the interactions. Let us define the Dirac spinor in a representation given by

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad \psi_{L,R} = \hat{P}_{L,R}\psi = \frac{1}{2}\left(1 \mp \gamma^5\right)\psi, \tag{3.1.1}$$

where ψ_L and ψ_R are two-component spinors, corresponding to the left-handed and right-handed components of the Dirac spinor. For a massless neutrino, the Dirac equation decouples into two equations for ψ_L and ψ_R,

$$\mathrm{i}\bar{\sigma}^\mu \partial_\mu \psi_L - m_D\psi_R = 0 \xrightarrow{m_D=0} \mathrm{i}\bar{\sigma}^\mu \partial_\mu \psi_L = 0, \tag{3.1.2a}$$

$$\mathrm{i}\sigma^\mu \partial_\mu \psi_R - m_D\psi_L = 0 \xrightarrow{m_D=0} \mathrm{i}\sigma^\mu \partial_\mu \psi_R = 0, \tag{3.1.2b}$$

which are equations that describe massless particles of half-spin. Substituting ψ_L, as a plane wave with $u(\mathbf{p})$ for positive energy, into eq. (3.1.2a), we get $(\sigma\mathbf{p} + E)\, u(\mathbf{p}) = 0$, i.e.

$$\hat{h}u(\mathbf{p}) = -u(\mathbf{p}), \tag{3.1.3}$$

where $\hat{h}$ represents the helicity operator. The eigenvalue, -1, indicates that ψ_L, which satisfies eq. (3.1.2a), corresponds to left-handed neutrinos and right-handed antineutrinos. The case with Dirac spinor component, ψ_R, corresponding to neutrinos and antineutrinos, of the opposite handedness, have not been observed in experiments so far.

Above we made an assumption that neutrino is massless, hence assuming no distinction between the so-called Majorana and Dirac neutrinos. However, it has been confirmed that neutrinos have a mass, hence one possibility is of them being Majorana fermions, which are their own antiparticles, i.e.

$$\psi = \psi^c = C\psi^*, \tag{3.1.4}$$

where ψ^c is the charge-conjugated field and C is the charge-conjugation matrix.

3.2 Neutrino mass generation and seesaw mechanism

Fermion masses in the SM are generated through their interaction with the Higgs field via Yukawa coupling,

$$\mathcal{L}_{\text{Yuk}} = -y_l \bar{L}_l \Phi_H l_R + \text{h.c.}, \tag{3.2.1}$$

when the Higgs field develops a vacuum expectation value following spontaneous symmetry breaking. y_l is the Yukawa coupling, Φ_H is the Higgs field, L_l and l_R are fermion doublet and singlet accordingly, representing the left-handed and right handed components of fields:

$$L_l = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}, \quad l_R. \tag{3.2.2}$$

As it was mentioned in previous section, the SM does not contain right-handed neutrinos. If we allow lepton number violation, right handed neutrinos, as singlets under $SU(2) \times U(1)$, can be included and we could write Yukawa term as

$$\mathcal{L}'_{\text{Yuk}} = -y_l \bar{L}_l \Phi'_H \mathcal{N}_R + \text{h.c.}, \tag{3.2.3}$$

where $\mathcal{N}_R$ – right-handed neutrino. This would lead to a Dirac mass $m_D = y_l \Phi_H^0$ (Φ_H^0 is the expectation value), thus

$$\begin{aligned} \mathcal{L}_{\text{D. mass}} &= -m_D \overline{\mathcal{N}}_R \nu_L + \text{h.c.} \\ &= \frac{m_D}{2} \left[\left(\overline{\mathcal{N}}_R^C \right)^T C^{-1} \nu_L + \nu_L^T C^{-1} N_R^C + \text{h.c.} \right], \end{aligned} \tag{3.2.4}$$

where we used that

$$\overline{\mathcal{N}}_R \nu_L = \left(\overline{\mathcal{N}}_R^C \right)^T C^{-1} \nu_L = \frac{1}{2} \left[\left(\overline{\mathcal{N}}_R^C \right)^T C^{-1} \nu_L + \nu_L^T C^{-1} N_R^C \right]. \tag{3.2.5}$$

Another way is to add a Majorana mass term, at the expense of lepton number violation $\Delta L = 2$. Then the Lagrangian interaction term would be of form

$$\mathcal{L}_{\text{int}} = \frac{g_{\text{int}}}{\Lambda} (L_l \Phi_H)^T C^{-1} (L_l \Phi_H) + \text{h.c.}, \tag{3.2.6}$$

giving

$$\mathcal{L}_{\text{Maj. mass}}^{\nu_L} = m_L \nu_L^T C^{-1} \nu_L + \text{h.c.}, \tag{3.2.7}$$

where Λ is the high-energy scale beyond SM. Such mass generation also generates Majorana mass to right-handed neutrino $\mathcal{N}_R$,

$$\mathcal{L}_{\text{Maj. mass}}^{\mathcal{N}_R} = m_R (\mathcal{N}_R^C)^T C^{-1} \mathcal{N}_R^C + \text{h.c.} \tag{3.2.8}$$

Such mass terms are allowed by SM rules due to the fact that right-handed neutrinos are singlets under $SU(2)_L$, hence electroweak isospin $T_3 = 0$. Looking at mass terms in eqs. (3.2.4), (3.2.7) and (3.2.8), one can write, in component form,

$$\mathcal{L}_{mass} = \begin{pmatrix} \nu_L^T \ (\mathcal{N}_R^C)^T \end{pmatrix} C^{-1} \begin{pmatrix} m_L & \frac{m_D}{2} \\ \frac{m_D}{2} & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \mathcal{N}_R^C \end{pmatrix}. \tag{3.2.9}$$

Diagonalizing mass matrix (3.2.9) in so called limit of the Seesaw mechanism [18], i.e. $m_R \gg m_D$

and $m_L \to 0$, we obtain,

$$\begin{aligned} \nu'_L &= \nu_L - \frac{m_D}{2m_R}\mathcal{N}_L^C, \\ \mathcal{N}_R'^C &= \frac{m_D}{2m_R}\nu_L + \mathcal{N}_R^C \end{aligned} \tag{3.2.10}$$

where ν'_L and $\mathcal{N}_R'^C$ are basis after diagonalization. Finding the eigenvalues of the mass matrix (3.2.9), we get neutrino masses:

$$\begin{aligned} m_{\mathcal{N}} &\approx m_R, \\ m_\nu &\approx \frac{m_D^2}{4m_R}. \end{aligned} \tag{3.2.11}$$

So the seesaw mechanism essentially introduced a right-handed neutrino and coupling to left-handed neutrinos in this case. That resulted in a mass matrix, which we then diagonalized to find masses, leading to small mass for the left-handed neutrinos and heavy right-handed neutrino.

3.3 Spontaneous Symmetry Breaking

The generation of fermion masses, as explored in the case of neutrinos in the previous chapter, hinges on spontaneous symmetry breaking (SSB), where the Higgs field acquires a non-zero vacuum expectation value (VEV). In this section, we introduce SSB in the context of global and local $U(1)$ symmetry breaking, leading to the Higgs mechanism.

The following subsections are based on [19, 20]

3.3.1 Global $U(1)$ symmetry breaking

Consider a Lagrangian of complex scalar bosons Φ and $\Phi^\dagger$ that has a $U(1)$ symmetry:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\partial^\mu\Phi^\dagger\partial_\mu\Phi - \frac{M^2}{2}\Phi^\dagger\Phi \\ &= \frac{1}{2}\partial^\mu\Phi^\dagger\partial_\mu\Phi - V(\Phi), \end{aligned} \tag{3.3.1}$$

where $V(\Phi) = \frac{M^2}{2}\Phi^\dagger\Phi$ is the potential with a minimum at $|\Phi|^2 = 0$. Now consider replacing the potential V with a potential V' of form

$$V'(\Phi) = \frac{\kappa M^2}{2}(\Phi^\dagger\Phi - \phi^2)^2, \tag{3.3.2}$$

where κ and ϕ – real valued constants. Such vacuum of this potential at $|\Phi| = \phi$ is parameterized by a global $U(1)$ and represented as a circle at $|\Phi| = \phi$, leading to infinite number of vaccua. Hence one has to pick a vacuum. The Lagrangian (3.3.1) is invariant under $\mathrm{e}^{\mathrm{i}\alpha}$, so we pick a specific α to have our vacuum at $|\Phi| = \phi$. We have therefore gauge-fixed the symmetry in the Lagrangian, and the $U(1)$ symmetry is no longer explicitly present.

We can now rewrite the Lagrangian (3.3.1) in terms of the new vacuum, by expanding around the vaccuum value with a new field $\eta + \mathrm{i}\lambda$:

$$\Phi = \phi + \eta + \mathrm{i}\lambda, \tag{3.3.3}$$

where η and λ are real fields. Then Lagrangian in eq. (3.3.1) becomes

$$\begin{aligned}\mathcal{L} =& -\frac{1}{2}\partial^\mu[\eta - i\lambda]\partial_\mu[\eta + i\lambda] - \frac{M^2}{2}\kappa\left[(\phi + \eta - i\lambda)(\phi + \eta + i\lambda) - \phi^2\right]^2 \\ =& -\frac{1}{2}\partial^\mu\eta\partial_\mu\eta - 2\kappa M^2\phi^2\eta^2 - \frac{1}{2}\partial^\mu\lambda\partial_\mu\lambda \\ & - \frac{M^2}{2}\kappa\left[4\phi\eta^3 + 4\phi\eta\lambda^2 + \eta^4 + 2\eta^2\lambda^2 + \lambda^4\right].\end{aligned} \tag{3.3.4}$$

So the Lagrangian above describes the dynamics of a scalar field η and massless scalar field λ. It is worth noting that breaking global continuous symmetries, like $U(1)$, will result in a massless boson.

3.3.2 Local $U(1)$ symmetry breaking and Higgs mechanism

Now let's assume we have already gauged $U(1)$ symmetry and have a Lagrangian of form

$$\mathcal{L} = -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}(D^\mu\Phi)^\dagger D_\mu\Phi - V(\Phi^\dagger\Phi). \tag{3.3.5}$$

Again, we "shift" the potential like in eq. (3.3.2) and then fix gauge parameter $\alpha = \alpha(x)$ so that the vacuum at $|\Phi| = \phi$ is real, and expand around vacuum value ϕ with real field H,

$$\Phi = \phi + H. \tag{3.3.6}$$

Then the Lagrangian in eq. (3.3.5) becomes

$$\begin{aligned}\mathcal{L} =& -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\left[(\partial^\mu - \mathrm{i}gA^\mu)(\phi + H)\right]\left[(\partial_\mu + \mathrm{i}gA_\mu)(\phi + H)\right] \\ & - \frac{M^2}{2}\kappa\left[(\phi + H)(\phi + H) - \phi^2\right]^2 \\ =& -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\partial^\mu H\partial_\mu H - 2\kappa M^2\phi^2 H^2 - \frac{1}{2}g^2\phi^2 A^\mu A_\mu \\ & - 2\kappa M^2(\phi H^3 + H^4) - \frac{1}{2}g^2\phi A^\mu A_\mu H + \dots\end{aligned} \tag{3.3.7}$$

So after breaking local $U(1)$, we have dynamics describing scalar boson H with mass $M_H = 2\sqrt{\kappa}M\phi$ and gauge field A_μ that gained mass $m_A = g\phi$. Such procedure of mass generation for gauge bosons via spontaneous symmetry breaking is called Higgs mechanism [21].

3.4 The Grimus–Neufeld Model

Grimus–Neufeld Model (GNM) is a 2 Higgs Doublet Model (2HDM) that extends the Standard Model to explain small neutrino masses and mixing. It introduces a right-handed neutrino, which is a singlet under the Standard Model gauge group, and another Higgs doublet. The right-handed neutrinos do not interact via the weak force, but couple to the Higgs field through Yukawa interactions, enabling the generation of neutrino masses.

The overall GN Lagrangian can be written as

$$\mathcal{L}_{GN} = (D_\mu H_1)^\dagger D^\mu H_1 + (D_\mu H_2)^\dagger D^\mu H_2 - V_{\text{2HDM}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{SM}}, \tag{3.4.1}$$

where H_1 and H_2 are Higgs doublets and $\mathcal{L}_{SM}-$ Lagrangian of the SM without Higgs doublet terms. The general 2HDM potential [22] is of form

$$\begin{aligned} V_{\text{2HDM}} =& \mu_1 H_1^\dagger H_1 + \mu_2 H_2^\dagger H_2 + \left(\mu_3 H_1^\dagger H_2 + \text{h.c.}\right) + \lambda_1 \left(H_1^\dagger H_1\right)^2 + \lambda_2 \left(H_2^\dagger H_2\right)^2 \\ &+ \lambda_3 \left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + \lambda_4 \left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right) + \left(\lambda_5 \left(H_1^\dagger H_2\right)^2 + \text{h.c.}\right) \\ &+ H_1^\dagger H_1 \left(\lambda_6 \left(H_1^\dagger H_2\right) + \text{h.c.}\right) + H_2^\dagger H_2 \left(\lambda_7 \left(H_1^\dagger H_2\right) + \text{h.c.}\right), \end{aligned} \tag{3.4.2}$$

where couplings constants μ_3, $\lambda_{5,6,7}$, are real if CP is conserved.

After symmetry breaking, we choose that only the first Higgs doublet H_1 acquires VEV ϕ, thus having Higgs doublets of form

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(\phi + h_1 + iG^0\right) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(h_2 + i\sigma\right) \end{pmatrix}, \tag{3.4.3}$$

where h_1, h_2 are neutral scalar degrees of freedom and σ – pseudo-scalar one; H^+ is a real charged Higgs boson, G^0 and G^+ and neutral and charged Goldstone bosons accordingly. Having only one doublet with vacuum expectation value v (VEV) is called Higgs basis and it is useful due to the fact that it avoids mixing angle between Higgs doublets. Then the first doublet H_1 is identical to SM Higgs doublet and contains all Goldstone Bosons.

Then the neutral scalar fields of both doublets (3.4.3) can be rotated to mass eigenstate basis via orthogonal rotation matrix $\mathcal{R}$,

$$\mathcal{R} = \begin{pmatrix} c_\alpha & -s_\alpha & 0 \\ s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.4.4}$$

that is

$$\begin{pmatrix} h_1 \\ h_2 \\ \sigma \end{pmatrix} = \mathcal{R} \begin{pmatrix} h \\ H \\ A \end{pmatrix}, \tag{3.4.5}$$

where $c_\alpha = \cos(\alpha)$, $s_\alpha = \sin(\alpha)$ and α is the mixing angle. Rotation matrix components $\mathcal{R}_{31}$, $\mathcal{R}_{32}$, $\mathcal{R}_{13}$ and $\mathcal{R}_{23}$ are zero due to non-mixing of pseudo-scalar and scalar fields (CP-conserving Higgs sector). In case of stepping out of GNM and having additional Higgs doublets (with CP conservation), rotation matrix $\mathcal{R}$ would be block-diagonal – one block mixing scalar fields and the other mixing pseudo-scalar fields (for example, in 3HDM case, $\mathcal{R}$ would be a 5×5 matrix with 3×3 and 2×2 mixing blocks for scalars and pseudo-scalars).

Going back to right-handed Majorana neutrinos $\mathcal{N}$, they enter the Lagrangian in eq. (3.4.1) with a complex Majorana mass m (which becomes real once one adjusts phase of $\mathcal{N}$) and a Yukawa coupling $Y_j^{(i)}$ to the $SU(2)$ leptons L_j doublets [5]:

$$\mathcal{L}_{GN} \ni -\frac{1}{2} m \mathcal{N}\mathcal{N} - Y_l^{(i)} L_l \epsilon H_i \mathcal{N} + \text{h.c.}, \tag{3.4.6}$$

where ϵ in the second term of eq. (3.4.6) combines the doublets to an $SU(2)$ invariant product. When Higgs H_1 acquires VEV due to symmetry breaking, eq. (3.4.6) will lead to the Dirac-Majorana mass matrix. For its diagonalization, we rotate neutrino flavour eigenstates as well as Majorana neutrino into mass eigenstates, which leads to

$$\tilde{m} = \mathcal{U}^T M_\nu \mathcal{U} = \text{diag}(0, 0, m_3, M_4), \tag{3.4.7}$$

The other non-vanishing small masses of SM neutrinos in GNM are generated via interactions of H_2 at one-loop level [5].

As it was mentioned above, neutrinos ν_i' in flavour state, including the Majorana neutrino $\mathcal{N}$ field, can be rotated into mass eigenstates,

$$\begin{pmatrix} \nu_i' \\ \mathcal{N} \end{pmatrix} = \mathcal{U} \begin{pmatrix} \nu_i \\ \nu_4 \end{pmatrix}, \tag{3.4.8}$$

where index $i = 1, 2, 3$ labels generation and the rotation matrix $\mathcal{U}$ can be partitioned into two blocks,

$$\mathcal{U} = \begin{pmatrix} U_L \\ U_R^* \end{pmatrix}. \tag{3.4.9}$$

U_L and U_R are 3×4 and 1×4 matrices mixing SM neutrinos and Majorana neutrino with all eigenstates respectively. Then, in the mass basis defined above, the relation between the Majorana neutrino mass matrix to the neutrino Yukawa coupling is given by

$$y_\nu = \frac{\sqrt{2}}{v} U_L^* \tilde{m}_\nu U_R^T. \tag{3.4.10}$$

U_L and U_R have restrictions on their products due to unitarity of $\mathcal{U}$,

$$U_L U_L^\dagger = 1_{3\times 3}, \tag{3.4.11a}$$
$$U_R^* U_L^\dagger = 0_{1\times 3}, \tag{3.4.11b}$$
$$U_R^* U_R^T = 1_{1\times 3}, \tag{3.4.11c}$$
$$U_L^\dagger U_L + U_R^T U_R^* = 1_{1\times 4}, \tag{3.4.11d}$$
$$U_L U_R^T = 0, \tag{3.4.11e}$$
$$U_L^* \tilde{m} U_L^\dagger = 0, \tag{3.4.11f}$$

which could be useful when deriving BRST transformations of neutrino mass eigenstates in the upcoming section.

4 BRST transformations of the GNM

To be able to utilize NI for 2-point or 3-point correlation functions of GNM, the BRST Lagrangian [17], consisting of BRST–exact terms, that we used in section 2.1,

$$\mathcal{L}_{\rm BRST} = \sum_i \gamma_i \delta\Phi_i, \tag{4.1}$$

must be extended with extra Higgs doublet and Majorana neutrino in mass eigenstates. In this section, we explicitly repeat known BRST transformations for (which are essentially gauge transformations with ghost fields as transformation parameter) SM Higgs doublet and neutrinos, as well as deriving such transformations for the second Higgs doublet, and Majorana neutrinos in mass eigenstates; we also explicitly show BRST-exact terms of scalar fields h and H, as well as pseudoscalar field A and neutrinos ν.

4.1 BRST transformations of the Higgs sector

For the SM Higgs doublet, the gauge transformation under $SU(2)_L$ and $U(1)_Y$ is

$$H_1 \to H_1 + i\left[g\delta\theta^a\frac{\sigma^a}{2} + g'\delta\theta^Y Y\right]H_1, \tag{4.1.1}$$

where g, g' are couplings, $\delta\theta^a$ and $\delta\theta^Y$ are gauge transformation parameters, σ^a are Pauli matrices and Y — hypercharge of field (in this case, $Y = 1/2$). Then, expanding $SU(2)_L$ term, transformation (4.1.1) becomes

$$\begin{aligned}
\delta H_1 &= i\left[g\delta\theta^a\frac{\sigma^a}{2} + g'\delta\theta^Y Y\right]H_1 \\
&= i\left[\frac{g}{2}\begin{pmatrix}\delta\theta^3 & \sqrt{2}\delta\theta^+ \\ \sqrt{2}\delta\theta^- & -\delta\theta^3\end{pmatrix} + g'\frac{\delta\theta^Y}{2}\right]\begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(\phi + h_1 + iG^0\right)\end{pmatrix} \\
&= \frac{ig}{2}\begin{pmatrix}\delta\theta^3 G^+ + \sqrt{2}\delta\theta^+\frac{1}{\sqrt{2}}\left(\phi + h_1 + iG^0\right) \\ \sqrt{2}\delta\theta^- G^+ - \delta\theta^3\frac{1}{\sqrt{2}}\left(\phi + h_1 + iG^0\right)\end{pmatrix} + \frac{ig'}{2}\begin{pmatrix}\delta\theta^Y G^+ \\ \delta\theta^Y\frac{1}{\sqrt{2}}\left(\phi + h_1 + iG^0\right)\end{pmatrix},
\end{aligned} \tag{4.1.2}$$

where in second line, we wrote gauge parameters $\delta\theta^{1,2}$ in terms of lowering and raising operators, $\delta\theta^\pm = (\delta\theta^1 \mp i\delta\theta^2)/\sqrt{2}$. Writing transformations for each field separately, we have

$$\begin{aligned}
\delta G^+ &= \frac{ig}{2}\left(\delta\theta^3 G^+ + \delta\theta^+\left(\phi + h_1 + iG^0\right)\right) + \frac{ig'}{2}\left(\delta\theta^Y G^+\right) \\
&= \frac{ig}{2}\left(\left(s_w\delta\theta^A + c_w\delta\theta^Z\right)G^+ + \delta\theta^+\left(\phi + h_1 + iG^0\right)\right) + \frac{ig'}{2}\left(\left(C_w\delta\theta^A - s_w\delta\theta^Z\right)G^+\right) \\
&= ie\left(\delta\theta^A - \frac{s_w^2 - c_w^2}{2s_wc_w}\delta\theta^Z\right)G^+ + \frac{ie}{2s_w}\delta\theta^+\left(\phi + h_1 + iG^0\right),
\end{aligned} \tag{4.1.3}$$

$$
\begin{aligned}
\delta G^0 &= \frac{g}{2}\mathrm{Re}(\delta\theta^- G^+) - \frac{g}{2}\delta\theta^3(\phi + h_1) + \frac{g'}{2}\delta\theta^Y(\phi + h_1) \\
&= \frac{e}{2s_w}\mathrm{Re}(\delta\theta^- G^+) - \frac{e}{2s_w}\left(s_w\delta\theta^A + c_w\delta\theta^Z\right)(\phi + h_1) + \frac{e}{2c_w}\left(c_w\delta\theta^A - s_w\delta\theta^Z\right)(\phi + h_1) \\
&= \frac{e}{2s_w}\left(\delta\theta^- G^+ + \delta\theta^+ G^-\right) - \frac{e}{2s_w c_w}\delta\theta^Z(\phi + h_1)
\end{aligned}
\tag{4.1.4}
$$

and

$$
\begin{aligned}
\delta h_1 &= \frac{ig}{2}\left(\mathrm{Im}\left(\delta\theta^- G^+\right) + \delta\theta^3 G^0\right) - \frac{g'}{2}\delta\theta^Y G^0 \\
&= \frac{ie}{2s_w}\left(\delta\theta^- G^+ - \delta\theta^+ G^-\right) + \frac{e}{2s_w c_w}\delta\theta^Z G^0,
\end{aligned}
\tag{4.1.5}
$$

where relations

$$
\delta\theta^3 = s_w\delta\theta^A + c_w\delta\theta^Z \tag{4.1.6}
$$

and

$$
\delta\theta^Y = c_w\delta\theta^A - s_w\delta\theta^Z, \tag{4.1.7}
$$

as well as $e = gs_w$ and $e = g'c_w$, were used [23].

By the same analogy, the transformation of the second Higgs doublet,

$$
\delta H_2 = \frac{ig}{2}\begin{pmatrix} \delta\theta^3 H^+ + \delta\theta^+(h_2 + i\sigma) \\ \sqrt{2}\delta\theta^- H^+ - \delta\theta^3\frac{1}{\sqrt{2}}(h_2 + i\sigma)\end{pmatrix} + \frac{ig'}{2}\begin{pmatrix} \delta\theta^Y H^+ \\ \delta\theta^Y\frac{1}{\sqrt{2}}(h_2 + i\sigma)\end{pmatrix}, \tag{4.1.8}
$$

where

$$
\delta G^+ = ie\left(\delta\theta^A - \frac{s_w^2 - c_w^2}{2s_w c_w}\delta\theta^Z\right)H^+ + \frac{ie}{2s_w}\delta\theta^+(h_2 + \sigma), \tag{4.1.9}
$$

$$
\delta\sigma = \frac{e}{2s_w}\left(\delta\theta^- H^+ + \delta\theta^+ H^-\right) - \frac{e}{2s_w c_w}\delta\theta^Z h_2, \tag{4.1.10}
$$

$$
\delta h_2 = \frac{ie}{2s_w}\left(\delta\theta^- H^+ - \delta\theta^+ H^-\right) + \frac{e}{2s_w c_w}\delta\theta^Z\sigma. \tag{4.1.11}
$$

Since in BRST transformation case, the gauge parameters $\delta\theta^a$ are replaced by the corresponding ghost fields c^a, we have $\delta\theta^{\pm,A,Z} = c^{\pm,A,Z}$.

Now, rotating the transformations of fields δh_1, δh_2 and δG^0 into mass eigenstates δh, δH and $\delta\sigma$, we have

$$
\begin{aligned}
\begin{pmatrix}\delta h \\ \delta H \\ \delta A\end{pmatrix} &= \mathcal{R}^{-1}\begin{pmatrix} h_1 \\ h_2 \\ \sigma\end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha & 0 \\ -s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\delta h_1 \\ \delta h_2 \\ \delta\sigma\end{pmatrix} \\
&= \begin{pmatrix} c_\alpha\delta h_1 + s_\alpha\delta h_2 \\ -s_\alpha\delta h_1 + c_\alpha\delta h_2 \\ \delta\sigma\end{pmatrix},
\end{aligned}
\tag{4.1.12}
$$

were, as we already mentioned, due to CP-conservation, $\delta A = \delta\sigma$. Inserting field transformation expressions of δh_1 and δh_2 into mass eigenstate transformations of δh and δH, we obtain

$$
\delta h = \mathcal{R}^{-1}_{11}\delta h_1 + \mathcal{R}^{-1}_{12}\delta h_2
$$

$$
\begin{aligned}
&= \mathcal{R}_{11}^{-1}\left(\frac{ie}{2s_w}\left(c^- G^+ - c^+ G^-\right) + \frac{e}{2s_w c_w} c^Z G^0\right) \\
&+ \mathcal{R}_{12}^{-1}\left(\frac{ie}{2s_w}\left(c^- H^+ - c^+ H^-\right) + \frac{e}{2s_w c_w} c^Z \sigma\right) \\
&= \frac{e}{2s_w c_w} c^Z \left(\mathcal{R}_{11}^{-1} G^0 + \mathcal{R}_{12}^{-1}\sigma\right) \\
&+ \frac{ie}{2s_w}\left[c^-\left(\mathcal{R}_{11}^{-1} G^+ + \mathcal{R}_{12}^{-1} H^+\right) - c^+\left(\mathcal{R}_{11}^{-1} G^- + \mathcal{R}_{12}^{-1} H^-\right)\right]
\end{aligned}
\tag{4.1.13}
$$

and

$$
\begin{aligned}
\delta H &= \mathcal{R}_{21}^{-1}\delta h_1 + \mathcal{R}_{22}^{-1}\delta h_2 \\
&= \frac{e}{2s_w c_w} c^Z \left(\mathcal{R}_{21}^{-1} G^0 + \mathcal{R}_{22}^{-1}\sigma\right) \\
&+ \frac{ie}{2s_w}\left[c^-\left(\mathcal{R}_{21}^{-1} G^+ + \mathcal{R}_{22}^{-1} H^+\right) - c^+\left(\mathcal{R}_{21}^{-1} G^- + \mathcal{R}_{22}^{-1} H^-\right)\right].
\end{aligned}
\tag{4.1.14}
$$

We see that BRST transformations of fields h and H are almost identical — the difference lies in components of the inverse rotation matrix $\mathcal{R}^{-1}$ in eq. (4.1.12), which results in different mixing angles for the fields σ, $G^{0,\pm}$ and $H^{\pm}$.

Then the BRST–exact terms, excluding terms of Goldstone bosons, are

$$
\begin{aligned}
\mathcal{L}_{\text{BRST}}^{\text{GN}} \ni \gamma^h &\left\{ \frac{e}{2s_w c_w} c^Z \left(R_{11}^{-1} G^0 + R_{21}^{-1}\sigma\right) \right. \\
&\left. + \frac{ie}{2s_w}\left[c^-\left(R_{11}^{-1} G^+ + R_{21}^{-1} H^+\right) - c^+\left(R_{11}^{-1} G^- + R_{21}^{-1} H^-\right)\right] \right\} \\
+ \gamma^H &\left\{ \frac{e}{2s_w c_w} c^Z \left(R_{12}^{-1} G^0 + R_{22}^{-1}\sigma\right) \right. \\
&\left. + \frac{ie}{2s_w}\left[c^-\left(R_{12}^{-1} G^+ + R_{22}^{-1} H^+\right) - c^+\left(R_{12}^{-1} G^- + R_{22}^{-1} H^-\right)\right] \right\} \\
+ \gamma^A &\left\{ \frac{e}{2s_w}\left(c^- H^+ + c^+ H^-\right) - \frac{e}{2s_w c_w} c^Z \left(\mathcal{R}_{21} h + \mathcal{R}_{22} H\right) \right\} \\
+ \text{h.c.},&
\end{aligned}
\tag{4.1.15}
$$

where we expressed h_2 in terms of h and H as

$$
h_2 = \mathcal{R}_{21} h + \mathcal{R}_{22} H,
\tag{4.1.16}
$$

using eq. (4.1.12).

4.2 BRST transformations of the neutrino sector

Similarly to eq. (4.1.1), the transformation for the left–handed fermion doublet is

$$
L_l \to L_l + i\left[g\delta\theta^a \frac{\sigma^a}{2} + g'\delta\theta^Y Y\right] L_l;
\tag{4.2.1}
$$

Here, we relabel the fermion doublet from eq. (3.2.2) as

$$L_l = \begin{pmatrix} \nu'_{l,L} \\ l_L, \end{pmatrix} \tag{4.2.2}$$

where $l = e, \mu, \tau$ labels generations, for convenience. Then the transformation of L_l,

$$\begin{aligned} \delta L_l &= i\left[g\delta\theta^a \frac{\sigma^a}{2} + g'\delta\theta^Y Y\right] L_l \\ &= i\left[\frac{g}{2}\begin{pmatrix} \delta\theta^3 & \sqrt{2}\delta\theta^+ \\ \sqrt{2}\delta\theta^- & -\delta\theta^3 \end{pmatrix} - g'\frac{\delta\theta^Y}{2}\right]\begin{pmatrix} \nu'_{l,L} \\ l_L \end{pmatrix}, \end{aligned} \tag{4.2.3}$$

leads to

$$\begin{aligned} \delta\nu'_{l,L} &= \frac{ig}{2}\left(\delta\theta^3\nu'_{l,L} + \sqrt{2}\delta\theta^+ l_L\right) - \frac{ig'}{2}\delta\theta^Y\nu'_{l,L} \\ &= \frac{ie}{\sqrt{2}s_w}\delta\theta^+ l_L + \frac{ie}{2}\left(\frac{1}{s_w}\delta\theta^3 - \frac{1}{c_w}\delta\theta^Y\right)\nu'_{l,L} \\ &= \frac{ie}{\sqrt{2}s_w}\delta\theta^+ l_L + \frac{ie}{2s_w c_w}\delta\theta^Z\nu'_{l,L}, \end{aligned} \tag{4.2.4}$$

where we again used relations (4.1.6) and (4.1.7) in second line.

Now we can rotate the left–handed transformations of SM and Majorana neutrinos into mass eigenstates,

$$\begin{pmatrix} \delta\nu_1 \\ \delta\nu_2 \\ \delta\nu_3 \\ \delta\nu_4 \end{pmatrix} = \mathcal{U}^\dagger \begin{pmatrix} \delta\nu'_e \\ \delta\nu'_\mu \\ \delta\nu'_\tau \\ \delta\mathcal{N} \end{pmatrix}; \tag{4.2.5}$$

since Majorana neutrino $\mathcal{N}$ is a singlet, its gauge (BRST) transformation is zero. In more compact form, we have

$$\begin{aligned} \delta\nu_{j,L} &= U^*_{lj}\delta\nu'_{l,L} \\ &= U^*_{lj}\left(\frac{ie}{\sqrt{2}s_w}c^+ l_l + \frac{ie}{2s_w c_w}c^Z\nu'_{l,L}\right) \\ &= \frac{ie}{\sqrt{2}s_w}U^*_{lj}c^+ l_l + \frac{ie}{2s_w c_w}U^*_{lj}U_{li}c^Z\nu_i, \end{aligned} \tag{4.2.6}$$

where $i, j = 1, 2, 3, 4$ labels the neutrino mass eigenstates, elements U^*_{lj} and U_{li} are of U_L and its complex conjugate. The flavor states ν'_l were also rotated into the mass eigenstates, that is, $\nu'_l = U_{li}\nu_i$.

Since ν_j are Majorana fields, they satisfy the condition $\nu_j = (\nu_j)^c$, and the right–handed components are related to the left–handed ones by charge conjugation, that is

$$\nu_{j,R} = (\nu_{j,L})^c. \tag{4.2.7}$$

Then the full BRST transformation of ν_j can be written as

$$\delta\nu_j = P_L\delta\nu_{j,L} + P_R(\delta\nu_{j,L})^c. \tag{4.2.8}$$

Thus, while Majorana neutrino $\mathcal{N}$ itself has no BRST transformation (as a singlet), its mass eigenstate ν_4 obtains non-zero BRST transformation through mixing with active neutrinos; typically, in type-I Seesaw mechanism, the mixing elements U_{l4} are suppressed by small ratio of m_D/M_R (approximately zero), but to keep track of how full mass basis transforms, we set it as non-zero. In the next section, we will see that non-zero U_{l4} allows ν_4 to behave like a weekly coupled neutral fermion at loop level.

Finally, the Lagrangian BRST–exact terms of the neutrino sector,

$$\begin{aligned}\mathcal{L}_{\text{BRST}}^{\text{GN}} \ni \bar{\eta}_{\nu_j} \Bigg\{ & P_L \left(\frac{ie}{\sqrt{2}s_w} (U_L^\dagger)_{jl} c^+ l_l + \frac{ie}{2s_w c_w} (U_L^\dagger U_L)_{ji} c^Z \nu_i \right) \\ & - P_R \left(\frac{ie}{\sqrt{2}s_w} (U_L^T)_{jl} c^- l_l^c + \frac{ie}{2s_w c_w} (U_L^T U_L^*)_{ji} c^Z \nu_i \right) \Bigg\} + \text{h.c.}, \end{aligned} \tag{4.2.9}$$

where $U_{lj}^* = (U_L^\dagger)_{jl}$ and $U_{lj}^* U_{li} = (U_L^\dagger U_L)_{ji}$.

5 Nielsen Identity and the neutrino self-energy of the GNM

Following section 2, we now provide the NI for the neutrino self-energy. Differentiating the NI in eq. (2.1.17) with respect to ν_j and $\bar{\nu}_i$, we obtain

$$- \sum_{\beta=W,Z} \partial_{\xi_\beta} \Gamma_{\nu_j \bar{\nu}_i} = \sum_{\beta=W,Z} \left(\Gamma_{\bar{\nu}_i \nu_k} \Gamma_{\nu_j \bar{\eta}_{\nu_k} \chi_\beta} + \Gamma_{\bar{\nu}_i \eta_{\bar{\nu}_k} \chi_\beta} \Gamma_{\nu_j \bar{\nu}_k} \right) . \tag{5.1}$$

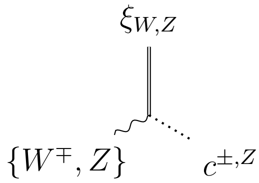
At 1-loop level, this leads to

$$\begin{aligned} - \sum_{\beta=W,Z} \partial_{\xi_\beta} \Gamma^{(1)}_{\nu_j \bar{\nu}_i} &= \sum_{\beta=W,Z} \left(\Gamma^{(0)}_{\bar{\nu}_i \nu_k} \Gamma^{(1)}_{\nu_j \bar{\eta}_{\nu_k} \chi_\beta} + \Gamma^{(1)}_{\bar{\nu}_i \eta_{\bar{\nu}_k} \chi_\beta} \Gamma^{(0)}_{\nu_j \bar{\nu}_k} \right) \\ &= \sum_{\beta=W,Z} \left((\not{p} - m_{\nu_i}) \Gamma^{(1)}_{\nu_j \bar{\eta}_{\nu_i} \chi_\beta} + \Gamma^{(1)}_{\bar{\nu}_i \eta_{\bar{\nu}_j} \chi_\beta} (\not{p} - m_{\nu_j}) \right) , \end{aligned} \tag{5.2}$$

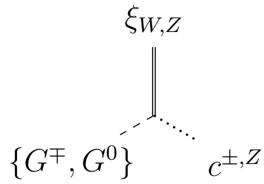
where $\Gamma^{(0)}_{\nu_j \bar{\nu}_k} = (\not{p} - m_{\nu_k}) \delta_{jk}$. To evaluate NI for the neutrino self-energy in eq. (5.2), 3-point functions of 1-loop $\Gamma^{(1)}_{\nu_j \bar{\eta}_{\nu_i} \chi_\beta}$ and $\Gamma^{(1)}_{\bar{\nu}_i \eta_{\bar{\nu}_j} \chi_\beta}$ must be constructed. In the following, we provide the Feynman rules obtained from the χ-extended Lagrangian [17],

$$\begin{aligned} \mathcal{L}_{\chi_{W,Z}} = &- \frac{\chi_Z}{2\xi_Z} \bar{c}^Z \left(\partial_\mu Z^\mu + \xi_Z M_Z G^0 \right) \\ &- \frac{\chi_W}{2\xi_W} \left[\bar{c}^+ \left(\partial^\mu W^-_\mu - i \xi_W M_W G^- \right) + \bar{c}^- \left(\partial^\mu W^+_\mu + i \xi_W M_W G^+ \right) \right] , \end{aligned} \tag{5.3}$$

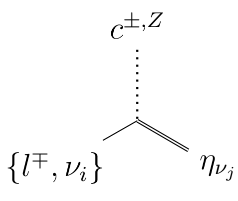
as well as BRST-exact Lagrangian terms of the neutrino sector in eq. (4.2.9):



$$= \left\{ \frac{-k^\mu}{2\xi_W}, \frac{-k^\mu}{2\xi_Z} \right\} ,$$



$$= \left\{ \pm \frac{m_W}{2}, \frac{i m_Z}{2} \right\} ; \tag{5.4}$$

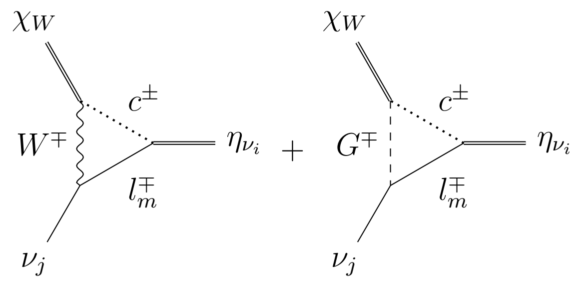


$$= \left\{ \frac{g_W}{\sqrt{2}} (U_L^\dagger)_{jl} P_L, - \frac{g_W}{\sqrt{2}} (U_L^T)_{jl} P_R, \frac{g_Z}{2} \left((U_L^\dagger U_L)_{ji} P_L - (U_L^T U_L^*)_{ji} P_R \right) \right\} , \tag{5.5}$$

where $e/s_w = g_W$ and $e/(s_w c_w) = g_Z$.

The 1-loop Feynman diagrams, contributing to 3–point functions $\Gamma^{(1)}_{\nu_j \bar{\eta}_{\nu_i} \chi_W}$ and $\Gamma^{(1)}_{\nu_j \bar{\eta}_{\nu_i} \chi_Z}$,

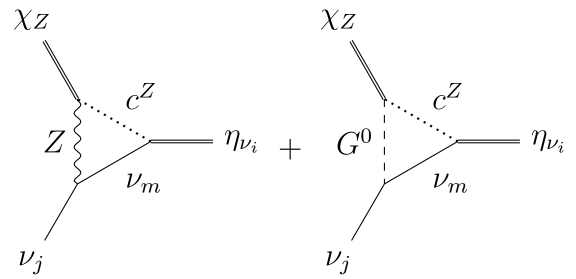
$$\Gamma^{(1)}_{\nu_j \bar{\eta}_{\nu_i} \chi_W} =$$



(5.6)

and

$$\Gamma^{(1)}_{\nu_j \bar{\eta}_{\nu_i} \chi_Z} =$$



, (5.7)

where two Feynman diagrams in eq. (5.7) represent four diagrams (two possible ways of the fermion flow); $\Gamma^{(1)}_{\bar{\nu}_i \eta_{\bar{\nu}_j} \chi_W}$ and $\Gamma^{(1)}_{\bar{\nu}_i \eta_{\bar{\nu}_j} \chi_Z}$ are constructed analogously to the 3-point vertex functions shown above. Thus, we complete the setup for calculating NI of neutrino self–energy.

In the following sections, we aim to confirm the implementation of NI of the neutrino self-energy by: 1) evaluating the 3-point vertex functions and inserting them into NI; 2) directly computing the ξ_β-dependent part of the neutrino self-energy via `FeynCalc` (and taking derivatives with respect to ξ_β), and comparing to the NI result. We express the results explicitly for the case $\beta = W$; For case $\beta = Z$, we provide some of the expressions explicitly, analogously to case $\beta = W$.

5.1 ξ_W-dependent part of Nielsen Identity and neutrino self-energy

The constructed 3-point vertex function $\Gamma^{(1)}_{\nu_j \bar{\eta}_{\nu_i} \chi_W}$ from eq. (5.2) (see appendix A for full derivation),

$$\begin{aligned}\Gamma^{(1)}_{\nu_j \bar{\eta}_{\nu_i} \chi_W} = &\frac{g_W^2}{4}(U_L^\dagger)_{im}(U_L)_{mj} P_L \int \frac{d^4 k}{(2\pi)^4} \frac{(\not{p} - \not{k})(\not{k} + \tilde{m}_{\nu_j}) - m_{l_m}^2}{\left[(p-k)^2 - m_{l_m}^2\right]\left[k^2 - \xi_W m_W^2\right]^2} \\ &+ \Big((U_L^\dagger)_{im}(U_L)_{mj} P_L \leftrightarrow (U_L^\dagger)_{jm}(U_L)_{mi} P_R \Big)\,;\end{aligned} \tag{5.1.1}$$

in case of $\Gamma^{(1)}_{\bar{\nu}_i \eta_{\bar{\nu}_j} \chi_W}$, we see that, diagrammatically, it corresponds to the same Feynman diagrams as of $\Gamma^{(1)}_{\nu_j \bar{\eta}_{\nu_i} \chi_W}$, but with reversed fermion lines. This means that, below particle production threshold, we have pseudo–Hermiticity:

$$\Gamma^{(1)}_{\bar{\nu}_i \eta_{\bar{\nu}_j} \chi_W} = \gamma^0 (\Gamma^{(1)}_{\nu_j \bar{\eta}_{\nu_i} \chi_W})^\dagger \gamma^0; \tag{5.1.2}$$

Expressing the vertex function in eq. (5.1.1) in terms of Passarino–Veltman (PaVe) functions [24] (see Appendix D for definitions), using `FeynCalc`, we have

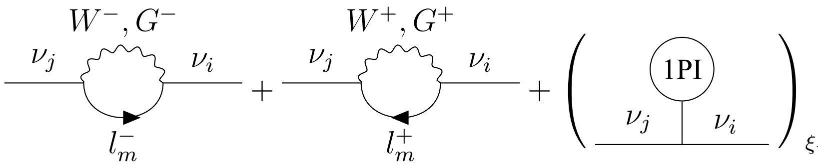
$$\Gamma^{(1)}_{\nu_j \bar{\eta}_{\nu_i} \chi_W} = -\tau \frac{g_W^2}{8}(U_L^\dagger)_{im}(U_L)_{mj} P_L \frac{\not{p}}{p^2}(\not{p} - \tilde{m}_{\nu_j}) B_0(0, \xi_W m_W^2, \xi_W m_W^2)$$

$$
\begin{aligned}
&- \tau \frac{g_W^2}{8} (U_L^\dagger)_{im} (U_L)_{mj} P_L \frac{\not{p}}{p^2} (\not{p} + \tilde{m}_{\nu_j}) B_0(p^2, m_{l_m}^2, \xi_W m_W^2) \\
&- \tau \frac{g_W^2}{8} (U_L^\dagger)_{im} (U_L)_{mj} P_L (3m_{l_m}^2 + \xi_W m_W^2 - p^2) \\
&\times C_0(0, p^2, p^2, \xi_W m_W^2, \xi_W m_W^2, m_{l_m}^2) \\
&+ \tau \frac{g_W^2}{8} (U_L^\dagger)_{im} (U_L)_{mj} P_L \frac{\not{p} \tilde{m}_{\nu_j}}{p^2} (m_{l_m}^2 - \xi_W m_W^2 + p^2) \\
&\times C_0(0, p^2, p^2, \xi_W m_W^2, \xi_W m_W^2, m_{l_m}^2) \\
&+ \Big((U_L^\dagger)_{im} (U_L)_{mj} P_L \leftrightarrow (U_L^\dagger)_{jm} (U_L)_{mi} P_R \Big) ,
\end{aligned} \tag{5.1.3}
$$

where $\tau = i/(16\pi)$ is coefficient that we end up with from the definition of PaVe functions; $\Gamma^{(1)}_{\bar{\nu}_i \eta_{\bar{\nu}_j} \chi_W}$ is expressed in PaVe basis analogously to 3-point vertex function in eq. (5.1.3).

To have a comparison, we also need to calculate neutrino self–energy part that depends on ξ_W, and take derivative w.r.t to this gauge parameter. The 1-loop Feynman diagrams, contributing to this part of the neutrino self-energy,

$\Sigma_{ji}(\not{p}, \xi_W) =$



, (5.1.4)

where tadpole contribution is shown in appendix C. The first four Feynman diagrams of $\Sigma_{ji}(\not{p}, \xi_W)$,

$$
\begin{aligned}
\Sigma_{ji}(\not{p}, \xi_W) \supset &- \int \frac{d^4k}{(2\pi)^4} \frac{g_W^2}{2} (U_L^\dagger)_{im} (U_L)_{mj} \frac{P_L(\not{p} - \not{k})(4(k^2 - \xi_W m_W^2) - (1 - \xi_W \not{k}\not{k}))}{[(p-k)^2 - m_{lm}^2]\,[k^2 - m_W^2]\,[k^2 - \xi_W m_W^2]} \\
&+ \frac{2}{v^2} (U_L^\dagger)_{im} (U_L)_{mj} \int \frac{d^4k}{(2\pi)^4} \frac{P_L \tilde{m}_{\nu_i}(-\tilde{m}_{\nu_j}(\not{p} - \not{k}) + m_{l_m}^2) + P_R m_{lm}^2 (\tilde{m}_{\nu_j} - (\not{p} - \not{k}))}{\left[(p-k)^2 - m_{l_m}^2\right]\,[k^2 - \xi_W m_W^2]} \\
&- \int \frac{d^4k}{(2\pi)^4} \frac{g_W^2}{2} (U_L^\dagger)_{jm} (U_L)_{mi} \frac{P_R(\not{p} - \not{k})(4(k^2 - \xi_W m_W^2) - (1 - \xi_W \not{k}\not{k}))}{[(p-k)^2 - m_{lm}^2]\,[k^2 - m_W^2]\,[k^2 - \xi_W m_W^2]} \\
&+ \frac{2}{v^2} (U_L^\dagger)_{jm} (U_L)_{mi} \int \frac{d^4k}{(2\pi)^4} \frac{P_R \tilde{m}_{\nu_i}(-\tilde{m}_{\nu_j}(\not{p} - \not{k}) + m_{l_m}^2) + P_L m_{lm}^2 (\tilde{m}_{\nu_j} - (\not{p} - \not{k}))}{\left[(p-k)^2 - m_{l_m}^2\right]\,[k^2 - \xi_W m_W^2]} ;
\end{aligned} \tag{5.1.5}
$$

Then, with the tadpole contribution (see Appendix B for tadpole contribution in PaVe basis), we express it in PaVe basis using `FeynCalc` and take the derivative with respect to ξ_W:

$$
\begin{aligned}
\partial_{\xi_W} \Sigma_{ji}(\not{p}, \xi_W) = \tau \frac{g_W^2}{4} \Bigg(&\not{p} P_L (U_L^\dagger)_{im} (U_L)_{mj} + \frac{\not{p}}{p^2} \tilde{m}_{\nu_j} \tilde{m}_{\nu_i} P_L (U_L^\dagger)_{jm} (U_L)_{mi} \\
&- P_L (\mathcal{Y}_1)_{jmi} \Big) \times B_0(0, \xi_W m_W^2, \xi_W m_W^2) \\
&+ \tau \frac{g_W^2}{4} \Bigg(\not{p} P_L (U_L^\dagger)_{im} (U_L)_{mj} - \frac{\not{p}}{p^2} \tilde{m}_{\nu_j} \tilde{m}_{\nu_i} P_L (U_L^\dagger)_{jm} (U_L)_{mi} \Bigg) \\
&\times B_0(p^2, m_{l_m}^2, \xi_W m_W^2) \\
&+ \tau \frac{g_W^2}{4} \Big(\not{p} \left(3m_{l_m}^2 + \xi_W m_W^2 - p^2\right) P_L (U_L^\dagger)_{im} (U_L)_{mj}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\not{p}}{p^2}\tilde{m}_{\nu_j}\tilde{m}_{\nu_i}\left(m_{l_m}^2-\xi_W m_W^2+p^2\right)P_L(U_L^\dagger)_{jm}(U_L)_{mi}\\
&-2m_{l_m}^2 P_L(\mathcal{Y}_1)_{jmi}\Big)\times C_0(0,p^2,p^2,\xi_W m_W^2,\xi_W m_W^2,m_{l_m}^2)\\
&+\Big((U_L^\dagger)_{im}(U_L)_{mj}P_L\leftrightarrow(U_L^\dagger)_{jm}(U_L)_{mi}P_R\Big),
\end{aligned}
\tag{5.1.6}
$$

where, for convenience, we define factors

$$
(\mathcal{Y}_1)_{jmi}=\tilde{m}_{\nu_j}(U_L^\dagger)_{jm}(U_L)_{mi}+\tilde{m}_{\nu_i}(U_L^\dagger)_{im}(U_L)_{mj}
\tag{5.1.7}
$$

and

$$
(\mathcal{Y}_2)_{jmi}=(U_L^\dagger)_{jm}(U_L)_{mi}\tilde{m}_{\nu_i}+(U_L^\dagger)_{im}(U_L)_{mj}\tilde{m}_{\nu_j}.
\tag{5.1.8}
$$

Inserting the 1PI 3-point vertex functions $\Gamma^{(1)}_{\bar{\nu}_i\eta_{\bar{\nu}_j}\chi_W}$ and $\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_W}$ into the NI of the neutrino self-energy in eq. (5.2) gives exactly the same gauge dependence of the neutrino self-energy as obtained by direct computation of $\partial_{\xi_W}\Sigma_{ji}(\not{p},\xi_W)$ in eq. (5.1.6)

5.2 ξ_Z–dependent part of Nielsen Identity and neutrino self energy

Similarly to the previous section, the first two Feynman diagrams, with the same fermion flow, that contribute to the vertex function $\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_Z}$ (see appendix A for full derivation):

$$
\begin{aligned}
\left(\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_Z}\right)_{1,2}=&\frac{g_Z^2}{4}\int\frac{d^4k}{(2\pi)^4}\frac{(P_L(U_L^\dagger U_L)_{im,mj}+P_R(U_L^\dagger U_L)_{mi,jm})(\not{p}\not{k}-\not{k}\not{k})}{\left[(p-k)^2-m_{\nu_m}^2\right]\left[k^2-\xi_Z m_Z^2\right]^2}\\
&-\frac{g_Z^2}{4}\int\frac{d^4k}{(2\pi)^4}\frac{(P_L(U_L^\dagger U_L)_{im,jm}+P_R(U_L^\dagger U_L)_{mi,mj})\tilde{m}_{\nu_m}\not{k}}{\left[(p-k)^2-m_{\nu_m}^2\right]\left[k^2-\xi_Z m_Z^2\right]^2}\\
&+\frac{g_Z^2}{8}\int\frac{d^4k}{(2\pi)^4}\frac{(P_L(U_L^\dagger U_L)_{im}(\mathcal{Y}_2)_{mfj}+P_R(U_L^\dagger U_L)_{mi}(\mathcal{Y}_1)_{mfj})(\not{p}-\not{k})}{\left[(p-k)^2-m_{\nu_m}^2\right]\left[k^2-\xi_Z m_Z^2\right]^2}\\
&-\frac{g_Z^2}{8}\int\frac{d^4k}{(2\pi)^4}\frac{(P_L(U_L^\dagger U_L)_{im}(\mathcal{Y}_1)_{mfj}+P_R(U_L^\dagger U_L)_{mi}(\mathcal{Y}_2)_{mfj})\tilde{m}_{\nu_m}}{\left[(p-k)^2-m_{\nu_m}^2\right]\left[k^2-\xi_Z m_Z^2\right]^2},
\end{aligned}
\tag{5.2.1}
$$

where $(U_L^\dagger U_L)_{im,mj}=(U_L^\dagger U_L)_{im}(U_L^\dagger U_L)_{mj}$.

Feynman diagrams with reversed fermion flow orientation (contributing to $\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_Z}$) are not independent and are related to the first two diagrams in eq. (5.2.1) via charge conjugated transpose:

$$
\left(\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_Z}\right)_{3,4}=C\left(\left(\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_Z}\right)_{1,2}\right)^T C^{-1};
\tag{5.2.2}
$$

then, the two remaining Feynman diagrams contribute

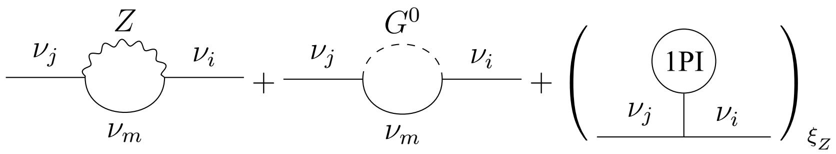
$$
\begin{aligned}
\left(\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_Z}\right)_{3,4}=&\frac{g_Z^2}{4}\int\frac{d^4k}{(2\pi)^4}\frac{(\not{p}\not{k}-\not{k}\not{k})(P_L(U_L^\dagger U_L)_{jm,mi}+P_R(U_L^\dagger U_L)_{mj,im})}{\left[(p-k)^2-m_{\nu_m}^2\right]\left[k^2-\xi_Z m_Z^2\right]^2}\\
&-\frac{g_Z^2}{4}\int\frac{d^4k}{(2\pi)^4}\frac{\not{k}(P_R(U_L^\dagger U_L)_{jm,im}+P_L(U_L^\dagger U_L)_{mj,mi})\tilde{m}_{\nu_m}}{\left[(p-k)^2-m_{\nu_m}^2\right]\left[k^2-\xi_Z m_Z^2\right]^2}
\end{aligned}
$$

$$+\frac{g_Z^2}{8}\int\frac{d^4k}{(2\pi)^4}\frac{(\not p-\not k)(P_R(U_L^\dagger U_L)_{mi}(\mathcal{Y}_1)_{mfj}+P_L(U_L^\dagger U_L)_{im}(\mathcal{Y}_2)_{mfj}}{\left[(p-k)^2-m_{\nu_m}^2\right]\left[k^2-\xi_Z m_Z^2\right]^2}$$
$$-\frac{g_Z^2}{8}\int\frac{d^4k}{(2\pi)^4}\frac{(P_L(U_L^\dagger U_L)_{im}(\mathcal{Y}_1)_{mfj}+P_R(U_L^\dagger U_L)_{mi}(\mathcal{Y}_2)_{mfj})\tilde{m}_{\nu_m}}{\left[(p-k)^2-m_{\nu_m}^2\right]\left[k^2-\xi_Z m_Z^2\right]^2}. \qquad (5.2.3)$$

Feynman diagrams with reversed fermion propagation, $\left(\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_Z}\right)_{3,4}$, were expressed in PaVe functions, in the same manner as $\left(\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_Z}\right)_{1,2}$. The second proper vertex function $\Gamma^{(1)}_{\bar{\nu}_i\eta_{\bar{\nu}_j}\chi_Z}$ is obtained using $\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_Z}$ via pseudo-hermiticity, as in eq. (5.1.2).

Again, to check the consistency of the implementation and construction of vertex functions above, we calculate the dependence of the neutrino self-energy on ξ_Z. The 1-loop Feynman diagrams, which are part of neutrino self-energy and depends on ξ_Z, are

$$\Sigma_{ji}(\not p,\xi_Z) =$$



. (5.2.4)

Due to the length of the expression, we provide the derivative of part of the neutrino self-energy with respect to ξ_Z in the appendix C . The dependence of the neutrino self-energy on ξ_Z, as obtained by direct calculation, matches the result of the NI of the self-energy in eq. (5.2), in the case $\beta = Z$.

6 Towards Nielsen Identity automatization in the GNM

As we see in section 5, calculations of 1PI 3-point vertex functions and NI at 1-loop are tedious if done by hand, especially in the case $\beta = Z$. Also worth noting that, expanding NI to second order, i.e.

$$\begin{aligned} -\sum_{\beta=W,Z} \partial_{\xi_\beta}\Gamma^{(2)}_{\nu_j\bar{\nu}_i} = \sum_{\beta=W,Z} \Big(& \Gamma^{(0)}_{\bar{\nu}_i\nu_k}\Gamma^{(2)}_{\nu_j\bar{\eta}_{\nu_k}\chi_\beta} + \Gamma^{(2)}_{\bar{\nu}_i\eta_{\bar{\nu}_k}\chi_\beta}\Gamma^{(0)}_{\nu_j\bar{\nu}_k} \\ & + \Gamma^{(1)}_{\bar{\nu}_i\nu_k}\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_k}\chi_\beta} + \Gamma^{(1)}_{\bar{\nu}_i\eta_{\bar{\nu}_k}\chi_\beta}\Gamma^{(1)}_{\nu_j\bar{\nu}_k} \Big) , \end{aligned} \tag{6.1}$$

means that one has to evaluate more than two 3-point vertex functions! This leads to the necessity of automating NI using `FeynCalc` [25–28] and `FeynArts` [29], to have a method to obtain these results.

The initial concept is to complement the already existing GNM model files [6, 22], so that we could generate 1PI vertex functions by: 1) adding an additional class description file with all BRST sources involved in $\mathcal{L}^{\text{GN}}_{\text{BRST}} + \mathcal{L}_\chi$; 2) introducing the BRST transformations of the fields and gauge parameters; 3) extending the Lagrangian of the GNM with the Lagrangians mentioned previously; 4) Generating the `FeynRules` model files, as well as topologies of the new Feynman diagrams using `FeynArts`.

In the following, we provide the code example for defining BRST sources of the scalars h, H, pseudo-scalar A and gauge parameter ξ:

```
M$BRSTsources = {

(* BRST source for the scalar h field *)
U[101] == {
   ClassName          -> BRSTgammah,
   ParticleName       -> "gammah",
   QuantumNumbers     -> {GhostNumber -> 1},
   Mass               -> 0,
   Width              -> 0,
   PropagatorLabel    -> Subscript["\[gamma]", "h"],
   SelfConjugate      -> False,
   PropagatorType     -> Double,
   PropagatorArrow    -> Forward,
   InsertOnly         -> External
},

(* BRST source for the scalar H field *)
U[102] == {
   ClassName          -> BRSTgammaH,
   ParticleName       -> "gammaH",
   QuantumNumbers     -> {GhostNumber -> 1},
```

```
  Mass              -> 0,
  Width             -> 0,
  PropagatorLabel -> Subscript["\[gamma]", "H"],
  SelfConjugate   -> False,
  PropagatorType  -> Double,
  PropagatorArrow -> Forward,
  InsertOnly      -> External
},

(* BRST source for the pseudo-scalar A field *)
U[103] == {
  ClassName       -> BRSTgammaA,
  ParticleName    -> "gammaA",
  QuantumNumbers  -> {GhostNumber -> 1},
  Mass            -> 0,
  Width           -> 0,
  PropagatorLabel -> Subscript["\[gamma]", "A"],
  SelfConjugate   -> False,
  PropagatorType  -> Double,
  PropagatorArrow -> Forward,
  InsertOnly      -> External
},

(*BRST source for gauge parameters \xi*)
U[104] == {
  ClassName       -> chiW,
  ParticleName    -> "chiW",
  QuantumNumbers  -> {GhostNumber -> 1},
  PropagatorLabel -> Subscript["\[chi]", "W"],
  Mass            -> 0,
  Width           -> 0,
  SelfConjugate   -> False,
  PropagatorType  -> Straight,
  PropagatorArrow -> Forward,
  InsertOnly -> External
},
U[105] == {
  ClassName       -> chiZ,
  ParticleName    -> "chiZ",
  QuantumNumbers  -> {GhostNumber -> 1},
```

```
    PropagatorLabel -> Subscript["\[chi]", "Z"],
    Mass            -> 0,
    Width           -> 0,
    SelfConjugate   -> False ,
    PropagatorType  -> Straight ,
    PropagatorArrow -> Forward ,
    InsertOnly -> External
}
}
```

Code 1. Class description file example of BRST sources for the GNM

For the scalar sector, as shown above, we define each BRST source with a ghost field-type class `U[...]` (substitute would be a scalar field-type `S[...]`) with a positive ghost number (such field with negative ghost number would be anti-ghost). We ensure that these sources are auxiliary and external by choosing `SelfConjugate -> False`.

The problem arises when it comes to defining BRST sources of neutrinos. Fermionic sources must be defined as a ghost field-like class types that take into account the spinorial nature, which it seems that neither fermion, nor ghost field-type classes frame it correctly (attempts based on fermion and ghost field-type classes lead to unsuccessful implementation).

We also provide an example of defining the BRST transformation, χ-extended and the BRST-exact Lagrangian of the neutrinos, which, mentioned earlier, are needed to generate `FeynRules` model files:

```
(*BRST transformation of neutrinos in mass eigenstate basis*)
dbrst[F[1, j_]] :=
  ProjM*(I*e/(Sqrt[2]*sw)*Conj[UL[1, j]]*U[31]*F[2, 1] +
      I*e/(2*sw*cw)*Conj[UL[1, j]]*UL[1, i]*U[2]*F[1, i]) -
   ProjP*(I*e/(Sqrt[2]*sw)*UL[1, j]*U[32]*CC[F[2, 1]] +
      I*e/(2*sw*cw)*UL[1, j]*Conj[UL[1, i]]*U[2]*F[1, i]);

(*BRST-exact Lagrangian term involving neutrinos*)
LbrstExact :=
  anti[U[101]][j]*dbrst[F[1, j]] + HC[anti[U[101]][j]*dbrst[F[1,
     j]]];

(*Chi-extended Lagrangian*)
Lchi := -U[104]/(2*GaugeXi[Z])*
    anti[U[2]]*(del[V[2][mu]] + GaugeXi[Z]*MZ*S[21]) -
   U[105]/(2*
       GaugeXi[W])*(anti[
        U[31]]*(del[anti[V[3]][mu]] - I*GaugeXi[W]*MW*anti[S
           [22]]) +
```

```
    anti[U[32]]*(del[V[3][mu]] + I*GaugeXi[W]*MW*S[22]));

(*Combine everything*)
Ltotal = LbrstExact + Lchi ;
}
```

Code 2. Example of BRST transformation and the BRST Lagriangan definition

where `ProjP` and `ProjM` are chiral projectors $P_{L,R}$, `F[...]` is a fermion class and `U[...]` being BRST sources and ghost fields — we keep the fermionic BRST source classified as a ghost field-type source for the sake of example; We define BRST transformation of the neutrinos `dbrst[F[1, j]]`, as shown in eq. (4.2.8), as well as `Ltotal`, which is a sum of Lagrangians in eq. (4.2.9) and (5.3).

At this point, the implementation to construct 1PI vertex functions needed to evaluate NI for the neutrino self-energy is limited to what is shown above. Mainly because the classification of fermionic BRST source being incomplete. A possible alternative would be to approach the construction of the Feynman rules of the mentioned vertex functions via `FeynArts` directly.

Conclusion

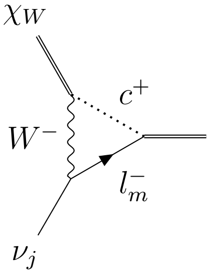
In summary, we conclude the following results from the final thesis work:

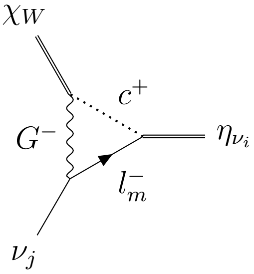
1. The topics and methods required to implement Nielsen Identities of neutrino self energy in the Grimus–Neufeld Model were studied: derivation of Nielsen Identities from the BRST invariance condition of the effective action; construction 1PI 3-point vertex functions, from the derived BRST Lagrangian, required to evaluate Nielsen Identities.

2. Nielsen Identity of the neutrino self-energy explicitly evaluated and compared to the direct computation of the neutrino self-energy dependence of gauge. The results confirm the implementation of the Nielsen Identity of the neutrino self-energy in the model.

3. The concept towards automating the Nielsen Identities. The full automatization of these identities in the Grimus–Neufeld model has not been achieved.

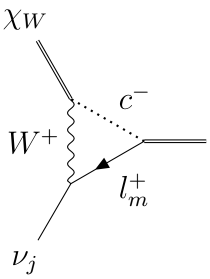
A 1PI 3-point vertex functions of ξ_W and ξ_Z dependent cases

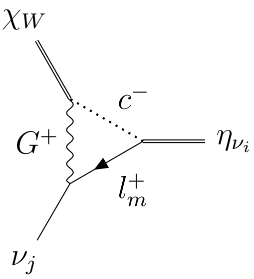
$\left(\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_W}\right)_{1,2} =$ 

$+$ 

,

$$
\begin{aligned}
\left(\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_W}\right)_{1,2} &= \int \frac{d^4k}{(2\pi)^4}\frac{g_W(U_L^\dagger)_{im}}{\sqrt{2}}P_L\frac{i(\not{p}-\not{k}+m_{l_m})}{(p-k)^2-m_{l_m}^2}\frac{-ig_W}{\sqrt{2}}\gamma^\mu(U_L)_{mj} \\
&\times P_L\frac{-i}{k^2-m_W^2}\left(g_{\mu\nu}-(1-\xi_W)\frac{k_\mu k_\nu}{k^2-\xi_W m_W^2}\right)\left(\frac{-k^\nu}{2\xi_W}\right)\frac{-i}{k^2-\xi_W m_W^2} \\
&+\int \frac{d^4k}{(2\pi)^4}\frac{g_W(U_L^\dagger)_{im}}{\sqrt{2}}P_L\frac{i(\not{p}-\not{k}+m_{l_m})}{(p-k)^2-m_{l_m}^2}\left(\frac{i\sqrt{2}}{v}P_L m_{lm}(U_L)_{mj}\right. \\
&\left.-iP_R(U_R^T y_\nu^\dagger)_{jm}\right)\times\frac{i}{k^2-\xi_W m_W^2}\left(\frac{m_W}{2}\right)\frac{-i}{k^2-\xi_W m_W^2} \\
&= \frac{g_W^2}{4}(U_L^\dagger)_{im}(U_L)_{mj}P_L\int\frac{d^4k}{(2\pi)^4}\frac{\not{p}\not{k}-\not{k}\not{k}}{\left[(p-k)^2-m_{l_m}^2\right]\left[k^2-\xi_W m_W^2\right]^2} \\
&-\frac{g_W m_W}{2\sqrt{2}}(U_L^\dagger)_{im}P_L\int\frac{d^4k}{(2\pi)^4}\frac{\frac{\sqrt{2}m_{l_m}^2}{v}(U_L)_{mj}-(\not{p}-\not{k})(U_L)_{mj}\frac{\sqrt{2}}{v}\tilde{m}_{\nu_j}}{\left[(p-k)^2-m_{l_m}^2\right]\left[k^2-\xi_W m_W^2\right]^2};
\end{aligned}
\tag{A.1}
$$

$\left(\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_W}\right)_{3,4} =$ 

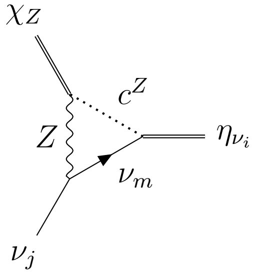
$+$ 

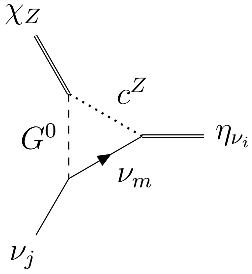
,

$$
\begin{aligned}
\left(\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_W}\right)_{3,4} &= \int \frac{d^4k}{(2\pi)^4}\frac{-g_W(U_L^T)_{im}}{\sqrt{2}}P_R\frac{i(\not{p}-\not{k}+m_{l_m})}{(p-k)^2-m_{l_m}^2}\frac{ig_W}{\sqrt{2}}(U_L^*)_{mj}\gamma^\mu P_R \\
&\times\frac{-i}{k^2-m_w^2}\left(g_{\mu\nu}-(1-\xi_w)\frac{k_\mu k_\nu}{k^2-\xi_W m_W^2}\right)\left(\frac{-k^\nu}{2\xi_w}\right)\frac{-i}{k^2-\xi_w m_w^2} \\
&+\int\frac{d^4k}{(2\pi)^4}(-P_R\frac{g_w}{\sqrt{2}}(U_L^T)_{im})\frac{i(\not{p}-\not{k}+m_{l_m})}{(p-k)^2-m_{l_m}^2}\left(\frac{i\sqrt{(}2)}{v}m_{lm}(U_L^*)_{mj}P_R\right. \\
&\left.-i(y_\nu U_R^*)_{mj}P_L\right)\frac{i}{k^2-\xi_W m_W^2}\left(-\frac{m_W}{2}\right)\frac{-i}{k^2-\xi_W m_W^2} \\
&= \frac{g_W^2}{4}(U_L^T)_{im}(U_L^*)_{mj}P_R\int\frac{d^4k}{(2\pi)^4}\frac{(\not{p}\not{k}-\not{k}\not{k})}{\left[(p-k)^2-m_{l_m}^2\right]\left[k^2-\xi_W m_W^2\right]^2}
\end{aligned}
$$

$$
-\frac{g_W m_W}{2\sqrt(2)}(U_L^T)_{im}P_R\int\frac{d^4k}{(2\pi)^4}\frac{\frac{\sqrt{2}}{v}m_{l m}^2(U_L^*)_{mj}-(\not p-\not k)(U_L^*)_{mj}\frac{\sqrt{2}}{v}\tilde{m}_{\nu_j}}{\left[(p-k)^2-m_{l_m}^2\right]\left[k^2-\xi_W m_W^2\right]^2}. \tag{A.2}
$$

As it was mentioned previously, one obtains $\Gamma^{(1)}_{\bar{\nu}_i\eta_{\bar{\nu}_j}\chi_W}$ using $\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_W}$ via a pseudo-hermitian relation.

$$\left(\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_Z}\right)_{1,2} =$$ 

$+$ 

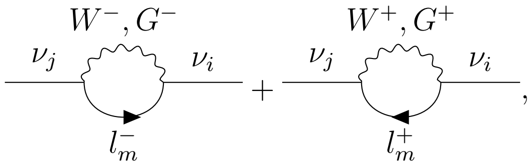
, (A.3)

$$
\begin{aligned}
\left(\Gamma^{(1)}_{\nu_j\bar{\eta}_{\nu_i}\chi_Z}\right)_{1,2} &= \int\frac{d^4k}{(2\pi)^4}\frac{g_Z}{2}\left(P_L(U_L^\dagger U_L)_{im}-P_R(U_L^T U_L^*)_{im}\right)\frac{i(\not p-\not k+m_{\nu_m})}{(p-k)^2-m_{\nu_m}^2}\\
&\times\left(\frac{i\gamma^\mu}{2}\right)g_Z\left(P_L(U_L^T U_L^*)_{mj}-P_R(U_L^\dagger U_L)_{mj}\right)\frac{-i}{k^2-m_Z^2}\\
&\times\left(g_{\mu\nu}-(1-\xi_Z)\frac{k_\mu k_\nu}{k^2-\xi_Z m_Z^2}\right)\left(-\frac{k^\nu}{2\xi_Z}\right)\frac{-i}{k^2-\xi_Z m_Z^2}\\
&+\int\frac{d^4k}{(2\pi)^4}\frac{g_Z}{2}\left(P_L(U_L^\dagger U_L)_{im}-P_R(U_L^T U_L^*)_{im}\right)\frac{i(\not p-\not k+m_{\nu_m})}{(p-k)^2-m_{\nu_m}^2}\\
&\times\frac{1}{v}\left(P_L(\mathcal{Y}_1)_{mfj}-P_R(\mathcal{Y}_2)_{mfj}\right)\frac{i}{k^2-\xi_Z m_Z^2}\left(\frac{im_Z}{2}\right)\frac{-i}{k^2-\xi_Z m_Z^2}\\
&= +\frac{g_Z^2}{4}\int\frac{d^4k}{(2\pi)^4}\frac{(P_L(U_L^\dagger U_L)_{im,mj}+P_R(U_L^\dagger U_L)_{mi,jm})(\not p\not k-\not k\not k)}{\left[(p-k)^2-m_{\nu_m}^2\right]\left[k^2-\xi_Z m_Z^2\right]^2}\\
&-\frac{g_Z^2}{4}\int\frac{d^4k}{(2\pi)^4}\frac{(P_L(U_L^\dagger U_L)_{im,jm}+P_R(U_L^\dagger U_L)_{mi,mj})\tilde{m}_{\nu_m}\not k}{\left[(p-k)^2-m_{\nu_m}^2\right]\left[k^2-\xi_Z m_Z^2\right]^2}\\
&+\frac{g_Z^2}{8}\int\frac{d^4k}{(2\pi)^4}\frac{(P_L(U_L^\dagger U_L)_{im}(\mathcal{Y}_2)_{mfj}+P_R(U_L^\dagger U_L)_{mi}(\mathcal{Y}_1)_{mfj})(\not p-\not k)}{\left[(p-k)^2-m_{\nu_m}^2\right]\left[k^2-\xi_Z m_Z^2\right]^2}\\
&-\frac{g_Z^2}{8}\int\frac{d^4k}{(2\pi)^4}\frac{(P_L(U_L^\dagger U_L)_{im}(\mathcal{Y}_1)_{mfj}+P_R(U_L^\dagger U_L)_{mi}(\mathcal{Y}_2)_{mfj})\tilde{m}_{\nu_m}}{\left[(p-k)^2-m_{\nu_m}^2\right]\left[k^2-\xi_Z m_Z^2\right]^2}.
\end{aligned}\tag{A.4}
$$

B ξ_W-dependent part of the neutrino self-energy

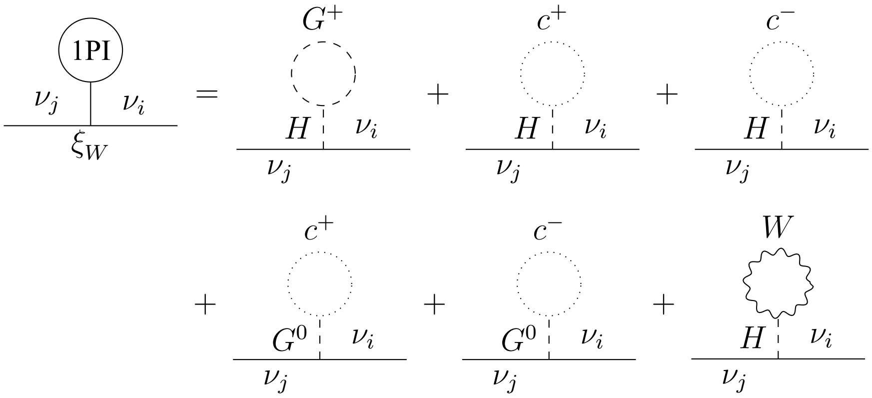
Explicit derivation of the first four Feynman diagrams, contributing to the neutrino self-energy, that depends on gauge parameter ξ_W:

$$\Sigma_{ji}(\not{p}, \xi_W) \ni \tag{B.1}$$



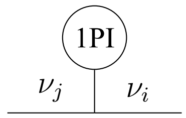
$$\begin{aligned}
\Sigma_{ji}(\not{p}, \xi_W) \ni & \int \frac{d^4k}{(2\pi)^4} \left(-\frac{i g_W^2 \gamma^\nu P_L (U_L^*)_{mi}}{\sqrt{2}} \right) \frac{i(\not{p} - \not{k} + m_{lm})}{(p-k)^2 - m_{lm}^2} \\
& \times \left(\frac{i g_W^2 \gamma^\mu P_L (U_L)_{mj}}{\sqrt{2}} \right) \frac{-i}{k^2 - m_W^2} \left(g_{\mu\nu} - (1-\xi_W) \frac{k_\mu k_\nu}{k^2 - \xi_W m_W^2} \right) \\
& + \int \frac{d^4k}{(2\pi)^4} \left(-\frac{i g_W^2 P_L \gamma^\nu (U_L)_{mi}}{\sqrt{2}} \right) \frac{i(\not{p} - \not{k} + m_{lm})}{(p-k)^2 - m_{lm}^2} \\
& \times \left(\frac{i g_W^2 P_L \gamma^\mu (U_L^*)_{mj}}{\sqrt{2}} \right) \frac{-i}{k^2 - m_W^2} \left(g_{\mu\nu} - (1-\xi_W) \frac{k_\mu k_\nu}{k^2 - \xi_W m_W^2} \right) \\
& + \int \frac{d^4k}{(2\pi)^4} i \left(P_L \frac{\sqrt{2}}{v} \tilde{m}_{\nu_i} (U_L^\dagger)_{im} - P_R \frac{\sqrt{2}}{v} m_{lm} (U_L^\dagger)_{im} \right) \frac{i(\not{p} - \not{k} + m_{lm})}{(p-k)^2 - m_{lm}^2} \\
& \times i \left(P_L \frac{\sqrt{2}}{v} m_{l_m} (U_L)_{mj} - P_R \frac{\sqrt{2}}{v} \tilde{m}_{\nu_j} (U_L)_{mj} \right) \frac{i}{k^2 - \xi_W m_W^2} \\
& + \int \frac{d^4k}{(2\pi)^4} i \left(-P_R \frac{\sqrt{2}}{v} \tilde{m}_{\nu_i} (U_L)_{im} + P_L \frac{\sqrt{2}}{v} m_{lm} (U_L)_{im} \right) \frac{i(\not{p} - \not{k} + m_{lm})}{(p-k)^2 - m_{lm}^2} \\
& \times i \left(-P_R \frac{\sqrt{2}}{v} m_{l_m} (U_L^\dagger)_{jm} + P_L \frac{\sqrt{2}}{v} \tilde{m}_{\nu_j} (U_L^\dagger)_{jm} \right) \frac{i}{k^2 - \xi_W m_W^2} \\
& = - \int \frac{d^4k}{(2\pi)^4} \frac{g_W^2}{2} (U_L^\dagger)_{im} (U_L)_{mj} \frac{P_L(\not{p} - \not{k})(4(k^2 - \xi_W m_W^2) - (1 - \xi_W \not{k}\not{k}))}{[(p-k)^2 - m_{lm}^2]\,[k^2 - m_W^2]\,[k^2 - \xi_W m_W^2]} \\
& + \frac{2}{v^2} (U_L^\dagger)_{im} (U_L)_{mj} \int \frac{d^4k}{(2\pi)^4} \frac{P_L \tilde{m}_{\nu_i} (-\tilde{m}_{\nu_j}(\not{p} - \not{k}) + m_{l_m}^2) + P_R m_{lm}^2 (\tilde{m}_{\nu_j} - (\not{p} - \not{k}))}{\left[(p-k)^2 - m_{l_m}^2\right]\,[k^2 - \xi_W m_W^2]} \\
& - \int \frac{d^4k}{(2\pi)^4} \frac{g_W^2}{2} (U_L^\dagger)_{jm} (U_L)_{mi} \frac{P_R(\not{p} - \not{k})(4(k^2 - \xi_W m_W^2) - (1 - \xi_W \not{k}\not{k}))}{[(p-k)^2 - m_{lm}^2]\,[k^2 - m_W^2]\,[k^2 - \xi_W m_W^2]} \\
& + \frac{2}{v^2} (U_L^\dagger)_{jm} (U_L)_{mi} \int \frac{d^4k}{(2\pi)^4} \frac{P_R \tilde{m}_{\nu_i} (-\tilde{m}_{\nu_j}(\not{p} - \not{k}) + m_{l_m}^2) + P_L m_{lm}^2 (\tilde{m}_{\nu_j} - (\not{p} - \not{k}))}{\left[(p-k)^2 - m_{l_m}^2\right]\,[k^2 - \xi_W m_W^2]} ;
\end{aligned} \tag{B.2}$$

Tadpole Feynman diagrams, that depend on gauge parameter ξ_W, are shown below:



(B.3)

Taking the derivative with respect to the gauge parameter, we get the following gauge dependence of tadpoles in the PaVe basis:



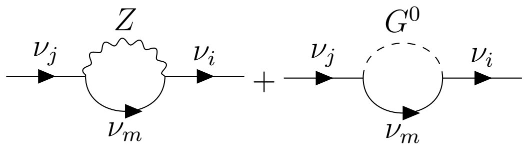
$$\partial_{\xi_W}\left(\cdots \right)_{\xi_W} = \Big(-\tau\frac{g_W^2}{4}(U_L^\dagger)_{im}(U_L)_{mj}(P_L\tilde{m}_{\nu_i}+P_R\tilde{m}_{\nu_j}) \\ -\tau\frac{g_W^2}{4}(U_L^\dagger)_{jm}(U_L)_{mi}(P_L\tilde{m}_{\nu_j}+P_R\tilde{m}_{\nu_i})\Big) \\ \times B_0(0,\xi_W m_W^2,\xi_W m_W^2). \tag{B.4}$$

C ξ_Z-dependent neutrino self-energy

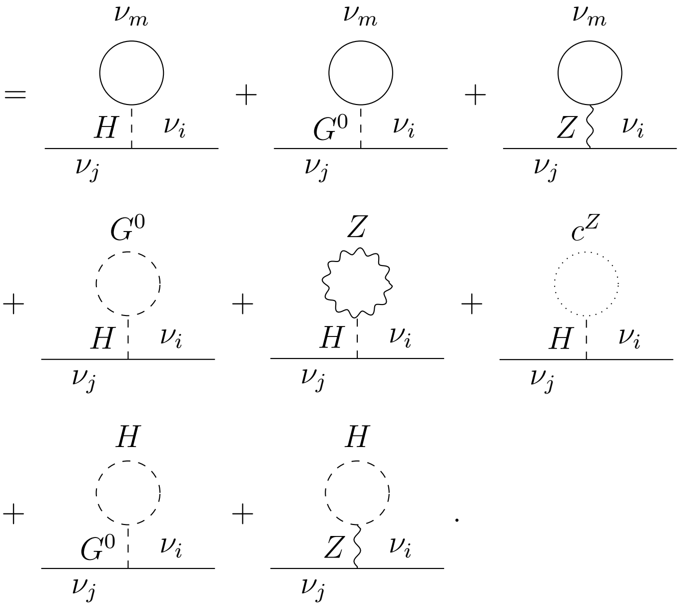
For convenience, we decompose the ξ_Z-dependent neutrino self-energy,

$$\begin{aligned}\partial_{\xi_Z}\Sigma_{ji}(\not{p},\xi_Z) &= \partial_{\xi_Z}(\Sigma_{ji}(\not{p},\xi_Z)_1 + \Sigma_{ji}(\not{p},\xi_Z)_2 + \Sigma_{ji}(\not{p},\xi_Z)_3) \\ &= \partial_{\xi_Z}\Sigma_{ji}(\not{p},\xi_Z)_1 + \partial_{\xi_Z}C(\Sigma_{ji}(\not{p},\xi_Z)_1)^T C^{-1} + \partial_{\xi_Z}\Sigma_{ji}(\not{p},\xi_Z)_3,\end{aligned} \tag{C.1}$$

where

$\Sigma_{ji}(\not{p},\xi_Z)_1 =$  (C.2)

and

$\Sigma_{ji}(\not{p},\xi_Z)_3 =$  (C.3)

$\partial_{\xi_Z}\Sigma_{ji}(\not{p},\xi_Z)_1$ and $\partial_{\xi_Z}\Sigma_{ji}(\not{p},\xi_Z)_3$ expressed in PaVe basis as shown below:

$$\begin{aligned}\partial_{\xi_Z}\Sigma_{ji}(\not{p},\xi_Z)_1 = \tau\Bigg(&\frac{g_Z^2}{8}(U_L^\dagger U_L)_{jm,mi}P_L\left(\not{p} - \frac{\not{p}\tilde{m}_{\nu_m}}{p^2}\right) \\ &+\frac{g_Z^2}{8}\frac{P_L\not{p}}{p^2}\left((U_L^\dagger U_L)_{jm,im}\tilde{m}_{\nu_m}\tilde{m}_{\nu_i} + (U_L^\dagger U_L)_{jm,mi}\tilde{m}_{\nu_m}^2\right) \\ &+\frac{g_Z^2}{8}\frac{P_L\not{p}}{p^2}\left((U_L^\dagger U_L)_{im,mj}\tilde{m}_{\nu_i}\tilde{m}_{\nu_j} + (U_L^\dagger U_L)_{mi,mj}\tilde{m}_{\nu_j}\tilde{m}_{\nu_m}\right) \\ &+(P_L \leftrightarrow P_R, j \leftrightarrow i)\Bigg) \times B_0(0,\xi_Z m_Z^2,\xi_Z m_Z^2) \\ +\tau\Bigg(&\frac{g_Z^2}{8}\left(\not{p} + \frac{\not{p}\tilde{m}_{\nu_m}^2}{p^2}\right)P_L(U_L^\dagger U_L)_{im,mj} \\ &-\frac{g_Z^2}{8}\frac{P_R\not{p}}{p^2}\tilde{m}_{\nu_m}\tilde{m}_{\nu_i}(U_L^\dagger U_L)_{mj,mi}\end{aligned}$$

$$
\begin{aligned}
&-\frac{g_Z^2}{8}\frac{P_L\not{p}}{p^2}\left((U_L^\dagger U_L)_{jm,im}\tilde{m}_{\nu_m}\tilde{m}_{\nu_i}+(U_L^\dagger U_L)_{jm,mi}\tilde{m}_{\nu_m}^2\right)\\
&-\frac{g_Z^2}{8}\frac{P_R\not{p}}{p^2}(U_L^\dagger U_L)_{jm,mi}\tilde{m}_{\nu_j}\tilde{m}_{\nu_i}\\
&+(P_L\leftrightarrow P_R, j\leftrightarrow i)\\
&+\frac{g_Z^2}{4}\tilde{m}_{\nu_m}\left(P_L(U_L^\dagger U_L)_{mi,mj}+P_R(U_L^\dagger U_L)_{im,jm}\right)\Bigg)\times B_0(p^2,\tilde{m}_{\nu_m},\xi_Z m_Z^2)\\
+\tau\Bigg(&-\frac{g_Z^2}{8}\frac{P_L\not{p}}{p^2}\left(\tilde{m}_{\nu_m}^2(\tilde{m}_{\nu_m}^2-\xi_Z m_Z^2-2p^2)+p^2(p^2-\xi_Z m_Z^2)\right)(U_L^\dagger U_L)_{jm,mi}\\
&+\frac{g_Z^2}{8}\frac{\not{p}}{p^2}\left(\tilde{m}_{\nu_m}^2-\xi_Z m_Z^2+p^2\right)\tilde{m}_{\nu_m}\tilde{m}_{\nu_i}(P_L(U_L^\dagger U_L)_{mj,mi}+P_R(U_L^\dagger U_L)_{jm,im})\\
&+\frac{g_Z^2}{8}\frac{\not{p}}{p^2}\left(\tilde{m}_{\nu_m}^2-\xi_Z m_Z^2+p^2\right)(\tilde{m}_{\nu_j}\tilde{m}_{\nu_i}P_L(U_L^\dagger U_L)_{jm,mi}+\tilde{m}_{\nu_m}^2P_R(U_L^\dagger U_L)_{jm,mi})\\
&(P_L\leftrightarrow P_R, j\leftrightarrow i)\\
&+\frac{g_Z^2}{4}\left(\xi_Z m_Z^2\tilde{m}_{\nu_m}-\tilde{m}_{\nu_m}^3\right)\left(P_L(U_L^\dagger U_L)_{mi,mj}+P_R(U_L^\dagger U_L)_{im,jm}\right)\\
&-\frac{g_Z^2}{4}\tilde{m}_{\nu_m}^2\tilde{m}_{\nu_i}\left(P_L(U_L^\dagger U_L)_{im,mj}+P_R(U_L^\dagger U_L)_{jm,mi}\right)\\
&-\frac{g_Z^2}{4}\tilde{m}_{\nu_m}^2\tilde{m}_{\nu_j}\left(P_L(U_L^\dagger U_L)_{jm,mi}+P_R(U_L^\dagger U_L)_{im,mj}\right)\\
&-\frac{g_Z^2}{4}\tilde{m}_{\nu_m}\tilde{m}_{\nu_j}\tilde{m}_{\nu_i}\left(P_L(U_L^\dagger U_L)_{im,jm}+P_R(U_L^\dagger U_L)_{mi,mj}\right)\Bigg)\\
\times\, & C(0,p^2,p^2,\xi_Z m_Z^2,\xi_Z m_Z^2,\tilde{m}_{\nu_m}),
\end{aligned}
\tag{C.4}
$$

$$
\begin{aligned}
\partial_{\xi_Z}\Sigma_{ji}(\not{p},\xi_Z)_3 = -\tau\Bigg(&\frac{g_W^2}{4}(U_L^\dagger)_{im}(U_L)_{mj}(P_L\tilde{m}_{\nu_i}+P_R\tilde{m}_{\nu_j})\\
&\frac{g_W^2}{4}(U_L^\dagger)_{jm}(U_L)_{mi}(P_L\tilde{m}_{\nu_j}+P_R\tilde{m}_{\nu_i})\Bigg)\\
\times\, & B_0(0,\xi_W m_W^2,\xi_W m_W^2).
\end{aligned}
\tag{C.5}
$$

D PaVe reduction method

The amplitudes, as those derived and calculated in sections 5.1 and 5.2, are of the tensorial integral form. One of the methods to evaluate such integrals faster (especially using mathematical software) is the Passarino-Veltman (PaVe) reduction scheme — reducing complex tensorial integrals in terms of scalar integrals.

In general, integrals of the form

$$T^N_{\mu\nu\ldots\rho} = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \mathrm{d}^D k \frac{q_\mu \ldots q_\rho}{D_0 \ldots D_{N-1}}, \tag{D.1}$$

(where D is the dimension of the integral, μ is the parameter of the mass dimension; $D_0 = q^2 - m_0$, $D_1 = (q + p_1)^2 - m_1$ etc.), are defined in PaVe notation based on momentum in the numerator and terms of the denominator:

$$A_0(m_0^2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \mathrm{d}^D q \frac{1}{D_0}, \tag{D.2}$$

$$B_0; B_\mu; B_{\mu\nu}; B_{\mu\nu\lambda}(p_1, m_0^2, m_1^2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \mathrm{d}^D q \frac{1; q_\mu; q_\mu q_\nu; q_\mu q_\nu q_\lambda}{D_0 D_1}, \tag{D.3}$$

$$C_0; C_\mu; C_{\mu\nu}; C_{\mu\nu\lambda}(p_1, p_2, m_0^2, m_1^2, m_2^2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \mathrm{d}^D q \frac{1; q_\mu; q_\mu q_\nu; q_\mu q_\nu q_\lambda}{D_0 D_1 D_2} \tag{D.4}$$

and so on. Then, tensorial integrals such as B_μ, C_μ, $B_{\mu\nu}$ and $C_{\mu\nu}$ can be reduced via tenzor basis as follows:

$$B_\mu = p_{1\mu} B_1, \tag{D.5}$$

$$B_{\mu\nu} = g_{\mu\nu} B_{00} + p_{1\mu} p_{1\nu} B_{11}, \tag{D.6}$$

$$C_\mu = p_{1\mu} C_1 + p_{2\nu} C_2, \tag{D.7}$$

$$C_{\mu\nu} = g_{\mu\nu} C_{00} + \sum_{i,j=1}^{2} p_{i\mu} p_{j\nu} C_{ij}, \tag{D.8}$$

where B_{00}, B_{11}, C_1, C_{00} etc. are expressed in terms of scalar integrals A_0, B_0 etc. by contracting decomposed tensorial integrals with tensors, such as metric $g^{\mu\nu}$, p^μ or p^ν.

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Nielsen tapatybių automatizavimas Grimus–Neufeld Modeliui

Nojus Danyla

Santrauka

Šito darbo tikslas buvo patvirtinti Nielsen tapatybės pritaikymą bei jas automatizuoti Grimus–Neufeld Modelyje.

Apibrežimai ir sąvokos, BRST formalizmo bei Grimus–Neufeld Modelio supratimui, yra aptariami pirmoje baigiamojo darbo dalyje. Skyriuose 1 ir 2 yra aptariamas Faddeev–Popov metodas, kuris lydi į BRST simetriją bei formalizmą. Taip pat yra aptariamos modifikuotas Ward–Takashi tapatybės neabelinėse teorijose; esant BRST simetrijai, parodoma kaip išvedimos Lee tapatybės, iš kurių taip pat galima išvesti Nielsen tapatybes (šios tapatybės išvedamos kvantinės elektrodinamikos (angl. quantum electrodynamics) ir kvantinės chromodinamikos (angl. quantum chromodynamics) teorijose kaip pavyzdžiai).

Literatūros apžvalgos pabaigoje, t.y. 3 skyriuje, aptariami neutrinai, kaip yra sugeneruojamos jų masės, sverto (angl. seesaw) mechanizmą bei Higgs mechanizmą, t.y. kalibruotės bozonų masės generavimą per spontanišką simetrijos lūžį. Taip pat aptariamas Grimus–Neufeld modelis — Standartinio modelio papildinys, skirtas paiškinti mažas neutrinų mases, įvedant papildomą Higgs dubletą bei dešinio chirališkumo Majorana neutriną.

4 skyriuje išvedama skaliarinių laukų h, H, pseudo-skaliarinio lauko A bei neutrinų BRST transformacijos masės tikrinių verčių bazėje, siekiant sukonstruoti tikslios BRST formos Lagranžianą, kuris yra reikalingas Nielsen tapatybių skaičiavimui Grimus-Neufeld Modelyje.

Paskutiniojoje darbo dalyje, išreikštai išvedame bei apskaičiuojame Nielsen tapatybę neutrinų savąjai energijai bei išreikškiame rezultatą per Passarino-Veltman funkcijų bazę. Taip pat tiesiogiai apskaičiuojame neutrinų savosios energijos priklausomybę nuo kalibruotes bei palyginame rezultatus su Nielsen tapatybės gauta išraiška. Rezultatų sutapimas tarp šių nepriklausomų metodų patvirtina, jog neutrinų savosios energijos Nielsen tapatybės implementacija Grimus–Neufeld Modelyje buvo sėkminga.

Taip pat nagrinėjama Nielsen tapatybių automatizacija Grimus-Neufeld modelyje. Yra aptariamas pradinis planas kaip automatizuoti 3-taškų funkcijas, pasinaudojus `FeynRules` paketu, kurios yra reikalingos Nielsen tapatybės neutrinų savąjai energijai apskaičiuoti: apibrėžiamos skaliarinių laukų bei kalibruotės parametro BRST srovių laukų klasifikacija, aptariama problema dėl fermioninių laukų BRST srovių apibrėžimo; taip pat apžvelgiamas pavyzdys kaip apibrėžti neutrino lauko BRST transformaciją bei tikslios BRST formos Lagranžianas, kuris yra reikalingas Feynman taisyklių generavimui minėtoms 3-taškų funkcijoms. Nielsen tapatybių automatizavimas Grimus–Neufeld modelyje nebuvo pasiektas.