

Vilniaus universitetas  
Fizikos fakultetas  
Teorinės fizikos ir astronomijos institutas

Dominykas Malinauskas

**YUKAWA SEKTORIAUS RENORMALIZACIJA GRIMUS–NEUFELD MODELyje**

MAGISTRO STUDIJŲ BAIGIAMASIS DARBAS

Teorinės fizikos ir astrofizikos studijų programa

|                       |                               |
|-----------------------|-------------------------------|
| Studentas             | Dominykas Malinauskas         |
| Leista ginti          | 2026-05-19                    |
| Darbo vadovas         | dr. Simonas Draukšas          |
| Konsultantas          | doc. dr. Thomas Gajdosik      |
| Instituto direktorius | prof. dr. Egidijus Anisimovas |

Vilnius 2026

Vilnius University  
Faculty of Physics  
Institute of Theoretical Physics and Astronomy

Dominykas Malinauskas

**YUKAWA SECTOR RENORMALIZATION IN THE GRIMUS–NEUFELD MODEL**

MASTER’S THESIS

Theoretical Physics and Astrophysics study program

|                    |                                  |
|--------------------|----------------------------------|
| Student            | Dominykas Malinauskas            |
| Approved           | 2026-05-19                       |
| Thesis Advisor     | dr. Simonas Draukšas             |
| Consultant         | assoc. prof. dr. Thomas Gajdosik |
| Institute director | prof. dr. Egidijus Anisimovas    |

Vilnius 2026

# Contents

|                                                                                        |           |
|----------------------------------------------------------------------------------------|-----------|
| <b>Introduction</b>                                                                    | <b>4</b>  |
| <b>1 Theoretical framework</b>                                                         | <b>6</b>  |
| 1.1 The Standard Model . . . . .                                                       | 6         |
| 1.2 Symmetry breaking . . . . .                                                        | 6         |
| 1.2.1 Spontaneous Symmetry Breaking . . . . .                                          | 6         |
| 1.2.2 Electroweak Symmetry Breaking . . . . .                                          | 8         |
| 1.3 The Seesaw Mechanism . . . . .                                                     | 10        |
| 1.4 The Grimus–Neufeld Model . . . . .                                                 | 12        |
| 1.4.1 The Gauge–Higgs Sector . . . . .                                                 | 13        |
| 1.4.2 The Fermion–Gauge Sector . . . . .                                               | 14        |
| 1.4.3 The Fermion–Higgs Sector . . . . .                                               | 15        |
| 1.4.4 The Fermion–Majorana Sector . . . . .                                            | 17        |
| 1.4.5 Seesaw Mechanism in the Grimus–Neufeld Model . . . . .                           | 18        |
| 1.5 Renormalization . . . . .                                                          | 18        |
| 1.5.1 Concept of Renormalization . . . . .                                             | 19        |
| 1.5.2 Example of Renormalization . . . . .                                             | 21        |
| <b>2 Yukawa sector counterterm Feynman rules</b>                                       | <b>26</b> |
| <b>3 On-Shell renormalization and S.Draukšas scheme</b>                                | <b>31</b> |
| <b>4 Renormalization of two-point functions in the quark sector</b>                    | <b>34</b> |
| <b>5 Renormalization of Higgs two-point function</b>                                   | <b>37</b> |
| <b>6 Yukawa counterterm</b>                                                            | <b>38</b> |
| <b>Conclusion</b>                                                                      | <b>42</b> |
| <b>A Wick rotations</b>                                                                | <b>43</b> |
| <b>B Dimensional regularization</b>                                                    | <b>44</b> |
| <b>C Explicit computational results for the Yukawa counterterm with up-type quarks</b> | <b>46</b> |
| <b>References</b>                                                                      | <b>53</b> |
| <b>Santrauka lietuvių kalba</b>                                                        | <b>55</b> |

# Introduction

The study of fundamental particles and their interactions is deeply rooted in the framework of quantum field theory (QFT), of which the Standard Model (SM) of particle physics is the best model so far. It has correctly predicted the physical properties of many particles, the most recent example being the Higgs boson [1]. Despite its success, the SM does not address several phenomena, including the origins of neutrino masses [2], the existence of dark matter, gravity, etc. Thus particle physicists have been constructing extensions for the Standard Model to try to explain the discrepancies. This thesis delves into one such extension of the Standard Model—the Grimus–Neufeld Model (GNM) [3]. The Grimus–Neufeld Model adds an additional Higgs doublet, which is why it is considered a two Higgs doublet model (2HDM), and a Majorana fermion [4] is used to explain the light neutrino masses.

For a model to be physically applicable, its predictions must be compared with experimental data. This means the model should predict physical observables. However, because QFT is done perturbatively, when computing the amplitudes of the particle processes at higher orders, the results diverge, giving non-physical infinities. This happens because the model attempts to describe processes beyond its range of validity, thus affecting the physical quantities and yielding numerical results that are not consistent with experimental data. This is an issue in both SM and GNM. Thus we need to apply a procedure that deals with such divergences and redefines our parameters as physical ones, such a procedure is called renormalization. So to use a model effectively, we need to renormalize it.

In this thesis we deal with the renormalization of the Yukawa sector in GNM, which includes the Yukawa interactions between the fermions and the Higgs doublets. Specifically, we want to renormalize the Yukawa constants that come from the Higgs doublet. However, we will work with an extremely simplified version of GNM. Specifically, we will work only with the SM part of it, not considering the effects of the second Higgs doublet or the Majorana fermion. However, in the thesis we will use the Simonas Draukšas [5] (SD) scheme, which is a modified on-shell renormalization scheme and was constructed with GNM in mind. Therefore the main objective of this thesis is to have a first check if the SD scheme can be used for Yukawa constant renormalization.

The hypothesis is that, by renormalizing the Higgs–fermion vertex and fixing particle propagator counterterms it consist of, we can derive the Yukawa constant counterterm. This because the Yukawa sector gives us the interactions of Higgs boson and fermions, by deriving the Higgs–fermion vertex, also called the three-point function, counterterms, we can see that it is a combination of the fermion and Higgs propagator, also called two-point functions, counterterms and the Yukawa counterterm. This is where the SD scheme is used. The scheme has been already used for the renormalization of two-point functions [5–7] in GNM. As seen in the derived three-point function counterterms, the vertex can be renormalized by two-point function field counterterms and the Yukawa counterterm. Thus we get the Yukawa counterterm by computing the higher order perturbations, otherwise known as one-loop corrections, of the interactions and using, specifically, the field counterterms of the two-point functions to deal with the divergences present. To check the validity of the Yukawa counterterm, we then use it’s connection to the two-

point function mass counterterms. This is supported by the connection of the Yukawa constants to the fermion masses via the Dirac mass term in the Lagrangian, due to Spontaneous Symmetry breaking (SSB) and the Higgs mechanism, giving that:

$$m_{ji} = \frac{Y_{ji}v}{\sqrt{2}}.$$

where  $m_f$  is the fermion mass,  $Y_{hf_j\bar{f}_i}$  is it's respective Yukawa constant from the Higgs doublet and  $v$  is the vacuum expectation value.

This thesis is structured as follows: In section 1 we give a detailed overview of the theoretical foundation needed for the thesis; In section 2 we derive the counterterm Feynman rules for the Higgs–fermion vertex; In section 3 we describe how the On-shell renormalization scheme is structured and describe the SD scheme; In section 4 by using the SD scheme we derived the counterterms for the fermion propagator; In section 5 we derived the counterterms for the Higgs propagator; In section 6 we discuss the Yukawa counterterm.

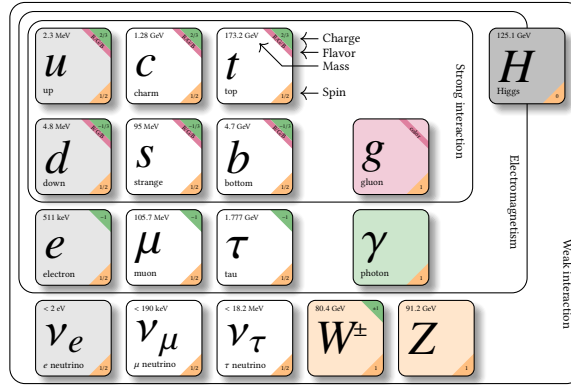
# 1 Theoretical framework

## 1.1 The Standard Model

As stated in the introduction section of this thesis, the SM is the established model on which particle physics rests. Thus before delving into it's extensions or any additions, it is necessary to give a quick overview of how is it structured.

First, the SM is a gauge theory that has  $SU(3) \times SU(2) \times U(1)$  gauge symmetry. The  $SU(3)$  symmetry governs strong interactions, which are important for the interactions of quarks and gluons. It has 8 group generators that correspond to 8 gluons that carry colour charge, which affects the quarks and allows them to change their colour as well as is responsible for keeping the quarks together within hadrons. Thus giving way for protons, neutrons, and other hadrons to form. The  $SU(2)$  symmetry governs weak interactions, which are responsible for the existence of beta decay as well as the interactions of neutrinos with other leptons, which are important for processes like neutrino oscillations. It has 3 group generators that, in tandem with  $U(1)_Y$ , which governs hypercharge interactions, give rise to the 3 weak bosons,  $W^+$ ,  $W^-$  and  $Z^0$ , and the photon  $\gamma$ . Together they are called the electroweak (EW) sector. After the process called Spontaneous Symmetry Breaking (SSB), which we will discuss in section 1.2.1, the generators from both symmetries form the before mentioned bosons. Also the  $SU(2) \times U(1)_Y$  symmetry breaks down to  $U(1)_{EM}$ , which governs the electromagnetic force [8].

Schematically the field content of the Standard model is depicted in Figure 1.



1 pav. Field (particle) content of the Standard Model.

## 1.2 Symmetry breaking

A full grasp of SM cannot be achieved without discussing Spontaneous Symmetry Breaking and the Higgs mechanism. They are needed to explain how the gauge bosons acquire their masses. We will discuss the general formalism, but specifically go in depth talking about electroweak symmetry breaking that happens in SM. This section is adapted from [9–11]

### 1.2.1 Spontaneous Symmetry Breaking

Symmetry breaking can generally be divided into two categories:

- Explicit symmetry breaking (ESB);
- Spontaneous symmetry breaking (SSB).

In ESB, the symmetry is broken by terms in the Lagrangian or Hamiltonian that are not invariant under the symmetry transformation. An example of this would be adding a disturbance term to an equation that governs the movement of air particles in a room, e.g., adding an effect of a ceiling fan. In contrast, in SSB the Lagrangian remains invariant under the symmetry, while the vacuum state does not.

A key consequence of spontaneous breaking of a continuous global symmetry is the appearance of massless scalar excitations known as Goldstone bosons. Such modes do not arise for broken discrete symmetries, since discrete symmetries do not generate continuous directions in field space.

The way that Goldstone bosons appear is defined by the Goldstone theorem. To derive the theorem, consider a set of fields  $\phi_i$  with a potential  $V(\phi_i)$  invariant under a continuous symmetry transformation. For an infinitesimal transformation [9],

$$\phi_i \rightarrow \phi_i + \delta\phi_i = \phi_i + i\theta^a (T^a)_{ij} \phi_j, \quad (1.1)$$

where  $T^a$  are the generators of the symmetry group and  $\theta^a$  are infinitesimal transformation parameters.

Invariance of the potential implies

$$V(\phi_i + \delta\phi_i) = V(\phi_i). \quad (1.2)$$

Expanding to first order gives

$$\frac{\partial V}{\partial \phi_i} \delta\phi_i = 0. \quad (1.3)$$

Differentiating with respect to  $\phi_j$  yields

$$\frac{\partial^2 V}{\partial \phi_j \partial \phi_i} \delta\phi_i + \frac{\partial V}{\partial \phi_i} \frac{\partial(\delta\phi_i)}{\partial \phi_j} = 0. \quad (1.4)$$

After spontaneous symmetry breaking, the fields are expanded around the vacuum expectation value (VEV),

$$\phi_i(x) = v_i + \chi_i(x), \quad (1.5)$$

where  $v_i$  minimises the potential and  $\chi_i(x)$  describes fluctuations around the vacuum. At the minimum,

$$\left. \frac{\partial V}{\partial \phi_i} \right|_{\phi=v} = 0, \quad (1.6)$$

so the second term in Eq. (1.4) vanishes. Defining the mass matrix by

$$(M^2)_{ij} \equiv \left. \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \right|_{\phi=v_i}, \quad (1.7)$$

we obtain

$$(M^2)_{ij} \delta v_j = 0, \quad (1.8)$$

where

$$\delta v_i = i\theta^a (T^a)_{ij} v_j. \quad (1.9)$$

If the vacuum is invariant under a generator  $T^a$ , then  $\delta v_i = 0$  for all  $i$  and the symmetry remains unbroken. For broken generators,  $\delta v$  is non-vanishing and therefore constitutes a zero-eigenvalue eigenvector of the mass matrix. Therefore, each broken generator of a continuous symmetry gives rise to a massless mode, namely a Goldstone boson.

An important example is the electroweak sector of the Standard Model, where the symmetry  $SU(2) \times U(1)_Y$  is spontaneously broken to  $U(1)_{EM}$ . The original symmetry group has four generators, while the unbroken electromagnetic subgroup has one. Therefore, three generators are broken, which would correspond to three Goldstone bosons according to Goldstone's theorem. Since the electroweak symmetry is local rather than global, these Goldstone modes are absorbed by the gauge fields through the Higgs mechanism, becoming the longitudinal degrees of freedom of the  $W^\pm$  and  $Z$  bosons.

### 1.2.2 Electroweak Symmetry Breaking

As discussed in the previous section, electroweak symmetry breaking occurs when the  $SU(2)_L \times U(1)_Y$  gauge symmetry is spontaneously broken to the electromagnetic symmetry  $U(1)_{EM}$ . This mechanism explains how the weak gauge bosons acquire mass while the photon remains massless. The symmetry breaking is generated by the vacuum expectation value (VEV) of the Higgs field.

The Higgs doublet is a complex scalar field transforming as a doublet under  $SU(2)_L$  with weak hypercharge  $y = 1/2$ ,

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \quad (1.10)$$

where  $H^+$  and  $H^0$  are complex scalar fields carrying electric charge +1 and 0, respectively. Under a gauge transformation with  $SU(2)_L$  parameters  $\alpha^a(x)$  and  $U(1)_Y$  parameter  $\beta(x)$ , the doublet transforms as

$$H \rightarrow e^{i\alpha^a(x)T^a} e^{i\beta(x)y} H, \quad (1.11)$$

where  $T^a = \sigma^a/2$  are the  $SU(2)_L$  generators with  $\sigma^a$  the Pauli matrices.

The electroweak sector of the Standard Model is described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}(W_{\mu\nu}^a)^2 - \frac{1}{4}B_{\mu\nu}^2 + (D_\mu H)^\dagger (D^\mu H) + m^2(H^\dagger H) - \lambda(H^\dagger H)^2, \quad (1.12)$$

where  $m^2$  is the mass parameter of the Higgs field,  $\lambda > 0$  is the quartic self-coupling constant,  $B_{\mu\nu}$  is the  $U(1)_Y$  field strength tensor

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (1.13)$$

and  $W_{\mu\nu}^a$  are the  $SU(2)_L$  field strength tensors. The covariant derivative acting on the Higgs doublet is

$$D_\mu = \partial_\mu - igW_\mu^a\tau^a - ig'yB_\mu, \quad (1.14)$$

where  $g$  and  $g'$  are the  $SU(2)_L$  and  $U(1)_Y$  gauge couplings, respectively,  $\tau^a = \sigma^a/2$ , and  $y$  is the hypercharge of the field on which  $D_\mu$  acts.

The Higgs potential,

$$V(H) = -m^2(H^\dagger H) + \lambda(H^\dagger H)^2, \quad (1.15)$$

has a non-trivial minimum at  $H^\dagger H = v^2/2$ , where  $v = m/\sqrt{\lambda}$  is the VEV. This non-zero minimum triggers spontaneous symmetry breaking. Expanding the Higgs field around the VEV and using the  $SU(2)_L$  gauge freedom to work in the unitary gauge, where the Goldstone fields are set to zero, gives

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad (1.16)$$

where  $h$  is the physical Higgs field. The general expansion before fixing the gauge is

$$H = \exp\left(\frac{2i\pi^a\tau^a}{v}\right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad (1.17)$$

where  $\pi^a$  ( $a = 1, 2, 3$ ) are the three Goldstone fields. These correspond to the three broken generators of the  $SU(2)_L \times U(1)_Y$  symmetry: by Goldstone's theorem, each broken generator gives rise to a massless scalar, and since three of the four generators are broken (as discussed in Section 1.2.1), there are precisely three such fields.

Substituting Eq. (1.16) into the kinetic term  $(D_\mu H)^\dagger (D^\mu H)$  and retaining only the terms quadratic in the gauge fields yields the gauge boson mass matrix. Evaluating the covariant derivative on the VEV gives

$$(D_\mu H)^\dagger (D^\mu H) \supset \frac{g^2 v^2}{8} \left[ (W_\mu^1)^2 + (W_\mu^2)^2 + \left( W_\mu^3 - \frac{g'}{g} B_\mu \right)^2 \right]. \quad (1.18)$$

In the basis  $(W_\mu^1, W_\mu^2, W_\mu^3, B_\mu)$ , the mass-squared matrix for the neutral sector  $(W_\mu^3, B_\mu)$  is

$$\mathcal{M}^2 = \frac{v^2}{4} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}. \quad (1.19)$$

This matrix has eigenvalues

$$m_Z^2 = \frac{v^2}{4}(g^2 + g'^2), \quad m_A^2 = 0, \quad (1.20)$$

corresponding to a massive neutral boson and a massless one. The matrix is diagonalised by the

orthogonal rotation

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}, \quad (1.21)$$

where the Weinberg angle  $\theta_W$  is defined through

$$\tan \theta_W = \frac{g'}{g}, \quad (1.22)$$

so that  $\cos \theta_W = g/\sqrt{g^2 + g'^2}$  and  $\sin \theta_W = g'/\sqrt{g^2 + g'^2}$ .

In the charged sector, the mass eigenstates with electric charge  $\pm 1$  are

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left( W_\mu^1 \mp i W_\mu^2 \right), \quad (1.23)$$

with mass

$$m_W = \frac{gv}{2}. \quad (1.24)$$

The electromagnetic coupling is related to the gauge couplings by

$$e = g \sin \theta_W = g' \cos \theta_W. \quad (1.25)$$

Expanding the full Lagrangian around the VEV in unitary gauge gives the Higgs interactions

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} = & -\frac{1}{2} h(\square + m_h^2)h - \frac{gm_h^2}{4m_W} h^3 - \frac{g^2 m_h^2}{32m_W^2} h^4 \\ & + \frac{2h}{v} \left( m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \right) \\ & + \left( \frac{h}{v} \right)^2 \left( m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \right), \end{aligned} \quad (1.26)$$

where  $m_h = \sqrt{2} m$  is the Higgs boson mass and  $\square = \partial^\mu \partial_\mu$ .

To summarise the effect of electroweak symmetry breaking: the three Goldstone bosons  $\pi^a$ , which arise from the three broken generators, are not independent physical particles. In the unitary gauge they are removed from the spectrum entirely; in a general gauge they reappear as the longitudinal polarisation states of the  $W^\pm$  and  $Z$  bosons, providing these massive gauge bosons with their third (longitudinal) degree of freedom. The photon  $A_\mu$  remains massless, retaining only its two transverse polarisations. The sole remaining scalar degree of freedom is the physical Higgs boson  $h$ , with mass  $m_h$ .

### 1.3 The Seesaw Mechanism

The seesaw mechanism [12] provides an explanation for why neutrino masses are significantly smaller than those of charged leptons and quarks. In this framework, the Standard Model is extended by introducing right-handed neutrino singlets  $N_R$ , which allow both Dirac and Majorana

Majorana mass terms for neutrinos. This construction is commonly referred to as the Type-I seesaw mechanism.

A Majorana mass term for a fermion  $\psi$  takes the form  $\frac{1}{2}M\bar{\psi}^C\psi + \text{h.c.}$ , where  $\psi^C = C\bar{\psi}^T$  is the charge-conjugate field, with  $C$  the charge conjugation matrix satisfying  $C\gamma^\mu C^{-1} = -(\gamma^\mu)^T$ . Unlike a Dirac mass term, which couples two distinct Weyl spinors, a Majorana mass term couples a field to its own conjugate, identifying the particle with its antiparticle. Such a term violates lepton number by two units.

For simplicity, we consider a single generation of neutrinos; the generalisation to multiple generations is straightforward. The relevant terms in the Lagrangian are

$$-\mathcal{L}_\nu = Y_\nu \bar{\ell}_L \tilde{H} N_R + \frac{1}{2} \overline{N_R^C} M_R N_R + \text{h.c.}, \quad (1.27)$$

where the first term is the neutrino Yukawa interaction,  $Y_\nu$  is the neutrino Yukawa coupling,  $\ell_L$  is the left-handed lepton doublet, and the second term is the Majorana mass term for the right-handed singlet with mass parameter  $M_R$ . Here

$$\tilde{H} = i\sigma^2 H^* = \begin{pmatrix} (H^0)^* \\ -(H^+)^* \end{pmatrix} \quad (1.28)$$

is the conjugate Higgs doublet.

Since  $N_R$  is a singlet under the Standard Model gauge group, the Majorana mass term  $\overline{N_R^C} M_R N_R$  is gauge invariant and can therefore appear at any scale, independently of electroweak symmetry breaking. The Yukawa interaction, on the other hand, couples  $N_R$  to the Higgs doublet; evaluating the Higgs field at its VEV,  $\langle H^0 \rangle = v/\sqrt{2}$ , generates a Dirac mass term  $M_D = Y_\nu v/\sqrt{2}$ . The neutrino mass Lagrangian then takes the form

$$-\mathcal{L}_{\text{mass}} = \overline{\nu}_L M_D N_R + \frac{1}{2} \overline{N_R^C} M_R N_R + \text{h.c.} \quad (1.29)$$

It is convenient to write the mass terms in matrix form. Defining the field vector

$$n_L = \begin{pmatrix} \nu_L \\ N_R^C \end{pmatrix}, \quad (1.30)$$

the Lagrangian can be written as

$$-\mathcal{L}_{\text{mass}} = \frac{1}{2} \overline{n}_L M_\nu n_L + \text{h.c.}, \quad (1.31)$$

where the neutrino mass matrix is

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix}. \quad (1.32)$$

Diagonalising this matrix gives the mass eigenvalues

$$m_{1,2} = \frac{1}{2} \left( M_R \mp \sqrt{M_R^2 + 4M_D^2} \right). \quad (1.33)$$

In the seesaw limit  $M_R \gg M_D$ , the eigenvalues reduce approximately to

$$m_\nu \simeq \frac{M_D^2}{M_R}, \quad m_N \simeq M_R. \quad (1.34)$$

The mixing angle between the light and heavy neutrino states is approximately

$$\theta \simeq \frac{M_D}{M_R} \simeq \sqrt{\frac{m_\nu}{m_N}} \ll 1. \quad (1.35)$$

The seesaw mechanism therefore naturally produces a light neutrino state with mass  $m_\nu \ll M_D$  alongside a heavy sterile state with mass  $m_N \approx M_R$ . The name refers to the inverse relationship between the two scales: a larger Majorana mass  $M_R$  suppresses the light neutrino mass  $m_\nu$ . The Majorana nature of the mass eigenstates is a prediction of this framework that can be tested experimentally, for example through neutrinoless double beta decay [13].

## 1.4 The Grimus–Neufeld Model

The Grimus–Neufeld model [3, 5, 14] is an extension of the Standard Model in which the scalar sector is enlarged by a second Higgs doublet and the fermion sector is extended by right-handed neutrino singlets  $N_R$ . The key properties of right-handed neutrinos — their gauge singlet nature and the resulting Majorana mass term — were discussed in Section 1.3.

The purpose of this section is to introduce the structure of the model by constructing its Lagrangian. As in the Standard Model, explicit Dirac mass terms for fermions are not present before spontaneous symmetry breaking, because such terms would violate the electroweak gauge symmetry. The fermion masses are generated through Yukawa interactions after the neutral scalar fields acquire vacuum expectation values as discussed in Section 1.2.2. The exception is the Majorana mass term for the right-handed neutrino singlets, which is allowed already before symmetry breaking.

We divide the Lagrangian into the following sectors:

- the gauge–Higgs sector, containing the gauge kinetic terms, scalar kinetic terms, and scalar potential;
- the fermion–gauge sector, containing the fermion kinetic terms and their gauge interactions;
- the fermion–Higgs sector, containing the Yukawa interactions between fermions and the Higgs doublets, which generate fermion masses after symmetry breaking;

- the Majorana sector, containing the Majorana mass term for the right-handed neutrino singlets, which is present independently of symmetry breaking.

The neutral neutrino mass Lagrangian combines contributions from the last two sectors and is discussed together in Section 1.4.4. This section is based on [3], [14] and [5].

#### 1.4.1 The Gauge–Higgs Sector

The Grimus–Neufeld model contains two Higgs doublets,

$$H_a = \begin{pmatrix} H_a^+ \\ H_a^0 \end{pmatrix}, \quad a = 1, 2, \quad (1.36)$$

each transforming as a doublet under  $SU(2)_L$  with weak hypercharge  $y_H = \frac{1}{2}$ , identically to the Standard Model Higgs doublet introduced in Section 1.2.2. Under a gauge transformation parametrised by  $\phi_B$  and  $\phi_W^k$ , each doublet transforms as

$$H_a \rightarrow H'_a = \exp \left\{ i y_H \phi_B + i \frac{1}{2} \sigma^k \phi_W^k \right\} H_a, \quad (1.37)$$

and the covariant derivative is therefore the same as in Eq. (1.14), with  $H$  replaced by  $H_a$ .

Throughout this section we use the index convention that a barred index denotes the complex-conjugated field,  $H_{\bar{a}} \equiv H_a^*$ . Barred and unbarred indices are independent and both are summed over 1, 2 wherever they appear as repeated indices. Since conjugation is already implied by the bar, no  $\dagger$  appears on barred fields.

The gauge–Higgs sector of the Lagrangian is

$$\begin{aligned} \mathcal{L}_{\text{Gauge-Higgs}} = & -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^j W^{j\mu\nu} + (D_\mu H_a)^\dagger (D^\mu H_a) \\ & - \left[ L_{a\bar{b}} H_{\bar{a}} H_b + \frac{1}{2} Z_{a\bar{b}c\bar{d}} (H_{\bar{a}} H_b) (H_{\bar{c}} H_d) \right], \end{aligned} \quad (1.38)$$

where  $B_{\mu\nu}$  and  $W_{\mu\nu}^j$  are the  $U(1)_Y$  and  $SU(2)_L$  field strength tensors defined in Section 1.2.2, with  $W_{\mu\nu}^j$  including the non-Abelian self-interaction term,

$$W_{\mu\nu}^j = \partial_\mu W_\nu^j - \partial_\nu W_\mu^j - g \epsilon^{jk\ell} W_\mu^k W_\nu^\ell. \quad (1.39)$$

The tensor  $L_{a\bar{b}}$  contains the bilinear couplings of the scalar potential and satisfies  $L_{a\bar{b}} = (L_{b\bar{a}})^*$ , while  $Z_{a\bar{b}c\bar{d}}$  contains the quartic scalar couplings and satisfies  $Z_{a\bar{b}c\bar{d}} = Z_{c\bar{d}a\bar{b}} = (Z_{d\bar{c}b\bar{a}})^*$ .

After spontaneous symmetry breaking it is convenient to work in the Higgs basis, in which only the first doublet acquires a non-zero vacuum expectation value. In this basis,

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} G^+ \\ v + H_{1r} + i G^0 \end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \\ H_{2r} + i H_{2i} \end{pmatrix}, \quad (1.40)$$

where  $G^+$  and  $G^0$  are the charged and neutral Goldstone bosons, which become the longitudinal degrees of freedom of the  $W^\pm$  and  $Z$  bosons as discussed in Section 1.2.2. The fields  $H^+$ ,  $H_{1r}$ ,  $H_{2r}$ , and  $H_{2i}$  are scalar degrees of freedom before diagonalization of the scalar mass matrices; the physical neutral Higgs mass eigenstates are obtained from  $H_{1r}$ ,  $H_{2r}$ , and  $H_{2i}$  by a further orthogonal rotation.

#### 1.4.2 The Fermion–Gauge Sector

The fermion representations are the same as in the Standard Model, with the addition of right-handed neutrino singlets. The left-handed quarks and leptons are arranged into  $SU(2)_L$  doublets,

$$q'_L = \begin{pmatrix} u'_L \\ d'_L \end{pmatrix}, \quad \ell'_L = \begin{pmatrix} \nu'_L \\ e'_L \end{pmatrix}, \quad (1.41)$$

and the right-handed fields  $u'_R$ ,  $d'_R$ ,  $e'_R$ , and  $N'_R$  are singlets under  $SU(2)_L$ . The superscript prime denotes fields in the weak interaction basis, before rotation to the mass eigenstate basis.

The fermion–gauge sector is

$$\mathcal{L}_{\text{Fermion–Gauge}} = \sum_{\psi} \bar{\psi} i \not{D} \psi. \quad (1.42)$$

For an  $SU(2)_L$  doublet, the covariant derivative has the form already introduced in Eq. (1.14),

$$D_\mu \psi = \partial_\mu \psi - ig W_\mu^k \frac{\sigma^k}{2} \psi - ig' y_\psi B_\mu \psi. \quad (1.43)$$

For an  $SU(2)_L$  singlet, the  $SU(2)_L$  term is absent,

$$D_\mu \psi = \partial_\mu \psi - ig' y_\psi B_\mu \psi. \quad (1.44)$$

The hypercharges are fixed by requiring consistency with the observed electric charges via the relation

$$Q = T_3 + y, \quad (1.45)$$

where  $T_3$  is the third component of weak isospin. For the upper component of an  $SU(2)_L$  doublet  $T_3 = +\frac{1}{2}$ , and for the lower component  $T_3 = -\frac{1}{2}$ . For singlets,  $T_3 = 0$ . Applying this to each field:

- quark doublet:  $Q(u) = \frac{2}{3}$  and  $T_3(u) = +\frac{1}{2}$  give  $y_q = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$ ;
- lepton doublet:  $Q(\nu) = 0$  and  $T_3(\nu) = +\frac{1}{2}$  give  $y_\ell = 0 - \frac{1}{2} = -\frac{1}{2}$ ;
- right-handed up-type quark:  $Q(u_R) = \frac{2}{3}$  and  $T_3 = 0$  give  $y_u = \frac{2}{3}$ ;
- right-handed down-type quark:  $Q(d_R) = -\frac{1}{3}$  and  $T_3 = 0$  give  $y_d = -\frac{1}{3}$ ;
- right-handed charged lepton:  $Q(e_R) = -1$  and  $T_3 = 0$  give  $y_e = -1$ ;

- right-handed neutrino singlet:  $Q(N_R) = 0$  and  $T_3 = 0$  give  $y_N = 0$ , reflecting the fact that  $N'_R$  is a complete gauge singlet.

This gives the hypercharge assignments

$$y_q = \frac{1}{6}, \quad y_\ell = -\frac{1}{2}, \quad y_u = \frac{2}{3}, \quad y_d = -\frac{1}{3}, \quad y_e = -1, \quad y_N = 0. \quad (1.46)$$

Using the definitions of  $W_\mu^\pm$ ,  $Z_\mu$ ,  $A_\mu$ , and the Weinberg angle from Section 1.2.2, the doublet covariant derivative can be written in terms of the physical gauge fields. With the abbreviations

$$g_Z = \sqrt{g^2 + g'^2}, \quad s_w = \sin \theta_W = \frac{g'}{g_Z}, \quad c_w = \cos \theta_W = \frac{g}{g_Z}, \quad (1.47)$$

the covariant derivative acting on a generic doublet  $(\psi_\uparrow, \psi_\downarrow)^T$  takes the explicit form:

$$D_\mu \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = \begin{pmatrix} \left[ \partial_\mu - \frac{ig}{2c_w} ((c_w^2 - 2y_\psi s_w^2)Z_\mu + s_w c_w (2y_\psi + 1)A_\mu) \right] \psi_\uparrow - \frac{ig}{\sqrt{2}} W_\mu^+ \psi_\downarrow \\ -\frac{ig}{\sqrt{2}} W_\mu^- \psi_\uparrow + \left[ \partial_\mu + \frac{ig}{2c_w} ((c_w^2 + 2y_\psi s_w^2)Z_\mu + s_w c_w (2y_\psi - 1)A_\mu) \right] \psi_\downarrow \end{pmatrix}. \quad (1.48)$$

For singlet fields, the terms involving the charged gauge bosons are absent.

### 1.4.3 The Fermion–Higgs Sector

The Yukawa sector of the Grimus–Neufeld model is

$$\begin{aligned} -\mathcal{L}_{\text{Fermion-Higgs}} = & \bar{\ell}'_{Lj} H_a Y_{Ljk}^{\bar{a}} e'_{Rk} + \bar{\ell}'_{Lj} \tilde{H}_{\bar{a}} \Gamma_{Ljn}^a N'_{Rn} \\ & + \bar{q}'_{Lj} H_a Y_{Qjk}^{\bar{a}} d'_{Rk} + \bar{q}'_{Lj} \tilde{H}_{\bar{a}} \Gamma_{Qjk}^a u'_{Rk} + \text{h.c.}, \end{aligned} \quad (1.49)$$

where the indices  $j, k$  denote fermion generations and  $a = 1, 2$  labels the Higgs doublets. The conjugate Higgs doublet  $\tilde{H}_a = i\sigma^2 H_a^*$  was introduced in Section 1.3. The matrices  $Y_L^{\bar{a}}$  and  $Y_Q^{\bar{a}}$  are the Yukawa coupling matrices for the charged leptons and down-type quarks respectively, while  $\Gamma_L^a$  and  $\Gamma_Q^a$  are the Yukawa coupling matrices involving the conjugate Higgs doublet, coupling to the right-handed neutrino singlets and up-type quarks respectively. The use of distinct symbols  $Y$  and  $\Gamma$  reflects the fact that these matrices couple to different Higgs doublets ( $H_a$  and  $\tilde{H}_{\bar{a}}$  respectively) and have different transformation properties under the Higgs doublet symmetry.

Since only  $H_1$  has a non-zero vacuum expectation value in the Higgs basis, the fermion mass terms generated after symmetry breaking are

$$\begin{aligned} -\mathcal{L}_{\text{Fermion mass}} = & \frac{v}{\sqrt{2}} \left( \bar{\ell}'_{Lj} Y_{Ljk}^{\bar{1}} e'_{Rk} + \bar{\nu}'_{Lj} \Gamma_{Ljn}^1 N'_{Rn} \right. \\ & \left. + \bar{d}'_{Lj} Y_{Qjk}^{\bar{1}} d'_{Rk} + \bar{u}'_{Lj} \Gamma_{Qjk}^1 u'_{Rk} \right) + \text{h.c.} \end{aligned} \quad (1.50)$$

The charged-lepton, up-quark and down-quark mass matrices are defined by

$$(M_e)_{j\ell} := \frac{v}{\sqrt{2}} Y_{Lj\ell}^\dagger = U_{jk}^{eL\dagger} \text{diag}(m_e, m_\mu, m_\tau)_{kk} U_{k\ell}^{eR}, \quad (1.51)$$

$$(M_{\text{up}})_{j\ell} := \frac{v}{\sqrt{2}} \Gamma_{Qj\ell}^1 = U_{jk}^{uL\dagger} \text{diag}(m_u, m_c, m_t)_{kk} U_{k\ell}^{uR}, \quad (1.52)$$

and

$$(M_{\text{down}})_{j\ell} := \frac{v}{\sqrt{2}} Y_{Qj\ell}^\dagger = U_{jk}^{dL\dagger} \text{diag}(m_d, m_s, m_b)_{kk} U_{k\ell}^{dR}, \quad (1.53)$$

where  $U^{eL}, U^{eR}, U^{uL}, U^{uR}, U^{dL}, U^{dR}$  are unitary matrices that bring each mass matrix to diagonal form via the singular value decomposition. The corresponding field redefinitions are:

$$\begin{aligned} e'_{Lj} &= U_{jk}^{eL\dagger} e_{Lk}, & e'_{Rj} &= U_{jk}^{eR\dagger} e_{Rk}, \\ u'_{Lj} &= U_{jk}^{uL\dagger} u_{Lk}, & u'_{Rj} &= U_{jk}^{uR\dagger} u_{Rk}, \\ d'_{Lj} &= U_{jk}^{dL\dagger} d_{Lk}, & d'_{Rj} &= U_{jk}^{dR\dagger} d_{Rk}. \end{aligned} \quad (1.54)$$

Since the left-handed neutrino and charged lepton form an  $SU(2)_L$  doublet, they must undergo the same rotation. The neutrino weak basis field is therefore redefined as

$$\nu'_{Lj} = U_{jk}^{eL\dagger} \nu_{Lk}, \quad (1.55)$$

which defines the neutrino flavour states  $\nu_e, \nu_\mu, \nu_\tau$  as the interaction eigenstates in the charged current.

With these redefinitions, the charged-fermion mass terms become diagonal,

$$\begin{aligned} -\mathcal{L}_{\text{charged-fermion mass}} &= m_{e_k} (\bar{e}_{Lk} e_{Rk} + \bar{e}_{Rk} e_{Lk}) \\ &\quad + m_{d_k} (\bar{d}_{Lk} d_{Rk} + \bar{d}_{Rk} d_{Lk}) \\ &\quad + m_{u_k} (\bar{u}_{Lk} u_{Rk} + \bar{u}_{Rk} u_{Lk}). \end{aligned} \quad (1.56)$$

The neutral Dirac mass term — the analogue of the Dirac mass introduced in Section 1.3, now extended to three generations — is

$$-\mathcal{L}_{\nu D} = \bar{\nu}_{Lj} (M_D)_{jn} N'_{Rn} + \text{h.c.}, \quad (1.57)$$

where

$$(M_D)_{jn} = \frac{v}{\sqrt{2}} \left( U^{eL} \Gamma_L^1 \right)_{jn}. \quad (1.58)$$

The field redefinitions also affect the charged-current interactions. In the quark sector, the mismatch between the left-handed up- and down-quark rotations gives rise to the Cabibbo–Kobayashi–Maskawa [15, 16] matrix,

$$V^{\text{CKM}} = U^{uL} U^{dL\dagger}. \quad (1.59)$$

It is useful to define the Yukawa matrices associated with the second Higgs doublet in the fermion mass basis:

$$Y_L^2 = U^{eL\dagger} Y_E^{(2)} U^{eR}, \quad \Gamma_Q^2 = U^{uL\dagger} Y_U^{(2)} U^{uR}, \quad Y_Q^2 = U^{dL\dagger} Y_D^{(2)} U^{dR}. \quad (1.60)$$

For the neutrino Yukawa matrices, we write

$$\Gamma_L^1 = U^{eL\dagger} \frac{\sqrt{2}}{v} M_D, \quad \Gamma_L^2 = U^{eL\dagger} Y_N^{(2)}. \quad (1.61)$$

#### 1.4.4 The Fermion–Majorana Sector

The Grimus–Neufeld model contains a single right-handed neutrino singlet  $N'_R$ . This is not merely a simplification but a defining feature of the model: it is precisely the interplay between one right-handed neutrino at tree level and radiative corrections that generates the two non-zero light neutrino masses, as discussed in Section 1.3. Since  $N'_R$  is a gauge singlet, the Lagrangian may contain a Majorana mass term independently of electroweak symmetry breaking. As introduced in Section 1.3, this takes the form:

$$-\mathcal{L}_{\text{Majorana}} = \frac{1}{2} \overline{N'_R{}^C} M_R N'_R + \text{h.c.}, \quad (1.62)$$

where  $M_R$  is the Majorana mass parameter.

Combining this with the neutral Dirac mass term from Section 1.4.3, the full neutral fermion mass Lagrangian is:

$$-\mathcal{L}_{\text{neutral mass}} = \bar{\nu}_{Lj} (M_D)_j N'_R + \frac{1}{2} \overline{N'_R{}^C} M_R N'_R + \text{h.c.}, \quad (1.63)$$

where  $(M_D)_j$  is the three-component Dirac mass vector in flavour space defined in Section 1.4.3. Note that the Dirac mass term originates from the Fermion–Higgs sector while the Majorana mass term is independent of symmetry breaking; they are combined here because together they constitute the complete neutral fermion mass Lagrangian.

Collecting the neutral fields into the vector:

$$n_L = \begin{pmatrix} \nu_{L1} \\ \nu_{L2} \\ \nu_{L3} \\ N'_R{}^C \end{pmatrix}, \quad (1.64)$$

the neutral mass Lagrangian can be written as:

$$-\mathcal{L}_{\text{neutral mass}} = \frac{1}{2} \overline{n_L} M_\nu n_L^C + \text{h.c.}, \quad (1.65)$$

with the  $4 \times 4$  neutral mass matrix:

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & (M_D)_1 \\ 0 & 0 & 0 & (M_D)_2 \\ 0 & 0 & 0 & (M_D)_3 \\ (M_D)_1 & (M_D)_2 & (M_D)_3 & M_R \end{pmatrix}. \quad (1.66)$$

#### 1.4.5 Seesaw Mechanism in the Grimus–Neufeld Model

The neutral mass matrix in Eq. (1.66) generalises the one-generation seesaw matrix introduced in Section 1.3. For one generation, the corresponding matrix was:

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix}. \quad (1.67)$$

In the Grimus–Neufeld model with three active neutrinos and one right-handed neutrino,  $M_D$  becomes a three-component vector in flavour space, and the mass matrix takes the form shown in Eq. (1.66).

In the seesaw limit,

$$M_R \gg |(M_D)_j|, \quad (1.68)$$

the heavy-neutrino mass is approximately

$$m_N \simeq M_R, \quad (1.69)$$

while the effective light-neutrino mass matrix is

$$(m_\nu)_{ij} \simeq -\frac{(M_D)_i(M_D)_j}{M_R}. \quad (1.70)$$

This matrix is of the form  $-\vec{M}_D \vec{M}_D^T / M_R$ , which is the outer product of a single vector with itself, and therefore has rank one. Consequently, only one active neutrino obtains a non-zero mass at tree level. However, the rank of the full  $4 \times 4$  matrix  $M_\nu$  in Eq. (1.66) is two, since it has two non-zero eigenvalues: the heavy state  $m_N \simeq M_R$  and one light state. The remaining two light neutrino masses are generated radiatively through the Grimus–Neufeld mechanism, which is one of the central features of the model [3].

## 1.5 Renormalization

While the Lagrangian of a theory is a useful tool to describe how particles interact and from which we can derive the Feynman rules, the end goal is for the theory to coincide with experimental data. Upon computing the amplitudes of particle processes at higher orders, the results start to diverge, giving non-physical infinities known as UV divergences. These divergences arise because the theory attempts to describe processes beyond its range of validity. They affect physical quantities and yield values inconsistent with experiment. This is resolved by renormalization,

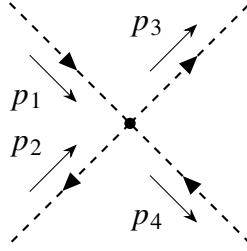
which is a procedure that deals with unphysical quantities, that appear in theories, when they try to predict effects outside their energy scales. This section is loosely based on [8, 9, 11, 17].

### 1.5.1 Concept of Renormalization

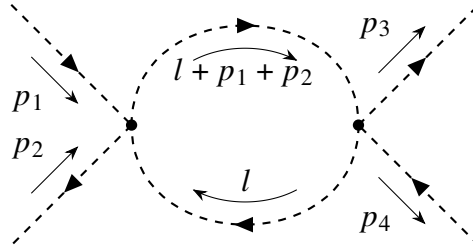
To understand the need for renormalization, we consider a scalar quartic interaction described by the Lagrangian

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2, \quad (1.71)$$

where  $\phi$  is a complex scalar field,  $m$  is its mass, and  $\lambda$  is the quartic coupling constant. This Lagrangian describes a process in which two scalar particles enter a vertex and two exit, as shown in Figure 2. A full example of a renormalization procedure will be given in the next section.



2 pav. Leading order scalar quartic interaction for  $\phi\phi^* \rightarrow \phi\phi^*$  scattering.



3 pav. One-loop contribution to scalar quartic interaction for  $\phi\phi^* \rightarrow \phi\phi^*$  scattering.

By using the Feynman rules for this interaction, the leading order diagram shown in Figure 2 gives the amplitude

$$i\mathcal{M}^{(0)} = -i\lambda, \quad (1.72)$$

where  $\lambda$  is the coupling constant of the interaction vertex. If we were to only look at these types of interactions, it would be simple enough to carry out the relevant computations. For more precise results, however, we must include higher order contributions. We therefore include the one-loop diagram shown in Figure 3. It should be noted that an explicit check at order  $\mathcal{O}(\lambda^2)$  requires also including permutations of the outgoing momenta, but these add only additional terms with shifted kinematics that do not affect the overall argument, so we omit them for brevity. Including the one-loop correction gives

$$i\mathcal{M}^{(1)} = -i\lambda + \lambda^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 - m^2)((l + p_1 + p_2)^2 - m^2)}. \quad (1.73)$$

Focusing on the integral term, in the limit  $l \rightarrow \infty$  the integrand behaves as  $1/l^4$ , so the integral reduces to

$$\begin{aligned} i\mathcal{M}^{(1-\text{loop})} &= \lambda^2 \int_0^\infty \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 - m^2)((l + p_1 + p_2)^2 - m^2)} \xrightarrow{l \rightarrow \infty} \lambda^2 \int_0^\infty \frac{d^4 l}{(2\pi)^4} \frac{1}{l^4} \\ &\sim \lambda^2 \int_0^\infty \frac{dl}{l} = \infty. \end{aligned} \quad (1.74)$$

This is an example of a UV divergence: the integral diverges because we integrate over arbitrarily large loop momenta. The fundamental issue is that the theory, as formulated, is sensitive to arbitrarily high energy scales that lie beyond its domain of validity.

A natural way to address this is to assume that the theory does not extend to arbitrarily large momenta, and to introduce an energy cut-off  $\Lambda$  above which the theory is no longer applicable. Restricting the integration to  $l < \Lambda$  gives

$$\lambda^2 \int_0^\Lambda \frac{dl}{l} \sim \lambda^2 \log \Lambda. \quad (1.75)$$

This is now finite, but it remains large and highly sensitive to the value of  $\Lambda$ . Evaluating the full one-loop amplitude with the cut-off gives

$$i\mathcal{M}^{(1)} = -i\lambda + i\lambda^2 \left[ A \log \left( \frac{\Lambda^2}{q^2} \right) + B \right], \quad (1.76)$$

where  $A$  and  $B$  are numerical coefficients that depend on the parameters of the integral,  $q = p_1 + p_2$  is the momentum transfer, and the additional factor of  $i$  in the second term arises from the Wick rotation used in the integration. This result is still arbitrary, as it depends on  $\Lambda$  and diverges as  $\Lambda \rightarrow \infty$ .

The key observation is that the cut-off  $\Lambda$  is not a physical quantity. Rather than relating the amplitude to  $\Lambda$ , we can instead relate the amplitude at one momentum scale  $q_1$  to the amplitude at another scale  $q_2$ :

$$i\mathcal{M}^{(1)}(q_2^2) = i\mathcal{M}^{(1)}(q_1^2) + i\lambda^2 A \log \left( \frac{q_1^2}{q_2^2} \right). \quad (1.77)$$

This relation is finite and well-defined, as the dependence on  $\Lambda$  cancels. This is the central idea of renormalization: instead of relating the parameters of the theory to a non-physical cut-off, we relate observable quantities to other observable quantities at different energy scales.

A more systematic way to implement this is to recognise that the coupling constant  $\lambda$  appearing in the Lagrangian is not directly the quantity measured in experiment. The measured amplitude receives contributions from the tree-level coupling as well as from loop corrections at all orders. We therefore define the renormalized coupling constant  $\lambda_R$  as the value extracted from a measurement of the amplitude at a specific scale  $q_1$ :

$$-i\lambda_R := i\mathcal{M}^{(1-\text{loop})}(q_1^2) = -i\lambda_0 + i\lambda_0^2 \left[ A \log \left( \frac{\Lambda^2}{q_1^2} \right) + B \right] + \mathcal{O}(\lambda^3), \quad (1.78)$$

where  $\lambda_0$  is the bare coupling constant appearing in the Lagrangian, which is not directly related to any measured quantity. The condition that fixes  $\lambda_R$  at the scale  $q_1$  is called a renormalization condition. In this case, it is the requirement that the full one-loop amplitude evaluated at  $q^2 = q_1^2$  equals  $-i\lambda_R$ , i.e.:

$$i\mathcal{M}^{(1)}|_{q^2=q_1^2} = -i\lambda_R. \quad (1.79)$$

This fixes the value of the renormalized coupling at the scale  $q_1$  and absorbs the cut-off dependence into the definition of  $\lambda_0$ . The renormalized amplitude is then represented diagrammatically as

$$i\mathcal{M} = \text{blob with } \lambda_R \text{ inside} , \quad (1.80)$$

where the blob represents the sum of all loop contributions with loop momenta  $q_1^2 < l^2 < \Lambda^2$ .

At leading order,

$$\lambda_0 = \lambda_R, \quad (1.81)$$

so in situations where loop corrections are negligible the bare and renormalized coupling constants coincide. This is useful for classical or semi-classical approximations like Coulomb's law in Quantum Electrodynamics and any calculations that do not require precision measurements [17]. At first order in the loop expansion,

$$\lambda_0 = \lambda_R + \lambda_R^2 \left[ A \log \left( \frac{\Lambda^2}{q_1^2} \right) + B \right] + O(\lambda_R^3). \quad (1.82)$$

Substituting this into Eq. (1.76), the amplitude at a second scale  $q_2$  becomes

$$i\mathcal{M}^{(1)}(q_2^2) = -i\lambda_R - i\lambda_R^2 \left[ A \log \left( \frac{\Lambda^2}{q_1^2} \right) + B \right] + i\lambda_R^2 \left[ A \log \left( \frac{\Lambda^2}{q_2^2} \right) + B \right] + O(\lambda_R^3) \quad (1.83)$$

$$= -i\lambda_R + i\lambda_R^2 A \log \left( \frac{q_1^2}{q_2^2} \right) + O(\lambda_R^3). \quad (1.84)$$

This is consistent with Eq. (1.77), confirming that by expressing the theory in terms of  $\lambda_R$ , the predictions are finite, cut-off independent, and physically meaningful.

In practice, the renormalization procedure is more involved, as one must account for multiple renormalizable quantities and higher orders of loop corrections. Furthermore, the cut-off regularisation used here breaks translational invariance in momentum space,  $l \rightarrow l + a$ , and can violate gauge symmetry. This becomes problematic in gauge theories such as the Standard Model, where gauge invariance must be preserved at every stage of the computation. A regularisation scheme that maintains these symmetries is therefore required.

### 1.5.2 Example of Renormalization

To round out our understanding of renormalization, we will go through a toy model, which has similar interactions to the ones we talked about in the previous sections, and apply the

renormalization procedure. Let's say we have a theory with a Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_0)^2 - \frac{1}{2}m_0^2 \phi_0^2 - \frac{\lambda_0}{4!} \phi_0^4, \quad (1.85)$$

where the index 0 denotes them as being bare. We can then apply the renormalization constants mentioned in the previous section. Specifically, we will apply the renormalization constant  $Z$  to the field  $\phi$ :

$$\phi_0 = Z^{\frac{1}{2}} \phi_R. \quad (1.86)$$

This rewrites the Lagrangian as:

$$\mathcal{L} = \frac{1}{2}Z_\phi(\partial_\mu \phi_R)^2 - \frac{1}{2}Z_m m_R^2 \phi_R^2 - \frac{\lambda_R}{4!} Z_\lambda^2 \phi_R^4. \quad (1.87)$$

Now we define the counterterms for each renormalizable parameter:

$$\delta_\phi = Z_\phi - 1, \quad \delta_{m^2} = m_R^2 Z_m - m_R^2, \quad \delta_\lambda = \lambda_R Z_\lambda^2 - \lambda_R. \quad (1.88)$$

By doing so our Lagrangian becomes:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_R)^2 - \frac{1}{2}m_R^2 \phi_R^2 - \frac{\lambda_R}{4!} \phi_R^4 + \frac{1}{2}\delta_\phi(\partial_\mu \phi_R)^2 - \frac{1}{2}\delta_{m^2} \phi_R^2 - \frac{\delta_\lambda}{4!} \phi_R^4. \quad (1.89)$$

From the Lagrangian, we get the Feynman rules of the theory:

$$\text{---}\overrightarrow{p}\text{---} = \frac{i}{p^2 - m^2 + i\epsilon}, \quad (1.90)$$

$$\text{---}\bullet\text{---} = -i\lambda, \quad (1.91)$$

$$\text{---}\times\text{---} = i(p^2 \delta_\phi - \delta_{m^2}), \quad (1.92)$$

$$\text{---}\bigotimes\text{---} = -i\delta_\lambda, \quad (1.93)$$

where the last two equations give the counterterm Feynman rules.

In the next steps we will only show the steps of how we renormalize the field propagator. This choice was made to keep the example simpler. But all the other counterterm calculations are analogous. For example, for a coupling constant counterterm, one would have to calculate the loop corrections for an interaction vertex. Such calculations would include more complex diagrams, but they would follow the same steps. But, as mentioned, we only need an example

of how one would calculate a parameters counterterm. Moving along, we now use the Feynman rules we have established from the Lagrangian to construct the amplitude. This is achieved in a similar way as we did in Eq. (1.80), where we can define the amplitude of the inverse field propagator as:

$$i\mathcal{M} = \text{---} \text{---} \text{---} \text{---} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} + \dots, \quad (1.94)$$

where 1PI stands for One-particle irreducible. A 1PI diagram is a Feynman diagram that cannot be disconnected into two separate diagrams by cutting a single internal line. Because we will be calculating one-loop corrections, we will only need the 1PI diagrams, however, if we wanted to calculate higher order loops, we could chain them together to do so. We will denote the 1PI diagram as:

$$\text{---} \text{---} \text{---} \text{---} = i\Sigma(p^2), \quad (1.95)$$

which is referred to as the self-energy. By having these definitions, we get that the full Green's is:

$$\text{---} \text{---} \text{---} \text{---} = \frac{1}{p^2 - m^2 + \Sigma(p^2)}. \quad (1.96)$$

Now we calculate the self-energy of the propagator to get the mass counterterm. Firstly, the self-energy of the propagator consists of the diagrams:

$$i\Sigma(p^2) = \text{---} \text{---} \text{---} \text{---} + \text{---} \times \text{---}. \quad (1.97)$$

By applying the Feynman rules, we get that:

$$i\Sigma(p^2) = -\frac{i\lambda}{2}\mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{i}{k^2 - m^2 + i\epsilon} + i(p^2\delta_\phi - \delta_{m^2}), \quad (1.98)$$

where  $k$  is the loop momentum of the first diagram in Eq. (1.97), commonly called a seagull diagram, and  $\mu^{4-D}$  is the mass scale we mentioned in Appendix B.

Now we proceed with the evaluation of the integral in the first term. Firstly, we apply a Wick rotation to it, which gives us:

$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 - m^2 + i\epsilon} = \int \frac{d^D k_E}{(2\pi)^D} \frac{-i}{k_E^2 + m^2}. \quad (1.99)$$

After the application of a Wick rotation, we now get a suitable form of the integral, on which we can apply the master formula given in Eq. (B.10). For this integral, specifically,  $X = m^2$ ,  $a = 0$  and  $b = 1$ . Therefore, our integral is:

$$\int \frac{d^D k_E}{(2\pi)^D} \frac{-i}{k_E^2 + m^2} = -\frac{1}{(4\pi)^{\frac{D}{2}}} \frac{1}{-(m^2)^{1-\frac{D}{2}}} \frac{\Gamma(\frac{D}{2})\Gamma(1-\frac{D}{2})}{\Gamma(1)\Gamma(\frac{D}{2})} = -\frac{1}{(4\pi)^{\frac{D}{2}}} \frac{1}{-(m^2)^{1-\frac{D}{2}}} \Gamma\left(1-\frac{D}{2}\right). \quad (1.100)$$

As stated in section B, we denote that  $D = 4 - \epsilon$ . This gives us that:

$$-\frac{1}{(4\pi)^{\frac{D}{2}}} \frac{1}{-(m^2)^{1-\frac{D}{2}}} \Gamma\left(1 - \frac{D}{2}\right) = -\frac{1}{(4\pi)^{2-\frac{\epsilon}{2}}} \frac{1}{-(m^2)^{\frac{\epsilon}{2}-1}} \Gamma\left(\frac{\epsilon}{2} - 1\right). \quad (1.101)$$

By having our expression in this form, we can move to evaluating the Gamma function. To do so for the function given in Eq. (1.101), we will use the Gamma function property  $\Gamma(z+1) = z\Gamma(z)$ . Applying this property as well as Eq. (B.11), we get:

$$\Gamma\left(\frac{\epsilon}{2} - 1\right) = \left(-\frac{2}{\epsilon} - 1 + \gamma + O(\epsilon)\right). \quad (1.102)$$

Thus giving us:

$$-\frac{1}{(4\pi)^{2-\frac{\epsilon}{2}}} \frac{1}{-(m^2)^{\frac{\epsilon}{2}-1}} \Gamma\left(1 - \frac{D}{2}\right) = -\frac{1}{(4\pi)^{2-\frac{\epsilon}{2}}} \frac{1}{-(m^2)^{\frac{\epsilon}{2}-1}} \left(-\frac{2}{\epsilon} - 1 + \gamma + O(\epsilon)\right). \quad (1.103)$$

However, having the equation in this form does not give us insight in how the mass counterterm looks, so we have to massage it in to a more suitable form also not forgetting to bring back the mass scale  $\mu$ , which now will be  $\mu^{4-D} = \mu^\epsilon$ :

$$-\mu^\epsilon \frac{1}{(4\pi)^{2-\frac{\epsilon}{2}}} \frac{1}{-(m^2)^{\frac{\epsilon}{2}-1}} \left(-\frac{2}{\epsilon} - 1 + \gamma + O(\epsilon)\right) = \frac{m^2}{16\pi^2} \left[\frac{2}{\epsilon} + 1 + \ln\left(\frac{4\pi\mu^2 e^{-\gamma}}{m^2}\right)\right], \quad (1.104)$$

where we dropped  $O(\epsilon)$  for simplicity. Now we multiply out all the terms:

$$\frac{m^2}{16\pi^2} \left[\frac{2}{\epsilon} + 1 + \ln\left(\frac{4\pi\mu^2 e^{-\gamma}}{m^2}\right)\right] = \frac{m^2}{8\pi^2\epsilon} + \frac{m^2}{16\pi^2} + \frac{m^2}{16\pi^2} \ln\left(\frac{4\pi\mu^2 e^{-\gamma}}{m^2}\right). \quad (1.105)$$

Now we plug this expression back into Eq. (1.98) and get:

$$\begin{aligned} i\Sigma(p^2) &= -\frac{i\lambda}{2} i \left[ \frac{m^2}{8\pi^2\epsilon} + \frac{m^2}{16\pi^2} + \frac{m^2}{16\pi^2} \ln\left(\frac{4\pi\mu^2 e^{-\gamma}}{m^2}\right) \right] - (p^2\delta_\phi - \delta_{m^2}) \\ &= \frac{m^2\lambda}{16\pi^2\epsilon} + \frac{m^2\lambda}{16\pi^2} + \frac{m^2\lambda}{16\pi^2} \ln\left(\frac{4\pi\mu^2 e^{-\gamma}}{m^2}\right) - p^2\delta_\phi + \delta_{m^2}. \end{aligned} \quad (1.106)$$

By having Eq. (1.106), we can distinguish the terms that will cancel each other. Because we are using the Minimal subtraction (MS) scheme, we only look at the term that has the physical pole  $\epsilon$  for the mass counterterm. The field counterterm  $\delta_\phi$  vanishes at one-loop because the tadpole integral is independent of the external momentum  $p^2$ , producing no  $p^2$ -divergent contribution that are needed for the mass counterterm. Now to calculate the mass counterterm, the counterterm and the term it is contracted with need to satisfy the equation:

$$\delta_{m^2} + \frac{m^2\lambda}{16\pi^2\epsilon} = 0. \quad (1.107)$$

By rearranging the equation, we get that the mass counterterm is:

$$\delta_{m^2} = -\frac{m^2\lambda}{16\pi^2\epsilon}. \quad (1.108)$$

As we have calculated the mass counterterm, we now have an example of how one applies the renormalization procedure. Of course, this example is only a part of the renormalization of the chosen model, however, it explicitly shows how one applies the ideas discussed in the sections beforehand. If one would want to be extremely strict and have the need to renormalize the theory in full, then they should apply the steps shown to all the parameters.

## 2 Yukawa sector counterterm Feynman rules

Having gone through the necessary theoretical foundation in section 1, we start the derivation of the Yukawa sector counterterm Feynman rules. As stated in the Introduction, we will limit our work to the SM part of the GNM. We will not take into account the effects of the second Higgs doublet, the Higgs bosons mixing with other bosons and the Majorana fermion. This is done to have a simpler evaluation of the hypotheses that we can renormalize the Yukawa constants by renormalizing the Higgs–fermion interaction vertex, before going in to the complexity of 2HDM.

Having restricted our framework, we thus can take the simpler form of the Yukawa sector Lagrangian of GNM, which in section 1.4.3 we called the Fermion–Higgs sector. This simpler form is analogous to the Yukawa sector in SM:

$$\mathcal{L}_{\text{Yukawa}} = -Y_{ij}^u \bar{q}_{Li} \tilde{H} u_{Rj} - Y_{ij}^d \bar{q}_{Li} H d_{Rj} - Y_{ij}^e \bar{\ell}_{Li} H e_{Rj} + \text{h.c.}, \quad (2.1)$$

where  $Y^u$ ,  $Y^d$  and  $Y^e$  are  $3 \times 3$  Yukawa matrices in flavour space. The Higgs doublet  $H$ , the conjugate doublet  $\tilde{H}$ , and the fermion doublets  $q_L$  and  $\ell_L$  were introduced in the previous sections.

After spontaneous symmetry breaking, the Higgs doublet and its conjugate take the form

$$H = \begin{pmatrix} h_3^+ \\ v + h_1 + ih_2 \\ \sqrt{2} \end{pmatrix}, \quad \tilde{H} = i\sigma_2 H^* = \begin{pmatrix} \frac{v + h_1 - ih_2}{\sqrt{2}} \\ -h_3^- \end{pmatrix}, \quad (2.2)$$

where  $v$  is the vacuum expectation value,  $h_1$  is the physical Higgs boson,  $h_2$  is the neutral Goldstone boson and  $h_3^\pm$  are the charged Goldstone bosons. Only  $h_3^\pm$  has the superscript  $\pm$ , because it is the only one that has charge associated to it, thus the only one to have a positive or negative charge distinction. As explained in Section 1.2.2, the Goldstone bosons are absorbed by the gauge bosons through the Higgs mechanism.

Expanding the Yukawa Lagrangian gives

$$\mathcal{L}_{\text{Yukawa}} = \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{FH}} + \mathcal{L}_{\text{G}}. \quad (2.3)$$

The fermion mass terms are

$$\mathcal{L}_{\text{mass}} = -\frac{v}{\sqrt{2}} Y_{ij}^u \bar{u}_{Li} u_{Rj} - \frac{v}{\sqrt{2}} Y_{ij}^d \bar{d}_{Li} d_{Rj} - \frac{v}{\sqrt{2}} Y_{ij}^e \bar{e}_{Li} e_{Rj} + \text{h.c.}. \quad (2.4)$$

The physical Higgs interaction terms are

$$\mathcal{L}_{\text{FH}} = -\frac{1}{\sqrt{2}} Y_{ij}^u h_1 \bar{u}_{Li} u_{Rj} - \frac{1}{\sqrt{2}} Y_{ij}^d h_1 \bar{d}_{Li} d_{Rj} - \frac{1}{\sqrt{2}} Y_{ij}^e h_1 \bar{e}_{Li} e_{Rj} + \text{h.c.}. \quad (2.5)$$

Finally, the Goldstone interaction terms are

$$\begin{aligned}\mathcal{L}_G = & -\frac{i}{\sqrt{2}}Y_{ij}^u h_2 \bar{u}_{Li} u_{Rj} - \frac{i}{\sqrt{2}}Y_{ij}^d h_2 \bar{d}_{Li} d_{Rj} - \frac{i}{\sqrt{2}}Y_{ij}^e h_2 \bar{e}_{Li} e_{Rj} \\ & + Y_{ij}^u h_3^- \bar{d}_{Li} u_{Rj} - Y_{ij}^d h_3^+ \bar{u}_{Li} d_{Rj} - Y_{ij}^e h_3^+ \bar{\nu}_{Li} e_{Rj} + \text{h.c.}\end{aligned}\quad (2.6)$$

The relative sign difference between the up-type neutral Goldstone term and the other neutral Goldstone terms follows from the definition of the conjugate Higgs doublet  $\tilde{H}$ .

Now we define the renormalization counterterms that will be used:

$$\begin{aligned}\psi_i^0 &= \sum_j (Z_\psi)_{ij} \psi_j, & (Z_\psi)_{ij} &= \delta_{ij} + (\delta Z^\psi)_{ij}, \\ \bar{\psi}_i^0 &= \sum_j \bar{\psi}_j \gamma^0 \left( (Z_\psi)_{ji}^\dagger \right) \gamma^0, & (Z_\psi)_{ji}^\dagger &= \delta_{ji} + (\delta Z^\psi)_{ji}^\dagger, \\ h_k^0 &= \sum_l (Z_h)_{kl} h_l, & (Z_h)_{kl} &= \delta_{kl} + (\delta h)_{kl}, \\ Y_{ij}^{\psi,0} &= Y_{ij}^\psi + \delta Y_{ij}^\psi.\end{aligned}\quad (2.7)$$

Here  $\psi$  denotes the fermion fields, while  $h_k$  denotes the scalar components introduced above. The scalar renormalization constant is written in matrix form in order to account for possible Higgs–Goldstone mixing.

Next, the counterterms are inserted into the Yukawa Lagrangian. Keeping only terms linear in the counterterms, we obtain the mass counterterm Lagrangian. For up-type quarks,

$$\begin{aligned}\delta \mathcal{L}_{\text{mass(up-type)}} &= -\frac{v}{\sqrt{2}} \delta Y_{ij}^u \bar{u}_{Li} u_{Rj} \\ &\quad - \frac{v}{\sqrt{2}} Y_{ij}^u \bar{u}_{Li} \sum_b (\delta Z^u)_{jb} u_{Rb} \\ &\quad - \frac{v}{\sqrt{2}} Y_{ij}^u \sum_a \bar{u}_{La} \gamma^0 (\delta Z^u)_{ai}^\dagger \gamma^0 u_{Rj} + \text{h.c.}\end{aligned}\quad (2.8)$$

For down-type quarks,

$$\begin{aligned}\delta \mathcal{L}_{\text{mass(down-type)}} &= -\frac{v}{\sqrt{2}} \delta Y_{ij}^d \bar{d}_{Li} d_{Rj} \\ &\quad - \frac{v}{\sqrt{2}} Y_{ij}^d \bar{d}_{Li} \sum_b (\delta Z^d)_{jb} d_{Rb} \\ &\quad - \frac{v}{\sqrt{2}} Y_{ij}^d \sum_a \bar{d}_{La} \gamma^0 (\delta Z^d)_{ai}^\dagger \gamma^0 d_{Rj} + \text{h.c.}\end{aligned}\quad (2.9)$$

For charged leptons,

$$\begin{aligned}\delta \mathcal{L}_{\text{mass(lepton)}} &= -\frac{v}{\sqrt{2}} \delta Y_{ij}^e \bar{e}_{Li} e_{Rj} \\ &\quad - \frac{v}{\sqrt{2}} Y_{ij}^e \bar{e}_{Li} \sum_b (\delta Z^e)_{jb} e_{Rb} \\ &\quad - \frac{v}{\sqrt{2}} Y_{ij}^e \sum_a \bar{e}_{La} \gamma^0 (\delta Z^e)_{ai}^\dagger \gamma^0 e_{Rj} + \text{h.c.}\end{aligned}\quad (2.10)$$

The calculation for the Fermion–Higgs Lagrangian is analogous. The counterterm contributions to the Fermion–Higgs Lagrangian are:

$$\begin{aligned}
\delta \mathcal{L}_{\text{FH(up-type)}} = & -\frac{1}{\sqrt{2}} Y_{ij}^u \sum_n (\delta_h)_{1n} h_n \bar{u}_{Li} u_{Rj} \\
& -\frac{1}{\sqrt{2}} \delta Y_{ij}^u h_1 \bar{u}_{Li} u_{Rj} \\
& -\frac{1}{\sqrt{2}} Y_{ij}^u h_1 \bar{u}_{Li} \sum_b (\delta Z^u)_{jb} u_{Rb} \\
& -\frac{1}{\sqrt{2}} Y_{ij}^u h_1 \sum_a \bar{u}_{La} \gamma^0 (\delta Z^u)_{ai}^\dagger \gamma^0 u_{Rj} + \text{h.c.},
\end{aligned} \tag{2.11}$$

$$\begin{aligned}
\delta \mathcal{L}_{\text{FH(down-type)}} = & -\frac{1}{\sqrt{2}} Y_{ij}^d \sum_n (\delta_h)_{1n} h_n \bar{d}_{Li} d_{Rj} \\
& -\frac{1}{\sqrt{2}} \delta Y_{ij}^d h_1 \bar{d}_{Li} d_{Rj} \\
& -\frac{1}{\sqrt{2}} Y_{ij}^d h_1 \bar{d}_{Li} \sum_b (\delta Z^d)_{jb} d_{Rb} \\
& -\frac{1}{\sqrt{2}} Y_{ij}^d h_1 \sum_a \bar{d}_{La} \gamma^0 (\delta Z^d)_{ai}^\dagger \gamma^0 d_{Rj} + \text{h.c.},
\end{aligned} \tag{2.12}$$

and

$$\begin{aligned}
\delta \mathcal{L}_{\text{FH(lepton)}} = & -\frac{1}{\sqrt{2}} Y_{ij}^e \sum_n (\delta_h)_{1n} h_n \bar{e}_{Li} e_{Rj} \\
& -\frac{1}{\sqrt{2}} \delta Y_{ij}^e h_1 \bar{e}_{Li} e_{Rj} \\
& -\frac{1}{\sqrt{2}} Y_{ij}^e h_1 \bar{e}_{Li} \sum_b (\delta Z^e)_{jb} e_{Rb} \\
& -\frac{1}{\sqrt{2}} Y_{ij}^e h_1 \sum_a \bar{e}_{La} \gamma^0 (\delta Z^e)_{ai}^\dagger \gamma^0 e_{Rj} + \text{h.c.}.
\end{aligned} \tag{2.13}$$

The Goldstone counterterm Lagrangian is obtained in the same way. For the neutral Goldstone terms, we get

$$\begin{aligned}
\delta \mathcal{L}_{\text{G(up-type)}} = & -\frac{i}{\sqrt{2}} Y_{ij}^u \sum_n (\delta_h)_{2n} h_n \bar{u}_{Li} u_{Rj} \\
& -\frac{i}{\sqrt{2}} \delta Y_{ij}^u h_2 \bar{u}_{Li} u_{Rj} \\
& -\frac{i}{\sqrt{2}} Y_{ij}^u h_2 \bar{u}_{Li} \sum_b (\delta Z^u)_{jb} u_{Rb} \\
& -\frac{i}{\sqrt{2}} Y_{ij}^u h_2 \sum_a \bar{u}_{La} \gamma^0 (\delta Z^u)_{ai}^\dagger \gamma^0 u_{Rj} + \text{h.c.},
\end{aligned} \tag{2.14}$$

$$\begin{aligned}
\delta \mathcal{L}_{\text{G(down-type)}} = & -\frac{i}{\sqrt{2}} Y_{ij}^d \sum_n (\delta_h)_{2n} h_n \bar{d}_{Li} d_{Rj} \\
& -\frac{i}{\sqrt{2}} \delta Y_{ij}^d h_2 \bar{d}_{Li} d_{Rj} \\
& -\frac{i}{\sqrt{2}} Y_{ij}^d h_2 \bar{d}_{Li} \sum_b (\delta Z^d)_{jb} d_{Rb} \\
& -\frac{i}{\sqrt{2}} Y_{ij}^d h_2 \sum_a \bar{d}_{La} \gamma^0 (\delta Z^d)_{ai}^\dagger \gamma^0 d_{Rj} + \text{h.c.},
\end{aligned} \tag{2.15}$$

and

$$\begin{aligned}
\delta \mathcal{L}_{\text{G(lepton)}} = & -\frac{i}{\sqrt{2}} Y_{ij}^e \sum_n (\delta_h)_{2n} h_n \bar{e}_{Li} e_{Rj} \\
& -\frac{i}{\sqrt{2}} \delta Y_{ij}^e h_2 \bar{e}_{Li} e_{Rj} \\
& -\frac{i}{\sqrt{2}} Y_{ij}^e h_2 \bar{e}_{Li} \sum_b (\delta Z^e)_{jb} e_{Rb} \\
& -\frac{i}{\sqrt{2}} Y_{ij}^e h_2 \sum_a \bar{e}_{La} \gamma^0 (\delta Z^e)_{ai}^\dagger \gamma^0 e_{Rj} + \text{h.c.}.
\end{aligned} \tag{2.16}$$

The charged Goldstone terms mix different fermion types. Their counterterm Lagrangians are

$$\begin{aligned}
\delta \mathcal{L}_{\text{Gdu}} = & Y_{ij}^u (\delta h_3^\pm) h_3^- \bar{d}_{Li} u_{Rj} + \delta Y_{ij}^u h_3^- \bar{d}_{Li} u_{Rj} \\
& + Y_{ij}^u h_3^- \bar{d}_{Li} \sum_b (\delta Z^u)_{jb} u_{Rb} \\
& + Y_{ij}^u h_3^- \sum_a \bar{d}_{La} \gamma^0 (\delta Z^d)_{ai}^\dagger \gamma^0 u_{Rj} + \text{h.c.},
\end{aligned} \tag{2.17}$$

$$\begin{aligned}
\delta \mathcal{L}_{\text{Gud}} = & -Y_{ij}^d (\delta h_3^\pm) h_3^+ \bar{u}_{Li} d_{Rj} - \delta Y_{ij}^d h_3^+ \bar{u}_{Li} d_{Rj} \\
& -Y_{ij}^d h_3^+ \bar{u}_{Li} \sum_b (\delta Z^d)_{jb} d_{Rb} \\
& -Y_{ij}^d h_3^+ \sum_a \bar{u}_{La} \gamma^0 (\delta Z^u)_{ai}^\dagger \gamma^0 d_{Rj} + \text{h.c.},
\end{aligned} \tag{2.18}$$

and

$$\begin{aligned}
\delta \mathcal{L}_{\text{Gve}} = & -Y_{ij}^e (\delta h_3^\pm) h_3^+ \bar{\nu}_{Li} e_{Rj} - \delta Y_{ij}^e h_3^+ \bar{\nu}_{Li} e_{Rj} \\
& -Y_{ij}^e h_3^+ \bar{\nu}_{Li} \sum_b (\delta Z^e)_{jb} e_{Rb} \\
& -Y_{ij}^e h_3^+ \sum_a \bar{\nu}_{La} \gamma^0 (\delta Z^e)_{ai}^\dagger \gamma^0 e_{Rj} + \text{h.c.}.
\end{aligned} \tag{2.19}$$

Thus, we have obtained the counterterm Lagrangian for the Standard Model Yukawa sector. From these terms we can derive the counterterm Feynman rules. Since the Lagrangian has the structure

$$\mathcal{L}^0 = \mathcal{L} + \delta \mathcal{L}, \tag{2.20}$$

the counterterm terms are read from  $\delta \mathcal{L}$ .

For the mass counterterm, the Feynman rule is

$$b \text{ --- } \times \text{ --- } a = -i \frac{v}{\sqrt{2}} \left( \delta Y_{ab}^\psi + \sum_i Y_{ib}^\psi \gamma^0 (\delta Z^\psi)_{ai}^\dagger \gamma^0 + \sum_j Y_{aj}^\psi (\delta Z^\psi)_{jb} \right) P_R + \text{h.c.} \quad (2.21)$$

For the physical Higgs and neutral Goldstone counterterms, the Feynman rule can be written in the combined form

$$\begin{aligned}
 h_k \text{ --- } \times \begin{array}{l} \nearrow \bar{\psi}_a \\ \searrow \psi_b \end{array} &= -\frac{1}{\sqrt{2}} \sum_{n=1,2} C_n \left( \delta Y_{ab}^\psi \delta_{nk} + \sum_i Y_{ib}^\psi \gamma^0 (\delta Z^\psi)_{ai}^\dagger \gamma^0 \delta_{nk} \right. \\
 &\quad \left. + \sum_j Y_{aj}^\psi (\delta Z^\psi)_{jb} \delta_{nk} + Y_{ab}^\psi (\delta_h)_{nk} \right) P_R + \text{h.c.}, \quad (2.22)
 \end{aligned}$$

where  $C_n = \{1, i\}$ . The first and second element applies to the physical Higgs and the neutral Goldstone counterterms, respectively.

For the charged Goldstone counterterms, the Feynman rule can be written in the compact form

$$h_3^\pm \text{ --- } \times \begin{array}{l} \nearrow \bar{\psi}_a \\ \searrow \chi_b \end{array} = \mp i \left( Y_{ab}^\chi \delta_{h_3^\pm} + \delta Y_{ab}^\chi + \sum_i Y_{ib}^\chi \gamma^0 (\delta Z^\psi)_{ai}^\dagger \gamma^0 + \sum_j Y_{aj}^\chi (\delta Z^\chi)_{jb} \right) P_R + \text{h.c.} \quad (2.23)$$

Here  $\psi$  and  $\chi$  denote different fermion types, while  $\delta Z^\psi$  and  $\delta Z^\chi$  are the field counterterms of the outgoing and incoming fermion fields, respectively. The upper or lower sign is chosen according to the corresponding charged Goldstone term in the expanded Yukawa Lagrangian.

### 3 On-Shell renormalization and S.Draukšas scheme

In section 1.5.1, we talked about the general concept of renormalization. However, doing proper calculations for our theories becomes much harder in such a general form. Therefore, the next step in renormalization is to choose renormalization conditions that help do this, these combinations of conditions are also called renormalization schemes. In the aforementioned section, we used the  $\overline{\text{MS}}$  scheme. But for our calculations, the scheme we will use is the On-Shell (OS) renormalization scheme and this sections will detail it's conditions [5, 6].

The OS renormalization schemes main feature is that the parameters of a quantum field theory are defined directly in terms of physical observables. In particular, particle masses are identified with the physical pole masses of propagators. The fields then are normalized such that propagator residues equal unity. This is necessary, because the field describes a physical asymptotic particle, whilst a physical one-particle state must have unit probability amplitude to be created or annihilated by the field operator. The couplings are fixed by matching scattering amplitudes to experimentally measurable quantities.

From the statements above, we can get the renormalization conditions for the propagators. First, these conditions depend on the particles involved, this is because there is a possibility of particle mixing, for example, there might be a change in flavour of the particles etc. Therefore, we must account for this and we do so by separating the conditions into two types, one that works with cases where there is no mixing involved, called the diagonal case, and one that deals with possible mixing, which is called the off-diagonal case. The diagonal case has the diagonal pole mass condition that is used to renormalize the mass. This condition states:

$$\sum_{k,l} \bar{\mathcal{Z}}_{ik} \Sigma_{kl}^{Rn}(\not{p}) \mathcal{Z}_{li} u_i = 0 \quad (3.1)$$

where  $i, j, k$  and  $l$  are flavour indices, with  $k$  and  $l$  being summed over and  $\bar{\mathcal{Z}}$  and  $\mathcal{Z}$  are the Lehmann–Symanzik–Zimmermann [18] (LSZ) factors for the outgoing and incoming particles respectively, as well as  $u_i$  is the Dirac spinor. Also  $\Sigma^{Rn}$  is the renormalized self-energy, which comes from the inverse propagator of the particle.

Having the mass renormalization condition, we can arrive at the field renormalization condition for the diagonal case. As stated at the start of the section, the field terms must have a residue that approaches unity, therefore we get the condition:

$$\lim_{p \rightarrow m_i} \frac{1}{\not{p} - m_i} \sum_{k,l} \bar{\mathcal{Z}}_{ik} \Sigma_{kl}^R(\not{p}) \mathcal{Z}_{li} u_i = u_i, \quad (3.2)$$

the LSZ factors are there to ensure the unit residue condition.

Then moving on to the off-diagonal conditions, where one would see the before mentioned particle mixing. However, we want our particle propagator to be diagonal, thus this in turn gives us that the off-diagonal condition should cancel the possible terms that would give this mixing.

This condition is:

$$\sum_{k,l} \bar{\mathcal{Z}}_{jk} \Sigma_{kl}^{Rn}(\not{p}) \mathcal{Z}_{li} u_i = 0 \quad (3.3)$$

and

$$\sum_{k,l} \bar{u}_j \bar{\mathcal{Z}}_{jk} \Sigma_{kl}^{Rn}(\not{p}) \mathcal{Z}_{li} = 0. \quad (3.4)$$

Now by having these conditions, one would think that we just apply them and derive our counterterms, however, there arises a problem. Usually the OS renormalization scheme requires that the LSZ factors be set to be unit matrices so their role would be fulfilled by the field renormalization constants. But absorptive parts from the loop functions appear, when internal particles in the loop integrals hit physical thresholds, which means that the external particle is unstable or shows that there is a dampening effect on the propagator. If we were to arbitrarily apply this condition, meaning we set  $\bar{\mathcal{Z}} = \mathcal{Z} = \mathbb{I}$  and  $\delta m_{ji} = \delta m_{ii} \delta_{ji}$ , we would get that our field renormalization constants  $Z_{L,R}$ ,  $Z_{L,R}^\dagger$  and  $Z_{L,R}^\dagger$  are written in terms of self-energy scalar functions. However, these renormalization terms would imply that they are hermitian conjugates of one another. But when we account for the absorptive parts we mentioned before, we see that the field renormalization terms are not related by hermitian conjugation. Simply put, this is an over-specification problem.

This over-specification problem has a lot of possible solutions, from relaxing the non-mixing conditions we stated before or to use two independent field renormalization constants that separately associated to particle and anti-particle fields. However, non of the possible solutions are satisfactory, thus from this point on we will use a variant of OS renormalization - the S.Draukšas (SD) [5]. By using this scheme, we include the absorptive parts as much as possible as well as keeping the Lagrangian of our sector hermitian. This is achieved by only setting  $\mathcal{Z} = \mathbb{I}$  and keeping  $\bar{\mathcal{Z}} \neq \mathbb{I}$ , thus having the absorptive parts fully in the latter part. This choice is also usually called the incoming renormalization scheme [19], because we focus mainly on the incoming propagator leg. Another interesting point is that if the incoming particles mass is below the production threshold, the problem arising from the absorptive parts is void, due to them not existing in such a case. For the mass counterterm conditions do not have the same problem. This is due to them being fully hermitian thus fully appropriating the absorptive parts irrespective of our choice of the LSZ factors.

However, we must point out, that the SD scheme use is not specifically chosen because of the absorptive parts. It is an advantage of the scheme that these parts are accounted for. But the main reason we choose this scheme is because it explicitly deals with the off-diagonal contributions to our counterterms. Meaning, it does not set the off-diagonal mass counterterms to zero, like other similar OS schemes [20].

In summary, We will use the SD scheme. This means we will choose the incoming variant of renormalization and set the incoming LSZ factor to unity  $\mathcal{Z} = \mathbb{I}$ . By doing so we get updated renormalization conditions, which we will use in the next section. The diagonal mass condition becomes:

$$\Sigma_{ii}^{Rn}(\not{p}) u_i = 0. \quad (3.5)$$

Then the diagonal residue condition becomes:

$$\lim_{p \rightarrow m_i} \frac{1}{\not{p} - m_i} \Sigma_{ii}^{Rn}(\not{p}) u_i = u_i. \quad (3.6)$$

Finally, the off-diagonal renormalization condition becomes:

$$\Sigma_{ji}^{Rn}(\not{p}) u_i = 0. \quad (3.7)$$

From these equations, one might think that we also set  $\tilde{\mathcal{Z}} = \mathbb{I}$ , however this is not the case. Because we chose to use the incoming renormalization scheme, we only work with the incoming propagator and leave the outgoing propagator untouched. Meaning, all the renormalization conditions are only applied to the incoming propagator.

## 4 Renormalization of two-point functions in the quark sector

As stated in previous sections, to renormalize the Yukawa constants, we need to renormalize the three-point function for the Higgs boson. This is due to the fact that the Yukawa constants come into play through the Higgs doublets interactions with the particles. This can be seen in Eq. (2.1). We could derive the renormalization constant explicitly, however this would be a laborious and error-prone. We avert it by analysing the structure of Eq. (2.21), where we can see that the counterterm Feynman rules for the Higgs doublet elements have the particle field and mass renormalization counterterms in them. These counterterms come from the two-point functions, also called particle inverse propagators.

Thus we proceed with two-point function renormalization. As we presented about in section 1.5.2, we need the self-energy for the quark propagators. The self-energies of the fermion propagators can be shown diagrammatically as:

$$i\Sigma_{ji}(p^2) = \text{diagram 1} + \text{diagram 2} . \quad (4.1)$$

We then decompose this self-energy into:

$$\Sigma_{ji}(\not{p}) = \Sigma_{ji}^L(p^2) \not{p} P_L + \Sigma_{ji}^R(p^2) \not{p} P_R + \Sigma_{ji}^{sL}(p^2) P_L + \Sigma_{ji}^{sR}(p^2) P_R. \quad (4.2)$$

The next step is to renormalize this self-energy. By doing so we will connect the self-energy we get from loop correction with the renormalization counterterms. From the way we define the Lagrangian of the Yukawa sector of the SM, we can already see what form the renormalized self-energy will take. However, unlike in the previous section, we will not decompose the field and mass counterterms by chirality, but keep them combined. By doing so we have that our renormalized self-energy is:

$$\Sigma_{ji}^{Rn}(\not{p}) = \Sigma_{ji}(\not{p}) + \gamma^0 \delta Z_{ji}^\dagger \gamma^0 (\not{p} - m_i) + (\not{p} - m_j) \delta Z_{ji} - \delta m_{ji}. \quad (4.3)$$

By having the renormalized self-energy, we can now apply the conditions we defined in section 3. Starting with the diagonal mass condition in Eq. (3.5). By applying this condition on the renormalized self-energy in Eq. (4.3), we get that our diagonal mass counterterm is:

$$\delta m_{ii} = \frac{1}{2} \text{Re} \left[ m_i \Sigma_{ii}^R(m_i^2) + m_i \Sigma_{ii}^R(m_i^2) + \Sigma_{ii}^{sL}(m_i^2) + \Sigma_{ii}^{sR}(m_i^2) \right]. \quad (4.4)$$

Then to derive the diagonal field counterterms, we apply the residue condition on the renorma-

lized self-energy. By doing so we get:

$$\begin{aligned} \delta Z_{ii} = & \left( -\Sigma_{ji}^L(m_i^2) - \partial_{p^2} \left( m_i^2 \Sigma_{ji}^L(p^2) + m_i^2 \Sigma_{ji}^R(p^2) + m_i \Sigma_{ji}^{sL}(p^2) + m_i \Sigma_{ji}^{sR}(p^2) \right) \right) \Big|_{p^2=m_i^2} P_L \\ & + \left( -\Sigma_{ji}^R(m_i^2) - \partial_{p^2} \left( m_i^2 \Sigma_{ji}^L(p^2) + m_i^2 \Sigma_{ji}^R(p^2) + m_i \Sigma_{ji}^{sL}(p^2) + m_i \Sigma_{ji}^{sR}(p^2) \right) \right) \Big|_{p^2=m_i^2} P_R. \end{aligned} \quad (4.5)$$

Now we move to the off-diagonal case. We apply the off-diagonal renormalization condition and get:

$$[(m_i^2 - m_j^2) \delta Z_{ji} - m_j \delta m_{ji} - \delta m_{ji}^\dagger m_i] u_i = -(\not{p} + m_j) \Sigma_{ji}(\not{p}) u_i, \quad (4.6)$$

which gives us the relation between off-diagonal field counterterms and the off-diagonal mass [5, 7]. If we apply hermitian conjugation, we can derive the hermitian and anti-hermitian field counterterm relations:

$$[(m_i^2 - m_j^2) \delta Z_{ji}^A - m_j \delta m_{ji} - \delta m_{ji}^\dagger m_i] u_i = -(\not{p} + m_j) \Sigma_{ji}(\not{p}) u_i + \text{h.c.} \quad (4.7)$$

$$[(m_i^2 - m_j^2) \delta Z_{ji}^H - m_j \delta m_{ji} - \delta m_{ji}^\dagger m_i] u_i = -(\not{p} + m_j) \Sigma_{ji}(\not{p}) u_i - \text{h.c.} \quad (4.8)$$

, where the h.c on the right-hand sides of the equations gives the hermitian conjugate of the first terms.

By applying the renormalization condition for the off-diagonal case, we should get the mass and field counterterms. But when we look at Eq. (4.7), a complication emerges. When we do so, the condition can only be used to fix the off-diagonal mass or the off-diagonal field counterterms but not both. Meaning, we have that the equation has two variables we need to solve for simultaneously, but we only have one constraint. This problem shows why most renormalization schemes set the off-diagonal mass to zero, but this only moves the problem to the CKM matrix renormalization side, which again is not ideal. Thus to go around this problem, as given in the SD scheme, we define the anti-hermitian part of the off-diagonal field counterterms as:

$$\begin{aligned} \delta Z_{ji}^A \equiv & -\frac{1}{2} \left( m_i^2 \Sigma_{ji}^L(m_i^2) + m_i m_j \Sigma_{ji}^R(m_i^2) + m_j \Sigma_{ji}^{sL}(m_i^2) + m_i \Sigma_{ji}^{sR}(m_i^2) + \text{h.c.} \right) P_L \Big|_{m_i^2-m_j^2} \\ & -\frac{1}{2} \left( m_i^2 \Sigma_{ji}^R(m_i^2) + m_i m_j \Sigma_{ji}^L(m_i^2) + m_j \Sigma_{ji}^{sR}(m_i^2) + m_i \Sigma_{ji}^{sL}(m_i^2) + \text{h.c.} \right) P_R \Big|_{m_i^2-m_j^2}, \end{aligned} \quad (4.9)$$

where they are defined as the coefficient of the  $(m_i^2 - m_j^2)$  mass structure. This definition is supported by the structure of UV divergences and Nielsen identities [21, 22].

Now by having the definition of the anti-hermitian part of the off-diagonal field counterterm, we can also define the hermitian part as well:

$$\begin{aligned} \delta Z_{ji}^H = & -\frac{1}{2} \frac{1}{m_i^2 - m_j^2} \left( m_i^2 \Sigma_{ji}^L(m_i^2) + m_i m_j \Sigma_{ji}^R(m_i^2) + m_j \Sigma_{ji}^{sL}(m_i^2) + m_i \Sigma_{ji}^{sR}(m_i^2) - \text{h.c.} \right) P_L \\ & -\frac{1}{2} \frac{1}{m_i^2 - m_j^2} \left( m_i^2 \Sigma_{ji}^R(m_i^2) + m_i m_j \Sigma_{ji}^L(m_i^2) + m_j \Sigma_{ji}^{sR}(m_i^2) + m_i \Sigma_{ji}^{sL}(m_i^2) - \text{h.c.} \right) P_R. \end{aligned} \quad (4.10)$$

By having the off-diagonal field counterterms defined, we then only need to use the relation in Eq. (4.6) to fix the off-diagonal mass counterterm. By doing so, we get:

$$\begin{aligned}
\delta m_{ji} = & \frac{1}{2} \left( m_j \Sigma_{ji}^L(m_j^2) + m_i \Sigma_{ji}^R(m_i^2) + \Sigma_{ji}^{sL}(m_i^2) + \Sigma_{ji}^{sL}(m_j^2) \right) P_L \\
& + \frac{1}{2} \left( m_j \Sigma_{ji}^R(m_j^2) + m_i \Sigma_{ji}^L(m_i^2) + \Sigma_{ji}^{sR}(m_i^2) + \Sigma_{ji}^{sR}(m_j^2) \right) P_R \\
& - m_j \delta Z_{ji}^A + m_i \gamma^0 \delta Z_{ji}^A \gamma^0.
\end{aligned} \tag{4.11}$$

## 5 Renormalization of Higgs two-point function

In the previous section, we renormalized the two-point functions of the quark sector, however to fully renormalize the Higgs–fermion vertex we also need to renormalize the Higgs two-point function. The self-energy of the Higgs propagator, can be shown diagrammatically:

$$i\Pi_{hh}(p^2) = \text{---} h \text{---} \textcircled{\text{IPI}} \text{---} h + \begin{array}{c} \textcircled{\text{IPI}} \\ | \\ \text{---} h \text{---} h \text{---} \end{array}. \quad (5.1)$$

This gives us that the renormalized Higgs two-point function is:

$$\Pi_{hh}^R(p^2) = \Pi_{hh}(p^2) + 2 \left( p^2 - m_h^2 \right) \delta Z_h - \delta m_h, \quad (5.2)$$

Now we move to the traditional On-shell renormalization conditions. Firstly, we again require that the renormalized masses be the On-shell pole masses. This means we use the On-shell mass condition:

$$\Pi_{hh}^R(m_h^2) = 0, \quad (5.3)$$

which fixes the Higgs mass counterterm:

$$\delta m_h = \Pi_{hh}(m_h^2). \quad (5.4)$$

Having fixed the Higgs mass counterterm, we move to the unit residue renormalization condition:

$$\left. \frac{\partial \Pi_{hh}^R(p^2)}{\partial p^2} \right|_{p^2=m_h^2} = 0. \quad (5.5)$$

This results in the Higgs field counterterm being fixed as:

$$\delta Z_h = -\frac{1}{2} \frac{\partial \Pi_{hh}(p^2)}{\partial p^2} \Big|_{p^2=m_h^2}. \quad (5.6)$$

Having derived the mass and field counterterms for our simplified GNM Yukawa sector, we see that these counterterms are diagonal. Meaning, there is no off-diagonal mixing for the Higgs boson in this case. If we were to work in the full GNM Yukawa sector, we would also need to account for the off-diagonal contributions, which for the Higgs would be it's mixing with the  $Z$  and neutral Goldstone  $G^0$  bosons. This would be done in a similar way that was done for the gauge bosons in [7].

## 6 Yukawa counterterm

By renormalizing the two-point functions and deriving the required counterterms, we can move on Yukawa counterterm derivation. However, by looking at the Yukawa counterterm Feynman rule in Eq. (C), we need to have the full field counterterms of the two-point functions, but in the previous sections we have our counterterms split in diagonal, off-diagonal, hermitian and anti-hermitian parts. Thus first we need to combine them to get the full counterterm. Note that: we will relabel the external leg indices in the counterterm Feynman rule from  $a$  and  $b$  to  $j$  and  $i$  respectively moving forward to match the later mentioned explicit computations.

We start with the combination of the fermion off-diagonal field hermitian and anti-hermitian counterterms. The hermitian and anti-hermitian field counterterm follow the decomposition rules:

$$\delta Z_{ji}^A = \frac{1}{2} \left( \delta Z_{ji} - \delta Z_{ji}^\dagger \right), \quad \delta Z_{ji}^H = \frac{1}{2} \left( \delta Z_{ji} + \delta Z_{ji}^\dagger \right), \quad (6.1)$$

where  $\delta Z_{ji}$  is the full off-diagonal field counterterm and  $\delta Z_{ji}^\dagger$  is its hermitian conjugate. By using these rules, we get that our full off-diagonal field counterterm:

$$\delta Z_{ji} = \delta Z_{ji}^H + \delta Z_{ji}^A. \quad (6.2)$$

Now by having the full off-diagonal field contribution, we can derive the full field counterterm by connecting the diagonal and off-diagonal contributions:

$$\delta Z_{ji} = \delta_{ji} \delta Z_{ii} + (1 - \delta_{ji}) \delta Z_{ji}. \quad (6.3)$$

Having fixed the fermion field counterterms, we move to the Higgs field counterterm. However because the counterterm is diagonal, the field counterterm in Eq. (5.6) is already the full field counterterm.

Now having the full field counterterm for our propagators, we now renormalize the Higgs-fermion vertex. The renormalized vertex receives contributions from the bare vertex, the one-loop triangle diagram, and the counterterm insertion, which can be shown diagrammatically as:

$$i\mathcal{M}_{h\bar{f}_j f_i}^R = \text{diagram 1} + \text{diagram 2} + \text{diagram 3}. \quad (6.4)$$

The blob denotes the one-loop 1PI contribution and the crossed dot denotes the counterterm vertex. By using the Feynman rules derived from the Yukawa sector Lagrangian in Eq. (2.1), the

tree-level  $h f \bar{f}$  vertex is

$$\begin{array}{c} \bar{f}_j \\ \swarrow \\ h \text{ --- } \bullet \\ \searrow \\ f_i \end{array} = -i \frac{m_f}{v} \delta_{ji}, \quad (6.5)$$

where  $v$  is the vacuum expectation value and  $m_f$  is the physical fermion mass. The Yukawa coupling is therefore not an independent parameter but is fixed entirely by  $m_f$  and  $v$ , as discussed in Section 1.4.3.

From Eq. (6.4), we can derive the renormalized vertex, which in a compact form is:

$$\Gamma_{h\bar{f}_j f_i}^R = \Gamma_{h\bar{f}_j f_i}^{\text{tree}} + \Gamma_{h\bar{f}_j f_i}^{\text{loop}} - \frac{1}{\sqrt{2}} \delta Y_{h\bar{f}_j f_i} - \frac{1}{\sqrt{2}} Y_{h\bar{f}_j f_i} \left( \delta Z_h + \frac{1}{2} \gamma^0 \delta Z_{\bar{f}_j f_i}^\dagger \gamma^0 + \frac{1}{2} \delta Z_{\bar{f}_j f_i} \right). \quad (6.6)$$

By having defined the renormalized vertex, we see the structure that we talked about in previous sections, on how the Yukawa counterterm can be extracted from renormalizing the vertex and fixing the propagator field counterterms. The propagator counterterms then must cancel the UV divergent terms of the renormalized vertex, so that:

$$\Gamma_{h\bar{f}_j f_i}^R \Big|_{\text{UV}} = 0. \quad (6.7)$$

This allows us to define an equation for the Yukawa counterterm:

$$\left[ \Gamma_{h\bar{f}_j f_i}^{\text{loop}} - \frac{1}{\sqrt{2}} \delta Y_{h\bar{f}_j f_i} - \frac{1}{\sqrt{2}} Y_{h\bar{f}_j f_i} \left( \delta Z_h + \frac{1}{2} \gamma^0 \delta Z_{\bar{f}_j f_i}^\dagger \gamma^0 + \frac{1}{2} \delta Z_{\bar{f}_j f_i} \right) \right] \Big|_{\text{UV}} = 0. \quad (6.8)$$

From this equation we get that the Yukawa counterterm is:

$$\delta Y_{h\bar{f}_j f_i} = \left[ \sqrt{2} \Gamma_{h\bar{f}_j f_i}^{\text{loop}} - Y_{h\bar{f}_j f_i} \left( \delta Z_h + \frac{1}{2} \gamma^0 \delta Z_{\bar{f}_j f_i}^\dagger \gamma^0 + \frac{1}{2} \delta Z_{\bar{f}_j f_i} \right) \right] \Big|_{\text{UV}}. \quad (6.9)$$

For the Yukawa constant we see in this equation, we use it's relation to the fermion mass:

$$m_f = \frac{Y_{h\bar{f}_j f_i} v}{\sqrt{2}}. \quad (6.10)$$

To check if the computed Yukawa counterterm is valid, we first check if all the appropriate UV divergent poles cancelled out and compare this counterterm with the Yukawa counterterm we get from the mass relation – the mass counterterm and the vacuum expectation value:

$$\delta Y_{h\bar{f}_j f_i} = \frac{\sqrt{2} \delta m_f}{v}. \quad (6.11)$$

To do so, we did all the theoretical computations using the *Wolfram Mathematica* [23] software with the *FeynCalc* [24–27] package. The *FeynCalc* package allows us to do all the necessary

mathematics like Dirac algebra etc. We explicitly computed this for up-type quarks. All the explicit derivations for the two-point function counterterms, Higgs–up-type quark vertex and their UV divergent terms are given in Appendix C. The procedure was as follows: We computed the two-point function counterterms, then we computed the one-loop corrections  $\Gamma_{hu\bar{u}}^{\text{loop}}$ , then we evaluate the results for UV divergencies and use our Yukawa counterterm definition in Eq. (6.9). This gives us the Yukawa counterterm for the up-types quarks:

$$\begin{aligned} \delta Y_{ji}^u = & \sum_{k=1}^3 \left( - \frac{\pi^2 e^3 m_Z^2 (m_k^d)^2 \left( (5P_R - 2P_L)m_i^u + (5P_L - 2P_R)m_j^u \right) V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^*}{16\sqrt{2} m_W^3 \varepsilon_{\text{UV}} s_w (m_Z^2 - m_W^2)} \right. \\ & - \frac{\pi^2 e^3 m_Z^2 m_i^u \delta_{ij}}{16\sqrt{2} m_W^3 \varepsilon_{\text{UV}} s_w (m_Z^2 - m_W^2)} \\ & \times \left[ (m_k^d)^2 (3V_{ik}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* - 4) - 4((m_k^\ell)^2 + (m_k^u)^2) \right] \Bigg) \\ & + \delta_{ij} \left( \frac{\pi^2 e^3 m_Z^2 m_i^u (9(m_i^u)^2 - 10m_W^2 - 17m_Z^2)}{24\sqrt{2} m_W^3 \varepsilon_{\text{UV}} s_w (m_Z^2 - m_W^2)} - \frac{3\pi^2 e (N_c^2 - 1) g_s^2 m_i^u}{2\sqrt{2} N_c m_W \varepsilon_{\text{UV}} s_w} \right) \end{aligned} \quad (6.12)$$

where  $\frac{1}{\varepsilon_{\text{UV}}}$  is the UV divergent pole and  $V^{\text{CKM}}$  is the Cabibbo–Kobayashi–Maskawa [15, 16] matrix.

To be able to compare this counterterm to the Yukawa counterterm we get from the mass counterterm, as stated in Eq. (6.11), we also need the full mass counterterm, thus we have to give the diagonal mass and off-diagonal mass counterterms the same treatment we did to the field counterterms, meaning:

$$\delta m = \delta_{ji} \delta m_{ii} + (1 - \delta_{ji}) \delta m_{ji}. \quad (6.13)$$

Now we can compute the Yukawa counterterm we would get from the mass relation:

$$\begin{aligned} \delta Y_{ji}^{u,\text{mass}} = & \sum_{k=1}^3 \left( - \frac{3\pi^2 e^3 m_Z^2 (m_k^d)^2 \left( P_R m_i^u + P_L m_j^u \right) V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^*}{8\sqrt{2} m_W^3 \varepsilon_{\text{UV}} s_w (m_Z^2 - m_W^2)} \right. \\ & + \frac{\pi^2 e^3 m_Z^2 \delta_{ij} m_i^u}{m_h^2 m_W^3 \varepsilon_{\text{UV}} \sqrt{2} s_w (m_Z^2 - m_W^2)} \sum_{c=1}^{N_c} \left( (m_k^d)^4 + (m_k^u)^4 \right) \\ & + \frac{\pi^2 e^3 m_Z^2 \delta_{ij} m_i^u (m_k^\ell)^4}{m_h^2 m_W^3 \varepsilon_{\text{UV}} \sqrt{2} s_w (m_Z^2 - m_W^2)} \Bigg) \\ & + \delta_{ij} \left( \frac{\pi^2 e^3 m_Z^2}{24\sqrt{2} m_h^2 m_W^3 \varepsilon_{\text{UV}} m_i^u m_j^u s_w (m_Z^2 - m_W^2)} \right. \\ & \times \left[ 3(3P_R + 2P_L) m_h^2 (m_i^u)^4 m_j^u + 3P_L m_h^2 m_i^u (m_W^2 (m_j^u)^2 + (m_j^u)^4 - 2m_W^4) \right. \\ & - (m_i^u)^2 m_j^u (m_h^2 (8m_Z^2 - (8P_R + 5P_L) m_W^2) + 9m_h^4 + 18(2m_W^4 + m_Z^4)) \\ & \left. \left. + 6P_L m_h^2 m_W^4 m_j^u \right] - \frac{3\pi^2 e (N_c^2 - 1) g_s^2 m_i^u}{2\sqrt{2} N_c m_W \varepsilon_{\text{UV}} s_w} \right). \end{aligned} \quad (6.14)$$

Comparing the two expressions for the Yukawa counterterm, we can see several differences: the mass relation counterterm  $\delta Y_{ji}^{u,\text{mass}}$  includes the Higgs boson mass  $m_h$  throughout, while  $\delta Y_{ji}^u$  does not. The mass relation also involves fourth powers of the fermion masses, such as  $(m_i^u)^4$ ,  $(m_j^u)^4$ ,  $(m_k^d)^4$ , and  $(m_k^u)^4$ , whereas the vertex expression uses only second powers. Additionally, the term that includes the CKM matrix in both expressions differs by a factor of  $\frac{1}{2}$ . However, both expressions share an identical QCD term, which involves  $(N_c^2 - 1)g_s^2$ .

By our hypothesis, this should not happen and we should have gotten identical expressions, but we did not. However, this does not mean our hypothesis is inherently wrong. The issue is in our way of renormalization. As stated in section 1.5, to fully renormalize the theory, we need to renormalize all of the connected parameter, which we did not do. This means the difference in the Yukawa counterterm expressions should come from the lack of the renormalization for the vacuum expectation value.

## Conclusion

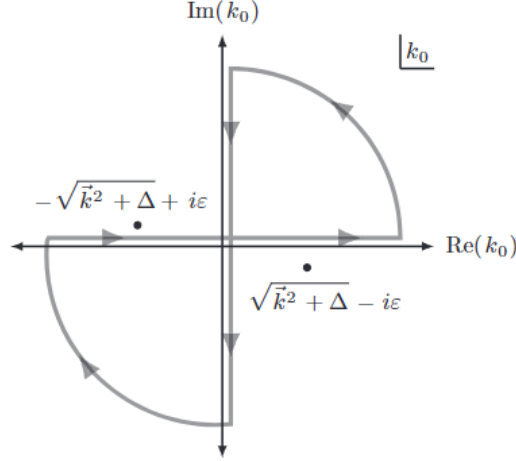
This thesis' goal was to determine whether the SD scheme could be used for the renormalization of the Yukawa sector in the Grimus–Neufeld model's Standard Model section. In order to achieve this, we used the SD scheme to renormalize the quark and Higgs two-point functions, derived the counterterm Feynman rules for the Higgs–fermion vertex, and explicitly computed the Yukawa counterterm for up-type quarks using *FeynCalc* [24–27].

The idea was to use the field counterterms of the two-point functions to cancel the UV divergences of the one-loop vertex correction in order to express the Yukawa counterterm from the renormalized Higgs–fermion vertex. The Yukawa counterterm would therefore consist of the residual UV-divergent structure, and it should coincide with the counterterm derived from the mass relation  $\delta Y_{ji} = \frac{\sqrt{2}\delta m_{ji}}{v}$ .

The explicit computations show the structural similarity but also the differences between the two expressions for the Yukawa counterterm, found in Eqs. (6.12) and (6.14). Only quark masses, CKM matrix elements, and gauge boson masses are present in the vertex-derived counterterm. These are also a part of the mass-relation counterterm, however it also includes higher powers of the fermion masses, the Higgs boson mass  $m_h$ , etc. Also terms that involve the CKM matrix in the expressions differ by a factor of  $\frac{1}{2}$ . But the term with  $(N_c^2 - 1)g_s^2$ , which is the QCD term, in both expressions is identical.

The mismatch may be due to the renormalization carried out in this thesis being incomplete. As discussed in Section 1.5, a fully consistent renormalization requires that all connected parameters be renormalized simultaneously. In the thesis, only the fermion field and mass counterterms and the Higgs field and mass counterterms were renormalized. The terms present in the mass-relation counterterm but absent from the vertex counterterm likely correspond to the contributions that would arise from the missing vacuum expectation value  $v$  renormalization.

The key result of this thesis is therefore: the SD scheme could be applied to compute the two-point function field counterterms, and these counterterms could be inserted into the renormalized Higgs–fermion vertex to extract the Yukawa counterterm. The two expressions in Eqs. (6.12) and (6.14) having identical QCD terms as well as most terms having their denominators differ by a factor  $m_h$ , show that this procedure has potential and these expressions should agree when the renormalization is done in full. However, further derivations and computations were beyond the scope of this thesis.



4 pav. Integration contour

## A Wick rotations

This section is based on [11].

Before moving to analysing the integration process and the problems arising from using the cut-off, we first need to talk about Wick rotations. When we work with integrals that we get from applying Feynman rules for loop corrections, we get them in the form:

$$\int \frac{d^4 k}{(2\pi)^4} \frac{i}{(k^2 - X + i\epsilon)^n}, \quad (\text{A.1})$$

where  $i\epsilon$  is required to account for time ordering in particle correlation functions in Minkowski space-time. With it, our integral has poles at  $k_0 = \sqrt{\mathbf{k}^2 + X} - i\epsilon$  and  $k_0 = -\sqrt{\mathbf{k}^2 + X} + i\epsilon$ . We can see this in Figure 4. These poles are in the top-left and bottom-right quadrants of the  $k_0$  complex plane. Because of this, when integrating over the figure-eight contour shown in Figure 4, the integral vanishes. Therefore the integrals over the real and imaginary axis are equal and opposite, so we can apply a substitution  $k_0 \rightarrow ik_0$ . With this, we moved our integration from Minkowski space to Euclidean space. Thus we get that  $k^2 \rightarrow -k_0^2 - \mathbf{k}^2 = -k_E^2$ , where  $k_E^2 = k_0^2 + \mathbf{k}^2$  is the Euclidean momentum. This is called a Wick rotation. After applying it,  $i\epsilon$  will no longer have an effect and we can simply set it  $\epsilon = 0$ .

When the integral is Wick-rotated, we can evaluate it straightforwardly. We will use the formula for the surface area of a Euclidean 4-sphere:

$$\int d\Omega_4 = 2\pi^2. \quad (\text{A.2})$$

By applying it to the integration, we get:

$$\int \frac{d^4 k_E}{(2\pi)^4} f(k_E^2) = \frac{1}{16\pi^4} \int d\Omega_4 \int_0^\infty k_E^3 dk_E f(k_E^2) = \frac{1}{8\pi^4} \int d\Omega_4 \int_0^\infty k_E^3 dk_E f(k_E^2). \quad (\text{A.3})$$

## B Dimensional regularization

This section is based on [9] and [11].

As mentioned previously, due to the cut-off, there arises a problem with the integration process due to breaking of necessary symmetries. This can lead to problems when calculating amplitudes with external photons because it would give us incorrect predictions. Therefore it is common to use a method called dimensional regularization.

As we have seen integrals like:

$$\int_l \frac{d^4 l}{l^4} \quad (\text{B.1})$$

are UV divergent, however if we were to shift the dimension of such integrals in to  $D$  dimensions, we get that:

$$\int \frac{d^D l}{l^4}, \quad (\text{B.2})$$

which is finite when  $D < 4$ . To stay consistent, because our experiments are being done in 4 dimensional space-time, we also add a so-called mass scale  $\mu$ . This allows us to shift the integration:

$$\int_l \frac{d^4 l}{l^4} \rightarrow \mu^{4-D} \int \frac{d^D l}{l^4}. \quad (\text{B.3})$$

We could now evaluate the integral, however in this form it would be a complicated task. But as we showed in section A, we can apply a Wick rotation, to be able to evaluate the integrals more easily. After the application of the rotation, we are still left with Euclidean space terms. To move forward we need to evaluate them. This is achieved by expressing the integration as a function of only the magnitude of momentum  $k$ :

$$\int d^D k = \int d\Omega_D \int k^{D-1} dk, \quad (\text{B.4})$$

where  $d\Omega_D$  is the differential solid angle of the  $D$ -dimensional unit sphere. Explicitly, the differential solid angle can be written as:

$$d\Omega_D = \left( \prod_{i=1}^{d-2} \sin^{d-1-i}(\phi_{i+1}) \right) \prod_{i=1}^{d-1} d\phi_i, \quad (\text{B.5})$$

where  $\phi_i$  is the angle of the  $i$ -th axis. Now we integrate this expression to evaluate the surface area of the Euclidean  $D$ -sphere:

$$\int d\Omega_D = 2\pi \prod_{n=2}^{D-1} \left[ \int_0^\pi d\phi_n \sin^{n-1}(\phi_n) \right] = 2\pi^{\frac{D}{2}} \prod_{n=2}^{D-1} \left( \frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n+1}{2})} \right) = 2\pi^{\frac{D}{2}} \frac{1}{\Gamma(\frac{D}{2})}. \quad (\text{B.6})$$

The integrals over Euclidean momentum  $k_E$  are undemanding:

$$\int dk_E \frac{k_E^a}{(k_E^2 + X)^b} = X^{\frac{a+1}{2}-b} \frac{\Gamma\left(\frac{a+1}{2}\right) \Gamma\left(b - \frac{a+1}{2}\right)}{2\Gamma(b)}. \quad (\text{B.7})$$

By applying Eq. (B.4), Eq. (B.6) and Eq. (B.7), we get that the integral can be evaluated as a combination of Gamma functions, which come from the evaluation of the surface area of a Euclidean 4-sphere as shown in Eq. (B.6):

$$\Gamma(z) = \int_0^\infty dt t^{z-1} e^{-t} \quad (\text{B.8})$$

and

$$\Gamma(n) = (n-1)! . \quad (\text{B.9})$$

Using them we get a so-called master formula that gives us the ability to calculate all variations of such integrals:

$$\int \frac{d^D l}{(2\pi)^D} \frac{l^{2a}}{(l^2 + X)^b} = i(-1)^{a-b} \frac{1}{(4\pi)^{\frac{D}{2}}} \frac{1}{-X^{b-a-\frac{D}{2}}} \frac{\Gamma(a + \frac{D}{2}) \Gamma(b - a - \frac{D}{2})}{\Gamma(b) \Gamma(\frac{D}{2})}, \quad (\text{B.10})$$

where the factor  $i$  comes from the Wick rotation.

A useful way to evaluate the gamma functions in the master formula is to apply a replacement  $\epsilon = \frac{4-D}{2}$  or variations of it and expand the functions around  $\epsilon = 0$ . This gives us:

$$\Gamma(\epsilon) = \frac{1}{\epsilon} - \gamma + \mathcal{O}(\epsilon), \quad (\text{B.11})$$

where  $\gamma$  is the Euler-Mascheroni constant. This replacement also gives us that:

$$\mu^{2\epsilon} = 1 + \epsilon \log \mu^2 + \dots \quad (\text{B.12})$$

But for dimensional regularization to work, the integrals need to be in the form shown in the master formula which is not usually the case. When we analyse the diagrams and apply the Feynman rules, we get the integrals in the form of:

$$\int_l \frac{1}{A_1 A_2 \dots A_n}, \quad (\text{B.13})$$

where  $A_n$  is the loop function. To shift this to the expression we see in the master formula, we use Feynman parameters:

$$\frac{1}{A_1 A_2 \dots A_n} = (n-1)! \int_0^1 dx_1 \dots dx_n \frac{\delta(x_1 + \dots + x_n - 1)}{(x_1 A_1 + \dots + x_n A_n)}. \quad (\text{B.14})$$

One can also use Schwinger parameters, however, Feynman parameters are used more commonly.

## C Explicit computational results for the Yukawa counterterm with up-type quarks

In this section we will give explicit computational results for the Yukawa counterterm by computing and renormalizing the Higgs interaction vertex with up-type quarks. The procedure is analogous for other fermion types. For these explicit computations we used the *Wolfram Mathematica* [23] software with the *FeynCalc* package. The *FeynCalc* package allows us to do all the necessary mathematics like Dirac algebra etc.

Firstly, we give the explicit results for the two-point function counterterms. We start with the diagonal mass counterterm for the quark sector, which is given by Eq. (4.4). Explicitly for the up-type quarks, the diagonal mass counterterm is:

$$\begin{aligned}
\delta m_{ii}^u = & \frac{\pi^2 A_0(m_W^2) \left( (D-2)m_h^2 m_W^2 - (m_i^u)^2 (m_h^2 + 4(D-1)m_W^2) \right) e^2}{8s_w^2 m_i^u m_h^2 m_W^2} \\
& + \frac{\pi^2 A_0(m_Z^2) \left( (D-2)m_h^2 (32m_W^4 - 40m_Z^2 m_W^2 + 17m_Z^4) - 36(D-1)(m_i^u)^2 m_Z^4 \right) e^2}{144 s_w^2 m_i^u m_h^2 m_W^2 m_Z^2} \\
& - \frac{\pi^2 B_0((m_i^u)^2, (m_i^u)^2, m_Z^2) e^2}{144 s_w^2 m_i^u m_W^2 m_Z^2} \\
& \times \left[ 2(96m_W^4 + 8(-15 + 4s_w^2)m_Z^2 m_W^2 + (51 - 9D - 8s_w^2)m_Z^4)(m_i^u)^2 \right. \\
& \left. + (D-2)m_Z^2 (32m_W^4 - 40m_Z^2 m_W^2 + 17m_Z^4) \right] \\
& - \frac{\pi^2 m_i^u A_0(m_h^2) e^2}{4 s_w^2 m_W^2} + \frac{\pi^2 B_0((m_i^u)^2, (m_i^u)^2, m_h^2) (4(m_i^u)^3 - m_i^u m_h^2) e^2}{8 s_w^2 m_W^2} \\
& + \frac{\pi^2 m_i^u B_0((m_i^u)^2, 0, (m_i^u)^2)}{9} \left[ e^2 \left( \frac{12m_W^2}{s_w^2 m_Z^2} - \frac{12}{s_w^2} + 4 \right) - \frac{9(N_c^2 - 1)g_s^2}{N_c} \right] \\
& - \frac{\pi^2 A_0((m_i^u)^2) (36 N_c (m_i^u)^2 e^2 + (D-2)((17N_c m_Z^2 - 8N_c m_W^2)e^2 + 36(N_c^2 - 1)s_w^2 g_s^2 m_W^2))}{144 N_c s_w^2 m_i^u m_W^2} \\
& + \sum_{k=1}^3 \left( - \frac{\pi^2 V_{ik}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* A_0((m_k^d)^2) ((m_k^d)^2 + (m_i^u)^2 + (D-2)m_W^2) e^2}{8 s_w^2 m_i^u m_W^2} \right. \\
& + \frac{\pi^2 V_{ik}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* B_0((m_i^u)^2, (m_k^d)^2, m_W^2) e^2}{8 s_w^2 m_i^u m_W^2} \\
& \times \left[ (m_k^d)^4 + ((D-3)m_W^2 - 2(m_i^u)^2)(m_k^d)^2 + ((m_i^u)^2 - m_W^2)((m_i^u)^2 + (D-2)m_W^2) \right] \\
& + \frac{\pi^2 V_{ik}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* (m_k^d)^2 A_0(m_W^2) e^2}{8 s_w^2 m_i^u m_W^2} + \frac{\pi^2 (m_k^\ell)^2 m_i^u A_0((m_k^\ell)^2) e^2}{s_w^2 m_h^2 m_W^2}
\end{aligned}$$

$$+ \sum_{c=1}^{N_c} \left( \frac{\pi^2 (m_k^d)^2 m_i^u A_0((m_k^d)^2) e^2}{s_w^2 m_h^2 m_W^2} + \frac{\pi^2 m_i^u (m_k^u)^2 A_0((m_k^u)^2) e^2}{s_w^2 m_h^2 m_W^2} \right), \quad (C.1)$$

where structures like  $A_0(\dots)$ ,  $B_0(\dots)$  and so on, are called the Passarino–Veltman functions [28], that are used to evaluate one-loop diagrams. Then following Eq. (4.5), we get that for up-type quarks diagonal field counterterm is:

$$\begin{aligned} \delta Z_{ii}^u = & - \frac{2\pi^2 C_0(0, (m_i^u)^2, (m_i^u)^2, (m_i^u)^2, 0) (8N_c e^2 + 9(N_c^2 - 1)g_s^2) (m_i^u)^2}{9N_c} \\ & + \frac{\pi^2 ((48c_w^2 - 23)P_R + 7P_L + 24s_w^2 - 25) \xi_Z B_0((m_i^u)^2, (m_i^u)^2, \xi_Z m_Z^2) e^2}{72c_w^2 s_w^2} \\ & + \frac{\pi^2 B_0(0, (m_i^u)^2, (m_i^u)^2) (8N_c e^2 + 9(N_c^2 - 1)g_s^2)}{9N_c} \\ & + \frac{\pi^2 B_0((m_i^u)^2, 0, (m_i^u)^2) (4N_c (3 + c_w^2(D - 3) - 5s_w^2 + D(-1 + s_w^2)) e^2 - 9(N_c^2 - 1)s_w^2 g_s^2)}{9N_c s_w^2} \\ & - \frac{\pi^2 B_0(0, 0, 0) (8N_c(\xi_A - 1)e^2 + 9(N_c^2 - 1)(\xi_G - 1)g_s^2)}{18N_c} \\ & + \frac{\pi^2 A_0((m_i^u)^2)}{72} \left[ - \frac{8(D - 2)(P_R - 2P_L)e^2}{s_w^2 (m_i^u)^2} + \frac{(D - 2)(16P_R + P_L - 1)e^2}{c_w^2 s_w^2 (m_i^u)^2} \right. \\ & \left. - \frac{16c_w^2(D - 2)e^2}{s_w^2 (m_i^u)^2} - \frac{16(D - 2)e^2}{c_w^2 (m_i^u)^2} - \frac{18e^2}{c_w^2 s_w^2 m_Z^2} - \frac{2(D - 2)(8N_c e^2 + 9(N_c^2 - 1)g_s^2)}{N_c (m_i^u)^2} \right] \\ & + \frac{\pi^2 A_0(m_h^2) e^2}{8c_w^2 s_w^2 m_Z^2} + \frac{\pi^2 (1 + 2c_w^2 - 2s_w^2) A_0(\xi_Z m_Z^2) e^2}{9c_w^2 m_Z^2} \\ & + \frac{\pi^2 B_0((m_i^u)^2, (m_i^u)^2, m_h^2) e^2 (2(m_i^u)^2 - m_h^2)}{4c_w^2 s_w^2 m_Z^2} - \frac{\pi^2 B_0(0, m_h^2, m_h^2) e^2 (4(m_i^u)^2 - m_h^2)}{8c_w^2 s_w^2 m_Z^2} \\ & + \frac{\pi^2 C_0(0, (m_i^u)^2, (m_i^u)^2, m_h^2, m_h^2, (m_i^u)^2) e^2 (4(m_i^u)^2 m_h^2 - m_h^4)}{8c_w^2 s_w^2 m_Z^2} \\ & + \sum_{k=1}^3 \left( - \frac{\pi^2 V_{ik}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* (m_k^d)^2 B_0(0, c_w^2 m_Z^2, c_w^2 m_Z^2) ((m_k^d)^2 - 2(m_i^u)^2 + c_w^2(D - 3)m_Z^2) e^2}{8c_w^2 s_w^2 (m_i^u)^2 m_Z^2} \right. \\ & + \frac{\pi^2 V_{ik}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* A_0((m_k^d)^2) ((P_L - 1)(m_k^d)^2 + (P_R - 1)(m_i^u)^2 - c_w^2(D - 2)P_R m_Z^2) e^2}{4c_w^2 s_w^2 (m_i^u)^2 m_Z^2} \\ & \left. - \frac{\pi^2 V_{ik}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* C_0(0, (m_i^u)^2, (m_i^u)^2, c_w^2 m_Z^2, c_w^2 m_Z^2, (m_k^d)^2) e^2}{8c_w^2 s_w^2 (m_i^u)^2 m_Z^2} \right) \\ & \times ((m_k^d)^2 - (m_i^u)^2 - c_w^2 m_Z^2) \\ & \times \left[ (m_k^d)^4 + (c_w^2(D - 3)m_Z^2 - 2(m_i^u)^2)(m_k^d)^2 + ((m_i^u)^2 - c_w^2 m_Z^2)((m_i^u)^2 + c_w^2(D - 2)m_Z^2) \right] \end{aligned}$$

$$\begin{aligned}
& - \frac{\pi^2 V_{ik}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* B_0 \left( (m_i^u)^2, (m_k^d)^2, c_w^2 m_Z^2 \right) e^2}{8c_w^2 s_w^2 (m_i^u)^2 m_Z^2} \\
& \times \left[ (2P_L - 3)(m_k^d)^4 + ((2 - 4P_L)(m_i^u)^2 + c_w^2 (D - 3)(2P_L - 3)m_Z^2)(m_k^d)^2 \right. \\
& \left. + (2P_L + 1)(m_i^u)^4 - c_w^4 (D - 2)(2P_L - 3)m_Z^4 + c_w^2 (D - 3)(2P_L - 1)(m_i^u)^2 m_Z^2 \right] \\
& + \frac{\pi^2 V_{ik}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* B_0 \left( (m_i^u)^2, (m_k^d)^2, c_w^2 \xi_W m_Z^2 \right) e^2}{4c_w^2 s_w^2 m_Z^2} \\
& \times \left[ -(P_R + 3P_L - 1)(m_k^d)^2 - (P_R - P_L - 1)((m_i^u)^2 - c_w^2 \xi_W m_Z^2) \right] \\
& - \frac{\pi^2 V_{ik}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* (P_L - 1)(m_k^d)^2 A_0(c_w^2 m_Z^2) e^2}{4c_w^2 s_w^2 (m_i^u)^2 m_Z^2} \Bigg) \\
& - \frac{\pi^2 B_0(0, c_w^2 m_Z^2, c_w^2 m_Z^2) e^2 ((m_i^u)^2 - c_w^2 m_Z^2) ((m_i^u)^2 + c_w^2 (D - 2)m_Z^2)}{8c_w^2 s_w^2 m_Z^2 (m_i^u)^2} \\
& + \frac{\pi^2 C_0(0, (m_i^u)^2, (m_i^u)^2, m_Z^2, m_Z^2, (m_i^u)^2) e^2 m_Z^2}{144c_w^2 s_w^2 (m_i^u)^2} \\
& \times \left[ 2(-3 + 16c_w^4 (D - 3) + D - 40s_w^2 + 8Ds_w^2 + 8c_w^2 (3 + 2s_w^2 + D(-1 + 2s_w^2)))(m_i^u)^2 \right. \\
& \left. - (D - 2)(1 + 16c_w^4 + 16s_w^2 - 8c_w^2 (1 + 2s_w^2))m_Z^2 \right] \\
& + \frac{\pi^2 B_0(0, m_Z^2, m_Z^2) e^2}{144c_w^2 s_w^2 (m_i^u)^2} \\
& \times \left[ (D - 2)(1 + 16c_w^4 + 16s_w^2 - 8c_w^2 (1 + 2s_w^2))m_Z^2 \right. \\
& \left. - 2(-3 + 16c_w^4 (D - 3) + D - 40s_w^2 + 8Ds_w^2 + 8c_w^2 (3 + 2s_w^2 + D(-1 + 2s_w^2)))(m_i^u)^2 \right] \\
& + \frac{\pi^2 A_0(m_Z^2) e^2}{72c_w^2 s_w^2 m_Z^2 (m_i^u)^2} \\
& \times \left[ (1 + 16c_w^4 + 16s_w^2 - 8c_w^2 (1 + 2s_w^2))(m_i^u)^2 \right. \\
& \left. + (D - 2)(8(-2 + 3c_w^2)P_R - P_L + 16s_w^4 + 1)m_Z^2 \right] \\
& + \frac{\pi^2 A_0(c_w^2 m_Z^2) e^2 (P_L(m_i^u)^2 - c_w^2 (D - 2)(P_L - 1)m_Z^2)}{4c_w^2 s_w^2 m_Z^2 (m_i^u)^2} \\
& + \frac{\pi^2 B_0((m_i^u)^2, (m_i^u)^2, m_Z^2) e^2}{144c_w^2 s_w^2 (m_i^u)^2} \\
& \times \left[ 4(-72c_w^2 P_R + 48P_R + D(8(-2 + 3c_w^2)P_R - P_L + 12s_w^2 + 1) \right. \\
& \left. + 3P_L + 4(-11 + 8c_w^2)s_w^2 - 3)(m_i^u)^2 \right. \\
& \left. + (D - 2)(32P_R + 2P_L + 8c_w^2((-5 + 6s_w^2)P_R + (1 + 6s_w^2)P_L) - 48s_w^2 - 16c_w^4 - 3)m_Z^2 \right].
\end{aligned} \tag{C.2}$$

Then by using Eq. (4.9), we get:

$$\delta Z_{ji}^{A,u} = \sum_{k=1}^3 \left( \frac{\pi^2 e^2 P_L V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* \left( (m_k^d)^2 - (m_j^u)^2 + \xi_W m_W^2 \right) B_0 \left( (m_j^u)^2, (m_k^d)^2, \xi_W m_W^2 \right)}{8s_W^2 m_W^2} \right. \\ \left. - \frac{\pi^2 e^2 P_L V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* \left( (m_k^d)^2 - (m_i^u)^2 + \xi_W m_W^2 \right) B_0 \left( (m_i^u)^2, (m_k^d)^2, \xi_W m_W^2 \right)}{8s_W^2 m_W^2} \right). \quad (\text{C.3})$$

Next by using Eq. (4.10)

$$\delta Z_{ji}^{H,u} = \sum_{k=1}^3 \left( - \frac{\pi^2 e^2 \left( P_L m_i^u + P_R m_j^u \right) V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* B_0 \left( (m_i^u)^2, (m_k^d)^2, m_W^2 \right)}{8s_W^2 m_W^2 \left( (m_i^u)^3 - m_i^u (m_j^u)^2 \right)} \right. \\ \times \left[ (m_k^d)^2 ((D-3)m_W^2 - 2(m_i^u)^2) + (m_k^d)^4 + ((m_i^u)^2 - m_W^2)((D-2)m_W^2 + (m_i^u)^2) \right] \\ + \frac{\pi^2 e^2 \left( P_R m_i^u + P_L m_j^u \right) V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* B_0 \left( (m_j^u)^2, (m_k^d)^2, m_W^2 \right)}{8s_W^2 m_W^2 m_j^u \left( (m_i^u)^2 - (m_j^u)^2 \right)} \\ \times \left[ (m_k^d)^2 ((D-3)m_W^2 - 2(m_j^u)^2) + (m_k^d)^4 + ((m_j^u)^2 - m_W^2)((D-2)m_W^2 + (m_j^u)^2) \right] \\ - \frac{\pi^2 e^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* B_0 \left( (m_i^u)^2, (m_k^d)^2, \xi_W m_W^2 \right)}{8s_W^2 m_W^2 (m_i^u + m_j^u)} \\ \times \left[ (m_k^d)^2 (3P_L m_i^u + (2P_R + P_L)m_j^u) - P_L (m_i^u + m_j^u)((m_i^u)^2 - \xi_W m_W^2) \right] \\ - \frac{\pi^2 e^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* B_0 \left( (m_j^u)^2, (m_k^d)^2, \xi_W m_W^2 \right)}{8s_W^2 m_W^2 (m_i^u + m_j^u)} \\ \times \left[ (m_k^d)^2 ((2P_R + P_L)m_i^u + 3P_L m_j^u) - P_L (m_i^u + m_j^u)((m_j^u)^2 - \xi_W m_W^2) \right] \\ - \frac{\pi^2 e^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* A_0 \left( (m_k^d)^2 \right) \left[ P_R (m_k^d)^2 + (D-2)P_R m_W^2 + P_L m_i^u m_j^u \right]}{8s_W^2 m_W^2 m_i^u m_j^u} \\ + \frac{\pi^2 e^2 P_R V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* A_0 \left( m_W^2 \right) (m_k^d)^2}{8s_W^2 m_W^2 m_i^u m_j^u} \Bigg). \quad (\text{C.4})$$

By having the explicit off-diagonal anti-hermitian field counterterm for the up-type quarks, we use the Eq. (4.11) to compute the off-diagonal mass counterterm:

$$\delta m_{ji}^u = \sum_{k=1}^3 \left( \left[ \frac{\pi^2 e^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* B_0 \left( (m_i^u)^2, (m_k^d)^2, c_w^2 m_Z^2 \right)}{8c_w^2 s_w^2 m_Z^2 m_i^u} \right. \right. \\ \left. \times \left[ (m_k^d)^2 (c_w^2 (D-3)m_Z^2 - 2(m_i^u)^2) + (m_k^d)^4 + ((m_i^u)^2 - c_w^2 m_Z^2)(c_w^2 (D-2)m_Z^2 + (m_i^u)^2) \right] \right.$$

$$\begin{aligned}
& - \frac{\pi^2 e^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* A_0 \left( (m_k^d)^2 \right) \left[ (m_k^d)^2 + c_w^2 (D-2) m_Z^2 + (m_i^u)^2 \right]}{8 c_w^2 s_w^2 m_Z^2 m_i^u} \\
& + \frac{\pi^2 e^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* (m_k^d)^2 A_0 (c_w^2 m_Z^2)}{8 c_w^2 s_w^2 m_Z^2 m_i^u} \Bigg] P_R \\
& + \left[ \frac{\pi^2 e^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* B_0 \left( (m_j^u)^2, (m_k^d)^2, c_w^2 m_Z^2 \right)}{8 c_w^2 s_w^2 m_Z^2 m_j^u} \right. \\
& \times \left[ (m_k^d)^2 (c_w^2 (D-3) m_Z^2 - 2 (m_j^u)^2) + (m_k^d)^4 + ((m_j^u)^2 - c_w^2 m_Z^2) (c_w^2 (D-2) m_Z^2 + (m_j^u)^2) \right] \\
& - \frac{\pi^2 e^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* A_0 \left( (m_k^d)^2 \right) \left[ (m_k^d)^2 + c_w^2 (D-2) m_Z^2 + (m_j^u)^2 \right]}{8 c_w^2 s_w^2 m_Z^2 m_j^u} \\
& \left. + \frac{\pi^2 e^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* (m_k^d)^2 A_0 (c_w^2 m_Z^2)}{8 c_w^2 s_w^2 m_Z^2 m_j^u} \right] P_L \Bigg). \tag{C.5}
\end{aligned}$$

The UV divergent part of the Higgs propagator field counterterm:

$$\begin{aligned}
\delta Z_h|_{\text{UV}} = & \sum_{k=1}^3 \left( - \frac{\pi^2 e^2}{4 s_w^2 m_W^2 \varepsilon_{\text{UV}}} N_c \left( (m_k^d)^2 + (m_k^u)^2 \right) - \frac{\pi^2 e^2 (m_k^\ell)^2}{4 s_w^2 m_W^2 \varepsilon_{\text{UV}}} \right) \\
& - \frac{\pi^2 e^2 (2 m_W^2 (\xi_W - 3) + m_Z^2 (\xi_Z - 3))}{8 s_w^2 m_W^2 \varepsilon_{\text{UV}}}. \tag{C.6}
\end{aligned}$$

The UV divergent part of the up-type quark full field counterterm:

$$\begin{aligned}
\delta Z_{ji}^u|_{\text{UV}} = & \left( \sum_{k=1}^3 \left[ \frac{\pi^2 e^2 \delta_{ij} (m_k^d)^2 (V_{ik}^{\text{CKM}})^* \left( (m_i^u)^3 V_{jk}^{\text{CKM}} - 2 m_W^2 (m_i^u + m_j^u) V_{ik}^{\text{CKM}} \right)}{4 s_w^2 m_W^2 \varepsilon_{\text{UV}} (m_i^u)^2 (m_i^u + m_j^u)} \right. \right. \\
& - \frac{\pi^2 e^2 m_i^u (m_k^d)^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^*}{4 s_w^2 m_W^2 \varepsilon_{\text{UV}} (m_i^u + m_j^u)} \Bigg] \\
& + \frac{\pi^2 e^2 \delta_{ij}}{72 s_w^2 m_W^2 \varepsilon_{\text{UV}} (m_i^u)^2} \left[ (m_i^u)^2 (2 m_W^2 (9 \xi_W + 24 \xi_Z + 16) - 23 m_Z^2 \xi_Z) \right. \\
& - 18 (m_i^u)^4 + 8 (6 s_w^2 + 1) m_W^2 m_Z^2 \Bigg] P_R \\
& + \left( \sum_{k=1}^3 \left[ \frac{\pi^2 e^2 \delta_{ij} (m_k^d)^2 (V_{ik}^{\text{CKM}})^* \left( (m_i^u)^2 (3 m_i^u + m_j^u) V_{jk}^{\text{CKM}} \right)}{8 s_w^2 m_W^2 \varepsilon_{\text{UV}} (m_i^u)^2 (m_i^u + m_j^u)} \right. \right. \\
& - \frac{\pi^2 e^2 \delta_{ij} (m_k^d)^2 (V_{ik}^{\text{CKM}})^* \left( 2 (m_i^u + m_j^u) ((m_i^u)^2 + 2 m_W^2) V_{ik}^{\text{CKM}} \right)}{8 s_w^2 m_W^2 \varepsilon_{\text{UV}} (m_i^u)^2 (m_i^u + m_j^u)} \\
& \left. \left. - \frac{\pi^2 e^2 (3 m_i^u + m_j^u) (m_k^d)^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^*}{8 s_w^2 m_W^2 \varepsilon_{\text{UV}} (m_i^u + m_j^u)} \right] \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{\pi^2 e^2 \delta_{ij}}{72 s_w^2 m_W^2 \varepsilon_{UV} (m_i^u)^2} \left[ (m_i^u)^2 (m_W^2 (32 - 18 \xi_W) + 7 m_Z^2 \xi_Z) + 8 (6 s_w^2 + 1) m_W^2 m_Z^2 \right] \Big) P_L \\
& + \sum_{k=1}^3 \left[ \frac{\pi^2 e^2 \delta_{ij} (m_k^d)^2 (V_{ik}^{\text{CKM}})^* \left( 2 m_W^2 (m_i^u + m_j^u) V_{ik}^{\text{CKM}} + (m_i^u)^2 m_j^u V_{jk}^{\text{CKM}} \right)}{4 s_w^2 m_W^2 \varepsilon_{UV} (m_i^u)^2 (m_i^u + m_j^u)} \right. \\
& \quad \left. - \frac{\pi^2 e^2 m_j^u (m_k^d)^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^*}{4 s_w^2 m_W^2 \varepsilon_{UV} (m_i^u + m_j^u)} \right] \\
& - \frac{\pi^2 \delta_{ij}}{72 N_c s_w^2 m_W^2 \varepsilon_{UV} (m_i^u)^2} \\
& \times \left[ (m_i^u)^2 \left( e^2 N_c (2 m_W^2 (16 s_w^2 \xi_A - 8 s_w^2 \xi_Z + 9 \xi_W + 16) + (16 s_w^4 - 32 s_w^2 + 25) m_Z^2 \xi_Z) \right. \right. \\
& \quad \left. \left. + 36 (N_c^2 - 1) s_w^2 g_s^2 \xi_G m_W^2 \right) + 18 e^2 N_c (m_i^u)^4 \right. \\
& \quad \left. + 8 e^2 N_c m_Z^2 ((2 s_w^2 + 1) m_W^2 - 4 s_w^2 (s_w^2 - 1) m_Z^2) \right]. \tag{C.7}
\end{aligned}$$

The UV divergent part of the up-type quark full mass counterterm:

$$\begin{aligned}
\delta m_{ji}^u|_{\text{UV}} &= \sum_{k=1}^3 \left( \frac{\pi^2 e^2 \delta_{ij}}{8 s_w^2 m_h^2 m_W^2 \varepsilon_{UV}} \right. \\
& \times \left[ 3 m_h^2 (m_k^d)^2 (V_{ik}^{\text{CKM}})^* ((P_R m_i^u + P_L m_j^u) V_{jk}^{\text{CKM}} - m_i^u V_{ik}^{\text{CKM}}) + 8 m_i^u (m_k^\ell)^4 \right] \\
& - \frac{3 \pi^2 e^2 (m_k^d)^2 (P_R m_i^u + P_L m_j^u) V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^*}{8 s_w^2 m_h^2 m_W^2 \varepsilon_{UV}} \\
& + \frac{\pi^2 e^2 \delta_{ij} m_i^u}{s_w^2 m_h^2 m_W^2 \varepsilon_{UV}} \sum_{c=1}^{N_c} \left( (m_k^d)^4 + (m_k^u)^4 \right) \\
& + \frac{\pi^2 \delta_{ij} m_i^u}{72 N_c s_w^2 m_h^2 m_W^2 \varepsilon_{UV}} \\
& \times \left[ 27 e^2 N_c m_h^2 (m_i^u)^2 - e^2 N_c (-8 m_h^2 ((s_w^2 - 4) m_Z^2 + 4 m_W^2) + 27 m_h^4 + 54 (2 m_W^4 + m_Z^4)) \right. \\
& \quad \left. - 108 (N_c^2 - 1) s_w^2 g_s^2 m_h^2 m_W^2 \right]. \tag{C.8}
\end{aligned}$$

The UV divergent part of the Higgs-up-type quark vertex:

$$\begin{aligned}
\Gamma_{h\bar{u}j u_i}^{\text{loop}}|_{\text{UV}} &= \left( - \frac{\pi^2 e^3 m_i^u}{4 s_w^3 m_W^3 \varepsilon_{UV}} \sum_{k=1}^3 (m_k^d)^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* \right. \\
& - \frac{\pi^2 e \delta_{ij}}{72 N_c s_w^3 m_W^3 m_Z^2 \varepsilon_{UV}} \\
& \times \left[ 2 m_Z^2 \left( s_w^2 m_j^u (2 e^2 N_c ((4 \xi_A + 3) m_W^2 + 3 s_w^2 m_Z^2) + 9 (N_c^2 - 1) g_s^2 (\xi_G + 3) m_W^2) \right. \right. \\
& \quad \left. \left. + 9 e^2 N_c m_W^2 \xi_W m_i^u \right) + e^2 N_c \xi_Z m_j^u (-6 s_w^2 m_W^2 m_Z^2 + s_w^2 (s_w^2 + 12) m_Z^4 + 9 m_W^4) \right] \Big) P_R
\end{aligned}$$

$$\begin{aligned}
& + \left( - \frac{\pi^2 e^3 m_j^u}{4s_w^3 m_W^3 \varepsilon_{UV}} \sum_{k=1}^3 (m_k^d)^2 V_{jk}^{\text{CKM}} (V_{ik}^{\text{CKM}})^* \right. \\
& - \frac{\pi^2 e \delta_{ij}}{72 N_c s_w^3 m_W^3 m_Z^2 \varepsilon_{UV}} \\
& \times \left[ 2m_Z^2 \left( s_w^2 m_i^u (2e^2 N_c ((4\xi_A + 3)m_W^2 + 3s_w^2 m_Z^2) + 9(N_c^2 - 1)g_s^2 (\xi_G + 3)m_W^2) \right. \right. \\
& \left. \left. + 9e^2 N_c m_W^2 \xi_W m_j^u \right) + e^2 N_c \xi_Z m_i^u (-6s_w^2 m_W^2 m_Z^2 + s_w^2 (s_w^2 + 12)m_Z^4 + 9m_W^4) \right] \Big) P_L. \quad (\text{C.9})
\end{aligned}$$

## References

- [1] S. Chatrchyan, V. Khachatryan, A. Sirunyan, *et al.*, Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, *Physics Letters B*, Sept. 2012, **716**, 30–61.
- [2] B. Batell, A. Bhoonah, and W. Huang, Right-Handed Neutrino Masses from the Electroweak Scale, 2024.
- [3] W. Grimus and H. Neufeld, Radiative neutrino masses in an  $SU(2) \times U(1)$  model, *Nuclear Physics B*, Oct. 1989, **325**, 18–32.
- [4] E. Majorana, Teoria simmetrica dell’elettrone e del positrone, *Il Nuovo Cimento*, Apr. 1937, **14**, 171–184.
- [5] S. Draukšas, *Renormalization of the Grimus-Neufeld model*. Natural Sciences, Vilnius University, Lithuania, June 2024.
- [6] S. Draukšas, On-shell renormalization of fermion masses, fields, and mixing matrices at 1-loop, *The European Physical Journal Plus*, July 2023, **138**, 671.
- [7] S. Draukšas, Renormalization of mixing angles and computation of the hadronic  $W$  decay widths, Dec. 2025. arXiv:2512.11717 Read\_Status: New Read\_Status\_Date: 2026-01-04T08:28:29.989Z.
- [8] M. B. Robinson, *Symmetry and the standard model: mathematics and particle physics*. New York: Springer, 2011.
- [9] M. A. Srednicki, *Quantum field theory*. Cambridge ; New York: Cambridge University Press, 2007.
- [10] A. Beekman, L. Rademaker, and J. Van Wezel, An introduction to spontaneous symmetry breaking, *SciPost Physics Lecture Notes*, Dec. 2019, 11. arXiv:1909.01820.
- [11] M. D. Schwartz, *Quantum field theory and the standard model*. New York: Cambridge University Press, 2014.
- [12] J. M. González, Phenomenological Aspects of Models with Low Scale Seesaw, 2024.
- [13] J. Schechter and J. W. F. Valle, Neutrinoless double- $\beta$  decay in  $SU(2) \times U(1)$  theories, *Physical Review D*, June 1982, **25**, 2951–2954.
- [14] V. Dūdėnas, *Renormalization of neutrino masses in the Grimus-Neufeld model*. Vilnius: Vilniaus universiteto leidykla, 2019.
- [15] N. Cabibbo, Unitary Symmetry and Leptonic Decays, *Physical Review Letters*, June 1963, **10**, 531–533.

- [16] M. Kobayashi and T. Maskawa,  $C P$  -Violation in the Renormalizable Theory of Weak Interaction, *Progress of Theoretical Physics*, Feb. 1973, **49**, 652–657.
- [17] M. E. Peskin and D. V. Schroeder, *An Introduction to quantum field theory*. Reading, USA: Addison-Wesley, 1995.
- [18] H. Lehmann, K. Symanzik, and W. Zimmermann, Zur Formulierung quantisierter Feldtheorien, *Il Nuovo Cimento*, Jan. 1955, **1**, 205–225. Read\_Status: New Read\_Status\_Date: 2026-01-21T10:13:36.286Z.
- [19] D. Espriu and J. Manzano, CP Violation and Family Mixing in the Effective Electroweak Lagrangian, 2000. Version Number: 2 Read\_Status: New Read\_Status\_Date: 2026-01-21T10:13:35.533Z.
- [20] A. Denner and S. Dittmaier, Electroweak radiative corrections for collider physics, *Physics Reports*, June 2020, **864**, 1–163. arXiv: 1912.06823.
- [21] N. Nielsen, On the gauge dependence of spontaneous symmetry breaking in gauge theories, *Nuclear Physics B*, Dec. 1975, **101**, 173–188.
- [22] P. Gambino and P. A. Grassi, The Nielsen Identities of the SM and the definition of mass, 1999. Version Number: 3.
- [23] W. R. Inc., “Mathematica, Version 14.3.” Champaign, IL, 2025.
- [24] R. Mertig, M. Böhm, and A. Denner, Feyn Calc - Computer-algebraic calculation of Feynman amplitudes, *Computer Physics Communications*, June 1991, **64**, 345–359.
- [25] V. Shtabovenko, R. Mertig, and F. Orellana, New Developments in FeynCalc 9.0, 2016. Version Number: 2.
- [26] V. Shtabovenko, R. Mertig, and F. Orellana, FeynCalc 9.3: New features and improvements, 2020. Version Number: 2.
- [27] V. Shtabovenko, R. Mertig, and F. Orellana, FeynCalc 10: Do multiloop integrals dream of computer codes?, 2023. Version Number: 2.
- [28] G. Passarino and M. Veltman, One-loop corrections for  $e^+e^-$  annihilation into  $\mu^+\mu^-$  in the Weinberg model, *Nuclear Physics B*, Nov. 1979, **160**, 151–207.

# Yukawa sektoriaus pernормavimas Grimus–Neufeld modelyje

Dominykas Malinauskas

## Santrauka

Elementariųjų dalelių ir jų sąveikų tyrimai yra paremti kvantinio lauko teorija (QFT), kurios sėkmingiausias modelis yra Standartinis modelis (SM). Jis teisingai nuspėjo daugelio dalelių ir jų fizines savybes, iš kurių naujausia yra Higso bozonas [1]. Nepaisant to, SM negali nepaaiškinti keleto reiškinių, tokių kaip neutrinų masės kilmė [2], tamsiosios medžiagos egzistavimas, gravitacija ir kt. Todėl dalelių fizikai kuria Standartinio modelio plėtinius, siekdami paaiškinti šiuos neatitikimus. Šiame darbe gilinamasi į vieną iš tokių Standartinio modelio plėtinių – Grimus–Neufeld modelį (GNM) [3]. Grimus–Neufeld modelis prideda papildomą Higso dubletą, todėl jis laikomas dviejų Higso dubletų modeliu (2HDM), bei Majorana fermioną [4], kuris naudojamas mažoms neutrinų masėms paaiškinti.

Mūsų hipotezė yra kad pernормuodami Higso–fermionų sąveiką ir fiksuodami jį sudarančius dalelių propagatorių atsvaros narius, galime išvesti Yukawa konstantos atsvaros narį. Tai grindžiama tuo, kad Yukawa sektorius aprašo Higso bozono ir fermionų sąveikas – išvedę Higso–fermionų sąveikos, dar vadinamo trijų taškų funkcija, atsvaros narių Feinmano taisyklės, matome, jog tai yra fermionų ir Higso propagatorių, dar vadinamų dviejų taškų funkcijomis, atsvaros narių bei Yukawa atsvaros nario derinys, kaip matoma lygyje:

$$\begin{aligned}
 \begin{array}{c} \bar{\psi}_a \\ \swarrow \\ h_k \text{ --- } \otimes \\ \searrow \\ \psi_b \end{array} &= -\frac{1}{\sqrt{2}} \sum_{n=1,2} C_n \left( \delta Y_{ab}^{\psi} \delta_{nk} + \sum_i Y_{ib}^{\psi} \gamma^0 (\delta Z^{\psi})_{ai}^{\dagger} \gamma^0 \delta_{nk} \right. \\
 &\quad \left. + \sum_j Y_{aj}^{\psi} (\delta Z^{\psi})_{jb} \delta_{nk} + Y_{ab}^{\psi} (\delta_h)_{nk} \right) P_R + \text{h.c.},
 \end{aligned}$$

kur  $C_n = \{1, i\}$ . Čia tuomet taikome SD schemą, kuri jau buvo pritaikyta dviejų taškų funkcijų pernормavimui [5–7] GNM modelyje. Taigi Yukawa atsvaros narį gauname apskaičiuodami aukštesnės eilės trikdžius, kitaip vadinamus vienos kilpos pataisomis, ir naudodami dviejų taškų funkcijų lauko atsvaros narius UV divergencijoms pašalinti. Yukawa atsvaros narį patikriname naudodami jo ryšį su dviejų taškų funkcijų masės atsvaros nariais. Tai grindžiama Yukawa konstantų ryšiu su fermionų masėmis per Dirako masės narį Lagranžiane, atsirandantį dėl spontaniško simetrijos lūžimo ir Higso mechanizmo, iš ko išplaukia, kad:

$$m_f = \frac{Y_{h\bar{f}_j f_i} v}{\sqrt{2}}.$$

kur  $m_f$  yra fermiono masė,  $Y_{h\bar{f}_j f_i}$  – atitinkama Yukawa konstanta iš Higso dubleto, o  $v$  – vakuomo lūkesčio vertė.

Šio darbo tikslas buvo nustatyti, ar SD schema gali būti taikoma Yukawa sektoriaus pernормavimui GNM Standartinio modelio dalyje. Tiesioginiai skaičiavimai parodė struktūrinį panašumą tarp iš sąveikos ir iš masės ryšio išvestų Yukawa atsvaros narių, tačiau taip pat atskleidė neatitikimų. Yukawa atsvaros narys, išvestas naudojant sąryšį su mase, turi masių narius, kurie yra aukštenių laipsnių nei sąveikos atsvaros naryje esantys, taip pat masės sąryčio vertė turi daugumoje narių Higso bosono masę. Taip pat, išraiškų nariai, kuriuose yra CKM matricą, skiriasi per  $\frac{1}{2}$  daugiklį. Tačiau narys su  $(N_c^2 - 1)g_s^2$ , kitaip vadinamas QCD nariu, yra identiškas abiejose išraiškose. Tikėtina, kad šie neatitikimai atsirado dėl neišsamaus pernормavimo – vakuumo lūkesčio vertės  $v$  pernормavimas nebuvo įtrauktas, o visiškai nuoseklus pernормavimas reikalauja, kad visi tarpusavyje susiję parametrai būtų pernормuojami vienu metu. Nepaisant to, gautas rezultatas rodo, kad darbe pateikta procedūra yra perspektyvus metodas Yukawa sektoriaus pernормavimui GNM, o tolesni išvedimai ir skaičiavimai galėtų patvirtinti šį ryšį.