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Particles and quantum waves diffusion in physical vacuum

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A B S T R A C T

The new quantum waves' diffusion equation based on Heisenberg uncertainties and De Broglie's frequencies of waves is presented. The free movement and quantum diffusion through a rectangular barrier are considered. We find a quantum diffusion coefficient and radii of bound systems. The obtained formula connecting radii and bound energies of simple quantum systems, such as a hydrogen atom, deuteron and mesons, consisted of quarks.

Introduction

Usually, in quantum mechanical investigations of properties of elementary particle systems are provided. Only the quantum field theory of interaction of real particles with virtual particles and antiparticles represents different fields in physical vacuum. Using quantum field interaction with particles we can include generation of particles and antiparticles, and also reactions which can be investigated experimentally. According to Sokolov and Tumanov [1], vacuum oscillations of the quantum field require to introduce for electrons the effective radius what can help to explain Lamb shift of atomic levels $2S_{1/2}$ and $2P_{1/2}$ [2] in hydrogen. The vacuum oscillations can spread the dot-electron in some region with the radius R_e proportional to Compton wavelength [1] λ_e and square root of the fine-structure constant α

$$R_e = \sqrt{\alpha}\frac{\hbar}{m_0 c}, \quad \alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} = 7.29735257\cdot 10^{-3}, \quad \lambda_e = \frac{\hbar}{m_0 c} = 2.42\cdot 10^{-10}\ \text{cm}. \tag{1.1}$$

The presented statistical model of quantum mechanics represents an electron's movement inside atom like Brownian particle [3] interacting with fluctuations of electromagnetic vacuum. Taking into consideration that quantum phenomena have a stochastic character, we propose a new equation of quantum waves' diffusion [3] based on Heisenberg uncertainties and de Broglie waves. The link between uncertainties and non-locality [2] holds for all physical theories. Heisenberg observed that quantum mechanics [4] have restrictions of accuracy of incompatible measurements, such as position and momentum whose results cannot be simultaneously predicted. These restrictions are known as uncertainty relations. Applications of these uncertainties mainly to measurements are misleading because they suggest that the restrictions occur only when one makes measurements, but in our case it is not necessary. Taking into consideration the Lamb shift and Eq. (1.1), we can say that the problem is more general than quantum mechanics suggests. The definition of duality of wave-particle and physical parameters by probabilities require modifying the classical Schrödinger equation based on the wave equation and de Broglie waves. We have proposed the quantum equation connecting stochastic quantum diffusion in physical vacuum and de Broglie waves representing a guiding field for direction of moving quantum particles.

Diffusion of quantum waves

We assume that the equation of quantum mechanics diffusion can be derived from the diffusion equation [5]

$$\frac{\partial\psi_J}{\partial t} = D_C\frac{\partial^2\psi_J}{\partial x^2} \tag{2.1}$$

applied to the wave function

$$\psi_J = Ae^{-i\omega t+\lambda x}. \tag{2.2}$$

In this case, we obtain

$$\lambda^2 = -\frac{i\omega}{D_C}, \quad \lambda_{1,2} = \pm i\sqrt{\frac{\omega}{2D_C}} \mp \sqrt{\frac{\omega}{2D_C}}. \tag{2.3}$$

Requiring that the solution must represent some kind of linearly independent physical ψ_{J1} and nonphysical ψ_{J2} solutions

$$\psi_{J1} = Ae^{-i\omega t+ikx-k|\Delta x|}, \quad \psi_{J2} = Be^{-i\omega t-ikx+k|\Delta x|}, \quad \Delta x \geqslant x-x_n, \quad k = \sqrt{\frac{\omega}{2D_C}}, \quad x_n = n\frac{\lambda}{2} \quad n = 0, 1, 2 \tag{2.4}$$

Free solutions can be presented by introduction [2] of maximum $x = x_0$ or minimum $|x-x_0| = \pi/\Delta k$ of amplitude for a wave pack or the following superposition of quantum oscillations (2.4)

$$\psi_J(x) = Ae^{ikx-k|x-x_0|} + Be^{-ikx+k|x-x_0|}, \tag{2.5}$$

where we can take for the coordinates $x = x_0$ at the maximum $x_{n0\max}$ or

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minimum $x_{n0\min}$ oscillations

$$kx_{n0_{\max}} = n\pi, \quad n = 0,\ 1,\ 2... \ |x - x_{n0_{\max}}| = 0, \tag{2.6}$$

$$kx_{n0\min} = (2n+1)\frac{\pi}{2}, \quad n = 0,\ 1,\ 2..., \quad k|x_{n0\min} - x_{n0\max}| = \frac{\pi}{2}. \tag{2.7}$$

of the real parts of wave function (2.5). We can represent this wave function in the point x by decreasing oscillations generated in maximum point $x_{n0\max}$. Here we have some train of decreasing waves such as in a wave packet.

Now we will try to consider the spreading wave in x direction

$$\psi_J(x,\ t) = Ae^{-i\omega t + ikx - k|x-x_0|} = A\exp\left[\frac{i}{\hbar}(-Et + px + ip|x-x_0|)\right] \tag{2.8}$$

This plain wave can be rewritten in the following way as follows

$$\psi_J(x_0,\ t) = Ae^{-i(\omega\cdot t - kx_0) - k|x_0 - x|} \tag{2.9}$$

which satisfies the simple quantum wave Eq. (2.1)

$$\psi_J(x_0,\ t) = Ae^{-i(\omega\cdot t - kx_0)} \tag{2.10}$$

when $x = x_0$. We can represent this wave function for the point x_0 by spreading oscillations generated in maximum point (2.6) $x_{n0\max}$ where we can find a moving particle with maximum probability in point x_0. Here we have some train of decreasing waves like in a wave packet where proposed wave function (2.9) can be normalized by integrating probability density $\psi^*\psi$

$$dP = A^2e^{-2k|x-x_0|}d|x-x_0|, \quad P = 2A^2\int_0^\infty e^{-2ky}dy = A^2\frac{1}{k} = 1, \quad A = \sqrt{k}. \tag{2.11}$$

The probability of a freely moving particle to be in interval $d|x-x_0|$ is a proportional to $k = \frac{2\pi}{\lambda}$ which is an important result of the scattering theory in quantum mechanics [2]. If a low energy beam of particles is incident on a sphere with radius $r_0 = d|x-x_0|$, then from (2.11) we obtain $k\cdot r_0 < 1$ or $\hbar k\cdot r_0 < \hbar$. Only partial waves with orbital quantum numbers $l = 0$ take part in interaction with a sphere and freely moving particle represented by wave function (2.9) also located in this region. The velocity of the spreading of these waves can be evaluated requiring

$$-Et + px + ip|x-x_0| = -Et_1 + px_1 + ip|x_1 - x_0| \tag{2.12}$$

of equally complex phases when

$$t_1 = t_0 + \Delta t, \quad x_1 = x_0 + \Delta x. \tag{2.13}$$

Substituting (2.13) in (2.12) we obtain

$$E\Delta t - p\Delta x - ip|\Delta x| = 0. \tag{2.14}$$

For maximum movement $\Delta x_m > 0$ of the waves packet connected with particles generated by quantum diffusion in physical vacuum at wave maximum point x_0, we have $\Delta x_m = \frac{1}{2}\Delta x = \frac{1}{2}|\Delta x|$. Then from the last equation for a nonrelativistic case $E = p^2/2m$, we obtain

$$v_J = \frac{\Delta x_m}{\Delta t} = \frac{2E}{p + ip} = \frac{v}{2}(1-i). \tag{2.15}$$

Calculating the square of modulus, $v_J^* v_J$, we obtain that quantum diffusion stochastic waves train free spreading

$$v_J^* v_J = \frac{v^2}{2} \tag{2.16}$$

satisfies the conservation of kinetic energy

$$mv_J^* v_J = \frac{mv^2}{2} \tag{2.17}$$

and momentum mv for a freely moving quantum particle with the average velocity v and mass m. From here, we can define an assumption that

$$vt \approx x_{n0\max} = n\frac{\lambda}{2}, \quad |\Delta x_n| = |x - x_{n0\max}| = \left|x - n\frac{\lambda}{2}\right| \tag{2.18}$$

Finding the minimum difference $\Delta x_n = vt - n\frac{\lambda}{2} = x - n\frac{\lambda}{2} < \frac{\lambda}{2}$ from (4.15), we can determine n, $x_{n0\max}$ and $|\Delta x_n|$. Also, the free solutions (2.8) and (2.5) of the quantum diffusion Eq. (2.1) are defined.

From this, for a free space [2], when $\omega = ck$, $E = \hbar\omega$, we can obtain wave function

$$\psi_{J1} = A\exp\left[-i\omega\cdot t + \frac{1}{c\hbar}(iEx - E|\Delta x_n|)\right], \tag{2.19}$$

which include oscillations in physical vacuum. The free particle with mass m moving with velocity v by action of classical forces and quantum forces [6] depending on wave function or in our case on stochastic waves' packet generated in physical vacuum at points $n\frac{\lambda}{2}$ whose maximums of amplitudes are spreading with velocity v.

Comparing a standard formula $E = \frac{k^2\hbar^2}{2m}$ and (2.4)

$$k^2 = \frac{2mE}{\hbar^2} = \frac{\omega}{2D_C}, \quad E = \hbar\omega \tag{2.20}$$

we obtain the quantum stochastic wave diffusion equation [3]

$$\frac{\partial\psi_J}{\partial t} = D_C\frac{\partial^2\psi_J}{\partial x^2}, \quad D_C = \frac{\hbar}{4m} = \frac{\omega}{2k^2}. \tag{2.21}$$

For free moving particles $k^2 = \frac{2mE}{\hbar^2}$ from the last formula, we obtain the standard nonrelativistic expression

$$E = \hbar\omega = \frac{p^2}{2m}, \quad p = \hbar k. \tag{2.22}$$

From the expression of the quantum diffusion coefficient we can get that a photon is reducible to virtual particles and antiparticles [7]. When an important expressions (2.4), (2.21) are satisfied, a connection with relativistic virtual processes [7] in physical vacuum

$$m = \frac{\hbar k^2}{2\omega} = \frac{h\nu}{2c^2}, \quad h\nu = 2mc^2 \tag{2.23}$$

can be obtained. The last equation shows that a photon can produce both particle and antiparticle with common mass $2m$ or annihilation by virtual processes.We also can obtain the expression of diffusion coefficient D_C from Heisenberg uncertainties for oscillations in physical vacuum [3]

$$2mc\cdot\Delta r = \hbar, \quad 2mc^2\cdot\Delta t = \hbar, \tag{2.24}$$

$$D_C = \frac{1}{2}\Delta r^2\frac{1}{\Delta t} = \frac{\hbar}{4m}, \quad \Delta t = D_C\frac{2}{c^2} \tag{2.25}$$

From (2.23) we get

$$\omega\psi_{J1} = \frac{\hbar k^2}{2m}\psi_{J1}. \tag{2.26}$$

Multiplying the last equation for $\hbar$, if de Broglie equation $\lambda = h/p$ is satisfied, we obtain

$$E\psi_{J1} = \frac{p^2}{2m}\psi_{J1},\ p = \hbar k. \tag{2.27}$$

After introducing operators to wave processes

$$\hat{E} = i\hbar\frac{\partial}{\partial t}, \quad \hat{p} = -i\hbar\nabla \tag{2.28}$$

and including the potential energy $V(r)$ and new functions, depending on wave $\vec{r_v}$ and diffusion $\vec{r_d}$ coordinates

$$\psi_{JS}(\vec{r} = \vec{r_v} + \vec{r_d}) = \psi_{JS}(\vec{r_v},\ \vec{r_d}) = \psi_S(\vec{r_v})\psi_J(\vec{r_d}), \tag{2.29}$$

we obtain the Schrodinger equation [1,2] for bound states:

$$i\hbar\frac{\partial\psi_S}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi_S + V(r)\psi_S \tag{2.30}$$

Free solutions (2.4) did not satisfy the Schrödinger equation when $V(r) = 0$ and for a coincidence free solution of (2.5) and (2.30), we must separate the diffusion processes with different diffusion waves' coordinates $\vec{r_d}$ (2.29).

For bound systems, coordinates of wave $\vec{r_v}$ and diffusion $\vec{r_d}$ coincide.

The quantum diffusion of an electron in the hydrogen atom

The quantum diffusion equation in a three-dimensional case can be obtained using (2.21)

$$\frac{\partial \psi_J}{\partial t} = \frac{\hbar}{4m}\nabla^2\psi_J. \tag{3.1}$$

Here, the diffusion coefficient depends only on mass and is significant only for elementary particles like electrons and protons [7]

$$D_{C,e} = \frac{\hbar}{4m_e} = 0.2893\frac{\text{cm}^2}{s}, \quad D_{C,p} = \frac{\hbar}{4m_p} = 0.1575\cdot 10^{-3}\frac{\text{cm}^2}{s}. \tag{3.2}$$

This diffusion happens for photons, free particles (2.5) and bound particles. In the center of forces and in the region [2] where kinetic energy $T = E - V(r)$ is negative the wave function is decreasing, like $\exp[-r/R_n]$ with a decay length [2]

$$R_n = k_n^{-1} = \frac{\lambda n}{2\pi} = \sqrt{\frac{2D_{Cn}}{\omega}} = \frac{\hbar n}{p} = \frac{\hbar n}{\sqrt{2m|E_n|}}. \tag{3.3}$$

For an electron bounded in the hydrogen atom [2,3], we obtain a Bohr radius

$$R_n = \frac{\hbar^2 n^2}{k_e Z m e^2}, \quad n = 1, 2, 3 \ldots, k_e = 9.10^9\frac{\text{N}\cdot\text{m}^2}{C^2} \tag{3.4}$$

for a principal quantum number n and energy levels [2,3]

$$E_n = -\frac{k_e^2 Z^2 m e^4}{2\hbar^2 n^2}. \tag{3.5}$$

The last formula can be obtained from (3.3) and (3.4). Now substituting (3.5) by (3.3) and taking into consideration that $\hbar\omega = E_{n=\infty} - E_n$ we can find the quantum diffusion coefficient D_{Cn} and the connection between Bohr radius R_n and energy levels E_n for the hydrogen atom

$$D_{Cn} = \frac{\hbar}{4m}n^2 = D_C n^2, \quad R_n^2|E_n| = 2\hbar D_{Cn} = 6.1\cdot 10^{-39} n^2 \tag{3.6}$$

with principal quantum number n. We have the smallest diffusion coefficient for the stable ground state when $n = 1$ and it is rapidly increasing for excited states like n^2. The obtained formula (3.6) can be used for approximate evaluation of quantum systems: atoms, ions, molecules and point defect's parameters in solids.

Now radii of some atoms defined by diffusion of electrons in physical vacuum according to the formula (3.6) can be calculated. Using the bound energies E_n of electrons [8] in free atoms H, He, Li, Be in external shells with energies (13.60; 39.47; 5.39; 9.32) in eV, radii (3.6) of these atoms in Angstroms (10^{-10} m) are obtained

$$R_H = 0.529\ \text{Å}, \quad R_{He} = 0.310\ \text{Å}, \quad R_{Li} = 1.68\ \text{Å}, \quad R_{Be} = 1.28\ \text{Å} \tag{3.7}$$

which were compared with calculations [9,10]

$$R_H = 0.529\ \text{Å}, \quad R_{He} = 0.31\ \text{Å}, \quad R_{Li} = 1.67\ \text{Å}, \quad R_{Be} = 1.28\ \text{Å}. \tag{3.8}$$

Taking into consideration the presented values, we can suppose that energies of the external subshell electrons define atomic radii with high accuracy and depend on the quantum diffusion coefficient D_C like some constant of physical vacuum for an electron and a square of a principal quantum number n.

Applying this conclusion, we will calculate some nuclear radii. For a deuteron, where a neutron and a proton are diffusing in region $R_d = 2R$ of physical vacuum defined by radius R in the coordinates of the center of mass, the formula (3.3) must be modified

$$R_d = 2R = \frac{\hbar}{\sqrt{2m|E_B|}}. \tag{3.9}$$

For the deuteron bound energy [11,12] $E_B = -2.225$ MeV and reduced mass

$$m = \frac{m_n m_p}{m_n + m_p}, \tag{3.10}$$

we obtained $R = 2.158\cdot 10^{-15}$ m or 2.158 *fm*. The charge radius [12] of a deuteron is 2.095 *fm*.

It is interesting to note that if the radius R is known for the energy of the bound system consisting of two equal particles from (3.6) for $n = 1$

$$E_B = \frac{-\hbar^2}{8mR^2}. \tag{3.11}$$

can be determined. Using (3.11) we can find Bohr's radii for 1S states of charmed and bottom mesons using quarks masses and bound energies of quark and antiquark from the paper [13,7]

$$R_{C\overline{C}} = 0.144\ fm, \quad R_{b\overline{b}} = 0.08151\ fm \tag{3.12}$$

$$R_{C\overline{C}} = 0.1704\ fm, \quad R_{b\overline{b}} = 0.05561\ fm. \tag{3.13}$$

For toponium [7] with ground state mass $M_t = 347.4$ GeV and top quark mass $m_t = 179.25$ GeV from (3.3)

$$R_{t\bar{t}} = 0.2212\cdot 10^{-2}\ fm. \tag{3.14}$$

Taking masses m_q of u and d quarks [7] $m_u = 0.35$ GeV, $m_d = 0.35$ GeV and the proton root-mean-charge radius [14] $R_p = 0.8621\ fm$ from (3.3) we obtained bound energy of quarks and mass of a nucleon

$$E_B = 0.08416\ \text{GeV}, \quad M_n = 3m_q - E_B = 0.9658\ \text{GeV}. \tag{3.15}$$

Here we used the approach that every quark is diffusing according to mass center with reduced mass

$$m = \frac{0.5 m_q m_q}{0.5 m_q + m_q}. \tag{3.16}$$

The obtained mass of a nucleon is in approximate coincidence with masses of a neutron

$$m_n = 0.9396\ \text{GeV} \quad \text{and} \quad m_p = 0.9382\ \text{GeV}. \tag{3.17}$$

Quantum diffusion equation solution for tunnel effect for rectangular barrier

From (3.1) separating variable t in the expression of the wave function [2]

$$\psi_J(\vec{r}, t) = e^{\mp i\omega\cdot t}\psi_J(\vec{r}) \tag{4.1}$$

we obtain the quantum waves' diffusion equations

$$\mp i\omega\psi_J(\vec{r}) = \frac{\hbar}{4m}\nabla^2\psi_J, \tag{4.2}$$

$$E\psi_J = \pm i\frac{\hbar}{4m}\nabla^2\psi_J, \ E = \hbar\omega \tag{4.3}$$

for the free movement.

It is possible to present the quantum diffusion equation with potential $V(r)$

$$E\psi_J(\vec{r}) = \frac{i}{2}\left[-\frac{\hbar^2}{2m}\nabla^2\psi_J(\vec{r}) + V(r)\psi_J(\vec{r})\right] \tag{4.4}$$

This equation can be applied for the quantum diffusion in the case of bound states [14,15]. Now we will obtain the solution of the time-independent quantum diffusion equation for a rectangular well of an infinite depth.

Introducing rectangular barrier

$$V(0) = V(b) = V_0 \tag{4.5}$$

and remarking solutions of (4.4) such as falling and reflected waves at barrier $x_0 = 0$ we get

$$\psi_{J1} = A_1 e^{-kx+ikx} + B_1 e^{kx-ikx}. \quad (4.6)$$

In the barrier region $0 \leqslant x \leqslant b$ of a small extension, we have solution of (4.4)

$$\psi_{J1} = A_2 e^{-kx+ikx} + B_2 e^{kx-ikx}, \quad \varkappa^2 = \frac{2m}{\hbar^2}V_0 - k^2, \quad k^2 = \frac{2mE}{\hbar^2}. \quad (4.7)$$

The solution (4.6) for a transmitted wave at $x = b$ can be expressed in the following way

$$\psi_{J3} = A_3 e^{(-k+ik)\cdot(x-b)}. \quad (4.8)$$

Requiring the equalities

$$\psi_{J1}(0) = \psi_{J2}(0), \quad (4.9)$$

$$\frac{d\psi_{J1}}{dx} = \frac{d\psi_{J2}}{dx}, \quad x = 0, \quad (4.10)$$

$$\psi_{J2}(x = b) = \psi_{J3}(x = b), \quad (4.11)$$

$$\frac{d\psi_{J2}}{dx} = \frac{d\psi_{J3}}{dx}, \quad x = b, \quad (4.12)$$

we obtain the following equations

$$A_1 + B_1 = A_2 + B_2, \quad (4.13)$$

$$(-k + ik)A_1 + (k - ik)B_1 = (-\varkappa + i\varkappa)A_2 + (\varkappa - i\varkappa)B_2, \quad (4.14)$$

$$A_2\exp[(-\varkappa + i\varkappa)b] + B_2\exp[(\varkappa - i\varkappa)b] = A_3, \quad (4.15)$$

$$(-\varkappa + i\varkappa)A_2\exp[(-\varkappa + i\varkappa)b + (\varkappa - i\varkappa)B_2\exp(\varkappa - i\varkappa)b = (-k + ik)A_3. \quad (4.16)$$

Solving the presented equations we get

$$\frac{A_3}{A_1} = \frac{4k\varkappa\cdot\exp[(-\varkappa + i\varkappa)b]}{k^2 + \varkappa^2 - (k-\varkappa)^2\exp[2(-\varkappa + i\varkappa)b]}, \quad k^2 = \frac{2mE}{\hbar^2}, \quad k^2 = \frac{2m}{\hbar^2}(V_0 - E) \quad (4.17)$$

or

$$\frac{A_3}{A_1} = \frac{4n\cdot\exp[(-\varkappa + i\varkappa)b]}{1 + n^2 - (n-1)^2\exp[2(-\varkappa + i\varkappa)b]}, \quad n = \frac{k}{\varkappa}, \quad n = \frac{k}{\varkappa} = \sqrt{\frac{E}{V_0 - E}}. \quad (4.18)$$

Now, the transmission (diffusion) coefficient can be obtained

$$D = \frac{|A_3|}{|A_1|}, \quad D = \frac{16k^2\varkappa^2\exp[-2\varkappa b]}{s^2 - 2s\delta^2\exp[-2\varkappa b]\cos(\varkappa b) + \delta^2\exp(-4\varkappa b)} \quad (4.19)$$

where $s = k^2 + \varkappa^2$, $\delta = k - \varkappa$, or

$$D = \frac{16n^2\exp[-2\varkappa b]}{N^2 + \Delta N^4\exp[-4\varkappa b] - N\Delta N^2\cos[2\varkappa b]\exp[-2\varkappa b]}. \quad (4.20)$$

where $N = 1 + n^2$, $\Delta N = n - 1$.

Taking an approximate value of the denominator, where the width b of the barrier is large compared to the wave length $\lambda = 2\pi/\varkappa$, $\varkappa b \gg 1$, we get

$$D \cong D_0\exp[-2\varkappa b], \quad D_0 = \frac{16n^2}{(1 + n^2)^2} \quad (4.21)$$

the same expression as in [2]. For this case, from (4.20) we can get the equivalent expression [10]

$$D = \frac{16k^2\varkappa^2\exp[-2\varkappa b]}{(k^2 + \varkappa^2)^2}. \quad (4.22)$$

For the obtained solution, we have essential differences of the wave functions in the barrier region where we have the oscillating function (4.7) and the essentially different classical solution [2]

$$\psi_{J2} = A_2 e^{-\varkappa\cdot x} + B_2 e^{\varkappa\cdot x}, \quad \varkappa^2 = \frac{2m}{\hbar^2}V_0 - k^2, \quad k^2 = \frac{2mE}{\hbar^2}. \quad (4.23)$$

Conclusions

The Heisenberg uncertainty principle connected with the diffusion equation is a powerful method for explaining of differences between classical and quantum physics and expand theoretical and practical applications. According to (2.25) we obtained that frequency of disappearance of an electron in the hydrogen atom $1/\Delta t = 0.156\cdot10^{22}\ \mathrm{s}^{-1}$ by annihilation interacting with virtual positrons in physical vacuum. They can appear at the distances $c\Delta t = 0.192\cdot10^{-2}$ Å.

In this case, the electrons cannot move in circular Bohr orbits. Free movement of quantum particles is stochastic processes and is correlated with quantum diffusion in physical vacuum (3.2) with decreasing stochastic waves' packets (2.8). The obtained formulas (3.3), (3.4) can be used for evaluation of radii of spherical defects in solids. This is important for the analysis of point defects by X-ray diffraction experiments [16,17], and application to new superdiffusion technologies of semiconductors production [18]. The wave functions in the potential region (4.7) are oscillating in different way like for free movement represented by Eqs. (2.15) and (2.16). In the potential barrier region we cannot use the assumption that here we have not a spreading of real particle, but there are virtual processes [19] of particle's quantum diffusion in physical vacuum. This comment is interesting because in paper [20] the hypothesis that a particle with mass m takes part in Brownian motion with diffusion coefficient $\hbar/2m$ obtained from the second Newton's law was applied. The wave functions of particles having continuous trajectories [19] cannot be used to describe the quantum states.

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