Nonlinear Analysis: Modelling and Control, Vilnius, IMI, 1999, No 4, pp.11-22

MODELLING OF WOOD DRYING AND AN INFLUENCE OF LUMBER GEOMETRY ON DRYING DYNAMICS

R. Baronas
Vilnius University, Naugarduko 24, LT-2006 Vilnius, Lithuania

F. Ivanauskas
Vilnius University, Naugarduko 24, LT-2006 Vilnius, Lithuania
Institute of Mathematics and Informatics, Akademijos 4, LT-2600 Vilnius, Lithuania

M. Sapagovas
Institute of Mathematics and Informatics, Akademijos 4, LT-2600 Vilnius, Lithuania

Abstract

Modelling of wood drying is analyzed. Wood drying involves moisture transfer from the interior of the wood to the surface, then from the wood surface to the surrounding air. These processes can be characterized by the internal and surface moisture transfer coefficients. A model of the two-dimensional moisture transfer is suggested to determine these coefficients in contrast to the one-dimensional model which was proposed in [12]. The model is based on a diffusion equation with a variable diffusion coefficient. The insufficiency of the one-dimensional model is considered. The influence of the geometry of a lumber on determination of the surface emission and diffusion coefficients and on the dynamics of drying is investigated.
Key words: wood drying, diffusion, surface emission, modelling

INTRODUCTION

Wood drying involves two types of moisture transfer. The moisture moves from the interior of the wood to the surface, then from the wood surface to the surrounding air [7], [12], [13]. In initial drying of permeable species, the wood surface is wet and is supplied with moisture from the interior at the rate greater than, or nearly as great as, the evaporation rate from the surface. In this case, the surface evaporation rate, characterized by the surface emission (transfer) coefficient, controls (and is a major factor in) the rate of drying. As drying progresses, the supply of moisture can no longer keep pace with the rate of surface evaporation. Then, the internal transfer, characterized by the moisture diffusion (internal transfer) coefficient, controls the rate of drying.

Recently, an optimization technique to determine these coefficients was proposed in [12]. The technique is based on the least squares method, used in conjunction with standard mathematics of diffusion analysis resulting in good agreement between calculated and experimental drying curves. The mathematical model involves a diffusion equation with initial and boundary conditions. Red oak surface emission coefficients at various drying conditions ([12]) determined by this technique were considerably greater than the values determined by other analysis methods [9], but the reason for the difference was not known in [12]. A model of two-dimensional moisture transfer is suggested to determine surface emission and diffusion coefficients in contrast to the one-dimensional model which was used in [12]. The insufficiency of the one-dimensional model has been considered. The influence of the ratio of the width to the thickness of a lumber on determination of the surface emission and diffusion coefficients and on the dynamics of drying is investigated.

MODEL DEFINITION AND REALIZATION

Let Ω be a restricted area of a wood piece and Γ is the surface of the area Ω. The moisture moving in the direction normal to the surface of the wood piece can be expressed in the diffusion equation with variable coefficient:

$$\frac{\partial u}{\partial t} = \operatorname{div}(D(u)\operatorname{grad} u) \qquad (1)$$

where u is the moisture content of wood as mass of water/mass of dry wood, t the time and $D(u)$ the variable diffusion coefficient. The diffusion equation can be used to predict moisture distribution in wood in a isothermal drying process because the thermal conductivity for wood is orders of magnitude higher then the diffusion coefficient [8].

The wood piece is assumed to be homogeneous and isotropic. This is used in the initial condition given by

$$u|_{\Omega} = u_0 \qquad (2)$$

The boundary condition that describes surface evaporation is

$$D(u)\frac{\partial u}{\partial n}\bigg|_{\Gamma} = S\left(u_e - u|_{\Gamma}\right) \qquad (3)$$

where $(\partial u/\partial n)|_{\Gamma}$ is a derivative of u with respect to the internal normal direction to the surface Γ, S is the surface emission coefficient, u_e is the equilibrium moisture content (EMC) with the ambient air climate at any time. Values of D, S and u_e depend on environment (drying) conditions, and in addition the internal moisture transfer coefficient D depends on the moisture content in the wood. The moisture content influences on the coefficient D only if the moisture content is below the fiber saturation point (fsp) (typically 20%-30% for softwoods):

$$D(u) = \begin{cases} f_D(u) \ \text{ if } u < u_{fsp}\,, \\ f_D(u_{fsp}) \ \text{ if } u \geq u_{fsp}\,, \end{cases} \qquad (4)$$

where u_{fsp} denotes the fsp and f_D is a function which expresses diffusion coefficient in moisture content, temperature and may be some other parameters of ambient air climate [5], [11]. The expression of f_D depends on variety of wood.

The average moisture content is measured at various time in order to understand the dynamics of drying in a real experiment. The calculated average moisture content $\overline{u}(t)$ values at any time t can be determined as

$$\overline{u}(t) = \frac{1}{V_{\Omega}} \iiint_{\Omega} u(t)\,d\Omega \qquad (5)$$

where V_{Ω} denotes the volume of the area Ω.

A wood piece (lumber) was modelled as rectangular parallelepiped. The lumber is assumed to be relatively long. In reality, the physical problem is three-dimensional, however, due to the assumption the three-dimensional task (1)-(4) may be reduced to a two-dimensional task. In that case, the area Ω is to be a rectangle.

The finite-difference technique has been used for discretisation of the model (1)-(4) [10]. We introduced a non-uniform discrete grid to avoid an overload of calculations due to the boundary condition (3). An exponentially increasing step of the grid was used in the space directions across the lumber transverse section from the surface to center, while a constant step was used in t direction. Due to symmetry of rectangle (transverse section of lumber), the moisture content values were calculated only for a quarter of the rectangle. Let us assume the following:

$$x_0 = 0,\ x_i = x_{i-1} + h_{x_i},\ x_n = a/2,\ y_0 = 0,\ y_j = y_{j-1} + h_{y_j},\ y_m = b/2,$$
$$u_{ij} = u(x_i, y_j, t),\ \hat{u}_{ij} = u(x_i, y_j, t+t),\ D_{ij} = D(u_{ij}),\ i = 1,\ldots,n,\ j = 1,..,m\ . \qquad (6)$$

The finite-difference solutions to (1) and (2) can be expressed in two parts used sequentially for all $i = 1,..,n$, $j = 1,..,m$:

1) interior moisture content

$$\hat{u}_{ij} = u_{ij} + $$
$$+ t\Big(\Big(D_{i+0.5,j}\big(u_{i+1,j} - u_{ij}\big)\big/h_{x_{i+1}} - D_{i-0.5,j}\big(u_{ij} - u_{i-1,j}\big)\big/h_{x_i}\Big)\Big/h_{x_{i+0.5}} + \qquad (7)$$
$$+\Big(D_{i,j+0.5}\big(u_{i,j+1} - u_{ij}\big)\big/h_{y_{j+1}} - D_{i,j-0.5}\big(u_{ij} - u_{i,j-1}\big)\big/h_{y_j}\Big)\Big/h_{y_{j+0.5}}\Big)$$

2) surface moisture content

$$\hat{u}_{0j} = \hat{u}_{1j} + h_{x_{0.5}}\left(u_e - \hat{u}_{0j}\right)S\big/D_{0.5,j} \qquad (8)$$
$$\hat{u}_{i0} = \hat{u}_{i1} + h_{y_{0.5}}\left(u_e - \hat{u}_{i0}\right)S\big/D_{i,0.5} \qquad (9)$$

where

$$h_{x_{i+0.5}} = (h_{x_i} + h_{x_{i+1}})/2,\ h_{y_{j+0.5}} = (h_{y_j} + h_{y_{j+1}})/2,$$
$$h_{x_0} = h_{x_1},\ h_{y_0} = h_{y_1},\ h_{x_{n+0.5}} = h_{x_{n-0.5}},\ h_{y_{m+0.5}} = h_{y_{m-0.5}},$$
$$u_{n+1,j} = u_{n-1,j},\ u_{i,m+1} = u_{im-1},\ D_{n+0.5,j} = D_{n-0.5,j}, D_{i,m+0.5} = D_{i,m-0.5}$$

A solution of the finite-difference formulas requires both D and S values. It was assumed that the diffusion coefficient bellow fsp can be represented by

$$f_D(u) = A \cdot e^{-5280/T} \cdot e^{B \cdot u/100} \qquad (10)$$

where T is the temperature in Kelvin, u is percent moisture content, A and B are experimentally determined [12]. The same equation of diffusion as (1) with various boundary conditions has been used in modelling of diffusion of ingredients in semiconductors [3], [4].

Two drying cases (conditions) were investigated:

a) air velocity 1.5 m/s: A = 10.3 cm^2/s, B = 3.20, S = $0.479 \cdot 10^{-5}$cm/s;

b) air velocity 5.1 m/s: A = 11.7 cm^2/s, B = 3.14, S = $0.787 \cdot 10^{-5}$cm/s.

Let a and b be the thickness and width of the lumber, respectively. The model (1)-(4) was used in the numerical experiments with the following values of the parameters:

$$\begin{aligned} &u_0 = 82.5\%, \quad u_e = 16.2\%, \quad T = 316.15\,\mathrm{K} = 43^0\,\mathrm{C}, \\ &a = 2.9\,\mathrm{cm}, \quad b = 10.2\ \mathrm{cm}. \end{aligned} \tag{11}$$

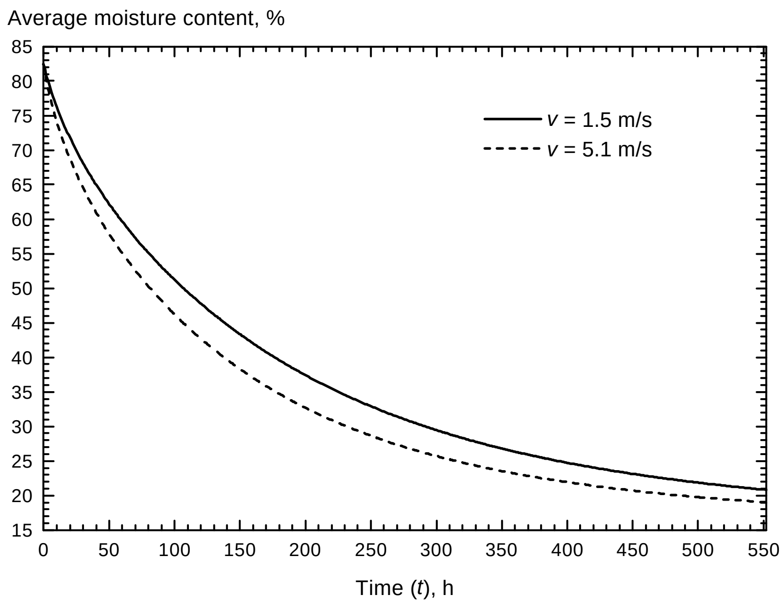

Fig. 1. Dynamics of the average moisture content at 1.5 and 5.1 m/s air velocity

The average moisture content-time curves are shown in fig. 1. Apparently, the values of moisture content for higher air velocity (v = 5.1 m/s) are less than the values for lower air velocity (v = 1.5 m/s), i.e. wood dries faster if air velocity is higher.

Fig. 2 shows the moisture content throughout the lumber at 50 percent average moisture content for the air velocity of 1.5 m/s ($t \approx 106$ h). The numerical results are presented in the form of a three-dimensional surface plot, where the x and y axes represent the thickness and width directions, and the z axis represents moisture content. The point $x = 0$, $y = 0$ conforms to the corner of transverse section of the lumber. As it is possible to notice, the moisture within the lumber is forced to the surface of the lumber, where it evaporates into the airstream, and it is particularly significant for the corner of lumber. The similar three-dimensional plots for the moisture content are presented in [6] where values of the moisture content are reflected about the axis of symmetry in the longitudinal direction.

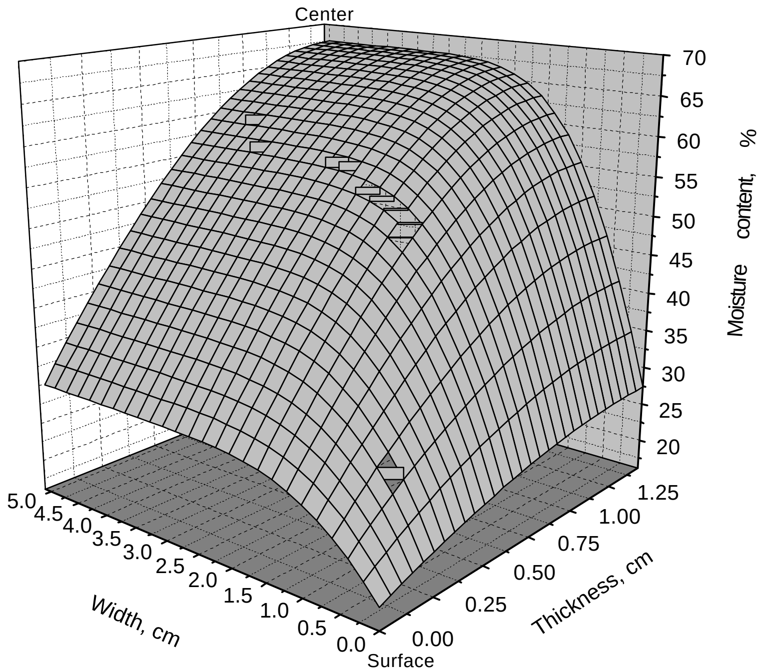

Fig. 2. Moisture content at 50 percent average moisture content for the air velocity of 1.5 m/s.

INFLUENCE OF A GEOMETRY OF LUMBER ON DRYING DYNAMICS

The fraction of total moisture content (the relative amount of the remaining moisture content) in the lumber can be defined as

$$E(t) = \frac{\overline{u}(t) - u_e}{u_0 - u_e} \qquad (12)$$

Let $t_{0.5}$ be the time when the drying process reaches the medium, called halfdrying time, i.e. $E(t_{0.5}) = 0.5$ [13]. The dynamics of drying was considered in a parametrization of the thickness and width of the lumber. In each case of lumber geometry the drying until the time $t_{0.5}$ was simulated to determine dynamics of the drying.

Let k be a ratio of the width to the thickness of the lumber. The transverse section of lumber was modelled as a rectangle with various values of the ratio k (k = 1,..,100) keeping the area of every rectangle equal to ab. The results of calculation are presented in fig. 3. Apparently, the ratio of the width to the thickness important enough for the drying dynamics: a wide lumber dries faster than a narrow lumber with the same volume. Fig. 3 shows that the time $t_{0.5}$ decays exponentially if the ratio k increases.

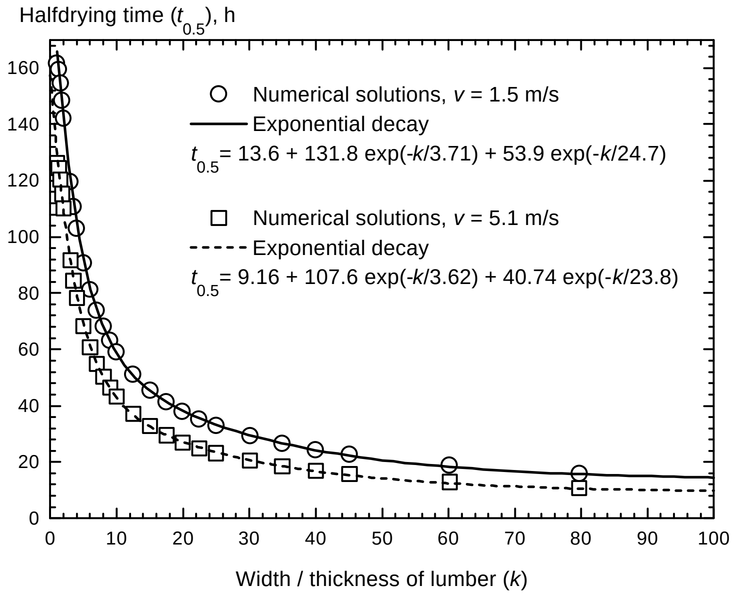

Fig. 3. Dependence of halfdrying time on the ratio of the width to the thickness of lumber keeping the area of the transverse section of the lumber constant.

Rectangles with various values of the ratio k of the width to the thickness of lumber keeping the thickness equal to a was investigated as well. The results of calculation are depicted in fig. 4. The relative thickness of the lumber appears to be important for the drying time. The halfdrying time $t_{0.5}$ increases perceptibly for small values of the ratio k ($k < \approx 10$), and it stays almost unchanged for large values of the ratio k ($k > \approx 30$). So, the usage of the two-dimensional (in space directions) model for the lumber with transverse section of 2.9 cm to 10.2 cm (11) is important because the width of the lumber must be taken into account.

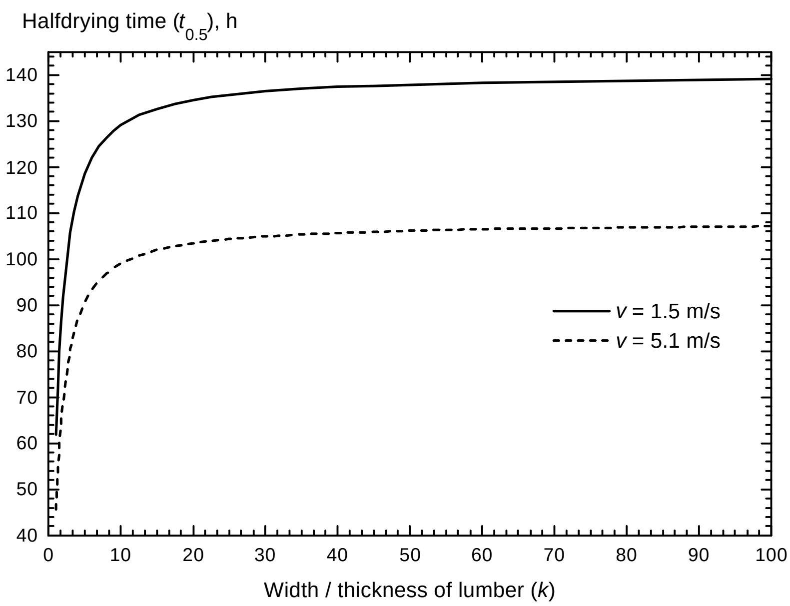

Fig. 4. Dependence of halfdrying time on the ratio of the width to the thickness of lumber keeping the thickness of the lumber constant.

The similar investigation of the influence of the geometry on an operation of the microreactor as a glucose biosensor was presented in [2].

INFLUENCE OF A GEOMETRY OF LUMBER ON DETERMINATION OF COEFFICIENTS

The two-dimensional model (1)-(4) has been applied to the optimization technique to determine red oak surface and internal moisture transfer coefficients during drying, [12]. The values for *A*, *B*, and *S* in the difference solutions were determined by a technique where the optimum values were the ones that minimized the sum of squares of the differences between calculated and experimental moisture content values at various times during drying:

$$\min_{A,B,S}\left(\sum_{i=1}^{n}\left(\overline{u}[A,B,S](t_i)-u_{\exp}(t_i)\right)^2\right) \tag{13}$$

where $u_{exp}(t_i)$ an experimental moisture content value at time t_i (i=1,...,n), n a number of the experimental values. The sums of squares were minimized by a technique that sequentially adjusted each of the three coefficients until the sum of squares was minimized. This procedure was iterated until changes in the third significant digit of the coefficients resulted in no further reduction in the sum of squares. Initial estimates of the coefficients are required to initiate the technique and were available in [12]. The experimental moisture content values were taken from [12] where the values are presented as the moisture content-time curves. Authors of [12] presented these experimental data us in a numerical form to have more exact result. The calculated values of *A*, *B* and *S* are shown in table 1. Table 1 contains values determined by using the one-dimensional model (1D model) ([12]) as well as values determined by using the two-dimensional model (2D model) (1)-(4).

Coefficient	v = 1.5 m/s		v = 5.1 m/s	
	1D model	2D model	1D model	2D model
A (cm^2/s)	12.9	12.9	14.8	13.9
B	2.32	2.41	2.48	2.55
S *10^5 (cm/s)	0.927	0.483	1.51	0.786

Table 1. Values of *A*, *B* (10) and *S* (3) that minimize the sum of squared deviations between experimental and calculated moisture content values

A noticeable difference between the values of the surface emission coefficient *S* is the most important difference between the values determined by the one and two-dimensional models. The values of the coefficient *S* determined by using the two-dimensional model are almost two times less in comparison with the values determined by using the one-dimensional model. The less values of the coefficient *S* compare more favorable with the values found by different analysis methods ([1], [9]) than the greater values determined by one-dimensional model and presented in [12]. The reason for the difference can be the relatively small ratio $k \approx 3.52$ of the width to the thickness of the lumber used in the experiments (11). Due to the influence of the lumber geometry on the dynamics of drying discussed above, usage of the one-dimensional model for the lumber with the ratio $k \approx 3.52$ is not enough for determination of the surface and internal moisture transfer coefficients. The usage of the one-dimensional model may not be justified for a lumber with the ratio $k < \approx 10$. The usage of the one-dimensional model to determine the surface and internal moisture transfer coefficients for a rather narrow lumber may be a reason of inexact values of the coefficients.

REFERENCES

1. S. Avramidis, J. F. Siau, *An investigation of the external and internal resistance to moisture diffusion in wood, Wood Sci. Technol.* 21, p.249-256, 1987.

2. R. Baronas, F. Ivanauskas, J. Kulys, *Modelling a biosensor based on the heterogeneous microreactor, J. Math. Chem.*, 25 , p. 245-252, 1999.

3. V. Buda. R. Èiegis, M. Sapagovas. *A model of multiple diffusion from the restrict origin, Differential equations and their application,* [in Russian], Institute of Mathematics and Cybernetics, Vilnius, 38, p. 9-14, 1986.

4. V. Buda. M. Sapagovas, R. Èiegis. *A two dimensional model of non-linear diffusion, Mathematical and mechanical modelling in microelectronics,* [in Russian], Institute of Physics of Semiconductors, Vilnius, p. 36-43, 1985.

5. Y. Chen, E. T. Choong, D. W. Wetzel, *A numerical analysis technique to evaluate the moisture dependent diffusion coefficient on moisture movement during drying, Wood Fiber Sci.,* 28(3), p. 338-345, 1996.

6. W. J. Ferguson, I. W. Turner, *A comparison of the finite element and control volume numerical solution techniques applied to timber drying problems below the boiling point, Int, J. Num. Methods Eng.*, 38(3), p. 451-467, 1995.

7. A. J. Hunter, *Equilibrum moisture content and the movement of water through wood above fibre saturation, Wood Sci. Technolog.*, 29, p. 129-135, 1995.

8. J. Y. Liu, J. Y. Simpson, *Two-stage moisture diffusion in wood with constant transport coefficients, Drying Technol.,* 17(1&2), p. 257-269, 1999.

9. H. N. Rosen, *The influence of external resistance on moisture adsorption rates in wood, Wood Fiber Sci.*, 10(3), p. 218-228, 1978.

10. A. Samarskii, *Difference Scheme Theory,* [in Russian], Nauka, Moskow, 1989.

11. W. T. Simpson, *Determination and use of moisture diffusion coefficient to characterize drying of northern red oak (Quercus rubra), Wood Sci. Technolog.*, 27, p. 409-420, 1993.

12. W. T. Simpson, J. Y. Liu, *An optimization technique to determine red oak surface and internal moisture transfer coefficients during drying, Wood Fiber Sci.,* 29(4), p. 312-318, 1997.

13. O. Söderstrom, J.-G. Salin, *On determination of surface emission factors in wood drying, Holzforschung,* 47(5), p. 391-397, 1993.

Medienos dþ iovinimo modeliavimas ir bandinio geometrijos átaka dþ iûvimo dinamikai

R. Baronas, F. Ivanauskas, M. Sapagovas

Šiame straipsnyje yra analizuojama medienos dž iovinimo modeliavimas. Medienai dþ iûstant, drëgmë, esanti bandinio viduje, juda link pavirðiaus, o po to á aplinkà. Ðie procesai charakterizuojami drëgmës laidumo ir drëgmës atidavimo koeficientais. Medienos dž iovinimo modeliavimui yra taikoma difuzijos lygtis su kintamu difuzijos koeficientu, pradine ir kraðtine sàlygomis. Ðiame straipsnytje yra pasiûlytas dvimatis drëgmës sklidimo medienoje modelis, kuris sëkmingai gali bûti taikomas drëgmës laidumo ir atidavimo koeficientams nustatyti panaudojant eksperimentinius duomenis. Straipsnyje yra iðtirta bandinio geometrijos átaka medienos dþ iûvimo dinamikai bei modelio taikymui drëgmës laidumo ir atidavimo koeficientams rasti. Medienos dþ iovinimo uþ davinys yra sprendþ iamas baigtiniø skirtumø metodu.

Received: September 30, 1999
Accepted: October 27, 1999