

## Determination of Eye Torsion by Videoculography Including Cornea Optics

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### Introduction

In videoculography torsional eye rotations are calculated using image of iris signature [1, 2]. Previous research works [3, 4] have considered the influence of cornea optics throw linear transformation of iris image.

Analysing pictures taken using videoculography we observed changes in the shape of pupil and iris in them [5]. Distortion of the image of iris can be caused by cornea serving as an optic lens, that causes non-linear distortion [6, 7]. The above-mentioned facts can influence the quality and accuracy in registration of torsional eye movements in cases when iris is used to calculate torsion.

The purpose of this research is to determinate eye torsion by videoculography including cornea optics.

### Method

The influence of corneal optics to the iris and pupil picture we will reach using mathematical model of corneal optics [6, 7]. The eye is made of mediums having different refraction indices. The rays refract getting over the boundary of mediums with different refraction indices. After the ray propagation analysis, we obtain that the boundary between the cornea and anterior chamber has little influence to ray propagation direction. Bigger changes are when the rays go from the air into the cornea. The iris image has non-linear transformation. We estimated this iris transformation [6, 7]. After the calculation, there are got the iris image points.

In previous work [5] we developed mathematical model of eye iris image formation on the plane. In that model there were not estimated corneal optics influence, so we will correct the model using the results of other researches [6, 7]. We established what influence cornea has on location of the iris image points, when eye makes rotations (see Fig. 1).

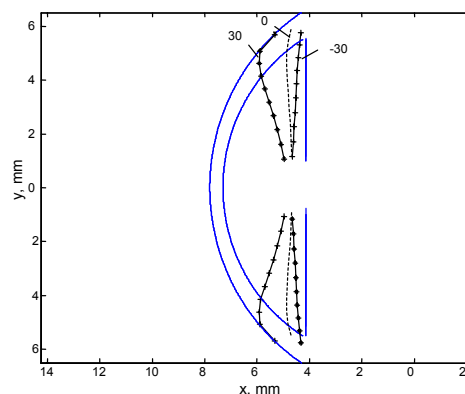


Fig. 1. The location of iris image points (cross-section view) in primary eye position (0 deg.), after eye rotation up (30 deg.) and down (-30 deg)

The surface of iris image could be approximated by the third degree polynomial using least squares method:

$$z(x, y) = a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 + a_6x^3 + a_7x^2y + a_8xy^2 + a_9y^3; \quad (1)$$

here  $x, y, z$  – coordinates of the iris image points,  $a_i$  – coefficients, which depend on  $\theta$  (horizontal rotation angle of the eye) and  $\varphi$  (vertical rotation angle of the eye). We can find polynomial approximation of  $a_i$  coefficients on eye rotation angles also by the third degree polynomial:

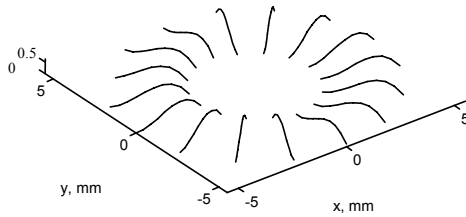
$$a_i(\theta, \varphi) = b_{i0} + b_{i1}\theta + b_{i2}\varphi + b_{i3}\theta^2 + b_{i4}\theta\varphi + b_{i5}\varphi^2 + b_{i6}\theta^3 + b_{i7}\theta^2\varphi + b_{i8}\theta\varphi^2 + b_{i9}\varphi^3. \quad (2)$$

When eye makes rotations, changes are symmetric in the image of the iris and they originate in direction of the rotations [6, 7]. We can use dependence on  $\theta$  (horizontal

rotation angle of the eye) only, because eyelids cover iris in vertical direction and this disturb to calculate torsional eye rotations, when we use iris signature for it. So, it's possible equation (2) express in simpler way:

$$a_i(\theta) = b_{i0} + b_{i1}\theta + b_{i2}\theta^2 + b_{i3}\theta^3. \quad (3)$$

We use MATLAB™ for all calculations. Three-dimensional image of the iris is presented in Fig. 2



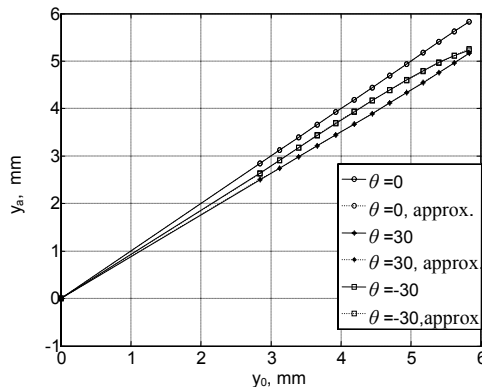
**Fig. 2.** Three – dimensional image of the iris, including cornea optics, when eye is in primary position (0 deg.)

We used a mathematical model of three – dimensional eyeball and derived from it a mathematical model of formation of image of the iris on the plane [5].

Changes of distance between iris signature points and centre of the pupil, when eye makes rotations of  $\theta$  angle are presented in Fig. 3. Here  $y_0$  (axis X) – distance between centre of pupil and points of image of the iris in primary position (0 deg.) and  $y_a$  (axis Y) – distance between centre of the pupil and points of image of the iris eye makes rotations of  $a$  angle. We calculated approximations of these functions dependence using third degree polynomial:

$$y_a(y_0) = a_0 + a_1y_0 + a_2y_0^2 + a_3y_0^3; \quad (4)$$

here  $a_i$  – coefficients, which depend on  $\theta$  (horizontal rotation angle of the eye: left (30 deg.) and right (-30 deg.)).



**Fig. 3.** Changes of distance between iris signature points and centre of the pupil, when eye makes rotations of  $a$  angle

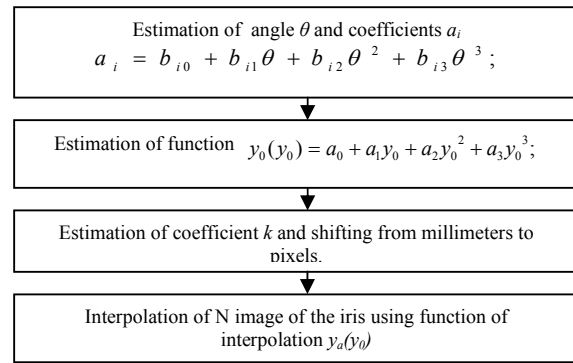
So, we can make conclusion that when eye makes rotations dependence of function  $y_a(y_0)$  becomes non-

linear. Nonlinearity depends on distance between centre of pupil and points of image of the iris and it increase from centre of the pupil to limbus (cornea-sclera junction).

If we want to determinate eye torsion by videooculography using iris signature and including cornea optics, we must make the interpolation of image of the iris. We can use dependence  $y_a(y_0)$  as function for interpolation of it. Algorithm of the interpolation of image of the iris is presented in Fig. 4. We create model of the eye using parameters of real eye. In videooculography these parameters are expressed in pixels, but we use millimetres in model of the eye. So, we must transform model from millimetres to pixels. It's needed to calculate coefficient:

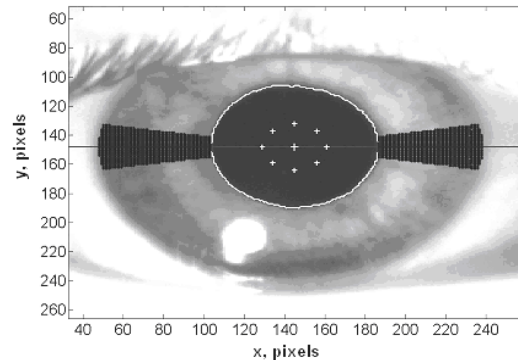
$$k = \frac{R_{mm}}{R_{px}}; \quad (5)$$

where  $R_{mm}$  – parameters of the eye in millimetres and  $R_{px}$  – parameters of the eye in pixels.



**Fig. 4.** Algorithm of the interpolation of image of the iris

Suggested method for determination of eye torsion by videooculography use sections from image of the iris (see Fig. 5).



**Fig. 5.** Image of eye and selected sections of iris, which are used for torsion evaluation

We rotate eye ball (see Fig. 6) by angle  $\alpha$  (range –  $\alpha = \pm 10$  deg. and step –  $\alpha = 2.5$  deg.) using quaternions mathematics around the axis, which go throw centre of the pupil and is perpendicular to pupil plane. After we calculate correlation coefficients of sections from image, in which do torsion evaluation, with sections of simulated image. We assume, that torsional angle of eye is the angle  $\alpha$ , where correlation coefficient has peak. Reference sections of the iris, which is used for calculating

correlation coefficients, are selecting during calibration procedure, when eye is in primary position (0 deg.).

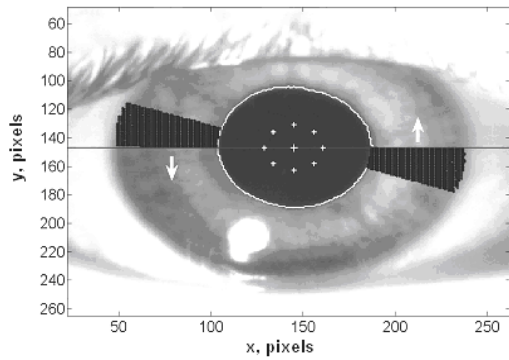


Fig. 6. Image of eye and rotations of selected sections of iris, which are used for torsion evaluation

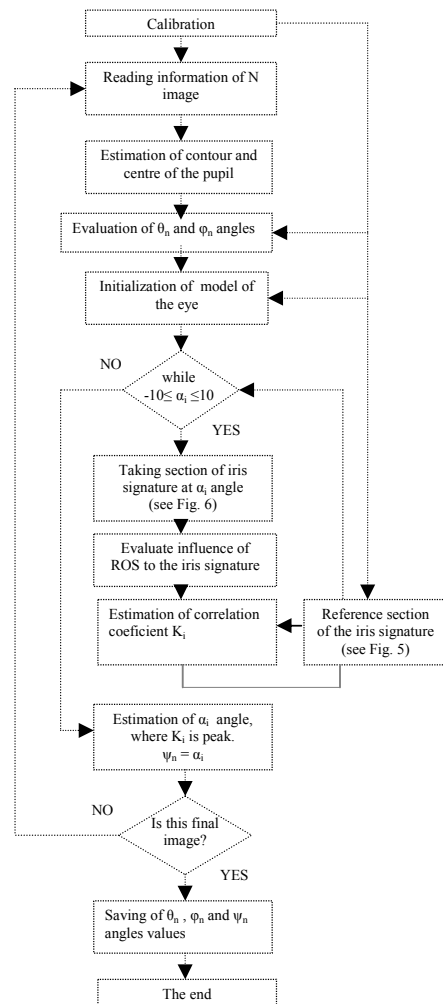


Fig. 7. Algorithm of torsional eye movements' evaluation system

## Results

The models, with taking and without taking cornea optics into consideration, were applied to simulate the images of eye, when it performed horizontal rotations.

Simulated images were compared with experimental eye images by testing position of pupil centre, forms of contours of iris and pupil, iris segments and elements of iris signature. During experiment nine patients (six male and three female) sited before screen with red light LEDs. The LEDs were switched on by random order in horizontal, vertical, and diagonal direction.

The results of comparison of the pupil centre position show that when cornea optics was considered the error decreases in average by 0.7 degree. If influence of cornea is not taken into consideration, the error increases together with the increase of rotation angle. Performing the rotation every 5 degrees, the error of pupil centre increases every 0.23 degrees. It indicates that contribution of cornea optics can be considered if the radius of eyeball is increased by 4.6%

Errors of model, when influence of cornea optics was taken into consideration, are of similar value, when horizontal, vertical and diagonal movements are performed. This is the proof, that a model with an average error  $0.55 \pm 0.25$  degrees along the movement direction simulate the position of pupil centre is suitable for any type of movement. In all fixations considerable large dynamic ranges (average 0.51 degrees) and standard deviation of result (average 0.25 degrees) are observed and this shows influence of micro movements during fixations and individual visual system characteristics of subjects participating in experiments. We observed no considerable difference between male and female subjects, but we noticed differences of individual character. Construction of the model is based on average parameters of human eye. Inadequacy obtained may be caused by accuracy of the experiment itself and the videoculography registration system as well as by head movements during the experiment.

Comparison of suggested videoculography method based on Listing planes with that of „Chronos“ proves that, the new method reduces standard deviation (STD) by  $\approx 15\%$ . An STD of Listings plane of two patients (EP and GD) is presented in Fig. 8.

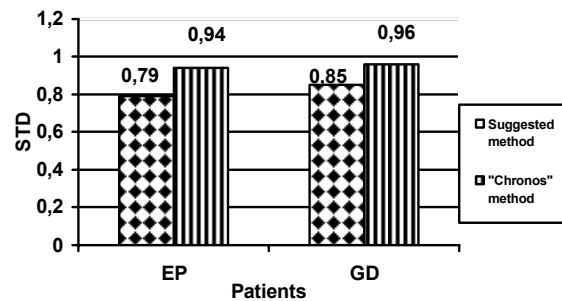


Fig. 8. STD of Listing planes, when suggested and „Chronos“ methods are used

## Conclusions

Influence of optical characteristics of cornea on formation of image of iris and its signature is evaluated. A mathematical model of optical characteristics of cornea is

created for this purpose. The nature of changes in geometric parameters of pupil, iris and its signature images, when the rotation of eye is performed, is analysed. We concluded that optical characteristics of cornea influence iris and its signature images and those distortions should be considered when calculating torsion by the method of correlation of signature of iris.

Mathematical model of formation of image of iris on the plane is created. It evaluated the influence of optical characteristics of cornea on the process. We drew the conclusion that the results of modelling fairly well correspond the experimental results ( $0.55 \pm 0.25$  degrees) and this model can be used for calibration of 3D eye movements and calculation of torsion. This enabled us to develop new method for calibration of videooculography systems.

More robust videooculographical method to register torsional movements of human is created. It evaluates the influence of optical characteristics of cornea on the image of iris. This model reduces standard deviation of points on Listing planes on average by  $\approx 14\%$  in comparison with previously used methods. Thus, it is significant to consider the influence of optical characteristics of cornea on iris image formation, when calculating torsional eye movements by the method of correlation of signature of iris.

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### **E. Paliulis, G. Daunys. Determination of Eye Torsion by Videooculography Including Cornea Optics // Electronics and Electrical Engineering. – Kaunas: Technologija, 2006. – No. 5(69). – P. 83–86.**

Influence of optical characteristics of cornea on formation of image of iris and its signature is evaluated. A mathematical model of optical characteristics of cornea is created for this purpose. The nature of changes in geometric parameters of pupil, iris and its signature images, when the rotation of eye is performed, is analyzed. We concluded that optical characteristics of cornea influence iris and its signature images and those distortions should be considered when calculating torsion by the method of correlation of signature of iris. Original videooculography method to register torsional movements of human is created. It evaluates the influence of optical characteristics of cornea on the image of iris. This model reduces standard deviation of points on Listing planes on average by approximately 14% in comparison with previously used methods. Thus, it is significant to consider the influence of optical characteristics of cornea on image of iris when calculating torsional eye movements by the method of correlation of signature of iris. III. 8, bibl. 7 (in English; summaries in English, Russian and Lithuanian).

### **E. Paliulis, G. Daunys. Определение торцевых движений глаза в видеоокулографии с учетом оптических свойств роговицы // Электроника и электротехника. – Каунас: Технология, 2006. – № 5(69). – С. 83–86.**

Оценено влияние оптических свойств роговицы при формировании изображения радужной оболочки и ее сигнатуры. Математическая модель оптических свойств роговицы была создана для этой цели. Проведен анализ закономерностей изменения геометрических параметров изображений зрачка, радужной оболочки и ее сигнатуры, когда глаз совершает повороты. Определено, что при выполнении глазом поворотов, изображение радужной оболочки, а тем самым и ее сигнатура, искажаются и это надо учитывать при проведении измерений торцевых движений глаза методом корреляции сигнатуры радужной оболочки. Создан универсальный видеоокулографический метод для регистрации торцевых движений человеческого глаза, который учитывает влияние оптических свойств роговицы. Эта модель уменьшает разброс точек плоскости Листинга по сравнению с раньше применяемыми методами в среднем на 14%. Таким образом становится ясно, что необходимо учитывать влияние оптических свойств роговицы при моделировании изображения радужной оболочки. Ил. 8, bibl. 7 (на английском языке; резюме на английском, русском и литовском яз.).

### **E. Paliulis, G. Daunys. Akies suktyjes judesių nustatymas videookulografijoje atsižvelgiant į ragenos optines savybes // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2006. – Nr. 5(69). – P. 83–86.**

Įvertinta ragenos optinių savybių įtaka rainelės ir jos signatūros atvaizdo susidarymui. Šiam tikslui sukurtas ragenos optinių savybių matematinis modelis. Išnagrinėti vyzdžio, rainelės bei jos signatūros geometrinų parametrų kitimo dėsningumai, kai akis daro sukčius. Nustatyta, kad dėl ragenos optinių savybių įtakos rainelė bei jos signatūra iškraipomi ir, skaičiuojant suktyjes judesius rainelės signatūros koreliacijos metodu, į tai reikėtų atsižvelgti. Sukurtas originalus žmogaus akies suktyjes judesių registravimo VO metodas, įvertinantis ragenos optinių savybių įtaką rainelės atvaizdui. Toks modelis vidutiniškai apie 14% sumažina Listingo plokštumų taškų sklaidą, palyginti su anksčiau naudotais modeliais. Taigi, skaičiuojant akies suktyjes judesius rainelės signatūros koreliacijos metodu, svarbu įvertinti ragenos optinių savybių įtaką rainelės atvaizdui. Il. 8, bibl. 7 (anglų kalba; santraukos anglų, rusų ir lietuvių k.).