396. DAMPING OF SMALL ELECTRONIC EQUIPMENT VIBRATIONS

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Abstract. The present paper is aimed at developing methods of damping (decreasing) the amplitudes of random mechanical vibrations of moving equipment by using the approach suggested by the authors. In the description of random vibrations, the theory of random processes is applied, the energy dissipation of mechanisms is determined. Internal and constructional losses are analyzed. The role of viscosity at the propagation of wave energy in certain systems is specified. A level of random vibrations is evaluated. A concrete sample illustrating the role of constructive absorption of energy is analyzed. The paper presents calculation methods, which may be easily applied for the creation of a computer programme.

Keywords: random vibrations, damping.

INTRODUCTION

At present, electronic equipment installed in moving objects (e.g. vehicles, etc.) are being produced and widely used. Therefore, the fixing of individual elements of sensing devices to the body of mechanisms strongly affects their performance and service life.

For analysis of random vibrations of electronic apparatus and in solving other technical problems, where the theory of random processes is applied, of key importance is the installation of the mechanisms of energy dissipation. It is known that heat conductivity plays an extremely important role during vibrations of the elements of the type of bars and plates. [1, 2]

In the complex mechanical systems, alongside inner losses in the material of the construction parts, the so-called constructional losses are of special importance. Any part of the construction interacts with other parts through the greater or lesser number of contacts, receiving or releasing the energy of vibrations. It is that interaction, characteristic of the complex systems, which plays an important role in random vibrations and leads to the effect of constructional friction. On the example of random vibrations of the liquid-metal contact, the classical case of viscous dissipation is examined [3, 4].

Since those issues are quite extensive and are related to special mechanical problems, we shall restrict ourselves to giving the typical examples, illustrating the role of the most important mechanisms of dissipation at random vibrations of electronic apparatus.

For description of inner losses in the substance, certain approaches exist in the mechanics. Most often the components, reflecting dissipation, are introduced into the equation. This method may be best traced on the example of Navier-Stokes equations, used for viscous liquid small bodies, which are employed in the capacity of the working substance in the devices.

Any excitation of a sufficiently small amount of viscous liquid is described by the system of linearized differential equations [5].

By applying a computer program, a vibration model with the smallest number of free vibrations may be constructed by the considered statistical theory method and provide for the effective damping of constructional vibrations.

In developing statistical models of mechanical vibrations, small dimensions of the vibrating system compared to the wave length are taken into consideration.

RANDOM VIBRATIONS ON THE BASIS OF ENERGY DISSIPATION OF NATURAL MECHANISMS

In the case of low viscosity in the capillary tube, the solution of a dispersion equation for a symmetrical wave may be searched in the form of $k_1 h = \pi n + \delta_n$, where δ_n is a small correction, conditioned by viscosity.

In the presence of the layer of liquid, laying on the solid base and having the free upper surface, the components of the tensor of tensions σ_{yy} and σ_{yx} are turning into zero:

$$\sigma_{yy} = \rho_0 \frac{\partial \Phi}{\partial t} - 2\eta \frac{\partial^2 \Phi}{\partial x^2} - 2\eta \frac{\partial^2 \phi}{\partial x \partial y}, u_y = \frac{\partial \Phi}{\partial y} - \frac{\partial \phi}{\partial x},$$

$$\sigma_{yx} = \eta \left(2\frac{\partial^2 \Phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right), u_x = \frac{\partial \Phi}{\partial x} + \frac{\partial \phi}{\partial y}.$$

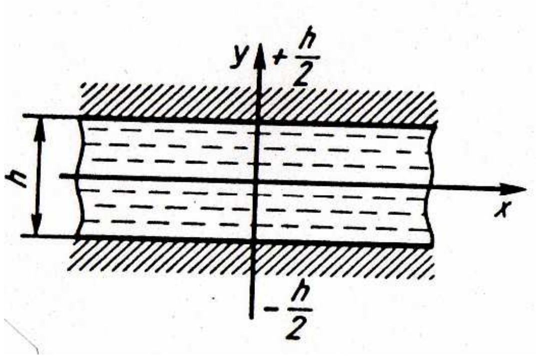


Fig. 1. The calculation diagram for the capillary with the natural viscosity of the medium

The presented method of calculation may be used in the case when it is necessary to take into account the losses, resulting from the heat conductivity. The full system of linear equations has the form:

$$\rho_0 \frac{\partial u}{\partial t} = -grad\,\rho + \eta \Delta u + \left(\xi + \frac{1}{3}\eta \right) graddivu,$$

$$\frac{ds}{dt} + divu = 0, \frac{\partial T}{\partial t} = a\frac{\partial s}{\partial t} + \beta \Delta T, \qquad (1)$$

$$\frac{\partial \rho}{\partial t} = \rho_0 c_0^2 \left(\frac{\partial s}{\partial t} + \frac{\partial T}{\partial t} \right).$$

The appearance of new solutions in a wave equation means the existence of such waves in the real liquid, the existence of which is connected with thermal processes, accompanying the compression and rarefaction of the medium. That type of waves is realized only close to the boundary of the wave field, as they turn to be the quickly damped. Correspondingly, the number of boundary conditions increases. Setting of the heat regime onto the edges of the wave field is an additional condition, necessary for solving the system.

The solutions found may be used for investigating the physical properties of wave fields in the viscous media of capillary tubes – the sensitive elements of the sensors and devices of automation. It is important, for example, to define the role of viscosity during the propagation of wave energy in the indicated systems. From our investigations [6] it is well known that in the waveguide with solid walls at low frequencies the so-called zero normal wave, which is capable of carrying energy along the waveguide to any big distances, may propagate. It is shown below that at the accounting of viscosity, energy propagation deep into the capillary tube of small section is impossible, as the corresponding constant of wave propagation will be the complex number. Let's go back to the problem of propagation of a symmetrical wave in the waveguide with the solid surface, studied in the works [6]. As it was shown, the constant f wave propagation k must satisfy the dispersion equation

$$\frac{ctg\frac{h}{2}\sqrt{k_L^2 - k^2}}{ctg\frac{h}{2}\sqrt{k_t^2 - k^2}} = -\frac{\sqrt{k_L^2 - k^2}\sqrt{k_t^2 - k^2}}{k^2}$$

Since we speak of a zero wave, having almost the plane front for which $k_t h \le 1; k_L \le 1$, it is possible to replace $tg\frac{h}{2}\sqrt{k_L^2 - k^2} \cong \frac{h}{2}\sqrt{k_L^2 - k^2}$ (due to the small thickness of the capillary h). Thus, $k = \sqrt{i\frac{12\eta\omega}{\rho c^2 h^2}}$, i.e. instead of energy propagation we observe the extinction of excitations close to the entrance of the capillary tube. This result is also obtained for the capillary tube of round form. The examples show that in each concrete case, the choice of the design diagram of the phenomenon requires a careful approach and justification.

Inner losses in the material of the solid bodies are also conditioned by friction and heat conductivity. For description of internal friction in the material instead of Hooke's law [7] the more complex correlation of the kind is used

$$\sigma_{ik} + \alpha \frac{\partial \sigma_{ik}}{\partial t} + \ldots = 2\mu \in_{ik} + \lambda \delta_{ik} \in_{kk} +$$

$$\beta \frac{\partial}{\partial t} \in_{ik} + \lambda \delta_{ik} \frac{\partial}{\partial t} \in_k + \ldots$$

The significant part of inner losses in the material of the solid body is related to heat conductivity. For description of those losses, the temperature component in introduced into Hooke's law, i.e.

$$\sigma_{ik} = -\alpha \delta_{ik} KT + 2\mu \in_{ik} + \lambda \delta_{ik} \in_{kk},$$

where α is coefficient of linear expansion; K is the module of comprehensive compression.

The appearance of the unknown variable – temperature T must be accompanied by the corresponding additional equation of heat conductivity:

$$\frac{\partial T}{\partial t}+\frac{c_p-c_v}{\alpha c_v}div\frac{\partial \bar{u}}{\partial t}=k\Delta_3 T.$$

Here α, c_p, c_v, k – heat parameters of the material (coefficient of temperature expansion, heat capacity at constant pressure and volume and correspondingly the coefficient of heat conductivity).

At the boundary of the body under deformation, alongside the ordinary boundary conditions for dynamic variables, the additional boundary conditions for temperature should be set.

For equipment and the parts of apparatus, it is interesting to analyse the preset problem of thermoelasticity in the case of longitudinal and flexural waves. At longitudinal vibrations we get the following system:

$$\frac{\partial^2 u}{\partial x^2}-\alpha\frac{\partial T}{\partial x}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2},$$

$$\frac{\partial T}{\partial t}+\gamma\frac{\partial^2 u}{\partial x\partial t}=\kappa\frac{\partial^2 T}{\partial x^2}.$$

Here introduced is the designation

$$\gamma=\left(\frac{c_p}{c_y}-1\right)/\alpha.$$

The more complex discussions are necessary for introduction of the system in the case of flexural vibrations. The equations of elasticity with the temperature component are reduced to the form:

$$\frac{Eh^3}{12\left(1-v^2\right)}\Delta\Delta u+\rho h\frac{\partial^2 u}{\partial t^2}+\frac{\Delta M_T}{1-v}=0,$$

where temperature moment M_T is determined by the drop in temperature:

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}, M_T=\alpha E\int_{-\frac{h}{2}}^{+\frac{h}{2}}T\left(z\right)zdz.$$

While calculating the divergence of displacement vector at the bending with the help of hypothesis of plane sections, we reduce the additional equation (of heat conductivity) to the form:

$$\partial\frac{\partial T}{\partial t}-\gamma\frac{\partial}{\partial t}z\Delta u=\kappa\frac{\partial^2 T}{\partial z^2}.$$

By multiplying both parts of the equation by zαE and integrating by thickness, we receive the following correlation:

$$\frac{\partial M_T}{\partial t}+\frac{12\kappa}{h^2}M_t=\frac{h^3}{12}E\left(\frac{c_p}{c_v}-1\right)\Delta\frac{\partial u}{\partial t}.$$

The second member in this equation is received by integrating in parts of the derivative from temperature.

The second component in the right part of that equation usually exceeds significantly the first one; therefore the connected system of equation of heat conductivity is reduced to the usual equation of bending with the correction on the losses:

$$\frac{Eh^3}{12\left(1-\nu^2\right)}\left\{1+\frac{h^2}{12}\frac{\left(c_p-c_v\right)\left(1+\nu\right)}{\kappa c_p}\frac{\partial}{\partial t}\right\}\Delta\Delta u+\rho h\frac{\partial^2 u}{\partial t^2}=0$$

In the work [3] the following coefficients of temporary damping of vibrations δ are given for longitudinal waves:

$$\delta=\lambda T_0\rho\alpha^2\omega^2/18c_p^2$$

and for flexural waves

$$\delta=\frac{h^2}{24}\left(\frac{c_p}{c_v}-1\right)\frac{\left(1+\nu\right)}{k}\omega^2$$

Of special importance are mechanisms of energy dissipation, essential for non-linear processes. The energy consumption for the change in the structure of the medium (in particular, for the accumulation of damages in the medium [4], dissipative mechanisms, accompanying the phenomena of the blow [3], are reflected in the most essential way at the level of the random vibrations.

From the work performed, it is necessary to note that non-linear vibrations in their turn are accompanied by the transformation of frequencies, since the wave energy of high-frequency vibrations may be absorbed more intensively at the expense of aforementioned linear mechanisms.

RANDOM VIBRATIONS OF THE ELASTIC PANEL AT RADIATION-RELATED ENERGY LOSSES

For engineering supplements, the results of the approximated Rayleigh theory of radiation [8] may be used. According to that theory, the loading impedance for the body, vibrating in the screen, makes the value:

$$Z(-i\omega) = p/u = (-i\omega)^2 \rho_0 \left\{ -\frac{i\omega\pi R^4}{2c_0} + \frac{8R^3}{3} \right\} / \pi R^2. \quad (2)$$

Thus, it is possible to produce the following equation of the movement of the plate, loaded with the medium:

$$\rho h \frac{\partial^2 u}{\partial t^2} - \frac{\rho_0 R^2}{2c_0}\frac{\partial^3 u}{\partial t^3} + \frac{8\rho_0 R}{3\pi}\frac{\partial^2 u}{\partial t^2} + D\Delta\Delta u = 0$$

That equation is supplemented with the ordinary boundary conditions, reflecting the excitation of the elastic element from the body of the article,

$$u(R,t) = u_0 e^{-i\omega t}, \frac{\partial u}{\partial r} = 0, (r = R)$$

The solution of the boundary condition, reflecting the frequency properties of the system, may be written in the form

$$u = u_0 e^{-i\omega t} \left\{ \frac{I_0(kR)J_0(kr) - J_0(kR)I_0(kr)}{I_0(kR)J_0(kR) - J_0(kR)I_0(kR)} \right\} \quad (3)$$

where the wave number of flexural vibrations k is determined now by the correlation

$$k = \left\{ \frac{12(1-\nu^2)}{Eh^3} \left[(\rho h + 0{,}82\rho_0 R)\omega^2 + i\omega^3 \frac{\rho_0 R^2}{2c_0} \right] \right\}^{\frac{1}{4}} \quad (4)$$

It is easy to note that the effect of the medium is expressed by the appearance of the connected mass (at the expense of the component $0{,}82\rho_0 R$) and, what is most important for the random vibrations, to the dissipation, which is expressed by the complex value of the wave number *k*. Now, with quite full justification it is possible to pass over to the problem of random stationary vibrations of the panel, radiating the energy, and to record the spectrum of vibrations in the form

$$S_u(\omega, r) = S_0(\omega) \left| \frac{I_0(kR)J_0(kr) - J_0(kR)I_0(kr)}{I_0(kR)J_0(kR) - J_0(kR)I_0(kR)} \right|^2 \quad (5)$$

However, for the quite extended body elements of construction, the character of energy dissipation changes essentially due to radiation. Let, for example, the plate make vibrations according to the law $u = u_0 e^{-i\omega t}\sin\kappa_n x$, where κ_n – wave number. On both sides from the plate, the plane sound waves will emerge, for the determination of the amplitudes of which the boundary condition $\rho_0 \frac{\partial^2 u}{\partial t^2} = -\frac{\partial \rho}{\partial y}$ may be used. Thus, the amplitude of the pressure in each of the four plane waves will be $\rho_0 = \rho_0 \omega^2 u_0 / \sqrt{\frac{\omega^2}{c_0^2} - k_n^2}$, and the value of the vector Umov-Poynting U is in the form $U = p_0^2 / \rho_0 c_0$, i.e.

$$\text{t.e. } U = \frac{\rho_0}{c_0}\omega^4 u_0^2 / \left[\frac{\omega^2}{c_0^2} - k_n^2 \right].$$

Energy radiation means that the amplitude of free vibrations u_0 will decay slowly. The energy of the plate $W = \rho h \omega^2 u_0^2 / 2$ will actually diminish according to the law $\exp\{-2\delta t\}$, where $\delta = \frac{4\rho_0\omega}{\rho h\sqrt{-k_n^2 + \omega^2/c_0^2}}$. From here it follows that ostensible part of the wave number, necessary for the evaluation of the level of random vibrations, will make $\delta / c = k_n \delta / \omega$, where *c* is the velocity of wave propagation of the corresponding type in the plate. At high frequencies it is possible to consider that $k = k_n \left(1 + i4\rho_0 / \rho h \sqrt{\frac{\omega^2}{c_0^2} - k_n^2} \right)$.

A concrete example, illustrating the role of constructional absorption of energy, is considered. Linear oscillatory models, constructions are given, i.e. mass *m* on the elastic spring *k* gets excited from the wall, onto which the spring is fixed (see Figure 2). Energy, stored up in such oscillatory circuit, will radiate partly in the form of flexural waves into the plate. It is shown that dissipating links, which may be calculated, will appear in the oscillatory circuit. In order to find a dissipating member in the equation of motion, the excitation of the plate under the action of the concentrated force $F_0 e^{-i\omega t}$, applied at the beginning of coordinates, is studied.

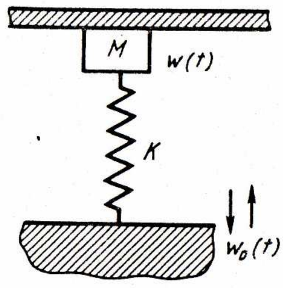


Fig. 2. Estimated model of constructional damping at the account of excitation of flexural waves in the plate

For obtaining an answer to the set task we search for solution of an equation of the axiosymmetrical vibrations of the plate

$$D\left\{\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial \omega}{\partial r}\right\}-ph\frac{\partial^2\omega}{\partial t^2}=0, \qquad (6)$$

which is limited at zero and has the specificity of such order that

$$\lim_{\in\to 0}\left\{-D\oint_{C_\in}\left[\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial\omega}{\partial r}\right]dS\right\}=F_0e^{-i\omega t}. \qquad (7)$$

This condition physically means that the sum of the dissecting forces in the plate on the circuit of the circle, laid around the concentrated force, equals that force, if the radius of the circuit is sufficiently small. The solution of such a type is called the Green's function [3].

Alongside the indicated condition, the Green's function must satisfy the conditions at infinity with $r \to \infty$, i.e. conditions of the absence of the sources at infinity. For deriving of the necessary solution, we shall write the equation of the plate movement for the case of the established harmonious vibrations with frequency ω

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial\omega}{\partial r}-k^4\omega=0, \qquad (8)$$

where $k^4 = ph\omega^2 / D$.

Of the four independent solutions, only two Hankel's functions of the first class from the real and purely putative arguments satisfy the conditions at infinity.

$$H_0^{(1)}(kr)\cong\sqrt{\frac{2}{\pi kr}}e^{i\left(kr-\frac{\pi}{4}\right)},$$

$$H_0^{(1)}(ikr)\cong\sqrt{\frac{2}{\pi kr}}e^{-kr-\frac{i\pi}{4}}$$

Since at $r \to 0$ both those functions have the logarithmic specificity, their difference should be taken as the solution. That solution will be restricted

$$w(r,t)=Ae^{-i\omega t}\left[H_0^{(1)}(kr)-H_0^{(1)}(ikr)\right] \qquad (9)$$

The constant of integration A we shall find from the condition (7). Performing the operations of differentiation and making use of (7), we shall obtain

$$\dot{w}(0,t)\approx i\omega A\left[1nkr-1nikr\right]e^{-i\omega t}=-\frac{\pi}{2}\omega Ae^{-i\omega t},$$

$$\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial w}{\partial r}\approx\frac{\partial}{\partial r}\left[k^2 1nkr+k^2 1nikr\right]Ae^{-i\omega t}.$$

Thus, we find

$$A\lim_{e\to 0}\left\{k^2De^{-i\omega t}\oint_{c_e}\frac{2ds}{e}\right\}=4\pi A\omega\sqrt{\rho hDe^{-i\omega t}}. \qquad (10)$$

It is possible to write the solution obtained in the following form:

$$\dot{w}(0,t)=-\frac{F_0e^{-i\omega t}}{8\sqrt{\rho hD}} \qquad (11)$$

In this case the equation of the movement of the mass, adjacent to the plate, will be

$$\frac{e^2w}{dt^2}+\omega_0^2(w-w_0)=-\frac{8\sqrt{\rho hD}}{m}\frac{dw}{dt}. \qquad (12)$$

This equation equivalent to the equation of the movement of the oscillator with viscous friction is indicated in the work, i.e. all statistical characteristics of which have already been found [1]. At the spectral approach we have the following formula for spectral density of the intensity of the fixed vibrations:

$$S_w(\omega)=\frac{S_{w_0}(\omega)}{\left[\left(\frac{\omega}{\omega}\right)^2-1\right]^2+\left[\frac{8h^2\rho c\omega}{\sqrt{12}m\omega_0^2}\right]^2}. \qquad (13)$$

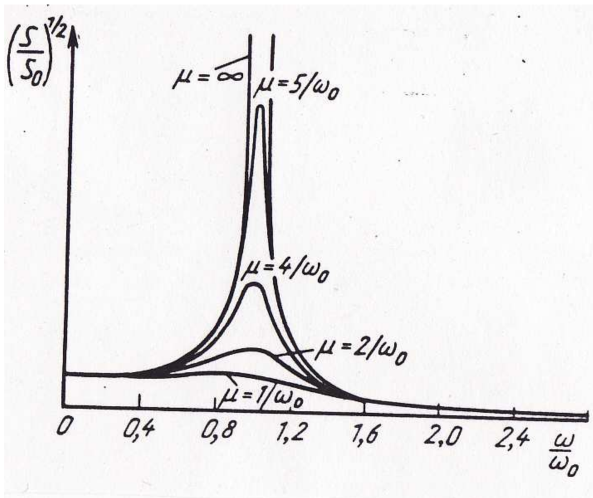


Fig. 3. The resonant characteristic of the linear vibratory model at different values of the decrement of the constructional dissipation

It is interesting that the decrement of attenuation in the system is defined by the fully elastic properties of the plate. Fig. 3. presents the relation characterizing the

$$\frac{S_w^{\frac{1}{2}}(\omega)}{S_{w0}^{\frac{1}{2}}(\omega)} = \left\{\left[\left(\frac{\omega}{\omega_0}\right)^2 - 1\right]^2 + \left[\frac{8h^2\rho c\omega}{\sqrt{12}m\omega_0^2}\right]^2\right\}^{-\frac{1}{2}} \qquad (14)$$

transmission qualities of the system at different meanings of the coefficient $\mu\omega_0 = \frac{8h^2\rho c\omega_0}{\sqrt{12}m\omega_0^2}$ with accuracy up to, coinciding with the logarithmic decrement of attenuation. For stationary vibrations it is also possible to define the function of correlation for the response of the linear oscillator

$R_u(\tau)=$

$$\frac{Q}{2\omega_0\delta}e\frac{\mu|\tau|}{2}\left\{\cos\sqrt{\omega_0^2 - \frac{\delta^2}{4}}\tau + \frac{2\delta\sin\sqrt{\omega_0^2 - \frac{\delta^2}{4}}|\tau|}{\sqrt{\omega_0^2 + \frac{\delta^2}{4}}}\right\} \qquad (15)$$

Non-stationary vibrations may be studied in the closed manner only for the case of delta-correlative process of excitation

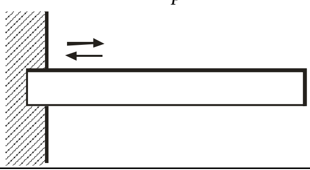
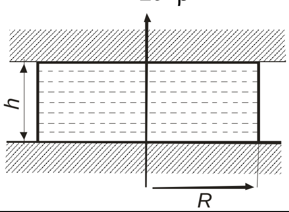
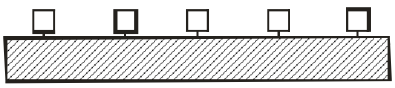
$$M\left\{w_0(t_1)w_0(t_2)\right\} = Q\delta(t_1 - t_2).$$

In this case as shown in the Markov's processes [5], we obtain the following statistical characteristics for the intensity of vibrations [3]:

$$\sigma_w^2 = \frac{Q}{4\omega_0^2}\left\{\frac{2}{\mu}\left(1 - e^{-\mu t}\right) - \frac{e^{-\mu t}}{\omega^2}\left(\mu\sin^2\omega t + \omega\sin 2\omega t\right)\right\}, \qquad (16)$$

where $\omega = \left(\omega_0^2 - \mu^2/4\right)^{\frac{1}{2}}$, which exhausts the whole of statistics for the normal process.

MODELS OF EVALUATION OF DISSIPATION COEFFICIENTS IN THE APPARATUS PARTS

Apparatus elements	**Mechanical model, temporary damping factor**	**The corresponding complex Young's modulus *E* for calculation of structures**
1.1. Parts of electronic devices flexurally vibrating bar *h* thickness, *a* thermal diffusivity, ω angular (circular) frequency	$\delta = \frac{\gamma h^2\omega^2}{24a}$ 	$E = E_0(1 + i\eta)$, $E = E_0\left(1 + \mu\frac{\partial}{\partial t}\right)$, where $\eta = \frac{\gamma h^2\omega}{12a}$, or $\mu = \frac{\gamma h^2}{12a}$, where $\gamma = -1 + c_p/c_v$, c_p and c_v – thermal capacities
1.2. Longitudinally vibrating bar λ heat capacity, ρ density, *a* coefficient of linear expansion, *T* temperature	$\delta = \frac{\gamma T_0\rho\alpha^2}{18c_p^2}\omega^2$ 	$E = E_0(1 + i\eta)$, where $\eta = 2\delta/\omega$, or $E = E_0\left(1 + \mu\frac{\partial}{\partial t}\right)$, where $\mu = \frac{\gamma T_0\rho\alpha^2}{9c_p^2}$
1.3. Molten metal element η viscosity, β coefficient of surface tension, *R* drop radius	$\delta = \frac{\eta\omega^2}{2c^2\rho}$ 	$c^2 = \beta/R\rho$
1.4. Constructional losses in the chassis		$E = E_0(1 + i\eta)$, where $\eta = 0,02 \div 0,05$

CONCLUSIONS

1. It has been established that apart of the internal losses in the mechanical system shown, constructional losses are of special importance.
2. It was established that the interaction force of the fixed mass with the plate permits one to decide about the energy absorption (dissipation) of the construction.
3. By means of Green's function, it is established that the sum of the forces released in the plate is equal to the concentrated force in the circuit, if that circuit is sufficiently small.
4. For spectral approach, a formula is deduced for determination of spectral density.
5. It is shown that decrement of attenuation is determined by the elastic properties of the plate.
6. For revealing the intensity of vibration, the statistical characteristic, which for the investigated process exhausts all statistics, is obtained
7. With the help of a computer program it is possible to create a vibration model of elements, which will give the opportunity with the least number of free vibrations to determine the efficient damping of vibrations of equipment construction.

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