Vilnius University
Center for Physical Sciences and Technology

Vytautas
Dūdėnas

Renormalization of neutrino masses in the Grimus-Neufeld model

Doctoral dissertation

Physical sciences,
Physics 02P

VILNIUS 2019

This dissertation was prepared at Vilnius University in 2014 – 2018.
The research was supported by Research Council of Lithuania

Scientific supervisor:
dr. Thomas Gajdosik (Vilnius University, Physical sciences, Physics – 02P).

This doctoral dissertation will be defended in a public meeting of the Dissertation Defence Panel:
Chairman – prof. dr. Egidijus Anisimovas (Vilnius University, Physical sciences, Physics – 02P).

Members:
dr. Jevgenij Chmeliov (Vilnius University, Physical sciences, Physics – 02P).
dr. Viktor Novičenko (Vilnius University, Physical sciences, Physics – 02P).
dr. Vidas Regelskis (Center for Physical Sciences and Technology, Physics – 02P).
prof. dr. Dominik Stoeckinger (TU Dresden, Physical sciences, Physics – 02P).

The dissertation shall be defended at a public meeting of the Dissertation Defence Panel at 15.00 on 18 02 2019 in Room D401 of the Center for Physical Sciences and Technology.
Address: Saulėtekio av. 3, LT-10257, Vilnius, Lithuania,
tel. no. +37052648884; email office@ftmc.lt.

The text of this dissertation can be accessed at Vilnius University library, as well as on the website of Vilnius University:
www.vu.lt/lt/naujienos/ivykiu-kalendorius

Vilniaus Universitetas
Fizinių ir Technologijos mokslų centras

Vytautas
DŪDĖNAS

Neutrino masių pernormavimas Grimus-Neufeld modelyje

Daktaro disertacija

Fiziniai mokslai,
Fizika 02P

VILNIUS 2019

Disertacija rengta 2014 – 2018 metais Vilniaus universitete.
Mokslinius tyrimus rėmė Lietuvos mokslo taryba

Mokslinis vadovas:
dr. Thomas Gajdosik (Vilniaus universitetas, Fiziniai mokslai, Fizika – 02P).

Gynimo taryba:
Pirmininkas – prof. dr. Egidijus Anisimovas (Vilniaus universitetas, fiziniai mokslai, fizika – 02P).

Nariai:
dr. Jevgenij Chmeliov (Vilniaus universitetas, fiziniai mokslai, fizika – 02P),
dr. Viktor Novičenko (Vilniaus universitetas, fiziniai mokslai, fizika – 02P),
dr. Vidas Regelskis (Fizinių ir technologijos mokslų centras, fiziniai mokslai, fizika – 02P),
prof. dr. Dominik Stoeckinger (Dresdeno technikos universitetas, fiziniai mokslai, fizika – 02P).

Disertacija ginama viešame Gynimo tarybos posėdyje 2019 m. vasario mėn. 18 d. 15 val. Fizinių ir technologijos mokslų centro D401 auditorijoje.
Adresas: Saulėtekio al. 3, LT-10257, Vilnius, Lietuva,
tel. +37052648884; el. paštas office@ftmc.lt.

Disertaciją galima peržiūrėti Vilniaus universiteto bibliotekoje ir Vilniaus universiteto interneto svetainėje adresu:
https://www.vu.lt/naujienos/ivykiu-kalendorius

Acknowledgments

First of all, I would like to thank my scientific supervisor Thomas Gajdosik, who was first to introduce and guide me into the subject of Particle Physics. He also gave me enough freedom to experience the excitement of independent research, which helped me to mature as a scientist. If not him, this thesis would simply not exist.

I would also like to thank my referees, Vidas Regelskis and Jevgenij Chmeliov for their comments on this thesis.

I thank Joao Silva for inviting to the conference on multi Higgs doublet models and to the discussion sessions he invited me.

I thank Maximilian Löschner and Johan Lövgren for valuable discussions.

I thank my colleague Augustinas Stepšys for sharing the ups and downs in a not so easy life as a PhD student. I also thank him for all the practical helping that he did during this period. This also includes the thorough reading of the Lithuanian summary of this thesis.

I thank my colleagues that directly or indirectly contributed to my academic career: Lukas Razinkovas, Gintaras Kerevičius, Vladimir Chorošajev, Andrius Gelžinis.

I thank my friends: Vladas Dautartas, Stanislovas Marmokas, Žygimantas Šimoliūnas. I am privileged to have such long lasting and strong friendships, which are immune to any kind of changes in life.

I thank my parents for all kinds of emotional, practical and financial support.

I thank my son Jonas for the motivation to finish this thesis faster. I cannot imagine a better motivation than the birth of Jonas.

I am most grateful to my wife Vėjūnė. I could not do this work without her love and support. Hence I dedicate this work to her.

Contents

List of abbreviations

SM Standard model

2HDM Two Higgs doublet model

H.c. Hermitian conjugate

c.t. Counterterms

FJ Fleischer–Jegerlehner

CMS Complex mass scheme

OS on–shell scheme

CP Charge–parity

GN Grimus–Neufeld

MS Minimal subtraction

$\overline{\text{MS}}$ Modified minimal subtraction

VEV Vacuum expectation value

LH Left handed

RH Right handed

EWSB Electroweak symmetry breaking

1PI One particle irreducible

UV Ultraviolet

PT Pinch technique

SSB Spontaneous symmetry breaking

Introduction

The Standard model (SM) is a remarkably successful theory able to explain a vast range of phenomena in particle physics to an unprecedented accuracy. Yet there is no doubt, that the SM is not a complete theory and we need to look forward for a new physics beyond the SM. Neutrino mixing is an undeniable hint of physics beyond the SM of particle physics. The experimental evidences for neutrino oscillations [1–4] prove that neutrinos have non–vanishing masses. The SM alone cannot explain this phenomenon, hence numerous extensions of the SM to incorporate masses of neutrinos were proposed. Among the first and most straightforward extensions to account for massive neutrinos are the so-called seesaw mechanisms [5–8]. They naturally give rise to small masses for neutrinos in a relatively simple way, making them one of the most appealing theoretical resolution of the neutrino mass puzzle. For more recent reviews on the seesaw models we refer to [9–12].

The neutrinos is not the only possible physics beyond the SM. The discovery of the Higgs boson [13, 14] which completed the SM, also inspired further research in the scalar sector, which is possibly larger than the one of the SM. Since it is the first and the only scalar particle detected so far, the natural question arises if there are more scalar particles in nature. Theoretical motivations for more than a single Higgs doublet have been around for quite some time already [15–18]. Many of the extensions can be described as special cases of the general two Higgs doublet model (2HDM) comprehensively reviewed in [19] for the non–supersymmetric cases.

The model we study consists of the general charge–parity (CP) conserving 2HDM, extended with one neutral Weyl spinor that enables the seesaw mechanism together with a radiative mass generation. We call it the Grimus–Neufeld (GN) model, since they were the first to propose it to explain neutrino masses [20]. An elegant feature of this model is that it gives a loop ordered masses for neutrinos: at tree level, we get only one non–zero light neutrino mass via seesaw mechanism, at one loop level, second light neutrino gets a mass radiatively. So the seesaw mass and the radiative mass is naturally separated by different orders in perturbation theory. One of the neutrino stays massless at one loop level, but can get a non–vanishing mass at two loops. To explain two experimentally measured mass differences, it is enough that two of the light neutrinos are massive. Hence, the GN model at one loop level is sufficient to fit these experimental values.

So far, there is no way to tell which extension of the SM is preferred by nature. However, as the experimental research of particle physics advances (for a review of experimental data see [21]), the parameter spaces of different models are being narrowed with the hope to rule out some of the most constraining models. In order to do the analysis of the parameter space of the model, one needs to relate measured cross sections and decay rates to the parameters of the theory. This relation is done by calculating scattering amplitudes in a perturbation series from a specific Lagrangian. In the zeroth order approximation, the parameters that appear in the Lagrangian stands for the parameters that we measure in the decay rates and cross sections, such as masses and couplings. However, the zeroth order approximation of the model is not enough, especially when it comes to neutrino masses [22, 23]. One needs to go at least to loop level calculations.

At loop level, the original (bare) parameters of the model does not represent the physical values anymore, as the loop corrections modifies the trivial relations of the tree level theory. Hence at loop level, we need to renormalize the model to define how physical parameters are related to the parameters of the model. This is done by requiring the renormalized fields and parameters to satisfy the specific renormalization conditions, which defines the renormalization scheme. Physics does not depend on the choice of the renormalization scheme. However, every scheme has its strengths and weaknesses when it is applied, hence choosing the most appropriate scheme for the studied problem is always an important decision to make. In this thesis, we will concentrate on the gauge dependence in the renormalization procedure.

Gauge symmetries are the guiding principle of contemporary particle physics. However, a symmetry on the classical level might not be satisfied after the quantization of the theory. This phenomenon is known as anomaly [24]. Gauge invariance of the theory is ensured to hold if the theory is anomaly–free [25–27]. Checking if the model is anomaly–free, is an easy check of a few algebraic expressions. However, this does not mean that we are safe from accidentally introducing gauge dependences in parameters of the model, i.e. masses or couplings. These gauge dependent terms, of course, cancel out in the scattering matrix, but the interpretation of gauge dependent parameters then become ambiguous, as they cannot stand for true physical parameters. Hence it is theoretically desirable to define gauge independent physical parameters in the renormalization procedure.

We study the mass renormalization of the neutrinos in the GN model. In order to define gauge invariant renormalized masses, we employ the complex mass scheme (CMS). As the masses are a consequence of the electroweak symmetry breaking, their renormalization is closely related to the renormalization of tadpoles. We show how the mass counterterm, fixed by CMS together with the usual tadpole scheme is gauge dependent. Since the CMS is proven to give gauge invariant renormalized masses, it follows that bare masses become gauge dependent. A way to avoid these gauge dependences is to apply the Fleischer–Jegerlehner (FJ) scheme [28]. We apply this scheme to the neutrino mass renormalization and verify with computer algebra systems that the gauge dependences cancel for all neutrino mass parameters. Hence we succeed to define gauge invariant bare and renormalized neutrino masses in the GN model.

The main goal and tasks of the research work

The main goal of the research presented in the thesis is to formulate a consistent scheme for renormalizing neutrino masses in the Grimus–Neufeld model at one loop level. This was done by completing the following tasks:

- Formulate the model in terms of Weyl spinors and implement it in computer software packages for automated calculations.

- Decide on the renormalization scheme for the neutrino mass renormalization and adapt it to the mixed Majorana fermion system in Weyl spinor notation.

- Check the gauge invariance of the masses and the mass counterterms.

- Find a gauge invariant prescription for renormalizing neutrinos.

Novelty and relevance of the results

The model we study in this paper was first proposed by Grimus and Neufeld in [20]. However, the renormalization and the issue of the gauge invariance of the renormalization of neutrino masses was not studied for this model until very recently [29], where the $\overline{\mathrm{MS}}$ scheme was used for the renormalization. We use different approach. We define the renormalized masses as physical parameters, so that they can be used as an input of the theory, rather than an output. The usual scheme for this purpose is the OS, but it is known to give gauge dependent mass definitions for unstable particles [30]. The CMS however, is proven to give a gauge invariant definition of mass for all loops [31]. As neutrinos of the GN model are mixed particles which include unstable ones, we apply the CMS in order to have conceptually consistent and gauge invariant renormalized mass parameters.

Using the CMS solves the problems of the gauge dependencies for renormalized masses. However, bare masses of the theory are not necessarily gauge independent, since the counterterms that relate them can acquire gauge dependent contributions. This becomes problematic if one wants to make a meaningful and gauge invariant comparison to other renormalization schemes such as the $\overline{\mathrm{MS}}$. A way to avoid these gauge dependencies is to use the FJ procedure [28], which recently got some renewed attention in order to use the $\overline{\mathrm{MS}}$ scheme in a gauge invariant way [29, 32–35]. We apply the FJ procedure in the CMS, so that the CMS renormalized mass is related to bare mass gauge invariantly. The correct application of the FJ scheme also allows for additional consistency checks of the model implementation, as all the gauge dependencies of mass parameters have to be separated by this procedure.

To summarize, this is a first attempt to set up a consistent and gauge invariant renormalization procedure in a physical basis for the masses of the GN model. We also describe how to reproduce our expressions of counterterms using SARAH, FeynArts and FormCalc.

Statements of the thesis

1. The radiatively generated neutrino mass is finite and gauge independent in the one loop approximation.

2. The FJ scheme is directly applicable for the renormalization of the neutrino masses in the seesaw extended 2HDM at one loop level.

3. The CMS can be applied together with the FJ scheme for neutrino masses at one loop and is algebraically equivalent to the OS.

Author's contribution and approbation of the results

This thesis is based on 3 research publications:

1. V. Dūdėnas and T. Gajdosik. Lith. J. Phys. **56**, 149–163, 2016,

2. V. Dūdėnas and T. Gajdosik. Acta Phys. Pol. B **48**, 2243–2249, 2017,

3. V. Dūdėnas and T. Gajdosik. Phys. Rev. D **98**, 035034, 2018.

The author of this thesis is also the main author and the main contributor in all of these three papers. The author is also a co-author of the work that is not directly related to the results of this thesis:

1. V. Dūdėnas, T. Gajdosik, A. Juodagalvis, and D. Jurčiukonis. Acta Phys. Pol. B **48**, 2235, 2017.

This paper is nevertheless related to the content of this thesis in the sense that it studies the same model, but uses a different formulation and emphasizes on the different aspects of the model by using numerical analysis. The thesis, however, presents purely analytical results.

The results were also presented in the following conferences:

1. Vytautas Dūdėnas, Thomas Gajdosik, Developing Weyl spinor formalism for seesaw neutrinos, Open readings 2015, March 24-27, Vilnius, Lithuania.

2. Vytautas Dūdėnas, Thomas Gajdosik, Weyl'o spinorių formalizmo plėtojimas sūpuokliniams neutrinams, 41 Nacionalinė lietuvos fizikos konferencija, 2015, May 17-19, Vilnius, Lithuania.

3. Vytautas Dūdėnas, Thomas Gajdosik, Renormalization of propagators of Weyl spinors in the seesaw extension of the standard model, Open readings 2016, March 15-18, Vilnius, Lithuania.

4. Vytautas Dūdėnas, Thomas Gajdosik, Using techniques of algebraic renormalization for the two Higgs doublet model with one heavy neutrino, Open readings 2017, March 14-17, Vilnius, Lithuania.

5. Vytautas Dūdėnas, Thomas Gajdosik, On the renormalization of neutrinos of the seesaw extended 2HDM, Matter to the deepest, 2017, September 3-8, Podlesice, Poland.

6. Vytautas Dūdėnas, Thomas Gajdosik, Renormalization of the neutrino sector of Grimus–Neufeld model, 42 Nacionalinė lietuvos fizikos konferencija, 2017, October 4-6, Vilnius, Lithuania.

7. Vytautas Dūdėnas, Thomas Gajdosik, Gauge parameter dependence of the neutrino mass renormalization, Open readings 2018, March 20-23, Vilnius, Lithuania.

8. Vytautas Dūdėnas, Thomas Gajdosik, Gauge dependence of tadpole and mass renormalization for a seesaw extended 2HDM, Workshop on multi–Higgs models, 2018, September 4-7, Lisbon, Portugal.

Structure of the thesis

In the first chapter of this dissertation we give an introduction to the GN model. We list all the particles, present the scalar potential and the Yukawa sectors and show the Lagrangian part that is responsible for neutrino masses. We also present the basis choices that we will use in the scalar and Yukawa sectors. In the second chapter we present the CMS scheme and its application to mixed Majorana fermion systems in terms of Weyl spinors. We discuss the algebraic equivalence of the OS and the CMS, which will be continued in the fourth chapter in the context of loop calculations. We also present how radiative masses are incorporated in the framework of the CMS and are interpreted as the CMS–renormalized masses. The second chapter is mostly based on [37] and partly on [38]. The third chapter is based on [38] and is the main chapter of the thesis, where we present the gauge independent renormalization of neutrino masses in the GN model. As the renormalization of masses depend also on the renormalization of tadpoles, we start the chapter by presenting the renormalization of tadpoles. Then we present the full expression of the one loop radiative neutrino mass, which is finite and gauge invariant, hence proves the first statement of the thesis. Then we continue the application of the CMS to the renormalization of the other two non–vanishing neutrino masses and show that this application in the usual tadpole scheme leads to the gauge dependent bare mass parameters for the neutrinos. In order to define bare masses gauge invariantly, we apply the FJ scheme and then modify the CMS–fixed counterterms. We prove that these CMS+FJ fixed counterterms are gauge invariant by explicitly calculating them, leading to the second statement of the thesis. In the fourth chapter we present how we arrived at this result with the help of SARAH, FeynArts and FormCalc. We also continue the discussion about the algebraic equivalence of the OS and CMS scheme, which leads to the third statement of the thesis. We summarize all the results in the concluding section of the thesis. We include additional material in the appendices that should help the reader to trace back all the conventions and definitions used throughout the text.

Chapter 1

The Grimus-Neufeld model

In this chapter we will introduce the scalar and the Yukawa sectors of the GN model together with the particle content and the necessary definitions. The scalar sector is the general CP conserving 2HDM, extensively studied in [19, 40–43]. The Yukawa sector of the model is presented and parametrized as in the original paper of Grimus and Neufeld [20]. Since the pole masses will become the renormalized masses, the relations to the physical basis, i.e. mass eigenstate basis, for both Yukawa and scalar sectors are presented. The presentation of a physical parametrization of 2HDM can also be found in [44, 45].

Note that in this section we will only talk about the construction of the tree level or the bare theory. Hence all parameters and bare fields in this section are bare parameters and fields. This is not to be confused with the renormalized parameters introduced in Chapter 2, where we distinguish bare parameters by adding an additional index 0 to the subscript. Since we do not talk about renormalized parameters in this section, we will not add this index in the expressions of this section in order not to overcrowd the notation where it is not needed.

1.1 Particles of the model

Any model in quantum field theory is described by its Lagrangian, from which we can deduce Feynman rules and hence calculate amplitudes. So, in order to understand the model, we need to understand how the Lagrangian is constructed first. We briefly present how to construct the model of QFT by postulating particle content of the theory. These basic principles of constructing the model can also be found in almost every serious introductory book on quantum field theory, such as [46–49]. We present the particle content of the SM, the usual SM relations and then we present what additional particles there are in the GN extension of the SM.

The Lagrangian is described by the gauge symmetries it satisfies. For instance, the GN Lagrangian, just as well as the SM Lagrangian, is symmetric under a local $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group, where the index C stands for colour charge, L stands for left handiness of fermions and Y stands for hypercharge. Gauge bosons of the theory transform under the adjoint representation of these gauge groups. They are: photons, W/Z bosons and gluons. All fermions and scalar bosons are grouped according to the charges they have with respect to these gauge symmetries. If all the particles with their charges under the gauge groups are given, then the Lagrangian is given by all possible terms that are allowed by the gauge symmetries. The usual way to describe the model is to tell the dimension of the representation of $SU(3)_C$ and $SU(2)_L$ and the $U(1)$ hypercharge Y in the form of $\left(D\left(R_{SU(3)}\right), D\left(R_{SU(2)}\right), Y\right)$ for each particle. Hence the representation of 2 is the fundamental representation of $SU(2)$, 3 is the fundamental representation of $SU(3)$ and $\bar{3}$ is the antifundamental representation of $SU(3)$. All the fermionic degrees of freedom of the model are written as left handed Weyl spinors. Then the SM is fully described by three copies of left handed (LH) Weyl spinors multiplets[1] (three copies stands for electronic, muonic and taonic families), which transform in the representations [47, 48]:

$$\left(1, 2, -\frac{1}{2}\right) \oplus (1, 1, 1) \oplus (3, 2, 1/6) \oplus (\bar{3}, 1, -2/3) \oplus (\bar{3}, 1, 1/3) \tag{1.1}$$

and one complex scalar, which transform under:

$$(1, 2, -1/2) \ . \tag{1.2}$$

In order for a model to be invariant under local gauge symmetries, a derivative of a field with respect to spacetime is promoted to a covariant derivative of a field. The covariant derivative acting on a field gives interaction terms of fermions and scalars with gauge bosons. For the SM, 2HDM and the GN model, it is:

[1] See the Appendix B for more on Weyl spinors as the fundamental representation of an $SU(2)$ subgroup of a Lorentz group.

$$D_\mu = \partial_\mu - ig_1 Y B_\mu - ig_2 T_a W^a_\mu - ig_3 \frac{\lambda_a}{2} G^a_\mu\,, \tag{1.3}$$

with $T_a = \frac{1}{2}\sigma_a$, where σ_a are the Pauli matrices, λ_a are Gell–Mann matrices, μ is a spacetime index and the Einstein summation convention is assumed[2]. B_μ, W^a_μ and G^a_μ are the gauge bosons of $U(1)$, $SU(2)$ and $SU(3)$ group. The sign convention in covariant derivative Eq. (1.3) is consistent with the most usual quantum field theory textbooks (i.e. [46, 47, 49]) and with the conventions in [50, 51]. Some other sign conventions are occasionally used in the literature. For a resource on the relationships to other sign conventions we refer to [52].

How the covariant derivative, shown in Eq. (1.3), acts on a field, depends on the charges the field carries. For example, let us write ℓ as the particle that transforms under the representations that are shown as the first entry of Eq. (1.1). ℓ is a color singlet, a doublet under $SU(2)_L$ and has a hypercharge $Y = -\frac{1}{2}$. It has two eigenvalues of $SU(2)$ generator T_3. Let us separate these two states of T_3 into $+\frac{1}{2}$ state and $-\frac{1}{2}$ state as ℓ_+ and ℓ_-. The covariant derivative, written explicitly with the summation over indices expanded and the Pauli matrices written in, acting on ℓ field is:

$$\begin{aligned} D_\mu \ell = \partial_\mu \ell &+ \frac{i}{2}\left(g_1 B_\mu - g_2 W^3_\mu\right)\ell_+ + \frac{i}{2}\left(g_1 B_\mu + g_2 W^3_\mu\right)\ell_- \\ &- g_2 \frac{i}{2}\left(W_1 - iW_2\right)\ell_- - g_2\frac{i}{2}\left(W_1 + iW_2\right)\ell_+\,. \end{aligned} \tag{1.4}$$

The expression in the first brackets of this equations is identified as the Z boson, Z_μ, second brackets stands for the photon, A_μ, and the other two brackets stand for $W^\pm$ bosons. We introduce g_e as the electromagnetic coupling, s_W and c_W as the sine and cosine of the weak mixing angle:

$$g_e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}\,, \quad c_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}\,, \quad s_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}\,, \tag{1.5}$$

which allows us to write the gauge fields as:

$$A_\mu = s_W W^3_\mu + c_W B_\mu, \tag{1.6}$$

$$Z_\mu = c_W W^3_\mu - s_W B_\mu\,, \tag{1.7}$$

[2]We use Einstein summation convention everywhere in the thesis, unless explicitly stated otherwise.

$$W^{\pm} = \frac{1}{\sqrt{2}} \left(W_1 \mp i W_2 \right) . \tag{1.8}$$

With these definitions and Eq. (1.4), the electromagnetic charge of a particle is described by

$$Q = T_3 + Y , \tag{1.9}$$

from which we can see that ℓ_+ stands for the SM neutrinos and ℓ_- stands for the electron, muon and tau. In the same way, one can identify other representations from Eq. (1.1) as being the representations of LH antileptons, LH quark doublets, LH down type antiquarks and LH up type antiquarks respectively[3]. The scalar particle, which transforms under Eq. (1.2) is the SM Higgs doublet. In this work we will not consider the $SU(3)$ gauge group anymore, which is responsible for strong interactions. This is because all the particles we consider are singlets under the $SU(3)$ gauge group, hence the quark sector is separated from the analysis presented in this thesis. However, one should keep in mind that in the full implementation of the model into the computer algebra systems, the quark sector has to be included.

The Higgs doublet of the SM, Eq. (1.2), spontaneously breaks the electroweak symmetry, $SU(2)_L \times U(1)_Y \to U(1)_Q$, by going into the minimum of its potential. The value of the field at the minimum is the vacuum expectation value (VEV) of the Higgs. The Higgs VEV gives masses to all the particles of the theory, which otherwise would violate gauge invariance of the theory. W and Z bosons of the broken theory get masses, which are related to the electromagnetic coupling, weak mixing angle and the VEV of the Higgs by

$$m_W = c_W m_Z = \frac{g_2 v}{2} = \frac{g_e v}{2 s_W} . \tag{1.10}$$

The equivalent directions of the spontaneous symmetry breaking (SSB) are parametrized by gauge fixing conditions. The usual gauge fixing condition is the R_ξ gauge, in which, the unphysical Goldstone bosons in the broken theory have masses of:

$$m^2_{\chi^\pm} = \xi_W m^2_W , \; m^2_{\chi_Z} = \xi_Z m^2_Z . \tag{1.11}$$

[3] We postulate all the Weyl spinors in the LH representation under a Lorentz group. The RH representations of the corresponding Weyl spinors appear in the Lagrangian as a hermitian conjugate fields. For more details on this, see Appendix B.

Table 1.1: Naming of particles, grouped according to the representations under the gauge groups of $U(1)_Y$ and $SU(2)_L$. Y stands for a hypercharge of the $U(1)_Y$ and $D\left(R_{SU(2)}\right)$ stands for the dimension of the representation under $SU(2)$. The $SU(3)$ representation of all these particles is trivial.

label	families	$\left(D\left(R_{SU(2)_L}\right), Y\right)$	name
$\ell = \begin{pmatrix} \nu \\ e \end{pmatrix}$	3	$\left(2, -\frac{1}{2}\right)$	lepton = $\begin{pmatrix} \text{neutrino} \\ \text{electron} \end{pmatrix}$
N	1	$(1, 0)$	singlet neutrino
E	3	$(1, 1)$	positron
H	2	$\left(2, +\frac{1}{2}\right)$	Higgs doublet

Table 1.2: Naming of particles in their mass eigenstates.

label	families	name
ν	4	neutrino
e	3	electron
E	3	positron
H^+	1	charged Higgs boson
h	1	SM Higgs
H	1	heavy Higgs
A	1	Axial Higgs
χ_W	1	W Goldstone
χ_Z	1	Z Goldstone

Goldstone bosons $\chi_W^{\pm}$ and χ_Z are the components of the Higgs doublet of the broken directions of $SU(2)_L \times U(1)_Y$. They become unphysical after the SSB takes an effect and are "eaten up" by the longitudinal polarizations of W and Z bosons.

The GN model, as the extension of the SM, have all the particles, presented in Eq. (1.1) and Eq. (1.2). In addition, it has one copy of the complex scalar, transforming in the representation of Eq. (1.2) and the Weyl fermion, which is a singlet under all gauge groups, i.e. transform as $(1, 1, 0)$. All the particles of the GN model (except the $SU(3)$ part, which we ignore in the presentation), are listed in Table 1.1.

The electroweak symmetry breaking gives rise to mass parameters, hence particles can be listed as mass eigenstates. They are presented in Table 1.2. In terms of Weyl spinors, the Dirac fermion can be thought of as a doubly degenerate mass eigenstate, i.e. two Weyl spinors that have the same mass. This is the case for the Weyl electron e and the Weyl positron E[4]. This degeneracy is due to a gauge symmetry: neither e nor E cannot have a gauge invariant Majorana mass term, but they can have a Dirac mass term that couples e and E together. In contrast, the gauge singlet N can have a mass term for itself, which is called a Majorana mass term, and Dirac mass terms that couple N with ν states. At the end, this results in 4 different mass eigenstates labeled by ν, which are called Majorana mass eigenstates[5].

The additional scalar and fermion in the GN model extends the Scalar and the Yukawa[6] parts of the Lagrangian. The scalar potential is the scalar potential of the 2HDM. Second Higgs doublet allows to write additional Yukawa couplings for all the fermions of the SM. Additional singlet neutrino allows for neutrino Yukawa couplings, which are absent in the SM. We will now present the extended Lagrangian parts that will be important in the study of neutrino masses.

1.2 Scalar sector

The general 2HDM has the potential [19, 42]

$$\begin{aligned} V &= m_{11}^2 H_1^\dagger H_1 + \frac{1}{2}\lambda_1 \left(H_1^\dagger H_1\right)^2 + m_{22}^2 H_2^\dagger H_2 + \frac{1}{2}\lambda_2 \left(H_2^\dagger H_2\right)^2 \\ &+\lambda_3 \left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + \lambda_4 \left(H_2^\dagger H_1\right)\left(H_1^\dagger H_2\right) \\ &+\Big[- m_{12}^2 H_1^\dagger H_2 + \frac{1}{2}\lambda_5 \left(H_2^\dagger H_1\right)\left(H_2^\dagger H_1\right) \\ &+\lambda_6 \left(H_1^\dagger H_1\right)\left(H_1^\dagger H_2\right) + \lambda_7 \left(H_2^\dagger H_2\right)\left(H_2^\dagger H_1\right) + H.c.\Big], \end{aligned} \tag{1.12}$$

where H_i is the field, listed in Table 1.1 with the family index $i = 1, 2$. We restrict the potential to have a CP symmetry, which restricts the

[4]Since we define all spinors as LH, we are led to a convention where we write a LH positron as E. Then the RH positron and the RH electron is $e^\dagger$ and $E^\dagger$, respectively. Later, we will need to generalize the hermitian conjugation operation. With this generalization, the antiparticles of given particles will be written with bars i.e. $\bar{e}$ and $\bar{E}$ instead of $e^\dagger$ and $E^\dagger$.

[5]In the GN model, four distinct mass eigenstates can be identified only after the loop corrections, as will be explained in detail a bit later.

[6]By Yukawa sector we mean the Lagrangian part in which fermions are coupled to scalars. These couplings are called the Yukawa couplings.

parameters to be [19]:

$$m_{ij}, \lambda_k \in \mathbb{R}\,,\ i,j = 1,2\,, \quad k = 1,2,3,4,5,6,7. \tag{1.13}$$

As the CP conserving case makes all the bare parameters real, both of the vacuum expectation values (VEVs) in a generic basis are real as well.

The 2HDM potential has a family symmetry $U(2)$, which means that we cannot distinguish between H_1 and H_2 from the potential alone [40]. The overall phase is irrelevant, hence we can transform between the two Higgs doublets with a unitary transformation $U \in SU(2)$, getting the new basis with the parameters m'_{ij} and λ'_k. We parametrize this transformation by

$$U = \begin{pmatrix} e^{i\chi} c_\psi & e^{i(\chi-\xi)} s_\psi \\ -e^{i(\xi-\chi)} s_\psi & e^{-i\chi} c_\psi \end{pmatrix}, \tag{1.14}$$

where χ, ξ are phases and c_ψ, s_ψ are cosine and sine of an angle ψ. In the following section we will introduce the Higgs basis, which will be the main basis that we use throughout the rest of the manuscript.

1.2.1 Minimum and the Higgs basis

Minimizing the potential, Eq. (1.12), in the general basis gives two non-vansinshing vacuum expectation values v_1 and $v_2 e^{i\delta}$ for H_1 and H_2 respectively, were v_1 and v_2 are real [19]. The ratio of these two VEVs is conventionally parametrized by

$$\tan\beta \equiv \frac{v_1}{v_2}\,. \tag{1.15}$$

By making the unitary transformation, parametrized in Eq. (1.14), between the two Higgs doublets, we can choose a basis in which, only the first Higgs doublet acquires a VEV, $v^2 = v_1^2 + v_2^2$. This is achieved by setting:

$$\psi = \beta\,, \quad \xi = \frac{\chi}{2} \equiv \frac{\delta}{2} \tag{1.16}$$

in Eq. (1.14). This is called the Higgs basis. As we work in the CP conserving case, where all the parameters of the potential are real, both of the VEVs are real as well, hence $\delta = 0$. Therefore the general basis transformation, Eq. (1.14), reduces to an orthogonal transformation

$$U = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \tag{1.17}$$

which relates the CP conserving Higgs basis to the general basis.

An easier way to define the Higgs basis is just by choosing the parametrization of the components of the Higgs doublets from the beginning as:

$$H_1 = \begin{pmatrix} \chi_W^+ \\ \frac{1}{\sqrt{2}}\left(v + h + i\chi_Z\right) \end{pmatrix}, \; H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(H + iA\right) \end{pmatrix}. \tag{1.18}$$

One needs to remember that with this parametrization, the m_{ij} and λ_i of the potential Eq. (1.12) are the Higgs basis parameters and not of the general basis anymore. Note that the β parameter is hidden in the definition of the Higgs basis parameters and does not appear in the Lagrangian anywhere. This is due to the fact that for a general 2HDM potential, the β parameter is basis dependent, while the 2HDM potential preserves the $U(2)$ Higgs family symmetry [40]. Hence β does not have a physical meaning in the 2HDM potential alone.

The Higgs basis is well defined only at the minimum, where it gets a non vanishing VEV, so that the parametrization in the Eq. (1.18) makes sense. To get the minimum conditions of the Higgs basis, one simply inserts the Eq. (1.18) into the Higgs potential Eq. (1.12), computes derivative with respect to h, H and A, respectively, and sets all the fields to zero to obtain the three expressions that have to vanish at the minimum. These three expressions are called the tadpole functions. They are:

$$T_h = -v\left(m_{11}^2 + \frac{1}{2}\lambda_1 v^2\right), \quad T_H = v\left(m_{12}^2 - \frac{1}{2}v^2\lambda_6\right), \quad T_A = 0\,, \tag{1.19}$$

Simply put, the tadpole functions are the coefficients of the linear terms in h, H and A, respectively[7]. The so–called tadpole conditions are the requirement that these tadpole functions vanish. The third equation of Eq. (1.19) is trivially zero because of the imposed CP symmetry, which constrained the parameters to satisfy Eq. (1.13). Hence we have only two non-trivial tadpole conditions that minimize the potential by fixing

[7]The expressions in Eq. (1.19) are the tree level tadpole functions, when we talk about the tree level theory only. Higher order tadpole functions are defined as coefficients of linear terms in h, H and A of a corrected effective potential. The notation for tadpole functions and other Green's functions in the framework of the path integral and effective action are presented in Appendix A.

the values of v and m_{12}:

$$T_h = 0\,,\; T_H = 0$$
$$\Rightarrow m_{11}^2 = -\frac{1}{2}v^2\lambda_1\,,\; m_{12}^2 = \frac{1}{2}v^2\lambda_6\,. \tag{1.20}$$

The second equation of Eq. (1.20) can also be interpreted as the Higgs basis condition. That is, it ensures that H_2 does not get a vacuum expectation value at the minimum by relating the parameters.

1.2.2 Mass matrix and mixing

In the CP conserving case, A does not mix neither with h nor with H. Its mass is given by the coefficient of the bilinear term in A in the potential:

$$m_A^2 = m_{22}^2 + (\lambda_3 + \lambda_4 - \lambda_5)\,v^2. \tag{1.21}$$

In the Higgs basis, the charged Higgs boson is distinguished from the charged Goldstone bosons by the R_ξ gauge fixing conditions, so that the mixing between chared Higgs and Goldstone bosons vanish at the minimum, defined by Eq. (1.20). The mass of the charged Higgs is

$$m_{H+}^2 = \frac{1}{2}\lambda_3 v^2 + m_{22}^2\,. \tag{1.22}$$

In the general CP conserving 2HDM, a mixing between h and H occurs. The mass matrix for h and H is:

$$\begin{pmatrix} \frac{3}{2}\lambda_1 v^2 + m_{11}^2 & \frac{3}{2}\lambda_6 v^2 - m_{12}^2 \\ \frac{3}{2}\lambda_6 v^2 - m_{12}^2 & \frac{1}{2}\left(\lambda_3+\lambda_4+\lambda_5\right) v^2 + m_{22}^2 \end{pmatrix}. \tag{1.23}$$

After imposing the minimum conditions of Eq. (1.20), the mass matrix becomes:

$$\begin{pmatrix} \lambda_1 v^2 & \lambda_6 v^2 \\ \lambda_6 v^2 & \frac{1}{2}\left(\lambda_3+\lambda_4+\lambda_5\right) v^2 + m_{22}^2 \end{pmatrix}. \tag{1.24}$$

To obtain the mass eigenstate basis, we rotate between h and H with an orthogonal matrix:

$$O^\phi = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix}\,,\; \phi_i^{\rm mass} = O_{ij}^\phi \phi_j^{\rm Higgs}\,,\; \phi_i^{\rm Higgs} = (h, H)_i\,, \tag{1.25}$$

where sine and cosine of the mixing angle α are written as s_α and c_α, respectively. We can reparametrize the potential by the physical basis expressing five of the parameters in terms of the masses and the mixing. Using the definitions of the tadpole functions Eq. (1.19), we arrive at:

$$\begin{aligned}
\lambda_1 &= \frac{c_\alpha^2 m_h^2 + s_\alpha^2 m_H^2}{v^2} - \frac{T_h}{v^3}\,,\\
\lambda_6 &= -\frac{c_\alpha s_\alpha \left(m_H^2 - m_h^2\right)}{v^2} + \frac{T_H}{v^3}\,,\\
m_{22}^2 &= m_{H^+}^2 - \frac{\lambda_3 v^2}{2},\\
\lambda_4 &= \frac{m_A^2 + s_\alpha^2 \left(m_h^2 - m_H^2\right) + m_H^2 - 2m_{H^+}^2}{v^2}\,,\\
\lambda_5 &= \frac{-m_A^2 + s_\alpha^2 \left(m_h^2 - m_H^2\right) + m_H^2}{v^2}.
\end{aligned} \tag{1.26}$$

Together with the minimum conditions Eq. (1.20), seven parameters are reexpressed in terms of masses and mixing. Masses for Goldstone bosons come from the gauge fixing functions only. That is, the contribution to their masses from the scalar potential are zero at the minimum:

$$\frac{\delta^2}{\delta\chi_W \delta\chi_W^\dagger} V = \frac{\delta^2}{\delta^2\chi_Z} V = m_{11}^2 + \frac{\lambda_1 v^2}{2} = -\frac{1}{v} T_h\,. \tag{1.27}$$

Since v can be expressed by the SM relations of Eq. (1.10):

$$s_{2W} = 2 s_W c_W\,, \quad m_Z = \frac{g_e v}{s_{2W}}\,, \quad m_W = m_Z c_W\,, \tag{1.28}$$

we are only left with λ_2, λ_3 and λ_7, which cannot be expressed by masses and mixing. They can be defined by quartic vertices. Before rotating into the mass eigenstate basis, let us write the quartic vertex functions that define these couplings in the Higgs basis (see the notation in Appendix A):

$$\Gamma_{H^+H^-HH} = -\lambda_2\,,\ \Gamma_{H^+H^-Hh} = -\lambda_7\,,\ \Gamma_{H^+H^-hh} = -\lambda_3\,. \tag{1.29}$$

To relate to the vertex functions in the mass eigenstate basis, which we will now label by (m) in the subscript, we use chain rule for the derivatives with respect to fields together with Eq. (1.25):

$$\frac{\delta}{\delta h} = \frac{\delta h_{(m)}}{\delta h} \frac{\delta}{\delta h_{(m)}} + \frac{\delta H_{(m)}}{\delta h} \frac{\delta}{\delta H_{(m)}} = c_\alpha \frac{\delta}{\delta h_{(m)}} - s_\alpha \frac{\delta}{\delta H_{(m)}}\,, \tag{1.30}$$

$$\frac{\delta}{\delta H} = \frac{\delta h_{(m)}}{\delta H} \frac{\delta}{\delta h_{(m)}} + \frac{\delta H_{(m)}}{\delta H} \frac{\delta}{\delta H_{(m)}} = s_\alpha \frac{\delta}{\delta h_{(m)}} + c_\alpha \frac{\delta}{\delta H_{(m)}}\,. \tag{1.31}$$

The parameters, expressed in mass eigenstate vertex functions are:

$$\lambda_2 = -s_\alpha^2 \Gamma_{H^+H^-h_{(m)}h_{(m)}} - 2s_\alpha c_\alpha \Gamma_{H^+H^-H_{(m)}h_{(m)}} - c_\alpha^2 \Gamma_{H^+H^-H_{(m)}H_{(m)}} \,, \tag{1.32}$$

$$\lambda_3 = -c_\alpha^2 \Gamma_{H^+H^-h_{(m)}h_{(m)}} + 2c_\alpha s_\alpha \Gamma_{H^+H^-h_{(m)}H_{(m)}} - s_\alpha^2 \Gamma_{H^+H^-H_{(m)}H_{(m)}} \,, \tag{1.33}$$

$$\begin{aligned} \lambda_7 = -s_\alpha c_\alpha (\Gamma_{H^+H^-h_{(m)}h_{(m)}} - \Gamma_{H^+H^-H_{(m)}H_{(m)}}) \\ - \left(c_\alpha^2 - s_\alpha^2\right) \Gamma_{H^+H^-h_{(m)}H_{(m)}} \,. \end{aligned} \tag{1.34}$$

The fields $H^\pm$ are already in the mass eigenstate basis by choosing the Higgs basis. Tadpole functions, expressed in mass eigenstates rather than in the Higgs basis, are also related with the same rotation of Eq. (1.25):

$$\begin{aligned} &T_h = c_\alpha T_{h_{(m)}} - s_\alpha T_{H_{(m)}} \,, \; T_H = c_\alpha T_{H_{(m)}} + s_\alpha T_{h_{(m)}} \,, \\ &T_A = T_{A_{(m)}} \,. \end{aligned} \tag{1.35}$$

As we see, for the minimum conditions, the Higgs basis is far more comfortable than the mass eigenstate basis, as it gives simpler expressions for tadpole conditions. However, the usual framework to calculate loop corrections is in the mass eigenstate basis. Mass eigenstates are used in FeynArts and FormCalc Mathematica packages. Hence having the formulation in mass eigenstates is useful for the implementation in computer algebra systems. Keeping in mind the relations between the two basis, let us employ the benefits of both of them.

1.3 Yukawa sector

In comparison to the SM, the 2HDM has additional coupling parameters that couple fermions to the second Higgs doublet that is absent in the SM. As the number of parameters in this case increases significantly, various global symmetries can be employed to restrict them [19]. In the SM, where the neutrinos are massless, Yukawa couplings for leptons can be made diagonal to define the flavor basis, hence giving only three real parameters directly related to the masses. Even with no additional Yukawa couplings for neutrinos, the addition of the second Higgs doublet, not restricted by any symmetry, leads to an additional 3×3 complex matrix of parameters in the Yukawa sector that cannot be absorbed into redefinitions of the fields. The Higgs basis, however, has a comfortable feature of isolating the Yukawa couplings, that are directly related to masses of fermions from the couplings with additional scalar fields. The flavor basis is then defined in accordance to H_1 in the analogous way as in the SM. Starting from the general basis, the interactions of H_1 with leptons are:

$$\mathcal{L} = -\tilde{Y}^{e1}_{ij}\tilde{\ell}_i H_1^* \tilde{E}_j + H.c. \tag{1.36}$$

We make a unitary transformation from the general basis to the flavor basis:

$$\tilde{\ell}_i = U^{\ell}_{ij}\ell_j \ , \tilde{E}_i = U^{E}_{ij}E_j , \ \tilde{Y}^{e1}_{ij}U^{\ell}_{ik}U^{E}_{jl} = Y^{e1}_{kl} \tag{1.37}$$

to make the couplings diagonal:

$$Y^{e1}_{ii} = \sqrt{2}\frac{m_i}{v} \ \text{ and } \ Y^{e1}_{ij} = 0 \ \text{ for } i \neq j \tag{1.38}$$

The index i now can be called a flavor index and stands for electron, muon, or tau. After making this definition of the flavor basis, we look at other couplings in this particular basis. Once we write the Lagrangian in the flavor basis, these unitary transformations, Eq. (1.37), do not appear anywhere, since they get absorbed into the definition of the fields and couplings. The coupling of the charged leptons to the second Higgs doublet is still an arbitrary complex 3×3 matrix:

$$\mathcal{L} = -Y^{e1}_{ij}\ell_i H_1^* E_j - Y^{e2}_{ij}\ell_i H_2^* E_j + H.c. \tag{1.39}$$

From now on, we will introduce other parameters of the Lagrangian, starting from the flavor basis defined by Eq. (1.37).

Quarks also have additional couplings with the second Higgs doublet. However, for the case of neutrino masses and mixing, they do not give any contribution at one loop, hence they are ignored.

1.3.1 Yukawa couplings for neutrinos and a choice of a basis

Recall that the product ℓH_i^* is an invariant under the $SU(2)$ gauge group, where $i = 1, 2$ stands for the first or the second Higgs doublet in the Higgs basis. Formally, this product can be written in a group theoretical notation as a symmetric product in $2 \otimes \bar{2}$, where 2 stands for the dimension of the representation and the bar stands for the conjugate representation (see e.g. [53]). We write this product in components as:

$$\ell H_i^* = \ell_{(1)} H_{i(1)}^* + \ell_{(2)} H_{i(2)}^* \,, \tag{1.40}$$

where we write the components of the representation are written in the parenthesis in order to avoid the possibility to confuse them with the family indices. The $SU(2)$ has another bilinear invariant product, which is an antisymmetric product in $2 \otimes 2$:

$$\ell_{(1)} H_{i(2)} - \ell_{(2)} H_{i(1)} \,. \tag{1.41}$$

In order not to change the summation convention, we sum over indices symmetrically all the time, but introduce the antisymmetric symbol

$$\epsilon_{(12)} = -\epsilon_{(21)} = 1 \tag{1.42}$$

to write the same product as[8]:

$$\ell\,(\epsilon H_1) = \ell_{(1)} H_{i(2)} - \ell_{(2)} H_{i(1)} \tag{1.43}$$

In order to have a gauge invariant Yukawa term for the neutrino, the addition of the gauge group singlet to the product, shown in Eq. (1.43), is needed. In the GN model, a single gauge group singlet neutrino N (Table 1.1) is postulated. Then the Yukawa term in the flavor basis reads:

$$\mathcal{L} = -Y_i^{\nu 1} \ell_i \epsilon H_1 N - Y_i^{\nu 2} \ell_i \epsilon H_2 N + H.c. \,, \tag{1.44}$$

Since we introduced only one gauge singlet fermion N, the Yukawa couplings are two complex vectors in the neutrino family space. As we are in the flavor basis, the family index i stands for electron, muon or tau ($i = e, \mu, \tau$).

Now we will look only at the neutrinos from the lepton doublet. The neutrinos are in the first component of a doublet ℓ, hence it couples to the second component of the Higgs doublet, as shown in Eq. (1.43). Writing explicitly only the components for neutrinos from Eq. (1.44) gives

$$\mathcal{L} = -Y_i^{\nu 1} \nu_i^F H_{1(2)} N - Y_i^{\nu 2} \nu_i^F H_{2(2)} N + H.c\,. \,, \tag{1.45}$$

[8]Note that in SARAH, this symbol is omitted and this product is written as $\ell.H_1$. Hence if one interchanges the order in SARAH model file, one should also not forget the minus sign that pops out because of this convention.

where F stands for neutrino in flavor basis. The $H_{i(2)}$ include only neutral scalars. We can now reparametrize these Yukawa couplings by making unitary transformation on neutrinos[9]. First, let us absorb the phases of $Y_i^{\nu 1}$ into the fields ν_i^F, so that all three entries of $Y_i^{\nu 1}$ are real parameters. Then we can achieve a zero in the second entry of the first Yukawa coupling, by an orthogonal rotation:

$$O_{2j}^{23} Y_j^{\nu 1} = 0\,. \tag{1.46}$$

The superscript of the orthogonal operator tells the components of the Yukawa coupling vectors between which the rotation is made. Similarly, we rotate between the first and the third component to get the singular value out of the 3 entries of $Y^{\nu 1}$ by:

$$O_{1k}^{13} O_{kj}^{23} Y_j^{\nu 1} = 0\,,\ O_{3k}^{13} O_{kj}^{23} Y_j^{\nu 1} \equiv y\,. \tag{1.47}$$

We will now absorb the parameters of the second Yukawa coupling. As the first and the second entry of the reparametrized Yukawa couplings to the first Higgs doublet are zeros, we can rephase and rotate between the first and the second component freely, without altering anything else, except the second Yukawa coupling. We first adjust the phase of the first element of $Y^{\nu 2}$, to match the second element of $Y^{\nu 2}$ by a phase shift U^α:

$$\arg(U_{1l}^{\alpha} O_{lk}^{13} O_{kj}^{23} Y_j^{\nu 2}) = \arg(U_{2l}^{\alpha} O_{lk}^{13} O_{kj}^{23} Y_j^{\nu 2})\,. \tag{1.48}$$

Then we rotate between these two components to get zero in the first entry:

$$O_{1m}^{12} U_{ml}^{\alpha} O_{lk}^{13} O_{kj}^{23} Y_j^{\nu 2} = 0\,. \tag{1.49}$$

Finally we absorb the phase of the second entry to make it real and positive:

$$U_{2n}^{\beta} O_{nm}^{12} U_{ml}^{\alpha} O_{lk}^{13} O_{kj}^{23} Y_j^{2} \in \mathbb{R}^+\,. \tag{1.50}$$

We are left with 2 real and one complex parameters in the Yukawa sector. The other parameters were absorbed into 3 rotations and 2 phases of the unitary transformation. This is not yet the mass eigenstate of the fields, but merely a comfortable reparametrization of the Yukawa sector.

[9]Since we started from the flavor basis after the electroweak symmetry breaking, the 3×3 block of this Unitary matrix for light neutrinos can be identified with the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix.

To summarize this parametrization, let us name this whole matrix by $V = U^\beta O^{12} U^\alpha O^{13} O^{23}$ and write [38]:

$$\begin{aligned} &V_{1j} Y_j^{\nu 1} = 0\,, \quad V_{2j} Y_j^{\nu 1} = 0\,, \quad V_{3j} Y_j^{\nu 1} = y\,, \\ &V_{1j} Y_j^{\nu 2} = 0\,, \quad V_{2j} Y_j^{\nu 2} = d\,, \quad V_{3j} Y_j^{\nu 2} = d'\,, \\ &d, y \in \mathbb{R}^+\,, \quad d' \in \mathbb{C}\,. \end{aligned} \tag{1.51}$$

The fields transform as

$$\nu'_i = V^*_{ij} \nu^F_j\,. \tag{1.52}$$

The basis ν' with the parametrization Eq. (1.51) is also used in [20]. In this basis, the EWSB gives the Dirac mass term that couples ν'_4 with ν'_3. This "turns on" the seesaw mechanism and the two Majorana mass eigenstates ν_3 and ν_4 can then be defined at tree level. The states ν'_1 and ν'_2 are both massless at tree level. However, the parametrization of Eq. (1.51) distinguishes between the ν'_1 and ν'_2 by the interaction with the second Higgs doublet. As we will shortly see, only ν'_2 acquires a mass term at one loop level, while ν'_1 stays massless, which is the main reason for this parametrization.

1.3.2 Seesaw mechanism

The seesaw mechanism in the GN model is the simplest and earliest seesaw mechanism of type I [5]. After reparametrizing the Yukawa sector by Eq. (1.51), the seesaw mechanism is done only between two states: ν'_3 and ν'_4. First, recall that we chose the Higgs basis, hence the Dirac mass terms after the EWSB only appear from the interactions with H_1, which is represented by $Y^{\nu 1}$ for neutrinos. By Eq. (1.51), we chose such a basis that there is only a single interaction between neutrinos and H_1, parametrized by y. That is, the Yukawa coupling to the H_1 is described by:

$$\mathcal{L} = -y\, \nu'_3 H_{1(2)} N + H.c. \tag{1.53}$$

Then the seesaw transformation is just the 2×2 rotation matrix between N and ν'_3, which is caused by the non diagonal mass matrix, just as it would be the case in the 1 family analysis. After the EWSB, the non vanishing VEV of the Higgs generates the Dirac mass term $yv/\sqrt{2}$:

$$\mathcal{L} = -\left(\frac{1}{\sqrt{2}} y v\right) \nu'_3 N + \frac{1}{\sqrt{2}} y\, \nu'_3 \left(h + i\chi^0\right) N + H.c. \tag{1.54}$$

N has a Majorana mass term that is not protected by any gauge symmetry, hence it is not involved in the EWSB, which is

$$\mathcal{L} = -\frac{1}{2} M N N + H.c. \tag{1.55}$$

We can absorb the phase of the mass term M into the redefinition of the field N, so that the parameter M is real, similarly as we did for ν_i^F when arriving at the parametrization Eq. (1.51). Then the mass terms can be written in the matrix form as:

$$\mathcal{L}_{M_\nu} = -\frac{1}{2} n M_\nu n^T \,, \; n \equiv \begin{pmatrix} \nu_3' & N \end{pmatrix} , \tag{1.56}$$

where the mass matrix is

$$M_\nu = \begin{pmatrix} 0 & \sqrt{2} y v \\ \sqrt{2} y v & M \end{pmatrix} . \tag{1.57}$$

This matrix has two positive singular values, m_3 and m_4, with the relations:

$$M = m_4 - m_3 \quad \text{and} \quad y^2 v^2 = 2 m_3 m_4 \,, \, m_3 < m_4 \,. \tag{1.58}$$

We obtain the mass eigenstates by diagonalizing this mass matrix with the seesaw transformation parametrized by:

$$U^{34} = \begin{pmatrix} -i c_{34} & i s_{34} \\ s_{34} & c_{34} \end{pmatrix} , \tag{1.59}$$

$$U^{34} M_\nu U^{34T} = \text{diag}(m_3, m_4) \,, \tag{1.60}$$

where the sine and cosine parameters are related to the mass values by:

$$s_{34}^2 = \frac{m_3}{m_4 + m_3} \quad \text{and} \quad c_{34}^2 = \frac{m_4}{m_4 + m_3} . \tag{1.61}$$

The phase shift i comes from the requirement that m_3 and m_4 are positive, assuming M and y are positive. Hence the 4×4 mixing matrix, relating the mass the eigenstate basis and the flavor basis is:

$$U = U^{34} U^{\beta} O^{12} U^{\alpha} O^{13} O^{23} \,, \tag{1.62}$$

which relates ν and ν^F by:

$$\nu_i = U_{ij}^* \nu_j^F \,. \tag{1.63}$$

The Yukawa Lagrangian part of the interactions between neutrinos and neutral scalars together with the mass terms in the mass eigenstate is:

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} = &-\frac{1}{2}m_3\,\nu_3\nu_3 - \frac{1}{2}m_4\,\nu_4\nu_4 - \frac{1}{\sqrt{2}}d\,(H+iA)\,\nu_2\,(-is_{34}\nu_3 + c_{34}\nu_4)\\ &- \frac{1}{\sqrt{2}}\left[y\,(h+i\chi_Z) + d'\ (H+iA)\right]\\ &\times \left[c_{34}s_{34}\nu_3\nu_3 + i\left(c_{34}^2 - s_{34}^2\right)\nu_3\nu_4 + c_{34}s_{34}\nu_4\nu_4\right] + H.c.\,. \end{aligned} \tag{1.64}$$

The Lagrangian part, shown in Eq. (1.64), presents all the main characteristics of the mass generation in the GN. That is, we see from Eq. (1.64) that ν_3 and ν_4 have tree level Majorana masses and ν_2 interacts with neutral scalars of the second doublet, hence it can acquire a radiative mass at one loop. Finally, ν_1 does not appear in Eq. (1.64) at all. This makes it impossible for ν_1 to get a mass–like term at one loop level. This is the main reason of choosing the parametrization of Eq. (1.51). Eq. (1.64) presents the only Lagrangian term that we will need to consider in order to explain the renormalization of neutrino masses for ν_3 and ν_4, and construct the expression for the radiative mass of ν_2.

Summary

In comparison to the SM, the additional particles in the GN model are: one neutral scalar, one neutral pseudoscalar, one charged scalar and one heavy neutrino. We presented the Lagrangian parts of the GN model and presented the different basis choices that we will use in the following chapters of the thesis. In the scalar sector we introduced the general basis, the Higgs basis and the mass eigenstate basis. In the Yukawa sector we introduced the flavor basis, the parametrization used in [20], and the mass eigenstate basis. The mass eigenstate basis will be needed in order to use the complex mass scheme for the renormalization of masses. We introduced tadpole functions in Eq. (1.19). As we will see in Chapter 2, the renormalization of tadpoles interplays with the renormalization of masses. The part of the Yukawa sector, shown in Eq. (1.64), is the main expression of this section, since these terms are responsible for neutrino mass generation.

Chapter 2

Complex mass scheme

We use multiplicative renormalization constants for parameters and fields to derive counterterms. All the parameters and fields in Chapter 1 were presented as bare parameters and fields. In this chapter, we add zeros in the subscript everywhere to distinguish the bare parameters and fields from the renormalized ones. That is, every bare parameter of the theory p_0 and every bare field of the theory ϕ_0 is related to the renormalized parameter p and field ϕ by:

$$p_0 = (1 + \delta_p)\, p\,,\ \phi_0 = \left(1 + \frac{1}{2}\delta_\phi\right)\phi\,, \tag{2.1}$$

where δ_p and δ_ϕ are renormalization constants. Note that the renormalization constants are dimensionless. When fields mix, the field renormalization constants are promoted to matrices:

$$\phi_{0i} = \sum_j \left(1_{ij} + \frac{1}{2}\delta_{\phi ij}\right)\phi_j\,, \tag{2.2}$$

where the Kronecker delta is written as 1_{ij} and $\delta_{\phi ij}$ stands for the renormalization constant.

In this chapter, we will extensively use the notation presented in Appendix A, where the notion of one particle irreducible n–point functions in the context of effective quantum action is introduced.

2.1 Complex parameters in the Lagrangian

Before going into the presentation of the complex mass scheme (CMS), we present some modifications on the Lagrangian level, that arise due to the complex parameters. The CMS is the analytic continuation of the OS into the complex domain, which separates the bare parameters into complex counterterms and complex renormalized parameters. The hermicity of the bare Lagrangian ensures the unitarity of the theory. As far as we consistently set up the renormalized complex parameters and counterterms in the connection with real bare parameters, we do not violate unitarity, as it was shown in [54]. Hence the CMS is a viable consistent scheme. We now present the relations between the bare and the renormalized Larangians in the CMS, which was also discussed in [38]. First, consider the Majorana mass term

$$\mathcal{L}_{m_0} = -\frac{1}{2} m_0 \nu_0 \nu_0 - \frac{1}{2} m_0^\dagger \nu_0^\dagger \nu_0^\dagger \,. \tag{2.3}$$

We can absorb the phase of the mass parameter into the field, so that $m_0 \in \mathbb{R}$:

$$\mathcal{L}_{m_0} = -\frac{1}{2} m_0 \left(\nu_0 \nu_0 + \nu_0^\dagger \nu_0^\dagger \right) \,. \tag{2.4}$$

We can also write the same term as:

$$\mathcal{L}_{m_0} = -\frac{1}{2} m_0 \nu_0 \nu_0 + H.c. \,. \tag{2.5}$$

Now we introduce complex multiplicative renormalization constants for all parameters and fields:

$$m_0 = (1 + \delta_m)\, m \,,\; \nu_0 = \left(1 + \frac{1}{2}\delta_\nu\right) \nu \,,\; \nu_0^\dagger = \left(1 + \frac{1}{2}\bar{\delta}_\nu\right) \bar{\nu} \,, \tag{2.6}$$

and insert into Eq. (2.4) to get:

$$\mathcal{L}_{m_0} = -\frac{1}{2} m \left(\nu\nu + \bar{\nu}\bar{\nu} \right) + c.t. \,, \tag{2.7}$$

where *c.t.* stands for counterterms. Note that δ_m, δ and $\bar{\delta}$ are complex parameters, hence in general, m is complex and $\bar{\nu} \neq \nu^\dagger$. Renormalized fields are related [37] with the consistency relation from Eq. (2.6):

$$\begin{aligned} \nu_0^\dagger &= \left(1 + \frac{1}{2}\bar{\delta}_\nu\right) \bar{\nu} = \left(1 + \frac{1}{2}\delta_\nu^\dagger\right) \nu^\dagger \\ \Rightarrow \nu^\dagger &= \left(1 + \frac{1}{2}\left(\bar{\delta}_\nu - \delta_\nu^\dagger\right)\right) \bar{\nu} + O\left(\delta^2\right) \,. \end{aligned} \tag{2.8}$$

Having this in mind we can see that the neutrino mass term in the renormalized Lagrangian (taking the part of Eq. (2.7) with no counterterms):

$$\mathcal{L}_m = -\frac{1}{2} m \left(\nu\nu + \bar{\nu}\bar{\nu} \right) \neq -\frac{1}{2} m\nu\nu + H.c. \tag{2.9}$$

is not Hermitian in general. The mass term in the bare Lagrangian, shown in Eq. (2.7) or Eq. (2.4), however, is. In the CMS, the antihermitian part of the mass term or the phase difference of the field comes from the imaginary part of the two point correlation functions[1] (or self–energies). That is, the antihermitian parts reflect the instability of the particle and is directly related to the decay width of the particle. These antihermitian terms, however, do not affect the algebraic structure of the effective Lagrangian. That is, the renormalized Lagrangian can still be expressed as a sum of terms that are conjugate to each other, except that the conjugation operation does not act on the phase that appears due to an instability. To account for the "renormalized" hermitian conjugation, we define a new symbol $H.c.^*$[38] which allows us to write:

$$\mathcal{L}_m = -\frac{1}{2}m\left(\nu\nu + \bar{\nu}\bar{\nu}\right) = -\frac{1}{2}m\nu\nu + H.c.^* \tag{2.10}$$

Note that $H.c.^*$ term is not the hermitian conjugate of $-\frac{1}{2}m\nu\nu$ in the usual sense, but

$$H.c.^* : m\nu\nu \to m\bar{\nu}\bar{\nu}\,, \tag{2.11}$$

that is, the $H.c.^*$ stands for the terms in the renormalized Lagrangian, that were written as $H.c.$ in the bare Lagrangian. Simply put, it stands for the renormalized $H.c.$ terms.

In Eq. (2.7) we were assuming that the phase of the bare mass parameter is absorbed into the bare fields as in Eq. (2.4), so we wrote a single complex mass renormalization constant for the mass term in Eq. (2.6). Let us now loosen this assumption and consider the Eq. (2.3) in general. Then there is a phase difference between m_0 and $m_0^\dagger$, which we will now call 2α. We can write:

$$|m_0|\,e^{i\alpha} = me^{i\alpha}\left(1+\delta_m\right)\,,\;\; |m_0|\,e^{-i\alpha} = me^{-i\alpha}\left(1+\delta_m\right)\,. \tag{2.12}$$

As m is complex, we have $\left(me^{i\alpha}\right)^\dagger \neq me^{-i\alpha}$. This is similar case to renormalization of fields ν and $\bar{\nu}$, i.e. one of the phases comes from the initial field phase and another phase comes from the instability of the particle. The phase difference $e^{2i\alpha}$ can be absorbed into the definition of the fields and become a contribution to the phase of Eq. (2.8), which equals to $\bar{\delta}_\nu - \delta_\nu^\dagger$. The imaginary part of m accounts for the instability

[1]The definitions for the two point correlation functions are given in the Appendix A.

of the particle and is fixed by the pole position condition in the CMS. In contrast, α cancels in the pole position condition of the CMS and comes only in the field renormalization, i.e. the residue condition in the CMS. In the general setting, the CMS renormalized mass constant relates the CMS renormalized mass with the absolute value of the bare mass:

$$|m_0| = m\,(1+\delta_m)\ . \tag{2.13}$$

If the phase were not absorbed in the fields from the start, the $H.c.^*$ function would act as:

$$H.c.^* : me^{i\alpha}\nu\nu \to me^{-i\alpha}\bar{\nu}\bar{\nu}\,. \tag{2.14}$$

However, the redefinition of the fields by absorbing the phase as in Eq. (2.4) is always possible and comfortably separates the field phase from the mass renormalization. There is no real benefit from carrying around this phase in our expressions, hence we will always absorb them from the start to have simpler expressions.

2.2 Complex mass scheme for mixed fermions

The complex mass scheme was first proposed as a scheme to define a gauge invariant Z boson mass [30], and studied afterwards extensively in [55–63]. The formal proof of gauge invariance of the definition of mass in the complex mass scheme to all orders was done in [31], with the help of functional identities introduced by Nielsen [64]. We adapt the formulation of the CMS to the two component spinor notation for mixed Majorana fermions.

First we rescale all parameters and fields by renormalization constants as in Eq. (2.1), so that:

$$\nu_{0i} = \left(1+\frac{1}{2}\delta_\nu\right)_{ij}\nu_j\ ,\ \nu^\dagger_{0i} = \left(1+\frac{1}{2}\bar{\delta}_\nu\right)_{ij}\bar{\nu}_j\ ,\ m_i\,(1+\delta_{mi}) = m_{0i}\in\mathbb{R}\,. \tag{2.15}$$

From the first two equations, the consistency condition, shown in Eq. (2.8), is generalized to include more fermions [37]:

$$\begin{aligned}\nu^\dagger_i + \frac{1}{2}\delta^*_{\nu ij}\nu^\dagger_j &= \bar{\nu}_i + \frac{1}{2}\bar{\delta}_{\nu ij}\bar{\nu}_j\\ \Rightarrow \nu^\dagger_i &= \bar{\nu}_i + \frac{1}{2}\left(\bar{\delta}_{\nu ij}-\delta^*_{\nu ij}\right)\bar{\nu}_j + O\left(\delta^2\right)\end{aligned} \tag{2.16}$$

Now we insert these renormalization constants in the Lagrangian to get the one loop counterterms that appear in the two point functions[2]. The mass term in the Lagrangian in terms of renormalized fields and masses is:

$$\begin{aligned}-\frac{1}{2}m_{0i}\nu_{0i}\nu_{0i} &= -\frac{1}{2}\left(1+\delta_{mi}\right)m_i\left(1_{ij}+\frac{1}{2}\delta_{\nu ij}\right)\left(1_{ik}+\frac{1}{2}\delta_{\nu ik}\right)\nu_j\nu_k \\ &= -\frac{1}{2}\left(1+\delta_{mi}\right)m_i\nu_i\nu_i-\frac{1}{2}\left(\frac{1}{2}m_j\delta_{\nu jk}+\frac{1}{2}m_k\delta_{\nu kj}\right)\nu_j\nu_k \\ &\quad +O\left(\delta^2\right)\,. \end{aligned} \tag{2.17}$$

For the hermitian conjugate part of the mass term we get:

$$\begin{aligned}-\frac{1}{2}m_{0i}\nu_{0i}^{\dagger}\nu_{0i}^{\dagger} &= -\frac{1}{2}\left(1+\delta_{mi}\right)m_i\bar{\nu}_i\bar{\nu}_i-\frac{1}{2}\left(\frac{1}{2}m_j\bar{\delta}_{\nu jk}+\frac{1}{2}m_k\bar{\delta}_{\nu kj}\right)\bar{\nu}_j\bar{\nu}_k \\ &\quad +O\left(\delta^2\right)\,. \end{aligned} \tag{2.18}$$

The free field term[3], up to the first order in δ is:

$$\begin{aligned}\nu_{0i}^{\dagger}\bar{\sigma}p\nu_{0i} &= \left(1_{ij}+\frac{1}{2}\bar{\delta}_{\nu ij}\right)\left(1_{ik}+\frac{1}{2}\delta_{\nu ik}\right)\bar{\nu}_j\bar{\sigma}p\nu_k \\ &= \bar{\nu}_i\bar{\sigma}p\nu_i+\left(\frac{1}{2}\bar{\delta}_{\nu kj}+\frac{1}{2}\delta_{\nu jk}\right)\bar{\nu}_j\bar{\sigma}p\nu_k+O\left(\delta^2\right)\,. \end{aligned} \tag{2.19}$$

For Majorana fermions, the opposite chirality term is equivalent:

$$\nu_{0i}\sigma p\nu_{0i}^{\dagger} = \nu_i\sigma p\bar{\nu}_i+\left(\frac{1}{2}\delta_{\nu kj}+\frac{1}{2}\bar{\delta}_{\nu jk}\right)\nu_j\sigma p\bar{\nu}_k+O\left(\delta^2\right)\,. \tag{2.20}$$

After all these shifts, the tree level two point functions for Majorana fermions in terms of renormalized parameters and fields are[4]:

$$\hat{\Gamma}^{[0]}_{\nu_i\nu_i}=\hat{\Gamma}^{[0]}_{\bar{\nu}_i\bar{\nu}_i}=-m_i\,,\;\hat{\Gamma}^{[0]}_{\nu_i\bar{\nu}_j}=p\sigma\,,\;\hat{\Gamma}^{[0]}_{\bar{\nu}_i\nu_j}=p\bar{\sigma}\,, \tag{2.21}$$

where we write a hat to denote the renormalized Green's functions. We define dimensionless renormalized scalar self–energy functions as:

$$m_i\hat{\Sigma}_{\nu_i\nu_i}=\hat{\Gamma}_{\nu_i\nu_i}\,,\;\; m_i\hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i}=\hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_i}\,,\;\; p\sigma\hat{\Sigma}_{\nu_i\bar{\nu}_j}=\hat{\Gamma}_{\nu_i\bar{\nu}_j}\,,\;\; p\bar{\sigma}\hat{\Sigma}_{\bar{\nu}_i\nu_j}=\hat{\Gamma}_{\bar{\nu}_i\nu_j}\,. \tag{2.22}$$

[2]See Appendix A for the definitions.
[3]We refer to Appendix B or [65] on the usage of Weyl spinor notation.
[4]For definitions of the renormalized Green's functions, see Appendix A.

The renormalized self-energy functions at the one loop level are:

$$\hat{\Sigma}^{[1]}_{\nu_i\nu_i} = -\delta_{mi} - \delta_{ii} + \Sigma^{[1]}_{\nu_i\nu_i}\,,\ \hat{\Sigma}^{[1]}_{\bar{\nu}_i\bar{\nu}_i} = -\delta_{mi} - \bar{\delta}_{ii} + \Sigma^{[1]}_{\bar{\nu}_i\bar{\nu}_i}\,,$$
$$\hat{\Sigma}^{[1]}_{\bar{\nu}_i\nu_i} = \frac{1}{2}\left(\bar{\delta}_{ii} + \delta_{ii}\right) + \Sigma^{[1]}_{\bar{\nu}_i\nu_i},\ \hat{\Sigma}^{[1]}_{\nu_i\bar{\nu}_i} = \frac{1}{2}\left(\bar{\delta}_{ii} + \delta_{ii}\right) + \Sigma^{[1]}_{\nu_i\bar{\nu}_i} \tag{2.23}$$

for $i = j$, and:

$$\hat{\Sigma}^{[1]}_{\bar{\nu}_i\nu_j} = \frac{1}{2}\left(\delta_{ij} + \bar{\delta}_{ji}\right) + \Sigma^{[1]}_{\bar{\nu}_i\nu_j}\,,\ \hat{\Sigma}^{[1]}_{\nu_i\bar{\nu}_j} = \frac{1}{2}\left(\bar{\delta}_{ij} + \delta_{ji}\right) + \Sigma^{[1]}_{\nu_i\bar{\nu}_j}\,,$$
$$\hat{\Gamma}^{[1]}_{\nu_i\nu_j} = -\frac{1}{2}\left(m_i\delta_{ij} + m_j\delta_{ji}\right) + \Gamma^{[1]}_{\nu_i\nu_j}\,,\ \hat{\Gamma}^{[1]}_{\bar{\nu}_i\bar{\nu}_j} = -\frac{1}{2}\left(m_i\bar{\delta}_{ij} + m_j\bar{\delta}_{ji}\right) + \Gamma^{[1]}_{\bar{\nu}_i\bar{\nu}_j} \tag{2.24}$$

for $i \neq j$.

Now we have the expressions for renormalized two point functions as counterterms plus unrenormalized two–point functions. To fix the counterterms, we need the renormalization conditions on the renormalized two–point functions. As the CMS is the analytical continuation of the OS scheme, the derivation of the conditions is the same as for the mixed fermions in the OS. One can look at these derivations for the OS in the Dirac spinor notation, for example, in [59, 66, 67]. Here we present the derivation of the same expressions in Weyl spinor notation.

In Weyl spinor formulation, the chiral structure is presented in Feynman diagrams, hence no projection operators or Majorana conditions are needed. In some calculations, this makes it easier to see the vanishing contributions immediately on the diagrammatic level. The price to pay is that we usually have more diagrams to consider. To derive the conditions, we will follow a straightforward procedure of summing loop diagrams for a propagator. The Feynman rules and the main relations for using Weyl spinors is presented in Appendix B. We write all four tree level propagators, shown in Figure B.1:

$$\frac{i\sigma p}{D_k}\,,\ \frac{i\bar{\sigma} p}{D_k}\,,\ \frac{im_k}{D_k}\,,\ \frac{im_k}{D_k}\,,\ D_k = p^2 - m_k^2\,, \tag{2.25}$$

where the usual $+i\epsilon$ in the denominators and the index structure as in Figure B.1 is understood, so we do not need to write them explicitly all the time. The corrections to these propagators can come in four Lorentz structures. These four Lorentz structures for one particle irreducible (1PI) contributions are listed in Figure 2.1. These 1PIs then can be treated as two–vertices, i.e. we can attach two propagators to a 1PI two–vertex. The rule to connect the propagator to the 1PIs of Figure 2.1 is

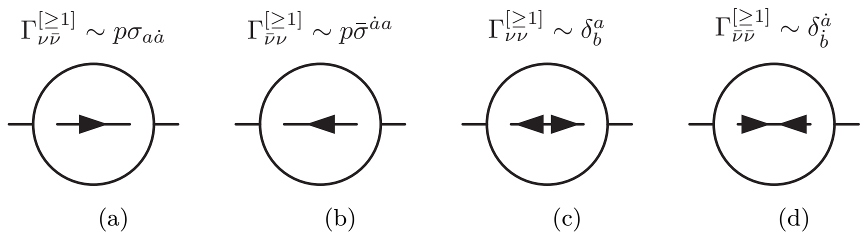


Figure 2.1: One particle irreducible diagrams, decomposed into four terms according to the Lorentz structure.

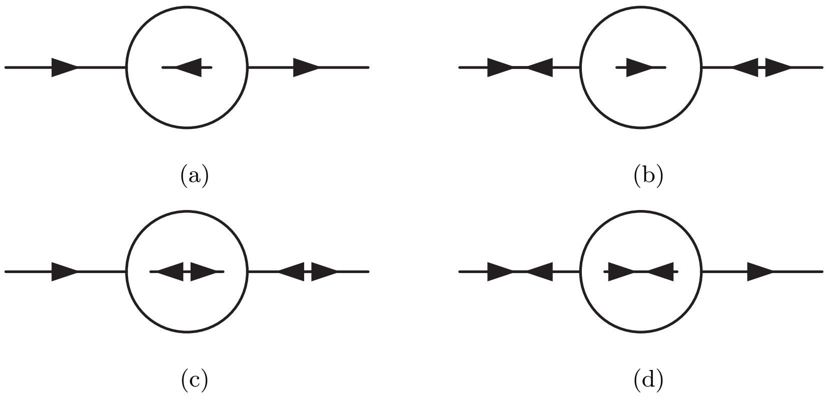


Figure 2.2: The insertions of the 1PI to the propagator of a propagator Figure B.1(a).

that the arrow inside the bubble of the 1PI that is closer to the connected propagator has to show to the opposite direction than the arrow of the connected propagator line that is closer to the 1PI insertion. That is, crossing the border of the 1PI bubble flips the arrow. Note that “crossing the border” of 1PI flips the arrow of a Weyl spinor propagator, just as interactions with scalars do, as is shown in Figure B.3.

Let us consider the first propagator in Eq. (2.25), which is the propagator that is also shown in Figure B.1(a). The corrections to this propagator must have the number of arrows directed to the right minus the number of arrows directed to the left equal to 1. Given the rule to connect the arrows, the considered propagator has four diagrams for 1PI insertions, which are shown in Figure 2.2.

Once we know the Lorentz structure of the diagrams, we can assign the expressions for them. For generality, let us say that we have the jth fermion going into the ith fermion. We now assign propagator expressions for every propagator, as shown in Figure B.1, and write the coefficients for 1PIs in. The sum of the expressions that we obtain from the diagrams Figure 2.2(a), Figure 2.2(b), Figure 2.2(c) and Figure 2.2(d) is:

$$\begin{aligned}\frac{i\sigma p}{D_i}\hat{\Gamma}_{\bar{\nu}_i\nu_j}\frac{i\sigma p}{D_j}+\frac{im_i}{D_i}\hat{\Gamma}_{\nu_i\bar{\nu}_j}\frac{im_j}{D_j}+\frac{i\sigma p}{D_i}\hat{\Gamma}_{\nu_i\nu_j}\frac{im_j}{D_j}+\frac{im_i}{D_i}\hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_j}\frac{i\sigma p}{D_j}\\ =-\frac{i\sigma p}{D_iD_j}\left(p^2\hat{\Sigma}_{\bar{\nu}_i\nu_j}+m_im_j\hat{\Sigma}_{\nu_i\bar{\nu}_j}+\hat{\Gamma}_{\nu_i\nu_j}m_j+m_i\hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_j}\right)\\ \equiv -i\sigma p\frac{1}{D_i}A_{ij}\frac{1}{D_j}\,. \qquad (2.26)\end{aligned}$$

Inserting the 1PIs two times, three times and continuing in this way, gives:

$$\begin{aligned}i\sigma p\Big(\frac{1_{ij}}{D_i}-\frac{1}{D_i}A_{ij}\frac{1}{D_j}+\sum_k\frac{1}{D_i}A_{ik}\frac{1}{D_k}A_{kj}\frac{1}{D_j}\\ -\sum_{k,l}\frac{1}{D_i}A_{ik}\frac{1}{D_k}A_{kl}\frac{1}{D_l}A_{lj}\frac{1}{D_j}+...\Big) \qquad (2.27)\end{aligned}$$

up to an infinite number of terms, giving the all loop level propagator. Equivalently, we get the contributions to the other three types of propagators:

$$i\bar{\sigma}p\left(\frac{1_{ij}}{D_i}-\frac{1}{D_i}\bar{A}_{ij}\frac{1}{D_j}+...\right)\,, \qquad (2.28)$$

$$im_i\left(\frac{1_{ij}}{D_i}-\frac{1}{D_i}B_{ij}\frac{1}{D_j}+...\right)\,, \qquad (2.29)$$

$$im_i\left(\frac{1_{ij}}{D_i}-\frac{1}{D_i}\bar{B}_{ij}\frac{1}{D_j}+...\right)\,, \qquad (2.30)$$

where:

$$\bar{A}_{ij}=p^2\hat{\Sigma}_{\nu_i\bar{\nu}_j}+m_im_j\hat{\Sigma}_{\bar{\nu}_i\nu_j}+\hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_j}m_j+m_i\hat{\Gamma}_{\nu_i\nu_j}\,, \qquad (2.31)$$

$$B_{ij}=m_im_j\hat{\Gamma}_{\nu_i\nu_j}+p^2\left[m_j\hat{\Sigma}_{\bar{\nu}_i\nu_j}+\hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_j}+m_i\hat{\Sigma}_{\nu_i\bar{\nu}_j}\right]\,, \qquad (2.32)$$

$$\bar{B}_{ij}=p^2\left[\hat{\Gamma}_{\nu_i\nu_j}+m_i\hat{\Sigma}_{\bar{\nu}_i\nu_j}+m_j\hat{\Sigma}_{\nu_i\bar{\nu}_j}\right]+m_im_j\hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_j}\,. \qquad (2.33)$$

The CMS condition for the ith particle is that at $p^2=m_i^2$, where m_i^2 in general is complex mass parameter, the propagator does not mix and has an exact pole with a residue 1. For the propagator of Eq. (2.27), we see that it is realized with $A_{ij}|_{p^2=m_i^2}=0$. As it is not independent from the other three types of propagators, all four conditions have to be satisfied simultaneously:

$$A_{ij}|_{p^2=m_i^2} = \bar{A}_{ij}|_{p^2=m_i^2} = B_{ij}|_{p^2=m_i^2} = \bar{B}_{ij}|_{p^2=m_i^2} = 0\,. \tag{2.34}$$

For $i \neq j$ we get

$$\left(\hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_j} + m_i\hat{\Sigma}_{\nu_i\bar{\nu}_j}\right)_{m_i^2} = 0\,, \;\left(\hat{\Gamma}_{\nu_i\nu_j} + m_i\hat{\Sigma}_{\bar{\nu}_i\nu_j}\right)_{m_i^2} = 0\,. \tag{2.35}$$

For $i = j$, all four conditions collapse into one condition for the position of the pole:

$$\begin{aligned} A_{ii}|_{p^2=m_i^2} &= \bar{A}_{ii}|_{p^2=m_i^2} = B_{ii}|_{p^2=m_i^2} = \bar{B}_{ii}|_{p^2=m_i^2} \\ &= m_i^3\left(\hat{\Sigma}_{\nu_i\nu_i} + \hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i} + \hat{\Sigma}_{\nu_i\bar{\nu}_i} + \hat{\Sigma}_{\bar{\nu}_i\nu_i}\right)\Big|_{p^2=m_i^2} = 0\,. \end{aligned} \tag{2.36}$$

Imposing the conditions of Eq. (2.35) gets rid of mixings, when the particle is at the pole. So, at the pole, we can formally do the Dyson resummation [68] ignoring the mixing terms:

$$\frac{i\sigma p}{D_i + A_{ii}}\,,\; \frac{i\bar{\sigma} p}{D_i + \bar{A}_{ii}}\,,\; \frac{i m_i}{D_i + B_{ii}}\,,\; \frac{i m_i}{D_i + \bar{B}_{ii}}\,. \tag{2.37}$$

The residue condition is then:

$$A'_{ii}|_{p^2=m_i^2} = \bar{A}'_{ii}|_{p^2=m_i^2} = B'_{ii}|_{p^2=m_i^2} = \bar{B}'_{ii}|_{p^2=m_i^2} = 0\,, \tag{2.38}$$

where the prime indicates the derivative with respect to p^2. In terms of the renormalized self energy functions, these relations give:

$$\hat{\Sigma}_{\bar{\nu}_i\nu_i}\left(m_i^2\right) + m_i^2\left(\hat{\Sigma}'_{\bar{\nu}_i\nu_i} + \hat{\Sigma}'_{\nu_i\bar{\nu}_i} + \hat{\Sigma}'_{\nu_i\nu_i} + \hat{\Sigma}'_{\bar{\nu}_i\bar{\nu}_i}\right)_{m_i^2} = 0\,, \tag{2.39}$$

$$\hat{\Sigma}_{\bar{\nu}_i\nu_i}\left(m_i^2\right) = \hat{\Sigma}_{\nu_i\bar{\nu}_i}\left(m_i^2\right) = -\hat{\Sigma}_{\nu_i\nu_i}\left(m_i^2\right) = -\hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i}\left(m_i^2\right)\,, \tag{2.40}$$

where the m_i^2 in the subscript of a bracket means that we evaluate the bracket at $p^2 = m_i^2$. Inserting the Eq. (2.23) and Eq. (2.24) into Eq. (2.35), Eq. (2.39) and Eq. (2.40), we get the expressions for counterterms:

$$\delta_{mi} = \frac{1}{2}\left(\Sigma_{\nu_i\bar{\nu}_i} + \Sigma_{\bar{\nu}_i\nu_i} + \Sigma_{\bar{\nu}_i\bar{\nu}_i} + \Sigma_{\nu_i\nu_i}\right)_{m_i^2}\,, \tag{2.41}$$

$$\frac{1}{2}\left(\bar{\delta}_{\nu ii} + \delta_{\nu ii}\right) = -\Sigma_{\bar{\nu}_i\nu_i}\left(m_i^2\right) - m_i^2\left(\Sigma'_{\bar{\nu}_i\nu_i} + \Sigma'_{\nu_i\bar{\nu}_i} + \Sigma'_{\nu_i\nu_i} + \Sigma'_{\bar{\nu}_i\bar{\nu}_i}\right)_{m_i^2}\,, \tag{2.42}$$

$$\bar{\delta}_{\nu ii} - \delta_{\nu ii} = \left(\Sigma_{\bar{\nu}_i\bar{\nu}_i} - \Sigma_{\nu_i\nu_i}\right)_{m_i^2}\,, \tag{2.43}$$

$$\delta_{\nu ij} = \frac{2}{m_i^2 - m_j^2}\left(m_j\Gamma_{\bar{\nu}_i\bar{\nu}_j} + m_i\Gamma_{\nu_i\nu_j} + m_j^2\Sigma_{\bar{\nu}_i\nu_j} + m_jm_i\Sigma_{\nu_i\bar{\nu}_j}\right)_{m_j^2}, \tag{2.44}$$

$$\bar{\delta}_{\nu ij} = \frac{2}{m_i^2 - m_j^2}\left(m_i\Gamma_{\bar{\nu}_i\bar{\nu}_j} + m_j\Gamma_{\nu_i\nu_j} + m_jm_i\Sigma_{\bar{\nu}_i\nu_j} + m_j^2\Sigma_{\nu_i\bar{\nu}_j}\right)_{m_j^2}, \tag{2.45}$$

with an additional consistency condition on the unrenormalized self energies:

$$\Sigma_{\bar{\nu}_i\nu_i}\left(m_i^2\right) = \Sigma_{\nu_i\bar{\nu}_i}\left(m_i^2\right). \tag{2.46}$$

In fact, this relation has to hold for every value of p^2 as it is a consequence of (see Appendix B)

$$\bar{\xi}\bar{\sigma}^\mu p_\mu \chi = \chi\sigma^\mu p_\mu \bar{\xi}. \tag{2.47}$$

A more general statement for any value of p^2 is:

$$\Sigma_{\bar{\nu}_i\nu_j} = \Sigma_{\nu_j\bar{\nu}_i}. \tag{2.48}$$

Other trivial relations are:

$$\Gamma_{\nu_i\nu_j} = \Gamma_{\nu_j\nu_i}, \; \Gamma_{\bar{\nu}_i\bar{\nu}_j} = \Gamma_{\bar{\nu}_j\bar{\nu}_i}. \tag{2.49}$$

All these relations also hold for the renormalized functions. In Eq. (2.44) and Eq. (2.45) we used these relations while relabeling indices $i \leftrightarrow j$ to match the definitions in standard references, such as [66, 69]. One can check that Eq. (2.44) and Eq. (2.45) can also be derived evaluating at $p^2 = m_j^2$ instead of $p^2 = m_i^2$.

These expressions are the same expressions as in [67, 70], except that they are written in two component spinor notation for mixed Majorana particles and we did not take the real part of the pole. This means that the equalities are in general complex.

Now we can return to the interpretation of these complex parameters. The complex mass parameter is a generalization of the real mass parameter including the decay width and is gauge dependent. The inclusion of the decay width into the definition is tricky, as it can be done in different ways [61]:

$$m^2 = \left(m_3 - \frac{i}{2}\Gamma_3\right)^2, \tag{2.50}$$

$$m^2 = m_2^2 - i m_2\Gamma_2, \tag{2.51}$$

$$m^2 = \frac{m_1^2 - im_1\Gamma_1}{1+\Gamma_1^2/m_1^2}\,. \tag{2.52}$$

$$m^2 \in \mathbb{C}, m_k, \Gamma_k \in \mathbb{R}\,, k = 1,2,3. \tag{2.53}$$

Note that Γ in these equations means the decay width and not the effective vertex functional, which should be clear from the context. The definition of Eq. (2.50) is used for fermions and Eq. (2.51) for bosons. They are different due to the fact that the mass dimension of a fermion ($[m]^{\frac{3}{2}}$) is different from the mass dimension of a boson($[m]^1$). So the self energies for bosons contribute to the mass parameter squared, while the self energies for fermions give a contribution to the mass to the first power. The third definition Eq. (2.52) is introduced in [30] where it was shown that the m_1 describes the Z boson mass at two loop level, hence one can think of it as an approximation of Eq. (2.51). Let us look how the definition of Eq. (2.50) enters at one loop.

Recall the discussion in Section 2.1 and take the definition of the mass renormalization constant in Eq. (2.13) and compare it to the definition Eq. (2.50). They give:

$$m = \left(m_3 - \frac{i}{2}\Gamma_3\right) = \frac{m_0}{(1+\delta_m)}\,. \tag{2.54}$$

Taking the real parts and imaginary parts:

$$m_3 = \frac{m_0}{(1+\mathrm{Re}\delta_m)} = m_0 - m_0\mathrm{Re}\delta_m + O\left(\delta^2\right)\,, \tag{2.55}$$

$$\Gamma_3 = -2\mathrm{Im}\frac{m_0}{(1+\delta_m)} = 2m_0\mathrm{Im}\delta_m + O\left(\delta^2\right)\,. \tag{2.56}$$

The renormalization constant δ_m is expressed by Eq. (2.41). At one loop:

$$\begin{aligned}\delta_m &= \frac{1}{2}\left(\Sigma_{\nu\bar{\nu}} + \Sigma_{\bar{\nu}\nu} + \Sigma_{\bar{\nu}\bar{\nu}} + \Sigma_{\nu\nu}\right)_{m^2=\left(m_3-\frac{i}{2}\Gamma_3\right)^2} \\ &= \frac{1}{2}\left(\Sigma_{\nu_i\bar{\nu}_i} + \Sigma_{\bar{\nu}_i\nu_i} + \Sigma_{\bar{\nu}_i\bar{\nu}_i} + \Sigma_{\nu_i\nu_i}\right)_{m_0^2} + O\left(\delta^2\right)\end{aligned} \tag{2.57}$$

so the definitions for mass and width in Eq. (2.55) and Eq. (2.56) give the same result at one loop as the OS scheme. The equation of Eq. (2.56) at one loop is then the usual Cutkosky rule that ensures the unitarity of the theory. How unitarity is maintained order by order in the CMS was studied in [54]. The only difference from the OS scheme at one loop is that we now include the width into the imaginary part of the mass counterterm. At two loops, however, we need to include the width also in the evaluation of loop integrals, which in turn will give the difference in the mass of the particle.

Let us now consider the effect of the complex field renormalization. At one loop level, we have the relation of Eq. (2.16):

$$\nu_i^\dagger = \bar{\nu}_i + \frac{1}{2}\left(\bar{\delta}_{\nu ij} - \delta^*_{\nu ij}\right)\bar{\nu}_j + O\left(\delta^2\right) . \tag{2.58}$$

In the OS, only the real part of the self energy functions is absorbed into the renormalization constants, hence in the OS we have:

$$\text{OS: } \bar{\delta}_{\nu ij} = \delta^*_{\nu ij} \Rightarrow \nu_i^\dagger = \bar{\nu}_i \,. \tag{2.59}$$

This can be seen by taking the real part only in Eq. (2.44) and Eq. (2.45). In the CMS, however, if particle j is unstable, we get contributions to this relations in the form of Eq. (2.16) or, at one loop, Eq. (2.58). First, consider the one loop case of a single fermion. In the mass eigenstate basis, we can always adjust the phase so that at one loop Eq. (2.43) is zero:

$$\bar{\delta}_{\nu ii} - \delta_{\nu ii} = \left(\Sigma^{[1]}_{\bar{\nu}_i\bar{\nu}_i} - \Sigma^{[1]}_{\nu_i\nu_i}\right)_{m_i^2} = 0 \,. \tag{2.60}$$

This is ensured by the hermicity requirement of the bare Lagrangian. Then the phase difference can be written as:

$$\nu_i^\dagger = \bar{\nu}_i + \frac{1}{2}\left(\delta_{\nu ii} - \delta^*_{\nu ii}\right)\bar{\nu}_i + O\left(\delta^2\right) = \bar{\nu}_i + i\text{Im}\delta_{\nu ii}\,\bar{\nu}_i + O\left(\delta^2\right) . \tag{2.61}$$

The imaginary part comes from Eq. (2.42):

$$\text{Im}\delta_{\nu ii} = -\text{Im}\Sigma^{[1]}_{\bar{\nu}_i\nu_i}\left(m_i^2\right) - m_i^2\text{Im}\left(\Sigma'^{[1]}_{\bar{\nu}_i\nu_i} + \Sigma'^{[1]}_{\nu_i\bar{\nu}_i} + \Sigma'^{[1]}_{\nu_i\nu_i} + \Sigma'^{[1]}_{\bar{\nu}_i\bar{\nu}_i}\right)_{m_i^2} . \tag{2.62}$$

For a mixed system, even if the ith particle is stable it can acquire the "instability phase" from the mixture with other unstable particles. Consider a two–particle system, in which the first particle is stable and the second is unstable. Then:

$$\text{Im}\delta_{11} = 0\,,\ \text{Im}\delta_{12} \neq 0\,. \tag{2.63}$$

Similarly as in Eq. (2.60), we can adjust the phase of the bare mass terms so that:

$$\Gamma^{[1]}_{\bar{\nu}_i\bar{\nu}_j} = \Gamma^{[1]}_{\nu_i\nu_j}\,,\ \Sigma^{[1]}_{\nu_i\bar{\nu}_j} = \Sigma^{[1]}_{\bar{\nu}_i\nu_j}\,, \tag{2.64}$$

where the second equation is ensured by the hermicity of the bare Lagrangian. This gives us:

$$\delta_{\nu ij} = \bar{\delta}_{\nu ij} \,, \tag{2.65}$$

so

$$\nu_1^\dagger = \bar{\nu}_1 + i\mathrm{Im}\delta_{\nu 12}\bar{\nu}_2 \,. \tag{2.66}$$

To understand this equation, consider that the renormalization in the usual OS scheme ignores the imaginary parts coming from loop functions. Hence in a mixed system, this imaginary part is also omitted for the non diagonal field renormalization constants in the OS scheme. In the CMS, this imaginary part is accounted by Eq. (2.66). The counter-intuitive feature is that even though ν_1 is stable, we still have $\nu_1^\dagger \neq \bar{\nu}_1$ due to a mixture with unstable particles. However, at the one loop level, this phase does not come in the diagonal two point function of a stable particle. To see this, let us write

$$\Gamma_{\bar{\nu}_j \nu_i} = \Gamma_{\nu_i \bar{\nu}_j} = \left(\frac{\delta \nu_k^\dagger}{\delta \bar{\nu}_j}\right) \Gamma_{\nu_i \nu_k^\dagger} \,, \; \Gamma_{\bar{\nu}_i \bar{\nu}_j} = \left(\frac{\delta \nu_l^\dagger}{\delta \bar{\nu}_i}\right) \left(\frac{\delta \nu_k^\dagger}{\delta \bar{\nu}_j}\right) \Gamma_{\nu_l^\dagger \nu_k^\dagger} \,. \tag{2.67}$$

At the one loop level:

$$\Gamma^{[\leq 1]}_{\nu_i \bar{\nu}_j} = \left(1_{kj} + \frac{1}{2}\left(\bar{\delta}_{\nu kj} - \delta^*_{\nu kj}\right)\right) \Gamma^{[0]}_{\nu_i \nu_k^\dagger} + \Gamma^{[1]}_{\nu_i \nu_j^\dagger} + O\left(\delta^2\right) \,, \tag{2.68}$$

$$\Gamma^{[\leq 1]}_{\bar{\nu}_i \bar{\nu}_j} = \Gamma^{[0]}_{\nu_i^\dagger \nu_j^\dagger} + \frac{1}{2}\left(\bar{\delta}_{\nu kj} - \delta^*_{\nu kj} + \bar{\delta}_{\nu ki} - \delta^*_{\nu ki}\right) \Gamma^{[0]}_{\nu_i^\dagger \nu_k^\dagger} + \Gamma^{[1]}_{\nu_i^\dagger \nu_j^\dagger} + O\left(\delta^2\right) \,. \tag{2.69}$$

In the mass eigenstates, we have:

$$\Gamma^{[0]}_{\nu_i^\dagger \nu_k^\dagger} = \Gamma^{[0]}_{\nu_i \nu_k^\dagger} = 0 \,, \; i \neq k \,. \tag{2.70}$$

Hence for $i = j$ (using Eq. (2.65)):

$$\Gamma^{[\leq 1]}_{\nu_i \bar{\nu}_i} = \left(1_{kj} + \frac{1}{2}\left(\delta_{\nu ii} - \delta^*_{\nu ii}\right)\right) \Gamma^{[0]}_{\nu_i \nu_i^\dagger} + \Gamma^{[1]}_{\nu_i \nu_i^\dagger} + O\left(\delta^2\right) \,, \tag{2.71}$$

$$\Gamma^{[\leq 1]}_{\bar{\nu}_i \bar{\nu}_i} = \Gamma^{[0]}_{\nu_i^\dagger \nu_i^\dagger} + \left(\delta_{\nu ii} - \delta^*_{\nu ii}\right) \Gamma^{[0]}_{\nu_i^\dagger \nu_i^\dagger} + \Gamma^{[1]}_{\nu_i^\dagger \nu_i^\dagger} + O\left(\delta^2\right) \,, \tag{2.72}$$

which shows that the "instability" phases do not come from the mixed terms for one loop diagonal two point functions, and only the stability of the particle in the consideration matters. Going back to an example of 2 particle mixing, for stable ν_1 we have:

$$\Gamma^{[\leq 1]}_{\nu_1\bar{\nu}_1} = (1 + \mathrm{Im}\delta_{\nu 11})\, \Gamma^{[0]}_{\nu_1\nu_1^\dagger} + \Gamma^{[1]}_{\nu_1\nu_1^\dagger} = \Gamma^{[\leq 1]}_{\nu_1\nu_1^\dagger}\,, \tag{2.73}$$

$$\Gamma^{[\leq 1]}_{\bar{\nu}_1\bar{\nu}_1} = (1 + 2\mathrm{Im}\delta_{\nu 11})\, \Gamma^{[0]}_{\nu_1^\dagger\nu_1^\dagger} + \Gamma^{[1]}_{\nu_1^\dagger\nu_1^\dagger} = \Gamma^{[\leq 1]}_{\nu_1^\dagger\nu_1^\dagger}\,, \tag{2.74}$$

which proves our statement about the absence of the "instability" phase in the two point functions of a stable particle at one loop. For unstable ν_2 we have:

$$\Gamma^{[\leq 1]}_{\nu_2\bar{\nu}_2} = (1 + \mathrm{Im}\delta_{\nu 22})\, \Gamma^{[0]}_{\nu_2\nu_2^\dagger} + \Gamma^{[1]}_{\nu_2\nu_2^\dagger} \neq \Gamma^{[\leq 1]}_{\nu_2\nu_2^\dagger}\,, \tag{2.75}$$

$$\Gamma^{[\leq 1]}_{\bar{\nu}_2\bar{\nu}_2} = (1 + 2\mathrm{Im}\delta_{\nu 22})\, \Gamma^{[0]}_{\nu_2^\dagger\nu_2^\dagger} + \Gamma^{[1]}_{\nu_2^\dagger\nu_2^\dagger} \neq \Gamma^{[1]}_{\nu_2^\dagger\nu_2^\dagger}\,. \tag{2.76}$$

The additional phases that enter the diagonal two–point functions due to a mixing with unstable particles like in Eq. (2.66), appear at higher loops than one. At the one loop level, the off–diagonal terms will have a contribution from the instability of ν_2, which ensures the cancellation of mixing at the exact poles of the particles. This means that the renormalization of mixing will have some contributions from unstable particles that are different than in the OS scheme already at one loop level.

2.2.1 Radiative masses in the CMS framework

When the Weyl spinor ν does not have a mass term, it has only two types of propagator, shown in Figure B.1(a) and Figure B.1(b). If there is no symmetry that forbids the mass term, then it can be generated at the loop level. Moreover, this generated mass term will give rise to a propagator that was absent at the tree level. That is, it will generate the correlation functions $\langle\nu\nu\rangle$ and $\langle\bar{\nu}\bar{\nu}\rangle$. For instance, at the loop level we have

$$\langle\nu\nu\rangle^{[\leq 1]} = \frac{i\sigma p}{p^2} i\Gamma^{[1]}_{\bar{\nu}\bar{\nu}} \frac{i\bar{\sigma} p}{p^2} = \frac{-i\Gamma^{[1]}_{\bar{\nu}\bar{\nu}}}{p^2} \equiv \frac{im^{[\mathrm{loop}]}}{p^2} - \frac{i\left(\Gamma^{[1]}_{\bar{\nu}\bar{\nu}}\left(p^2\right) - \Gamma^{[1]}_{\bar{\nu}\bar{\nu}}\left(0\right)\right)}{p^2}\,. \tag{2.77}$$

Here we identified the radiative mass $m^{[\mathrm{loop}]} = -\Gamma^{[1]}_{\bar{\nu}_2\bar{\nu}_2}(0)$ as being a one loop mass term. The diagram that corresponds to this correction is shown in Figure 2.3.

This definition of $m^{[\mathrm{loop}]}$ looks reasonable, since it gives the mass-like propagator in a similar form as for the usual massive particles. However, the pole of the one loop propagator, Eq. (2.77), still looks to be at $p^2 = 0$.

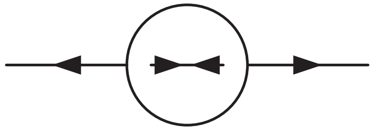


Figure 2.3: Diagram responsible for radiatively generated propagator of Eq. (2.77).

To see if this definition is consistent with a pole position of a particle, we need to go to higher loops. The propagators of all four types mix in a more complicated manner than for the massive particle case, since the absence of mass–like propagators at the tree level leads to a not so trivial loop ordering. Hence, to prove that this mass definition is consistent with the pole of the propagator in general, we will employ a slightly different approach that is based on a generalized on-shell equations of motion in the next section.

2.3 CMS as a generalization of the OS with asymptotic spinors

The other way to derive the CMS expressions is by formally generalizing the OS renormalization conditions for the asymptotic states given in e.g. [69]. To translate it for Weyl spinors, we can write the Majorana spinors as $\psi \equiv \begin{pmatrix} \bar{\nu} \\ \nu \end{pmatrix}$ $\bar{\psi} = \begin{pmatrix} \nu & \bar{\nu} \end{pmatrix}$. By Lorentz decomposition we find that the renormalized two point function in this notation:

$$\hat{\Gamma}_{\bar{\psi}\psi} = \begin{pmatrix} \sigma p \left(1 + \hat{\Sigma}_{\nu\bar{\nu}}\right) & -m\left(1 - \hat{\Sigma}_{\nu\nu}\right) \\ -\bar{m}\left(1 - \hat{\Sigma}_{\bar{\nu}\bar{\nu}}\right) & \bar{\sigma} p \left(1 + \hat{\Sigma}_{\bar{\nu}\nu}\right) \end{pmatrix} \tag{2.78}$$

The bar on the mass is to notify the chirality structure, otherwise, it has the same value as m, i.e.:

$$\bar{m} \to m\delta^{\dot{a}}_{\dot{b}}\,,\ m \to m\delta^{a}_{b}\,, \tag{2.79}$$

so that the spinor indices are implicitly understood, but not explicitly written. The on-shell Weyl spinor with Majorana mass term satisfies:

$$\bar{\sigma} p \nu = \bar{m}\nu^{\dagger}\,,\ \sigma p \nu^{\dagger} = m\nu\,. \tag{2.80}$$

We can formally say that the CMS Weyl spinors satisfies the modified equations at complex $p^2 = m^2$:

$$\bar{\sigma} p \nu = \bar{m} \bar{\nu}\,, \;\; \sigma p \bar{\nu} = m \nu\,, \tag{2.81}$$

which is the same as

$$\hat{\Gamma}^{[0]}_{\bar{\psi}\psi}|_{m^2} \psi = 0\,. \tag{2.82}$$

As a renormalization condition, following the expressions presented in [69] we require that the Eq. (2.81) or Eq. (2.82) are satisfied at all loops:

$$\begin{aligned} 0 = \Gamma_{\bar{\psi}\psi}|_{m^2}\psi &= \begin{pmatrix} \sigma p \left(1 + \hat{\Sigma}_{\nu\bar{\nu}}\right) \bar{\nu} - m \left(1 - \hat{\Sigma}_{\nu\nu}\right) \nu \\ \bar{\sigma} p \left(1 + \hat{\Sigma}_{\bar{\nu}\nu}\right) \nu - \bar{m} \left(1 - \hat{\Sigma}_{\bar{\nu}\bar{\nu}}\right) \bar{\nu} \end{pmatrix}_{m^2} \\ &= \begin{pmatrix} \left[m \left(1 + \hat{\Sigma}_{\nu\bar{\nu}}\right) - m \left(1 - \hat{\Sigma}_{\nu\nu}\right)\right] \nu \\ \left[\bar{m} \left(1 + \hat{\Sigma}_{\bar{\nu}\nu}\right) - \bar{m} \left(1 - \hat{\Sigma}_{\bar{\nu}\bar{\nu}}\right)\right] \bar{\nu} \end{pmatrix}_{m^2}, \end{aligned} \tag{2.83}$$

which gives:

$$\left(\hat{\Sigma}_{\bar{\nu}\nu} + \hat{\Sigma}_{\bar{\nu}\bar{\nu}}\right)_{m^2} = \left(\hat{\Sigma}_{\nu\bar{\nu}} + \hat{\Sigma}_{\nu\nu}\right)_{m^2} = 0\,. \tag{2.84}$$

The residue condition is:

$$\mathrm{Lim}_{p^2 \to m^2} \frac{\begin{pmatrix} \bar{\sigma} p & \bar{m} \\ m & \sigma p \end{pmatrix}}{p^2 - m^2} \hat{\Gamma}_{\bar{\psi}\psi} \psi = \psi\,. \tag{2.85}$$

The direct calculation gives:

$$1 + \hat{\Sigma}_{\nu\bar{\nu}}|_{m^2} + m^2 \left(\hat{\Sigma}'_{\nu\bar{\nu}} + \hat{\Sigma}'_{\bar{\nu}\bar{\nu}} + \hat{\Sigma}'_{\bar{\nu}\nu} + \hat{\Sigma}'_{\nu\nu}\right)_{m^2} = 1\,, \tag{2.86}$$

$$1 + \hat{\Sigma}_{\bar{\nu}\nu}|_{m^2} + m^2 \left(\hat{\Sigma}'_{\nu\bar{\nu}} + \hat{\Sigma}'_{\bar{\nu}\bar{\nu}} + \hat{\Sigma}'_{\bar{\nu}\nu} + \hat{\Sigma}'_{\nu\nu}\right)_{m^2} = 1\,, \tag{2.87}$$

hence we arrive at the expressions in Eq. (2.39) and Eq. (2.40).

Now let us assume that we have zero mass at tree level, but we generate the mass-like two point function at loop level. Eq. (2.78) is modified to

$$\hat{\Gamma}_{\bar{\psi}\psi} = \begin{pmatrix} \sigma p \left(1 + \hat{\Sigma}_{\nu\bar{\nu}}\right) & \hat{\Gamma}_{\nu\nu} \\ \hat{\Gamma}_{\bar{\nu}\bar{\nu}} & \bar{\sigma} p \left(1 + \hat{\Sigma}_{\bar{\nu}\nu}\right) \end{pmatrix}. \tag{2.88}$$

As the loop level mass term is non zero, we still use the relation Eq. (2.81) at $p^2 = m^2$ and write

$$\hat{\Gamma}_{\bar{\psi}\psi} \psi|_{p^2 = m^2} = 0\,. \tag{2.89}$$

This gives us two equations:

$$m = -\frac{\hat{\Gamma}_{\nu\nu}}{\left(1+\hat{\Sigma}_{\nu\bar{\nu}}\right)}\Big|_{p^2=m^2}\,,\ \bar{m} = -\frac{\hat{\Gamma}_{\bar{\nu}\bar{\nu}}}{\left(1+\hat{\Sigma}_{\bar{\nu}\nu}\right)}\Big|_{p^2=m^2}\,. \tag{2.90}$$

The residue condition, Eq. (2.85), in this case gives:

$$\hat{\Sigma}_{\nu\bar{\nu}}|_{m^2} = \hat{\Sigma}_{\bar{\nu}\nu}|_{m^2} = 0\,. \tag{2.91}$$

So the CMS renormalized radiative masses are just:

$$m = -\hat{\Gamma}_{\nu\nu}|_{p^2=m^2} = -\hat{\Gamma}^{[1]}_{\nu\nu}|_{p^2=0} + O\left(\delta^2\right)\,,\ \bar{m} = -\hat{\Gamma}^{[1]}_{\bar{\nu}\bar{\nu}}|_{p^2=0} + O\left(\delta^2\right)\,. \tag{2.92}$$

This one loop result is consistent with the radiative one loop mass definition also used in [71]. When we say “CMS renormalized mass”, we only mean that this renormalized mass is the position of the exact pole of the propagator. The cancellation of UV divergent terms has to be assured in this procedure by the counterterms that appear in the renormalized self energy functions. When the bare mass terms are non zero, the CMS condition fixes the counterterms to account for the mass shift due to the perturbative corrections and absorbs the UV divergences. For the massless case, however, we see from the Eq. (2.92), that the CMS conditions leads to an expression for the mass term rather then to a condition to fix the counterterms. In fact, as we assume the multiplicative renormalization Eq. (2.1), we see that there is no counterterm apparent in the one loop two point function in Eq. (2.92). This means that it has to be finite by itself i.e. the apparent divergences in loop functions have to cancel in the expression of Eq. (2.92) with no need of additional subtraction. Since it gives the exact pole, it has to be gauge invariant as well. We will see in further sections how this becomes evident in practical calculations for radiatively generated neutrino masses of the Grimus–Neufeld model.

Summary

The CMS is the analytical continuation of the OS, which defines renormalized mass parameters gauge invariantly at all loops [31]. We reformulated the CMS using Weyl spinors and presented the derivations of CMS conditions for a system of mixed Weyl spinors with Majorana masses [37]. We show that the CMS renormalized fields get the additional phase differences if any particle in the system of mixed particles is unstable. We also show that these phases do not enter the one loop two–point functions for stable particles, which was not done before. In the last section of the chapter we present another approach to derive the CMS conditions and show that the radiative mass is the CMS renormalized mass. To the best of our knowledge, the interpretation of radiative masses as the CMS renormalized masses is new. This leads to the conclusion that the radiatively generated masses for fermions are gauge independent. We check the gauge independence explicitly of the radiative neutrino mass in the GN model in the following chapter.

Chapter 3

Renormalization in the Grimus-Neufeld model

In this chapter we will derive renormalization constants for our specific model, by applying Eq. (2.1) to the parameters and fields of the GN model. The Lagrangian parts that we will need in the discussion are the scalar potential Eq. (1.12) and the Yukawa terms shown in Eq. (1.64). As it was noted in the introduction, we employ the CMS for mass renormalization to have a framework that gives the gauge invariant renormalized mass definitions at all loop levels for both stable and unstable particles. The CMS, in comparison to the OS, gives a significant difference only for unstable particles, where the narrow width approximation is not valid. In the GN model at one loop, all three neutrinos that are measured by the experiment are stable particles, hence it is natural to question the decision to use the CMS procedure for them. However, if the fourth neutrino has a large enough mass (which is the original assumption and motivation of the seesaw mechanism), it becomes unstable. Since all the neutrinos mix, it is reasonable to choose a single setup for the renormalization of all four neutrinos. As can be seen from Section 2.2, this is not only a convenient choice for stable particles, but it might actually give a difference in the field renormalization. As the field renormalization is related to the mixing, it affects the renormalization of the PMNS matrix. Furthermore, all the neutrinos except the lightest one might become unstable at some loop level, hence for full consistency, the CMS procedure for all mixed neutrinos is required.

As we will use the CMS, we use masses as the free parameters of the theory instead of the original, symmetry based parameters. However, it is worth to keep track of the relations to the parameters in other bases than the mass eigenstate basis. These relations between the renormalized parameters in different bases can also be represented with the help of Eq. (2.1) as relations between their counterterms. This allows us to work out the algebraic structure of the renormalization constants in more detail. For instance, as we will see, the relationship between the renormalization constants of the VEV of the Higgs and the masses of the neutrinos will help to identify the potentially gauge dependent term algebraically. This will become the main tool to separate the gauge dependences with the FJ scheme.

The renormalization of the minimum of the scalar sector is related to the renormalization of masses, because all of the masses in the theory, except for the Majorana mass of the heavy neutrino singlet, are given by the EWSB mechanism. Hence we first consider the renormalization of tadpoles, which define the minimum of the theory.

3.1 Renormalization of tadpoles

In this section, we derive the renormalized tadpole conditions to fix the counterterm for the Higgs VEV, which will later interplay with the renormalization of the masses of the neutrinos. For describing the minimum conditions, we will only need to consider the Higgs potential. In the CP conserving scalar potential, given in Eq. (1.12), we have all bare parameters real:

$$m_{0ij}, \lambda_{0i} \in \mathbb{R}\,. \tag{3.1}$$

We also have one parameter that is not an independent variable and is related to the minimum of the potential: the VEV of the Higgs $v_0 \in \mathbb{R}$. The renormalized tadpole function can be written as:

$$\hat{T} = T + \delta\hat{T}\,, \tag{3.2}$$

where hat stands for the renormalized functions. We get $\delta\hat{T}$ by replacing all the bare parameters in T by the renormalized parameters with the renormalization constants Eq. (2.1) including the arbitrarily introduced parameter v. At tree level, $\delta\hat{T}^{[0]} = 0$, hence the renormalized tree level tadpole functions are:

$$\hat{T}_h^{[0]} = -v\left(m_{11}^2 + \frac{1}{2}\lambda_1 v^2\right), \hat{T}_H^{[0]} = v\left(m_{12}^2 - \frac{1}{2}v^2\lambda_6\right), \ \hat{T}_A^{[0]} = 0\,. \quad (3.3)$$

Eq. (3.3) are the renormalized tadpole functions and the parameters in them are the renormalized parameters. Hence formally it is different from the tadpole functions introduced in Eq. (1.19) as they were the tadpole functions of the bare theory. However, the difference only make sense when going to the loop level. We will require, at every loop level i, the tadpole conditions for the renormalized tadpole functions:

$$\hat{T}^{[i]} = 0\,. \quad (3.4)$$

As we work at one loop level, we will look only at the following equations:

$$\hat{T}^{[0]} = 0\,, \ \left(T^{[1]} + \delta\hat{T}^{[1]}\right)|_{\hat{T}^{[0]}=0} = 0\,. \quad (3.5)$$

In practice, we insert Eq. (1.18) into the Eq. (1.12), do the redefinition of parameters and fields of Eq. (2.1), expand to the first order in the renormalization constants, collect terms near h, H and A and simplify with the tree level tadpole condition $\hat{T}^{[0]} = 0$ to get:

$$\begin{aligned}
\delta\hat{T}_h^{[1]} &= \frac{1}{2}\lambda_1 v^3\left(2\delta_{m11} - \delta_{\lambda 1} - 2\delta_v\right), \\
\delta\hat{T}_H^{[1]} &= \frac{1}{2}\lambda_6 v^3\left(2\delta_{m12} - \delta_{\lambda 6} - 2\delta_v\right), \\
\delta\hat{T}_A^{[1]} &= 0\,.
\end{aligned} \quad (3.6)$$

As v was introduced as a placeholder for the VEV of the field that minimizes the potential, it is not an independent variable on itself and is defined dynamically by the renormalization conditions. That means that one of the renormalization constants δ_{m11}, $\delta_{\lambda 1}$ or δ_v has to be redundant. We take the renormalization of m_{11} and λ_1 fixed together as[1]:

$$2\delta_{m11} - \delta_{\lambda 1} = 0\,. \quad (3.7)$$

Then the counterterm for the Higgs basis field h becomes:

$$\delta\hat{T}_h^{[1]} = -\lambda_1 v^3 \delta_v\,. \quad (3.8)$$

[1]This is motivated by the fact that this choice cancels all the divergences of the unbroken ϕ^4 theory, so that δ_v accounts for all the effects that arise due to a symmetry breaking.

The tadpole condition gives:

$$\delta_v = \frac{1}{\lambda_1 v^3} T_h^{[1]} \,. \tag{3.9}$$

As we chose the Higgs basis, we rotated out one of the VEVs of the fields and got a single value VEV v. This rotation is hidden in the definitions of the fields and we do not see any parameter directly from the potential that corresponds to this hidden VEV. Hence, there is no more redundancy in our formulation of the counterterms for tadpoles. The second tadpole condition gives:

$$\frac{1}{2}\lambda_6 v^3 \left(2\delta_{m12} - \delta_{\lambda 6} - 2\delta_v\right) + T_H^{[1]} = 0 \,. \tag{3.10}$$

Inserting Eq. (3.9) into Eq. (3.10) we get:

$$\left(\delta_{m12} - \frac{1}{2}\delta_{\lambda 6}\right) = \frac{1}{v^3}\left(\frac{1}{\lambda_1} T_h^{[1]} - \frac{1}{\lambda_6} T_H^{[1]}\right) \,. \tag{3.11}$$

In the expansion to the first order in multiplicative renormalization constants, we got that the pseudoscalar Higgs does not have any counterterm, i.e. $\delta \hat{T}_A^{[1]} = 0$. This is due to the CP conservation of the potential. If we set up everything correctly, we must have:

$$T_A^{[1]} = 0 \tag{3.12}$$

in our perturbative expressions. Otherwise, the potential could not be minimised at one loop level, which would be a contradictory result. In other words, this simple equation gives us a nice way to check the correctness of the implementation of the model and we checked that this is indeed the case for the one loop level with our setup. However, the two loop counterterm exists, i.e. $\delta \hat{T}_A^{[2]} \neq 0$, hence it allows for a consistent renormalization of CP odd tadpoles if the explicit violation of the CP symmetry appears at higher loops [72].

In this set up δ_v does not arise from the field renormalization and by absorbing all the tadpole functions, ensures that the v is the proper VEV of the Higgs. As we treated v as a separate parameter that is independent of the field, the field renormalization constants do not enter the expressions for the tadpole condition. All the field renormalization constants are fixed by the CMS conditions. It was shown explicitly in [73, 74] that δ_v coincides with δ_h only in some specific cases and that it is not the case for a general R_ξ gauge.

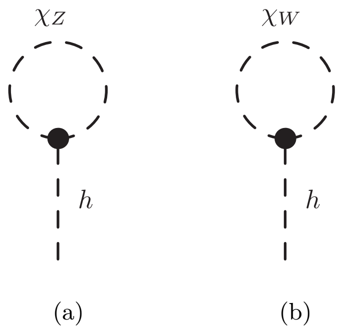


Figure 3.1: Tadpole diagrams that give gauge dependent contributions. h field here is the Higgs basis field, hence these tadpoles are related to the mass eigenstate basis by Eq. (1.35).

One last thing to note about expression Eq. (3.9) is that it is gauge dependent. The gauge dependences appear only in tadpole diagrams with Goldstone bosons, shown in Figure 3.1, as Goldstone boson masses explicitly depend on the gauge choices. This does not necessarily mean that v is gauge dependent. However, one of v_0 or v then definitely is. This is conceptually important in the FJ scheme that we are going to use. In that scheme, v as the VEV of the theory to all loops (the "proper VEV" as it is called in [28]) is expected to be gauge independent, while v_0 is merely a tree level approximated bare value, which might be gauge dependent.

3.2 Renormalization of neutrino masses

For renormalizing the neutrino masses, we will employ the CMS, presented in Section 2.2. First, let us look at the counterterms that we have for neutrinos. The bare mass Lagrangian for neutrinos includes only two mass terms: m_{03} and m_{04}. This means that neither ν_1 nor ν_2 has a mass term at tree level at all, hence no mass counterterms as well. So, if the two point correlation functions for ν_2 or for ν_1 give a non vanishing result, this whole result is a loop generated mass term as in Eq. (2.90). This kind of a mass term would be problematic if it would turn out to be not finite or not gauge independent: we would not be able to define a consistent mass parameter from it, meaning that the whole model is inconsistent. This is not the situation in the GN model, hence getting the finite and gauge invariant radiatively generated mass term serves as a first crosscheck of the implementation of the model. Also, the calculation of the mass term of ν_2 is not so complicated, so it can be done without computer algebra programs and then checked with them.

3.2.1 Radiative mass m_2

We now present the derivation of the expression for the mass of ν_2. As in the presentation of the renormalization of massive fermions in Section 2.2, we had mass parameters renormalized to coincide with the position of the pole. We found that for massless particles, the mass is given by the two point function $\Gamma_{\nu\nu}$ or $\Gamma_{\bar{\nu}\bar{\nu}}$. In that sense, the radiative mass for ν_2 is the renormalized mass, despite the fact that it does not have any renormalization constant and does not need any UV subtractions. At one loop level, Eq. (2.90) gives:

$$m_2 = -\Gamma^{[1]}_{\nu_2\nu_2}|_{p^2=0} = -\Gamma^{[1]}_{\bar{\nu}_2\bar{\nu}_2}|_{p^2=0} \,. \tag{3.13}$$

We first consider diagrams, that possibly can contribute, but give vanishing contributions. Possible contributions are shown in Figure 3.2. They all include propagators of type B.1(c) and B.1(d), for ν_2 and e. However, these bare propagators do not exist, neither for ν_2 nor for e, as they require non-vanishing bare Majorana mass terms, hence, all the diagrams shown in Figure 3.2 vanish. However, note that the Majorana mass terms and hence the Majorana type propagators vanish for ν_2 and e due to different reasons. The difference between ν_2 and e is that for ν_2, we chose a basis in which the bare mass vanishes and can generate the mass term at loop level. e, however, is a charged particle and has a similar type of propagator that is proportional to the Dirac mass term, which connects e with E, but cannot get a Majorana mass at any loop level. The Majorana mass term for e would violate the gauge symmetry. The diagrams that are proportional to a Dirac type of propagator do not appear at all for this mass term, since it would require the neutrino to couple to E, which is not the case in this model.

The only non-vanishing diagrams for m_2 are those that appear from the couplings that are proportional to the parameter d. These diagrams give contributions, proportional to the Majorana mases m_3 and m_4. This can be read out from the Lagrangian part given in Eq. (1.64). The Lagrangian term that gives Feynman rules for these contributions (written

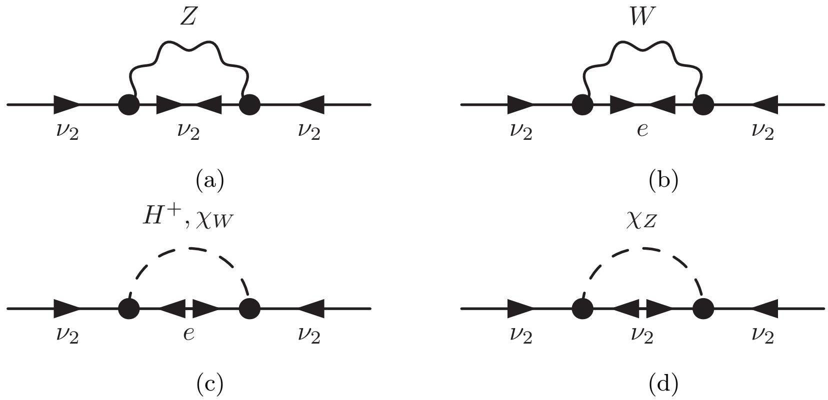


Figure 3.2: Diagrams that give vanishing results and do not contribute to $\Gamma^{[1]}_{\nu_2\nu_2}$. They all vanish, because the propagators for e and ν_2 that are shown in these pictures do not exist, since neither ν_2 nor e has bare Majorana mass.

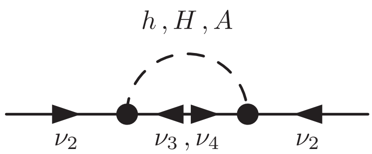


Figure 3.3: Six diagrams that give the radiatively generated mass m_2. h, H and A are in the mass eigenstate basis in the diagrams.

in the Higgs basis) is:

$$-\frac{1}{\sqrt{2}}d\,(H+iA)\,\nu_2\,(-is_{34}\nu_3+c_{34}\nu_4)\ . \tag{3.14}$$

The mass eigenstates of scalar particles are given by Eq. (1.25):

$$H=c_\alpha H_{(m)}+s_\alpha h_{(m)}\,. \tag{3.15}$$

In total there are six diagrams that give the one loop mass for ν_2 and are compactly shown as one diagram in Figure 3.3.

Let us get the loop with A and ν_3 first. The Feynman rule for the vertex is:

$$i\Gamma_{A\nu_2\nu_3}=-i\frac{1}{\sqrt{2}}ds_{34}A\nu_2\nu_3\,. \tag{3.16}$$

The Feynman rules give us the contribution for $i\Gamma^{[1]}_{\nu_2\nu_2}$ (inserting the dimensional regulator μ and setting $p^2 = 0$):

$$\left(-i\frac{1}{\sqrt{2}}ds_{34}\right)^2 (2\pi\mu)^{4-D} \int \frac{d^Dq}{(2\pi)^4} \langle\nu_3\nu_3\rangle \langle AA\rangle$$

$$= -\frac{1}{2}d^2\frac{m_3}{m_3+m_4}(2\pi\mu)^{4-D}\int \frac{d^Dq}{(2\pi)^4}\frac{im_3\cdot i}{\left(q^2-m_3^2\right)\left(q^2-m_A^2\right)} =$$

$$= \frac{1}{2}d^2\frac{i\pi^2 m_3^2}{(2\pi)^4(m_3+m_4)}\left(\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\frac{1}{\left(q^2-m_3^2\right)\left(q^2-m_A^2\right)}\right)$$

$$= id^2\frac{m_3^2}{32\pi^2(m_3+m_4)}B_0\left(0,m_3^2,m_A^2\right) . \tag{3.17}$$

In the first line we directly applied Feynman rules, writing the propagators for ν_3 and A as $\langle\nu_3\nu_3\rangle$ and $\langle AA\rangle$, in the second line we wrote the expressions for propagators and wrote the expression for s_{34}, given in Eq. (1.61), in terms of tree level masses, in the third line we factored out $i\pi^2$ to have the standard B_0 function in the parenthesis and in the last line we wrote the B_0 function as defined in Appendix C. The default multiplier for the FeynArts output is $-i$, in order to cancel the additional i from the expression $i\Gamma$. So the FeynArts output directly gives the n–point functions $\Gamma_{\phi_1 \ldots \phi_n}$. Hence, if the model is implemented correctly into FeynArts, we should get out in the output the same term, Eq. (3.17), without the i factor in front[2]. As all the couplings are the same for $\Gamma^{[1]}_{\bar\nu_2\bar\nu_2}$, the result is also the same as the result for $\Gamma^{[1]}_{\nu_2\nu_2}$.

The final result for the neutrino mass, which we will call m_2, with the CP conserving 2HDM potential is:

$$m_2 = -\Gamma^{[1]}_{\nu_2\nu_2}|_{p^2=0} = -\Gamma^{[1]}_{\bar\nu_2\bar\nu_2}|_{p^2=0} = -\frac{d^2}{32\pi(m_3+m_4)}\times$$
$$\times\left(m_3^2\left[B_0\left(0,m_3^2,m_A^2\right)-c_\alpha^2B_0\left(0,m_3^2,m_H^2\right)-s_\alpha^2B_0\left(0,m_3^2,m_h^2\right)\right]\right.$$
$$\left.-\,m_4^2\left[B_0\left(0,m_4^2,m_A^2\right)-c_\alpha^2B_0\left(0,m_4^2,m_H^2\right)-s_\alpha^2B_0\left(0,m_4^2,m_h^2\right)\right]\right) . \tag{3.18}$$

Note that the evaluation is done at $p^2 = 0$, the zeroth order mass term. m_2 now is the "renormalized" one loop mass. This mass can be used to

[2]Getting the factors right is a pain, but it is an essential and unavoidable step to get everything consistent. Hence we pay a lot of attention to explain where all the πs and is come from.

iterate to the next loop levels. This is the full one loop result without any other approximation and is finite and gauge dependent. It is an easy exercise to check the finiteness of the result. As B_0 functions have the same divergent constant c_∞, one just checks that it cancels out:

$$m_{2\infty} = -\frac{d^2}{32\pi\,(m_3+m_4)}\left(m_3^2\left[1-c_\alpha^2-s_\alpha^2\right]-m_4^2\left[1-c_\alpha^2-s_\alpha^2\right]\right)c_\infty = 0\,. \tag{3.19}$$

It is interesting to note some properties of m_2 that can be seen from the one loop expression Eq. (3.18). First we see that in the alignment limit of the 2HDM, i.e. where $\alpha = 0$, m_2 depends only on A and on H and does not depend on the SM Higgs h. In addition to the alignment limit, if we have degenerate scalar masses, i.e. where $\alpha = 0$ and $m_H = m_A$, m_2 vanishes at one loop.

3.2.2 Renormalization of m_3 and m_4

The masses m_{03} and m_{04}, are not zero at tree level, hence they have non vanishing counterterms that have to be defined in the renormalization procedure. The renormalization conditions given by the CMS, fix the mass counterterms by Eq. (2.36) at one loop. However, a direct calculation (by methods of Chapter 4) shows us that the mass counterterm is gauge dependent. As the CMS is rigorously proven to give gauge invariant mass definitions [31], the gauge dependent counterterms mean that the bare masses m_{03} and m_{04} are, in fact, gauge dependent. To see where these gauge dependencies appear, we compare the renormalization constants in two different bases. Then we will identify the problematic term, which naturally will lead to the FJ scheme.

We first start from the bare seesaw relation Eq. (1.58):

$$M_0 = m_{04} - m_{03} \quad \text{and} \quad y_0^2 v_0^2 = 2 m_{03} m_{04}\,. \tag{3.20}$$

These equations relate the parameters in two bases: the LH sides of Eq. (3.20) has the mass parameters in the basis defined by Eq. (1.51), while we have mass eigenvalues on the RH side. We replace every parameter with renormalized parameter and counterterm as in Eq. (2.1) and require Eq. (3.20) to hold for both, the renormalized and the bare parameters, together. In this way we can compare the renormalization constants in two different bases, with the relationships fixed as:

$$\delta_{m3} + \delta_{m4} = 2\left(\delta_v + \delta_y\right) , \tag{3.21}$$

$$m_4\delta_{m4} - m_3\delta_{m3} = \left(m_4 - m_3\right)\delta_M . \tag{3.22}$$

From the construction of Eq. (3.21) we see that the mass counterterms have a δ_v contribution, which is gauge dependent, as discussed in the end of Section 3.1. It is reasonable to expect that this is the only gauge dependent term, since v enters directly in the gauge fixing functions, but M and y do not. Hence one would expect to cancel gauge dependence just by subtracting the δ_v from the Eq. (3.21). One can do that, if the bare masses in Eq. (3.20) are defined with v instead of v_0, which leads to the FJ scheme.

3.3 Fleischer-Jegerlehner scheme

The conceptual reasoning of using the same VEV for both, renormalized and bare parameters lies on the different nature of the VEV compared to all other parameters: it is defined dynamically by the minimum conditions. The minimum of the all loop level bare theory and the renormalized theory has to be the same. As we define v to be all loop level VEV, and v_0 is merely a substitution for the bare tree level parameter expression, the "proper" VEV, that is responsible for all the masses in the theory, is v and not v_0. This identification allows a consistent subtraction of the gauge dependent tadpoles from mass renormalization constants. That the masses, defined by the proper VEV are gauge invariant then follows from the gauge invariance of the proper VEV. However, it is not an uncommon statement that the VEV is a gauge dependent quantity [73–75]. The roots of this confusion are the definitions of what we call the VEV. That is, the VEV that is defined as e.g.:

$$v = \frac{2 s_W M_W}{g_e} , \tag{3.23}$$

where M_W, g_e and s_W are pole mass, renormalized electromagnetic coupling and sine of Weinberg angle, is certainly gauge independent, because all these three parameters are gauge independent quantities. We call this VEV the "proper VEV", which has to coincide with the VEV that is given by all loops. If we use the R_ξ gauge in the renormalized perturbation theory with v being gauge invariant renormalized VEV as in Eq. (3.23), the v_0 becomes dependent on gauge parameters and not v. To understand the gauge dependence of v_0 consider that fixing the gauge is essentially choosing the direction in which the symmetry breaking is carried out. But loop corrections can rotate your fields out of a chosen direction, hence v_0 and v are not the values of exactly the same basis. As other equivalent directions are parametrized by ξ_Z and ξ_W, v_0 becomes dependent on these parameters as it gets contributions from those directions. The FJ scheme systematically separates this rotation by defining bare masses in terms of the proper renormalized VEV rather than the bare VEV. This can be thought of as choosing the correct basis from the start of the calculations. One can draw an analogy to the parametrization we use to "guess" one loop neutrino mass eigenstate for ν_2 already at tree level by Eq. (1.51). There we rotated the fields, so that ν_1 in that basis does not get loop corrections for a mass term. Analogy with the VEV is that we choose v to define bare masses, so that the loop corrections do not rotate the basis we chose by R_ξ to define the direction of EWSB. The VEV value shift due to the differences in these two bases are then conveniently separated in δ_v, which also enter in bilinear terms and let us separate the gauge dependences there to define gauge invariant bare and renormalized masses.

Considering the FJ scheme, some questions are still not adequately treated in the literature. For instance, an interesting question that we feel is still unclear is the FJ scheme's relationship to the pinch technique (PT) [76–78]. As the PT is used to define gauge invariant Green's functions, it identifies the tadpole contributions in two point Green's functions just as well as the FJ scheme. In fact, in [79], the method of attaching tadpoles to propagators were called "pinch technique inspired", but the resulting expression is the same as the FJ scheme. However, the PT is proven to coincide with the background field gauge with $\xi = 1$ [80], hence refs. [32, 33] point out that the PT cannot be called a gauge invariant prescription as it is just one of the gauge choices, in contrast to the FJ. While the PT is more of a technical tool, the FJ scheme has more of a physical interpretation that relies on the notion of the "proper VEV". Furthermore, the FJ scheme is used only in the context of the two point functions, but the contribution of different tadpole treatments appear also in vertices with scalar particles, which, for the best of our knowledge, have not got any attention in the literature in this context so far.

In the following section, we will present the application of the FJ scheme for the neutrino renormalization constants of the GN model.

3.3.1 Fleischer-Jegerlehner scheme applied

Following the procedure in [28], to define the gauge invariant mass counterterms we define the bare mass with the proper VEV. Thus the bare relation Eq. (3.20) is modified to:

$$M_0 = m'_{04} - m'_{03}\,, \quad y_0^2 v^2 = 2m'_{04}m'_{03}\,. \tag{3.24}$$

and the renormalized relation has the same VEV v:

$$M = m_4 - m_3\,, \quad y^2 v^2 = 2m_4 m_3\,. \tag{3.25}$$

From Eq. (3.24) and Eq. (3.25), we see that the mass counterterms after applying the FJ scheme for the neutrinos become:

$$\delta'_{m3} + \delta'_{m4} = 2\delta_y\,, \tag{3.26}$$

$$m_4\delta'_{m4} - m_3\delta'_{m3} = (m_4 - m_3)\,\delta_M\,, \tag{3.27}$$

where we use primes to define the counterterms of the FJ scheme to differ from the counterterms in the usual construction of Eq. (3.21) and Eq. (3.22). Note that Eq. (3.27) is not different from Eq. (3.22), because the Majorana mass of the neutrino singlet does not have any contribution from the EWSB mechanism. Bare masses in these two different schemes are related by the single VEV shift:

$$m_{0i} = m'_{0i} + \Delta_0\,, \quad \Delta_0 = 2\frac{m'_{04}m'_{03}\delta_v}{m'_{04} + m'_{03}}\,,\ i = 3, 4\,. \tag{3.28}$$

As the seesaw mixing parameters depend on the masses, they are shifted as well:

$$\begin{aligned} s_{034}^2 &\to s_{034}^2 + 2\delta_v c_{034}^2 s_{034}^2 \left(c_{034}^2 - s_{034}^2\right)\,, \\ c_{034}^2 &\to c_{034}^2 - 2\delta_v c_{034}^2 s_{034}^2 \left(c_{034}^2 - s_{034}^2\right)\,. \end{aligned} \tag{3.29}$$

However, these shifts of the mixing parameters become relevant only at higher loops than one, so we can drop them from our one loop expressions. At one loop level, everything is the same as in Eq. (1.64), except that the bare mass term Lagrangian for neutrinos becomes:

$$\begin{aligned}\mathcal{L}_{mass} &= -\frac{1}{2}\left(m'_{03}+\Delta_0\right)\,\nu_{03}\nu_{03} - \frac{1}{2}\left(m'_{04}+\Delta_0\right)\,\nu_{04}\nu_{04} \\ &= -\frac{1}{2}\left(m_3+\Delta\right)\,\nu_3\nu_3 - \frac{1}{2}\left(m_4+\Delta\right)\,\nu_4\nu_4 + c.t. \end{aligned} \tag{3.30}$$

Δ or Δ_0 are the parameters of a loop order, so formally, they are counterterms and should be written in terms of *c.t.*. For clarity, we wrote them here near the mass terms explicitly. The expression for the CMS fixed mass counterterms of Eq. (2.41) at one loop is modified to:

$$\delta'_{mi} = \frac{1}{2}\left(\Sigma^{[1]}_{\nu_i\nu_i} + \Sigma^{[1]}_{\bar{\nu}_i\bar{\nu}_i} + \Sigma^{[1]}_{\nu_i\bar{\nu}_i} + \Sigma^{[1]}_{\bar{\nu}_i\nu_i}\right)\Big|_{p^2=m_i^2} - \frac{\Delta}{m_i}\,, \text{ for } i=3,4\,. \tag{3.31}$$

We explicitly check this expression to be gauge independent for both neutrinos ($i = 3, 4$) in the GN model (see Section 4.3). Since δ'_{mi} are gauge independent, the masses in the CMS are also gauge independent, it follows from Eq. (3.21) and Eq. (3.22) that δ_M and δ_y are gauge independent, just as expected. Furthermore, both of the bare masses are shifted by the same value Δ_0, as shown in Eq. (3.28). This is a direct consequence of the fact that in the GN model, the interactions of neutrinos with the SM like Higgs have only one singular value, which we parametrized by y in Eq. (1.51) (in terms of mass eigenstates it is expressed in Eq. (3.24)). As Δ is essentially a tadpole function from Eq. (3.9) multiplied by couplings to neutrinos, the result of Eq. (3.31) is equivalent to the result, which we would get by including tadpoles attached to the propagators in self energy functions as is used in e.g. [79]. However, our construction has a more direct link to the interpretation of using the proper VEV, and in that sense, is closer to the original proposal of the scheme in [28].

Summary

We presented the renormalization of tadpoles and the gauge invariant renormalization of neutrino masses in the GN model [38]. We outlined the derivation of the one loop expression for the radiative neutrino mass and presented the final result. The expression for the radiative mass is finite and gauge invariant, proving the first statement of the thesis. We showed that the bare masses for neutrinos in the GN model depend on the gauge in the usual CMS scheme. We presented the FJ scheme and then applied it together with the CMS to get gauge invariant bare and renormalized masses for all neutrinos at one loop. We derived the FJ–modified CMS counterterms for neutrinos in the GN model, which we explicitly checked to be gauge invariant by the methods of the following chapter. This is the first time this scheme was applied for neutrino masses in the GN model. Also, to the best of our knowledge, it is the first time the FJ scheme was used together with the CMS.

Chapter 4

Setting up the calculations

4.1 Using SARAH for generating FeynArts model file

SARAH [50, 51] is a general tool for building a model, which automatically calculates mass matrices, vertices and self energies. It was originally created for supersymmetry (SUSY) calculations [50] and was adapted for non-SUSY models afterwards [51]. As using Weyl spinors in SUSY is more natural than using Dirac spinors, the SARAH model file is based on Weyl spinor representations of fermions rather than Dirac spinor representations. This feature is particularly comfortable for us, as we choose to work in Weyl spinors in our formulation.

The model building in SARAH is based on gauge group representations. In the model file, first one defines gauge fields by naming them, writing the gauge group name, naming the coupling and writing if SARAH should expand the sums over charge indices in the output. Then one defines scalars and Weyl spinors according to the gauge group representations. It is done by writing a name, writing how many identical particles that name will carry (family index), writing names for gauge group components and writing charges under all gauge groups in the same order as the gauge groups were defined. All the gauge interactions are automatically calculated by the SARAH, so there is no need to write the Lagrangian parts for that. The only Lagrangian terms that need to be written in are the Yukawa terms and the scalar potential.

The standard SARAH package comes with a variety of model files that are already set up. One of them is the general 2HDM, which we can use and modify it to our convenience. In the out of the box model file, all the general Yukawa terms and the scalar potential of the 2HDM is written already. So taking the main model file for the general 2HDM, we only need to add the singlet neutrino to have all the particle content of the GN model, presented in Section 1.1. We define the singlet neutrino in the model file [38]:

```
FermionFields[[6]] = {n, 1, conj[nR], 0, 1, 1}
```

where the first and the third entry is the name of the field and its component respectively, the second is the number of families and the last three entries are the charges under the gauge groups (singlets under all of them). The singlet in the theory allows for the Yukawa terms for neutrinos, which are analogous to the Yukawa terms for the up–type quarks. Additionally, the singlet neutrino has a Majorana mass term. Hence the neutrino Yukawa terms together with the Majorana mass terms are added:

```
LagYukawan = - ( - Yn1 H1.n.l - Yn2 H2.n.l + 1/2 M n.n )
```

After this inclusion, the SARAH model file has all the general parameters and all the particle content of the GN model. Up to this point, no parametrization is done, hence everything is written in the general basis. We implement the Higgs basis, Eq. (1.18), by writing in the model file the definitions of EWSB. That is, we set one of the VEVs to zero in the definition for VEVs that is in the original 2HDM model file:

```
DEFINITION[EWSB][VEVs]=
{ { H10,{v,1/Sqrt[2]},{sigma1,\[ImaginaryI]/Sqrt[2]},
    {phi1,1/Sqrt[2]} },
  { H20,{0,1/Sqrt[2]},{sigma2,\[ImaginaryI]/Sqrt[2]},
    {phi2,1/Sqrt[2]} } };
```

After the definitions of VEVs, there are definitions for the particle mixings in the “MatterSector”. As we work in the R_ξ gauge and in a CP conserving potential, we delete the mixing between the charged scalars and pseudoscalars. As the neutrinos now can have eigenstates, we introduce the neutrino mixing matrix:

```
{ {vL,conj[nR]}, {VL,Un} }
```

where the VL is the combined mass eigenstate neutrino four-vector and Un is the mixing matrix U^*, where U is shown in Eq. (1.62)[1].

[1]Note that the Un is complex conjugate of U and not U itself, since Eq. (1.63).

We will use this SARAH model file only for generating the FeynArts model file and it works fine for this matter. However, one should be careful when using this model file for other uses and check whether these simplifications do not pose any difficulties. We generate the FeynArts model file from the "Mathematica" interface for SARAH, applying the functions Start["modelfile"], ModelOutput[EWSB], and MakeFeynArts[].

4.2 Using FeynArts + FormCalc with Mathematica

After doing the steps described in Section 4.1, we get the FeynArts [81] model file that is written in the Higgs basis. The Yukawa terms are still general and no particular basis is chosen so far. We choose the flavor basis by setting all the mixing matrices of the charged leptons to the identity matrix, and identify the Yukawa matrix that couples charged leptons with the first Higgs doublet as a diagonal matrix, shown in Eq. (1.38). The neutrino mixing parametrization, Eq. (1.51), is done by the replacement:

$$\sum_{j=1}^{3} U_{ij} Y_j^1 \to (0,\, 0,\, -i\, c_{34} y,\, s_{34} y)_j \ , \tag{4.1}$$

$$\sum_{j=1}^{3} U_{ij} Y_j^2 \to \big(0,\, d,\, -i\, c_{34} d',\, s_{34} d'\big)_j \ , \tag{4.2}$$

The mixing matrix U is parametrized as a combination of orthogonal rotations and phase shifts just as presented in Eq. (1.62). These replacements can be done in the FeynArts model file itself, or later in the calculations. It is easier to implement it during the calculations as replacement rules in Mathematica interface, which also reduces the risk of accidentally introducing mistakes in the model file. However, we do the replacements Eq. (4.1) and Eq. (4.2) in the FeynArts file for the neutrino–neutrino–Higgs vertices as these replacements drastically reduce the time of calculations of neutrino self–energies. We did additional crosschecks, so that the parametrization is consistent with the one presented in the thesis. We leave other Yukawa couplings to be replaced afterwards in the Mathematica notebook interface, when the specific amplitude is considered.

After setting up the FeynArts model file, we use it with the FeynArts package to draw possible Feynman diagrams for the amplitude that we are interested in. We then use FormCalc [82] to calculate Feynman amplitudes in terms of standard Passarino–Veltman functions [83]. Further reduction and simplification of the results is done by creating functions with replacement rules that implement our chosen parametrization and relate the parameters with the tree level relations. As we work at one loop level, all tree level relations, presented in Chapter 1, can be used to simplify one loop expressions. In this way, we automate the algebraic simplifications of the one loop results for two–point and one–point correlation functions that we need for mass and tadpole counterterms.

Formally, when applying the CMS conditions, the mass parameters are complex. However, at one loop level, there is no need to implement complex parameters of the CMS in FeynArts directly. In fact, we use the assumption that they are real, when doing the algebra. One reason why this is consistent with CMS is that the effect of using complex parameters in the loop functions is actually of higher order that one loop. Hence the expressions for complex counterterms are evaluated at real poles at one loop. For consistency, one has to be able to argue the possibility of complex parameters in the implementation. The fact that the algebraic structure is unchanged by using the CMS allows us to do that. As an example, consider the SM–like Higgs contribution to the mass of ν_4. It is proportional to y, as can be read out from Eq. (1.64). We parametrized the Yukawa couplings by Eq. (1.51), so that $y_0 \in \mathbb{R}$, but suppose that the renormalized $y = y_0/\left(1+\delta_y\right)$ is not necessarily real. The contributions of the loop with the SM–like Higgs to the mass of ν_4 in the renormalized theory are:

$$\hat{\Gamma}_{\nu_4\nu_4} \sim y\,,\ \hat{\Gamma}_{\bar{\nu}_4\bar{\nu}_4} \sim y\,. \tag{4.3}$$

If we do not tell FeynArts+FormCalc that y is real, we will get in the output:

$$\hat{\Gamma}_{\nu_4\nu_4} \sim y\,,\ \hat{\Gamma}_{\bar{\nu}_4\bar{\nu}_4} \sim y^\dagger\,, \tag{4.4}$$

which is incorrect in the context of the CMS[2], but can be corrected by the replacement $y^\dagger \to y$. But in Mathematica interface, the same identification of $y^\dagger$ with y is achieved by imposing an assumption that

[2]Recall the discussion in Section 2.1 about the introduced $H.c.^*$

the parameter y is real. Hence imposing assumptions of real parameters in algebraic simplifications that are real only as bare parameters is a correct way to do the formal algebra simplifications with Mathematica. In this sense, the CMS keeps the algebra of the bare theory unchanged. So the assumptions of reality of masses and most of the couplings[3] are implemented in the Mathematica files, where the algebra between loop integrals is done. Note that after the simplifications are done, the parameters and loop functions can be consistently continued to the complex domain without any loss of generality and hence can be used in the CMS conditions. The collection of all the relations and parametrizations used is presented in Appendix D.

4.3 Gauge cancellation in the FJ scheme

When we set up the FeynArts model file and collected the parameter relations, we can check how the FJ scheme works for the neutrino masses in the GN model. That is, we want to see if the expression shown in Eq. (3.31) is gauge parameter independent. Using Eq. (2.41), we can write Eq. (3.31) as:

$$m_i \delta'_{mi} = m_i \delta_{mi} - \Delta\,, \ i = 3, 4\,, \tag{4.5}$$

where we write Eq. (2.41), using Eq. (2.60) and Eq. (2.46):

$$m_i \delta_{mi} = m_i \Sigma_{\bar{\nu}_i \nu_i} \left(m_i^2\right) + \Gamma_{\nu_i \nu_i} \left(m_i^2\right) \tag{4.6}$$

and the definition for Δ from Eq. (3.28) with Eq. (3.9):

$$\Delta = 2 \frac{m_4 m_3}{m_4 + m_3} \frac{1}{\lambda_1 v^3} T_h^{[1]}\,. \tag{4.7}$$

From now on, all the two loop functions are evaluated at $p^2 = m_i^2$, hence we will drop the momentum dependencies from the expressions for convenience.

To denote the gauge dependent term, we will add the gauge parameter ξ in the subscript at the end of the renormalization constants, self energies and tadpole functions; for example:

$$\delta_p \equiv \delta_{p\xi} + \text{gauge independent terms}, \ \delta_{p\xi} = \delta_{p\xi_W} + \delta_{p\xi_Z}\,. \tag{4.8}$$

[3] I.e. all the properties of the parameters of the bare theory are assumed. For convenience, we list them in Appendix D.

Now we can write the gauge dependent terms of Eq. (4.9) as:

$$0 \overset{!}{=} m_i \delta_{mi\xi} - \Delta_\xi \,, \tag{4.9}$$

where we used exclamation mark to note that this equality has to be checked. We explicitly check that Eq. (4.9) holds, using SARAH, FeynArts and FormCalc. Note that m_i is gauge independent as it is the pole mass of the particle by CMS conditions, in contrast to m_{0i}. We now outline how we separate the gauge dependent terms to check Eq. (4.9).

4.3.1 Getting Δ_ξ

To get Δ_ξ, we need to calculate $T_h^{[1]}$ in the Higgs basis. As FeynArts uses the mass eigenstate basis, we need to use the basis transformation, relation Eq. (1.35), and calculate $T_{h(m)}^{[1]}$ and $T_{H(m)}^{[1]}$. They are generated by FeynArts as diagrams of $h_{(m)}$ →nothing and $H_{(m)}$ →nothing. Gauge dependencies can come into the loops only via gauge dependent propagators. Hence we can consider only tadpoles with gauge bosons, ghosts and Goldstone bosons. The gauge dependent part of gauge boson contributions are exactly canceled by the contributions from ghosts, hence, only the Goldstone bosons give gauge dependencies in tadpoles, which are shown in Figure 3.1. This contribution is:

$$T_{h\xi}^{[1]} = \frac{\lambda_1 v}{32\pi^2} \left[A_0 \left(m_Z^2 \xi_Z \right) + 2\, A_0 \left(m_W^2 \xi_W \right) \right] \,. \tag{4.10}$$

Inserting it into Eq. (4.7) we get:

$$\Delta_\xi = \frac{m_4 m_3}{m_4 + m_3} \frac{1}{16\pi^2 v^2} \left[A_0 \left(m_Z^2 \xi_Z \right) + 2\, A_0 \left(m_W^2 \xi_W \right) \right] \,. \tag{4.11}$$

4.3.2 Getting $m_i \delta_{mi\xi}$

The FormCalc output is easy to use in Weyl spinor notation as the spinor products in the result of the amplitude appear in “WeylChains”. By collecting terms near those “WeylChains” we can take separately all four components presented in Eq. (2.22). The structure of the output of FeynArts for the two point function of ν_i going to ν_i at one loop is:

$$\langle \nu_i \nu_i \rangle \, \Gamma_{\nu_i \nu_i}^{[1]} + \langle \bar{\nu}_i \bar{\nu}_i \rangle \, \Gamma_{\bar{\nu}_i \bar{\nu}_i}^{[1]} + \langle \nu_i p \sigma \bar{\nu}_i \rangle \, \Sigma_{\nu_i \bar{\nu}_i}^{[1]} + \langle \bar{\nu}_i p \bar{\sigma} \nu_i \rangle \, \Sigma_{\bar{\nu}_i \nu_i}^{[1]} , \tag{4.12}$$

where the brackets denote the four different “WeylChains”. As the first consistency check, Eq. (2.60) and Eq. (2.46) should be satisfied in the output.

To make algebra simplifications easier and faster, we separated different one loop contributions to self energies according to the bosons that appear in the loop. Those contributions are from the neutral Higgs scalars, the charged scalar Higgs, the neutral Goldstone boson, the charged Goldstone boson, the W boson and the Z boson. We label them as Σ^{H0}, Σ^{H+}, Σ^{χ_Z}, Σ^{χ_W}, Σ^{W} and Σ^{Z}, respectively. The gauge dependence enters into the two point functions from propagators of gauge and Goldstone bosons. Hence Σ^{H0} and Σ^{H+} do not depend on any gauge parameter, as they do not have those propagators. We can also separate the terms that depends on ξ_W from the terms that depends on ξ_Z. That is, the terms that depends on ξ_Z will come from neutral loops Σ^{χ_Z} and Σ^{Z}, while Σ^{χ_W}, Σ^{W} will have ξ_W dependence.

Let us consider $m_3\delta_{m3\xi_W}$ as an example. We write potentially the ξ_W dependent terms of Eq. (4.6):

$$m_3\delta_{m3\xi_W} = m_3\left(\Sigma^{\chi_W}_{\bar{\nu}_3\nu_3} + \Sigma^{W}_{\bar{\nu}_3\nu_3}\right) + \Gamma^{\chi_W}_{\nu_3\nu_3} + \Gamma^{W}_{\nu_3\nu_3}\,. \tag{4.13}$$

We create the amplitude for $\nu_3 \to \nu_3$ in FeynArts, separating the diagrams according to the bosons that appear to the loops. In the FormCalc output, we get the structure of Eq. (4.12), so we take the needed self energies to put into Eq. (4.13) and then simplify them with the replacements and parametrizations of Appendix D. This direct calculation first gives:

$$\Gamma^{W}_{\nu_3\nu_3} = 0\,, \tag{4.14}$$

and the final result is:

$$m_3\delta_{m3\xi_W} = \frac{m_3 m_4}{(m_3+m_4)}\frac{g_e^2}{16\pi^2 m_Z^2 s_{2W}^2} 2\,A_0\left(m_W^2\xi_W\right)\,, \tag{4.15}$$

where $s_{2W} \equiv 2s_W c_W$ is the sine of the double Weinberg angle. Intermediate steps of going from Eq. (4.13) to Eq. (4.15) can be found in the Appendix B of [38]. Doing the analogous procedure for m_4 and ξ_Z dependent terms, we get:

$$\begin{aligned} m_3\delta_{m3\xi} &= m_4\delta_{m4\xi} \\ &= \frac{m_3 m_4}{(m_3+m_4)}\frac{g_e^2}{16\pi^2 m_Z^2 s_{2W}^2}\left[A_0\left(m_Z^2\xi_Z\right) + 2\,A_0\left(m_W^2\xi_W\right)\right]\,. \end{aligned} \tag{4.16}$$

Using the relation Eq. (1.10), we see that this term is exactly the same as Eq. (4.11), which proves that Eq. (4.9) holds at one loop. Thus we checked the validity of the FJ scheme in the GN model and supported all the claims of Section 3.3.1 by an explicit one loop calculation.

Summary

We use SARAH for generating a FeynArts model file. We use Feynarts to generate amplitudes of the process $\nu_i \to \nu_j$. We use FormCalc to express these amplitudes in terms of standard Passarino–Veltmann integrals. In this chapter, we described the technical steps we did, in order to explicitly cancel gauge dependent terms in the FJ+CMS fixed counterterms [38]. We also describe how the CMS–induced complex parameters behave like real parameters in the algebra of loop corrections. This feature allowed us to use the mentioned packages in a default configuration to get the expressions of the CMS–fixed counterterms. This was not described in the literature before.

Conclusions

In this thesis, we present a study of gauge dependence of the renormalization of the one loop neutrino masses of the GN model. To ensure the gauge dependence of the renormalized masses, we choose to use the CMS. In Section 2.3 we show how the radiative masses can be incorporated into the formalism of CMS and give arguments why the result for the radiative mass term should be gauge invariant and finite. This leads to the first statement of the thesis. In Section 3.2.1 we explicitly calculate the radiatively induced mass m_2 for neutrino ν_2 and show that it is indeed gauge invariant and finite.

In renormalizing m_3 and m_4, the CMS conditions alone do not give gauge independent counterterms. We apply the FJ scheme to define gauge independent counterterms. As this was not done in the GN model before, we checked whether the FJ scheme is applicable, i.e. if it cancels gauge dependencies in the neutrino two point functions. In Section 3.3.1, we show that the singlet neutrino does not affect the application of the FJ scheme as its Majorana mass term is not related to the EWSB. Thus the renormalization of the Majorana mass term for the singlet neutrino is gauge invariant by itself. We also show that the gauge dependent term for the renormalization constants of m_3 and m_4 is the same, due to a single value of the coupling to the first Higgs doublet in the Higgs basis. To prove that this construction works, we have explicitly checked the gauge cancellation in the GN model by employing SARAH, FeynArts and FormCalc. This is presented in Chapter 4.

The CMS introduces complex masses and couplings to the renormalized Lagrangian. As the CMS is an analytical continuation of the OS, the algebraic structure of the loop corrections is expected to be the same. If this is the case, the algebraic simplifications with assumptions that the parameters are real, should still give the correct algebraic expressions for counterterms in the CMS. This means that without loss of generality we can introduce the complex parameters after the expressions in consideration are algebraically simplified. In practice, we can take the usual FeynArts model file to calculate the expressions for counterterms in the CMS, with no additional modifications, as the complexity of parameters can be defined later. In Section 2.1, we present how this continuation can be understood on the Lagrangian level, by defining the renormalized hermitian conjugation symbol $H.c.^{*}$. In Section 4.2, we argue how this algebraic structure is maintained when calculating loop corrections in the GN model. As the application of the FJ scheme is an algebraic procedure, the CMS can be used together with the FJ scheme to define both the renormalized and the bare masses gauge invariantly, which we do for the neutrinos of the GN model in Eq. (3.31).

Appendix A

Quantum effective action

Here we present the derivation of the quantum effective action from the classical action $S(\phi)$ to clarify the sign conventions used and introduce the definitions for Green's functions that are used throughout this thesis. The derivations of Green's functions from functional methods can also be found in e.g. [46, 84–86]. However, since much confusion can arise due to differences in sign conventions, we believe that it is important to collect the derivation here, so that the consistency of conventions can be easily verified.

We will use a shorthand notation for the integral and frequently skip the indices that are summed over:

$$\int J\phi \equiv \int J_i\phi_i \equiv \int J_i(x)\,\phi_i(x)\,d^4x \equiv \int \sum_i J_i(x)\,\phi_i(x)\,d^4x\,. \tag{A.1}$$

In the cases when we have many integration measures, we will write indices in the subscript of the integral sign as:

$$\int_{1\ldots n} \equiv \int d^4x_1 \ldots d^4x_n\,. \tag{A.2}$$

We insert sources in the action $S(\phi)$ by:

$$S(\phi, J) = S(\phi) + \int J\phi\,. \tag{A.3}$$

The path integral is:

$$Z \equiv Z(J) = \int D\phi e^{iS(\phi,J)} = Ne^{iS(J)}\,, \tag{A.4}$$

where N is an integration constant. The connected[1] correlation functions then are:

$$\begin{aligned} \langle \phi_1..\phi_n \rangle_c &= \frac{\int D\phi\phi_1..\phi_n e^{iS(\phi,J)}}{\int D\phi e^{iS(\phi,J)}}\Big|_{J=0} = Z^{-1}\frac{\delta^n}{i\delta J_1...i\delta J_n} Z\Big|_{J=0} \\ &= \frac{\delta^n}{i\delta J_1...i\delta J_n}\ln Z\Big|_{J=0} = i\frac{\delta^n}{i\delta J_1...i\delta J_n}\left(-i\ln Z\right)\Big|_{J=0} \\ &= i\frac{\delta^n}{i\delta J_1...i\delta J_n} W\Big|_{J=0}, \end{aligned} \tag{A.5}$$

where we have defined the generating funtional:

$$W \equiv W(J) \equiv -i\ln Z(J) \text{ or } Z = e^{iW} \tag{A.6}$$

The important correlation function is the propagator, which is the 2 point correlation function $\langle \phi\phi \rangle_c$:

$$D_{\phi\phi} \equiv \langle \phi\phi \rangle_c = -i\frac{\delta^2}{\delta J\delta J} W\Big|_{J=0}. \tag{A.7}$$

The classical field, which is dependent on J, can be written as:

$$\langle \phi \rangle_J = \frac{\delta W}{\delta J}. \tag{A.8}$$

One can write:

$$W_n = -i\int_{1...n} \frac{iJ_1...iJ_n}{n!}\langle \phi_1..\phi_n \rangle_c, \tag{A.9}$$

so that:

$$W = \sum_{n=1}^{\infty} W_n = -i\sum_{n=1}^{\infty}\frac{i^n}{n!}\int_{1...n} J_1...J_n \langle \phi_1..\phi_n \rangle_c. \tag{A.10}$$

The Legendre transform of W is an effective action (or the effective vertex functional):

$$\Gamma\left(\langle \phi \rangle_J\right) = W - \int \langle \phi \rangle_J J. \tag{A.11}$$

From now on, we will use the classical fields $\langle \phi \rangle_J$ as variables, so let us drop the brackets and the source index as:

$$\langle \phi \rangle_J \to \phi. \tag{A.12}$$

[1] They are called connected because the Feynman diagrams describing them are connected.

Then writing the variation of the effective action with respect to the classical field gives:

$$\begin{aligned}\frac{\delta}{\delta\phi}\Gamma &= \frac{\delta}{\delta\phi}W - \frac{\delta}{\delta\phi}\int\phi J = \frac{\delta}{\delta\phi}W - J - \int\phi\frac{\delta}{\delta\phi}J \\ &= \int\frac{\delta J}{\delta\phi}\frac{\delta}{\delta J}W - \int\phi\frac{\delta}{\delta\phi}J - J = \int\frac{\delta J}{\delta\phi}\phi - \int\phi\frac{\delta}{\delta\phi}J - J \\ &= -J\,. \end{aligned} \tag{A.13}$$

From here we can relate propagator with its inverse (we indicate space-time variable in the subscript):

$$\begin{aligned}\delta_{xy} = \frac{\delta J_x}{\delta J_y} &= -\frac{\delta}{\delta J_y}\frac{\delta}{\delta\phi_x}\Gamma = -\frac{\delta\phi_z}{\delta J_y}\frac{\delta}{\delta\phi_z}\frac{\delta}{\delta\phi_x}\Gamma \\ &= -\frac{\delta}{\delta J_y}\frac{\delta W}{\delta J_z}\Gamma_{\phi\phi} = -iD_{\phi\phi}\Gamma_{\phi\phi}\end{aligned}$$

It follows that the propagator is:

$$D_{\phi\phi} = i\Gamma^{-1}_{\phi\phi}\,. \tag{A.14}$$

The effective vertex functional can be ordered by loop expansion:

$$\Gamma = \sum_{\text{loop}=0}^{\infty}\Gamma^{[\text{loop}]}\,. \tag{A.15}$$

Also, as Γ is a polynomial in ϕ, it can be expanded as:

$$\Gamma = \sum_{n=2}^{\infty}\frac{1}{n!}\int\phi_1...\phi_n\frac{\delta^n}{\delta\phi_1...\delta\phi_n}\Gamma\,. \tag{A.16}$$

The expansion coefficients then are the 1 particle irreducible functions with an i prefactor and Γ is the generating functional for 1PI vertices:

$$\langle\phi_1...\phi_n\rangle_{1PI} = i\frac{\delta^n\Gamma}{\delta\phi_1..\delta\phi_n}\Big|_{\phi_i=0} \equiv i\Gamma_{\phi_1...\phi_n}\,. \tag{A.17}$$

The Feynman diagrams, that represent the functions in Eq. (A.17), cannot be reduced into a two different disconnected diagrams by cutting a single line, hence they are called 1 particle irreducible functions. Each individual coefficient can be ordered by loops just as well as Eq. (A.15). The tree level effective action is just a classical action ($\Gamma^{[0]}(\phi) = S(\phi)$).

For example, the two point function of a scalar at tree level is (in the momentum space):

$$\Gamma^{[0]}_{\phi\phi} = p^2 - m^2_\phi\,, \tag{A.18}$$

which corresponds to the Lagrangian term $\mathcal{L}_{mass} = \frac{1}{2}\left(\partial_\mu\phi\right)^2 - \frac{1}{2}m^2_\phi\phi^2$. It is, of course, zero on-shell. The definitions of these Green's functions comply with the default definitions in FeynArts. That is, by default the FeynArts amplitude is multiplied by $-i$ to cancel the i factor of the RHS of Eq. (A.17), so that the calculated output would be $\Gamma_{\phi_{y1}\ldots\phi_{yn}}$. These are the same sign conventions used in standard textbooks e.g. [46, 86], in which the action can be identified with as negative energy function $S = -E$. We write the one particle VEV as the special case of Eq. (A.17):

$$T_\phi \equiv \Gamma_\phi\,, \tag{A.19}$$

which is called the tadpole function.

The renormalized Green's functions are given in terms of renormalized fields and parameters and they are:

$$\begin{aligned} \langle\phi_1\ldots\phi_n\rangle^{[loop]}_{1PI} &= i\frac{\delta^n\hat{\Gamma}^{[loop]}}{\delta\phi_1..\delta\phi_n}\Big|_{\phi_i=0} \equiv i\hat{\Gamma}^{[loop]}_{\phi_1\ldots\phi_n} \\ &\equiv i\Gamma^{[loop]}_{\phi_1\ldots\phi_n} + i\delta\hat{\Gamma}^{[loop]}_{\phi_1\ldots\phi_n}\,, \end{aligned} \tag{A.20}$$

where $\delta\Gamma^{[loop]}$ stands for the counterterm part of the renormalized effective action. Similarily,

$$\hat{T}^{[loop]}_\phi = T^{[loop]}_\phi + \delta\hat{T}^{[loop]}_\phi = \hat{\Gamma}^{[loop]}_\phi\,. \tag{A.21}$$

As spinors are anticommuting, the definition Eq. (A.17) might be confusing as the ordering of spinors is important. Since in this thesis we are interested in two point functions only, it is convenient to define:

$$\begin{aligned} \Gamma_{\xi\chi} &= \frac{\delta}{\delta\xi}\Gamma\frac{\overleftarrow{\delta}}{\delta\chi}|_{\xi,\chi=0}\,, \qquad & \Gamma_{\xi^\dagger\chi} &= \frac{\delta}{\delta\xi^\dagger}\Gamma\frac{\overleftarrow{\delta}}{\delta\chi}|_{\xi,\chi=0}\,, \\ \Gamma_{\xi\chi^\dagger} &= \frac{\delta}{\delta\xi}\Gamma\frac{\overleftarrow{\delta}}{\delta\chi^\dagger}|_{\xi,\chi=0}\,, \qquad & \Gamma_{\xi^\dagger\chi^\dagger} &= \frac{\delta}{\delta\xi^\dagger}\Gamma\frac{\overleftarrow{\delta}}{\delta\chi^\dagger}|_{\xi,\chi=0}\,, \end{aligned} \tag{A.22}$$

where ξ and χ are Weyl spinors. These definitions are easy to use, as the derivatives are visually in the same order. Other positions of derivatives give either a minus sign, or has opposite order than fields. For example:

$$\Gamma_{\xi\chi^\dagger} = \frac{\delta}{\delta\xi}\Gamma\frac{\overleftarrow{\delta}}{\delta\chi^\dagger}|_{\xi,\chi=0} = \frac{\delta}{\delta\chi^\dagger}\frac{\delta}{\delta\xi}\Gamma|_{\xi,\chi=0} = -\frac{\delta}{\delta\xi}\frac{\delta}{\delta\chi^\dagger}\Gamma|_{\xi,\chi=0}\,. \tag{A.23}$$

So it makes sense to use Eq. (A.22) in order to avoid confusion with orderings or additional minus signs. The easiest way to check the relations Eq. (A.23) and the Eq. (A.22) is to make sure it does not introduce any unwanted signs to the tree level free field action. This can be done using the definitions and relations, presented in Appendix B.

Appendix B

Weyl spinor notation

Here we collect the most important conventions and relations concerning Weyl spinor usage that are most extensively presented in [65]. We use refs. [36, 47, 48, 65] for this overview of Weyl spinors. We first recall that Weyl spinors transform in a fundamental representation of $SU(2)$. As is shown in many standard textbooks (e.g. [47, 48]), the Lorentz group can be decomposed into the direct product $SU(2)_L \times SU(2)_R$, were the $SU(2)_L$ is hermitian conjugate to $SU(2)_R$ by construction. To label the representation under these groups, we use the same notation as in 1, i.e. we will write them as $\left(D\left(R_{SU(2)_L}\right), D\left(R_{SU(2)_R}\right)\right)$, where D is a dimension of the representation R. Then Weyl spinors, are called left–handed if they are in the representation $(2,1)$ [1] and right–handed in the representation $(1,2)$ under the Lortentz group. We write undotted indices to denote the components of the LH Weyl spinor and we write dotted indices to denote the components of the RH spinor. Since the RH spinor transforms under the group that is the hermitian conjugate of the $SU(2)_L$ group, we have, for a LH spinor χ:

$$(\chi_a)^\dagger = \chi_{\dot{a}}\,,\ \ \chi : (2,1)\ ,\ \ \chi^\dagger : (1,2)\ . \tag{B.1}$$

Group theoretical relation for two dimensional representations of $SU(2)$:

$$2 \otimes 2 = 1_A \oplus 3_S\,, \tag{B.2}$$

[1] Here we use the notation from i.e. [47, 48], in which the dimension of the representation $D(R)$ is used, rather than spin value of the representation as in [65]. I.e. 2 dimensional representation of $SU(2)_L$ in [65] is written as $(\frac{1}{2}, 0)$, since $D = 2s+1$, but it means the same thing as $(2,1)$ in notation we use.

where the subscript A stands for antisymmetric combination and S for symmetric, says that the natural metric in Weyl spinor space is antisymmetric. Define [65]:

$$\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1\,,\ \left(\epsilon^{ab}\right)^{*} = \epsilon^{\dot{a}\dot{b}}\,,\ \left(\epsilon_{ab}\right)^{*} = \epsilon_{\dot{a}\dot{b}}\,. \tag{B.3}$$

Having this symbol, the singlet out of 2 Weyl spinors χ and ξ is given by:

$$\epsilon^{ab}\chi_b\xi_a\,,\ \left(\epsilon^{ab}\chi_b\xi_a\right)^{\dagger} = \epsilon^{\dot{a}\dot{b}}\xi^{\dagger}_{\dot{a}}\chi^{\dagger}_{\dot{b}}\,. \tag{B.4}$$

Defining raising and lowering the indices:

$$\chi^a = \epsilon^{ab}\chi_b\,,\ \chi^{\dagger\dot{a}} = \epsilon^{ab}\chi^{\dagger}_{\dot{b}}\,,\ \chi_a = \epsilon_{ab}\chi^b\,,\ \chi^{\dagger}_{\dot{a}} = \epsilon_{\dot{a}\dot{b}}\chi^{\dagger\dot{b}} \tag{B.5}$$

one can introduce the summation convention for spinors to have an index free notation:

$$\begin{aligned} \xi\chi &= \chi\xi = \chi^a\xi_a = \epsilon^{ab}\chi_b\xi_a\,, \\ \xi^{\dagger}\chi^{\dagger} &= \chi^{\dagger}\xi^{\dagger} = \xi_{\dot{a}}\chi^{\dot{a}} = \epsilon^{\dot{a}\dot{b}}\xi^{\dagger}_{\dot{a}}\chi^{\dagger}_{\dot{b}}\,, \end{aligned} \tag{B.6}$$

so that the LH spinors are summed when indices go down from left to right, and the RH spinors are summed when indices go up from left to right.

The usual Lorentz four–vector transform under the $SO\,(3,1)$ group as a four dimensional object. Under the product of $SU\,(2)_L \times SU\,(2)_R$, the Lorentz four vector is in the $(2,2)$ representation. Since for Lorentz vectors it is far more natural to work with the four dimensional $SO\,(3,1)$ representation, a map, relating $R_{SU(2)_L\times SU(2)_R} \leftrightarrow R_{SO(3,1)}$ is needed. Since now the spinors are defined with indices raised or lowered, the connections that takes two spinors into a vector now are of two types: the one which takes two spinors with indices down and makes a vector and the one that takes two spinors with indices up and makes a vector. Naturally, they are related by the "spinorial metric":

$$\bar{\sigma}^{\mu\dot{a}a} = \epsilon^{\dot{a}\dot{b}}\epsilon^{ab}\sigma^{\mu}_{b\dot{b}}\,. \tag{B.7}$$

Two spinors compose into a vector object as:

$$\xi^{\dagger}_{\dot{a}}\bar{\sigma}^{\mu\dot{a}a}\chi_a = -\chi^a\sigma^{\mu}_{a\dot{a}}\xi^{\dagger\dot{a}}\,, \tag{B.8}$$

where:

$$\sigma^{\mu} = \left(1,\sigma^i\right)\,,\ \bar{\sigma}^{\mu} = \left(1,-\sigma^i\right)\,, \tag{B.9}$$

where σ^i are the Pauli matrices. Note that the summation convention is always kept, so we can go into the index free notation:

$$\xi^\dagger \bar{\sigma}^\mu \chi = -\chi \sigma^\mu \xi^\dagger \,. \tag{B.10}$$

The other useful relation is:

$$i\partial_\mu \left(\chi^\dagger \bar{\sigma}^\mu \chi\right) = i\partial_\mu \chi^\dagger \bar{\sigma}^\mu \chi + i\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi = -i\chi \sigma^\mu \partial_\mu \chi^\dagger + i\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi \,. \tag{B.11}$$

In the action, the total derivative can be ignored as it is just a boundary term, hence in the momentum space, this identity becomes:

$$\chi^\dagger \bar{\sigma}^\mu p_\mu \chi = \chi \sigma^\mu p_\mu \chi^\dagger \,, \tag{B.12}$$

which is the usual kinetic term in the Lagrangian. The Majorana mass term in the Lagrangian is:

$$-\frac{1}{2} m \chi\chi + H.c. \tag{B.13}$$

The kinetic term together with the Majorana mass term give four types of the propagators that are shown in Figure B.1. The momentum in the diagrams is fixed to be from left to right in the diagrams, hence the rules for propagators Figure B.1(a) and Figure B.1(b) reflect the equality Eq. (B.12). I.e. Figure B.1(a) is different from Figure B.1(b) only in the assignment of momentum for spinors.

The Dirac fermion consists of two Weyl spinors, let us call them e and E. The Dirac fermion does not have a mass term, which is shown in Eq. (2.3), instead, it has a mass term in the form:

$$-meE + H.c. \tag{B.14}$$

which is called a Dirac mass term. Both e and E have the propagators shown in Figure B.1(a) and Figure B.1(b). The mass–like propagator has the same expression as in the Majorana case but, now it connects two different Weyl spinors e and E. We indicated the spinors explicitly in the diagrams shown in Figure B.2.

The interaction term with the gauge boson A_μ[2], is written as:

$$g\, \xi^\dagger \bar{\sigma}^\mu A_\mu \chi = -g\, \chi \sigma^\mu A_\mu \xi^\dagger \,. \tag{B.15}$$

[2] A_μ in this section stands for a generic gauge boson, not necessarily a photon.

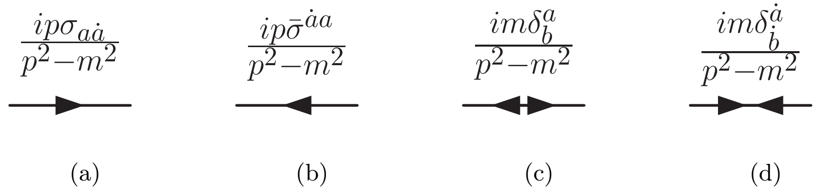


Figure B.1: Four types of propagators for a single Weyl fermion with a Majorana mass m. The expression for every propagator is written above the diagram. The direction of momentum is understood to be assigned from left to right. The arrow shows the direction from the spinor with dotted index, to the spinor with undotted index.

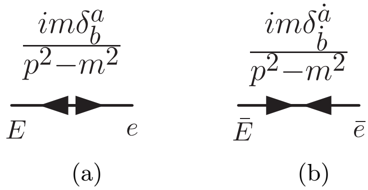


Figure B.2: Two mass–like propagators for Weyl spinors e and E that composes into a Dirac spinor. These propagators are different from those shown Figure B.1(c) and Figure B.1(d), since they connect two different Weyl spinors and are proportional to a Dirac mass term. The propagators of the type Figure B.1(a) and Figure B.1(b) are the same for e and E with m being their common Dirac mass term.

The interaction terms with some real scalar S:

$$\frac{1}{2}yS\chi\xi + \frac{1}{2}y^\dagger S\chi^\dagger\xi^\dagger\,. \tag{B.16}$$

The corresponding Feynman rules for the interaction terms of Eq. (B.15) and Eq. (B.16) are presented in Figure B.3.

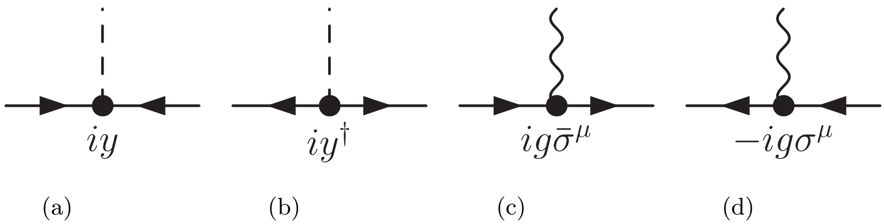


Figure B.3: The Feynman rules for interactions of Weyl spinors with Scalar, and Vector particle. They correspond to the Lagrangian terms Eq. (B.16) and Eq. (B.15). The momentum flow is understood as going from left to right. Note the similarity with propagators, shown in Figure B.1.

Appendix C

Standard PaVe functions

We present the standard loop integrals, which we encounter when calculating radiative corrections for masses. These standard one loop functions were first introduced in [83]. We will follow [69]. As in our calculations we encounter only one point and two point functions, we will present only those. For simpler expressions, the $i\epsilon$ term, that is responsible for time ordering in the propagators will be absorbed into the masses, i.e.

$$m^2 \to m^2 - i\epsilon\,. \tag{C.1}$$

The one point scalar function is defined as:

$$A_0\left(m_0^2\right) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^D q \frac{1}{q^2 - m_0^2}\,. \tag{C.2}$$

The two point scalar functions are:

$$B_0\left(p_1^2, m_0^2, m_1^2\right) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^D q \frac{1}{\left(q^2 - m_0^2\right)\left(\left(q+p_1\right)^2 - m_1^2\right)}\,, \tag{C.3}$$

$$B_\mu\left(p_1^2, m_0^2, m_1^2\right) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^D q \frac{q_\mu}{\left(q^2 - m_0^2\right)\left(\left(q+p_1\right)^2 - m_1^2\right)}\,. \tag{C.4}$$

B_μ can be decomposed by Lorentz decomposition into the scalar function and momentum four vector:

$$B_\mu = p_\mu B_1\,. \tag{C.5}$$

B_1 can be expressed as:

$$B_1\left(p_1^2, m_0^2, m_1^2\right) = \frac{1}{2p_1^2}\left[A_0\left(m_0^2\right) - A_0\left(m_1^2\right)\right] + \frac{1}{2p_1^2}\left(-p_1^2 + m_1^2 - m_0^2\right) B_0\left(p_1^2, m_0^2, m_1^2\right) .$$

When getting functions with FeynArts, the default overall factor for the amplitude is $\frac{-i}{(2\pi)^{D\times(\text{loop number})}}$ and default setting is $D = 4$[81]. The $-i$ cancels the i coming from the action $i\Gamma$.

Note about the measure

The measure near one loop integral in D dimensions is $\int \frac{d^D q}{(2\pi)^D}$. The problem with the integral, that it is no longer of a fixed dimensions, hence the couplings become with variate dimensions as well. To fix the mass dimension of the integral we use the regulator μ of mass dimension in the following manner:

$$\mu^{4-D} \int \frac{d^D q}{(2\pi)^D} = (2\pi\mu)^{4-D} \int \frac{d^D q}{(2\pi)^4} \tag{C.6}$$

Now we see that this measure is of fixed dimension four. Also, for every loop, there comes the additional factor of $(2\pi)^{-4}$ which is consistent with the $\frac{-i}{(2\pi)^{D\times(\text{loop number})}}$ factor of FeynArts with $D = 4$.

Appendix D

Parametrizations

For convenience, we list all the relations and parametrization used in the simplifying the Feynman amplitude. This list is also shown in the appendix of[38].

D.1 Scalar sector and the SM relations

The assumptions of the CP conservation:

$$m_{0ij}^2, \lambda_{0k} \in \mathbb{R}\,;\; i,j = 1,2\,,\, k = 1,...,7. \tag{D.1}$$

The minimum conditions:

$$m_{11}^2 = -\frac{1}{2}\lambda_1 v^2 \quad \text{and} \quad m_{12}^2 = \frac{1}{2}\lambda_6 v^2\,. \tag{D.2}$$

The Higgs basis:

$$H_1 = \left(\begin{array}{c} \chi_{0W}^+ \\ \frac{1}{\sqrt{2}}\left(v + h + i\chi_Z\right) \end{array} \right),\; H_2 = \left(\begin{array}{c} H_0^+ \\ \frac{1}{\sqrt{2}}\left(H + iA\right) \end{array} \right). \tag{D.3}$$

The mixing matrix for scalars is only between h and H:

$$O^\phi = \left(\begin{array}{cc} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{array} \right),\; \phi_i^{mass} = O_{ij}^\phi \phi_j^{Higgs}\,,$$
$$\phi_i^{Higgs} = (h, H)_i\,, \tag{D.4}$$

where s_α, c_α are sine and cosine functions of the mixing angle α. The relations of the Electroweak sector are:

$$s_{2W} \equiv 2 s_W c_W\,,\; m_Z = \frac{g_e v}{s_{2W}}\,, m_W = m_Z c_W\,, \tag{D.5}$$

where s_W and $c_W$are sine and cosine functions of Weinberg angle.

D.2 Yukawa sector

The first thing that we do after generating FeynArts model file is making the replacements Eq. (4.1) and Eq. (4.2) in it:

$$\sum_{j=1}^{3} U_{ij} Y_j^1 \to (0,\, 0,\, -i\, c_{34} y,\, s_{34} y)_j \ ,$$
$$\sum_{j=1}^{3} U_{ij} Y_j^2 \to \big(0,\, d,\, -i\, c_{34} d',\, s_{34} d'\big)_j \ . \tag{D.6}$$

The parametrization of Yukawa couplings:

$$\begin{aligned} &V_{1j} Y_j^1 = 0\,, \quad V_{2j} Y_j^1 = 0\,, \quad V_{3j} Y_j^1 = y_0\,, \\ &V_{1j} Y_j^2 = 0\,, \quad V_{2j} Y_j^2 = d_0\,, \quad V_{2j} Y_j^2 = d_0'\,, \\ &d_0, y_0 \in \mathbb{R}\,, \quad d_0' \in \mathbb{C}\,, \end{aligned} \tag{D.7}$$

where the neutrino mixing matrix is:

$$U = U^{34} V = U^{34} U^{\beta} O^{12} U^{\alpha} O^{13} O^{23} \tag{D.8}$$

with relations

$$\nu_{0i}^F = (\nu_{0e}, \nu_{0\mu}, \nu_{0\tau}, N_0)_i \ , \ \nu_{0i}^{mass} = U_{ij}^* \nu_{0j}^F\,. \tag{D.9}$$

The parametrization of mixing matrix is:

$$\begin{aligned} &s_{0ij}^2 + c_{0ij}^2 = 1\,,\ s_{0ij}, c_{0ij}, \sigma_0, \rho_0 \in \mathbb{R}\,; \\ &O_{ij}^{AB} = 1_{ij} \text{ for } i,j \neq A,B\,; \\ &O_{AB}^{AB} = -O_{BA}^{AB} = s_{0AB}\,;\ O_{AA}^{AB} = O_{BB}^{AB} = c_{0AB}\,; \\ &U_{ij}^{\sigma} = e^{i\sigma_0} \text{ for } i=j=1;\ U_{ij}^{\sigma} = 1_{ij} \text{ for } i,j \neq 1\,; \\ &U_{ij}^{\rho} = e^{i\rho_0} \text{ for } i=j=2;\ U_{ij}^{\rho} = 1_{ij} \text{ for } i,j \neq 2\,; \\ &U_{34}^{34} = i \cdot U_{43}^{34} = i \cdot s_{034};\ U_{33}^{34} = -i \cdot U_{44}^{34} = -i \cdot c_{034}; \\ &U_{ij}^{34} = 1_{ij} \text{ for } i,j \neq 3,4\,. \end{aligned} \tag{D.10}$$

The seesaw is realized with:

$$M_0 = m_{04} - m_{03}\,, \qquad y_0^2 v_0^2 = 2 m_{03} m_{04}\,, \tag{D.11}$$

$$s_{034}^2 = \frac{m_{03}}{m_{04} + m_{03}} \quad \text{and} \quad c_{034}^2 = \frac{m_{04}}{m_{04} + m_{03}}\,. \tag{D.12}$$

Bibliography

[1] C. Athanassopoulos et al. Phys. Rev. Lett. **81**, 1774–1777, 1998.

[2] Y. Fukuda et al. Phys. Rev. Lett. **81**, 1562–1567, 1998.

[3] K. Eguchi et al. Phys. Rev. Lett. **90**, 021802, 2003.

[4] Q. R. Ahmad et al. Phys. Rev. Lett. **89**, 011301, 2002.

[5] P. Minkowski. Phys. Lett. B **67**, 421–428, 1977.

[6] T. Yanagida. In *Proceedings: Workshop on the Unified Theories and the Baryon Number in the Universe, Tsukuba, Japan, 13-14 Feb 1979*, volume C7902131, pages 95–99 (1979).

[7] R. N. Mohapatra and G. Senjanovic. Phys. Rev. Lett. **44**, 912, 1980.

[8] R. N. Mohapatra and G. Senjanovic. Phys. Rev. D **23**, 165, 1981.

[9] S. F. King. Rept. Prog. Phys. **67**, 107–158, 2004.

[10] G. Senjanovic. Riv. Nuovo Cim. **34**, 1–68, 2011.

[11] A. de Gouvêa. Ann. Rev. Nucl. Part. Sci. **66**, 197–217, 2016.

[12] J. Orloff, S. Lavignac, and M. Cribier, editors. *Seesaw mechanism. Proceedings, International Conference, SEESAW25, Paris, France, June 10-11, 2004* (2005), 2005.

[13] S. Chatrchyan et al. Phys. Lett. B **716**, 30–61, 2012.

[14] G. Aad et al. Phys. Lett. B **716**, 1–29, 2012.

[15] H. E. Haber and G. L. Kane. Phys. Rept. **117**, 75–263, 1985.

[16] J. E. Kim. Phys. Rept. **150**, 1–177, 1987.

[17] T. D. Lee. Phys. Rev. D **8**, 1226–1239, 1973.

[18] R. D. Peccei and H. R. Quinn. Phys. Rev. Lett. **38**, 1440–1443, 1977.

[19] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva. Phys. Rept. **516**, 1–102, 2012.

[20] W. Grimus and H. Neufeld. Nucl. Phys. B **325**, 18–32, 1989.

[21] M. Tanabashi et al. Phys. Rev. D **98**, 030001, 2018.

[22] D. Aristizabal Sierra and C. E. Yaguna. JHEP **08**, 013, 2011.

[23] W. Grimus and L. Lavoura. Phys. Lett. B **546**, 86–95, 2002.

[24] S. L. Adler. Phys. Rev. **177**, 2426–2438, 1969.

[25] G. 't Hooft. Nucl. Phys. B **33**, 173–199, 1971.

[26] G. 't Hooft. Nucl. Phys. B **35**, 167–188, 1971.

[27] A. A. Slavnov. Theor. Math. Phys. **10**, 99–107, 1972.

[28] J. Fleischer and F. Jegerlehner. Phys. Rev. D **23**, 2001–2026, 1981.

[29] W. Grimus and M. Löschner. JHEP **11**, 087, 2018.

[30] A. Sirlin. Phys. Rev. Lett. **67**, 2127–2130, 1991.

[31] P. Gambino and P. A. Grassi. Phys. Rev. D **62**, 076002, 2000.

[32] A. Denner, L. Jenniches, J.-N. Lang, and C. Sturm. JHEP **09**, 115, 2016.

[33] A. Denner, S. Dittmaier, and J.-N. Lang. JHEP **2018**, 104, 2018.

[34] M. Krause. *On the Renormalization of the Two-Higgs-Doublet Model*. Master's thesis, KIT, Karlsruhe, TP, 2016.

[35] M. Krause, R. Lorenz, M. Muhlleitner, R. Santos, and H. Ziesche. JHEP **09**, 143, 2016.

[36] V. Dūdėnas and T. Gajdosik. Lith. J. Phys. **56**, 149–163, 2016.

[37] V. Dūdėnas and T. Gajdosik. Acta Phys. Pol. B **48**, 2243–2249, 2017.

[38] V. Dūdėnas and T. Gajdosik. Phys. Rev. D **98**, 035034, 2018.

[39] V. Dūdėnas, T. Gajdosik, A. Juodagalvis, and D. Jurčiukonis. Acta Phys. Pol. B **48**, 2235, 2017.

[40] H. E. Haber and D. O'Neil. Phys. Rev. D **74**, 015018. [Erratum: Phys. Rev.D74,no.5,059905(2006)], 2006.

[41] H. E. Haber and D. O'Neil. Phys. Rev. D **83**, 055017, 2011.

[42] S. Davidson and H. E. Haber. Phys. Rev. D **72**, 035004. [Erratum: Phys. Rev.D72,099902(2005)], 2005.

[43] L. Altenkamp, S. Dittmaier, and H. Rzehak. JHEP **09**, 134, 2017.

[44] B. Grzadkowski, H. E. Haber, O. M. Ogreid, and P. Osland , 2018.

[45] M. P. Bento, H. E. Haber, J. C. Romão, and J. P. Silva. JHEP **11**, 095, 2017.

[46] M. E. Peskin and D. V. Schroeder. *An Introduction to quantum field theory* (Reading, USA: Addison-Wesley, 1995).

[47] M. Srednicki. *Quantum field theory* (Cambridge University Press, 2007).

[48] M. Robinson. *Symmetry and the standard model: Mathematics and particle physics* (Springer, New York, USA, 2011).

[49] M. D. Schwartz. *Quantum Field Theory and the Standard Model* (Cambridge University Press, 2014).

[50] F. Staub. ArXiv:0806.0538, 2008.

[51] F. Staub. Comput. Phys. Commun. **185**, 1773–1790, 2014.

[52] J. C. Romao and J. P. Silva. Int. J. Mod. Phys. A **27**, 1230025, 2012.

[53] H. Georgi. Front. Phys. **54**, 1–320, 1999.

[54] A. Denner and J.-N. Lang. Eur. Phys. J. C **75**, 377, 2015.

[55] B. A. Kniehl and A. Pilaftsis. Nucl. Phys. B **474**, 286–308, 1996.

[56] B. A. Kniehl and A. Sirlin. Phys. Rev. D **77**, 116012, 2008.

[57] B. A. Kniehl. Phys. Rev. Lett. **112**, 071603, 2014.

[58] B. A. Kniehl. Phys. Rev. D **89**, 116010, 2014.

[59] B. A. Kniehl and A. Sirlin. Phys. Rev. D **85**, 036007, 2012.

[60] B. A. Kniehl and A. Sirlin. Phys. Rev. Lett. **81**, 1373–1376, 1998.

[61] B. A. Kniehl and A. Sirlin. Phys. Lett. B **440**, 136–140, 1998.

[62] B. A. Kniehl and A. Sirlin. Phys. Lett. B **530**, 129–132, 2002.

[63] D. Espriu, J. Manzano, and P. Talavera. Phys. Rev. D **66**, 076002, 2002.

[64] N. K. Nielsen. Nucl. Phys. B **101**, 173–188, 1975.

[65] H. K. Dreiner, H. E. Haber, and S. P. Martin. Phys.Rept. **494**, 1–196, 2010.

[66] K. I. Aoki, Z. Hioki, M. Konuma, R. Kawabe, and T. Muta. Prog. Theor. Phys. Suppl. **73**, 1–225, 1982.

[67] W. Grimus and M. Löschner. Int. J. Mod. Phys. A **31**, 1630038. [Erratum: Int. J. Mod. Phys.A32,no.13,1792001(2017)], 2017.

[68] F. J. Dyson. Phys. Rev. **75**, 1736–1755, 1949.

[69] A. Denner and S. Dittmaier. Nucl. Phys. Proc. Suppl. **160**, 22–26, 2006.

[70] A. Denner. Fortsch.Phys. **41**, 307–420, 1993.

[71] A. Pilaftsis. Z. Phys. C **55**, 275–282, 1992.

[72] A. Pilaftsis. Phys. Rev. D **58**, 096010, 1998.

[73] M. Sperling, D. Stöckinger, and A. Voigt. JHEP **07**, 132, 2013.

[74] M. Sperling, D. Stöckinger, and A. Voigt. JHEP **01**, 068, 2014.

[75] H. H. Patel and M. J. Ramsey-Musolf. JHEP **07**, 029, 2011.

[76] J. M. Cornwall, J. Papavassiliou, and D. Binosi. *The Pinch Technique and its Applications to Non-Abelian Gauge Theories* (Cambridge University Press, 2010).

[77] V. Mathieu. In *3rd Modave Summer School in Mathematical Physics Modave, Belgium, August 29-September 4, 2007* (2008).

[78] D. Binosi and J. Papavassiliou. Phys. Rept. **479**, 1–152, 2009.

[79] S. Liebler and W. Porod. Nucl. Phys. B **849**, 213–249. [Erratum: Nucl. Phys.B856,125(2012)], 2011.

[80] A. Denner, G. Weiglein, and S. Dittmaier. Phys. Lett. B **333**, 420–426, 1994.

[81] T. Hahn. Comput. Phys. Commun. **140**, 418–431, 2001.

[82] T. Hahn and M. Perez-Victoria. Comput. Phys. Commun. **118**, 153–165, 1999.

[83] G. Passarino and M. J. G. Veltman. Nucl. Phys. B **160**, 151–207, 1979.

[84] A. Jevicki and C. K. Lee. Phys. Rev. D **37**, 1485, 1988.

[85] G. A. Vilkovisky. Nucl. Phys. B **234**, 125–137, 1984.

[86] S. Weinberg. *The quantum theory of fields. Vol. 2: Modern applications* (Cambridge University Press, 2013).

Santrauka

Komentaras apie terminiją

Teorinėje elementariųjų dalelių fizikoje yra daugybė nusistovėjusių angliškų terminų, kurie skamba gal kiek neformaliai ir žargoniškai. Jie ne visada atitinka tikslią fizikinę prasmę, kaip pavyzdžiui, šioje disertacijoje vartojamas terminas „buožgalvis“ (angl. tadpole), tačiau yra asociatyvūs ir trumpi. Dėl šios priežasties jie yra patogūs profesinėse diskusijose ir plačiai vartojami angliškoje literatūroje. Kadangi Lietuvos teorinėje dalelių fizikoje ne visi tokie terminai yra nusistovėję, šių žargoninių terminų lietuvių kalboje vartojimas nėra įprastas, taigi tiesioginis terminų vertimas iš anglų kalbos be prasmės bei kilmės paaiškinimo padaro lietuvišką tekstą sunkiai skaitomu. Kita vertus, pernelyg sudėtingas, nors ir fizikine prasme tikslus terminas, negali turėti pasisekimo neformaliose mokslinėse diskusijose. Tokiais atvejais dažnai linkstama į pasiskolintus iš anglų kalbos barbarizmus. Šių barbarizmų neįmanoma išgyvendinti iš lietuvių kalbos, jeigu nėra jų tiksliai atitinkančių ir paprastai naudojamų terminų. Taigi, nereikėtų vengti profesinio žargono vartojimo, o, atvirkščiai, stengtis jį kurti ir puoselėti, tokiu būdu turtinant profesinę lietuvių kalbą. Todėl šios santraukos gale pateikiu kai kuriuos verstus terminus su vartojimo pasiūlymais bei trumpais fizikinės prasmės ir termino kilmės paaiškinimais. Tikiuosi, kad šis sąrašas leis skaitytojui lengvai susigaudyti tarp vartojamų terminų ir padarys šią santrauką lengviau suprantamą.

Įvadas

Standartinis elementariųjų dalelių modelis (SM) yra labiausiai eksperimentiškai pasisekęs modelis, beprecedenčiu tikslumu aprašantis dalelių fizikos fenomenus. Tačiau neabejotina yra ir tai, kad SM nėra visus reiškinius apimanti teorija, taigi fizika nesibaigia ties SM ribomis. Neutrino aromatų maišymasis yra aiškiausias to įrodymas: eksperimentiškai aptiktos neutrinų osciliacijos [1–4] įrodo, kad neutrinų masė nėra nulinė. SM nenumato masyvių neutrinų, tačiau yra pasiūlyta įvairių būdų kaip praplėsti SM juos įskaitant. Vieni iš pirmųjų ir labiausiai tiesmukų SM plėtinių yra sverto mechanizmai [5–8]. Šiuose modeliuose neutrinai yra natūraliai mažų masių bei su pakankamai paprastu masės suteikimo mechanizmu, todėl šie modeliai yra populiarūs teoretikų tarpe. Šiuolaikines sverto mechanizmų apžvalgas galima rasti čia [9–12].

Neutrinų aromatų maišymasis nėra vienintelis fenomenas gamtoje, kuris yra įmanomas už SM ribų. Higgs'o dalelės atradimas [13, 14], užbaigdamas eksperimentines SM aprašytų dalelių paieškas, tuo pačiu įkvepia tolesnius skaliarinių laukų tyrimus, kurie nebūtinai apsiriboja SM skaliariniu sektoriumi. Kol kas Higgs'o dalelė yra pirma ir vienintelė užfiksuota skaliarinė dalelė gamtoje. Kyla natūralus klausimas ar yra daugiau tokių dalelių, nes yra nemažai teorinių argumentų už didesnį skaliarinių dalelių skaičių [15–18]. Daugelis modelių, numatančių daugiau skaliarinių dalelių, gali būti aprašyti kaip atskiras apibendrinto dviejų Higgs'ų dubletų modelio (2HDM) atvejis. 2HDM modeliai, neturintys supersimetrijos, plačiai aprašyti čia [19].

Kol kas nėra galimybių pasakyti, kuris modelis vienareikšmiškai teisingai aprašo visus fizikinius gamtos reiškinius. Tačiau daugėjant eksperimentinių duomenų (jų apžvalgą galima rasti [20]) yra vis siaurinamos įvairių modelių parametrų erdvės. Tokiu būdu atmetama dalis modelių, kurių parametrų vertės yra apribotos taip, kad nepatenka į galimas eksperimentinių verčių ribas. Nulinis perturbacijų teorijos artinys nėra pakankamas norint atlikti tokią parametrų erdvės analizę. Neutrinų masėms aukštesnės eilės pataisos yra ypatingai svarios [21, 22], o joms yra reikalinga pernormavimo procedūra.

Pernormavimo procedūra apibrėžia teorijos parametrus, tokius kaip masė ir sąveikos konstantos, kilpose. Kalibruotinis teorijos invariantiškumas yra automatiškai tenkinamas, jeigu teorija yra be anomalijų [23–25], tačiau tai neapsaugo nuo netyčinių kalibruotės priklausomybių įvedimų modelio parametrų apibrėžimuose, t. y. suskaičiavus parametrus kilpose, gali paaiškėti, kad jie priklausomi nuo kalibruotės. Žinoma, šios kalibruotinės priklausomybės išsiprastina skaičiuojant sklaidos matricos elementus, tačiau parametrų, priklausomų nuo kalibracijos, fizikinė interpretacija tampa kebli.

Šiame darbe yra tiriamas Grimus–Neufeld (GN) modelio [26] neutrinų pernormavimas. Jis sudarytas iš CP simetriško 2HDM, praplėsto vienu neutraliu Weyl'o spinoriumi, kurio įvedimas sąlygoja sverto mechanizmą bei radiacinį masės generavimą. Norint, kad pernormuoti masės parametrai būtų kalibruotiškai invariantiški, mes pritaikome kompleksinės masės schemą (CMS). Kadangi masė yra elektrosilpnosiosios simetrijos pažeidimo pasekmė, masių pernormavimas yra glaudžiai susijęs su buožgalvių pernormavimu. Mes parodome, kaip masės kontranarys, išreikštas iš CMS sąlygų, tampa priklausomas nuo kalibruotės, jeigu CMS sąlygos yra naudojamos kartu su įprastine buožgalvių pernormavimo procedūra. Kadangi yra įrodyta [27], kad CMS apibrėžia kalibruotiškai invariantiškas pernormuotas mases visose kilpose, plikos masės standartinėje buožgalvių schemoje tampa kalibruotiškai priklausomos. Pritaikant Fleischer–Jegerlehner (FJ) schemą [28], šių priklausomybių nuo kalibruotės galima išvengti. Mes ją pritaikome neutrinų masių pernormavimui ir, kompiuterinės algebros paketų pagalba, parodome, kad priklausomybės nuo kalibruotės mūsų nagrinėjamame modelyje išnyksta. Taigi, mes apibrėžiame kalibruotiškai invariantiškas tiek plikas, tiek pernormuotas GN modelio neutrinų mases.

Pagrindinis tyrimo tikslas ir uždaviniai

Pagrindinis šioje disertacijoje pristatyto tyrimo tikslas yra suformuluoti korektišką Grimus–Neufeld modelio neutrinų masės pernormavimą vienoje kilpoje. Šiam tikslui pasiekti buvo išspręsti šie uždaviniai:

- Suformuluoti modelį Weyl'o spinorių formalizme ir įvesti jį į kompiuterines programas automatiniams skaičiavimams.

- Pasirinkti pernormavimo schemą neutrino masių pernormavimui ir pritaikyti ją sumaišytoms Majorana fermionų sistemoms Weyl'o spinorių formalizme.

- Patikrinti masių ir masių kontranarių kalibruotinį invariantiškumą.

- Rasti kalibruotiškai invariantišką neutrinų pernormavimo procedūrą.

Rezultatų naujumas ir aktualumas

Šiame darbe tiriamas modelis pirmą kartą yra pasiūlytas Grimus'o ir Neufeld'o [26]. Neseniai pasirodžiusiame darbe [29], šio modelio pernormavimas bei neutrinų masių pernormavimo kalibruotinio invariantiškumo klausimas buvo tirtas, pritaikant modifikuoto minimalaus sutraukimo ($\overline{\text{MS}}$) schemą. Iki minėto darbo šie klausimai GN modelyje nebuvo paliesti. Kitaip nei [29], mes apibrėžiame pernormuotas mases kaip fizikinius parametrus tam, kad jie galėtų būti naudojami kaip įvesties parametrai, o ne išvesties. Tokiu atveju yra įprasta naudoti antmasės schemą (OS), tačiau yra žinoma, kad OS apibrėžtos nestabilių dalelių masės yra priklausomos nuo kalibruotės [30]. Yra įrodyta [27], kad CMS, kitaip nei OS, apibrėžia kalibruotiškai invariantiškas pernormuotas mases visose kilpose. Kadangi GN modelio neutrinai sudaro sumaišytą dalelių sistemą, susidedančią tiek iš stabilių, tiek iš nestabilių dalelių, mes joms visoms pritaikome CMS tam, kad apibrėžtume konceptualiai korektiškas ir kalibruotiškai invariantiškas mases.

Nors CMS ir apibrėžia kalibruotiškai invariantiškas pernormuotas mases, plikos masės nebūtinai yra kalibruotiškai nepriklausomos, nes jas surišantys kontranariai gali įgauti kalibruotinę priklausomybę. Tai tampa problema, jeigu norima CMS mases palyginti su kitų pernormavimo schemų masėmis, tokių kaip $\overline{\text{MS}}$, nuo kalibruotės nepriklausomu būdu. Šių priklausomybių nuo kalibruotės galima išvengti pritaikant FJ procedūrą [28], kuri neseniai vėl sulaukė išaugusio susidomėjimo, pritaikant ją kartu su $\overline{\text{MS}}$ [29,31–34]. Mes pritaikome FJ procedūrą kartu su CMS, taigi, CMS apibrėžtos masės yra surištos su plikomis masėmis kalibruotiškai nepriklausomu būdu. FJ schemoje kalibruotinės priklausomybės skaičiavimuose yra sistematiškai atskirtos. Taigi, atsiranda galimybė papildomam skaičiavimų patikrinimams kompiuterinėse programose.

Šiame darbe pristatomas pirmas bandymas suformuluoti korektišką ir kalibruotiškai invariantišką pernormavimo procedūrą fizikinėje bazėje GN modeliui. Mes taip pat aprašome kaip pakartoti mūsų rezultatus, naudojantis SARAH, FeynArts ir FormCalc.

Ginamieji teiginiai

1. Vienos kilpos artinyje radiaciškai gauta neutrino masė yra baigtinė ir nepriklausanti nuo kalibruotės.

2. FJ schema yra tinkama svertu išplėsto 2HDM neutrino masių pernormavimui.

3. CMS yra algebriškai ekvivalenti OS ir ją galima taikyti kartu su FJ schema vienos kilpos neutrinų masėms.

Autoriaus indėlis ir rezultatų aprobacija

Šis darbas remiasi rezultatais, pristatytais trijuose publikuotuose straipsniuose:

1. V. Dūdėnas and T. Gajdosik. Lith. J. Phys. **56**, 149–163, 2016,

2. V. Dūdėnas and T. Gajdosik. Acta Phys. Pol. B **48**, 2243–2249, 2017,

3. V. Dūdėnas and T. Gajdosik. Phys. Rev. D **98**, 035034, 2018.

Šios disertacijos autorius yra pagrindinis šių straipsnių autorius. Taip pat yra bendraautorius darbo, kuris yra netiesiogiai susijęs su šios disertacijos rezultatais:

1. V. Dūdėnas, T. Gajdosik, A. Juodagalvis, and D. Jurčiukonis. Acta Phys. Pol. B **48**, 2235, 2017.

Šiame darbe taip pat pristatomas GN modelio tyrimas, tačiau jame naudojamos skirtingas formalizmas ir koncentruojamasi į kitus modelio aspektus naudojant skaitmeninius metodus. Tuo tarpu disertacijoje pristatomi tik analitiškai gauti rezultatai.

Rezultatai taip pat buvo pristatyti konferencijose:

1. Vytautas Dūdėnas, Thomas Gajdosik, Developing Weyl spinor formalism for seesaw neutrinos, Open readings 2015, March 24-27, Vilnius, Lithuania,

2. Vytautas Dūdėnas, Thomas Gajdosik, Weyl'o spinorių formalizmo plėtojimas sūpuokliniams neutrinams, 41 Lietuvos nacionalinė fizikos konferencija, 2015, May 17-19, Vilnius, Lithuania,

3. Vytautas Dūdėnas, Thomas Gajdosik, Renormalization of propagators of Weyl spinors in the seesaw extension of the standard model, Open readings 2016, March 15-18, Vilnius, Lithuania,

4. Vytautas Dūdėnas, Thomas Gajdosik, Using techniques of algebraic renormalization for the two Higgs doublet model with one heavy neutrino, Open readings 2017, March 14-17, Vilnius, Lithuania,

5. Vytautas Dūdėnas, Thomas Gajdosik, On the renormalization of neutrinos of the seesaw extended 2HDM, Matter to the deepest, 2017, September 3-8, Podlesice, Poland,

6. Vytautas Dūdėnas, Thomas Gajdosik, Renormalization of the neutrino sector of Grimus–Neufeld model, 42 Lietuvos nacionalinė fizikos konferencija, 2017, October 4-6, Vilnius, Lithuania,

7. Vytautas Dūdėnas, Thomas Gajdosik, Gauge parameter dependence of the neutrino mass renormalization, Open readings 2018, March 20-23, Vilnius, Lithuania,

8. Vytautas Dūdėnas, Thomas Gajdosik, Gauge dependence of tadpole and mass renormalization for a seesaw extended 2HDM, Workshop on multi–Higgs models, 2018, September 4-7, Lisbon, Portugal.

Grimus–Neufeld modelis

Grimus–Neufeld (GN) modelis [26] yra 2HDM [19], praplėstas svertiniu [5] neutrinu. Šis modelis turi du mechanizmus, suteikiančius neutrinams masę: sverto mechanizmą bei radiacinėmis pataisomis gaunamą neutrino masę. Sverto mechanizmas leidžia tik dviem iš keturių neutrinų turėti medžio mases, iš kurių viena masė yra didelė[1]. Dėl sąveikos su antruoju Higgs'o dubletu vienoje kilpoje sugeneruojama dar viena nenulinė masė neutrinui, vadinama radiacine mase. Taigi, vienoje kilpoje, vienas neutrinas yra su didele mase, du su nedidelėmis masėmis ir vienas su nuline mase.

[1] Jeigu tikėsimės Yukawa sąveikos konstantų panašios eilės, kaip ir krūvinių leptonų atžvilgiu, sunkioji masė turėtų būti apytiksliai ties Didžiojo Susijungimo (angl. Grand Unification) energijos skale [10].

$\nu_i^F = (\nu_e, \nu_\mu, \nu_\tau)_i$	Trys neutrinai aromato bazėje
N	Neutralus neutrinas
$H_1 = \begin{pmatrix} \chi_W^+ \\ \frac{1}{\sqrt{2}}(v + h + i\chi_Z) \end{pmatrix}$	Pirmasis Higgs'o dubletas Higgs'o bazėje.
$H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix}$	Antrasis Higgs'o dubletas Higgs'o bazėje.
$h\,, H$	Neutralūs Higgs'o laukai
H^+	Krūvį turintis Higgs'o laukas
A	Aksialinis Higgs'o laukas
χ_W	W Goldstone'o bozonas
χ_Z	Z Goldstone'o bozonas
$Y_i^{\nu 1}\,, Y_i^{\nu 2}\,, i = 1, 2, 3$	Yukawa koeficientai

L1 lentelė: Neutrinų ir skaliarinių dalelių žymėjimai aromato bei Higgs'o bazėse.

Šios GN modelio savybės, pasirinkus patogią bazę, gali būti identifikuotos jau medžio artinyje, kaip ir buvo padaryta [26]. Pirmiausia, skaliariniame sektoriuje mes pasirenkame Higgs'o bazę [35–37]. Higgs'o bazė, gaunama atliekant $U(2)$ transformaciją tarp dviejų Higgs'o dubletų. Higgs'o bazėje yra išskiramas vienas dubletas atsakingas už elektrosilpnosios simetrijos pažeidimą, t. y. nenulinė vakuumo tikėtiniausia vertė (VEV) priskiriama tik šiam dubletui. Kitas dubletas šioje bazėje, kuris turi nulinę VEV, simetrijos pažeidime nedalyvauja. Dėl to masės nariai medyje kyla tik iš Yukawa sąveikų su pirmuoju dubletu Higgs'o bazėje.

Nagrinėjant leptonus, paprasčiausia yra iš pradžių pasirinkti aromato bazę, t.y. kai krūvį turintys leptonai yra savo masės tikrinėse būsenose ir yra identifikuojami kaip elektronas, miuonas ir taonas. Aromato bazėje neutrinai yra atitinkamai elektroniniai, muoniniai arba taoniniai. Papildomas Majorana neutrinas nesąveikauja elektrosilpnąja sąveika ir yra neutralus visų simetrijų atžvilgiu. Trijų aromato bazės neutrinų, svertinio neutrino, Higgs'o dubletų bei laisvųjų Yukawa koeficientų pažymėjimai yra nurodyti lentelėje L1. Tada Yukawa nariai neutrinams yra užrašomi taip:

$$\mathcal{L} = -Y_i^{\nu 1} \nu_i^F H_{1(2)} N - Y_i^{\nu 2} \nu_i^F H_{2(2)} N + H.c.\,. \quad \text{(S.1)}$$

Skliausteliuose pažymėjome, kad sąveikose su neutrinais imama antroji dubleto komponentė[2]. Iš (S.1) matyti, kad tik pirmasis narys bus atsakingas už masės narį, nes tik H_1 turi nenulinę VEV. Masės parametras, gaunamas po elektrosilpnosios simetrijos pažeidimo (S.1) lygtyje, yra vadinamas Dirac'o masės parametru. N laukas taip pat turi Majorana masės parametrą M:

$$\mathcal{L} = -\frac{1}{2}M\left(NN + H.c.\right) . \tag{S.2}$$

(S.1) išraiškoje mes galime pakeisti bazę unitariomis transformacijomis ir atlikti SVD dekompoziciją Yukawa sąveikų vektoriams. Bazė, taip pat naudojama ir [22, 26], yra parametrizuojama:

$$\begin{aligned}
&\nu_i' = V_{ij}^* \nu_j^F \\
&V_{1j}Y_j^{\nu 1} = 0\,, \quad V_{2j}Y_j^{\nu 1} = 0\,, \quad V_{3j}Y_j^{\nu 1} = y\,, \\
&V_{1j}Y_j^{\nu 2} = 0\,, \quad V_{2j}Y_j^{\nu 2} = d\,, \quad V_{3j}Y_j^{\nu 2} = d'\,, \\
&d, y \in \mathbb{R}^+\,, \quad d' \in \mathbb{C}\,.
\end{aligned} \tag{S.3}$$

Šioje bazėje po elektrosilpnosios simetrijos pažeidimo Dirac'o masė, jungianti ν_3' ir N yra lygi $\frac{1}{\sqrt{2}}yv$, kas matyti, įstačius skaliarinių laukų išraiškas iš L1 lentelės į (S.1). Unitaria transformacija tarp ν_3' ir N pereiname prie masės singuliarių verčių bazės, gaudami du Majorana fermionus. Visa transformacija tarp pradinės aromato bazės ir masės singuliarių verčių bazės užrašoma kaip sandauga ortogonalių transformacijų O, fazės pasukimų U bei sverto transformacijos U^{34}:

$$\nu_i = U_{ij}^* \nu_j^F\,,\; U = U^{34}V = U^{34}U^{\beta}O^{12}U^{\alpha}O^{13}O^{23}\,. \tag{S.4}$$

Sverto transformacija parametrizuojama

$$U^{34} = \begin{pmatrix} -ic_{34} & is_{34} \\ s_{34} & c_{34} \end{pmatrix},\, s_{34}^2 = \frac{m_3}{m_4 + m_3}\,, \quad c_{34}^2 = \frac{m_4}{m_4 + m_3}\,, \tag{S.5}$$

kur m_3 ir m_4 Majorana masės nariai atitinkamai ν_3 ir ν_4 fermionams, susiję su ν' bazės parametrais:

$$M = m_4 - m_3 \quad \text{ir} \quad y^2v^2 = 2m_3m_4\,,\, m_3 < m_4\,. \tag{S.6}$$

[2]Iš lentelės L1: $H_{1(2)} = \frac{1}{\sqrt{2}}\left(v + h + i\chi_Z\right)$ ir $H_{2(2)} = \frac{1}{\sqrt{2}}\left(H + iA\right)$

Yukawa nariai, įskaitantys neutrinų sąveikas su neutraliais skaliariniais laukais, kartu su masės nariais, užrašomi:

$$\mathcal{L}_{\text{Yuk}} = -\frac{1}{2}m_3\,\nu_3\nu_3 - \frac{1}{2}m_4\,\nu_4\nu_4 - \frac{1}{\sqrt{2}}d\,(H+iA)\,\nu_2\,(-is_{34}\nu_3 + c_{34}\nu_4)$$
$$- \frac{1}{\sqrt{2}}\left[y\,(h+i\chi_Z) + d'\,(H+iA)\right]$$
$$\times \left[c_{34}s_{34}\nu_3\nu_3 + i\left(c_{34}^2 - s_{34}^2\right)\nu_3\nu_4 + c_{34}s_{34}\nu_4\nu_4\right] + H.c.\,. \tag{S.7}$$

Kadangi ν_1 ir ν_2 vis dar neturi masės parametro, jos yra išsigimusios būsenos. Iš (S.7) lygties matome, kad ν_2 sąveikauja su skaliarais iš antrojo Higgs'o dubleto, kai ν_1 lygtyje iš viso nefigūruoja. Dėl šios priežasties, skaičiuojant vienos kilpos pataisas, ν_2 įgaus radiacines pataisas, o ν_1 liks su nuline mase. Taigi, parametrizacija (S.3) dar medyje leidžia iš karto nuspėti visas keturias vienos kilpos masių tikrines būsenas.

Kompleksinė masės schema

Pernormuoti neutrinams mes naudojame kompleksinės masės schemą [30, 38–46] (CMS, angl. complex mass scheme). Ji yra analitinis tęsinys antmasės schemos (OS, angl. on-shell), į masės apibrėžimą įtraukiant dalelės skilimo plotį kaip menamą masės parametro dalį. Taigi, OS ir CMS masės apibrėžimai yra identiški stabilioms dalelėms, tačiau išsiskiria dalelėms, kurios nėra stabilios. OS masės apibrėžimas yra kalibruotiškai priklausomas nestabilioms dalelėms [27], pvz.: Z bozonui. Istoriškai CMS pirmą kartą ir buvo pristatyta kaip schema, išsprendžianti kalibruotės priklausomybės problemą Z bozonui dviejose kilpose [30]. Vėliau įrodyta, kad CMS masės apibrėžimas yra kalibruotiškai invariantiškas visose kilpose [27].

Nors GN modelio eksperimentiškai matuojami trys neutrinai yra stabilūs, jie yra maišyti su nestabiliu ketvirtuoju neutrinu. Taigi, norint matematiškai korektiškai pernormuoti neutrinus, mes taikome CMS kaip bendrą schemą dalelių masėms pernormuoti. CMS aprašymas Weyl'o spinorių formalizme maišytiems Majorana fermionams yra pristatytas [47]. CMS schema mes užtikriname, kad mūsų pernormuoti masių parametrai būtų kalibruotiškai nepriklausomi.

Naudojantis laisvo Weyl'o spinoriaus ν su Majorana mase m judėjimo lygtimis, mes galime bendrais bruožais apibūdinti OS ir CMS schemas.

Judėjimo lygtys yra:

$$\bar{\sigma} p \nu = m \nu^\dagger \,,\ \sigma p \nu^\dagger = m \nu \,. \tag{S.8}$$

Įskaičius trikdžių pataisas, tiek kairiosios, tiek dešiniosios šių lygčių pusės pasikeičia. OS schemoje pareikalaujama, kad reali šios lygties su pataisomis dalis būtų tenkinama ties realia $p^2 = m^2$ reikšme. CMS schemoje realumo reikalavimas išmetamas, todėl atsiranda papildoma fazė tiek masės parametrui, tiek laukui, t. y. (S.8) modifikuojasi į

$$\bar{\sigma} p \nu = m \bar{\nu} \,,\ \sigma p \bar{\nu} = m \nu \,, \tag{S.9}$$

kur m nebėra realus, o $\bar{\nu}$ skiriasi nuo $\nu^\dagger$ faze, atsirandančia dėl dalelės nestabilumo. Jeigu dalelė yra stabili, lygtis (S.9) redukuojasi į (S.8). Tačiau, jeigu stabilios dalelės maišosi su nestabiliomis, situacija nėra tokia paprasta ir bendru atveju „nestabilumo“ fazės gali įeiti ir į stabilių dalelių pernormavimą [47].

Jeigu plika dalelės masė yra lygi nuliui, dešiniosios (S.9) lygčių pusės gali būti sugeneruojamos trikdžių pataisose. Tokiu atveju vienos kilpos masė yra skaičiuojama ties $p^2 = 0$, t. y. nulinės eilės masės vertė įdedama į kilpos masės išraišką. Kadangi radiacinė masė tenkina CMS sąlygas, ji ir yra pernormuota CMS masė pagal apibrėžimą. Iš to, kad CMS pernormuotos masės yra kalibruotiškai nepriklausomos, išplaukia, jog radiacinė masė taip pat turi būti kalibruotiškai nepriklausoma. Šis pastebėjimas yra ypač svarbus, nes radiaciškai sugeneruota masė neturi kontranario. Taigi, jeigu atsirastų kalibruotinė priklausomybė radiacinei masei, jos nebūtų įmanoma panaikinti. Tai reikštų, kad teorija yra matematiškai nekorektiška. Taigi, kalibruotiškai nepriklausomos ir baigtinės radiacinės masės išraiškos gavimas yra pirminis modelio bei jo įvedimo į programinius paketus korektiškumo patikrinimas.

CMS schemoje įvedami pernormuoti kompleksiniai parametrai, tokie kaip masė ir sąveikų konstantos, tačiau jie yra susieti su plikos teorijos realiomis masėmis ir sąveikos konstantomis. Ši sąsaja leidžia išlaikyti modelio unitariškumą kiekvienoje kilpoje [48]. Taip pat algebrinė plikos teorijos struktūra yra išsaugojama ir pernormuotoje teorijoje, nes menamos pernormuotų parametrų dalys įeina tik į antiermitinę efektinio potencialo dalį. T. y., jeigu plikoje teorijoje turime operatorių O_0 kartu su ermitiškai jungtiniu:

$$\mathcal{L} = O_0 + H.c. \,, \tag{S.10}$$

tai pernormuotoje teorijoje turime:

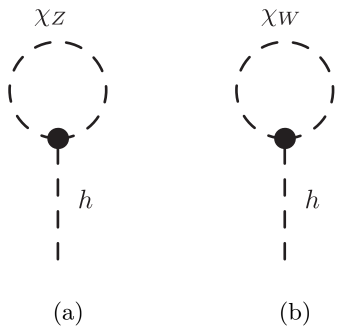
$$\mathcal{L} = O + H.c.^{*} + c.t. \qquad \text{(S.11)}$$

kur $H.c.^{*}$ ženklina iš (S.10) lygties pernormuotus $H.c.$ narius. Atkreipiame dėmesį, kad $H.c.^{*}$ nėra O operatoriaus ermitiškai jungtiniai nariai. Kita vertus, papildoma kompleksinė fazė, atsirandanti CMS schemoje, yra bendra abiems O ir $H.c.^{*}$ nariams. Tokiu būdu pernormuotas Lagranžianas atspindi pliko Lagranžiano realumą. Taip pat, esantys sąryšiai tarp parametrų išlieka tokie patys kaip ir taikant OS schemą, nors tie parametrai ir įgauna bendrą fazę. Skaičiuojant trikdžių pataisas, papildoma fazė nepakeičia algebrinių skaičiavimų. T.y., skaičiuojant pataisas, laikant, kad masės parametrai yra realūs, gaunama ta pati algebrinė išraiška kaip ir to nepadarius. Tai leidžia naudoti CMS atliekant algebrinius skaičiavimus su programiniais paketais, kuriuose nėra numatytas kompleksinių masių verčių naudojimas.

Plikų masių priklausomybė nuo kalibruotės bei FJ schema

Taikant CMS, pernormuoti masės parametrai yra kalibruotiškai nepriklausomi, tačiau tai dar nereiškia, kad pliki masės parametrai tos priklausomybės neturi. Nors pliki parametrai ir neatspindi matuojamų dydžių, jų priklausomybė nuo kalibruotės kelia keblumų, norint palyginti rezultatus su kitomis pernormavimo schemomis, tokiomis kaip $\overline{\mathrm{MS}}$ [29, 31]. Kadangi visos masės, išskyrus Majorana neutrino N masės parametrą M, yra kilusios iš elektrosilpnosios simetrijos pažeidimo, jų pernormavimas yra susijęs su skaliarinio sektoriaus pernormavimu, konkrečiai, su buožgalvių pernormavimu.

Įprastoje buožgalvių pernormavimo schemoje (pvz.: [49]), plikos masės kyla iš plikos Higgs'o VEV, o pernormuotos masės atitinkamai išreiškiamos iš pernormuotos Higgs'o VEV. Pernormuota Higgs'o VEV yra apibrėžiama buožgalvių sąlygose kaip reikšmė, minimizuojanti potencialą nagrinėjamame artinyje. Tuo tarpu plika VEV, surišta su pernormuota VEV per pernormavimo konstantą, minimizuoja tik medžio potencialą. Jau vienoje kilpoje Higgs'o VEV pernormavimo konstanta yra kalibruotiškai priklausoma. Diagramos, kurios lemia šią priklauso-



P1 pav.: Diagramos, lemiančios kalibruotinę priklausomybę. h laukas čia yra laukas Higgs'o bazėje.

mybę yra pavaizduotos P1 pav. Tai reiškia, kad arba pernormuota, arba plika VEV priklauso nuo kalibruotės.

Literatūroje yra dažnai teigiama [50–52], kad Higgs'o VEV yra kalibruotiškai priklausoma. Šis teiginys paremtas kalibruotine priklausomybe pataisų, pavaizduotų P1 diagramose. Tačiau taip pat akivaizdu, kad pernormuota VEV, apibrėžta buožgalvių sąlygose ir surišta su pernormuotais kalibruotiškai invariantiška W bozono mase, elektromagnetinės sąveikos konstanta bei Weinberg'o kampo sinusu:

$$v = \frac{2 s_W M_W}{g_e}\,, \tag{S.12}$$

yra kalibruotiškai nepriklausoma. Šis nesusipratimas dėl kalibruotinių priklausomybių kyla iš skirtingų VEV apibrėžimų: plika VEV, minimizuojanti potencialą medyje, yra kalibruotiškai priklausoma, tuo tarpu (S.12) yra pernormuota ir kalibruotiškai nepriklausoma VEV.

Kadangi plika VEV kilpoje tampa priklausoma nuo kalibruotės, standartinėje buožgalvių schemoje jų kalibruotinė priklausomybė perduodama plikiems masių parametrams. Kalibruotiškai invariantiškai apibrėžti plikas mases galima panaudojant FJ schemą buožgalviams [28]. Praktiškai FJ schemoje naudojama viena ir ta pati Higgs'o VEV, apibrėžiant tiek plikas, tiek pernormuotas konstantas. Tokiu būdu kalibruotiškai priklausomi nariai atskiriami nuo masės parametrų, taip apibrėžiant kalibruotiškai nepriklausomas tiek plikas, tiek pernormuotas mases. Ši procedūra remiasi prielaida, kad visų kilpų pernormuota trikdžių teorija bei plika trikdžių teorija turi sutapti. Taip pat pastebėjimu, kad skirtingai negu kiti modelio parametrai, Higgs'o VEV yra gaunama dinamiškai,

t. y. minimizuojant potencialą. Vadinasi, yra tik viena „teisinga“ (angl. „proper“) VEV, kuri minimizuoja pilną visų kilpų teoriją. Taigi „teisinga“ VEV yra (S.12), kai plika VEV yra tik apytikslė nulinio artinio vertė. Remiantis šiais argumentais, visi masės parametrai, tiek pliki, tiek pernormuoti, turi kilti iš teisingos VEV.

Sekant šiais argumentais, vienas klausimas lieka neaiškus. Jeigu plika ir pernormuotos teorijos turi duoti tuos pačius rezultatus, kodėl plika vertė, pataisyta vienoje kilpoje, yra kalibruotiškai priklausoma, o pernormuota – ne? Atsakymas į šį klausimą slypi kalibruotės parametrizavime ir bazės pasirinkime. Kalibruotės pasirinkimas apibendrintoje R_ξ kalibruotėje yra parametrizuojamas ξ_Z ir ξ_W parametrais, išskiriant Goldstone'o bozonus χ_Z ir χ_W kaip nefizikinius laisvės laipsnius, “suvalgomus” išilginių W ir Z bozonų poliarizacijų. Taigi, kalibruotės parametrizavimas taip pat susijęs su Higgs'o dubleto H_1 parametrizavimu, kuriame išskiriama kryptis, kurioje laukas įgauna nenulinę VEV (žr. L1 lentelę). Po kilpos pataisų bazė kiek pasisuka, taigi, kryptis, kurioje laukas įgauna nenulinę VEV taip pat pasisuka. Tuomet Higgs'o VEV, žiūrint iš pirminės bazės, įgauna ξ_W ir ξ_Z priklausomybes, t. y. indėlius į jos vertę iš kitų krypčių. FJ schemoje yra pasirenkama „teisinga“ VEV, taip išvengiant šių bazės nesutapimų ir kalibruotinių priklausomybių. T. y. galime sakyti, kad FJ schemoje bazė, kurioje išreiškiama Higgs'o VEV, yra pasirenkama ne prieš, bet po kilpos pataisų įskaitymų.

Toliau mes pritaikysime FJ schemą GN modelio neutrinams kartu su CMS.

Neutrinų masių pernormavimas

Skyriuje „Grimus–Neufeld modelis“ visi pristatyti parametrai bei laukai yra pliki. Nuo šiol mes pridėsime indeksą „0“, norint atskirti plikus parametrus nuo pernormuotų. Visi pernormuoti parametrai p ir laukai ϕ yra susiję su plikais parametrais p_0 bei laukais ϕ_0 per pernormavimo konstantas:

$$p_0 = (1+\delta_p)\, p\,,\ \phi_0 = \left(1+\frac{1}{2}\delta_\phi\right)\phi\,. \tag{S.13}$$

Plika Higgs'o vertė nėra laisvas parametras, nes jis apibrėžiamas per minimumo sąlygas. Tačiau mes jo pernormavimo konstantą apibrėžiame analogiškai kaip ir visiems kitiems parametrams p[3]. Vienos kilpos Higgs'o VEV pernormavimo konstanta yra išreiškiama iš buožgalvių sąlygų ir yra lygi:

$$\delta_v = \frac{1}{\lambda_1 v^3} T_h^{[1]}\,, \tag{S.14}$$

kur λ_1 yra Higgs'o sąveikos su savimi konstanta, ateinanti iš 2HDM potencialo, $T_h^{[1]}$ yra vienos kilpos VEV pataisa Higgs'o bazėje, atvaizduojama buožgalvių diagramomis. Taigi į $T_h^{[1]}$ įeina ir diagramos, pavaizduotos (P1) pav.

Įstačius (S.13) į (S.7), matome, kad nei ν_1, nei ν_2 neturi masės kontranarių. Tai reiškia, kad kilpos pataisose sugeneruotos masės privalo būti baigtinės bei kalibruotiškai invariantiškos, antraip visas modelis būtų nekorektiškas. Tiesiogiai suskaičiavus vienos kilpos mases laukams ν_1 bei ν_2, gauname, kad ν_1 vienoje kilpoje lieka su nuline mase, o ν_2 masė yra:

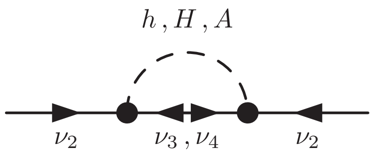
$$\begin{aligned} m_2 = &-\frac{d^2}{32\pi\left(m_3+m_4\right)}\times \\ &\times\Bigg(m_3^2\left[B_0\left(0,m_3^2,m_A^2\right)-c_\alpha^2 B_0\left(0,m_3^2,m_H^2\right)-s_\alpha^2 B_0\left(0,m_3^2,m_h^2\right)\right] \\ &-m_4^2\left[B_0\left(0,m_4^2,m_A^2\right)-c_\alpha^2 B_0\left(0,m_4^2,m_H^2\right)-s_\alpha^2 B_0\left(0,m_4^2,m_h^2\right)\right]\Bigg)\,, \end{aligned} \tag{S.15}$$

kur s_α ir c_α yra atitinkamai sinusas ir kosinusas laukų h ir H maišymosi kampo α, B_0 yra standartinis Passarino–Veltman integralas [53], o masių indeksai žymi daleles, kurių masės parametras yra užrašytas. Diagramatinis (S.15) išraiškos šešių narių atvaizdavimas yra parodytas P2 pav. Matome, kad rezultatas yra kalibruotiškai nepriklausomas bei yra baigtinis, t. y. diverguojantys integralo nariai susiprastina:

$$m_{2\infty} = -\frac{d^2}{32\pi\left(m_3+m_4\right)}\left(m_3^2\left[1-c_\alpha^2-s_\alpha^2\right]-m_4^2\left[1-c_\alpha^2-s_\alpha^2\right]\right)c_\infty = 0, \tag{S.16}$$

kur c_∞ žymi diverguojantį narį. Atkreipiame dėmesį į tai, kad (S.15) užrašyta masė yra pilna vienos kilpos CMS pernormuota masė. Tai, kad

[3]Šį papildomą laisvės laipsnį mes apribojame vėliau, po pernormavimo sąlygų išreiškimų.



P2 pav.: Šešios diagramos, kompaktiškai sudėtos viena ant kitos, atvaizduojančios m_2 kilpos pataisas, užrašytas (S.15).

šiai masei nėra kontranario, yra šio modelio ypatybė. Taip pat ši masė nepriklauso nuo FJ schemos pritaikymo, nes ν_2 neturi plikos masės. Rezultatas (S.15) yra gautas tiek tiesiogiai taikant Feynmano diagramas, tiek naudojantis SARAH, FeynArts bei FormCalc paketais. Tokiu būdu rezultatas (S.15) pasitarnauja kaip papildomas modelio įvedimo į programinius paketus patikrinimas prieš skaičiuojant sudėtingesnes išraiškas.

Visiškai kita situacija yra pernormuojant m_3 ir m_4. Plikos masės nėra nulinės – jos surištos su pernormuotomis masėmis per pernormavimo konstantas kaip parodyta (S.13). Pernormavimo konstantos yra išreiškiamos iš CMS sąlygų, kartu taikant FJ schemą. Kad lengviau suprastume kalibruotinių narių atskyrimą FJ schemoje, palyginkime ją su standartine buožgalvių schema GN modelyje. Standartinėje schemoje plikos m_{03} ir m_{04} tenkina (S.6):

$$M_0 = m_{04} - m_{03} \quad \text{ir} \quad y_0^2 v_0^2 = 2 m_{03} m_{04} \,. \tag{S.17}$$

Įvedus pernormavimo konstantas (S.13) kiekvienam iš šių parametrų, gauname, kad jos yra susietos:

$$\delta_{m3} + \delta_{m4} = 2\left(\delta_v + \delta_y\right) \,, \tag{S.18}$$

$$m_4 \delta_{m4} - m_3 \delta_{m3} = \left(m_4 - m_3\right) \delta_M \,. \tag{S.19}$$

Iš lygties (S.18) matome, kad masių kontranariai turi savo apibrėžime δ_v, kuri yra išreikšta iš buožgalvio sąlygų lygtyje (S.14). Vadinasi, masių kontranariai turi priklausomybę nuo kalibruotės, kurios pavaizduotos P1 diagramose. Lygtis (S.19) δ_v nario neturi, taigi, galima tikėtis, kad ji neturi ir kalibruotinės priklausomybės.

FJ schemoje vietoje S.17 mes užrašome:

$$M_0 = m'_{04} - m'_{03} \,, \quad y_0^2 v^2 = 2 m'_{04} m'_{03} \,. \tag{S.20}$$

T. y., apibrėžiame plikas mases per „teisingą“ Higgs'o VEV (šiuo atveju per vienoje kilpoje pernormuotą VEV). FJ schemoje masių kontranariai yra:

$$\delta'_{m3} + \delta'_{m4} = 2\delta_y \,, \tag{S.21}$$

$$m_4\delta'_{m4} - m_3\delta'_{m3} = (m_4 - m_3)\,\delta_M \,, \tag{S.22}$$

taigi, masių kontranariai nebeturi δ_v savo apibrėžimuose. Taip pat matome, kad (S.22) ir (S.19) išraiškos yra identiškos tiek FJ, tiek standartinėje schemoje. Tai galima paaiškinti tuo, kad Majorana masė M nekyla iš Higgs'o VEV, taigi, išraiškos, priklausančios tik nuo M, nesikeičia dėl skirtingo skaliarinio sektoriaus pernormavimo.

Išreiškiant masės kontranarius iš CMS sąlygų bei pasinaudojant FJ schema, mes gauname:

$$\begin{aligned} \delta'_{mi} &= \frac{1}{2}\left(\Sigma^{[1]}_{\nu_i\nu_i} + \Sigma^{[1]}_{\bar{\nu}_i\bar{\nu}_i} + \Sigma^{[1]}_{\nu_i\bar{\nu}_i} + \Sigma^{[1]}_{\bar{\nu}_i\nu_i}\right)\Big|_{p^2=m_i^2} - \frac{\Delta}{m_i}\,, \\ \Delta &= 2\frac{m_4 m_3 \delta_v}{m_4 + m_3}\,,\ \text{kai } i = 3,4\,, \end{aligned} \tag{S.23}$$

kur skliaustuose yra pažymėtos bedimensinės dviejų taškų Green'o funkcijos (savienergijos) visoms keturioms ν_i bei $\bar{\nu}_i$ kombinacijoms vienoje kilpoje, taške $p^2 = m_i^2$. Mes suskaičiavome, kad išraiška (S.23) vienoje kilpoje nepriklauso nuo kalibruotės pasirinkimo, t. y. kalibruotiškai priklausomi Δ nariai tiksliai susiprastina su savienergijų kalibruotiškai priklausomais nariais. Taigi, mes apibrėžiame kalibruotiškai nepriklausomas tiek pernormuotas, tiek plikas neutrinų mases GN modelyje. Iš (S.23) kalibruotinės nepriklausomybės išplaukia, kad Yukawa sąveikos konstantos y bei Majorana masės M pernormavimai yra taip pat kalibruotiškai nepriklausomi. Atkreipiame dėmesį į tai, kad FJ schemoje atsirandantis Δ yra tas pats abejoms masėms m_3 ir m_4. Taip yra dėl to, kad GN modelyje neutrinų sąveika su Higgs'o lauku aprašoma tik viena verte y.

Skaičiavimų atlikimui buvo pasinaudota SARAH, FeynArts ir FormCalc paketais. Generuojant GN modelio FeynArts failą, buvo pasinaudota SARAH. Su FeynArts buvo sugeneruotos reikalingos diagramos, o su FormCalc jos buvo išreikštos per standartinius Passarino–Veltman integralus. Algebriniai išraiškų suprastinimai, naudojantis modelyje apibrėžtomis parametrų lygybėmis, buvo atlikti Mathematica aplinkoje.

Išvados

Disertacijoje pristatome vienos kilpos GN modelio neutrinų masių pernormavimo kalibruotinės priklausomybės tyrimą. Užtikrinti pernormuotų masių nepriklausomumą nuo kalibruotės pasirinkta kompleksinės masės schema neutrinų masių pernormavimui. Parodome, kaip radiacinė masė įeina į bendrą CMS formalizmą bei pateikiame argumentus, kodėl ji būtinai turi būti kalibruotiškai nepriklausoma ir baigtinė. Iš to išplaukia pirmasis disertacijos ginamasis teiginys. Mes jį įrodome tiesiogiai apskaičiuodami GN modelio neutrino masę m_2 vienoje kilpoje.

Pernormuojant m_3 ir m_4 vien tik CMS sąlygos neužtikrina šių masių kontranarių kalibruotinę nepriklausomybę. Mes pritaikome FJ schemą kalibruotiškai nepriklausomiems kontranariams apibrėžti. Kadangi tai dar nebuvo atlikta GN modelyje, mes patikriname, ar FJ schema yra taikytina šiuo atveju, t. y. ar tikrai FJ schemos pritaikymas panaikina kalibruotines priklausomybes neutrinų kontranariams. Kadangi singletinio neutrino Majorana masė nėra susijusi su elektrosilpnosios simetrijos pažeidimu, šis neutrino singletas nepakeičia FJ schemos pritaikymo būdo. Taigi, singleto Majorana masės pernormavimas yra kalibruotiškai invariantiškas savaime. Mes taip pat parodome, kad m_3 ir m_4 pernormavime išskirta kalibruotiškai priklausoma dalis yra vienoda abejoms masėms. Taip yra dėl to, kad neutrinų sąveika su pirmuoju Higgs'o dubletu Higgs'o bazėje aprašoma vienintele konstanta y. Tam, kad įrodytume tokios konstrukcijos tinkamumą, mes patikriname, ar kalibruotinės priklausomybės išsiprastina iš išraiškų, naudojant SARAH, FeynArts bei FormCalc paketus.

Taikant CMS, pernormuotas Lagranžianas tampa priklausomas nuo kompleksinių masių ir sąveikos konstantų. Kadangi CMS yra analitinis OS tęsinys, tikėtina, kad algebrinė struktūra kilpos pataisose yra tokia pat taikant abejas schemas. Jeigu tai tiesa, algebriniai prastinimai, naudojant prielaidą, kad masės ir sąveikos konstantos yra realios, turėtų duoti korektiškas kontranarių algebrines išraiškas ir CMS schemoje. Vadinasi, mes galime analitiškai pratęsti mases ir sąveikos konstantas į kompleksinę plokštumą po to, kai išraiškų algebriniai prastinimai yra atlikti. Praktikoje tai reiškia, kad galime įprastai ir be papildomų modifikacijų naudoti FeynArts modelių failus kontranarių išraiškoms skaičiuoti, o vėliau jas naudoti CMS kontekste. Mes pristatome, kaip interpretuoti CMS per Lagranžiano narius, tam įvesdami pernormuotų ermitiškai sujungtinių narių simbolį $H.c.^*$. Taip pat pristatome, kaip algebrinė struktūra yra išlaikoma trikdžių pataisose GN modelyje. Kadangi FJ schemos pritaikymas yra algebrinė procedūra, CMS gali būti naudojama kartu su FJ schema. Tokiu būdu kalibruotiškai invariantiškai apibrėžiamos tiek plikos, tiek pernormuotos masės. Mes tai įgyvendiname GN modelio neutrinams.

Terminų žodynėlis

Medis, medžio lygmuo (angl. tree, tree level) – naudojamas nusakyti kvantinėje lauko teorijoje nulinės eilės trikdžių teorijos artinį. Nulinis artinys, vadinamas medžio lygmeniu, nes tame artinyje piešiamos Feynman'o diagramos atrodo kaip išsišakojęs medis. Trumpinant, siūlau nevengti sakyti tiesiog „medyje“ pvz.: skaičiavimai atlikti medyje, kas reikštų, kad skaičiavimai atlikti taikant nulinės eilės trikdžių teorijos artinį. Taip pat suskaičiuoti dydžiai gali būti „medžio“ dydžiai, pvz., neutrino medžio masė reikštų nulinės eilės artinį neutrino masei.

Kilpa, kilpos lygmuo (angl. loop, loop level) – nusako aukštesnės eilės pataisas negu medis. Pataisos vadinamos kilpos lygmeniu, nes Feynman'o diagramos, braižomos aukštesnės eilės pataisose negu medyje, turi kilpas. Pagal kilpų skaičių galima pasakyti, kurios eilės pataisa yra nagrinėjama. Taigi pirmos, antros, trečios ir t. t. eilės pataisos vadinamos atitinkamai vienos, dviejų, trijų ir t. t. kilpų pataisomis. Terminas galėtų būti vartojamas taip pat kaip ir medžio terminas.

Buožgalvis (angl. tadpole) – Higgs'o lauko vakuumo tikėtiniausios vertės pataisos. Ši funkcija vadinama buožgalviu, nes Higgs'o VEV vienos kilpos pataisų Feynman'o diagramos atrodo kaip buožgalviai. Terminas gali būti naudojamas ir nusakyti bet kurios eilės Higgs'o VEV pataisoms. Higgs'o VEV minimizuoja potencialą, taigi, buožgalvių funkcijos nusako potencialo minimumo sąlygas.

Kontranariai (angl. counterterms) – pernormavimo procedūroje atsirandantys nariai, kuriuos reikia apibrėžti per pernormavimo sąlygas. Kontranariai panaikina singuliarumus iš kilpų pataisų. Jie atsiranda iš parametrų bei laukų pernormavimo konstantų. Pernormavimo sąlygose jie yra apibrėžiami (užfiksuojami), kaip „atsveriantys“ diverguojančius narius, atsirandančius kilpose, taigi, priešdėlis „kontra“ nusako tą „atsvėrimą“.

Sverto mechanizmas (angl. seesaw mechanism) – sverto mechanizmai yra vieni iš neutrinų masių modelių. Juose postuluojamos dalelės, kurios sąveikaudamos su neutrinais suteikia neutrinams mases. Neutrinų masės šiose sąveikose tampa atvirkščiai proporcingos postuluotųjų dalelių masėms. Taigi, jei postuluojamos dalelės turi dideles mases, neutrinai tampa mažų masių. Ši priklausomybė yra vaizdžiai apibūdinama sverto įvaizdžiu, kur sunkios dalelės ant vienos sverto pusės „nusveria“ lengvus neutrinus esančius kitoje sverto pusėje.

Pliki parametrai / laukai (angl. bare masses / fields) – parametrai / laukai neapibrėžti per pernormavimo sąlygas. Pliki parametrai yra tiesiog laisvieji parametrai Lagranžiane, tačiau tik netiesiogiai atitinkantys fizikinius dydžius, skaičiuojamus kilpose. Tuo tarpu pernormavimo sąlygos suriša pernormuotus parametrus su fizikiniais dydžiais, tarsi „aprengia fizikine prasme“. Laukai taip pat vadinami plikais arba pernormuotais analogiškai.

Antmasės schema (angl. on-shell scheme arba OS) – pernormavimo schema, kurioje pernormavimo taškai yra eksperimentiškai matuojamos dalelių masės. Tuose taškuose yra apibrėžiamos pernormuotos masės bei laukai. Taigi, OS schemoje pernormavimas vyksta „ant“ masių (angl. on mass shell).

Literatūra

[1] C. Athanassopoulos et al. Phys. Rev. Lett. **81**, 1774–1777, 1998.

[2] Y. Fukuda et al. Phys. Rev. Lett. **81**, 1562–1567, 1998.

[3] K. Eguchi et al. Phys. Rev. Lett. **90**, 021802, 2003.

[4] Q. R. Ahmad et al. Phys. Rev. Lett. **89**, 011301, 2002.

[5] P. Minkowski. Phys. Lett. B **67**, 421–428, 1977.

[6] T. Yanagida. In *Proceedings: Workshop on the Unified Theories and the Baryon Number in the Universe, Tsukuba, Japan, 13-14 Feb 1979*, volume C7902131, pages 95–99 (1979).

[7] R. N. Mohapatra and G. Senjanovic. Phys. Rev. Lett. **44**, 912, 1980.

[8] R. N. Mohapatra and G. Senjanovic. Phys. Rev. D **23**, 165, 1981.

[9] S. F. King. Rept. Prog. Phys. **67**, 107–158, 2004.

[10] G. Senjanovic. Riv. Nuovo Cim. **34**, 1–68, 2011.

[11] A. de Gouvêa. Ann. Rev. Nucl. Part. Sci. **66**, 197–217, 2016.

[12] J. Orloff, S. Lavignac, and M. Cribier, editors. *Seesaw mechanism. Proceedings, International Conference, SEESAW25, Paris, France, June 10-11, 2004* (2005), 2005.

[13] S. Chatrchyan et al. Phys. Lett. B **716**, 30–61, 2012.

[14] G. Aad et al. Phys. Lett. B **716**, 1–29, 2012.

[15] H. E. Haber and G. L. Kane. Phys. Rept. **117**, 75–263, 1985.

[16] J. E. Kim. Phys. Rept. **150**, 1–177, 1987.

[17] T. D. Lee. Phys. Rev. D **8**, 1226–1239, 1973.

[18] R. D. Peccei and H. R. Quinn. Phys. Rev. Lett. **38**, 1440–1443, 1977.

[19] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva. Phys. Rept. **516**, 1–102, 2012.

[20] M. Tanabashi et al. Phys. Rev. D **98**, 030001, 2018.

[21] D. Aristizabal Sierra and C. E. Yaguna. JHEP **08**, 013, 2011.

[22] W. Grimus and L. Lavoura. Phys. Lett. B **546**, 86–95, 2002.

[23] G. 't Hooft. Nucl. Phys. B **33**, 173–199, 1971.

[24] G. 't Hooft. Nucl. Phys. B **35**, 167–188, 1971.

[25] A. A. Slavnov. Theor. Math. Phys. **10**, 99–107, 1972.

[26] W. Grimus and H. Neufeld. Nucl. Phys. B **325**, 18–32, 1989.

[27] P. Gambino and P. A. Grassi. Phys. Rev. D **62**, 076002, 2000.

[28] J. Fleischer and F. Jegerlehner. Phys. Rev. D **23**, 2001–2026, 1981.

[29] W. Grimus and M. Löschner. JHEP **11**, 087, 2018.

[30] A. Sirlin. Phys. Rev. Lett. **67**, 2127–2130, 1991.

[31] A. Denner, L. Jenniches, J.-N. Lang, and C. Sturm. JHEP **09**, 115, 2016.

[32] A. Denner, S. Dittmaier, and J.-N. Lang. JHEP **2018**, 104, 2018.

[33] M. Krause. *On the Renormalization of the Two-Higgs-Doublet Model*. Master's thesis, KIT, Karlsruhe, TP, 2016.

[34] M. Krause, R. Lorenz, M. Muhlleitner, R. Santos, and H. Ziesche. JHEP **09**, 143, 2016.

[35] S. Davidson and H. E. Haber. Phys. Rev. D **72**, 035004. [Erratum: Phys. Rev.D72,099902(2005)], 2005.

[36] H. E. Haber and D. O'Neil. Phys. Rev. D **74**, 015018. [Erratum: Phys. Rev.D74,no.5,059905(2006)], 2006.

[37] H. E. Haber and D. O'Neil. Phys. Rev. D **83**, 055017, 2011.

[38] B. A. Kniehl and A. Pilaftsis. Nucl. Phys. B **474**, 286–308, 1996.

[39] B. A. Kniehl and A. Sirlin. Phys. Rev. D **77**, 116012, 2008.

[40] B. A. Kniehl. Phys. Rev. Lett. **112**, 071603, 2014.

[41] B. A. Kniehl. Phys. Rev. D **89**, 116010, 2014.

[42] B. A. Kniehl and A. Sirlin. Phys. Rev. D **85**, 036007, 2012.

[43] B. A. Kniehl and A. Sirlin. Phys. Rev. Lett. **81**, 1373–1376, 1998.

[44] B. A. Kniehl and A. Sirlin. Phys. Lett. B **440**, 136–140, 1998.

[45] B. A. Kniehl and A. Sirlin. Phys. Lett. B **530**, 129–132, 2002.

[46] D. Espriu, J. Manzano, and P. Talavera. Phys. Rev. D **66**, 076002, 2002.

[47] V. Dūdėnas and T. Gajdosik. Acta Phys. Pol. B **48**, 2243–2249, 2017.

[48] A. Denner and J.-N. Lang. Eur. Phys. J. C **75**, 377, 2015.

[49] A. Denner. Fortsch.Phys. **41**, 307–420, 1993.

[50] H. H. Patel and M. J. Ramsey-Musolf. JHEP **07**, 029, 2011.

[51] M. Sperling, D. Stöckinger, and A. Voigt. JHEP **07**, 132, 2013.

[52] M. Sperling, D. Stöckinger, and A. Voigt. JHEP **01**, 068, 2014.

[53] G. Passarino and M. J. G. Veltman. Nucl. Phys. B **160**, 151–207, 1979.

Publications of doctoral dissertation

I V. Dūdėnas and T. Gajdosik. Lith. J. Phys. **56**, 149–163, 2016

II V. Dūdėnas and T. Gajdosik. Acta Phys. Pol. B **48**, 2243–2249, 2017

III V. Dūdėnas and T. Gajdosik. Phys. Rev. D **98**, 035034, 2018

I

Feynman rules for Weyl spinors with mixed Dirac and Majorana mass terms

Vytautas Dūdėnas and Thomas Gajdosik

Lithuanian Journal of Physics, **56**, 149–163, 2016.

Reprinted from the open access journal

Lithuanian Journal of Physics, Vol. **56**, No. 3, pp. 149–163 (2016)

FEYNMAN RULES FOR WEYL SPINORS WITH MIXED DIRAC AND MAJORANA MASS TERMS

V. Dūdėnas and T. Gajdosik

Department of Theoretical Physics, Faculty of Physics, Vilnius University, Saulėtekio 9, LT-10222 Vilnius, Lithuania
E-mail: vytautasdudenas@inbox.lt; tgajdosik@yahoo.com

Received 23 May 2016; revised 8 July 2016; accepted 23 September 2016

We present a basic formalism for using the Weyl spinor notation in Feynman rules. We focus on Weyl spinors with mixed Dirac and Majorana mass terms. To clarify the definitions we derive the Feynman rules from the path integral and present two examples: loop corrections for a fermion propagator and a tree level analysis of a seesaw toy model.

Keywords: Weyl spinors, Feynman rules, seesaw mechanism

PACS: 14.60.St, 14.60.Pq

1. Introduction

Despite the tremendous success of the Standard Model (SM), there is no doubt that it cannot be a complete theory due to numerous experimental evidences for which the SM fails to find an explanation. One of these experimental evidences is the observation of neutrino oscillations; for a short review of these experiments see references [1, 2]. The oscillations prove that at least two of the neutrinos have masses, but the original assumptions of the SM forbid these mass terms. So the neutrino sector and possibly the Higgs sector should be extended with some new degrees of freedom, i. e. new particles, to allow for the possibility that neutrinos have a mass.

The simplest "building blocks" for a fermionic particle content are Weyl spinors. Thus the model building is usually done in the Weyl spinor notation. However, if one looks at the standard textbooks on quantum field theory (QFT), like [3, 4], one can see that it is unusual to find a proper treatment for Weyl spinors. This is in contrast to supersymmetry (SUSY) references [5, 6]. As a result, non-SUSY calculations using Weyl spinors are somewhat absent in the literature, although the Weyl spinor formalism is known for an easier implementation on computer algebra systems [7]. This is not very surprising as we have only Dirac mass terms in the SM: the 4-component spinor formulation is way easier to deal with in this case. But considering a possible Majorana neutrino [8], theories with mixed Majorana and Dirac mass terms for fermions become relevant; for a review of seesaw mechanisms see [9]. Then the usual 4-component spinor techniques are not so transparent to understand the dynamics of mass mixing, whereas the Weyl spinor notation gives a natural diagrammatic approach to these cases, as we will see in Subsection 4.4.

The difficulty of using Weyl spinors also arises from having many possibilities of different conventions. We present the definitions, which are essential to understand these possibilities in Section 2. With the conventions from [5] we rederive Feynman rules in order to make these conventions visible by using the path integral approach in Section 3. We focus on the examples that are relevant for studying a seesaw model. This includes loop corrections for a Majorana particle in Subsubsection 3.4.2 and Subsection 4.3 and a diagrammatic approach of the seesaw itself in Subsection 4.4.

2. Weyl spinors

2.1. Definitions

The Weyl spinor [10, 11] is the fundamental representation of the group SU(2). The Lorentz group is homomorphic to $SU(2)_L \otimes SU(2)_R$ [12], where L and R are labels to distinguish the two subgroups. Particles fall into representations of these groups. A particle which is in the fundamental representation of the L subgroup and in the trivial representation of the R subgroup is called a "left-handed" spinor and has left chirality. The opposite is true for the "right-handed" spinor.

The two subgroups of the Lorentz group are related by Hermitian conjugation and by parity transformation [12]. So if we do a Hermitian conjugation or a parity transformation on a left-handed field, we get the field in the right-handed representation. Both of the two transformations flip the representations $L \leftrightarrow R$, but the Hermitian conjugation also makes a charge conjugation. For more detailed discussion see [11].

A Weyl spinor is anticommuting, so the symbols that give the spinor metric are antisymmetric. Denoting spinor indices by Latin letters, we write for left-handed spinors

$$\xi\chi \equiv \epsilon^{ba}\xi_a\chi_b , \tag{1}$$

where ϵ is the totally antisymmetric symbol, which takes a left-handed spinor into its dual space by the definition [5, 13]

$$\xi^a \equiv \epsilon^{ab}\xi_b,\ \xi_a \equiv \epsilon_{ab}\xi^b,\ \epsilon^{ab}\epsilon_{bc} \equiv \delta^a_c , \tag{2}$$

where δ is the Kronecker symbol. Note that the components of the spinor are Grassmannian (i. e. they anticommute). The Hermitian conjugation puts a spinor into the opposite handiness. The right-handed spinor index is written as a Latin letter with a dot, so $(\xi^a)^\dagger \equiv \xi^{\dagger\dot a}$. Doing Hermitian conjugation of the scalar product of left-handed spinors

$$(\xi\chi)^\dagger \equiv (\epsilon^{ba}\xi_a\chi_b)^\dagger = (\xi^b\chi_b)^\dagger = \chi^\dagger_{\dot b}\xi^{\dagger\dot b}, \tag{3}$$

and defining the raising and lowering of a right-handed index in a similar way as for the left-handed spinors

$$\xi^{\dot a} \equiv \epsilon^{\dot a\dot b}\xi_{\dot b},\ \xi_{\dot a} \equiv \epsilon_{\dot a\dot b}\,\xi^{\dot b},\ \epsilon_{\dot a\dot b}\,\epsilon^{\dot b\dot c} \equiv \delta^{\dot c}_{\dot a}, \tag{4}$$

we can write the definition of this metric:

$$\epsilon^{12} = \epsilon^{\dot 1\dot 2} = \epsilon_{21} = \epsilon_{\dot 2\dot 1} = 1 \text{ and } \epsilon^{21} = \epsilon^{\dot 2\dot 1} = \epsilon_{12} = \epsilon_{\dot 1\dot 2} = -1 . \tag{5}$$

Since the fields anticommute, we get

$$\xi\chi \equiv \xi^a\chi_a = -\chi_a\xi^a = -\epsilon^{ac}\epsilon_{ab}\chi^b\xi_c = \chi^a\xi_a = \chi\xi . \tag{6}$$

We define a summation convention for left-handed (undotted) spinor indices to sum from up to down. The Hermitian conjugation reverses this summation, hence dotted indices are summed from down to up; to conclude,

$$\xi\chi = \chi\xi \equiv \xi^a\chi_a = \chi^a\xi_a,\ \xi^\dagger\chi^\dagger = \chi^\dagger\xi^\dagger \equiv \xi^\dagger_{\dot a}\chi^{\dagger\dot a} = \chi^\dagger_{\dot a}\xi^{\dagger\dot a} . \tag{7}$$

2.2. Basic properties and the Lagrangian for Weyl spinors

The four components of a 4-vector can be written in the space of the direct product $SU(2)_L \otimes SU(2)_R$. Since the fundamental representation of SU(2) has 2 degrees of freedom, a 4-vector can be seen as the product of two fundamental representations of SU(2), i. e. two Weyl spinors. Hence we can find a connection that transforms two spinors from these two Lorentz's subgroups into a four-component vector in the Minkowski spacetime. The connection is

$$\xi^\dagger\bar\sigma^\mu\chi = \xi^\dagger_{\dot a}\bar\sigma^{\mu\dot a a}\chi_a = -\chi^a\sigma^\mu_{a\dot a}\xi^{\dot a} = -\chi\sigma^\mu\xi^\dagger \tag{8}$$

with the definition

$$\bar\sigma^{\mu\dot a a} = \epsilon^{ab}\epsilon^{\dot a\dot b}\sigma^\mu_{b\dot b} . \tag{9}$$

These connections can be written as

$$\bar\sigma^{\mu\dot a a} = (I, -\vec\sigma)^{\dot a a},\ \sigma^\mu_{a\dot a} \equiv (I, \vec\sigma)_{\dot a a} , \tag{10}$$

where $\vec\sigma$ is a 3-vector of Pauli matrices, and I is the 2×2 identity matrix. The product in Eq. (8) is a 4-vector composed of two Weyl spinors. If we multiply it with some other 4-vector, we will get a Lorentz scalar,

$$\xi^\dagger(A\cdot\bar\sigma)\chi = \xi^\dagger_{\dot a}A_\mu\bar\sigma^{\mu\dot a a}\chi_a = \epsilon_{\dot a\dot b}\xi^{\dagger\dot b}A_\mu\bar\sigma^{\mu\dot a a}\epsilon_{ab}\chi^b$$
$$= -\chi^b A_\mu\sigma^\mu_{b\dot b}\xi^{\dagger\dot b} = -\chi(A\cdot\sigma)\xi^\dagger , \tag{11}$$

where in the first and the last equality the summation convention is being used, which holds for these sigma symbols as well. Assuming A_μ represents a vector field, Eq. (11) forms a valid spinor–vector interaction term in a Lagrangian.

Free field terms in the Lagrangian must be bilinear and Hermitian. Given Eq. (7), it is easy to write down the mass term for a single Weyl spinor. We can write the mass term for a single left-handed spinor ξ^L as

$$\mathcal{L}_{\mathrm{M}} = -\frac{1}{2}M\left(\xi^L\xi^L + \xi^{L\dagger}\xi^{L\dagger}\right). \tag{12}$$

The parameter M is made real by absorbing its phase in the Weyl spinor. The term defined by Eq. (12) is called a Majorana mass term. A single Weyl spinor with such a mass term is called a Majorana particle, since from this Weyl spinor one can construct a four-component Majorana spinor. The factor of $\frac{1}{2}$ is conventional, to avoid additional numerical factors in amplitudes due to the symmetry of this coupling. Another possible invariant mass term can couple two Weyl spinors:

$$\mathcal{L}_D = -m_D \xi^{R\dagger}\chi^L - m_D^\dagger \chi^{L\dagger}\xi^R. \quad (13)$$

These terms are called Dirac mass terms. If two Weyl spinors share a Dirac mass term and do not have a Majorana mass term, they are usually combined into one Dirac spinor, which is nothing more than two Weyl spinors with the same mass. Such particles are called Dirac particles. It is also possible for Weyl spinors to have all those mass terms, with the result that the Weyl spinors are not in their mass eigenstates. In this case, the diagonalized mass matrix in general will give two different masses for two different Weyl spinors. Those are often called two Majorana particles, since the diagonalized mass terms can be written as in Eq. (12). The difference between Dirac and Majorana fermions is discussed in [11].

From Eq. (11) we see how a vector connects to spinors. The partial derivative ∂_μ is also a vector. So we can write Eq. (11) with ∂_μ in place of the vector to form the kinetic term

$$\mathcal{L}_K = \mathrm{i}\xi^{L\dagger}\bar{\sigma}^\mu \partial_\mu \xi^L, \quad (14)$$

which is Hermitian up to a total derivative, which does not affect dynamics:

$$\mathcal{L}_K = \mathrm{i}\xi^{L\dagger}\bar{\sigma}^\mu \partial_\mu \xi^L = \mathrm{i}\xi^L \sigma^\mu \partial_\mu \xi^{L\dagger} + \text{total derivative}. \quad (15)$$

Note the chirality structure of this term. If we recover indices, we see that

$$\bar{\sigma}^{\mu\dot{a}a}\partial_\mu \xi^L_a = \psi^{\dot{a}} \quad (16)$$

has only a dotted index, which means that acting on a spinor with $\sigma\partial$ or $\bar{\sigma}\partial$ gives a spinor that has the opposite chirality than the spinor the operators were acting on.

The superscript L that we used in ξ^L and χ^L is just a name of the field. We stick to this convention for naming left-handed spinors with superscript L and right-handed spinors with superscript R that correspond to particles and not to antiparticles. So $\xi^{L\dagger}$ is in the right-handed representation, but it is purely our convention that we call $\xi^{L\dagger}$ an antiparticle. All the results that are obtained for the charge conjugated left-handed spinor apply for a right-handed spinor and vice versa. The chirality is all what matters in taking care of the algebra in this formulation. Hence, keeping track of indices without suppressing them is often useful in order to make less mistakes. Whenever we do not use spinor indices, recall the summation conventions shown in Eq. (7).

A lot of spinor algebra relations consistent with these definitions can be found in [5]. For our purpose, we only need

$$[\sigma^\mu\bar{\sigma}^\nu + \sigma^\nu\bar{\sigma}^\mu]_\alpha^{\ \beta} = 2g^{\mu\nu}\delta_\alpha^{\ \beta}, \quad (17)$$

$$[\bar{\sigma}^\mu\sigma^\nu + \bar{\sigma}^\nu\sigma^\mu]^{\dot{\alpha}}_{\ \dot{\beta}} = 2g^{\mu\nu}\delta^{\dot{\alpha}}_{\ \dot{\beta}}. \quad (18)$$

The spinor indices are suppressed using the summation convention of Eq. (7). $g^{\mu\nu}$ is the usual Minkowski metric, taken to be diag(1,–1,–1,–1). When connecting these symbols with spinor indices, one can connect only barred to unbarred sigmas. As we will see, this knowledge helps in choosing the right rule for writing amplitudes.

3. Propagators

When dealing with the path integral formulation, it is convenient to go to the momentum space. For this we need to define Fourier transformations of the fields. For the left-handed Weyl spinor ξ^L we define

$$\xi^L(x) = \int_k \mathrm{e}^{-\mathrm{i}kx}\xi^L(k),\ \xi^{L\dagger}(x) = \int_k \mathrm{e}^{+\mathrm{i}kx}\xi^{L\dagger}(k), \quad (19)$$

where

$$\int_k \equiv \int \frac{\mathrm{d}^D k}{(2\pi)^D},\ \int_x \equiv \int \mathrm{d}^D x, \quad (20)$$

and D is the number of spacetime dimensions. In the spirit of dimensional regularization D is set to 4 at the end of the calculations. The Dirac delta function in D dimensions is represented by the integral

$$(2\pi)^D \delta(k-k') = \int_x \mathrm{e}^{-\mathrm{i}x(k-k')}. \quad (21)$$

Using Eqs. (19) and (21) in the action $S = \int_x \mathcal{L}_K$, with $\mathcal{L}_K$ given in Eq. (14), we arrive at the action in the momentum space:

$$S_K = \int_x \mathrm{i}\xi^{L\dagger}(x)(\bar{\sigma}\cdot\partial)\xi^L(x) = \int_p \xi^{L\dagger}(p)(\bar{\sigma}\cdot p)\xi^L(p). \quad (22)$$

The Majorana mass term in the momentum space becomes

$$S_M = \int_x \{M\xi^L(x)\xi^L(x) + M^\dagger \xi^{L\dagger}(x)\xi^{L\dagger}(x)\}$$

$$= \int_p \{M\xi^L(-p)\xi^L(p) + M^\dagger \xi^{L\dagger}(-p)\xi^{L\dagger}(p)\}. \quad (23)$$

For a Dirac particle, where we have 2 Weyl spinors, we could define L and R fields to Fourier transform in the same way. Then we would arrive at a Fourier transformed action, where all fields are expressed in the same p direction. But to have the same appearance of the p dependence as in the Majorana case, we rather keep a definition for all left-handed fields the same, i. e. we require $\xi^{R\dagger}$ to Fourier transform the same as ξ^L, and define

$$\xi^R(x) = \int_k e^{+ikx}\xi^R(k), \quad \xi^{R\dagger}(x) = \int_k e^{-ikx}\xi^{R\dagger}(k). \tag{24}$$

For two Weyl spinors ξ^L and ξ^R, sharing the same Dirac mass, we have the momentum dependences

$$S_D = \int_p \{\xi^{L\dagger}(p)\,(\bar\sigma\cdot p)\xi^L(p) + \xi^{R\dagger}(-p)\,(\sigma\cdot p)\,\xi^R(-p)$$
$$-m_D\xi^{R\dagger}(-p)\,\xi^L(p) - m^\dagger{}_D\xi^R(-p)\,\xi^{L\dagger}(p)\}. \tag{25}$$

Note that we do not have the freedom of choosing the definition of the Fourier transformation in the Majorana case, since we have twice less degrees of freedom.

Eq. (25) can also be written in an alternative form as is evident from Eq. (15). We have

$$\int_p \xi^{L\dagger}(p)\,(\bar\sigma\cdot p)\,\xi^L(p) = -\int_p \xi^L(p)\,(\sigma\cdot p)\,\xi^{L\dagger}(p)$$
$$= +\int_p \xi^L(-p)\,(\sigma\cdot p)\,\xi^{L\dagger}(-p). \tag{26}$$

Using Eq. (26), the action of Eq. (25) can be written as

$$S_D = \int_p \{\xi^L(-p)\,(\sigma\cdot p)\,\xi^{L\dagger}(-p) + \xi^R(p)\,(\bar\sigma\cdot p)\,\xi^{R\dagger}(p)$$
$$-m_D\xi^{R\dagger}(-p)\,\xi^L(p) - m_D^\dagger\xi^R(-p)\,\xi^{L\dagger}(p)\}. \tag{27}$$

As we will see, the fact that we can write the kinetic term for a single Weyl spinor in two different ways (Eqs. (25) and (27)) results in the freedom of choosing one of two rules for a single propagator line.

We introduce source functions to the Lagrangian density in the position space as

$$J^L(x)\,\xi^L(x) + J^R(x)\,\xi^R(x) + \text{H.c.} \tag{28}$$

and we define the Fourier transformation of the source functions for left and right fields:

$$J^L(x) = \int_k e^{+ikx}J^L(k),\; J^R(x) = \int_k e^{-ikx}\xi^R(k). \tag{29}$$

The Fourier-transformed version of Eq. (28) then becomes

$$J^L(p)\xi^L(p) + J^R(-p)\,\xi^R(-p) + J^{L\dagger}(p)\xi^{L\dagger}(p)$$
$$+ J^{R\dagger}(-p)\,\xi^{R\dagger}(-p). \tag{30}$$

The definition for the derivation with respect to the source function is

$$\frac{\delta J_a(p_1)}{\delta J_b(p_2)} = \delta(p_1-p_2)\delta_a^b,$$
$$\frac{\delta J_{\dot a}^\dagger(p_1)}{\delta J_{\dot b}^\dagger(p_2)} = \delta(p_1-p_2)\delta_{\dot a}^{\dot b},\quad \frac{\delta J_a(p_1)}{\delta J_{\dot b}^\dagger(p_2)} = 0\,. \tag{31}$$

Since all Weyl spinors anticommute, this is true for the sources as well, i. e. $\{J^a, J^b\} = \{J^a, J^{\dagger\dot b}\} = \{J^{\dagger\dot a}, J^{\dagger\dot b}\} = 0$. It also holds for their derivatives.

3.1. Propagator definitions

A propagator is a 2-point correlation function. Given the path integral $Z(J) = \int[D\phi]e^{iS(J)}$, where $[D\phi]$ stands for a formal measure of all possible field configurations, and the action $S(J) = \int_x(\mathcal{L} + J\phi)$, the 2-point correlation function of some scalar field ϕ is given by

$$\langle 0|\phi(x)\phi(y)|0\rangle = Z^{-1}(J=0)\int[D\phi]\phi(x)\phi(y)e^{iS(J=0)}$$
$$= Z^{-1}(J)\int[D\phi]\frac{\delta}{i\delta J(y)}\frac{\delta}{i\delta J(x)}e^{iS(J)}|_{J=0}. \tag{32}$$

Since the correlation functions are evaluated at vanishing sources and the path integral is a function of sources, we abbreviate

$$Z \equiv Z(J, J^\dagger) \text{ and } Z| \equiv Z(J, J^\dagger)|_{J=J^\dagger=0}. \tag{33}$$

Modifying the propagator definition for Weyl spinors poses some complications mainly because of their anticommutativity properties. We consider a left-handed spinor ξ with an effective action $S(J, J^\dagger) = \int_x(\mathcal{L} + J^a\xi_a + \xi^\dagger_{\dot a}J^{\dot a})$. The product of $J\xi = \xi J$ is invariant, but there is an ambiguity in the sign if we differentiate with respect to the source function. Since we defined $+J^a\xi_a$ in the action, we have the property

$$\frac{\delta}{\delta J^a}(J^b\xi_b) = -(J^b\xi_b)\frac{\overleftarrow{\delta}}{\delta J^a} = \xi_a, \tag{34}$$

where the arrow indicates the direction of acting. This arrow is introduced in order to compare the definitions with [5], where this opposite direction of acting for source derivatives is frequently used. Remembering the summation convention for dotted indices, we have

$$\frac{\delta}{\delta J_{\dot a}}(J^\dagger_{\dot b}\xi^{\dagger\dot b}) = -(J^\dagger_{\dot b}\xi^{\dagger\dot b})\frac{\overleftarrow{\delta}}{\delta J^\dagger_{\dot a}} = \xi^{\dagger\dot a}. \tag{35}$$

Given this, one can relate the definitions for propagators using source derivatives acting only from the left, with the definitions for propagators given in [5]. They are:

$$\langle 0|\,\xi^{\dagger\dot{a}}(x)\xi^{a}(y)|0\rangle = Z^{-1}\frac{\delta}{i\delta J_a(y)}\frac{\delta}{i\delta J^{\dagger}_{\dot{a}}(x)}Z\Big| = Z^{-1}\frac{\delta}{i\delta J_{\dot{a}}(x)}Z\frac{\overleftarrow{\delta}}{i\delta J^{\dagger}_{a}(y)}\Big| \quad (36)$$

$$\langle 0|\,\xi_a(x)\xi^{\dagger}_{\dot{a}}(y)|0\rangle = Z^{-1}\frac{\delta}{i\delta J^{\dagger\dot{a}}(y)}\frac{\delta}{i\delta J^{a}(x)}Z\Big| = Z^{-1}\frac{\delta}{i\delta J^{a}(x)}Z\frac{\overleftarrow{\delta}}{i\delta J^{\dagger\dot{a}}(y)}\Big|, \quad (37)$$

$$\langle 0|\,\xi_a(x)\xi^{b}(y)|0\rangle = Z^{-1}\frac{\delta}{i\delta J_b(y)}\frac{\delta}{i\delta J^{a}(x)}Z\Big| = Z^{-1}\frac{\delta}{i\delta J^{a}(x)}Z\frac{\overleftarrow{\delta}}{i\delta J_b(y)}\Big|, \quad (38)$$

$$\langle 0|\,\xi^{\dagger\dot{a}}(x)\xi^{\dagger}_{\dot{b}}(y)|0\rangle = Z^{-1}\frac{\delta}{i\delta J^{\dagger\dot{b}}(y)}\frac{\delta}{i\delta J^{\dagger}_{\dot{a}}(x)}Z\Big| = Z^{-1}\frac{\delta}{i\delta J_{\dot{a}}(x)}Z\frac{\overleftarrow{\delta}}{i\delta J^{\dot{b}}(y)}Z\Big|. \quad (39)$$

We use the full definition for propagators, where all corrections for a propagator are encoded in the path integral Z. The definitions for propagators in [5] are presented in the free field theory only, but the structure is the same. The only difference is the factor Z^{-1} in front of the expression, which we need to include to keep the right normalization of correlation functions in the presence of higher order corrections.

3.2. Propagator in momentum space

Since the momentum space is natural for Feynman diagram calculations, we will define the Fourier-transformed version of the previous propagator expression.

First consider a propagator for the left-handed field ξ of the form

$$\langle 0|\,\xi_a(x)\,\xi^{b}(y)|0\rangle = \left(\int[D\xi]\,\mathrm{e}^{\mathrm{i}S}\right)^{-1}\int[D\xi]\,\xi_a(x)\,\xi^{b}(y)\,\mathrm{e}^{\mathrm{i}S} = Z^{-1}\frac{\delta}{\mathrm{i}\delta J_b(y)}\cdot\frac{\delta}{\mathrm{i}\delta J^{a}(x)}Z\Big|. \quad (40)$$

We first take a look how derivatives with respect to source functions transform under Fourier transformations. Making use of the Fourier transformations defined in Eq. (19) and the chain rule for functional derivatives, which is just a generalization of $\frac{\partial}{\partial x_i}=\sum_j\frac{\partial y_j}{\partial x_i}\frac{\partial}{\partial y_j}$, we get

$$\frac{\delta}{\delta J^{L}(x)} = \int \mathrm{d}^{D}p\,\frac{\delta J^{L}(p)}{\delta J^{L}(x)}\frac{\delta}{\delta J^{L}(p)} = \int_p (2\pi)^{D}\frac{\int_{x_1}\mathrm{e}^{-\mathrm{i}px_1}\delta J^{L}(x_1)}{\delta J^{L}(x)}\frac{\delta}{\delta J^{L}(p)} = \int_p \mathrm{e}^{-\mathrm{i}px_1}\frac{(2\pi)^{D}\delta}{\delta J^{L}(p)}. \quad (41)$$

One can check that for the opposite chirality we have

$$\frac{\delta}{\delta J^{L\dagger}(x)} = \int_p e^{+ipx}\frac{(2\pi)^{D}\delta}{\delta J^{L\dagger}(p)}. \quad (42)$$

Putting Eq. (41) into Eq. (40) we get

$$\langle 0|\,\xi_a(x)\,\xi^{b}(y)|0\rangle = Z^{-1}\frac{\delta}{\mathrm{i}\delta J_b(y)}\cdot\frac{\delta}{\mathrm{i}\delta J^{a}(x)}Z = Z^{-1}\int_{p,p'}\mathrm{e}^{-\mathrm{i}px}\mathrm{e}^{-\mathrm{i}p'y}\frac{(2\pi)^{D}\delta}{\mathrm{i}\delta J_b(p')}\cdot\frac{(2\pi)^{D}\delta}{\mathrm{i}\delta J^{a}(p)}Z. \quad (43)$$

The propagator depends only on the spacetime difference $x–y$ and not on x and y separately. So the propagator should Fourier transform with a single factor of $x–y$. By rearranging exponents from Eq. (43) and adjusting the signs of the momentum to have the $\mathrm{e}^{-\mathrm{i}p(x-y)}$ factor in front, we get

$$\langle 0|\,\xi_a(x)\,\xi^{b}(y)|0\rangle = \int_p \mathrm{e}^{-\mathrm{i}p(x-y)} \times\left[Z^{-1}\int_{p'}\mathrm{e}^{-\mathrm{i}y(p-p')}\frac{(2\pi)^{D}\delta}{\mathrm{i}\delta J_b(-p')}\cdot\frac{(2\pi)^{D}\delta}{\mathrm{i}\delta J^{a}(p)}Z\Big|\right]. \quad (44)$$

This expression still depends on the spacetime point y, because we did not yet restrict the coordinate space propagator to depend only on the spacetime difference $x–y$. But the translational invariance of the action always gives this spacetime dependence for correlation functions, hence the correlation function is a translational invariant itself. To preserve this symmetry we need to have $p' = p$ in the momentum space; the additional exponent in the brackets of Eq. (44) will just give the identity all the time. There is also an additional integration $\int_{p'}$ which might seem strange at first glance. But actually, this integration is what is needed to set $p' = p$. To understand this, consider that we have two derivatives with respect to sources. Since the sources are set to zero, the terms that contribute from the action must also come as bilinear functions of sources. After differentiation with respect to the sources we should have two Dirac delta functions, recalling the definition of differentiation in Eq. (31). Because we have an action in the momentum space as $\int_\omega \mathcal{L}(\omega)$, we have only one integration measure coming from the action, which uses one Dirac delta function to fix one momentum. So the integration $\int_{p'}$ is needed to fix the other free momentum. Hence p' serves just as a dummy integration variable that matches dimensions and sets $p' = p$ at the end of the algebra. As a result of these considerations, we can safely omit the exponent in the brackets ($\mathrm{e}^{-\mathrm{i}y(p-p')} \to 1$) from our definitions for propagators. Furthermore, since we always have $p' = p$ in the end, there is no difference on what source function we put this integration variable: we can change $\frac{(2\pi)^D\delta}{\mathrm{i}\delta J_b(-p')}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^a(p)} \to \frac{(2\pi)^D\delta}{\mathrm{i}\delta J_b(-p)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^a(p')}$ in Eq. (44) without any consequence. However, the signs of the momenta depend on the choice of the Fourier transformation.

We define Fourier transformations for 2-point correlation functions to be

$$\langle 0|\,\xi_a(x)\,\xi^\dagger_{\dot a}(y)\,|0\rangle \equiv \int_p \mathrm{e}^{-\mathrm{i}p(x-y)}\langle 0|\,\xi_a\xi^\dagger_{\dot a}\,|0\rangle_{\mathrm{FT}(p)} \tag{45}$$

and

$$\langle 0|\,\xi_a(x)\,\xi^b(y)\,|0\rangle \equiv \int_p \mathrm{e}^{-\mathrm{i}p(x-y)}\langle 0|\,\xi_a\xi^b\,|0\rangle_{\mathrm{FT}(p)}, \tag{46}$$

where FT(p) labels that the Fourier transformed version of $\langle 0|\xi_a(x)\,\xi^b(y)|0\rangle$ depends only on the momentum p. The propagators in Eqs. (36) and (39) have an opposite chirality structure compared to those of Eqs. (45) and (46). We defined that fields that are of opposite chirality to each other transformed with the opposite momentum sign in Eq. (19). In order to be consistent with this definition, we have Fourier transformations for the propagators in Eqs. (36) and (39) with the opposite momentum sign relative to Eqs. (45) and (46). This is also consistent with momentum dependences in the free field actions of Eqs. (22), (23), (25) and (27). Given these definitions, we get the expressions for all four types of propagators:

$$\langle 0|\xi^{\dot a\dagger}\xi^a|0\rangle_{\mathrm{FT}(p)} = Z^{-1}\int_{p'}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J_a(p')}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^\dagger_{\dot a}(p')}Z\Big|, \tag{47}$$

$$\langle 0|\,\xi_a\xi^\dagger_{\dot a}\,|0\rangle_{\mathrm{FT}(p)} = Z^{-1}\int_{p'}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{\dagger\dot a}(-p)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^a(-p')}Z\Big|, \tag{48}$$

$$\langle 0|\,\xi_a\xi^b|0\rangle_{\mathrm{FT}(p)} = Z^{-1}\int_{p'}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J_b(-p)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^a(p')}Z\Big|, \tag{49}$$

$$\langle 0|\,\xi^{\dagger\dot a}\xi^\dagger_{\dot b}|0\rangle_{\mathrm{FT}(p)} = Z^{-1}\int_{p'}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{\dagger\dot b}(p)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^\dagger_{\dot a}(-p')}Z\Big|. \tag{50}$$

Now we take two Weyl spinors, ξ^L and ξ^R, with the same Dirac mass that couples them together. Since we introduced a Fourier transform in such a way that ξ^L transforms the same as $\xi^{R\dagger}$, we can already write propagators for this Dirac particle by just relabelling fields and without changing momentum dependences:

$$\langle 0|\,\xi^{L\dagger\dot a}\xi^{La}|0\rangle_{\mathrm{FT}(p)} = Z^{-1}\int_{p'}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^L_a(p)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{L\dagger}_{\dot a}(p')}Z\Big|, \tag{51}$$

$$\langle 0|\xi^{R\dagger}_a\xi^R_{\dot a}|0\rangle_{\mathrm{FT}(p)} = Z^{-1}\int_{p'}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{R\dot a}(-p)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{R\dagger a}(-p')}Z\Big|, \tag{52}$$

$$\langle 0|\xi^{Lb}\xi^{R\dagger}_a|0\rangle_{\mathrm{FT}(p)} = Z^{-1}\int_{p'}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{R\dagger a}(-p)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^L_b(p')}Z\Big|, \tag{53}$$

$$\langle 0|\xi^R_{\dot b}\xi^{L\dagger\dot a}|0\rangle_{\mathrm{FT}(p)} = Z^{-1}\int_{p'}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{L\dagger}_{\dot a}(-p)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{R\dot b}(p')}Z\Big|. \tag{54}$$

These propagators can be written differently, for example, one can use the propagator $\langle 0|\xi^L_a\xi^{L\dagger}_{\dot a}|0\rangle_{FT(p)}$ instead of $\langle 0|\xi^{L\dagger\dot a}\xi^{La}|0\rangle_{FT(p)}$. The changes should be clear from Eqs. (47) to (50).

3.3. Propagator for a free field

Considering free fields, it is always possible to shift fields in the action in such a way that the field dependent part is separated from the source dependent part. To be more precise, consider we have a field ξ and we shift it to ξ' such that the path integral becomes

$$Z(J) = \int[D\xi]\mathrm{e}^{\mathrm{i}S(\xi,J)} = \int[D\xi']\mathrm{e}^{\mathrm{i}S(\xi')+\mathrm{i}S(J)} = N\mathrm{e}^{\mathrm{i}S(J)},$$
$$N = \int[D\xi']\mathrm{e}^{\mathrm{i}S(\xi')}. \tag{55}$$

The integration over fields gives just a constant factor N to the path integral $Z(J)$.

Now let us consider the possible shift for Weyl spinors. We use left- and right-handed fields ξ^L and ξ^R sharing a Dirac mass term. Then the fields are shifted by a linear combination of sources, i. e. the left-handed field will be shifted by a linear combination of left- and right-handed sources. From Subsection 2.2 we know that constructing something that is left-handed from an originally right-handed spinor can be done with σp. It is easier to see this if we restore spinor indices. When two spinors have only a Dirac mass, the shift for the left-handed field will have the form

$$\xi^L_a(p) \to \xi^L_a(p) + x\cdot(\sigma p)_{a\dot a}J^{R\dot a}(-p) + y\cdot J^L_a(p)\,, \tag{56}$$

where x and y are just some unknown constants. The minus sign in the momentum dependence comes from the fact that the propagation of the right-handed field in the negative time direction is the left-handed field in the positive time direction. The important thing is keeping track of the chirality. Having in mind our labelling of R and L, the shift for ξ^R is the same as for ξ^L except for interchanging the labels $R \leftrightarrow L$ and the connection $\sigma \to \bar{\sigma}$ due to the opposite chirality of ξ^R. In the case of one left-handed spinor with the Majorana mass only, we can identify $R \to L\dagger$ in Eq. 56).

It is possible to calculate the coefficients x and y by straightforwardly inserting the shift, Eq. (56), into the action and requiring terms that couple sources with fields to cancel. However, since this form of the shift includes the transformation between the left and right chiral states, it makes sense to combine the left and right chiral states into one 2-component vector, where the components are Weyl spinors. This is just the usual 4-component spinor in the chiral representation. How this is done one can find in the appendix of [5]. The source dependent part of the action for two Weyl spinors sharing a Dirac mass term is

$$\mathrm{i}S(J) = -\int_p \left\{ J^{L\dagger}(p)\frac{\mathrm{i}(\bar{\sigma}\cdot p)}{p^2-m_\mathrm{D}^2}J^L(p) + J^{R\dagger}(-p)\frac{\mathrm{i}(\sigma\cdot p)}{p^2-m_\mathrm{D}^2}J^R(-p) + \frac{\mathrm{i}m_\mathrm{D}^\dagger}{p^2-m_\mathrm{D}^2}J^{R\dagger}(-p)J^L(p) + \frac{\mathrm{i}m_\mathrm{D}}{p^2-m_\mathrm{D}^2}J^R(-p)J^{L\dagger}(p) \right\}. \tag{57}$$

Remembering the definitions of propagators in Eqs. (51) to (54) we get

$$\langle 0|\xi^{L\dagger}\xi^L|0\rangle_{\mathrm{FT}(p)} = \mathrm{i}\frac{\bar{\sigma}\cdot p}{p^2-m_\mathrm{D}^2}, \tag{58}$$

$$\langle 0|\xi^{R\dagger}\xi^R|0\rangle_{\mathrm{FT}(p)} = \mathrm{i}\frac{\sigma\cdot p}{p^2-m_\mathrm{D}^2}, \tag{59}$$

$$\langle 0|\xi^L\xi^{R\dagger}|0\rangle_{\mathrm{FT}(p)} = \mathrm{i}\frac{m_\mathrm{D}^\dagger}{p^2-m_\mathrm{D}^2}, \tag{60}$$

$$\langle 0|\xi^R\xi^{L\dagger}|0\rangle_{\mathrm{FT}(p)} = \mathrm{i}\frac{m_\mathrm{D}}{p^2-m_\mathrm{D}^2}. \tag{61}$$

Comparing Eq. (57) with Eqs. (25) and (27), we see that the same action can be written as

$$\mathrm{i}S(J) = -\int_p \left\{ J^L(-p)\frac{\mathrm{i}(\sigma\cdot p)}{p^2-m_\mathrm{D}^2}J^{L\dagger}(-p) + J^R(p)\frac{\mathrm{i}(\bar{\sigma}\cdot p)}{p^2-m_\mathrm{D}^2}J^{R\dagger}(p) + J^L(p)\frac{\mathrm{i}m_\mathrm{D}^\dagger}{p^2-m_\mathrm{D}^2}J^{R\dagger}(-p) + J^R(-p)\frac{\mathrm{i}m_\mathrm{D}}{p^2-m_\mathrm{D}^2}J^{L\dagger}(p) \right\}. \tag{62}$$

Eqs. (58) and (59) can be written in alternative forms by changing $p \to -p$ and exchanging $\sigma \leftrightarrow \bar{\sigma}$. Hence we conclude that in this notation

$$\frac{\mathrm{i}\bar{\sigma}\cdot p}{p^2-m_\mathrm{D}^2} \text{ is equivalent to } \frac{-\mathrm{i}\sigma\cdot p}{p^2-m_\mathrm{D}^2}. \tag{63}$$

If we have a Weyl spinor with a Majorana mass term, the action can be written as

$$\mathrm{i}S(J) = -\frac{1}{2}\int_p \left\{ J^{L\dagger}(p)\frac{\mathrm{i}(\bar{\sigma}\cdot p)}{p^2-M^2}J^L(p) + J^L(-p)\frac{\mathrm{i}(\sigma\cdot p)}{p^2-M^2}J^{L\dagger}(-p) + \frac{\mathrm{i}M}{p^2-M^2}J^L(-p)J^L(p) + \frac{\mathrm{i}M}{p^2-M^2}J^{L\dagger}(-p)J^{L\dagger}(p) \right\}. \tag{64}$$

It is obvious that the same equivalence for propagators shown in Eq. (63) holds too. Using this action we get only two independent propagators instead of four, but all four forms, as seen from Eqs. (47) to (50), are present.

3.4. Propagator for the interacting theory

In the previous section we saw the free field terms of the action. If we consider an interacting theory, we have an additional term S_int and the path integral becomes

$$Z \sim \mathrm{e}^{\mathrm{i}S_\mathrm{int}+\mathrm{i}S_\mathrm{free}} = \mathrm{e}^{\mathrm{i}S_\mathrm{int}}\mathrm{e}^{\mathrm{i}S_\mathrm{free}}. \tag{65}$$

Most of the models in particle physics are built to describe the interactions as a perturbative series of this expression. The only case when the perturbation theory is not applicable is when we have a bound state. Since we are interested in models that should describe interactions with neutrinos (which do not participate in such states), treating $\mathrm{i}S_\mathrm{int}$ as a perturbation is general enough. The free field term is expressed in terms of the source functions, so the interaction term then can be expressed as derivatives with respect to sources acting on the free field action: S_int is promoted to an operator $\hat{S}_\mathrm{int}$. Then the path integral becomes

$$Z \sim \mathrm{e}^{\mathrm{i}\hat{S}_\mathrm{int}}\mathrm{e}^{\mathrm{i}S_\mathrm{free}} = \left[1 + \mathrm{i}\hat{S}_\mathrm{int} + \frac{1}{2}\left(\mathrm{i}\hat{S}_\mathrm{int}\right)^2 + \dots\right]\mathrm{e}^{\mathrm{i}S_\mathrm{free}}. \tag{66}$$

Given the Lagrangian of a theory and using the definitions for propagators of Eqs. (47) to (50) or Eqs. (51) to (54), we can calculate corrections for the tree level propagators to the desired order. The first term of this expansion is just a free field approximation that we discussed in the previous subsection. The second term becomes zero after setting sources to zero as will become clear after we work out the expressions for $\mathrm{i}\hat{S}_\mathrm{int}$. The third term in the expansion gives a loop correction for the propagators. In the following subsection we summarize the expressions of $\mathrm{i}\hat{S}_\mathrm{int}$ for possible interactions with Weyl spinors.

3.4.1. Vertices

We will consider possible couplings that appear in renormalizable models in four dimensions. We will leave, however, the letter D in the exponents of phase space integrals, denoting the number of dimensions as a free parameter in our expressions. This emphasizes that dimensional regularization can be used before setting $D = 4$.

In four dimensions a spinor can couple to a vector or a scalar. The spinor–vector coupling term in the action is

$$\mathrm{i}S^V_{int}=+\mathrm{i}g\int_x \xi^L(x)\big(A(x)\cdot\sigma\big)\xi^{L\dagger}(x)$$
$$=-\mathrm{i}g\int_x \xi^{L\dagger}(x)\big(A(x)\cdot\bar{\sigma}\big)\xi^L(x). \tag{67}$$

This is the same term written in two different ways, where g is some coupling constant. We promote this term to an operator $\mathrm{i}\hat{S}^V_{\mathrm{int}}$ by changing the fields to the corresponding derivatives with respect to the sources. By making use of Eqs. (41) and (42) we go to the momentum space. The expressions for $\mathrm{i}\hat{S}_{\mathrm{int}}$ are

$$\mathrm{i}\hat{S}^V_{int} = +\mathrm{i}g\int_{p_1,p_2,p_3}(2\pi)^D\delta(p_1+p_2+p_3)\sigma^\mu_{a\dot{a}}$$
$$\times\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^\mu_A(p_1)}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{L\dagger}_{\dot{a}}(-p_3)}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^L_a(p_2)} \tag{68}$$

or

$$\mathrm{i}\hat{S}^V_{int} = -\mathrm{i}g\int_{p_1,p_2,p_3}(2\pi)^D\delta(p_1+p_2+p_3)\,\bar{\sigma}^{\mu\dot{a}a}$$
$$\times\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^\mu_A(p_1)}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{La}(p_2)}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{L\dagger\dot{a}}(-p_3)}\,, \tag{69}$$

where we restored spinor and vector indices. We see that there is a freedom in choosing the connection between the vector and the spinors, i. e. we can choose either $\bar{\sigma}$ or $-\sigma$ to write down the same vertex. The minus sign for the momentum comes from the definition of the Fourier transformation presented in Eqs. (19) and (24).

The spinor scalar coupling can come in two forms:

$$\frac{1}{2}y_1\phi\xi^L\xi^L+\text{H.c., and } y_2\phi\xi^{R\dagger}\xi^L+\text{H.c.} \tag{70}$$

Here the first term couples some scalar ϕ to the same Weyl spinor and the second term couples it to two different spinors. We will always introduce the factor $\frac{1}{2}$ in the definition of the coupling of a scalar with two spinors of the same field in order to cancel additional combinatorial factors that appear due to the symmetry of this term. We take ϕ to be a complex scalar for generality, so that we have a complex coupling constant.

Using Eq. (41) we get $\mathrm{i}\hat{S}^S_{\mathrm{int}}$ for the scalar case in terms of the source functions

$$\mathrm{i}\hat{S}^S_{int}=\frac{1}{2}\mathrm{i}y_1\int_{p_1,p_2,p_3}(2\pi)^D\delta(p_1+p_2+p_3)$$
$$\frac{(2\pi)^D\delta}{\mathrm{i}\delta J_\phi(p_1)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{La}(p_2)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^L_a(p_3)} \tag{71}$$

or

$$\mathrm{i}\hat{S}^S_{\mathrm{int}} = \mathrm{i}y_2\int_{p_1,p_2,p_3}(2\pi)^D\delta(p_1+p_2+p_3)$$
$$\times\frac{(2\pi)^D\delta}{\mathrm{i}\delta J_\phi(p_1)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{R\dagger a}(p_2)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^L_a(p_3)}\,. \tag{72}$$

The Hermitian conjugate of $\mathrm{i}\hat{S}^S_{\mathrm{int}}$ just gives the Hermitian conjugate coupling constants and opposite signs for the momentum dependences in the source functions.

3.4.2. The spinor–vector loop

Now we can calculate corrections to all propagators defined in Eqs. (51) to (54). We see that the term linear in $\mathrm{i}S_{\mathrm{int}}$ vanishes for a propagator after setting sources to zero, since all possible interaction terms acting on the free field part of the path integral will leave an odd number of sources. Therefore the loop correction comes from $\frac{1}{2}(\mathrm{i}\hat{S}_{\mathrm{int}})^2$ in Eq. (66). To see how the path integral formalism applies, we work out the example for the loop correction to the propagator defined in Eq. (51).

Consider the loop correction to a propagator for a fermion with only a Majorana mass and a vector boson in the loop. Let us call the spinor ξ^L and the vector A_μ. The free field term for this fermion is given in Eq. (64). The interaction operator then is either Eq. (69) or Eq. (68). Let us use the form of Eq. (68). Then the interaction operator to order $O(g^2)$ is

$$\frac{1}{2}\big(\mathrm{i}\hat{S}_{\mathrm{int}}\big)^2=\frac{1}{2}(\mathrm{i}g)^2\int_{p_1,p_2,p_3,k_1,k_2,k_3}(2\pi)^D\delta(p_1+p_2+p_3)(2\pi)^D\delta(k_1+k_2+k_3)$$
$$\times\sigma^\mu_{a\dot{a}}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^\mu_A(p_2)}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{L\dagger}_{\dot{a}}(-p_3)}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^L_a(p_1)} \tag{73}$$
$$\times\sigma^\nu_{b\dot{b}}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^\nu_A(k_2)}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{L\dagger}_{\dot{b}}(-k_3)}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^L_b(k_1)}.$$

We abbreviate the fermion propagators of the free field as

$$\bar{P}(p)=\frac{\mathrm{i}\bar{\sigma}\cdot p}{p^2-M^2},\quad P(p)=\frac{\mathrm{i}\sigma\cdot p}{p^2-M^2},\quad P_M(p)=\frac{\mathrm{i}M}{p^2-M^2}, \tag{74}$$

suppressing the spinor indices. We will use the letter G as an abbreviation for the boson propagator, which is an even function of the momentum:

$$G(p) = G(-p)\,. \tag{75}$$

Let us take A_μ to be neutral. In this context, it is enough to say that a neutral field is self-conjugate, i. e. it is its own antiparticle. On the Lagrangian level we have a factor of $\frac{1}{2}$ in front of the bilinear terms due to this extra symmetry, hence the free field term for a neutral boson after completing the squares is

$$\mathrm{i}S = -\frac{1}{2}\int_p J(-p)G(p)J(p)\,. \tag{76}$$

For the propagator of the A_μ field we write $G_{A\mu\nu}(p)$, which is also symmetric under $\mu \leftrightarrow \nu$.

With the definition of the propagator in Eq. (51), the correction to one loop order is

$$\langle 0|\,\xi^{L\dagger\dot{a}}\xi^{La}\,|0\rangle^{[2]}_{\mathrm{FT}(p)} = Z^{-1}\int_{p'}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^L_a(p)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{L\dagger}_{\dot{a}}(p')} \times\frac{1}{2}\left(\mathrm{i}\hat{S}_{\mathrm{int}}\right)^2 \mathrm{e}^{\mathrm{i}S_{\mathrm{free}}}\Bigg|\,, \tag{77}$$

where the number in the brackets of the superscript denotes the order of the correction. In Eq. (77) we have 8 derivatives with respect to the sources in total. Since the propagator is evaluated at vanishing sources, only the term $\frac{1}{4!}(\mathrm{i}S_{\mathrm{free}})^4$ will contribute from the expansion of the free field part of the path integral. Looking at Eq. (73) we see that in Eq. (77) we have derivatives with respect to 3 $J^{L\dagger}$s, 3 J^Ls and 2 J_As. The only non-vanishing terms are those that have the same number of sources. These contributions from $\frac{1}{4!}(\mathrm{i}S_{\mathrm{free}})^4$ are

$$\int_{\omega_1,\omega_2,\omega_3,\omega_4}\frac{4}{4!}\frac{1}{2}J_A G_A J_A\left(J^{L\dagger}\bar{P}J^L\right)^3 \ \text{or}$$
$$\int_{\omega_1,\omega_2,\omega_3,\omega_4}\frac{4!}{4!}\frac{1}{2}J_A G_A J_A\left(J^{L\dagger}\bar{P}J^L\right)\frac{1}{2}\left(J^L P_M J^L\right)\frac{1}{2}\left(J^{L\dagger}P_M J^{L\dagger}\right), \tag{78}$$

where ω_1, ω_2... are the momenta of different pairs of source functions. All the indices are contracted. The momentum dependences of the source functions can be seen in Eqs. (76) and (64). Acting with source derivatives we get 8 Dirac delta functions that are integrated over with the momenta ω_1, ω_2.... In the end one arrives at an expression that can be diagrammatically expressed as a Feynman diagram.

To see explicitly how this is done, we take the first term of Eq. (78) as an example:

$$\Pi = Z^{-1}\int_{p'}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^L_a(p)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^{L\dagger}_{\dot{a}}(p')} \times\frac{1}{2}\left(\mathrm{i}\hat{S}_{\mathrm{int}}\right)^2\int_{\omega_1,\omega_2,\omega_3,\omega_4}\frac{4}{4!}\frac{1}{2}J_A G_A J_A\left(J^{L\dagger}\bar{P}J^L\right)^3\Bigg|\,. \tag{79}$$

The easiest start is to differentiate this term first with respect to the vector bosons, since it can be done independently: $\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^\mu_A(p_2)}\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^\nu_A(k_2)}$. Remembering the definition of differentiation, Eq. (31), we get

$$\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^\mu_A(p_2)}\cdot\frac{(2\pi)^D\delta}{\mathrm{i}\delta J^\nu_A(k_2)}\int_{\omega_1}\frac{1}{2}J^\rho(-\omega_1)G_{A\rho\sigma}(\omega_1)J^\sigma(\omega_1)$$
$$=\int_{\omega_1}(2\pi)^D\delta(p_2+\omega_1)\,G_{A\mu\nu}(\omega_1)(2\pi)^D\delta(\omega_1-k_2) \tag{80}$$
$$=(2\pi)^D\delta(p_2+k_2)G_{A\mu\nu}(k_2).$$

Differentiating in the same manner with respect to the spinor sources from Eq. (73) and integrating with respect to the momenta ω_2, ω_3, and ω_4, we are left with 6 Dirac delta functions in total (2 coming from Eq. (73)). The integrations over all momenta, coming from $\mathrm{i}S_{\mathrm{int}}$ in Eq. (73), will connect these momenta to preserve momentum conservation. The last integration over p' completes it with setting $p' = p$ as discussed in Subsection 3.2. Π, shown in Eq. (79), becomes the sum of

$$B_1 = \bar{P}^{\dot{f}f}(p)\Big\{+\frac{1}{2}(2\pi)^D\delta(0)(\mathrm{i}g)^2 \times\int_{p_1,p_2}\bar{P}^{\dot{a}a}(p_1)\sigma^\mu_{a\dot{a}}D_{A\mu\nu}(0)\sigma^\nu_{b\dot{b}}\bar{P}^{\dot{b}b}(p_2)\Big\}\,, \tag{81}$$

$$B_2 = \bar{P}^{\dot{f}f}(p)\Big\{-\frac{1}{2}(2\pi)^D\delta(0)(\mathrm{i}g)^2 \times\int_{p_1,p_2}D_{A\mu\nu}(p_2)\sigma^\mu_{a\dot{a}}\bar{P}^{\dot{a}b}(p_1+p_2)\sigma^\nu_{b\dot{b}}\bar{P}^{\dot{b}a}(p_1)\Big\}\,, \tag{82}$$

$$T = -(\mathrm{i}g)^2\left[\bar{P}^{\dot{e}a}(p)\sigma^\mu_{a\dot{a}}\bar{P}^{\dot{a}f}(p)\right]D_{A\mu\nu}(0)\int_k\left[\sigma^\nu_{b\dot{b}}\bar{P}^{\dot{b}b}(k)\right], \tag{83}$$

and

$$L = +\bar{P}^{\dot{f}e}(p)\left[(\mathrm{i}g)^2\int_k\sigma^\nu_{e\dot{a}}\bar{P}^{\dot{a}a}(p+k)\sigma^\mu_{a\dot{d}}D_{A\mu\nu}(k)\right]\bar{P}^{\dot{d}f}(p)\,. \tag{84}$$

These terms are represented as Feynman diagrams in Fig. 1. Arrows on the lines show the flow of the left chirality, i. e. they point from dotted to undotted indices. The momentum flow is taken from left to right as shown with additional arrows near the momenta.

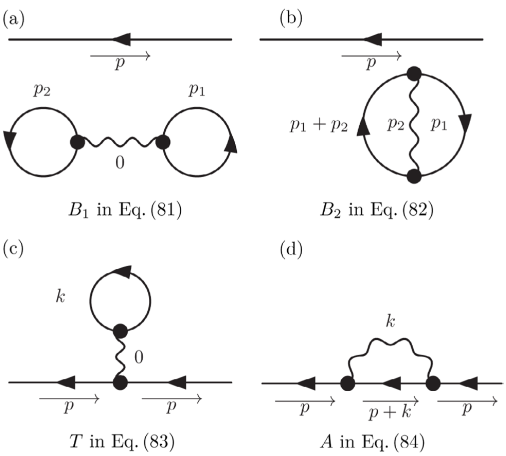


Fig. 1. Feynman diagrams showing the terms in Eqs. (81) to (84). The bubbles B_1 and B_2 shown in (a, b) are cancelled by the normalization of the path integral. T, the tadpole connected to the propagator, shown in (c), vanishes. A, shown in (d), is the contributing loop correction to the propagator.

The amplitudes in Eqs. (81) and (82) are the so-called vacuum bubbles. They are cancelled by Z^{-1} to all orders. To prove it for this order, see the $O(g^2)$ of Z at vanishing sources. By the same argument of matching the number of derivatives with the number of sources we get

$$\frac{1}{2}\left(\mathrm{i}\hat{S}_{\mathrm{int}}\right)^2 \mathrm{e}^{\mathrm{i}S_{\mathrm{free}}}\Big| = \frac{1}{2}\left(\mathrm{i}\hat{S}_{\mathrm{int}}\right)^2 \frac{1}{3!}\left(\mathrm{i}S_{\mathrm{free}}\right)^3 . \quad (85)$$

The only non-vanishing terms from $\frac{1}{2}\left(\mathrm{i}\hat{S}_{\mathrm{int}}\right)^2 \frac{1}{3!}\left(\mathrm{i}S_{\mathrm{free}}\right)^3$ are

$$\frac{1}{2}\left(\mathrm{i}\hat{S}_{\mathrm{int}}\right)^2 \int_{\omega_1,\omega_2,\omega_3} \frac{3}{3!}\frac{1}{2} J_A D_A J_A \left(J^{L\dagger}\bar{P}J^L\right)^2 \quad \text{and}$$

$$\frac{1}{2}\left(\mathrm{i}\hat{S}_{\mathrm{int}}\right)^2 \int_{\omega_1,\omega_2,\omega_3} \frac{3!}{3!}\frac{1}{2} J_A D_A J_A \frac{1}{2}\left(J^L P_M J^L\right)\frac{1}{2}\left(J^{L\dagger} P_M J^{L\dagger}\right). \quad (86)$$

Working out the first term, one arrives at the terms that are shown in the brackets of Eqs. (81) and (82).

There is also an interesting factor of $\frac{1}{2}$ in Eqs. (81) and (82), which stands for the symmetry factor of these diagrams, i. e. the diagram is identical if you change places of two identical fermion propagators or places of two vertices. Also, there is a minus sign in Eq. (82). This is due to the anticommutativity of fermions: each closed fermion loop gives a relative minus sign to the amplitude. Just as expected, the symmetry factors and the rule for closed fermion loops are the same as in the usual Feynman diagram calculus.

An interesting diagram is drawn from Eq. (83), which is a tadpole connected to a propagator. Note that it has a minus sign due to the closed fermion loop. This already looks strange from a physical perspective: the gauge boson of momentum 0 is vanishing into the vacuum. Since a vector has a Lorentz index, we might worry about the Lorentz invariance if this would contribute. But it does vanish: the propagator $\bar{P}(k)$ is an odd function of k, so the term $\int_k \left[\sigma^{\nu}_{b\dot{b}}\bar{P}^{\dot{b}b}(k)\right]$ gives 0 when integrating over all values of the momentum. Note that the propagator $P_M(k)$ is even: a tadpole diagram appearing with this propagator would give a contribution. This cannot happen with a gauge boson, but it appears in the interaction with scalars as will be discussed in Subsection 4.3. So we are left only with Eq. (84) contributing to the one loop correction.

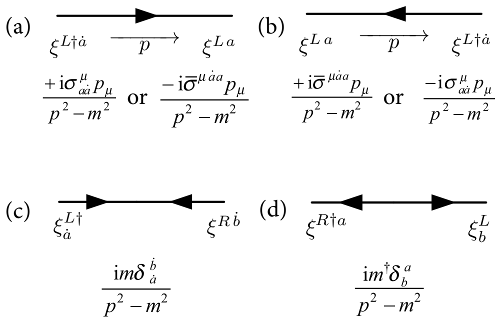


Fig. 2. Feynman diagrams and rules for propagators. These are all possible diagrams for propagators of Weyl spinors. The corresponding Feynman rule is written under each diagram. For the propagators shown in (a, b) one can choose between two possible rules. The mass term can be either Dirac or Majorana in these rules, but for the Majorana case one has to identify $\xi^R = \xi^{L\dagger}$ in (c, d). Propagators in (c, d) are even functions of the momentum, hence the direction of the momentum flow is irrelevant and not shown in the diagrams.

4. Feynman rules

4.1. Propagators and mass insertions

All the definitions in this paper are consistent with the definitions of [5]. To present Feynman rules for Weyl spinors, one has to include a chirality flow in the diagram. An arrow on the propagator line is defined to show the direction in which the left chirality flows, i. e. the arrow is directed from the dotted index towards the undotted index. Whenever a chiral symmetry breaking term appears (such as a mass), the directions of arrows indicate this by showing opposite directions in the diagram. This is in contrast with Feynman diagrams for 4-component spinors, where the direction of an arrow is defined as fermion flow, which has to be preserved all the time in order to have fermion number conservation. Feynman rules for propagators of Weyl spinors are shown in Fig. 2.

We first consider the propagators shown in Fig. 2(a, b). We draw an additional arrow near the propagator line showing the momentum flow. The definition of momentum flow is crucial for these propagators in order to assign the correct rule. To see why this is the case, recall Eq. (63). We have two alternative forms of writing down the expression for the same propagator and this form is related with the direction of momentum. This freedom can be understood comparing the two equivalent expressions for the same action shown in Eqs. (25) and (27) in terms of Weyl spinors or in Eqs. (57) and (62) in terms of sources functions. These alternative forms of writing down the same action are reflected in the rules shown in Fig. 2(a, b). We can go from one form to the other by either flipping the arrow of the propagator, or by changing the direction of the momentum. For Fig. 2(a), where the left chirality goes from left to right, we have the propagator $\sim \sigma p$ or by changing $p \to -p$ we have $\sim \bar{\sigma} p$. Equivalently, if we flip the direction of chirality as in Fig. 2(b), we have the propagator $\sim \bar{\sigma} p$ or $\sim -\sigma p$.

The propagators of Eqs. (60) and (61), shown in Fig. 2(c, d), exist only if the mass term is not zero. These propagators are even functions of the momentum, hence the direction of the momentum is not important. Since the mass term for fermions couples different chiral states, the direction of the arrow is not preserved along the propagator line for these propagators.

All the rules for propagators shown in Fig. 2 are obtained using the action of Eq. (57) and the definitions of the propagators from Eqs. (51) to (54). Alternatively, one could start from a chirality preserving action, where the mass terms are zero, and treat the mass terms as couplings. Then we have massless propagators as the first approximation in Fig. 2(a, b). Taking a Dirac mass term, Eq. (13), as a coupling, we get the Feynman rules shown in Fig. 3 with $m = m_\mathrm{D}$. Making an infinite sum of even numbers of mass insertions into the massless propagator for the Weyl spinor, we recover the mass term in the denominators of the propagators shown in Fig. 2(a, b). Making an infinite sum of odd numbers of mass insertions gives rise to the propagators of Fig. 2(c, d). If we have a Majorana mass as in Eq. (12) instead, we will have just the same rules of Fig. 3 with $m = M$. Making the infinite sums of these insertions will give all the same rules shown in Fig. 2 identifying $\xi^R = \xi^{L\dagger}$ and $m = M$.

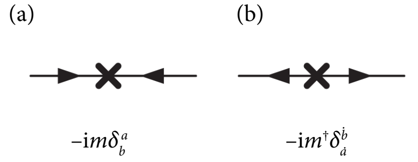


Fig. 3. Mass insertion diagrams and rules. These diagrams correspond to mass terms if they are treated as couplings. These rules can be used for either a Dirac or a Majorana mass term in the same way, i. e. $m = m_\mathrm{D}$ if we have a Dirac mass term as in Eq. (13) and $m = M$ if we have a Majorana mass term as in Eq. (12). The direction of arrows shows the chirality structure of the mass term. The momentum conservation along the line is understood. The direction of the momentum flow is irrelevant just as in Fig. 2(c, d).

4.2. Vertices

To define the set of rules for interactions with Weyl spinors, one just needs to understand the chirality structure of the interaction terms. The scalar–spinor interaction term changes chirality. Hence the arrows of the spinor lines point in opposite directions in the diagrams as shown in Fig. 4(b, c). The momenta are defined to flow into the vertex and the Dirac delta function of these momenta gives the momentum conservation at the vertex. In Fig. 4(b) we define the coupling constant y to come from the term that couples two left-handed spinors as in Eq. (70). Figure 4(c) is just the Hermitian conjugate of Fig. 4(b) with a coupling $y^\dagger$. If the scalar field is real, then one can define the phase of the spinors in such a way that $y^\dagger = y$.

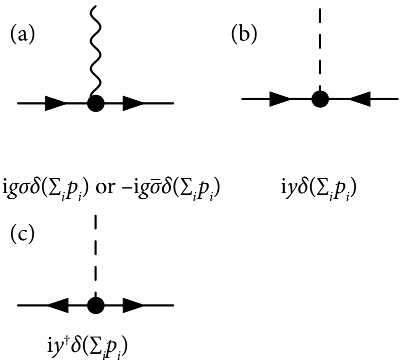


Fig. 4. Feynman diagrams and rules for vertices. The corresponding Feynman rule is written under each diagram. There are two possible rules for a vertex with a vector boson, as shown in Fig. 4(a). All momenta are defined to flow into the vertex, so that $\delta(\Sigma_i p_i)$ gives momentum conservation. The rules for vertices with scalars are shown in Fig. 4(b, c).

The spinor–vector interaction term, shown in Eq. (67), preserves chirality. Hence arrows on the propagator lines must show in the same direction for this coupling. There is also a freedom in choosing the connection: either σ or $\bar{\sigma}$, as seen in Eq. (67). These two rules are related by a relative minus sign. The rules, shown in Fig. 4(a), are consistent with this sign convention:

$$\mathcal{L}_{\text{int}} = +g\xi^{L}\sigma \cdot V\xi^{L\dagger} = -g\xi^{L\dagger}\bar{\sigma} \cdot V\xi^{L}. \tag{87}$$

Note that we also have a freedom in writing the expression for the propagator as shown in Fig. 4(a, b). But σ can only be connected with $\bar{\sigma}$ and vice versa as discussed in Subsection 2.2. Once we choose a rule for the vertex, we cannot choose the form of the propagator freely anymore. That means, if we choose a vertex as $\sim\bar{\sigma}$, both propagators must be $\sim\sigma$ to form a product $\sim\sigma\bar{\sigma}\sigma$. We illustrate this by an example in the next subsection.

4.3. Using Feynman rules: loop correction

To check the consistency of the rules and to present an example of using them, we derive Eq. (84) directly from the diagram shown in Fig. 5(b). This example helps to understand the property in Feynman rules for Weyl spinors that is not apparent in the usual Dirac spinor notation: the one to two correspondence between the diagram and the rules appearing in Fig. 4(a), Fig. 2(a) and Fig. 2(b). We use the same abbreviations for the free field propagators as in Eqs. (74) and (75). Taking the momentum flow from left to right, the rules presented in Fig. 2(b) tell us that we can choose either $\bar{P}(p)$ or $P(-p)$ for each fermion line. The rules for the vertex, shown in Fig. 4(a), give us the freedom to choose between $\mathrm{i}\sigma g\delta(\Sigma_i p_i)$ and $-\mathrm{i}\bar{\sigma} g\delta(\Sigma_i p_i)$. As noted at the end of the previous subsection, we can connect only barred to unbarred sigmas. We integrate over internal momenta of propagators which use up the delta functions that enforce momentum conservation at each vertex. So we are led to two possible ways to write this correction:

$$\bar{\Pi}(p) = \bar{P}(p)\left[\int_k (\mathrm{i}g\sigma)\,\bar{P}(p+k)D(k)(\mathrm{i}g\sigma)\right]\bar{P}(p) \tag{88}$$

or

$$\Pi(-p) = P(-p)\left[\int_k (-\mathrm{i}g\bar{\sigma})P(-p-k)D(k)(-\mathrm{i}g\bar{\sigma})\right]P(-p). \tag{89}$$

Because P and $\bar{P}$ are odd functions of the momentum, Π and $\bar{\Pi}$ are also odd functions. If we recover contracted indices, one can see that the functions Π and $\bar{\Pi}$ differ only by the index structure and this index structure is the same as for P and $\bar{P}$, respectively. The diagrams for these functions are presented in Fig. 5. The propagator shown in Fig. 2(b) together with its correction in Fig. 5(b) can be written as

$$\bar{P}(p) + \bar{\Pi}(p) \text{ or } -(P(p) + \Pi(p)), \tag{90}$$

whereas the diagram in Fig. 5(a) leads to a correction for a propagator shown in Fig. 2(a):

$$P(p) + \Pi(p) \text{ or } -(\bar{P}(p) + \bar{\Pi}(p)). \tag{91}$$

The corrections do not spoil the index structure and the properties under $p \to -p$ for correlation functions, which just means that we managed to consistently define Feynman rules. The freedom of choosing one of two rules for a vertex shown in Fig. 4(a) and for propagators shown in Fig. 2(a, b) at one loop order is reflected by the two functions for the same diagram as shown in Fig. 5. This justifies the freedom of choosing one of the two rules for the same propagator shown in Fig. 2(a, b) and for the vertex shown in Fig. 4(a) at one loop order.

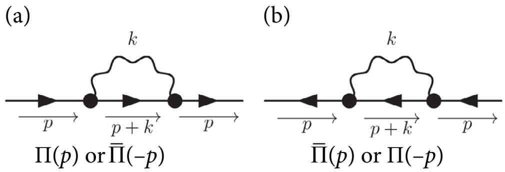


Fig. 5. Gauge loop corrections for the propagators shown in Fig. 2(a, b), respectively.

As discussed in Subsection 3.4.2, the vacuum bubbles do not contribute to the corrections. Also, the tadpole with a gauge boson connected to the propagator gives a vanishing result. The scalar tadpoles shown in Fig. 6 do not vanish. Usually one requires as a renormalization condition that these tadpoles cancel together with the tadpoles and counterterms arriving from corrections to the vacuum expectation value of the scalar field. However, it is important to note that other possibilities in defining renormalization conditions exist and, in principle, tadpoles can also be taken into the definition of a propagator.

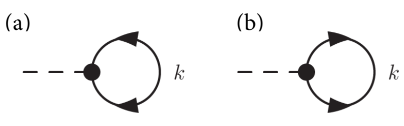


Fig. 6. Tadpole diagrams that give non-vanishing results.

The corrections for a single propagator shown in Fig. 2(a) has four possible forms. This is because there are four forms of propagators in the Weyl spinor notation, hence we are led to four possible combinations of external legs shown in Fig. 7.

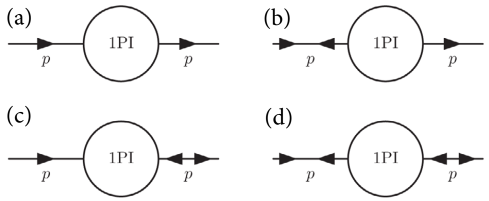


Fig. 7. All possible diagrams for correcting the propagator shown in Fig. 2(a). 1PI stands for the sum of all one particle irreducible diagrams.

4.4. Using the Feynman rules: example of the Seesaw

The seesaw mechanism [14–17] is an illustrative example for the usage of the sets of rules for Weyl spinors shown in Figs. 2, 3. Instead of looking at the seesaw extension of the SM, we consider a simplified toy model. We take two Weyl spinors ξ^L and ξ^R coupled with a Dirac mass term and we give a large Majorana mass to ξ^R, taking $M \gg m_\mathrm{D}$:

$$\mathcal{L}_m = -(m_\mathrm{D}\xi^L\xi^{R\dagger} + \mathrm{H.c.}) - \frac{1}{2}M(\xi^{R\dagger}\xi^{R\dagger} + \mathrm{H.c.})\,. \qquad (92)$$

We treat the Majorana mass term as a first approximation for the mass of ξ^R and the Dirac mass term as a coupling, shown in Fig. 3, which means that to the first approximation ξ^L is massless and does not have propagators like in Fig. 2(c, d).

The mass term m_D mixes the fields ξ^L and ξ^R. To estimate the size of this mixing consider the diagram shown in Fig. 3(a), which represents this mixing term. We can interpret this diagram as the field ξ^L transforming into ξ^R with the coupling of $(-\mathrm{i}m) = (-\mathrm{i}m_D)$. We take the positive momentum direction and assign propagators to external lines for ξ^L as in Fig. 2(a) with $m^2 = 0$ and for ξ^R as in Fig. 2(b) with $m^2 = M^2$. This correction reads

$$\frac{\mathrm{i}p\bar{\sigma}}{p^2-M^2}(-\mathrm{i}m_\mathrm{D})\frac{\mathrm{i}p\sigma}{p^2} = \frac{\mathrm{i}m_\mathrm{D}}{p^2-M^2} = \frac{m_\mathrm{D}}{M}\cdot\frac{\mathrm{i}M}{p^2-M^2}, \qquad (93)$$

where we used the property presented in Eq. (18) to get $(p\cdot\bar{\sigma})\,(p\cdot\sigma) = p^2$. Eq. (93) is an expression for the propagator for ξ^R, of the form shown in Fig. 2(c) with $m = M$ and an additional factor of $\frac{m_\mathrm{D}}{M}$. This means that the propagating field ξ^L transforms into ξ^R by a fraction $\sim\frac{m_\mathrm{D}}{M}$.

We further explore diagrams that give corrections to the ξ^L propagator. The correction arising from the Dirac mass term for a propagator of ξ^L is shown in Fig. 8(a). The diagram of Fig. 8(b) gives rise to a propagator of a form shown in Fig. 2(d) that is absent in the case when the Dirac mass term is neglected. Considering the case, where ξ^L is near its mass shell, we have $p^2 \ll M^2$. The diagram in Fig. 8(b), using the rules from Figs. 2, 3, gives

$$\frac{\mathrm{i}p\bar{\sigma}}{p^2}\cdot\mathrm{i}m_\mathrm{D}\cdot\frac{\mathrm{i}M}{-M^2}\cdot\mathrm{i}m_\mathrm{D}\cdot\frac{\mathrm{i}p\sigma}{p^2} = -\mathrm{i}\frac{m_\mathrm{D}^2}{M}\frac{1}{p^2} \equiv -\mathrm{i}m_\xi\frac{1}{p^2},\; m_\xi = \frac{m_\mathrm{D}^2}{M}, \qquad (94)$$

which is a new propagator for ξ^L. This expression is the first term in the infinite sum of

$$-\frac{\mathrm{i}m_\xi}{p^2-m_\xi^2} = -\frac{\mathrm{i}m_\xi}{p^2} - \frac{\mathrm{i}m_\xi}{p^2}\left(-\mathrm{i}m_\xi\right)\frac{\mathrm{i}m_\xi}{p^2} + \ldots, \qquad (95)$$

which we get when considering infinite copies of this diagram. Note that we have an opposite sign to the normal convention for this propagator.

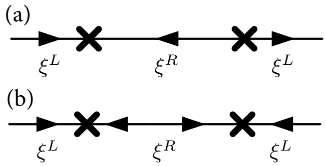


Fig. 8. Mass insertion diagrams for correcting the propagator of ξ^L.

Taking the diagram shown in Fig. 8(a) and using $p^2 \ll M^2$ we get

$$\frac{\mathrm{i}p\sigma}{p^2}\cdot\mathrm{i}m_\mathrm{D}\cdot\frac{\mathrm{i}p\bar{\sigma}}{-M^2}\cdot\mathrm{i}m_\mathrm{D}\cdot\frac{\mathrm{i}p\sigma}{p^2} = \mathrm{i}\frac{p\sigma}{p^2}\frac{(\mathrm{i}m_\mathrm{D})^2}{M^2}. \qquad (96)$$

Equivalently, considering the infinite sum of copies of this diagram, one gets

$$\mathrm{i}\frac{p\sigma}{p^2}\left(1+\left(\mathrm{i}\frac{m_\mathrm{D}}{M}\right)^2+\left(\mathrm{i}\frac{m_\mathrm{D}}{M}\right)^4\ldots\right) = \mathrm{i}\frac{p\sigma}{p^2\left(1+\frac{m_\mathrm{D}^2}{M^2}\right)}\,. \qquad (97)$$

From this propagator, we see that the field is rescaled by a factor of $\sqrt{1+\frac{m_\mathrm{D}^2}{M^2}}$.

Having the result from Eq. (93), we can define a new field, to include the admixture of ξ^R with a fraction of $-\frac{m_\mathrm{D}}{M}$:

$$\xi^L_\mathrm{new} = \xi^L - \frac{m_\mathrm{D}}{M}\xi^{R\dagger}. \qquad (98)$$

Then the transformation for $\xi^{R\dagger}$ reads

$$\xi^{R\dagger}_{\text{new}} = \frac{m_{\text{D}}}{M}\xi^{L} + \xi^{R\dagger} . \tag{99}$$

These fields are normalized up to the first order in $\frac{m_{\text{D}}}{M}$. This is evident from Eq. (97), which says that ξ^L rescales with a factor of

$$\sqrt{1+\frac{m_{\text{D}}^2}{M^2}} = 1 + O\left(\frac{m_{\text{D}}^2}{M^2}\right). \tag{100}$$

One can also check that inserting these redefinitions leaves the kinetic term unchanged up to the first order in $\frac{m_{\text{D}}}{M}$. Inserting the inverse transformation

$$\xi^{L} = \xi^{L}_{\text{new}} + \frac{m_{\text{D}}}{M}\xi^{R\dagger}_{\text{new}} , \tag{101}$$

$$\xi^{R\dagger} = -\frac{m_{\text{D}}}{M}\xi^{L}_{\text{new}} + \xi^{R\dagger}_{\text{new}} \tag{102}$$

into Eq. (92) we get

$$\begin{aligned}\mathcal{L}_m &= -\frac{1}{2}\left(-\frac{m_{\text{D}}^2}{M}\xi^{L}_{\text{new}}\xi^{L}_{\text{new}} + \text{H.c.}\right)\\ &-\frac{1}{2}M\left(\xi^{R\dagger}_{\text{new}}\xi^{R\dagger}_{\text{new}} + \text{H.c.}\right) + O\left(\frac{m_{\text{D}}^2}{M^2}\right).\end{aligned} \tag{103}$$

We see that the phase of ξ^{L}_{new} should be redefined in order to get the right sign for the mass term. This redefinition of the phase to get a positive mass term in the Lagrangian also cancels the minus sign in Eq. (95), which means that we recover the normal convention for a propagator. The phase of the parameter $\frac{m_{\text{D}}^2}{M}$ can also be absorbed into the field definition. So the final mass term for the redefined fields can be written as

$$\begin{aligned}\mathcal{L}_m &= -\frac{1}{2}m_{\xi}\left(\xi^{L}_{\text{new}}\xi^{L}_{\text{new}} + \text{H.c.}\right)\\ &-\frac{1}{2}M\left(\xi^{R\dagger}_{\text{new}}\xi^{R\dagger}_{\text{new}} + \text{H.c.}\right) + O\left(\frac{m_{\text{D}}^2}{M^2}\right),\end{aligned} \tag{104}$$

where m_ξ and M are real Majorana masses. By these redefinitions, we get rid of the mixing between the two spinors up to the first order in $\frac{m_{\text{D}}}{M}$. The mass parameter m_ξ is the same as in Eq. (94).

5. Conclusions

The main confusion in using Feynman rules in the Weyl spinor notation comes from keeping track of definitions. We see that in the Weyl spinor formulation we have an additional freedom of choosing between two equivalent rules for the same diagram. This one-to-two correspondence between diagrams and rules, as we see in Figs. 2 and 4, makes it even more complicated to follow where minus signs must appear. We try to ease this confusion by presenting explicit derivations of Feynman rules from the path integral and emphasizing on the definitions. We also define propagators in the momentum space rather than in the position space. This leads to the unusual looking propagator definitions presented in Eqs. (51) to (54). Concentrating on the momentum space we explore different choices of momentum dependences of the fields: Majorana spinors do not have the freedom in choosing momentum signs in the Fourier transformation, whereas the Dirac spinors do. In order to have the same definition for both cases, we introduce the convention to fix the momentum dependences of the Dirac spinor.

The examples presented here, loop corrections and the seesaw mechanism, are related to our future work. We plan to explore the nature of Weyl spinors with mixed mass terms in broken gauge field theories. The Standard Model with the seesaw mechanism for one family will be our next step. Later we will include mixings between families and a richer Higgs sector than in the Standard Model. The mixing terms then complicate the analysis and Weyl spinors, as the smallest representation for fermions can show their full advantage over the usual 4-component spinor notation.

The authors thank the Lithuanian Academy of Sciences for the support (Project DaFi2015).

References

[1] K.M. Heeger, Evidence for neutrino mass: A decade of discovery, in: *SEASAW 25: Proceedings of the International Conference on the Seesaw Mechanism*, eds. J. Orloff, S. Lavignac, and M. Cribier (World Scientific, Singapore, 2005) pp. 65–80, https://arxiv.org/abs/hep-ex/0412032

[2] *SEESAW 25: Proceedings of the International Conference on the Seesaw Mechanism,* eds. J. Orloff, S. Lavignac, and M. Cribier (World Scientific, Singapore, 2005).

[3] M.E. Peskin and D.V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, USA, 1995), http://www.slac.stanford.edu/spires/find/books/www?cl=QC174.45%3AP4

[4] I.J.R. Aitchison and A.J.G. Hey, *Gauge Theories in Particle Physics: A Practical Introduction. Volume 1: From Relativistic Quantum Mechanics to QED* (IOP, Bristol, UK, 2003), http://www-spires.fnal.gov/spires/find/books/www?cl=QC793.3.F5A34::2012

[5] H.K. Dreiner, H.E. Haber, and S.P. Martin, Two-component spinor techniques and Feynman

rules for quantum field theory and supersymmetry, Phys. Rept. **494**, 1–196 (2010), http://dx.doi.org/10.1016/j.physrep.2010.05.002

[6] A. Bilal, *Introduction to Supersymmetry*, lecture notes (2001), https://arxiv.org/abs/hep-th/0101055

[7] S. Dittmaier, Weyl–van der Waerden formalism for helicity amplitudes of massive particles, Phys. Rev. D **59**, 016007 (1998), http://dx.doi.org/10.1103/PhysRevD.59.016007

[8] J.B. Albert, D.J. Auty, P.S. Barbeau, D. Beck, V. Belov, M. Breidenbach, T. Brunner, A. Burenkov, G.F. Cao, C. Chambers, et al. (EXO-200 Collaboration), Search for Majorana neutrinos with the first two years of EXO-200 data, Nature **510**, 229–234 (2014), http://dx.doi.org/10.1038/nature13432

[9] G. Senjanovic, Neutrino mass: From LHC to grand unification, Riv. Nuovo Cim. **34**, 1–68 (2011), http://dx.doi.org/10.1393/ncr/i2011-10061-8

[10] H. Weyl, Electron and gravitation. 1, Z. Phys. **56**, 330–352 (1929) [in German], http://dx.doi.org/10.1007/BF01339504

[11] P.B. Pal, Dirac, Majorana and Weyl fermions, Am. J. Phys. **79**, 485–498 (2011), http://dx.doi.org/10.1119/1.3549729

[12] T. Ohlsson, *Relativistic Quantum Physics: From Advanced Quantum Mechanics to Introductory Quantum Field Theory* (Cambridge University Press, Cambridge, UK, 2011).

[13] M. Srednicki, *Quantum Field Theory* (Cambridge University Press, Cambridge, UK, 2007).

[14] P. Minkowski, $\mu \to e\gamma$ at a rate of one out of 10^9 muon decays? Phys. Lett. B **67**, 421–428 (1977), http://dx.doi.org/10.1016/0370-2693(77)90435-X

[15] R.N. Mohapatra and G. Senjanovic, Neutrino mass and spontaneous parity violation, Phys. Rev. Lett. **44**, 912 (1980), http://dx.doi.org/10.1103/PhysRevLett.44.912

[16] T. Yanagida, Horizontal symmetry and masses of neutrinos, Conf. Proc. C **7902131**, 95–99 (1979).

[17] M. Gell-Mann, P. Ramond, and R. Slansky, Complex spinors and unified theories, Conf. Proc. C **790927**, 315–321 (1979).

FEINMANO TAISYKLĖS VEILIO SPINORIAMS SU SUMAIŠYTAIS DIRAKO IR MAJORANOS MASĖS NARIAIS

V. Dūdėnas, T. Gajdosik

Vilniaus universiteto Teorinės fizikos katedra, Vilnius, Lietuva

Santrauka

Pristatome formalizmą, reikalingą norint naudoti Veilio spinorius remiantis Feinmano taisyklėmis. Pagrindinis dėmesys skiriamas Veilio spinoriams, sumaišytiems su Dirako ir Majoranos masės nariais. Tam, kad būtų aiškūs visi naudojami apibrėžimai, mes išvedame Feinmano taisykles iš trajektorijų integralo. Taip pat pristatome du paprastus Veilio spinorių naudojimo pavyzdžius: fermiono propagatoriaus kilpos pataisų integralų sukonstravimą ir žaislinio sūpuoklių modelio pirmojo artinio masės narių išvedimą.

II

On the Renormalization of Neutrinos in the Seesaw Extension of the Two-Higgs Doublet Model

Vytautas Dūdėnas and Thomas Gajdosik

Acta Physica Polonica B, **48**, 2243–2249, 2017

Vol. 48 (2017) *ACTA PHYSICA POLONICA B* No 12

ON THE RENORMALIZATION OF NEUTRINOS IN THE SEESAW EXTENSION OF THE TWO-HIGGS DOUBLET MODEL*

Vytautas Dūdėnas, Thomas Gajdosik

Faculty of Physics, Vilnius University
Universiteto 3, 01513, Vilnius, Lithuania

(Received November 6, 2017)

We present the complex mass renormalization scheme for mixed Majorana fermions using the Weyl spinor notation. Showing the expressions for field and mass renormalization constants, we discuss the differences to the on-shell renormalization scheme. Working in a seesaw extended two-Higgs doublet model, we apply the complex mass scheme for neutrino masses and mixings.

DOI:10.5506/APhysPolB.48.2243

1. Introduction

The most commonly used renormalization scheme is the on-shell scheme (OS). However, it is shown that for unstable particles this scheme leads to gauge non-invariant definitions of masses [1]. In the seesaw mechanism (for a review, see [2]), the heaviest neutrino is by no means a stable particle. Original assumptions of the seesaw mechanism put the heaviest neutrino beyond the reach of any possible experiment. This partially justifies the use of OS, since we are looking only at the light neutrinos. However, for a more precise study of the model, the assumptions on the unmeasured parameters should be relaxed and this justification is lost.

The extension of the OS for unstable particles is the complex mass scheme (CMS) [3–5]. It is the analytical continuation of the propagator to the complex domain. In that way, the information about the decay width of the particle is included in the renormalized mass as the imaginary part of self energies. One can formally prove gauge invariance of the definition of mass at all loop levels [1] with the help of Nielsen identities [6].

* Presented at the XLI International Conference of Theoretical Physics "Matter to the Deepest", Podlesice, Poland, September 3–8, 2017.

In Section 2, we present the main definitions used to define the renormalization scheme. In Section 3, we outline the derivation of mass and field renormalization constants for the Majorana fermions in the complex mass scheme and discuss the implications. Finally, in Section 4, we present the restrictions on the specific renormalization constants and one-loop Green's functions in the two-Higgs doublet model (2HDM) with one seesaw neutrino.

2. Definitions

We use the Weyl spinor notation in the chiral representation as in [7, 8]. Let us say we have left-handed Weyl spinors ν_{0i} with bare Majorana masses m_{0i}. We can always fix the phase of ν_{0i} so that the mass parameter m_{0i} is real. Then we can write the multiplicative renormalization constants as:

$$\begin{aligned} \nu_{0i} &= Z^{\frac{1}{2}}_{ij}\nu_j\,, \qquad \nu^{\dagger}_{0i} = Z^{\frac{1}{2}\dagger}_{ij}\nu^{\dagger}_j\,, \qquad m_{0i} = m_i Z_{m_i}\,, \\ Z_{m_i} &= 1+\delta_{m_i}\,, \qquad Z^{\frac{1}{2}}_{ij} = 1_{ij} + \tfrac{1}{2}\delta_{ij}\,. \end{aligned} \tag{1}$$

However, as we will see later, these multiplicative constants are not enough to absorb the imaginary parts coming from the loop functions for unstable particles. So we increase the degrees of freedom for the field renormalization part:

$$\nu_{0i} = Z^{\frac{1}{2}}_{ij}\nu_j\,, \qquad \nu^{\dagger}_{0i} = \bar{Z}^{\frac{1}{2}}_{ij}\bar{\nu}_j \Rightarrow \left(Z^{\frac{1}{2}}_{ij}\nu_j\right)^{\dagger} = \bar{Z}^{\frac{1}{2}}_{ij}\bar{\nu}_j\,. \tag{2}$$

This is equivalent to dropping the pseudohermicity requirement as suggested in [3].

The renormalized Green functions are

$$\langle\phi_1\ldots\phi_n\rangle^{[\mathrm{loop}]}_{1PI} = \frac{\delta^n\hat{\Gamma}^{[\mathrm{loop}]}}{\delta\phi_1\ldots\delta\phi_n} \equiv \hat{\Gamma}^{[\mathrm{loop}]}_{\phi_1\ldots\phi_n} \equiv \Gamma^{[\mathrm{loop}]}_{\phi_1\ldots\phi_n} + \delta\Gamma^{[\mathrm{loop}]}_{\phi_1\ldots\phi_n}\,, \tag{3}$$

where $\hat{\Gamma}$ is the renormalized effective action and $\delta\Gamma$ stands for counterterms. Then the tree-level Green's functions read as:

$$\hat{\Gamma}^{[0]}_{\nu_i\nu_i} = -m_i\,, \qquad \hat{\Gamma}^{[0]}_{\bar{\nu}_i\bar{\nu}_i} = -m_i\,, \qquad \hat{\Gamma}^{[0]}_{\bar{\nu}_i\nu_i} = p\bar{\sigma}\,, \qquad \hat{\Gamma}^{[0]}_{\nu_i\bar{\nu}_i} = p\sigma\,, \tag{4}$$

where σ and $\bar{\sigma}$ connect spinors to four vectors as defined in [7, 8]. Due to Lorentz invariance, we can factor out the scalar parts of Green's functions as:

$$\begin{aligned} \hat{\Gamma}_{\nu_i\nu_i} &= m_i\hat{\Sigma}_{\nu_i\nu_i}\,, \qquad \hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_i} = m_i\hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i}\,, \\ \hat{\Gamma}_{\nu_i\bar{\nu}_j} &= p\sigma\hat{\Sigma}_{\nu_i\bar{\nu}_j}\,, \qquad \hat{\Gamma}_{\bar{\nu}_i\nu_j} = p\bar{\sigma}\hat{\Sigma}_{\bar{\nu}_i\nu_j}\,. \end{aligned} \tag{5}$$

Then we can express the counterterms using Eqs. (1)–(5) and write the loop functions as:

$$\hat{\Sigma}_{\nu_i\nu_i} = -\delta_{m_i} - \delta_{ii} + \Sigma_{\nu_i\nu_i}\,, \qquad \hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i} = -\delta_{m_i} - \bar{\delta}_{ii} + \Sigma_{\bar{\nu}_i\bar{\nu}_i}\,, \tag{6}$$

$$\hat{\Sigma}_{\bar{\nu}_i\nu_j} = \tfrac{1}{2}\left(\delta_{ij} + \bar{\delta}_{ji}\right) + \Sigma_{\bar{\nu}_i\nu_j}\,, \qquad \hat{\Sigma}_{\nu_i\bar{\nu}_j} = \tfrac{1}{2}\left(\bar{\delta}_{ij} + \delta_{ji}\right) + \Sigma_{\nu_i\bar{\nu}_j}\,. \tag{7}$$

3. From on shell to complex mass shell

With these definitions, the resummed propagators are:

$$\langle\bar{\nu}_i\nu_i\rangle = i\bar{\sigma}p\left[p^2\left(1 + \hat{\Sigma}_{\nu_i\bar{\nu}_i}\right) - m_i^2\left(1 - \hat{\Sigma}_{\nu_i\nu_i} - \hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i} - \hat{\Sigma}_{\bar{\nu}_i\nu_i}\right)\right]^{-1}, \tag{8}$$

$$\langle\nu_i\nu_i\rangle = im_i\left[p^2\left(1 + \hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i} + \hat{\Sigma}_{\nu_i\bar{\nu}_i} + \hat{\Sigma}_{\bar{\nu}_i\nu_i}\right) - m_i^2\left(1 - \hat{\Sigma}_{\nu_i\nu_i}\right)\right]^{-1}, \tag{9}$$

together with analogous two propagators that can be obtained from Eq. (8) and Eq. (9) by changing $\nu_i \leftrightarrow \bar{\nu}_i$. Abbreviating $D_i \equiv p^2 - m_i^2$, the mixed two-point correlation functions ($i \neq j$) are:

$$\langle\nu_i\nu_j\rangle = -i(D_iD_j)^{-1}\left(m_im_j\hat{\Gamma}_{\nu_i\nu_j} + p^2\left[m_j\hat{\Sigma}_{\bar{\nu}_i\nu_j} + \hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_j} + m_i\hat{\Sigma}_{\nu_i\bar{\nu}_j}\right]\right), \tag{10}$$

$$\langle\bar{\nu}_i\nu_j\rangle = -i\bar{\sigma}p(D_iD_j)^{-1}\left(m_j\hat{\Gamma}_{\nu_i\nu_j} + m_im_j\hat{\Sigma}_{\bar{\nu}_i\nu_j} + m_i\hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_j} + p^2\hat{\Sigma}_{\nu_i\bar{\nu}_j}\right), \tag{11}$$

and $\nu_i \leftrightarrow \bar{\nu}_i$. The OS renormalization condition for a mass counterterm can be derived by requiring that the real part of the pole of the diagonal propagator coincides with the renormalized mass. The requirement that the mixed propagators vanish and that the residue of the diagonal propagator is equal to one gives the conditions for the wave function renormalization. Generalization from the OS to the CMS is obtained by just dropping the reality requirement and evaluating self energy functions at the exact complex pole of the propagator. Hence in the CMS, these conditions are:

$$\left(\hat{\Sigma}_{\nu_i\nu_i} + \hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i} + \hat{\Sigma}_{\nu_i\bar{\nu}_i} + \hat{\Sigma}_{\bar{\nu}_i\nu_i}\right)\Big|_{p^2=m_i^2} = 0\,, \tag{12}$$

$$\hat{\Sigma}_{\bar{\nu}_i\nu_i} = -m_i^2\frac{\partial}{\partial p^2}\left(\hat{\Sigma}_{\nu_i\nu_i} + \hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i} + \hat{\Sigma}_{\nu_i\bar{\nu}_i} + \hat{\Sigma}_{\bar{\nu}_i\nu_i}\right)\Big|_{p^2=m_i^2}\,, \tag{13}$$

$$\hat{\Sigma}_{\bar{\nu}_i\nu_i}\Big|_{p^2=m_i^2} = \hat{\Sigma}_{\bar{\nu}_i\nu_i}\Big|_{p^2=m_i^2} = -\hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i}\Big|_{p^2=m_i^2} = -\hat{\Sigma}_{\nu_i\nu_i}\Big|_{p^2=m_i^2}\,, \tag{14}$$

$$\left(\hat{\Gamma}_{\nu_i\nu_j} + m_j\hat{\Sigma}_{\nu_i\bar{\nu}_j}\right)\Big|_{p^2=m_i^2} = 0\,, \qquad \left(\hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_j} + m_j\hat{\Sigma}_{\bar{\nu}_i\nu_j}\right)\Big|_{p^2=m_i^2} = 0\,. \tag{15}$$

The condition of Eq. (12) comes from the requirement of the position of the pole for Eq. (8) and Eq. (9); the conditions of Eq. (13) and Eq. (14) come from the requirement that the residue of Eq. (8) and Eq. (9) is one and the conditions of Eq. (15) come from the requirement that the expressions in Eq. (10) and Eq. (11) vanish at $p^2 = m_i^2$ and $p^2 = m_j^2$. Inserting the expressions from Eq. (6) and Eq. (7) into these conditions leads to:

$$\delta_{m_i} = \frac{1}{2}\left(\Sigma_{\nu_i\nu_i} + \Sigma_{\bar{\nu}_i\bar{\nu}_i} + \Sigma_{\nu_i\bar{\nu}_i} + \Sigma_{\bar{\nu}_i\nu_i}\right)\Big|_{p^2=m_i^2}\,, \tag{16}$$

$$\frac{1}{2}\left(\bar{\delta}_{ii} + \delta_{ii}\right) = -\Sigma_{\bar{\nu}_i\nu_i} - m_i^2\frac{\partial}{\partial p^2}\left(\Sigma_{\nu_i\nu_i} + \Sigma_{\bar{\nu}_i\bar{\nu}_i} + \Sigma_{\nu_i\bar{\nu}_i} + \Sigma_{\bar{\nu}_i\nu_i}\right)\Big|_{p^2=m_i^2}\,, \tag{17}$$

$$\bar{\delta}_{ii} - \delta_{ii} = \left(\Sigma_{\nu_i\nu_i} - \Sigma_{\bar{\nu}_i\bar{\nu}_i}\right)\Big|_{p^2=m_i^2}\,, \tag{18}$$

$$\bar{\delta}_{ij} = \frac{2}{m_i^2 - m_j^2}\left(m_j\Gamma_{\nu_i\nu_j} + m_j^2\Sigma_{\nu_i\bar{\nu}_j} + m_i\Gamma_{\bar{\nu}_i\bar{\nu}_j} + m_i m_j\Sigma_{\bar{\nu}_i\nu_j}\right)\Big|_{p^2=m_j^2}\,, \tag{19}$$

$$\delta_{ij} = \frac{2}{m_i^2 - m_j^2}\left(m_i\Gamma_{\nu_i\nu_j} + m_i m_j\Sigma_{\nu_i\bar{\nu}_j} + m_j\Gamma_{\bar{\nu}_i\bar{\nu}_j} + m_j^2\Sigma_{\bar{\nu}_i\nu_j}\right)\Big|_{p^2=m_j^2}\,. \tag{20}$$

Equations (12)–(20) are consistent with the expressions in [3]. If we used the multiplicative constants only in the form of Eq. (1), without the field renormalization constants shown in Eq. (2), we could not absorb the imaginary parts from the loop functions. This can be easily seen from Eq. (17): using only constans from Eq. (1) would lead to an always real combination of constants $\delta_{ii}^\dagger + \delta_{ii}$ in the LHS of Eq. (17) instead of $\bar{\delta}_{ii} + \delta_{ii}$ which, in general, can be complex. On the other side, we see that the mass counterterm in Eq. (16), generalizes straightforwardly to the complex mass scheme by just dropping this reality condition. Actually, this would not be the case if we did not absorb the phase of the bare mass parameters in the Weyl spinors. Then we would have needed to introduce some new $\bar{m}_i$ and $\bar{\delta}_{m_i}$ in analogy to $\bar{\delta}_i$ and $\bar{\nu}_i$. However, there is no need for this additional complication, since we can always fix the phase of Majorana fermions.

Another interesting and somewhat odd feature of this scheme is that the renormalized field in the Lagrangian is not related to the corresponding antifield by Hermitian conjugation, whereas the bare fields are. The relation is altered by the wave function renormalization constants from Eq. (2):

$$\left(Z_{ij}^{\frac{1}{2}}\nu_j\right)^\dagger = \bar{Z}_{ij}^{\frac{1}{2}}\bar{\nu}_j \Rightarrow \nu_i^\dagger = \bar{\nu}_i + \tfrac{1}{2}\left(\bar{\delta}_{ij} - \delta_{ij}^\dagger\right)\bar{\nu}_j + O\left(\delta^2\right)\,. \tag{21}$$

From Eq. (19) and Eq. (20), we can see that if ν_j is stable, we have $\bar{\delta}_{ij} = \delta_{ij}^\dagger$. This means that the relation of Eq. (21) reduces to $\nu_i^\dagger = \bar{\nu}_i$ if every ν_j is stable and we recover the usual on-shell conditions. However, if at least one particle entering Eq. (21) is unstable, we get $\bar{\nu}_i \neq \nu_i^\dagger$ for all particles that mix, even if the particle under consideration is stable. This is not inconsistent: all particles mix at the Lagrangian level. To see how this is consistent, we should look at Green's functions instead. Let us assume that the particle ν_i is stable, then at one-loop level:

$$\begin{aligned} \frac{\delta^2}{\delta\nu_i\delta\bar{\nu}_i}\hat{\Gamma} &= \int\limits_j \frac{\delta\nu_j^\dagger}{\delta\bar{\nu}_i}\frac{\delta^2}{\delta\nu_i\delta\nu_j^\dagger}\hat{\Gamma}\,, \\ \frac{\delta\nu_j^\dagger}{\delta\bar{\nu}_i} &= 1_{ji} + \frac{1}{2}\left(\bar{\delta}_{ji} - \delta_{ji}^\dagger\right) = 1_{ij} \Rightarrow \hat{\Gamma}_{\nu_i\bar{\nu}_i} = \hat{\Gamma}_{\nu_i\nu_i^\dagger}\,. \end{aligned} \tag{22}$$

If ν_i is unstable, similar manipulations give:

$$\hat{\Gamma}^{[\geq 1]}_{\nu_i\bar{\nu}_i} = \left(1 + \frac{1}{2}\left(\bar{\delta}_{ii} - \delta_{ii}^\dagger\right)\right)\hat{\Gamma}^{[0]}_{\nu_i\nu_i^\dagger} + \hat{\Gamma}^{[1]}_{\nu_i\nu_i^\dagger}\,. \tag{23}$$

Here, we also used the assumption that the basis is chosen in such a way that there are no mixed terms at tree level. As an example, let us assume that all the couplings that go into the expression for $\bar{\delta}_{ii} - \delta_{ii}^\dagger$ are real. Then $\bar{\delta}_{ii} = \delta_{ii}$ and we can rewrite Eq. (23) as:

$$\hat{\Gamma}^{[\geq 1]}_{\nu_i\bar{\nu}_i} = e^{i\,\mathrm{Im}\,\delta_{ii}}\hat{\Gamma}^{[0]}_{\nu_i\nu_i^\dagger} + \hat{\Gamma}^{[1]}_{\nu_i\nu_i^\dagger}\,. \tag{24}$$

We see that the instability of ν_i is seen as the additional phase in its two-point Green's function, while a Green's function of a stable particle stays the same.

4. Renormalization constants for 4 neutrinos in the 2HDM

The Yukawa sector for neutrinos in the 2HDM in the Higgs basis includes four neutrinos, two neutral scalars h', H', one neutral pseudoscalar A', a charged scalar $H^\pm$ and Goldstone bosons. In general, all neutral scalars mix, giving the mass eigenstates h, H, and A. The seesaw mixing is defined between the third and the fourth neutrino ($s^2 = \frac{m_{03}}{m_{04}+m_{03}}$, $c^2 = \frac{m_{04}}{m_{04}+m_{03}}$). The full Yukawa Lagrangian for this model can be found in [9]. The Yukawa part that includes only the neutral scalars can be written as:

$$\begin{aligned} \mathcal{L}_\nu &= -\frac{1}{2}m_{03}\,\nu_{03}\nu_{03} - \frac{1}{2}m_{04}\,\nu_{04}\nu_{04} \\ &-\frac{1}{\sqrt{2}}\left[y\left(h' + i\chi^0\right) - d'\,\left(H' + iA'\right)\right]\left(cs\nu_{03}\nu_{03} + i\left(c^2 - s^2\right)\nu_{03}\nu_{04} + cs\nu_{04}\nu_{04}\right) \\ &-\frac{1}{\sqrt{2}}d\left(H' + iA'\right)\nu_{02}\left(-is\nu_{03} + c\nu_{04}\right) + \mathrm{h.c.} \end{aligned} \tag{25}$$

y is given by the neutrino masses and the vacuum expectation value, hence the only free parameters in this part of the Lagrangian are

$$m_{03}, m_{04}, d \in \mathbb{R} \quad \text{and} \quad d' \in \mathbb{C}\,. \tag{26}$$

There are no bare masses for ν_2 and ν_1, hence no mass counterterms and no counterterms for their mixing:

$$\delta_{m_1} = \delta_{m_2} = \delta_{12} = \bar{\delta}_{12} = \delta_{21} = \bar{\delta}_{21} = 0\,. \tag{27}$$

ν_1, ν_2 and ν_3 are stable at one-loop level, so the counterterms are the same as we would have in the OS scheme:

$$\nu_j^\dagger = \bar{\nu}_j\,, \quad \delta_{m_3} \in \mathbb{R}\,, \quad \delta_{jj}^\dagger = \bar{\delta}_{jj}\,, \quad \delta_{ij}^\dagger = \bar{\delta}_{ij}\,, \quad i = 1,2,3,4\,; \quad j = 1,2,3\,. \tag{28}$$

For an unstable ν_4, we have:

$$\delta_{m_4}, \delta_{i4}, \bar{\delta}_{i4}, \delta_{44}, \bar{\delta}_{44} \in \mathbb{C}\,, \quad \nu_4^\dagger = \left(1 - \tfrac{1}{2}\bar{\delta}_{44} + \tfrac{1}{2}\delta_{44}^\dagger\right)\bar{\nu}_4\,, \quad i = 1,2,3\,. \tag{29}$$

Since we chose a basis in such a way that ν_1 does not interact with any neutral scalar, it stays massless after loop corrections as well. Since the counterterms of Eq. (27) are zero, there should be no mixing between ν_1 and ν_2 after a loop correction, so:

$$\Gamma_{\nu_1\nu_1} = \Gamma_{\nu_1\nu_2} = 0\,. \tag{30}$$

Note that δ_{13} and δ_{14} are not equal to zero and they are used to absorb the mixing coming from $\Gamma_{\nu_1\nu_3}$ and $\Gamma_{\nu_1\nu_4}$, which are not zero due to a loop with a charged fermion and a charged scalar.

Finally, there is no counterterm for the mass term for ν_2, which means that the one-loop mass term

$$m_2 = -\Gamma_{\nu_2\nu_2}(0) \tag{31}$$

is finite and gauge invariant.

The authors thank the Lithuanian Academy of Sciences for the support (the project DaFi2017).

REFERENCES

[1] P. Gambino, P.A. Grassi, *Phys. Rev. D* **62**, 076002 (2000) [arXiv:hep-ph/9907254].

[2] G. Senjanović, *Riv. Nuovo Cim.* **34**, 1 (2011).

[3] D. Espriu, J. Manzano, P. Talavera, *Phys. Rev. D* **66**, 076002 (2002) [arXiv:hep-ph/9907254].

[4] A. Denner, S. Dittmaier, *Nucl. Phys. Proc. Suppl.* **160**, 22 (2006) [arXiv:hep-ph/0605312].

[5] B.A. Kniehl, A. Sirlin, *Phys. Rev. D* **77**, 116012 (2008) [arXiv:0801.0669 [hep-th]].

[6] N.K. Nielsen, *Nucl. Phys. B* **101**, 173 (1975).

[7] H.K. Dreiner, H.E. Haber, S.P. Martin, *Phys. Rep.* **494**, 1 (2010) [arXiv:0812.1594 [hep-ph]].

[8] V. Dūdėnas, T. Gajdosik, *Lith. J. Phys.* **56**, 149 (2016).

[9] W. Grimus, H. Neufeld, *Nucl. Phys. B* **325**, 18 (1989).

III

Gauge dependence of tadpole and mass renormalization for a seesaw extended 2HDM

Vytautas Dūdėnas and Thomas Gajdosik

Physics Review D, **98**, 035034, 2018

Gauge dependence of tadpole and mass renormalization for a seesaw extended 2HDM

Vytautas Dūdėnas* and Thomas Gajdosik

Institute of Theoretical Physics and Astronomy, Faculty of Physics, Vilnius University, Saulėtekio avenue 3, 10257 Vilnius, Lithuania

(Received 22 June 2018; published 24 August 2018)

We study the gauge dependence of the neutrino mass renormalization in a two Higgs doublet model, that is extended with one singlet seesaw neutrino. This model gives only one light neutrino a mass at tree level, while the second light mass is generated at loop level via the interaction with the second Higgs doublet. At one loop level, one neutrino stays massless. We use multiplicative renormalization constants to define counterterms. The renormalized mass parameters are defined as the complex poles of the propagators, using the complex mass scheme for mass renormalization. With this setup, we analytically get the expressions for the neutrino mass counterterms and isolate the gauge dependent part. We show, how relating this gauge dependent part with the tadpole renormalization leads to gauge independent counterterm definitions, hence gauge independent bare masses for neutrinos.

DOI: 10.1103/PhysRevD.98.035034

I. INTRODUCTION

Neutrino oscillations are known for more than 30 years [1]. They prove that neutrinos are not massless. However, how exactly neutrinos get their masses in the framework of quantum field theory is still unclear. Seesaw mechanisms [2,3] are by far the most popular attempts to extend the standard model with massive neutrinos. The type I seesaw mechanism [2] is the earliest and simplest such extension, which includes neutrino mass terms induced by the Higgs boson of the standard model (SM). In case there are more Higgs bosons than the single SM Higgs, the type I seesaw extension can be generalized as in [4]. This allows for a wider range of configurations in the seesaw and Yukawa sectors to generate the masses for neutrinos that are in agreement with the experimental values. Also, there are numerous theoretical motivations [5–8] suggesting a larger scalar sector. We restrict ourselves to a general *CP* conserving two Higgs doublet model (2HDM) [9], which can be viewed as a general class of more specific models that include two scalar doublets under the gauge group $SU(2)_{\mathrm{weak}}$.

The 2HDM paired with the seesaw mechanism gives a new way of generating masses for neutrinos that is absent in the usual SM seesaw extensions. That is, the mass terms that are absent at tree level arise at loop level due to the interactions with the second Higgs doublet. This radiative mass generation makes it possible to account for both experimentally measured mass differences at one loop level having only one sterile neutrino in the seesaw mechanism. This set up, with the 2HDM and one sterile neutrino at one loop was first proposed in [4] and we call it the Grimus-Neufeld model (GN model).

We look at the gauge parameter dependence of the neutrino mass renormalization in this GN model with a *CP* symmetric 2HDM potential. It is proven in general [10], that the position of the complex pole of the propagator is independent of the gauge. Hence one can extend the on-shell (OS) scheme to the complex domain to define gauge invariant masses as is done in the complex mass scheme (CMS) [11,12]. However, this does not mean that the mass counterterms are necessarily gauge parameter independent. In fact, at one loop there is the same gauge dependence of the mass counterterms in the CMS as in the OS scheme. This is because the one loop expressions for the OS are the same as in the CMS except for the required reality of loop functions in the OS scheme. As long as the mass is evaluated at the exact pole (as in the CMS), this gauge dependence of the counterterm does not bother the definition of mass since the exact pole is gauge independent anyway. Defining a gauge independent counterterm, however, is important in other schemes such as (modified) minimal subtraction, where the gauge dependence might occur in the running of parameters [13,14]. Some explicit examples of the gauge dependence in the $\overline{\mathrm{MS}}$ scheme are given in [15–17]. Hence it is worth to look at the possibilities to define gauge independent mass counterterms in the CMS or the OS, as well.

*vytautasdudenas@inbox.lt

In the GN model, we analytically check that the gauge dependent terms for the fermion two point function vanish if the tadpole diagrams are attached to the propagator as discussed in [18]. This way of dealing with gauge dependent parts originates from the pinch technique [19]. Hence applying this technique to define numerically gauge invariant counterterms seems rather straightforward. However, to analytically isolate these tadpole diagrams from the counterterms requires some effort. We present how we achieve this isolation of the gauge dependent terms for the neutrino mass counterterms in the GN model. We try to be as transparent as possible in showing our steps so that the reader can easily reproduce our results. All our renormalization constants arise from multiplicative renormalization and we use Weyl spinors for our expressions rather than Dirac spinors.

In Sec. II we present the main definitions and discuss the implications of using the complex mass scheme over the on-shell scheme. In Sec. III we introduce the scalar sector and present the tadpole renormalization conditions in the 2HDM. In Sec. IV we introduce the Yukawa sector of the GN model and show the expressions of mass counterterms for neutrinos. The relationship between tadpole conditions of Sec. III and mass counterterms is also explained in Sec. IV. In Sec. V we show how we set up the calculations using SARAH [20](version 4.12.0 [21]), FEYNARTS [22] (version 3.9) and FORMCALC [23] (version 9.4) and present the analytical results. Section V is accompanied by the Appendix B in which we present some intermediate steps of the derivations. We conclude the results in Sec. VI by discussing the cancellation of the gauge dependence of neutrino propagators in the GN model.

II. DEFINITIONS AND THE COMPLEX MASS SCHEME

We use the same definitions as in [24], where we presented the adaptation of the complex mass scheme [12] for Majorana fermions in Weyl spinor formalism. The renormalized Green functions are

$$\frac{1}{i}\langle\phi_1\ldots\phi_n\rangle^{[\mathrm{loop}]}_{1PI} = \left.\frac{\delta^n\hat{\Gamma}^{[\mathrm{loop}]}}{\delta\phi_1..\delta\phi_n}\right|_{\phi_i=0} \equiv \hat{\Gamma}^{[\mathrm{loop}]}_{\phi_1\ldots\phi_n}$$
$$\equiv \Gamma^{[\mathrm{loop}]}_{\phi_1\ldots\phi_n} + \delta\Gamma^{[\mathrm{loop}]}_{\phi_1\ldots\phi_n}, \qquad (1)$$

where $\delta\Gamma^{[\mathrm{loop}]}$ stands for the counterterm part of the renormalized effective action. The superscript denotes the loop order of the function in consideration. The tadpole function is defined as the special case of Eq. (1):

$$T^{[\mathrm{loop}]}_{\phi} \equiv \Gamma^{[\mathrm{loop}]}_{\phi}. \qquad (2)$$

The definitions for using Weyl spinors as the basis of Feynman diagram calculations can be found in [25]. The scalar parts of Green's functions of a left handed Weyl spinor ν_i and its Hermitian conjugate $\nu_i^\dagger$ can be separated by the Lorentz index structure:

$$\hat{\Gamma}_{\nu_i\nu_i} = m_i\hat{\Sigma}_{\nu_i\nu_i}, \qquad \hat{\Gamma}_{\nu_i^\dagger\nu_i^\dagger} = m_i\hat{\Sigma}_{\nu_i^\dagger\nu_i^\dagger},$$
$$\hat{\Gamma}_{\nu_i\nu_j^\dagger} = p\sigma\hat{\Sigma}_{\nu_i\nu_j^\dagger}, \qquad \hat{\Gamma}_{\nu_i^\dagger\nu_j} = p\bar{\sigma}\hat{\Sigma}_{\nu_i^\dagger\nu_j}. \qquad (3)$$

The definitions of Eq. (3) work well for the on-shell scheme, but have to be slightly modified for the complex mass scheme.

We work in renormalized perturbation theory, where the renormalized parameters p and the renormalized fields ϕ_j are related to bare parameters and bare fields by multiplicative renormalization constants:

$$p_0 = p(1+\delta_p), \qquad \phi_{0i} = \sum_j (1_{ij}+\delta_{ij})\phi_j. \qquad (4)$$

We use the subscript 0 to denote the bare quantities, 1_{ij} stands for the Kronecker delta, δ_p and δ_{ij} are one loop order renormalization constants. These redefinitions of parameters and fields give rise to the counterterms $\delta\Gamma^{[\mathrm{loop}]}_{\phi_1\ldots\phi_n}$ in Eq. (1).

We use the general R_ξ gauge for calculations. As we will look at the gauge parameter dependencies, we will frequently look at only the gauge parameter dependent part of the expressions. To denote the gauge dependent term, we will add the gauge parameter ξ in the subscript at the end of the renormalization constants, self-energies, and tadpole functions; e.g.:

$$\delta_p \equiv \delta_{p\xi} + \text{gauge independent terms}, \quad \delta_{p\xi} = \delta_{p\xi_W} + \delta_{p\xi_Z}. \qquad (5)$$

We use the complex mass scheme [12] (CMS) to renormalize masses and fields. The CMS for mixed fermions is presented in [26–28] and the adaptation to Weyl spinor formulation is presented in [24]. Here we mention the main differences that need to be considered when generalizing the OS framework to the CMS. Considering a Majorana mass term for the Weyl fermion ν:

$$\mathcal{L}_{m_0} = -\frac{1}{2}m_0\nu_0\nu_0 - \frac{1}{2}m_0^\dagger\nu_0^\dagger\nu_0^\dagger, \qquad (6)$$

we can absorb the phase of the mass parameter into the field, so that $m_0 \in \mathbb{R}$:

$$\mathcal{L}_{m_0} = -\frac{1}{2}m_0(\nu_0\nu_0 + \nu_0^\dagger\nu_0^\dagger). \qquad (7)$$

Renormalizing the mass parameter leads to

$$\mathcal{L}_{m_0} = -\frac{1}{2}m(\nu_0\nu_0 + \nu_0^\dagger\nu_0^\dagger) + \text{c.t.}, \qquad (8)$$

where $m \in \mathbb{C}$ and c.t. stands for the counterterms. Hence the CMS introduces an apparent nonhermiticity in the renormalized tree level Lagrangian (the full Lagrangian including all the counterterms is Hermitian). Also, the condition for the residue at the complex pole leads to an additional phase difference in the fields [26–28]. That means that the field renormalization constants are not Hermitian conjugate to each other either [24]:

$$\nu_0^\dagger = (1+\bar{\delta})\bar{\nu}, \quad \nu_0 = (1+\delta)\nu \Rightarrow \bar{\nu} \neq \nu^\dagger, \quad \delta^\dagger \neq \bar{\delta}, \tag{9}$$

where we use overbars as parts of the names of the renormalization constants and the fields. Hence the renormalized mass Lagrangian in the CMS is:

$$\mathcal{L}_m = -\frac{1}{2} m(\nu\nu + \bar{\nu}\bar{\nu}). \tag{10}$$

Comparing with the bare Lagrangian, we see that we could write Eq. (6) or Eq. (7) as:

$$\mathcal{L}_{m_0} = -\frac{1}{2} m_0 \nu_0 \nu_0 + \text{H.c.} \tag{11}$$

We cannot write Eq. (10) in the same way, since it is not hermitian. However, we can try to define a new symbol H.c.* to have the possibility to write:

$$\mathcal{L}_m = -\frac{1}{2} m(\nu\nu + \bar{\nu}\bar{\nu}) = -\frac{1}{2} m\nu\nu + \text{H.c.}^* \tag{12}$$

In this equation the symbol H.c.* makes the replacement for the field $\nu \to \bar{\nu}$ and leaves $m \to m$. The mass parameter is unchanged in the H.c.* since we found the basis, in which the bare parameter is real by absorbing the phase into ν_0 in Eq. (7). Hence the algebraic structure of Eq. (7) is kept in the renormalized version shown in Eq. (10). A similar thing happens in the CP conserving Higgs sector: the CP symmetry constrains the form of the Lagrangian, which has to be kept during the renormalization condition. Also, in the scalar and the vector case, if we have $\phi_0 \in \mathbb{R}$, then $\phi = \bar{\phi}$. The easiest way to generalize the H.c.* symbol is to say that we choose the basis in which the bare parameters that can be real are made real; then we can summarize:

$$\text{H.c.}^*: \begin{cases} p \to p, \phi \to \bar{\phi}; & p_0 \in \mathbb{R} \\ p \to p^\dagger, \phi \to \bar{\phi}; & p_0 \notin \mathbb{R}. \end{cases} \tag{13}$$

Normally, if a bare parameter is related to the bare mass term, that parameter can be made real by absorbing the phase into the field. Hence the second line of Eq. (13) assumes that there is no effect of the mass renormalization to the parameter p if p_0 cannot be related to the mass term. While this assumption is correct at one loop level, the definition Eq. (13) at higher loops should be treated with caution. Without going into too much technical details, one can think of H.c.* as a shorthand notation for the renormalized H.c. terms of the bare Lagrangian.

Now we can come back to the definitions of Eq. (3). As the CMS renormalized field is $\bar{\nu}$ and not $\nu^\dagger$, as can be seen from Eq. (9), we write [24]:

$$\begin{aligned} \hat{\Gamma}_{\nu_i\nu_i} &= m_i \hat{\Sigma}_{\nu_i\nu_i}, & \hat{\Gamma}_{\bar{\nu}_i\bar{\nu}_i} &= m_i \hat{\Sigma}_{\bar{\nu}_i\bar{\nu}_i}, \\ \hat{\Gamma}_{\nu_i\bar{\nu}_j} &= p\sigma \hat{\Sigma}_{\nu_i\bar{\nu}_j}, & \hat{\Gamma}_{\bar{\nu}_i\nu_j} &= p\bar{\sigma} \hat{\Sigma}_{\bar{\nu}_i\nu_j}. \end{aligned} \tag{14}$$

The difference between Eqs. (3) and (14) is rather formal: i.e., one does not really see the difference when calculating the Feynman diagrams. However, for using the CMS for field and mass renormalization, one should keep this difference in mind for the conceptual consistency.

After we have the consistent set up for renormalizing the fermions in the CMS, we continue to look at the gauge parameter dependencies of the renormalization constants in this scheme. The multiplicative renormalization constants Eq. (4) can be used for any renormalization condition. The algebra of the CMS is basically the same as in the OS, as the CMS is just the analytical continuation of the OS to the complex domain. In this paper, we study the algebraic relations that allow to isolate the gauge parameter term in the mass counterterm. As this procedure is purely algebraic, the expressions concerning the isolation of the gauge dependent part are the same as in the OS scheme apart from the reality requirement. We, however, do these manipulations with the CMS in mind, as the generalizations despite being rather straightforward are still needed for a full consistency. We now turn to the explicit expressions for the GN model.

III. SCALAR SECTOR AND TADPOLE CONDITIONS

The general 2HDM is an extension of the SM with a second Higgs doublet having the same charges as the SM Higgs doublet. The most general potential can be written as [9,29]:

$$\begin{aligned} \mathcal{V}_{\text{Higgs}} = {} & m_{011}^2 H_{01}^\dagger H_{01} + m_{022}^2 H_{02}^\dagger H_{02} - (m_{012}^2 H_{01}^\dagger H_{02} + \text{H.c.}) \\ & + \frac{1}{2}\lambda_{01}(H_{01}^\dagger H_{01})^2 + \frac{1}{2}\lambda_{02}(H_{02}^\dagger H_{02})^2 \\ & + \lambda_{03}(H_{01}^\dagger H_{01})(H_{02}^\dagger H_{02}) + \lambda_{04}(H_{02}^\dagger H_{01})(H_{01}^\dagger H_{02}) \\ & + \left[\frac{1}{2}\lambda_{05}(H_{02}^\dagger H_{01})(H_{02}^\dagger H_{01}) + \lambda_{06}(H_{01}^\dagger H_{01})(H_{01}^\dagger H_{02}) \right. \\ & \left. + \lambda_{07}(H_{02}^\dagger H_{02})(H_{02}^\dagger H_{01}) + \text{H.c.}\right], \end{aligned} \tag{15}$$

where H_{01} and H_{02} are the two Higgs doublets. In a general basis, they both develop VEVs: v_{01} and v_{02}, respectively. The VEV value that is responsible for the electroweak symmetry breaking is $v_0^2 = v_{01}^2 + v_{02}^2$. We choose to work in the Higgs basis, where we can parametrize the Higgs doublets as:

$$H_{01} = \begin{pmatrix} \chi_{0W}^{+} \\ \frac{1}{\sqrt{2}}(v_0 + h_0 + i\chi_{0Z}) \end{pmatrix},$$
$$H_{02} = \begin{pmatrix} H_0^{+} \\ \frac{1}{\sqrt{2}}(H_0 + iA_0) \end{pmatrix}. \quad (16)$$

In this basis, H_{02} is chosen to have 0 vacuum expectation value (VEV), v_0 is the VEV of H_{01}, χ_{0Z}, and χ_{0W} stand for Goldstone bosons, h_0, H_0, and A_0 are neutral scalars and H_0^+ is a charged scalar. Note that when we choose the Higgs basis by Eq. (16) and insert into the Eq. (15), the parameters in Eq. (15) are the Higgs basis parameters and not the ones of the general basis. The transformation of parameters between the Higgs and the general basis can be found in [9,30]. We consider the CP conserving case, where all the bare parameters are real,

$$m_{0ij}^2, \lambda_{0k} \in \mathbb{R}; \quad i,j = 1,2, \qquad k = 1,\ldots,7, \quad (17)$$

by an imposed CP symmetry on the bare Lagrangian.

After introducing the renormalization constants, Eq. (4), we write the zeroth order renormalized effective action (or the renormalized Lagrangian, ignoring the kinetic terms) of the Higgs sector as:

$$\begin{aligned}\Gamma_{\mathrm{Higgs}}^{[0]} = & -m_{11}^2 \bar{H}_1 H_1 - m_{22}^2 \bar{H}_2 H_2 + \{m_{12}^2 \bar{H}_1 H_2 + \mathrm{H.c}^*.\} \\ & -\frac{1}{2}\lambda_1 (\bar{H}_1 H_1)^2 - \frac{1}{2}\lambda_2 (\bar{H}_2 H_2)^2 - \lambda_3 (\bar{H}_1 H_1)(\bar{H}_2 H_2) \\ & -\lambda_4 (\bar{H}_2 H_1)(\bar{H}_1 H_2) \\ & -\Big[\frac{1}{2}\lambda_5 (\bar{H}_2 H_1)(\bar{H}_2 H_1) + \lambda_6 (\bar{H}_1 H_1)(\bar{H}_1 H_2) \\ & +\lambda_7 (\bar{H}_2 H_2)(\bar{H}_2 H_1) + \mathrm{H.c}^*.\Big], \end{aligned} \quad (18)$$

where we used the definitions of Eq. (13). As the bare fields h_0, H_0, A_0 are real, the renormalized fields are written as:

$$H_1 = \begin{pmatrix} \chi_W^{+} \\ \frac{1}{\sqrt{2}}(v + h + i\chi_Z) \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H^{+} \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix},$$
$$\bar{H}_i = H_i^T(\chi^+ \to \chi^-, H^+ \to H^-, i \to -i). \quad (19)$$

χ_W^+ and χ_W^- are related to χ_{0W}^+ as described by Eq. (9). The same holds for H^+ and H^-. The neutral fields appear in the barred doublets in the same way as in the unbarred doublets.

To get the minimum of the potential, Eq. (18), we need to solve three tadpole equations for the three neutral scalars. It is important to note that we will express the tadpole equations in the Higgs basis and not in the mass eigenstate basis as the expressions are simpler. The mass eigenstate basis for h and H and the Higgs basis is related by an orthogonal transformation parametrized by [9]:

$$O^{\phi} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix}, \qquad \phi_i^{\mathrm{mass}} = O_{ij}^{\phi} \phi_j^{\mathrm{Higgs}},$$
$$\phi_i^{\mathrm{Higgs}} = (h, H)_i, \quad (20)$$

where s_α and c_α are sine and cosine functions of a mixing angle α, respectively. In general, we would have 3×3 mixing matrix, but the imposed CP symmetry on the potential does not allow A to mix with h and H at tree level. Then the tadpole functions in different bases are related by:

$$T_h = c_\alpha T_{h_{(m)}} - s_\alpha T_{H_{(m)}}, \qquad T_H = c_\alpha T_{H_{(m)}} + s_\alpha T_{h_{(m)}},$$
$$T_A = T_{A_{(m)}}, \quad (21)$$

where we added the m in the subscript to indicate that the fields are in the mass eigenstates. At tree level, the tadpole functions are

$$\hat{T}_h^{[0]} = \frac{\delta \Gamma_{\mathrm{Higgs}}^{[0]}}{\delta h} = -v\left(m_{11}^2 + \frac{1}{2}\lambda_1 v^2\right),$$
$$\hat{T}_H^{[0]} = \frac{\delta \Gamma_{\mathrm{Higgs}}^{[0]}}{\delta H} = v\left(m_{12}^2 - \frac{1}{2}v^2 \lambda_6\right),$$
$$\hat{T}_A^{[0]} = \frac{\delta \Gamma_{\mathrm{Higgs}}^{[0]}}{\delta A} = 0. \quad (22)$$

We see that the third tadpole function is already zero in the CP conserving case. We require the tadpole conditions to hold for all loop levels:

$$\hat{T}_h^{[i]} = \hat{T}_H^{[i]} = \hat{T}_A^{[i]} = 0. \quad (23)$$

The tree level tadpole conditions $\hat{T}_h^{[0]} = \hat{T}_H^{[0]} = \hat{T}_{A_0}^{[0]} = 0$ give:

$$m_{11}^2 = -\frac{1}{2}\lambda_1 v^2 \quad \text{and} \quad m_{12}^2 = \frac{1}{2}\lambda_6 v^2. \quad (24)$$

Now we require the tadpole conditions Eq. (23) for tree and one loop level together:

$$\hat{T}^{[0]} = 0, \qquad (T^{[1]} + \delta\hat{T}^{[1]})\Big|_{\hat{T}^{[0]}=0} = 0, \quad (25)$$

where we indicate in the second equation that we use the relations from the first condition at the loop order after algebraically deriving counterterms from the multiplicative constants shown in Eq. (4). The one loop tadpole counterterms evaluated at $\hat{T}^{[0]} = 0$ for the CP conserving case then are

$$\delta\hat{T}_h^{[1]} = \frac{1}{2}\lambda_1 v^3(2\delta_{m11} - \delta_{\lambda 1} - 2\delta_v),$$
$$\delta\hat{T}_H^{[1]} = \frac{1}{2}\lambda_6 v^3(2\delta_{m12} - \delta_{\lambda 6} - 2\delta_v),$$
$$\delta\hat{T}_{A_0}^{[1]} = 0. \tag{26}$$

As v is defined dynamically by Eq. (24), it is not an independent parameter of the theory. This means that one of the counterterms δ_{m11}, $\delta_{\lambda 1}$, δ_v is redundant. This is because we did not yet choose which parameter is used over which from the tree level minimum condition Eq. (24). One of the choices is treating λ_1 and v as the independent ones so that the shift of m_{11} is given by:

$$\delta_{m11} = \frac{1}{2}\delta_{\lambda 1}. \tag{27}$$

Then the shift of the VEV yields the one loop tadpole counterterms, evaluated at $\hat{T}^{[0]} = 0$:

$$\delta\hat{T}_h^{[1]} = -\lambda_1 v^3 \delta_v, \tag{28}$$

$$\delta\hat{T}_H^{[1]} = \frac{1}{2}\lambda_6 v^3(2\delta_{m12} - \delta_{\lambda 6} - 2\delta_v). \tag{29}$$

The one loop tadpole conditions Eq. (25) give:

$$\delta_v = \frac{1}{\lambda_1 v^3} T_h^{[1]}, \tag{30}$$

$$\left(\delta_{m12} - \frac{1}{2}\delta_{\lambda 6}\right) = \frac{1}{v^3}\left(\frac{1}{\lambda_1}T_h^{[1]} - \frac{1}{\lambda_6}T_H^{[1]}\right). \tag{31}$$

The v now stands for a loop renormalized VEV or the "proper VEV" as in [31]. So far, the construction is similar to the β_t scheme of [32], "scheme 3" in [13] or [31] of the SM, but without the proper relation of the VEV to the mass terms, it is not yet complete. To complete it as in [13,31,32], one identifies the bare mass parameters arising from the proper VEV, rather than v_0, as also noted in [13,31–35]. The idea is to avoid the inclusion of the gauge dependence coming from δ_v into the definition of the mass counterterm δ_m as will be shown in the next sections.

IV. YUKAWA SECTOR

The GN model adds a single sterile neutrino N_0 to the general 2HDM. This sterile neutrino is a gauge singlet under all gauge groups of the SM and has a Majorana mass term M_0. To write the Yukawa couplings, we start in the flavor basis, in which the Yukawa coupling of the charged fermions to the first Higgs doublet in the Higgs basis is diagonal. Then the general Yukawa couplings for neutrinos can be seen as two three-vectors Y^1 and Y^2. The neutrino Yukawa Lagrangian together with the Majorana mass term then is written as:

$$\mathcal{L}_{\text{Yuk}} = -Y_i^1 n_{0i} N_0 H_{01} - Y_i^2 n_{0i} N_0 H_{02} - \frac{1}{2}M_0 N_0 N_0 + \text{H.c.} \tag{32}$$

where n_{0i} are neutrinos in the flavor basis with $i = e, \mu, \tau$. The Yukawa couplings Y_i^1 and Y_i^2 give in general 6 complex parameters and M_0 gives 1 complex parameter. We absorb four phases into the n_{0i} and N_0 to get Y_i^1, $M_0 \in \mathbb{R}$. By a singular value decomposition, we can parametrize the Yukawa couplings with only four real parameters:

$$d_0, y_0 \in \mathbb{R}, \qquad d_0' \in \mathbb{C}, \tag{33}$$

absorbing the other degrees of freedom into the Unitary mixing matrix. To make the parametrization easy, we decompose it into subsequent orthogonal rotations O and phase shifts U, so that O^{23} produces zero in the second position of Y^1 ($O_{2j}^{23} Y_j^1 = 0$), O^{13} in the first ($O_{1k}^{13} O_{kj}^{23} Y_j^1 = 0$). U^σ adjusts the phase of the first element of Y^2 to match it with the phase of the second element ($\arg(U_{1l}^\alpha O_{lk}^{13} O_{kj}^{23} Y_j^2) = \arg(U_{2l}^\alpha O_{lk}^{13} O_{kj}^{23} Y_j^2)$, O^{12} makes the first element of Y^2 zero ($O_{1m}^{12} U_{ml}^\alpha O_{lk}^{13} O_{kj}^{23} Y_j^2 = 0$) and U^ρ adjust the phase so that the second element of Y^2 is real ($U_{2n}^\beta O_{nm}^{12} U_{ml}^\alpha O_{lk}^{13} O_{kj}^{23} Y_j^2 \in \mathbb{R}$). Writing $V = U^\beta O^{12} U^\alpha O^{13} O^{23}$, the basis choice is summarized as:

$$V_{1j}Y_j^1 = 0, \qquad V_{2j}Y_j^1 = 0, \qquad V_{3j}Y_j^1 = y_0,$$
$$V_{1j}Y_j^2 = 0, \qquad V_{2j}Y_j^2 = d_0, \qquad V_{2j}Y_j^2 = d_0',$$
$$d_0, y_0 \in \mathbb{R}, \qquad d_0' \in \mathbb{C}. \tag{34}$$

Note that we are still free to adjust the phase of the first row of V. To combine these rotations with the seesaw transformation, we combine all neutrinos to a single vector, consisting of four flavor basis neutrinos:

$$\nu_{0i}^F = (n_{0e}, n_{0\mu}, n_{0\tau}, N_0)_i. \tag{35}$$

To account for the fourth component of this vector, the 3×3 matrix V is trivially extended to a 4×4 matrix by adding an identity element on the diagonal. As we work in the Higgs basis, only the first Higgs doublet gets the VEV. With the parametrization Eq. (34), the seesaw transformation acts on the third and fourth component yielding the whole 4×4 mixing matrix:

$$U = U^{34} V = U^{34} U^\beta O^{12} U^\alpha O^{13} O^{23} \tag{36}$$

and the relation between the mass eigenstate and the flavor basis becomes:

$$\nu_{0i}^{\text{mass}} = U_{ij}^* \nu_{0j}^F. \tag{37}$$

All the parametrization of neutrino mixing matrix is summarized in Appendix A.

In order to see the differences in the mass terms between the tadpole schemes, we first do the usual construction like in, e.g., [12], and then modify it according to the discussion at the end of Sec. III. After the electroweak symmetry breaking, the seesaw mechanism yields two bare mass eigenvalues m_{03} and m_{04} that have the relations:

$$M_0 = m_{04} - m_{03} \quad \text{and} \quad y_0^2 v_0^2 = 2m_{03}m_{04}. \tag{38}$$

The seesaw parameters are expressed in terms of masses:

$$s_{034}^2 = \frac{m_{03}}{m_{04}+m_{03}} \quad \text{and} \quad c_{034}^2 = \frac{m_{04}}{m_{04}+m_{03}}. \tag{39}$$

Note that as long as we stay at tree level, $v_0 = v$. In this basis we have four neutrino states ν_{0i}, where ν_{01} and ν_{02} have zero mass, but ν_{02} is distinguished from ν_{01} by its interaction with the second Higgs doublet, i.e., ν_{01} does not couple to any of the Higgses. By applying the rotation Eq. (37) in Eq. (32), using the parametrizations of Eq. (34), (38), and (39) and inserting the explicit Higgs basis Eq. (16), we write the Yukawa Lagrangian part that includes only neutral scalar fields together with the Majorana mass terms:

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} = &-\frac{1}{2}m_{03}\nu_{03}\nu_{03} - \frac{1}{2}m_{04}\nu_{04}\nu_{04}\\ &-\frac{1}{\sqrt{2}}d_0(H_0+iA_0)\nu_{02}(-is_{034}\nu_{03}+c_{034}\nu_{04})\\ &-\frac{1}{\sqrt{2}}[y_0(h_0+i\chi_{Z0})+d_0'(H_0+iA_0)]\\ &\times[c_{034}s_{034}\nu_{03}\nu_{03}+i(c_{034}^2-s_{34}^2)\nu_{03}\nu_{04}\\ &+c_{034}s_{034}\nu_{04}\nu_{04}] + \text{H.c.}\end{aligned} \tag{40}$$

We straightforwardly apply the multiplicative renormalization constants, Eq. (4), for all the parameters and fields. The tree level renormalized effective action is then written in the same way as the bare Lagrangian, except that the parameters and fields are the renormalized ones:

$$\begin{aligned}\hat{\Gamma}^{[0]}_{\text{Yuk}} = &-\frac{1}{2}m_3\nu_3\nu_3 - \frac{1}{2}m_4\nu_4\nu_4\\ &-\frac{1}{\sqrt{2}}d(H+iA)\nu_2(-is_{34}\nu_3+c_{34}\nu_4)\\ &-\frac{1}{\sqrt{2}}[y(h+i\chi_Z)+d'(H+iA)]\\ &\times[c_{34}s_{34}\nu_3\nu_3+i(c_{34}^2-s_{34}^2)\nu_3\nu_4+c_{34}s_{34}\nu_4\nu_4]\\ &+\text{H.c}^*.,\end{aligned} \tag{41}$$

where:

$$M = m_4 - m_3, \qquad y^2v^2 = 2m_3m_4. \tag{42}$$

$$s_{34}^2 = \frac{m_3}{m_4+m_3}, \qquad c_{34}^2 = \frac{m_4}{m_4+m_3}. \tag{43}$$

Having Eqs. (38) and (39) for the bare theory and Eqs. (42) and (43) for the renormalized one gives us the relations between the renormalization constants:

$$\delta_{m3} + \delta_{m4} = 2(\delta_v + \delta_y), \tag{44}$$

$$m_4\delta_{m4} - m_3\delta_{m3} = (m_4 - m_3)\delta_M. \tag{45}$$

The mass renormalization constants are fixed by the CMS condition [24]:

$$\delta_{mi} = \frac{1}{2}(\Sigma_{\nu_i\nu_i} + \Sigma_{\bar{\nu}_i\bar{\nu}_i} + \Sigma_{\nu_i\bar{\nu}_i} + \Sigma_{\bar{\nu}_i\nu_i})|_{p^2=m_i^2}, \qquad m_i \neq 0, \tag{46}$$

which is nothing more than the usual expression for the OS renormalized mass counterterm (as in [36]) extended to the complex domain and written in Weyl spinor formalism. The CMS condition gives us the renormalized mass parameters gauge independent, however from Eq. (44) we see that the mass counterterm has the δ_v contribution, which is gauge dependent. Hence in this way the bare masses become gauge dependent as well.

Recalling the discussion at the end of Sec. III: to define the gauge invariant mass counterterm we need to identify the bare mass with the proper VEV [31]. Thus the bare relation Eq. (38) is modified to:

$$M_0 = m_{04}' - m_{03}', \qquad y_0^2v^2 = 2m_{04}'m_{03}', \tag{47}$$

so that there is no δ_v in the definition of δ_m' s. From $v_0 = v(1+\delta_v)$ and comparing Eq. (38) with Eq. (47), we get the relationship between primed (FJ scheme) and unprimed (usual tadpole scheme) mass parameters:

$$m_{0i} = m_{0i}' + \Delta_0, \qquad \Delta_0 = 2\frac{m_{04}'m_{03}'\delta_v}{m_{04}'+m_{03}'}, \qquad i = 3,4. \tag{48}$$

As the seesaw mixing parameters depend on the masses, they are shifted as well:

$$\begin{aligned}s_{034}^2 &\to s_{034}^2 + 2\delta_v c_{034}^2 s_{034}^2(c_{034}^2 - s_{034}^2),\\ c_{034}^2 &\to c_{034}^2 - 2\delta_v c_{034}^2 s_{034}^2(c_{034}^2 - s_{034}^2).\end{aligned} \tag{49}$$

However, these shifts of the mixing parameters become relevant only at higher loops than one, so we can drop them from our one loop expressions. At one loop level, everything is the same as in Eq. (40), except that the bare mass term Lagrangian for neutrinos becomes:

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}(m'_{03} + \Delta_0)\nu_{03}\nu_{03} - \frac{1}{2}(m'_{04} + \Delta_0)\nu_{04}\nu_{04}. \qquad (50)$$

Starting from this bare Lagrangian, Eq. (46) is modified to:

$$\delta'_{mi} = \frac{1}{2}(\Sigma_{\nu_i\nu_i} + \Sigma_{\bar{\nu}_i\bar{\nu}_i} + \Sigma_{\nu_i\bar{\nu}_i} + \Sigma_{\bar{\nu}_i\nu_i})\Big|_{p^2=m_i^2} - \frac{\Delta}{m_i}, \quad \text{for } m_i \neq 0, \qquad (51)$$

where:

$$\Delta = 2\frac{m_3 m_4 \delta_v}{m_4 + m_3} \qquad (52)$$

is defined with the renormalized masses m_3 and m_4. We see that Δ is the same for ν_3 and ν_4. To check if Δ cancels the gauge invariance, we analytically calculate the gauge dependent parts of Eqs. (46) and (52) for ν_3 and ν_4. Note that in both tadpole schemes the renormalized masses are the same CMS masses, while the bare masses m_{0i} differ from m'_{0i} by Δ_0 as in Eq. (48).

V. ARRIVING AT THE EXPRESSIONS FOR RENORMALIZATION CONSTANTS

We use FeynArts [22] and FormCalc [23] to arrive at one loop expressions for self energies and tadpoles. For making the FeynArts model file we found the SARAH [20] package to be useful, which allows to quickly generate a model file from an input of the Lagrangian in terms of Weyl spinors and scalars in the user specified gauge group representations. It also has some built in functions to check the consistency of the model. We choose the Higgs basis by simply putting the VEV of the second Higgs doublet to zero in the input file. We leave all the other parameters arbitrary for generating the FeynArts model file and make replacement rules for the FeynArts model file parameters to implement our parametrization afterwards. As we work at the one loop level, tree level relations to simplify one loop diagrams can be used. As discussed in Sec. II, the CMS keeps the algebraic structure of the bare theory. This means that for the algebraic simplifications, all the properties and the relations of bare parameters can be used for the renormalized parameters in the CMS as well. Hence we can implement these properties and relations into the assumptions of the *Mathematica* file in which we do these simplifications. Then the results can be consistently continued to the complex domain afterwards. In the following subsection we show how we implemented the parametrizations into the FeynArts model file and the assumptions for the bare parameters that carry over to the algebraic one loop simplifications. Then we present the results that we got for the gauge dependent terms in mass and tadpole renormalization.

A. Getting FeynArts model file

(1) We generate a FeynArts model file using SARAH:

(i) We take a SARAH model file for a 2HDM, and define 1 additional gauge singlet like this:

```
FermionFields[[6]] =
{n, 1, conj[nR], 0, 1, 1}
```

where the last three entries are the charges under the gauge groups (singlets under all of them), the second is the number of families, the first and the third is the name of the field and its component, respectively (see [20]).

(ii) We modify the Yukawa Lagrangian of that model file to include the general Yukawa couplings of neutrinos with the first and the second Higgs doublet as in Eq. (32) in a direct analog to the quark sector and add the Majorana mass term for the sterile neutrino:

```
LagYukawan = -(-Yn1 H1.n.l
-Yn2 H2.n.l + 1/2 M n.n)
```

(iii) In the definitions for the "EWSB" phase, we set the VEV of the second Higgs doublet to zero to implement the Higgs basis as in Eq. (16):

```
DEFINITION[EWSB][VEVs] =
{{H10, {v, 1/Sqrt[2]},
{sigma1, \[ImaginaryI]/Sqrt[2]},
{phi1, 1/Sqrt[2]}},
{H20, {0, 1/Sqrt[2]},
{sigma2, \[ImaginaryI]/Sqrt[2]},
{phi2, 1/Sqrt[2]}}};
```

(iv) We leave the definition of mixing between Higgses h and H as in the 2HDM model, but omit mixings between the pseudoscalars and the charged scalars as they do not appear in the Higgs basis with *CP* conserved potential.

(v) We define an additional mixing matrix for neutrinos in the `DEFINITION[EWSB][MatterSector]`, combining the flavor basis SM neutrinos `vL` with the sterile neutrino `conj(nR)` as:

```
{{vL, conj[nR]}, {VL, Un}}
```

where the `VL` is the combined four-vector of the neutrino mass eigenstates and `Un` is the mixing matrix U^* from Eq. (37).

(vi) We generate the FeynArts model file by the SARAH command `MakeFeynArts[]`.

(2) We make modifications to the FeynArts model file:
   (i) To achieve the parametrization of Eq. (34) we make the replacements in the model file for the neutrino-neutrino—Higgs vertices:

$$\sum_{j=1}^{3} U_{ij} Y_j^1 \to (0, 0, -ic_{34} y, s_{34} y)_j, \tag{53}$$

$$\sum_{j=1}^{3} U_{ij} Y_j^2 \to (0, d, -ic_{34} d', s_{34} d')_j. \tag{54}$$

We do not replace the neutrino—electron—scalar vertices, hence they depend on Y^1 and Y^2 instead of the y, d and d' parameters in the model file. We leave them general, because it is easier to make algebraic simplifications of amplitudes in the general couplings for these vertices. After the expressions are simple enough, we invert Eqs. (53) and (54) to express Y^1 and Y^2 in terms of U, y, d and d' in the *Mathematica* notebook file.

After setting up the FeynArts model file, we generate 1 loop diagrams for the wanted correlation functions. The parametrizations and relations of Secs. III and IV are imposed as replacement rules during the algebraic simplifications of the expressions. The summary of the parameters and their relations is given in the Appendix A.

B. Mass renormalization

We construct the mass renormalization constants as in Eq. (46) to isolate the gauge dependent part so that we can later check if the definition in Eq. (51) really cancels it. The FormCalc output is easy to use in Weyl spinor notation as the spinor products in the result of the amplitude appear in "WeylChains". By collecting terms near those "WeylChains" we can take separately all four components presented in Eq. (3). The structure of the correction to a propagator is:

$$\langle \nu_i \nu_i \rangle \Gamma_{\nu_i \nu_i} + \langle \bar{\nu}_i \bar{\nu}_i \rangle \Gamma_{\bar{\nu}_i \bar{\nu}_i} + \langle \nu_i p\sigma \bar{\nu}_i \rangle \Sigma_{\nu_i \bar{\nu}_i} + \langle \bar{\nu}_i p\bar{\sigma} \nu_i \rangle \Sigma_{\bar{\nu}_i \nu_i}. \tag{55}$$

For Majorana particles only two of the scalar self energies are independent, since $\Sigma_{\nu\bar{\nu}}$ is the same as $\Sigma_{\bar{\nu}\nu}$ and $\Gamma_{\nu\nu}$ is related to $\Gamma_{\bar{\nu}\bar{\nu}}$. At one loop, this relation is just the Hermitian conjugation of couplings that enter the loop functions.

To make algebra simplifications easier and faster we separate different one loop contributions to self energies according to the particles that appear in the loop. Those contributions are from the neutral Higgs scalars, the charged scalar Higgs, the neutral Goldstone boson, the charged Goldstone boson, the W boson and the Z boson. We label them as Σ^{H0}, Σ^{H+}, $\Sigma^{\chi 0}$, $\Sigma^{\chi +}$, Σ^{W} and Σ^{Z}, respectively. Note that the Σs are the dimensionless one loop self energy functions defined in Eq. (14). Analogously, we write the dimensionful self energies as $\Gamma^{H0}_{\phi_1\phi_2}$, $\Gamma^{H+}_{\phi_1\phi_2}$, etc… Naturally, $\Gamma^{H0}_{\nu_i\nu_j}$ and $\Gamma^{H+}_{\nu_i\nu_j}$ do not depend on any gauge parameter. As the first results of the calculations give us:

$$\Gamma^{[1]}_{\nu_1\nu_1} = 0 \quad \text{and} \quad \Gamma^{[1]}_{\nu_2\nu_2} = \Gamma^{H0}_{\nu_2\nu_2}. \tag{56}$$

Note that ν_2 and ν_1 do not have mass renormalization constants coming from Eq. (4), since they do not have bare mass parameters. The nonvanishing contribution for the mass of ν_2 is gauge independent and finite. This is a good first crosscheck to see that the implementation of the model gives us expected results.

We are interested in the gauge dependent part of δ_{m3} and δ_{m4}, so we are interested only in $\Sigma^{\chi 0}$, $\Sigma^{\chi +}$, Σ^{W}, and Σ^{Z}. ξ_W will appear only in $\Sigma^{\chi +}$ and Σ^{W} and ξ_Z only in $\Sigma^{\chi 0}$ and Σ^{Z}. As one can check, the charged loop for masslike terms vanishes:

$$\Gamma^{W}_{\nu_3\nu_3} = \Gamma^{W}_{\nu_3^\dagger\nu_3^\dagger} = 0. \tag{57}$$

Hence the potentially ξ_W dependent contribution for $m_3\delta_{m3}$ is

$$\frac{1}{2}(\Gamma^{\chi +}_{\nu_3\nu_3} + \Gamma^{\chi +}_{\nu_3^\dagger\nu_3^\dagger}) + m_3 \Sigma^{W}_{\nu_3\nu_3^\dagger} + m_3 \Sigma^{\chi +}_{\nu_3\nu_3^\dagger}. \tag{58}$$

After some effort (see the Appendix B), we arrive at the ξ_W dependent part of the mass counterterm [recall Eq. (5)]:

$$m_3 \delta_{m_3\xi_W} = \frac{m_3 m_4}{(m_3+m_4)} \frac{g_e^2}{16\pi^2 m_Z^2 s_{2W}^2} 2A_0(m_W^2 \xi_W), \tag{59}$$

where $s_{2W} \equiv 2 s_W c_W$ is the sine of a double Weinberg angle Eq. (A5). For calculating $\delta_{m_3\xi_Z}$ one should note that $\Gamma^{Z}_{\nu_3\nu_3} \neq 0$. Apart from that, everything is analogous to the ξ_W case. At the end the full gauge dependence of the neutrino mass counterterms is

$$m_3\delta_{m_3\xi} = m_4 \delta_{m_4\xi} = \frac{m_3 m_4}{(m_3+m_4)} \frac{g_e^2}{16\pi^2 m_Z^2 s_{2W}^2} [A_0(m_Z^2\xi_Z) + 2A_0(m_W^2\xi_W)]. \tag{60}$$

C. VEV renormalization

When separating the gauge parameter dependent part of $T_h^{[1]}$ we first observe that tadpoles with physical Higgs bosons and fermions in the loop do not have any gauge dependence. The gauge dependent part of loops with gauge bosons and ghosts exactly cancel when these contributions are summed up. Hence the only gauge dependent terms in the tadpole contributions are the tadpoles with Goldstone bosons in the loops, which are

$$T^{[1]}_{h\xi} = \frac{\lambda_1 v}{32\pi^2} [A_0(m_Z^2\xi_Z) + 2A_0(m_W^2\xi_W)]. \tag{61}$$

This is exactly the same term that we would get for the Higgs tadpole in the SM. This again shows the convenience of the Higgs basis in the tadpole equations. From Eqs. (30) and (52) we have:

$$\Delta_\xi = \frac{m_3 m_4}{(m_3+m_4)} \frac{1}{16\pi^2 v^2} [A_0(m_Z^2\xi_Z) + 2A_0(m_W^2\xi_W)], \quad (62)$$

which, inserting the SM relations of Eq. (A5) gives exactly the same result as Eq. (60).

VI. DISCUSSION AND CONCLUSIONS

We analytically checked in the CMS or the OS scheme that the gauge dependent term of the mass counterterms for the neutrinos of the GN model comes only from the tadpole contributions, Eq. (60), as suggested in [18]. Using multiplicative renormalization constants and the relations between them, shown in Eqs. (44) and (45), we present how the gauge dependence of neutrino mass counterterms can be seen as a contribution coming from δ_v, the renormalization constant of the VEV in the usual tadpole renormalization (e.g., [36]). We also get that this tadpole contribution is the same for both neutrino counterterms:

$$m_3\delta_{m_3\xi} = m_4\delta_{m_4\xi} = \Delta_\xi. \quad (63)$$

This is one of the features of the GN model: the single sterile neutrino leads to the single value of the Yukawa coupling y to the first Higgs doublet in the Higgs basis. This single value is coupled to the VEV, hence only the single value Δ, related to the VEV shift δ_v, is possible for the neutrino mass counterterms in this setup.

The alternative tadpole scheme, or the FJ scheme [31], consistently omits this gauge dependence from the mass renormalization constants by identifying the bare masses with the proper VEV. Following this scheme, we modify the definition of the mass counterterms to include this tadpole contribution in Eq. (51). This definition now exactly cancels the gauge dependent contribution as can be seen from Eq. (63). The factor Δ gives the same contribution for the mass counterterms as if we would add the contribution of diagrams with tadpoles connected to the propagators as in [18]. The fact that the procedures of [31] works for the seesaw neutrinos just in the same way as with the Dirac particles is explained by the fact that only the Dirac mass [$\sim m_3 m_4$ from Eq. (38)] is directly related to the VEV. The other crosscheck is that the result of Eq. (63), using Eq. (45), gives

$$\delta_{M\xi} = 0, \quad (64)$$

or in other words, the Majorana mass term M, does not acquire gauge dependence in any of these schemes. This again confirms the statement that the Majorana mass term of the sterile part of the neutrino does not affect the application of the FJ scheme for mass counterterms for the neutrinos. Hence using the FJ scheme is straightforwardly applicable in the GN model.

ACKNOWLEDGMENTS

The authors thank the Lithuanian Academy of Sciences for the support (the Project No. DaFi2017).

APPENDIX A: PARAMETRIZATIONS, ASSUMPTIONS, AND RELATIONS

Here we collect all parameters and relations used in our 1 loop calculations. The assumption that some bare parameter p_0 is real, is reflected in the renormalized theory in the sense of Eq. (13). In the FormCalc output for one loop corrections for masses, we implement this assumption by the replacement rule $p^\dagger \to p$, for $p_0 \in \mathbb{R}$.

1. Scalar sector and the SM relations

The assumptions of CP conservation of the Higgs potential give:

$$m_{0ij}^2, \lambda_{0k} \in \mathbb{R}; \quad i,j = 1,2, \qquad k = 1,\ldots,7. \quad (A1)$$

The minimum conditions are

$$m_{11}^2 = -\frac{1}{2}\lambda_1 v^2 \quad \text{and} \quad m_{12}^2 = \frac{1}{2}\lambda_6 v^2. \quad (A2)$$

The Higgs basis is given by:

$$H_1 = \begin{pmatrix} \chi_{0W}^+ \\ \frac{1}{\sqrt{2}}(v+h+i\chi_Z) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_0^+ \\ \frac{1}{\sqrt{2}}(H+iA) \end{pmatrix}. \quad (A3)$$

The mixing matrix for scalars is only between h and H:

$$O^\phi = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix}, \qquad \phi_i^{\text{mass}} = O_{ij}^\phi \phi_j^{\text{Higgs}},$$
$$\phi_i^{\text{Higgs}} = (h, H)_i, \quad (A4)$$

where s_α, c_α are sine and cosine functions of the mixing angle α.

The relations of the electroweak sector are

$$s_{2W} \equiv 2 s_W c_W, \qquad m_Z = \frac{g_e v}{s_{2W}}, \qquad m_W = m_Z c_W, \quad (A5)$$

where s_W and c_W are sine and cosine functions of Weinberg angle.

2. Yukawa sector

As the first thing after generating the FeynArts model file we make the replacements Eqs. (53) and (54):

$$\sum_{j=1}^{3} U_{ij} Y_j^1 \to (0, 0, -ic_{34}y, s_{34}y)_j,$$
$$\sum_{j=1}^{3} U_{ij} Y_j^2 \to (0, d, -ic_{34}d', s_{34}d')_j. \quad (A6)$$

The parametrization of Yukawa couplings are summarized as:

$$V_{1j}Y_j^1 = 0, \quad V_{2j}Y_j^1 = 0, \quad V_{3j}Y_j^1 = y_0,$$
$$V_{1j}Y_j^2 = 0, \quad V_{2j}Y_j^2 = d_0, \quad V_{2j}Y_j^2 = d_0',$$
$$d_0, y_0 \in \mathbb{R}, \quad d_0' \in \mathbb{C}, \tag{A7}$$

where the neutrino mixing matrix is

$$U = U^{34}V = U^{34}U^{\beta}O^{12}U^{\alpha}O^{13}O^{23} \tag{A8}$$

with the relations

$$\nu_{0i}^F = (n_{0e}, n_{0\mu}, n_{0\tau}, N_0)_i, \qquad \nu_{0i}^{\text{mass}} = U_{ij}^* \nu_{0j}^F. \tag{A9}$$

The parametrization of the mixing matrix can be written as:

$$s_{0ij}^2 + c_{0ij}^2 = 1, \quad s_{0ij}, c_{0ij}, \sigma_0, \rho_0 \in \mathbb{R};$$
$$O_{ij}^{AB} = 1_{ij} \quad \text{for } i,j \neq A,B;$$
$$O_{AB}^{AB} = -O_{BA}^{AB} = s_{0AB}; \quad O_{AA}^{AB} = O_{BB}^{AB} = c_{0AB};$$
$$U_{ij}^{\sigma} = e^{i\sigma_0} \quad \text{for } i=j=1; \quad U_{ij}^{\sigma} = 1_{ij} \quad \text{for } i,j \neq 1;$$
$$U_{ij}^{\rho} = e^{i\rho_0} \quad \text{for } i=j=2; \quad U_{ij}^{\rho} = 1_{ij} \quad \text{for } i,j \neq 2;$$
$$U_{34}^{34} = i \cdot U_{43}^{34} = i \cdot s_{034}; \quad U_{33}^{34} = -i \cdot U_{44}^{34} = -i \cdot c_{034};$$
$$U_{ij}^{34} = 1_{ij} \quad \text{for } i,j \neq 3,4. \tag{A10}$$

The seesaw mechanism is realized with:

$$M_0 = m_{04} - m_{03}, \qquad y_0^2 v_0^2 = 2m_{03}m_{04}, \tag{A11}$$

$$s_{034}^2 = \frac{m_{03}}{m_{04} + m_{03}} \quad \text{and} \quad c_{034}^2 = \frac{m_{04}}{m_{04} + m_{03}}. \tag{A12}$$

APPENDIX B: ARRIVING AT EQ. (59)

Here we show some intermediate steps for arriving at the gauge parameter ξ_W dependent term for the δ_{m3} counterterm shown in Eq. (59). We start from Eq. (58):

$$\frac{1}{2}(\Gamma_{\nu_3\nu_3}^{\chi+} + \Gamma_{\nu_3^\dagger\nu_3^\dagger}^{\chi+}) + m_3\Sigma_{\nu_3\nu_3^\dagger}^{W} + m_3\Sigma_{\nu_3\nu_3^\dagger}^{\chi+}. \tag{B1}$$

Let us first look at the loop with the Goldstone boson $\Gamma_{\nu_3\nu_3}^{\chi+}(m_3^2)$. We set up the model file in FEYNARTS following the steps in Sec. V A. After generating diagrams with FeynArts, creating an amplitude with FORMCALC, implementing the parametrization that is summarised in Appendix A by the replacement rules, the standard *Mathematica* "Simplify" command should give:

$$\Gamma_{\nu_3\nu_3}^{\chi+}(m_3^2) = \frac{-\sqrt{m_3m_4}}{4\sqrt{2}\pi^2(m_3+m_4)v}[-m_\tau^2 Y_\tau^{1*} c_{13}c_{23}B_0(m_3^2, m_W^2\xi_W, m_\tau^3) + m_\mu^2 Y_\mu^{1*} c_{13}s_{23}B_0(m_3^2, m_W^2\xi_W, m_\mu^3)$$
$$+ m_e^2 Y_e^{1*} s_{13}B_0(m_3^2, m_W^2\xi_W, m_e^3)]. \tag{B2}$$

Expressing Y^1 from Eqs. (53) and (54) gives:

$$\Gamma_{\nu_3\nu_3}^{\chi+}(m_3^2) = \frac{-y\sqrt{m_3m_4}}{4\sqrt{2}\pi^2(m_3+m_4)v}[m_\tau^2 c_{13}^2c_{23}^2B_0(m_3^2, m_W^2\xi_W, m_\tau^3)$$
$$+ m_\mu^2 c_{13}^2 s_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\mu^3) + m_e^2 s_{13}^2 B_0(m_3^2, m_W^2\xi_W, m_e^3)]. \tag{B3}$$

Now we can express v in terms of Eq. (A5) and y in terms of Eq. (42) to get:

$$\Gamma_{\nu_3\nu_3}^{\chi+}(m_3^2) = \frac{-g_e^2 m_3m_4}{4\pi^2(m_3+m_4)m_Z^2s_{2W}^2}[m_\tau^2 c_{13}^2c_{23}^2B_0(m_3^2, m_W^2\xi_W, m_\tau^3)$$
$$+ m_\mu^2 c_{13}^2 s_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\mu^3) + m_e^2 s_{13}^2 B_0(m_3^2, m_W^2\xi_W, m_e^3)]. \tag{B4}$$

The result for $\Gamma_{\nu_3^\dagger\nu_3^\dagger}^{\chi+}$ is the same, as it should be, since ν_3 is a Majorana fermion and the couplings can be taken real for the one loop correction, hence we can write:

$$\frac{1}{2}(\Gamma_{\nu_3\nu_3}^{\chi+} + \Gamma_{\nu_3^\dagger\nu_3^\dagger}^{\chi+}) = \frac{-g_e^2 m_3m_4}{4\pi^2(m_3+m_4)m_Z^2s_{2W}^2} \times [m_\tau^2 c_{13}^2c_{23}^2B_0(m_3^2, m_W^2\xi_W, m_\tau^3)$$
$$+ m_\mu^2 c_{13}^2 s_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\mu^3) + m_e^2 s_{13}^2 B_0(m_3^2, m_W^2\xi_W, m_e^3)]. \tag{B5}$$

We follow exactly the same steps for $\Sigma^{\chi+}_{\nu_3\nu_3^\dagger}$ to get:

$$\frac{g_e^2 m_4}{8\pi^2(m_3+m_4)m_Z^2 s_{2W}^2}[m_3^2 c_{13}^2 c_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\tau^2) + m_3^2 c_{13}^2 s_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\mu^2) + m_3^2 s_{13}^2 B_0(m_3^2, m_W^2\xi_W, m_e^2)$$
$$+ m_\tau^2 c_{13}^2 c_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\tau^2) + m_\mu^2 c_{13}^2 s_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\mu^2) + m_e^2 s_{13}^2 B_0(m_3^2, m_W^2\xi_W, m_e^2)$$
$$+ m_3^2 c_{13}^2 c_{23}^2 B_1(m_3^2, m_W^2\xi_W, m_\tau^2) + m_3^2 c_{13}^2 s_{23}^2 B_1(m_3^2, m_W^2\xi_W, m_\mu^2) + m_3^2 s_{13}^2 B_1(m_3^2, m_W^2\xi_W, m_e^2)$$
$$+ m_\tau^2 c_{13}^2 c_{23}^2 B_1(m_3^2, m_W^2\xi_W, m_\tau^2) + m_\mu^2 c_{13}^2 s_{23}^2 B_1(m_3^2, m_W^2\xi_W, m_\mu^2) + m_e^2 s_{13}^2 B_1(m_3^2, m_W^2\xi_W, m_e^2)]. \quad \text{(B6)}$$

The loop with the W boson $\Sigma^{W}_{\nu_3\nu_3^\dagger}$ will have gauge invariant contributions from the transverse polarization of the W boson. These can be dropped out from the expression by formally differentiating and integrating with respect to ξ_W in Mathematica. Then every step for simplifying the expression is the same as before with the result:

$$\frac{g_e^2 m_4}{8\pi^2(m_3+m_4)m_Z^2 s_{2W}^2}[-m_3^2 c_{13}^2 c_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\tau^2) - m_3^2 c_{13}^2 s_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\mu^2) - m_3^2 s_{13}^2 B_0(m_3^2, m_W^2\xi_W, m_e^2)$$
$$+ m_\tau^2 c_{13}^2 c_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\tau^2) + m_\mu^2 c_{13}^2 s_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\mu^2) + m_e^2 s_{13}^2 B_0(m_3^2, m_W^2\xi_W, m_e^2)$$
$$- m_3^2 c_{13}^2 c_{23}^2 B_1(m_3^2, m_W^2\xi_W, m_\tau^2) - m_3^2 c_{13}^2 s_{23}^2 B_1(m_3^2, m_W^2\xi_W, m_\mu^2) - m_3^2 s_{13}^2 B_1(m_3^2, m_W^2\xi_W, m_e^2)$$
$$- m_\tau^2 c_{13}^2 c_{23}^2 B_1(m_3^2, m_W^2\xi_W, m_\tau^2) - m_\mu^2 c_{13}^2 s_{23}^2 B_1(m_3^2, m_W^2\xi_W, m_\mu^2) - m_e^2 s_{13}^2 B_1(m_3^2, m_W^2\xi_W, m_e^2)$$
$$+ c_{13}^2 c_{23}^2 A_0(m_W^2\xi_W) + c_{13}^2 s_{23}^2 A_0(m_W^2\xi_W) + s_{13}^2 A_0(m_W^2\xi_W)]. \quad \text{(B7)}$$

Comparing Eq. (B7) with Eq. (B6) we notice that the first, third and fourth lines of both expressions cancel and the second line of both equations is the same. Trigonometric functions near the A_0 integrals in Eq. (B7) sum to one. The sum of Eqs. (B7) and (B6) multiplied by m_3 then gives

$$\frac{g_e^2 m_4 m_3}{4\pi^2(m_3+m_4)m_Z^2 s_{2W}^2}[m_\tau^2 c_{13}^2 c_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\tau^2) + m_\mu^2 c_{13}^2 s_{23}^2 B_0(m_3^2, m_W^2\xi_W, m_\mu^2) + m_e^2 s_{13}^2 B_0(m_3^2, m_W^2\xi_W, m_e^2)]$$
$$+ \frac{g_e^2 m_4 m_3}{8\pi^2(m_3+m_4)m_Z^2 s_{2W}^2} A_0(m_W^2\xi_W). \quad \text{(B8)}$$

The second line cancels with the contribution of the Goldstone loop from Eq. (B5) giving exactly Eq. (59).

[1] K. M. Heeger, Evidence for neutrino mass: A decade of discovery, in *Seesaw 25: Proceedings of the International Conference on the Seesaw Mechanism* (World Scientific, Singapore, 2004), pp. 65–80, DOI 10.1142/9789812702210_0005.

[2] E. Molinaro, Type I seesaw mechanism, lepton flavour violation and Higgs decays, J. Phys. Conf. Ser. **447**, 012052 (2013).

[3] J. Orloff, S. Lavignac, and M. Cribier, eds., *Seesaw 25: Proceedings of the International Conference on the Seesaw Mechanism* (World Scientific, Singapore, 2005), DOI 10.1142/5717.

[4] W. Grimus and H. Neufeld, Radiative neutrino masses in an SU(2) X U(1) model, Nucl. Phys. **B325**, 18 (1989).

[5] H. E. Haber and G. L. Kane, The search for supersymmetry: Probing physics beyond the standard model, Phys. Rep. **117**, 75 (1985).

[6] J. E. Kim, Light pseudoscalars, particle physics and cosmology, Phys. Rep. **150**, 1 (1987).

[7] T. D. Lee, A theory of spontaneous t violation, Phys. Rev. D **8**, 1226 (1973).

[8] R. D. Peccei and H. R. Quinn, *CP* Conservation in the Presence of Pseudoparticles, Phys. Rev. Lett. **38**, 1440 (1977).

[9] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, Theory and phenomenology of two-Higgs-doublet models, Phys. Rep. **516**, 1 (2012).

[10] P. Gambino and P. A. Grassi, The Nielsen identities of the SM and the definition of mass, Phys. Rev. D **62**, 076002 (2000).

[11] A. Denner, S. Dittmaier, M. Roth, and D. Wackeroth, Predictions for all processes $e^+e^- \to$ fermions $+\gamma$, Nucl. Phys. **B560**, 33 (1999).

[12] A. Denner and S. Dittmaier, The complex-mass scheme for perturbative calculations with unstable particles, Nucl. Phys. B, Proc. Suppl. **160**, 22 (2006).

[13] A. Denner, L. Jenniches, J.-N. Lang, and C. Sturm, Gauge-independent $\overline{\text{MS}}$ renormalization in the 2HDM, J. High Energy Phys. 09 (2016) 115.

[14] L. Altenkamp, S. Dittmaier, and H. Rzehak, Renormalization schemes for the two-Higgs-doublet model and applications to $h \to WW/ZZ \to 4$ fermions, J. High Energy Phys. 09 (2017) 134.

[15] M. E. Machacek and M. T. Vaughn, Two loop renormalization group equations in a general quantum field theory. 1. Wave function renormalization, Nucl. Phys. **B222**, 83 (1983).

[16] M. Sperling, D. Stöckinger, and A. Voigt, Renormalization of vacuum expectation values in spontaneously broken gauge theories, J. High Energy Phys. 07 (2013) 132.

[17] M. Sperling, D. Stöckinger, and A. Voigt, Renormalization of vacuum expectation values in spontaneously broken gauge theories: Two-loop results, J. High Energy Phys. 01 (2014) 068.

[18] S. Liebler and W. Porod, Electroweak corrections to neutralino and chargino decays into a W-boson in the (N) MSSM, Nucl. Phys. **B849**, 213 (2011); Erratum, Nucl. Phys.**B856**, 125(E) (2012).

[19] J. M. Cornwall, J. Papavassiliou, and D. Binosi, *The Pinch Technique and its Applications to Non-Abelian Gauge Theories* (Cambridge University Press, Cambridge, England, 2010).

[20] F. Staub, SARAH, arXiv:0806.0538.

[21] F. Staub, SARAH 4: A tool for (not only SUSY) model builders, Comput. Phys. Commun. **185**, 1773 (2014).

[22] T. Hahn, Generating Feynman diagrams and amplitudes with FeynArts 3, Comput. Phys. Commun. **140**, 418 (2001).

[23] T. Hahn and M. Perez-Victoria, Automated one-loop calculations in four and D-dimensions, Comput. Phys. Commun. **118**, 153 (1999).

[24] V. Dūdėnas and T. Gajdosik, On the renormalization of neutrinos in the seesaw extension of the two-Higgs doublet model, Acta Phys. Pol. B **48**, 2243 (2017).

[25] H. K. Dreiner, H. E. Haber, and S. P. Martin, Two-component spinor techniques and Feynman rules for quantum field theory and supersymmetry, Phys. Rep. **494**, 1 (2010).

[26] B. A. Kniehl and A. Sirlin, Pole mass, width, and propagators of unstable fermions, Phys. Rev. D **77**, 116012 (2008).

[27] B. A. Kniehl, Propagator mixing renormalization for Majorana fermions, Phys. Rev. D **89**, 116010 (2014).

[28] D. Espriu, J. Manzano, and P. Talavera, Flavor mixing, gauge invariance and wave function renormalization, Phys. Rev. D **66**, 076002 (2002).

[29] S. Davidson and H. E. Haber, Basis-independent methods for the two-Higgs-doublet model, Phys. Rev. D **72**, 035004 (2005); Erratum, Phys. Rev. D**72**, 099902(E) (2005).

[30] H. E. Haber and D. O'Neil, Basis-independent methods for the two-Higgs-doublet model. II. The significance of $\tan\beta$, Phys. Rev. D **74**, 015018 (2006); Erratum, Phys. Rev. D**74**, 059905(E) (2006).

[31] J. Fleischer and F. Jegerlehner, Radiative corrections to Higgs decays in the extended Weinberg-Salam model, Phys. Rev. D **23**, 2001 (1981).

[32] S. Actis, A. Ferroglia, M. Passera, and G. Passarino, Two-loop renormalization in the standard model. Part I: Prolegomena, Nucl. Phys. **B777**, 1 (2007).

[33] M. Krause, On the renormalization of the two-Higgs-doublet model, Master's thesis, KIT, Karlsruhe, TP, 2016. https://www.itp.kit.edu/_media/publications/masterthesismarcel.pdf.

[34] M. Krause, R. Lorenz, M. Muhlleitner, R. Santos, and H. Ziesche, Gauge-independent renormalization of the 2-Higgs-doublet model, J. High Energy Phys. 09 (2016) 143.

[35] M. Krause, D. Lopez-Val, M. Muhlleitner, and R. Santos, Gauge-independent Renormalization of the N2HDM, J. High Energy Phys. 12 (2017) 077.

[36] A. Denner, Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200, Fortschr. Phys. **41**, 307 (1993).

UŽRAŠAMS

Vilniaus universiteto leidykla
Universiteto g. 1, LT-01513 Vilnius
El. p. info@leidykla.vu.lt,
www.leidykla.vu.lt
Tiražas 12 egz.