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Procedia Engineering 57 (2013) 1309 – 1318

Procedia Engineering

www.elsevier.com/locate/procedia

11th International Conference on Modern Building Materials, Structures and Techniques, MBMST 2013

Strength Calculation Method for Cross-Section of Reinforced Concrete Flexural Member Using Curvilinear Concrete Stress Diagram of EN-2

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Abstract

This article is continuation of earlier work of the author. Apractical method for cross-section strength calculation of flexural reinforced concrete members with rectangular compression zone using curvilinear concrete stress-strain diagrams of Eurocode 2 (EN-2) is presented in this article. The method can be used for strength calculation of members with flanges and reinforcement in compression zone as well. Curvilinear EN-2 stress diagrams are readily superseded by polynomials that can be easily integrated are proposed by the author. It gives opportunity for mathematically accurate, without changing of curvilinear diagram by arbitrary diagrams, determination of value and location for resultant of concrete compression zone stresses and location of neutral axes as well. Calculation is performed for any chosen strain of the layer subjected to the maximum compression. The said strain may be of greater or less value in comparison to that corresponding to the concrete strength. In this way the maximum strength of the member can be estimated when the compression zone strain value varies within chosen interval. The method is suitable in both cases for the members which are commonly (not abundantly) and abundantly reinforced if the neutral axes is located within the cross-section, i. e. within the interval up to the centre of the tensile reinforcement cross-sectional area. Strength of members reinforced with high strength reinforcement which is not sufficiently prestressed or not prestressed at all can be determined by calculation. Formulae can be used for calculation of stress-strain state in persistent situations (not ultimate ones) and for calculation of steel prestress value as well. Rectangular compression zone parameters required for member strength calculation using curvilinear stress diagram are presented in the article. Results obtained by the proposed method are compared with the results obtained using EN-2 formulae.

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Keywords: structural beam members, strength of flexural members, strains, stresses, reinforced concrete.

1. Introduction

In EN 1992-1-1, STR 2.05.05:2005, LST EN 1992-1-1:2005 (hereafter EN-2) [1–3] regulations for strength analysis several design compression concrete stress-strain diagrams are presented. The most general of them is curvilinear one. But its application is not simple because no convenient method of practical use is presented, therefore the curvilinear diagram is superseded by provisory simplified ones (above mentioned regulations: [4] and many others). Availability to use non linier stress diagrams is urgent [5–8].

This article is continuation of earlier work performed by the author [http://techno.su.lt/~zidonis/]. In [10, 11] articles a practical and pretty general engineering method making it possible using integrated technique and integrated formulas to determine by calculation the real values of stress-strain state parameters in cross-sections of members for any stage from the start of the loading up to the failure of the member [10, 11]. For the sake of simplicity this method thereafter will be referred to as ZI method. The ZI method is used here for strength calculation of commonly and abundantly reinforced cross-sections. Clear, easy to understand formulas, needed for calculations coefficients, calculation method and sequence for its application are presented.

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Selection and peer-review under responsibility of the Vilnius Gediminas Technical University
doi:10.1016/j.proeng.2013.04.165

Object of investigation. Commonly reinforced and prestressed concrete flexural members with rectangular compression zone and the neutral axis within the cross-section hereafter are referred to as *flexural* members or *beam type* members. The maximum compression zone depth value equals to d – see Fig. 1. The member can be subjected the action of longitudinal force *N*. It may be force due to loads, reinforcement prestressing force, force acting in compression reinforcement or that in compression flanges or the force equal to the sum of the both latter forces.

Goal of investigation – working up a method and formulas for cross-section strength calculation of commonly and abundantly reinforced concrete beam type members made of various concrete strength classes using curvilinear compression zone stress diagram and calculation and presentation of values required for it.

Tasks: 1) working out a method and formulas for application of curvilinear concrete stress diagram for beam type member cross-section strength calculation in accordance with the limit state (partial factors) method; 2) investigation of the effect of descendant part of stress-strain diagram on strength of cross-sections of beam type members; 3) presentation of method and formulas for calculation of the limit between commonly and abundantly reinforced concrete beams; 4) working out of enhanced method for strength calculation of abundantly reinforced beams [8]; 5) calculation and presentation of curvilinear stress diagram parameters for concretes of various strength classes needed for strength calculation of cross-sections of reinforced concrete beams, i. e. presentation of all information required for strength calculation using proposed method; 6) calculation strength of beams M_{Rd} using proposed method ZI and comparison of results with these obtained from calculations according to EN-2 method, i. e. using formulas recommended in EN-2 regulations where rectangular compression zone stress diagram for concrete is assumed.

Investigation method – theoretical analysis and comparison of results obtained by various theoretical calculations using different methods.

Scope of investigation– four versions of reinforcement ratios for tensile zone of reinforced concrete beams: low (reinforcement ratio $\rho_l \approx 0.44$ %), medium ($\rho_l \approx 1.01$% and $\rho_l \approx 1.60$ %), and high ($\rho_l \approx 2.13$ %). All strength classes of normal weight concrete included in regulations from $f_{ck} = 8$ MPa to $f_{ck} = 90$ MPa were considered. Cross-sectional dimensions of beams: $b = 0.20$ m, $h = 0.50$ m, $d = 0.46$ m (Fig. 1).

2. Investigation method and results

Strength $M = M_{Rd}$, i. e. design ultimate moment, of rectangular reinforced concrete beam type members (Fig. 1) is calculated in this article by two methods: EN-2 and ZI methods. In EN-2 method rectangular concrete compression zone stress diagram is applied while in ZI method – curvilinear one (Fig. 1). For the latter method five cases of the descending part of the curvilinear diagram are considered, i. e. five values of the concrete ultimate strain interval $\Delta\varepsilon_c = \varepsilon_{cu1} - \varepsilon_{c1}$ in the compression zone of reinforced concrete beams: $\varepsilon_{w1} = \varepsilon_{c1}$, $\varepsilon_{w2} = \varepsilon_{c1} + \Delta\varepsilon_c / 4$, $\varepsilon_{w3} = \varepsilon_{c1} + \Delta\varepsilon_c / 2$, $\varepsilon_{w4} = \varepsilon_{c1} + 3\Delta\varepsilon_c / 4$ and $\varepsilon_{w5} = \varepsilon_{cu1}$. Values of notations ε_{c1} and ε_{cu1} see EN-2 regulations and (Figs. 2 and 3).

Curvilinear diagram of stresses (Fig. 2) in EN-2 regulations is described by the following equation:

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k-2)\eta} \tag{1}$$

$\eta = \varepsilon_c / \varepsilon_{c1}$, $k = 1.05 E_{cm}\varepsilon_{c1} / f_{cm}$, $f_{cm} = f_{ck} + 8$, $E_{cm} = 22(f_{cm}/10)^{0.3}$, $\varepsilon_{c1} = 0.7 f_{cm}^{0.31}$; $\varepsilon_{cu1} = 3.5$, as $f_{ck} \le 50$ MPa, and $\varepsilon_{cu1} = 2.8 + 27[(98 - f_{cm})/100]^4$, when $f_{ck} \ge 50$ MPa.

If $E_c = 1.05 E_{cm}$ and $\nu_{c1} = f_{cm} / E_c\varepsilon_{c1}$, then $k = 1/\nu_{c1}$ and

$$\sigma_c = \frac{\eta/\nu_{c1} - \eta^2}{1 + (1/\nu_{c1} - 2)\eta} f_{cm} \tag{2}$$

Graph shown in Fig. 2 is changed by the graph in Fig. 3 [11], which is described by equations

$$\sigma_c = E_c\varepsilon_c(1 + c_1\eta + c_2\eta^2 + c_3\eta^3 + c_4\eta^4) = \nu_c E_c\varepsilon_c = \nu_c\sigma_{ce} \tag{3}$$

$$\nu_c = 1 + c_1\eta + c_2\eta^2 + c_3\eta^3 + c_4\eta^4 \tag{4}$$

Substitution of the graph in Fig. 2 by the graph in Fig. 3 is explained in article [11].

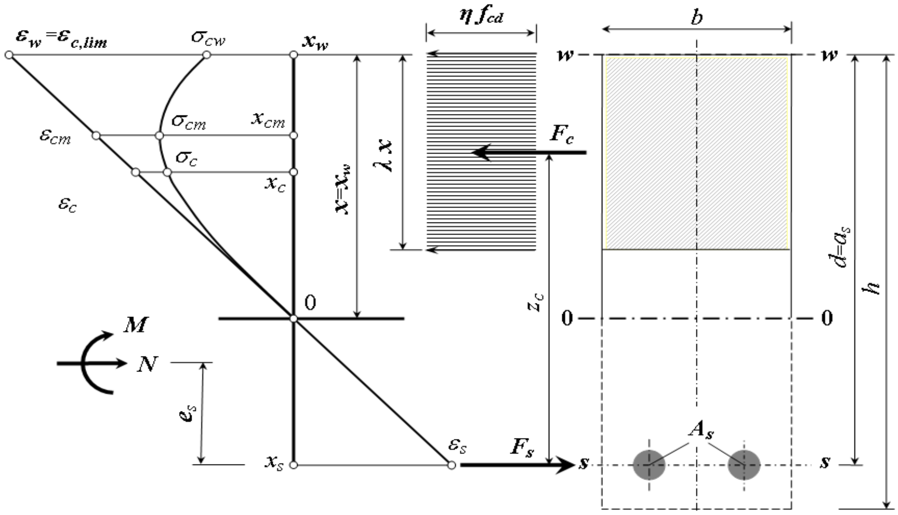

Fig. 1. Cross-section of flexural member with its actions and rectangular concrete stress distribution in compression zone of beams presented in EN-2 regulations

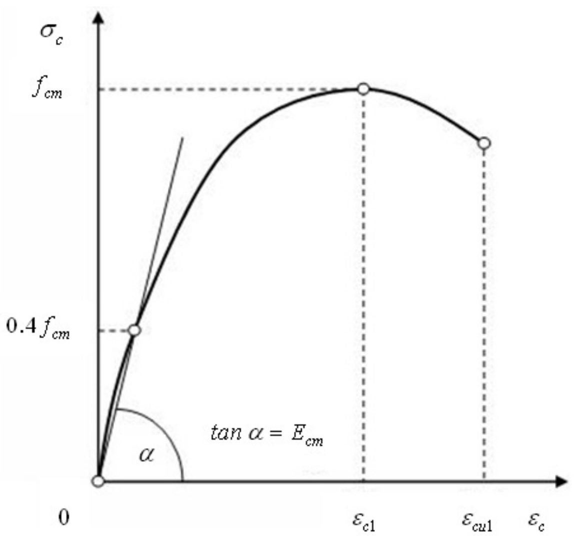

Fig. 2. Stress-strain relationship (1) and (2)

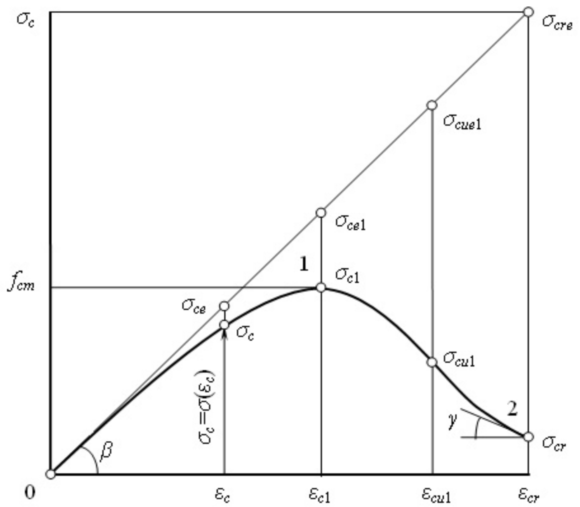

Fig. 3. Stress-strain relationship (3)

$E_c = \tan\beta$, $E_r = \tan(-\gamma)$, $e_r = E_r / E_c$; $\sigma_{ce} = E_c\varepsilon_c$; $\sigma_{ce1} = E_c\varepsilon_{c1}$; $\sigma_{c1} = f_{cm}$; $\nu_{c1} = \sigma_{c1}/\sigma_{ce1}$, $\nu_r = \sigma_{cr}/\sigma_{cre}$; $n_r = \nu_r/\nu_{c1} = \beta_r/\eta_r$, $\beta_r = \sigma_{cr}/\sigma_{c1}$, $\eta_r = \varepsilon_{cr}/\varepsilon_{c1}$ (Fig. 3).

Function (3) is easy to integrate. Resultant force of concrete compression zone stresses in the beams and moment of this force in relation to the neutral axis [11] are:

$$F_{cm} = \int_{x_w}^{0} \sigma_c b(dx_c) = \omega_{nc} E_c \varepsilon_w b x_w = \omega_{nc} E_c \varepsilon_w b d \xi_w , \tag{5}$$

$$M_{cm} = \int_{x_w}^{0} \sigma_c b x_c (dx_c) = \omega_{mc} E_c \varepsilon_w b x_w^2 = \omega_{mc} E_c \varepsilon_w b d^2 \xi_w^2 \tag{6}$$

Design values of the force and the moment F_{cd} and M_{cd} are obtained when F_{cm} and M_{cm} values are divided by the partial factor γ_{Fc} :

$$F_{cd} = \frac{F_{cm}}{\gamma_{Fc}} = \frac{\omega_{nc} E_c \varepsilon_w b x_w}{\gamma_{Fc}} = \frac{\omega_{nc} E_c \varepsilon_w b d \xi_w}{\gamma_{Fc}} \tag{7}$$

$$M_{cd} = \frac{M_{cm}}{\gamma_{Fc}} = \frac{\omega_{mc} E_c \varepsilon_w b x_w^2}{\gamma_{Fc}} = \frac{\omega_{mc} E_c \varepsilon_w b d^2 \xi_w^2}{\gamma_{Fc}} \tag{8}$$

In the proposed method partial factor γ_{Fc} for force and bending moment is used rather than the partial factor γ_C for concrete strength. It is a way to employ curvilinear concrete stress-stain relationship and concrete characteristics given in EN-2 which are used today for determination of design strength M_{Rd} of beams. Author of this article has analyzed and other ways but the proposed way appeared the most acceptable one. It seems that and another way is possible. The graph in Fig. 2 should be replaced by a graph in which instead of f_{cm} and E_{cm} values of $f_{ck} = f_{cm}/\gamma_{Ck}$ and $E_{ckm} = E_{cm}/\gamma_{Ck}$, i.e. characteristic values as in Figures 3.3 and 3.4 of Eurocode EN-2, have to be taken. Then the design values would be obtained using the partial factor γ_C rather than γ_{Fc}, i.e. $f_{cd} = f_{ck}/\gamma_C$ and $E_{cdm} = E_{ckm}/\gamma_C$. The way when $f_{cd} = f_{cm}/\gamma_{Fc}$ and $E_{cdm} = E_{cm}/\gamma_{Fc}$ might be suitable as well. One hopes to verify these versions in the nearest future using $\gamma_{Ck} = 1.3$ and $\gamma_{Fc} = \gamma_{Ck} \cdot \gamma_C = 1.3 \cdot 1.5 = 1.95$.

Reliability of calculated M_{Rd} is provided by concrete strength f_{cd} partial factor $\gamma_C = 1.5$, in the method proposed in this article it is obtained when $\gamma_{Fc} = 1.95$. Substantiation of the said is omitted because of limited extent of this article.

$$\omega_{nc} = \frac{1}{2} + \frac{c_1}{3}\eta_w + \frac{c_2}{4}\eta_w^2 + \frac{c_3}{5}\eta_w^3 + \frac{c_4}{6}\eta_w^4 \tag{9}$$

$$\omega_{mc} = \frac{1}{3} + \frac{c_1}{4}\eta_w + \frac{c_2}{5}\eta_w^2 + \frac{c_3}{6}\eta_w^3 + \frac{c_4}{7}\eta_w^4 \tag{10}$$

$\eta_w = \dfrac{\varepsilon_w}{\varepsilon_{c1}}$, c_i – from [11] or from the Table 2, $\xi_w = \dfrac{x_w}{d}$,

$$e_c = \frac{M_{cm}}{F_{cm}} = \frac{\omega_{mc}}{\omega_{nc}} x_w = \frac{\omega_{mc}}{\omega_{nc}} \xi_w d \tag{11}$$

$$a_c = x_w - e_c = \left(1 - \frac{\omega_{mc}}{\omega_{nc}}\right) \cdot \xi_w d \tag{12}$$

$$z_c = d - a_c = d - \left(1 - \frac{\omega_{mc}}{\omega_{nc}}\right) \cdot d\xi_w = \left[1 - \left(1 - \frac{\omega_{mc}}{\omega_{nc}}\right)\xi_w\right] \cdot d\,. \tag{13}$$

Equations of projections of forces and of moments in relation to the centre of tensile reinforcement cross-sectional area applied:

$$N - F_{cd} + F_{sd} = 0 \tag{14}$$

$$M_s = M + Ne_s = F_{cd} z_c \tag{15}$$

Values of notations used here become clear from Fig. 1. Subscript d indicates that the values are design ones.

Hypothesis of plane sections (Bernoulli) is applied. Effect of tension strength of concrete above the crack is neglected. Axial force N can be due to external loads, reinforcement prestress, compression reinforcement, not thick flanges or equal to the sum of mentioned above forces. Direction of force N shown in Fig. 1 is considered as positive.

Beams referred to as commonly (not abundantly) reinforced when $\varepsilon_s \geq \varepsilon_y$ then $\sigma_s = \sigma_y$ and abundantly reinforced ones – when $\varepsilon_s \leq \varepsilon_y$, then $\sigma_s \leq \sigma_y$.

Required reliability of calculations is provided using either characteristic values for parameters – subscript k is taken (reliability not less than 0.95), or design values – using subscript d (reliability close to 1.00).

2.1. Abundantly reinforced beam type members and such members reinforced with non prestressedor law-prestressed high strength reinforcement

Sometimes in manufacture of prestressed concrete structures short peaces of high strength steel are obtained which economically are not useful for prestressed structures. These peaces can be used for manufacture of non presstresed structures. The method and formulas for strength calculation of abundantly reinforced members which is presented below can be used for strength calculation of structures reinforced with non presstresed and low-prestresed high strength reinforcement as well. Formulas can be use for determination of the minimum value of prestress for high strength reinforcement needed to provide $\varepsilon_s \geq \varepsilon_y$, i.e. not abundant (common, normal, economical) reinforcing case is obtained.

When $\sigma_{sk} \leq \sigma_{yk}$ (case of abundantly reinforced members and other cases when concrete compression zone ultimate strain value is achieved earlier than the yield strain limit in tensile reinforcement, e. g. when non prestressed high strength reinforcement is used), the hypothesis of plain strains is applied for the whole cross-section at the crack (for concrete and reinforcement). Using notations $m = M/bd$, $n = N/bd$ and $\rho_l = A_s/bd$ one obtains

$$\varepsilon_w/\varepsilon_s = x_w/(d - x_w) = \xi_w/(1 - \xi_w) \tag{16}$$

$$F_{sd} = \frac{F_{sk}}{\gamma_S} = \frac{\nu_S E_s A_s}{\gamma_S}\frac{1-\xi_w}{\xi_w}\varepsilon_w \tag{17}$$

$$\left.N - \frac{\omega_{nc}E_c bd\xi_w}{\gamma_{Fc}}\varepsilon_w + \frac{\nu_S E_s A_s}{\gamma_S}\frac{1-\xi_w}{\xi_w}\varepsilon_w = 0, \qquad n - \frac{\omega_{nc}E_c\xi_w}{\gamma_{Fc}}\varepsilon_w + \frac{\nu_S E_s \rho_l}{\gamma_S}\frac{1-\xi_w}{\xi_w}\varepsilon_w = 0\right\} \tag{18}$$

$$\left.\xi_w^2 + \frac{\gamma_{Fc}(\nu_S E_s \varepsilon_w \rho_l - \gamma_S n)}{\gamma_S \omega_{nc} E_c \varepsilon_w}\xi_w - \frac{\gamma_{Fc}\nu_S E_s \rho_l}{\gamma_S \omega_{nc} E_c} = 0, \qquad \xi_w^2 + \left(s - \frac{r}{\varepsilon_w}\right)\xi_w - s = 0\right\} \tag{19}$$

$$\left.r = \frac{\gamma_{Fc} n}{\omega_{nc} E_c}, \qquad s = \frac{\gamma_{Fc}\nu_S E_s \rho_l}{\gamma_S \omega_{nc} E_c}\right\} \tag{20}$$

$$M_s = M + N \cdot e_s = F_{cd} z_s = \frac{\omega_{nc}\varepsilon_w E_c bd\xi_w}{\gamma_{Fc}} \cdot \left[1 - \left(1 - \frac{\omega_{mc}}{\omega_{nc}}\right)\xi_w\right] \cdot d = \frac{[\omega_{nc}\xi_w - (\omega_{nc} - \omega_{mc})\xi_w^2] \cdot E_c \varepsilon_w bd^2}{\gamma_{Fc}} \tag{21}$$

$$\left.\xi_w^2 - \frac{\omega_{nc}}{(\omega_{nc} - \omega_{mc})}\xi_w + \frac{\gamma_{Fc}(M + N \cdot e_s)}{(\omega_{nc} - \omega_{mc})E_c \varepsilon_w bd^2} = 0, \ \xi_w^2 - \frac{\omega_{nc}}{(\omega_{nc} - \omega_{mc})}\xi_w + \frac{\gamma_{Fc}(m + n \cdot e_s)}{(\omega_{nc} - \omega_{mc})E_c \varepsilon_w d} = 0, \ \xi_w^2 - u\xi_w + \frac{w}{\varepsilon_w} = 0\right\} \tag{22}$$

$$\left.u = \frac{\omega_{nc}}{(\omega_{nc} - \omega_{mc})}, \qquad w = \frac{\gamma_{Fc}(m + n \cdot e_s)}{(\omega_{nc} - \omega_{mc})E_c d}\right\} \tag{23}$$

2.2. Not abundantly (commonly) reinforced beam type members

If beam type members are not abundantly (normally, commonly) reinforced, as soon as yielding of tensile reinforcement starts one may assume that $\sigma_s = \sigma_y$ and

$$F_{sd} = \sigma_{yd} A_s = \frac{\nu_S E_s \varepsilon_{yk}}{\gamma_S} A_s = \frac{f_{yk}}{\gamma_S} A_s \tag{24}$$

Here f_{yk} is characteristic value of reinforcement strength. The use of equilibrium condition for the limit states of both compression and tensile zones of the beam – limit states of both zones – is made. When $\varepsilon_s > \varepsilon_y$ then there is no need to apply hypothesis of plain sections for the strain ε_s. From (14) one obtains

$$\left.\frac{\omega_{nc}\varepsilon_w E_c bd}{\gamma_{Fc}}\xi_w - \frac{f_{yk}}{\gamma_S}A_s - N = 0, \qquad \gamma_S\omega_{nc}E_c\varepsilon_w\xi_w - \gamma_{Fc}\left(\nu_S E_s\varepsilon_{yk}\rho_l + \gamma_S n\right) = 0\right\} \tag{25}$$

Equation of moments about F_{sd} is the same as (21) or (22) as for the case of abundantly reinforced members.

2.3. Calculation of the limit between abundant and not abundant (common) reinforcing

Equation (14) is general one and describes equilibrium condition for projections of both abundantly and not abundantly reinforced members. Formula (7) is general one as well. When $\sigma_s = \sigma_y$ then in formula (14) value of $F_{sd} = F_{yd}$ and is the same for both cases, only way of calculation for the cases differs. Then from $F_{sd} = F_{yd}$ equality in formulas (17) and (24)

$$F_{yd} = \frac{\nu_S E_s A_s}{\gamma_S}\frac{1-\xi_w}{\xi_w}\varepsilon_w = \frac{\nu_S E_s \varepsilon_{yk}}{\gamma_S}A_s \tag{26}$$

the value of relative compression zone depth $\xi_{w,\lim}$ corresponding to the limit between abundant and not abundant reinforcing cases is obtained:

$$\xi_{w,\lim} = \frac{\varepsilon_w}{\varepsilon_w + \varepsilon_{yk}} \tag{27}$$

Limit between abundant and not abundant reinforcing can by defined not only by $\xi_{w,\lim}$ but by the limit value of reinforcement ratio $\mu_{l,\lim}$ as well. On account that the value of $\xi_{w,\lim}$ in equations (19) and (25) is the same we obtain.

$$\rho_{l,\lim} = \rho_l = \frac{\gamma_S E_s \omega_{nc} E_c \varepsilon_w^2 - \gamma_{Fc}\gamma_S\left(E_s\varepsilon_w + f_{yk}\right)n}{\gamma_{Fc} f_{yk}\left(E_s\varepsilon_w + f_{yk}\right)} = \frac{\gamma_S\omega_{nc}E_c\varepsilon_w^2 - \gamma_{Fc}\gamma_S\left(\varepsilon_w + \varepsilon_{yk}\right)n}{\gamma_{Fc}E_s\varepsilon_{yk}\left(\varepsilon_w + \varepsilon_{yk}\right)} \tag{28}$$

Note that instead ε_{yk} in formulas (24)–(28) $\varepsilon_{yd} = \varepsilon_{yk}/\gamma_S$ cannot be taken since factor γ_S is already incorporated in equations (19) and (26) and it is not possible to do so for the second time.

Formulas of this article are applicable when $\xi_w \le 1$. For the case when $\xi_w = 1$, value of $\varepsilon_{yk} = 0$.

2.4. Influence of concrete compression zone limit strain and stress diagram on strength of beam type members

Mentioned above the ZI method allows calculation influence of concrete compression zone strain ε_w value on strength M_{Rd} of beams. Five length values of the descending part of curvilinear diagram, i. e. five compression zone limit strain ε_w values of beams within the interval $\Delta\varepsilon_c = \varepsilon_{cu1} - \varepsilon_{c1}$ are selected in the article: $\varepsilon_w = \varepsilon_{w1} = \varepsilon_{c1}$, $\varepsilon_w = \varepsilon_{w2} = \varepsilon_{c1} + \Delta\varepsilon_c/4$, $\varepsilon_w = \varepsilon_{w3} = \varepsilon_{c1} + \Delta\varepsilon_c/2$, $\varepsilon_w = \varepsilon_{w4} = \varepsilon_{c1} + 3\Delta\varepsilon_c/4$ and $\varepsilon_w = \varepsilon_{w5} = \varepsilon_{cu1}$. Beam strength M_{Rd} values corresponding these strains are noted as follows: $M_{Rd,c1,ZI} = M_{Rd,w1,ZI}$, $M_{Rd,w2,ZI}$, $M_{Rd,w3,ZI}$, $M_{Rd,w4,ZI}$ and $M_{Rd,cu1,ZI} = M_{Rd,w5,ZI}$. Subscript $c1$ is identical to $w1$ subscript and $cu1$ – to $w5$ subscript. While instead of subscripts $w1$ and $w5$ writing corresponding $c1$ and $cu1$ subscripts it is necessary to have in mind that these are limit, outside values corresponding beginning and the end of the descending part of analyzed stress diagram parameters (Figs. 2 and 3).

Ratios between $M_{Rd,wi,ZI}$ and $M_{Rd,wi,\lim}$ of abundantly reinforced beams are presented in Table 1. These moments are calculated using the ZI method. In calculations of $M_{Rd,wi,ZI}$ for abundantly reinforced beams the real value x_w of compression zone is taken from formula (19). In these formulas the real value of ε_s in failure stage of the beam is taken into account. In calculations of $M_{Rd,ci,\lim}$ $x_w = x_{w,\lim} = \xi_{w,\lim}d$ is assumed from (27), i. e. $M_{Rd,ci,\lim}$ is calculated in accordance with $x_{w,\lim}$ obtained from the maximum value of concrete compression zone force $N_{Rd,ci,\lim}$ corresponding to $\varepsilon_w = \varepsilon_{wi}$ and $x_w = x_{w,\lim} = \xi_{w,\lim}d$. Such way of calculation (used even in regulations) is illogic since $x_{w,\lim}$ according to formula (27) is independent of reinforcement ratio. Data in the Table 1 show that ratio between these moments depends on concrete strength, reinforcement ratio and on limit strain value $\varepsilon_w = \varepsilon_{c,\lim}$ of concrete compression zone as well and in the case presented here varies from 0.9477 to 1.2774 (in the Table 1 these numbers are accentuated). Thus, strength calculation of abundantly reinforced beams via limitation of compression zone depth irrespective of reinforcement ratio can be very inaccurate. True, this inaccuracy generally leads to reduction in economy rather than in reliability.

In this work M_{Rd} values has been calculated by various methods. Calculation of $M_{Rd,wi,ZI}$ values is explained above. Values of M_{ES-2} are calculated by EN-2 method using formulas recommended in EN-2 regulations in which rectangular concrete compression zone stress diagram is assumed.

Table 1. $M_{Rd,wi,ZI}$ to $M_{Rd,wi,\lim}$ ratios of abundantly reinforced beams. In calculations of $M_{Rd,wi,ZI}$ actual compression zone $x_w = \xi_w d$ value according formula (19) is used. In calculations of $M_{Rd,wi,\lim}$ $x_w = x_{w,\lim} = \xi_{w,\lim} d$ from (27) is used

f_{ck} (MPa)		8	12	16	20	25	30	35	40
ρ_l=1.01%	$M_{Rd,c1,ZI}/M_{Rd,c1,\lim}$	1.10277	1.03622						
	$M_{Rd,w2,ZI}/M_{Rd,w2,\lim}$	1.03998							
	$M_{Rd,w3,ZI}/M_{Rd,w3,\lim}$	**0.94771**							
ρ_l = 1.60%	$M_{Rd,c1,ZI}/M_{Rd,c1,\lim}$	1.20757	1.14090	1.08826	1.04448	1.00066			
	$M_{Rd,w2,ZI}/M_{Rd,w2,\lim}$	1.12941	1.08017	1.03936	1.00410				
	$M_{Rd,w3,ZI}/M_{Rd,w3,\lim}$	1.08696	1.04717	1.01279					
	$M_{Rd,w4,ZI}/M_{Rd,w4,\lim}$	1.06306	1.02982						
	$M_{Rd,cu1,ZI}/M_{Rd,cu1,\lim}$	0.99271	1.02185						
ρ_l = 2.13%	$M_{Rd,c1,ZI}/M_{Rd,c1,\lim}$	**1.27741**	1.21175	1.15963	1.11611	1.07230			
	$M_{Rd,w2,ZI}/M_{Rd,w2,\lim}$	1.18750	1.14130	1.10255	1.06873	1.03366	1.00304		
	$M_{Rd,w3,ZI}/M_{Rd,w3,\lim}$	1.13582	1.10034	1.06909	1.04083	1.01074			
	$M_{Rd,w4,ZI}/M_{Rd,w4,\lim}$	1.10374	1.07570	1.04971	1.02526				
	$M_{Rd,cu1,ZI}/M_{Rd,cu1,\lim}$	1.08204	1.06042	1.03901	1.01789				

Summary of results for calculated ratios of moments $M_{Rd,wi,ZI}/M_{Rd,c1,ZI}$ shows that the greatest value of $M_{Rd} = M_{Rd,ZI,\max}$ at various values of concrete strength and various reinforcement ratios is obtained at various values of $\varepsilon_w = \varepsilon_{wi}$. We regret that presentation of more detailed calculation results is not possible due to article size limitation.

Values of $M_{Rd,wi,ZI}/M_{Rd,c1,ZI}$ ratios of M_{Rd} moments calculated for various ε_{wi} values of the descending part of $\sigma_c - \varepsilon_c$ diagram vary from 0.9826 to 1.1775. These extreme values are obtained for the case of reinforcement factor $\rho_l \approx 1\%$: $M_{Rd,cu1,ZI}/M_{Rd,c1,ZI} = 0.9826$, when $f_{ck} = 16$ MPa, and $M_{Rd,w4}/M_{Rd,c1,ZI} = 1.1775$, when $f_{ck} = 8$ MPa. Due to variation of ε_{wi} value from ε_{c1} to ε_{cu1} $M_{Rd,ci,ZI}$ can be increased up to 17.75% or reduced up to 1.74% in relation to reinforcement percentage and concrete strength.

Seeking for simplification of calculations for design one can assume value of $\varepsilon_{w1} = \varepsilon_{c1}$ and $M_{Rd,c1,ZI}$ values to consider as the main ones. Such simplification will be at the expense of economy rather than of reliability. Data for accomplishment of these calculations are presented in the Table 2. The maximum values of $M_{Rd,ci,ZI}$ are to be considered as the most realistic ones.

Calculations executed in this work by EN-2 and ZI methods revealed that substitution of curvilinear diagram by rectangular one in EN-2 method can cause great errors for beams with high reinforcement ratio and low concrete strength. It was obtained that the ratio of $M_{ES-2}/M_{Rd,c1,ZI}$ varies from 0.7696 to 1.1467 in case of not abundantly reinforced beams and from 0.6644 to 1.0574 – for abundantly reinforced ones. Strength safety reaches even 41.69%: $M_{ES-2}/M_{Rd,w3,ZI} = = (M_{ES-2}/M_{Rd,c1,ZI})/(M_{Rd,w3,ZI}/M_{Rd,c1,ZI}) = 0.6644/1.1395 = 0.5831$.

Calculation of M_{Rd} for abundantly reinforced concrete beams using rectangular diagram and $\xi_c = \xi_{\lim}$ from (27) also is not acceptable because influence of reinforcement ratio is neglected.

Calculation of M_{Rd} by the ZI method presented in this article is not only correct, logical, obvious but and simple as well. The method is quite general. It allows not only to calculate M_{Rd} but and to investigate relationship between M_{Rd} and $\varepsilon_w = \varepsilon_c$ as well. Real assumptions are realized in mathematically correct way in the method. The same as in EN-2 parameters for concrete strength and strains are applied.

Table 2. Concrete strength and deformation properties

Parameters	Concrete strength class														
f_{ck} (MPa)	**8**	**12**	**16**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**70**	**80**	**90**
$f_{cm} = f_{ck} + 8$ (MPa)	16	20	24	28	33	38	43	48	53	58	63	68	78	88	98
$E_{cm} = 22(f_{cm}/10)^{0.3}$ (GPa) (f_{cm}, MPa)	25.33	27.09	28.61	29.96	31.48	32.84	34.08	35.22	36.28	37.28	38.21	39.10	40.74	42.24	43.63
$E_c = 1.05E_{cm}$ (GPa)	26.60	28.44	30.04	31.54	33.05	34.48	35.78	36.98	38.10	39.14	40.13	41.06	42.78	44.36	45.81
$\varepsilon_{c1} = 0.7 f_{cm}^{0,31} \leq 2.8$ (‰) (f_{cm}, MPa)	1.653	1.772	1.875	1.967	2.069	2.162	2.246	2.324	2.397	2.465	2.529	2.589	2.702	2.8	2.8
$\varepsilon_{cu1} = 3.5$ (‰), kai $f_{ck} \leq 50$ MPa $\varepsilon_{cu1} = 2.8 + 27[(98 - f_{cm})/100]^4$ (‰) kai $f_{ck} \geq 50$ MPa	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.205	3.019	2.843	2.803	2.8
$\nu_{c1} = f_{cm}/(1.05E_{cm}\varepsilon_{c1})$ (f_{cm}, MPa; E_{cm}, GPa; ε_{c1}, ‰)	0.364	0.397	0.426	0.453	0.483	0.510	0.535	0.558	0.580	0.601	0.621	0.640	0.675	0.709	0,764
c_1	–1.036	–0.889	–0.765	–0.661	–0.555	–0.471	–0.404	–0.344	–0.283	–0.207	–0.142	–0.072	0.105	0.263	0,669
c_2	0.550	0.379	0.241	0.134	0.040	–0.018	–0.055	–0.089	–0.148	–0.265	–0.376	–0.519	–0.967	–1.423	–2.698
c_3	–0.173	–0.107	–0.056	–0.023	–0.003	–0.001	–0.005	0.003	0.046	0.158	0.282	0.452	0.990	1.592	3.078
c_4	0.023	0.014	0.007	0.002	0.000	0.000	–0.002	–0.011	–0.035	–0.084	–0.143	–0.221	–0.455	–0.726	–1.311
$\omega_{nc} = \omega_{nc1}$ — $\varepsilon_w = \varepsilon_{c1}$,	**0.261**	**0.280**	**0.296**	**0.309**	**0.325**	**0.338**	**0.351**	**0.362**	**0.372**	**0.382**	**0.391**	**0.400**	**0.415**	**0.430**	**0.446**
$\omega_{mc} = \omega_{mc1}$ — $\eta_w = \varepsilon_w/\varepsilon_{c1} = 1$	**0.159**	**0.171**	**0.182**	**0.191**	**0.202**	**0.212**	**0.220**	**0.228**	**0.236**	**0.243**	**0.249**	**0.255**	**0.266**	**0.276**	**0.287**
$\omega_{mc1}/\omega_{nc1} = M_{c1}/F_{c1}/x_w =$ $= e_{0,c1}/x_w = e_{0,c1}/\xi_w d$	0.607	0.612	0.616	0.619	0.623	0.626	0.629	0.631	0.633	0.635	0.637	0.639	0.641	0.643	0.643
$\Delta\omega_{c1} = \omega_{nc1} - \omega_{mc1}$	0.102	0.108	0.113	0.118	0.122	0.126	0.130	0.134	0.136	0.139	0.142	0.145	0.149	0.153	0.159
$1 - \omega_{mc1}/\omega_{nc1} = \Delta\omega_{c1}/\omega_{nc1}$	0.393	0.388	0.384	0.381	0.377	0.374	0.371	0.369	0.367	0.365	0.363	0.362	0.360	0.357	0.357
$\sigma_{C1} = 10^3 E_c \varepsilon_{c1}$ (kPa)	43977	50389	56317	61869	68392	74538	80376	85954	91310	96472	10146 3	10630 2	11558 1	12440 7	13284 9
$10^3 \Delta\omega_{c1}\varepsilon_{c1}E_c$ (kPa)	4513	5458	6378	7276	8367	9427	10456	11464	12457	13448	14412	15365	17248	19059	21110
$10^3 \omega_{nc1}\varepsilon_{c1}E_c = F_{cm}/\xi_{c1}bd$ (kPa)	11496	14077	16618	19121	22193	25212	28171	31092	33981	36869	39697	42503	48012	53405	59193
$\frac{10^3}{1.95}\omega_{nc1}\varepsilon_{c1}E_c = \frac{F_{cm}}{\gamma_{Fc}\xi_{c1}bd} = \frac{F_{cd}}{\xi_{c1}bd}$ (kPa)	5896	7219	8522	9806	11381	12929	14447	15945	17427	18907	20358	21796	24622	27387	30356

When data in the Table 2 worked up by the author are used calculation by proposed ZI method is not more laborious than that by EN-2 method. It is proved by practical calculations of M_{Rd} and A_s for commonly and abundantly reinforced beams.

3. Examples of utilization of formulas used for calculation of abundantly reinforced members, examples of other possible use

3.1. Stress-strain state calculation in a cross-section at the crack for a beam type member subjected to the action of serviceability state bending moment M_{Ek}

During operation of building structures bending moment M_{Ek} value does not exceed $M_{Rd}/1.3$ and $\sigma_s \le \sigma_y$. Therefore formulas given in subsection 2.1 are applicable for calculations of parameters of stress-strain generated by bending moment M_{Ek} acting in persistent situation. Then value of M_{Ek} is known but value of neither ε_w, nor ε_s, nor ξ_w is known. Quite complicated empirical formula in STR 2.05.05:2005 is used for calculation of neutral axis location (ξ_w value). The ZI method does not need empirical formula since value of ξ_w is calculated in theoretical way using equation (29) obtained from solution of the set of equations (19) and (22):

$$r\xi_w^3 - (ru - w)\xi_w^2 + sw\xi_w - sw = 0 \tag{29}$$

For crack width calculation value of ε_s is very significant, while for curvature of member with cracks such is and value of ε_w as well. When value of ξ_w is already known calculation of ε_w value can be performed using equation (19) or (22):

$$\varepsilon_w = \frac{w}{(u - \xi_w)\xi_w} \tag{30}$$

Value of ε_s is calculated from equation (16).

When $n = 0$ then $r = 0$ as well – equation becomes simpler.

In calculation for the action of persistent situation loads (reliability 95%) the following values of partial factors are to be used: $\gamma_{Fck} = 1.3$ (but not $\gamma_{Fc} = 1.95$) and $\gamma_s = 1.0$.

Location of neutral axis can be determined by ZI method in cases of nonlinear $\sigma_s - \varepsilon_s$ reinforcement relationship as well. Then the method of consecutive approximation is used for the calculation.

Values of ν_S, ω_{nc} and ω_{mc} have to comply with the value of ε_w. If the values of ν_S, ω_{nc} and ω_{mc} need correction calculation is repeated.

Since in persistent situation $\varepsilon_w < \varepsilon_{c1}$ then values of ω_{nc} and ω_{mc} can be calculated using not only equations (9) and (10) but and much simpler [9] formulas of this article:

$$\left.\omega_{nc} = \frac{1}{2} + \frac{c_1}{3}\eta_w + \frac{c_2}{4}\eta_w^2, \qquad \omega_{nc} = \frac{1}{3} + \frac{c_1}{4}\eta_w + \frac{c_2}{5}\eta_w^2 \right\} \tag{31}$$

$$\left. c_1 = 3\nu_{c1} - 2, \qquad c_2 = 1 - 2\nu_{c1} \right\} \tag{32}$$

3.2. Calculation of reinforcement prestress

Prestress value σ_p of reinforcement in tensile zone of a flexural member with cracks in its tension zone and subjected to the action of particular value bending moment $M_{Ed} \le M_{Rd}$ can be determined such that desired stress value of $\Delta\sigma_{s,d} = \sigma_S - \sigma_p$ is provided. Here σ_S is value of reinforcement stress appearing from the beginning of its deformation. It requires using formulas for abundant reinforcing (subsections 2.1 and 3.1) to calculate $\sigma_s = \sigma_S$ assuming $\nu_S = 1$, $P = 0$ and provisional values of ω_{nc} and ω_{mc}. Then value of prestress $\sigma_{pI} = \sigma_S - \Delta\sigma_{s,d}$ is calculated. Force of prestress $P_I = \sigma_{pI} A_s$ is added to external forces, values of ω_{nc} and ω_{mc} are corrected and again the value of $\sigma_s = \Delta\sigma_{sI}$ is calculated. If accuracy of calculated $\Delta\sigma_{sI}$ value is insufficient, i. e. difference $\left|\Delta\sigma_{sI} - \Delta\sigma_{s,d}\right|$ is too great then calculation is repeated using $\sigma_{pII} = \sigma_{pI} + \Delta\sigma_{sI} - \Delta\sigma_{s,d}$, $P_{II} = \sigma_{pII} A_s$ and corrected values of ω_{nc} and ω_{mc}. Calculation is repeated until desired accuracy of calculated $\sigma_s = \Delta\sigma_{s,\,i}$ is obtained.

In this way σ_p can be calculated and for the action of $M_{Ed} = M_{Rd}$ moment, i. e. when $\Delta\sigma_{s,d} = \sigma_y - \sigma_p$. Then values of ω_{nc} and ω_{mc} are known in advance since they do not vary and do not need correction repeating calculations. In such a way the minimum value of prestress $\sigma_{p,\min}$ can be calculated.

4. Conclusions

Formulas of ZI method presented in this article for cross-section strength calculation of flexural concrete members not only commonly but and abundantly reinforced as well according to curvilinear Eurocode-2 compression concrete stress-strain diagrams enables solution of many problems as follows.

1. It is applicable not only for the cases of common and abundant reinforcing but and then when compression zone fails prior yielding of tensile reinforcement because high strength reinforcement is not prestressed or not sufficiently prestressed.
2. Prestress value σ_p of reinforcement in tensile zone of a flexural member with cracks in its tension zone and subjected to the action of particular value bending moment $M_{Ed} \le M_{Rd}$ can be determined such that desired stress value of $\Delta\sigma_{s,d} = \sigma_S - \sigma_p$ is provided. Here σ_S is value of reinforcement stress appearing from the beginning of its deformation.
3. One can calculate the minimum reinforcement prestress value $\sigma_{p,\min}$ which enables in calculation of M_{Rd} to consider that reinforcement yield limit is reached if prestress is not less than the said minimum, i. e. the calculation is carried out as for not abundantly (commonly) reinforced members.
4. Calculation of ultimate value of M_{Rd} moment corresponding any chosen deformation of descending part of stress diagram and as well as the possible maximum value of M_{Rd} can be calculated.
5. Formulas for rectangular members with rectangular compression zone are presented in the article when only tension zone is reinforced. Calculation in cases when compression zone is provided with reinforcement and/or flanges if the forces acting in such reinforcement and/or flanges at the ultimate stage are known. Forces acting in compression reinforcement and flanges can be corrected repeating calculation as well.
6. Calculation formulas are applicable for determination of stress-strain state parameters of cross-section at crack in *persistent situation* due to action of bending moment $M_{Ek} < M_{Rd}$, i. e. the value of M_{Ek} is given but neither value of ε_w nor ε_s and ξ_w are known while $\sigma_c < f_{cd}$ and $\sigma_s < \sigma_y$. Differently to the STR method, ξ_w value is calculated not from empirical formula but in theoretical way and compression stress diagram is not rectangular but curvilinear one.
7. Reliability specified in Eurocode-2 is provided using partial factor $\gamma_{Fc} = 1.95$ for resultant F_{cm} force of rectangular compression zone curvilinear stress diagram and bending moment M_{cm} about the neutral axes instead of partial factor for concrete strength $\gamma_C = 1.5$. It makes possible in proposed M_{Rd} calculation method using formula (3) and concrete strength and deformation characteristics given in the Table 2 of EN-2.
8. Calculation by ZI method is not more laborious than that by EN-2 method when data in Tables 2 worked up by the author are used.
9. ZI method and formulas presented in the article apparently could be used for crack width and member curvature calculations. It is the subjects of further investigations.

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