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FACULTY OF MATHEMATICS AND INFORMATICS

Master thesis

Nonstationarity Testing with Local Alternatives in AR(1) Model

Nestacionarumo tikrinimas su artimomis alternatyvomis AR(1) modelyje

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Nonstationarity testing with local alternatives in AR(1) model

Abstract

This master thesis examines unit root testing with the sequence of local alternatives. We analyse first order autoregressive process AR(1) and as test statistics we use $n(\hat{\phi}_n - 1)$, where $\hat{\phi}_n$ is a least squares estimator of an autoregressive parameter ϕ. As an alternative we use the definition and various parameterizations of nearly nonstationary AR(1) process. The empirical cumulative distribution functions and size-adjusted test power curves are analysed in the paper using the most famous parameterizations of coefficient of nearly nonstationary process. We prove limiting distribution of test statistics under alternative with root near unity, confirm the results graphically, with test power analysis we check the probability of making type II error. According to size-adjusted test power curves there is a big possibility to reject a null hypothesis that first order autoregressive model is nonstationary process when it is false.

Key words: Nearly nonstationary AR(1) process, nonstationarity, distribution, limit, test power analysis.

Nestacionarumo tikrinimas su artimomis alternatyvomis AR(1) modelyje

Santrauka

Magistriniame darbe nagrinėjamas vienetinės šaknies testavimas su lokalių alternatyvų seka. Analizuojant pirmos eilės autoregresinį procesą AR(1), testo statistikai naudojama $n(\hat{\phi}_n - 1)$, kur $\hat{\phi}_n$ yra parametro ϕ mažiausių kvadratų įvertinys. Kaip alternatyva darbe naudojamas beveik nestacionaraus AR(1) proceso apibrėžimas ir įvairios parametrizacijos. Naudojantis žymiausiomis beveik nestacionaraus proceso koeficiento parametrizacijomis, analizuojamos empirinės pasiskirstymo funkcijos bei testo galios kreivės. Pagrindinis darbo tikslas – įrodyti beveik nestacionaraus proceso testo statistikos konvergavimą su alternatyvia hipoteze apie vienetinę šaknį arti vieneto ir patvirtinti tai grafiškai, o pasinaudojus testo galios kreivėmis patikrinti, kokia yra tikimybė padaryti antros rūšies klaidą. Remiantis testo galios kreivėmis yra didelė tikimybė atmesti nulinę hipotezę, kuri teigia, kad pirmos eilės autoregresinis modelis yra nestacionarus, kai hipotezė yra klaidinga.

Raktiniai žodžiai: Beveik nestacionarus AR(1) procesas, nestacionarumas, pasiskirstymas, riba, testo galios analizė.

Contents

1 INTRODUCTION

Time series analysis is used in many fields including economic analysis or forecasting, financial and budgetary analysis. There are many models used for time series data. One of the most commonly used model is autoregressive (AR) model. Autoregressive processes are important in mathematics, econometrics and statistics. A lot of authors studied autoregressive time series with roots on or near the unit circle. The interest in nearly nonstationary processes is increasing over the time. Many financial time series data have trending behaviour or nonstationarity. The statisticians David Dickey and Wayne Fuller developed the Dickey–Fuller test [1] in 1979 which tests the null hypothesis of whether a unit root is present in an autoregressive model. The alternative hypothesis is the opposite – the model is stationary. Unit root test is useful to find out if the data needs to be transformed to avoid nonstationarity problem. The stationarity or nonstationarity can strongly influence the behaviour and properties of analysed data. Persistence of shocks will be infinite for nonstationary series.

Peter C. B. Phillips and Pierre Perron [2] have also analysed theory of unit root nonstationarity. The authors developed a similar test to Dickey–Fuller test but they incorporated an automatic correction to the Dickey–Fuller procedure to allow for autocorrelated residuals. Phillips–Perron test is used in time series analysis to test the null hypothesis that a time series is integrated of order 1.

Kwiatkowski–Phillips–Schmidt–Shin [3] developed KPSS test which checks a null hypothesis that time series is stationary around a deterministic trend against the alternative of a unit root. Unlike the other tests, the null hypothesis for the KPSS test is opposite to Dickey–Fuller test and Phillips–Perron test.

Autoregressive time series with a root near unity was studied by G. B. A. Evans and N. E. Savin [4]. By simulation experiment the authors found out that the statistical properties of the coefficient estimator and associated $t - test$ in a stationary AR(1) with a root near unity are close to those that are applied when the model is a random walk, even when the sample size is as large as 100. Also the authors reached similar results when AR(1) process was mildly explosive.

J. Kormos [5] analysed hypothesis for nearly nonstationary AR(1) model with Gaussian autoregressive innovations. The author proved that the limit distribution of test statistic remains unchanged when the first-order autoregressive model is replaced by a stationary p-th order AR process.

In this paper, big attention will be paid on the local power of unit root tests when alternative hypothesis is close to the null hypothesis, so-called "local alternatives".

The main purpose of master thesis is to test whether the selected test statistic can be used as a unit root test with local alternatives and to check at which parameter values we can distinguish between the null hypothesis and alternative hypothesis.

In master thesis first order autoregressive model will be analysed with different parameterization of coefficient ϕ_n. In Chapter 2 we will take a look how two parameterizations change limiting distribution theoretically and in Chapter 3 these results will be checked with simulated AR(1) process. Test power analysis will be analysed in Chapter 4.

2 FIRST ORDER AUTOREGRESSIVE AR(1) MODEL

In this chapter the first order autoregressive AR(1) model will be defined. We will take a look how model structure changes with different parameterizations of unknown parameter ϕ. Also the convergence of parameters will be explained in chapter 2.2.2.

An autoregressive model is used to describe certain time-varying processes in economics, finance, etc. A value from a time series is regressed on previous values from the same time series. In the AR(1) model the observations y_t at time t with starting point y_0 at time $t = 0$ are generated according to the scheme

$$y_t = \phi y_{t-1} + \epsilon_t,\ t \geq 1, \tag{1}$$

where the ϵ_t's are random disturbances or innovations, ϕ is an unknown parameter and ϵ_t is an i.i.d sequence with zero mean and variance σ_ϵ^2. There exist three cases:

- if $|\phi| < 1$, then AR(1) is stationary process;
- if $|\phi| > 1$, then AR(1) is explosive process;
- if $\phi = 1$, then AR(1) is nonstationary process.

If ϕ is close to 0, then the process behaves like white noise, but as ϕ approaches 1, the output gets a larger contribution from the previous term relative to the noise. For $|\phi| > 1$, the model (1) is called explosive. It means that the system is highly affected by the past information. The process (1) has a unit root if the coefficient ϕ is equal to 1. A nonstationary process has a variance that depends on time and diverges to infinity. If the process is nonstationary when shocks have permanent effect which does not decay contrary to stationary process.

The main task of master thesis is to test hypotheses:

$H_0 : \phi = 1 \quad$ alternatively $\quad H_0 : \gamma = 0$
$H_1 : \phi = \phi_n = e^{\gamma/n} \quad$ alternatively $\quad H_1 : \gamma/n \to 0, \quad$ as $\quad n \to \infty \quad$ and $\quad \gamma < 0$

and

$H_0 : \phi = 1$
$H_1 : \phi = \phi_n = 1 - \frac{\gamma_n}{n}$, where $\gamma_n \to \infty$, as $n \to \infty$ and $\frac{\gamma_n}{n} \to 0$, as $n \to \infty$ and γ_n tends to infinity slower than n

with test statistic $n(\widehat{\phi} - 1)$, when ϕ is unknown.

2.1 Least squares estimator (LSE)

The least square estimate of ϕ based on observations $y_1, ..., y_n$ is defined as

$$\widehat{\phi} = \frac{\sum_{t=1}^{n} y_t y_{t-1}}{\sum_{t=1}^{n} y_{t-1}^2}. \tag{2}$$

When $|\phi| < 1$ H. B. Mann and A. Wald (1943) [6] and T. W. Anderson (1959) [7] showed that the standardized LSE is asymptotically normal:

$$(\sum_{t=1}^{n} y_{t-1}^2)^{1/2}(\widehat{\phi} - \phi) \xrightarrow[n\to\infty]{\mathbb{D}} \mathcal{N}(0,1).$$

With another standardization:

$$\sqrt{n}(\widehat{\phi} - \phi) \xrightarrow[n\to\infty]{\mathbb{D}} \mathcal{N}(0, 1-\phi^2).$$

For $\phi > 1$ T. W. Anderson (1959)[7] showed that:

$$(\sum_{t=1}^{n} y_{t-1}^2)^{1/2}(\widehat{\phi} - \phi) \xrightarrow[n\to\infty]{\mathbb{D}} \mathcal{N}(0,1)$$

is true when ϵ_k's are i.i.d. For general ϵ_k's the author showed that limiting distribution of $(\sum_{t=1}^{n} y_{t-1}^2)^{1/2}(\widehat{\phi} - \phi)$ may not exist.

When $\phi = 1$ J. S. White (1958) [8] and M. M. Rao (1978) [9] showed that the limit distribution of the properly standardized sequence of the LSE is non-normal:

$$(\sum_{t=1}^{n} y_{t-1}^2)^{1/2}(\widehat{\phi} - \phi) \xrightarrow[n\to\infty]{\mathbb{D}} \frac{\frac{1}{2}(W^2(1) - 1)}{(\int_0^1 W^2(t)dt)^{\frac{1}{2}}}$$

or

$$n(\widehat{\phi} - 1) \xrightarrow[n\to\infty]{\mathbb{D}} \frac{\int_0^1 W(t)dW(t)}{\int_0^1 W^2(t)dt}, \tag{3}$$

where $(W(t), 0 \leq t \leq 1)$ is a standard Brownian motion.

2.2 Nearly nonstationary AR(1) process

Suppose we have first order autoregressive process defined as

$$y_{n,t} = \phi_n y_{n,t-1} + \epsilon_t,\ k = 1, ..., n;\ n = 1, 2, ... \quad , \tag{4}$$

where $\phi_n \to 1$, as $n \to \infty$, (ϵ_t) is a sequence of i.i.d random variables with zero mean and finite variance σ_ϵ^2, n is a sample size. The process generated by (4) is called nearly nonstationary AR(1) process.

2.2.1 Parameterization of ϕ_n

When ϕ_n is close to unity the limiting law of the standardized LSE of ϕ_n may not be an approximation of the exact distribution of the standardized LSE of ϕ. Nearly nonstationary AR(1) process (4) has a lot of parameterizations which makes ϕ_n be as close as possible to unity. P. C. B. Phillips (1987) [10] suggested to parameterize ϕ_n by $\phi_n = e^{\gamma/n}$ with constant $\gamma < 0$.

Another parameterization was suggested by P. C. B. Phillips and L. Giraitis (2006) [11]. The coefficient of nearly nonstationary AR(1) process the authors parameterized by $\phi_n = 1 - \frac{\gamma_n}{n}$, $\gamma_n \to \infty$, as $n \to \infty$ and $\frac{\gamma_n}{n} \to 0$, as $n \to \infty$ and γ_n tends to infinity slower than n.

2.2.2 Test statistic under alternatives

The limiting distribution of nearly nonstationary AR(1) process depends on the parameterizations of the coefficient ϕ_n.

Lemma 1: Let the first order autoregressive model be generated by (1) with test statistic $n(\widehat{\phi} - 1)$, where $\widehat{\phi}$ is least squares estimator of an autoregressive parameter ϕ.

Then under alternative that $\phi = \phi_n = e^{\gamma/n}, \quad \gamma/n \to 0$, as $n \to \infty$ and $\gamma < 0$:

$$n(\widehat{\phi} - 1) \xrightarrow[n\to\infty]{\mathbb{D}} \gamma + \frac{\int_0^1 U_\gamma(t)dW(t)}{\int_0^1 (U_\gamma)^2(t)dt}.$$

Proof:

Suppose that first order autoregressive process is generated by (1) and $\widehat{\phi}$ is estimated by (2).

Using the parameterization that suggested P. C. B. Phillips (1987) ($\phi_n = e^{\gamma/n}$ with constant $\gamma < 0$) the limit of $n(\widehat{\phi} - 1)$ is:

$n(\widehat{\phi} - 1) = n(\widehat{\phi}_n - \phi_n + \phi_n - 1) = n(\widehat{\phi}_n - \phi_n) + n(\phi_n - 1).$

- P. C. B. Phillips (1987) showed that

  $n(\widehat{\phi_n} - \phi_n) \xrightarrow[n\to\infty]{\mathbb{D}} \frac{\int_0^1 U_\gamma(t)dW(t)}{\int_0^1 (U_\gamma)^2(t)dt}$, as $n \to \infty$, when $\phi_n = e^{\gamma/n}, \gamma < 0$, where $U_\gamma(t)$ is Ornstein–Uhlenbeck process defined by $U_\gamma(t) = \int_0^s e^{\gamma(s-t)}dW(t)$.

- $n(\phi_n - 1) = n(e^{\gamma/n} - 1) = n((1 + \frac{\gamma}{n} + \frac{(\frac{\gamma}{n})^2}{2!} + \frac{(\frac{\gamma}{n})^3}{3!} + ... + \frac{(\frac{\gamma}{n})^n}{n!} + ...) - 1)$

  $= n(\frac{\gamma}{n} + \frac{(\frac{\gamma}{n})^2}{2!} + \frac{(\frac{\gamma}{n})^3}{3!} + ... + \frac{(\frac{\gamma}{n})^n}{n!} + ...)$

  $= (\gamma + \frac{1}{2!}n\frac{\gamma^2}{n^2} + \frac{1}{3!}n\frac{\gamma^3}{n^3} + \frac{1}{n!}n\frac{\gamma^n}{n^n} + ...)$

  $= (\gamma + \frac{1}{2!}\frac{\gamma^2}{n} + \frac{1}{3!}\frac{\gamma^3}{n^2} + \frac{1}{n!}\frac{\gamma^n}{n^{n-1}} + ...) \xrightarrow[n\to\infty]{} \gamma,$

  as $(\frac{1}{2!}\frac{\gamma^2}{n} + \frac{1}{3!}\frac{\gamma^3}{n^2} + \frac{1}{n!}\frac{\gamma^n}{n^{n-1}} + ...) \xrightarrow[n\to\infty]{} 0.$

  In Figure 1 red line indicates density plots of $\gamma + \frac{\int_0^1 U_\gamma(t)dW(t)}{\int_0^1 (U_\gamma)^2(t)dt}$ with $\gamma = -1, -2, -3, -5$ and black line indicates density plots of test statistic $n(\widehat{\phi} - 1)$ under null hypothesis that $\widehat{\phi} = 1$ and generated by (3). From figure below we can say that as γ increases density plots become more similar to the limiting distribution in Lemma 1.

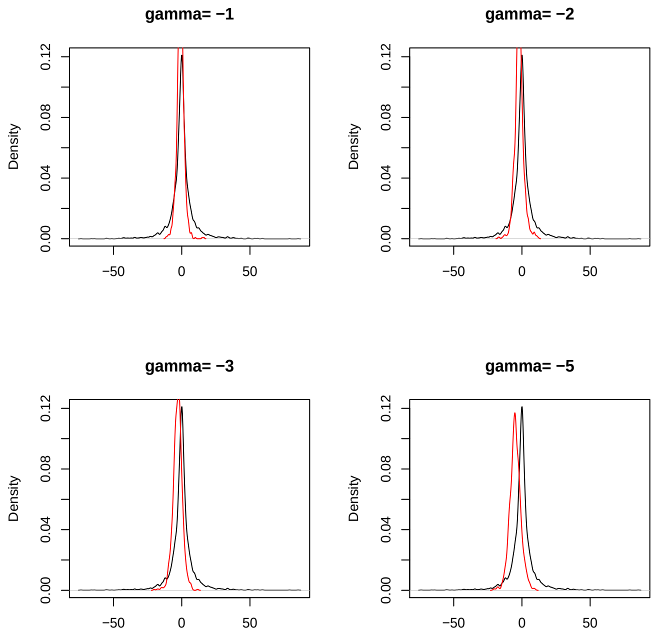

Figure 1: Plots of Kernel Density Estimations, when $\gamma = -1, -2, -3, -5$

Lemma 2: Let first order autoregressive model be generated by (1) with test statistic $n(\widehat{\phi} - 1)$, where $\widehat{\phi}$ is the least squares estimator of an autoregressive parameter ϕ. Then under alternative that $\phi = \phi_n = 1 - \frac{\gamma_n}{n}$, where $\gamma_n \to \infty$, as $n \to \infty$ and $\frac{\gamma_n}{n} \to 0$, as $n \to \infty$ and γ_n tends to infinity slower than n:

$$n(\widehat{\phi_n} - 1) \xrightarrow[n\to\infty]{\mathbb{D}} -\infty.$$

Proof:

Suppose that first order autoregressive process is generated by (1) and $\widehat{\phi}$ is estimated by (2).

Using the parameterization that was suggested P. C. B. Phillips and L. Giraitis (2006) ($\phi_n = 1 - \frac{\gamma_n}{n}$, $\gamma_n \to \infty$, as $n \to \infty$ and $\frac{\gamma_n}{n} \to 0$, as $n \to \infty$ and γ_n tends to infinity slower than n) the limit of $n(\widehat{\phi} - 1)$ is:

$n(\widehat{\phi} - 1) = n(\widehat{\phi}_n - \phi_n + \phi_n - 1) = n(\widehat{\phi}_n - \phi_n) + n(\phi_n - 1).$

- P. C. B. Phillips and L. Giraitis (2006) showed that

$$\frac{n^{\frac{1}{2}}}{(1-\phi_n^2)^{\frac{1}{2}}}(\widehat{\phi_n} - \phi_n) \xrightarrow[n\to\infty]{\mathbb{D}} \mathcal{N}(0,1), \tag{5}$$

when $\phi_n = 1 - \frac{\gamma_n}{n}$.

Using P. C. B. Phillips and L. Giraitis limit (5), we obtain
$n(\widehat{\phi}_n - \phi_n) = \frac{n^{\frac{1}{2}}}{(1-\phi_n^2)^{\frac{1}{2}}}(\widehat{\phi_n} - \phi_n)n^{\frac{1}{2}}(1-\phi_n^2)^{\frac{1}{2}}$

and $\frac{n^{\frac{1}{2}}}{(1-\phi_n^2)^{\frac{1}{2}}}(\widehat{\phi_n} - \phi_n) = Op(1)$.

Also, $n^{\frac{1}{2}}(1-\phi_n^2)^{\frac{1}{2}} = n^{\frac{1}{2}}((1-\phi_n)(1+\phi_n))^{\frac{1}{2}}$.

Next note that $(1+\phi_n) \leq 2$ and $(n(1-\phi_n))^{\frac{1}{2}} \xrightarrow[n\to\infty]{} \infty$.

Finally $n(\phi_n - 1) = n((1-\frac{\gamma_n}{n}) - 1) = -n\frac{\gamma_n}{n} = -\gamma_n \xrightarrow[n\to\infty]{\mathbb{D}} -\infty$.

Since $n^{\frac{1}{2}}(1-\phi_n^2)^{\frac{1}{2}}$ tends to infinity slower than $-\gamma_n$ tends to $-\infty$, so the result is proved.

3 TESTING NEARLY NONSTATIONARY AR(1) PROCESS

In this chapter empirical cumulative distribution functions will be analysed when parameters of nearly nonstationary AR(1) process change. The graphical results will be compared with theoretical analysis made in Chapter 2.

3.1 Nonstationary AR(1) process

When $\phi = 1$ J. S. White (1958) and M. M. Rao (1978) showed that the limit distribution of the properly standardized sequence of the LSE is non-normal and the limit (3) is valid. Here 1000 realizations of test statistic $n(\widehat{\phi}_n - 1)$ with the sample size 1000 of nonstationary AR(1) process were computed. The limit distribution was computed with 5000 realizations of test statistics and with sample size 5000. The values of the standard Brownian motion $(W(t))$ were approximated by

$$W(t) = \frac{1}{\sqrt{n}} \sum_{k=1}^{[nt]} \epsilon(k),\ t \in [0, 1],$$

where $\epsilon(k)$ are standard normally distributed random variables.

In Figure 2 red line indicates empirical cumulative distribution function of $\frac{\int_0^1 W(t)dW(t)}{\int_0^1 W^2(t)dt}$ and black line indicates empirical cumulative distribution function of $n(\widehat{\phi}_n - 1)$. From the figure below we can say that empirical cumulative distribution functions behaves very similarly.

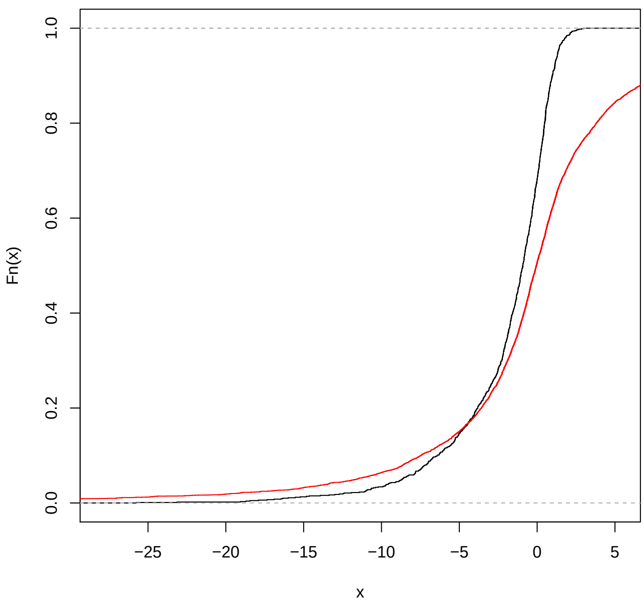

Figure 2: Empirical cumulative distribution functions

Next, we compute sample quantiles from the empirical distribution function, that might help with the hypothesis testing. If the value of test statistic $n(\widehat{\phi}_n - 1)$ is higher than value with selected probability from Table 1, then null hypothesis is accepted.

1%	2.5%	5%	10%	15%
-29.14975	-18.21477	-12.54647272	-7.49989742	-5.30053633

Table 1: Sample Quantiles

3.2 Nearly nonstationary AR(1) process when $\phi_n = e^{\gamma/n}$

P. C. B. Phillips (1987) showed that

$$n(\widehat{\phi_n} - \phi_n) \xrightarrow[n\to\infty]{\mathbb{D}} \frac{\int_0^1 U_\gamma(t)dW(t)}{\int_0^1 (U_\gamma)^2(t)dt},$$

as $n \to \infty$, when $\phi_n = e^{\gamma/n}, \gamma < 0$, where $U_\gamma(t)$ is Ornstein–Uhlenbeck process. Here 1000 realizations of test statistic $n(\widehat{\phi_n} - \phi_n)$ with the sample size $n = 100; 500; 1000$ of nonstationary AR(1) process and $\gamma = -0.5; -1; -2; -5; -20; -50$ were computed. The Ornstein–Uhlenbeck process was approximated by

$$S(j) = S(j-1)e^{\gamma/n} + \sqrt{\frac{1 - e^{2\gamma/n}}{-2\gamma}}\epsilon(j),$$

where $\epsilon(j)$'s are standard normal random variables with zero mean and variance equal to one. Black line indicates empirical cumulative distribution function of $n(\widehat{\phi}_n - \phi)$ and red line indicates empirical cumulative distribution function of $\frac{\int_0^1 U_\gamma(t)dW(t)}{\int_0^1 (U_\gamma)^2(t)dt}$. From the figures below (Figure 3 - Figure 8) we can say that empirical cumulative distribution functions behave very similarly, also when $\gamma \to -\infty$ empirical cumulative distribution functions become more similar.

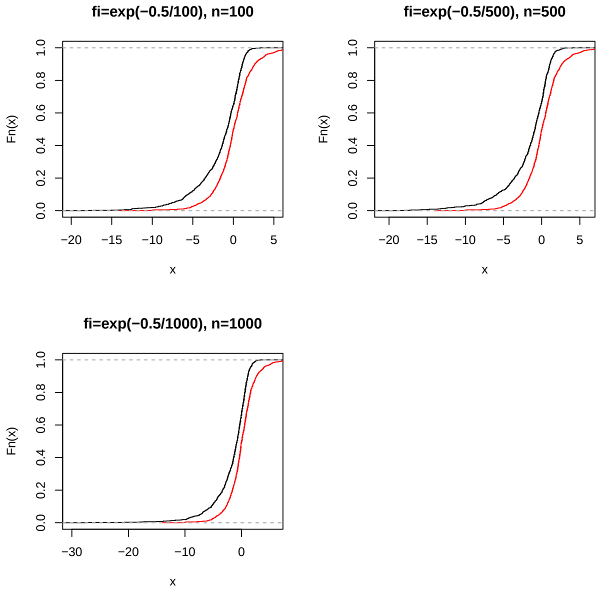

Figure 3: Empirical cumulative distribution functions

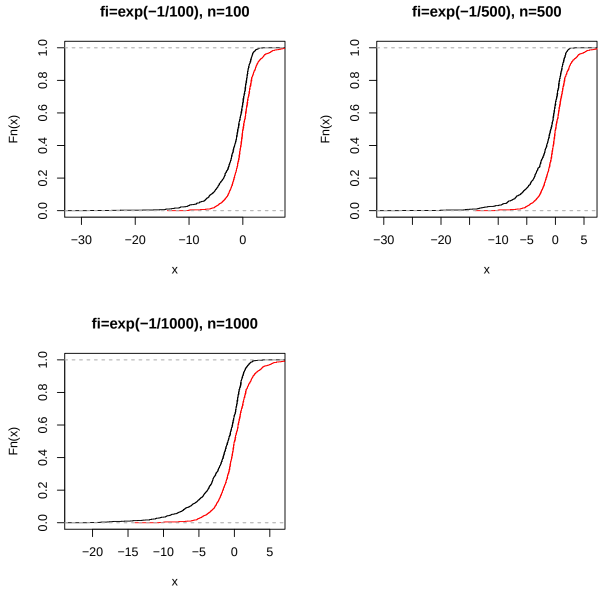

Figure 4: Empirical cumulative distribution functions

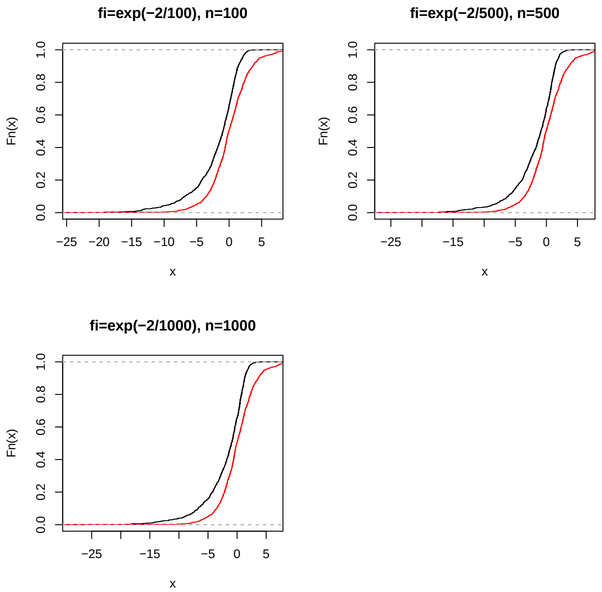

Figure 5: Empirical cumulative distribution functions

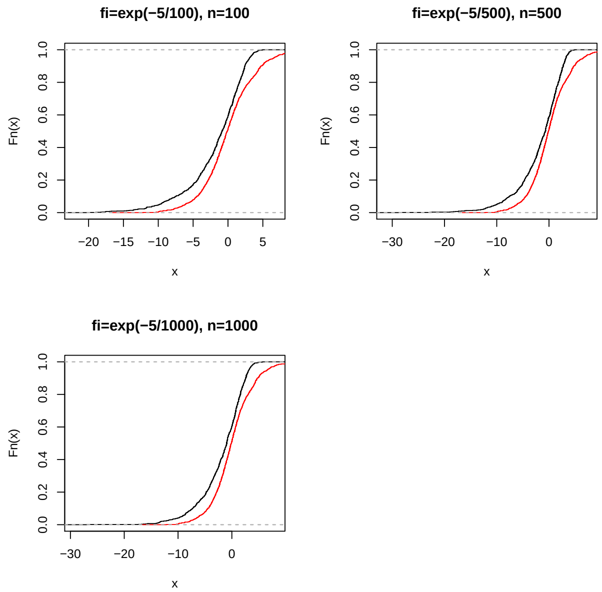

Figure 6: Empirical cumulative distribution functions

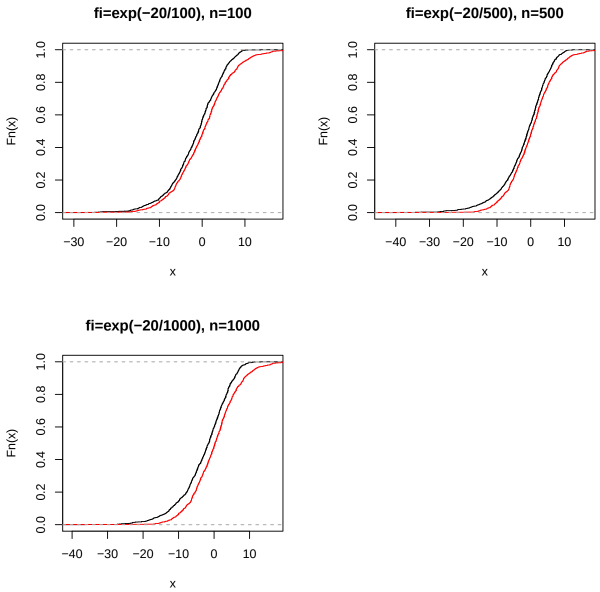

Figure 7: Empirical cumulative distribution functions

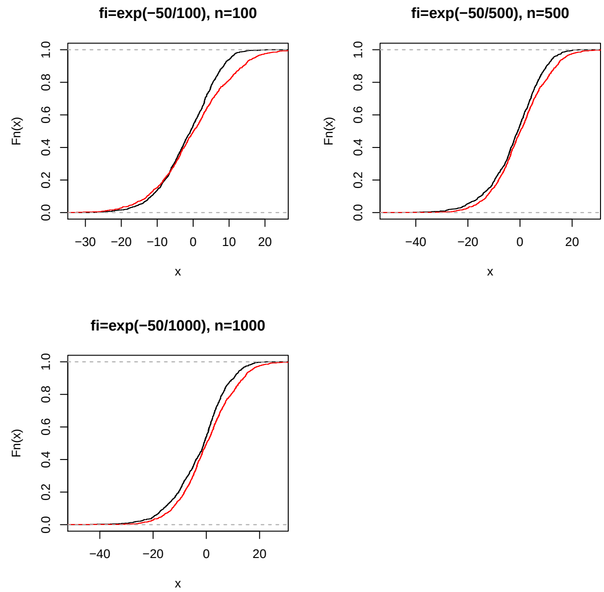

Figure 8: Empirical cumulative distribution functions

3.3 Nearly nonstationary AR(1) process when $\phi_n = 1 - \frac{\gamma_n}{n}$

P. C. B. Phillips and L. Giraitis (2006) showed that the limit (5) is valid , when $\phi_n = 1 - \frac{\gamma_n}{n}$. Here 1000 realizations of test statistic $\frac{n^{\frac{1}{2}}}{(1-\phi_n^2)^{\frac{1}{2}}}(\widehat{\phi_n} - \phi_n)$ with the sample size $n = 100; 500; 1000$ of nonstationary AR(1) process and $\gamma_n = \frac{n}{log(n)}; log(n); n^{3/4}$ were computed. The limit distribution is the standard normal distribution. In Figures 9-11 red lines indicate standard normal distribution and black lines indicate nearly nonstationary AR(1) processes with different sample size and three cases of constant γ. From the figures below (Figure 9 - Figure 11) we can say that empirical cumulative distribution functions behave very similarly, in all cases as sample size increases empirical cumulative distribution functions get closer to standard normal distribution functions.

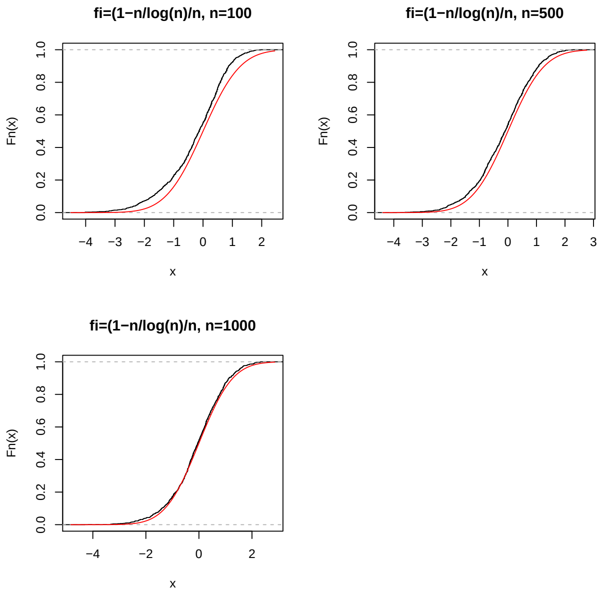

Figure 9: Empirical cumulative distribution functions

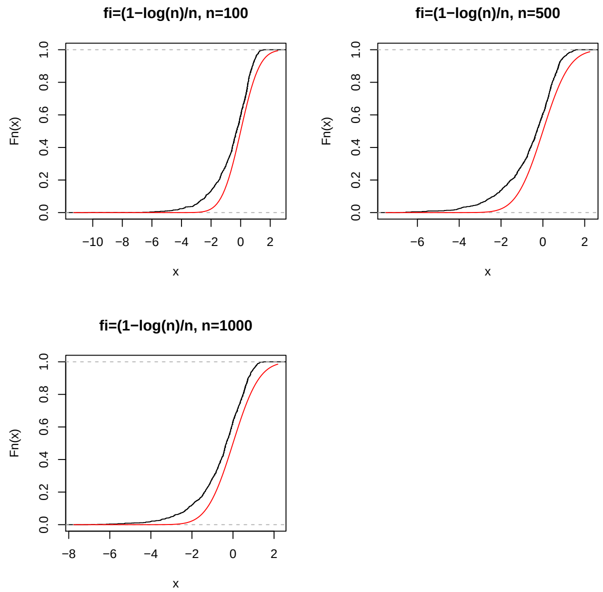

Figure 10: Empirical cumulative distribution functions

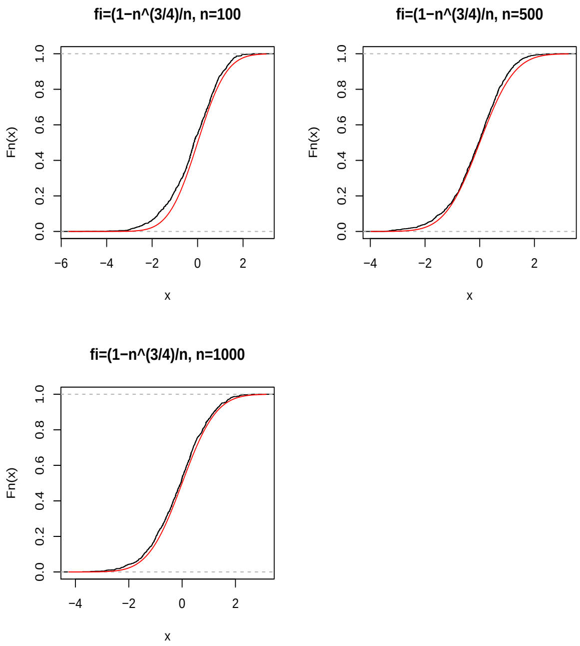

Figure 11: Empirical cumulative distribution functions

4 TEST POWER ANALYSIS

The hypothesis tests are used to make a decision to reject the null hypothesis then the p-value is below a fixed value α or to accept it. There are four possible decisions: two correct and two with possible errors. The decision making combinations are:

	Accept H_0	Reject H_0
H_0 is true	Correct decision	Type I error
H_0 is false	Type II error	Correct decision

Table 2: Errors in hypothesis tests

In Table 2 a type I error means that the null hypothesis is rejected when it is true. The probability of a type I error is called the significance level of a test and is denoted by α:

$$\alpha = \mathbb{P}(Type\, I\, error).$$

A type II error is not rejecting a null hypothesis when it is false. The power of a hypothesis test for a specified alternative hypothesis is

$$\beta = 1 - \mathbb{P}(Type\, II\, error).$$

In this chapter the power of test statistics will be checked using size-adjusted test power curves [12]. Here two experiments were performed using the same sequence of random numbers. In the first experiment the null hypothesis holds (first order autoregressive model is nonstationary ($\phi = 1$)) and in the second hypothesis it does not hold. Two cases with different parameterization of coefficient ϕ_n were analysed changing sample size and constant γ.

4.1 Size-adjusted power curves for nearly nonstationary AR(1) process when $\phi_n = e^{\gamma/n}$

Fifteen experiments were performed with hypothesis:

$H_0 : \gamma = 0$
$H_1 : \gamma/n \to 0, \quad \text{as} \quad n \to \infty \quad \text{and} \quad \gamma < 0,$

where $n = 100; 500; 1000$, $\gamma = -2; -5; -10; -20; -50$.

In Figures 12-16 black lines indicate size-adjusted test power when sample size is 100, blue and red lines indicate size-adjusted test power when sample size is 500 and 1000 accordingly. The closer the line to the left upper point – the higher the power of that test is. It means that the probability to make type II error is small. From figures we can say that the power of test does not depends drastically on sample size but there is a tendency that as sample size increases the power of test increases too. From Figures 12-14 we can conclude that when $\gamma = -2; -5; -10$ the probability to make type II error is big. When $\gamma = -20; -50$ the probability to accept the null hypothesis when it is false is very small.

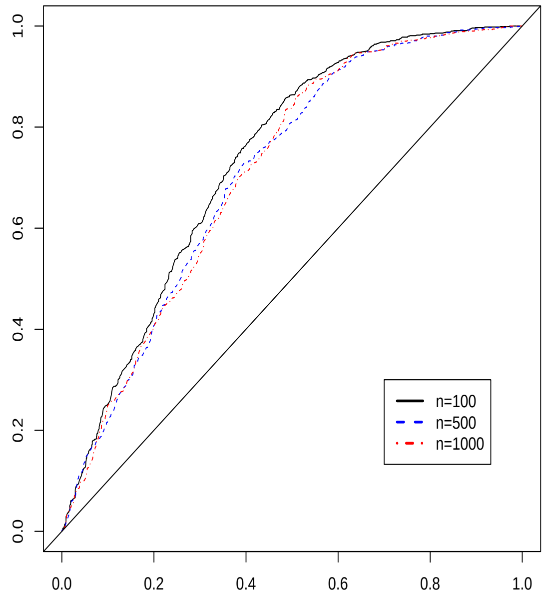

Figure 12: Size-power curve when $\phi_n = e^{-2/n}$

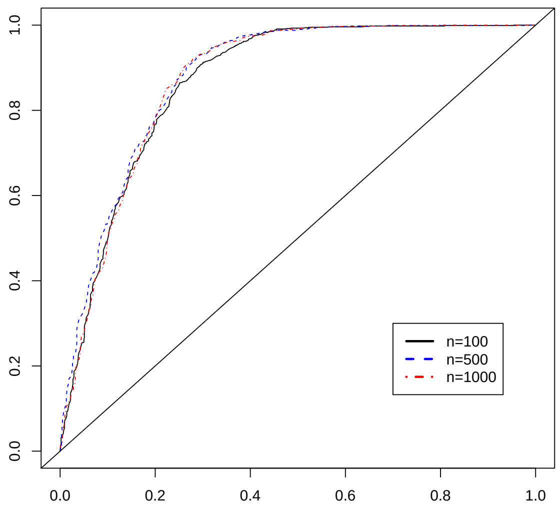

Figure 13: Size-power curve when $\phi_n = e^{-5/n}$

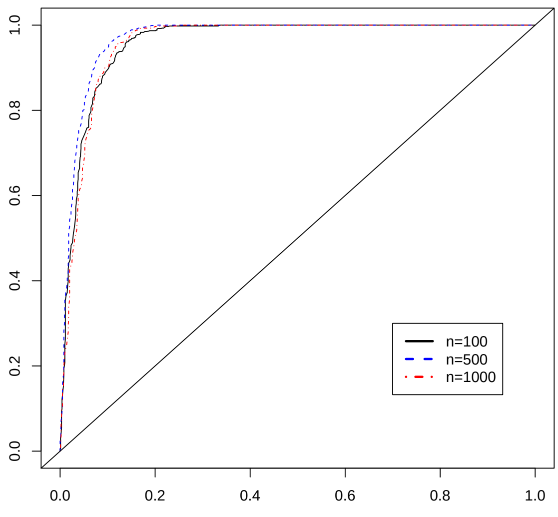

Figure 14: Size-power curve when $\phi_n = e^{-10/n}$

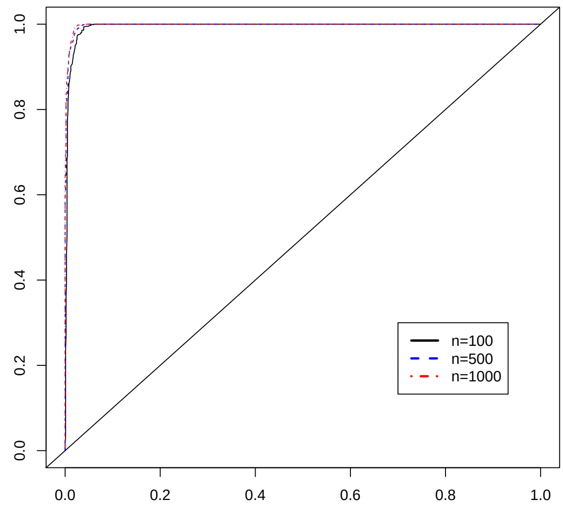

Figure 15: Size-power curve when $\phi_n = e^{-20/n}$

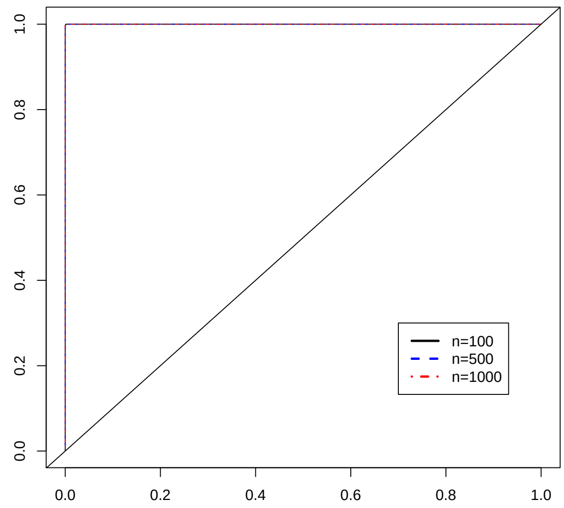

Figure 16: Size-power curve when $\phi_n = e^{-50/n}$

4.2 Size-adjusted power curves for nearly nonstationary AR(1) process when $\phi_n = 1 - \frac{\gamma_n}{n}$

Twelve experiments were performed with hypothesis:

$$H_0 : \phi = 1$$
$$H_1 : \phi = 1 - \frac{\gamma_n}{n}, \gamma_n/n \to 0, \quad \text{as} \quad n \to \infty \quad \text{and} \quad \gamma_n \to \infty,$$

where $n = 100; 500; 1000, \gamma = n^{3/4}; n^{1/4}; \frac{n}{log(n)}; log(n)$.

In Figures 17-20 black lines indicate size-adjusted test power when sample size is 100, blue and red lines indicate size-adjusted test power when sample size is 500 and 1000 accordingly. From Figures 17-18 we can say that as sample size increases, the power of test increases too but the probability to make type II error remains very big. When $\gamma_n = \frac{n}{log(n)}; n^{3/4}$ the probability of accepting null hypothesis when it is false is very small.

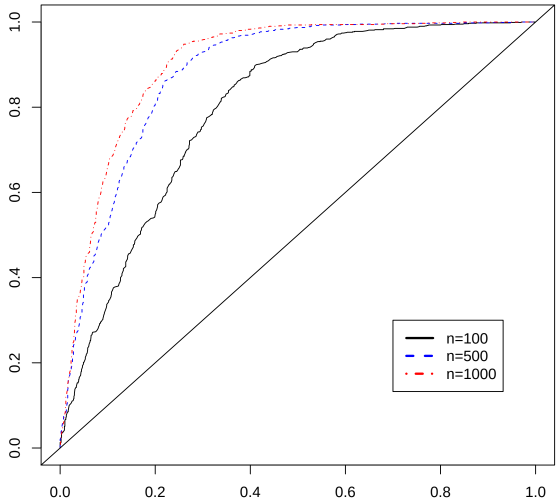

Figure 17: Size-power curve when $\phi_n = 1 - \frac{n^{1/4}}{n}$

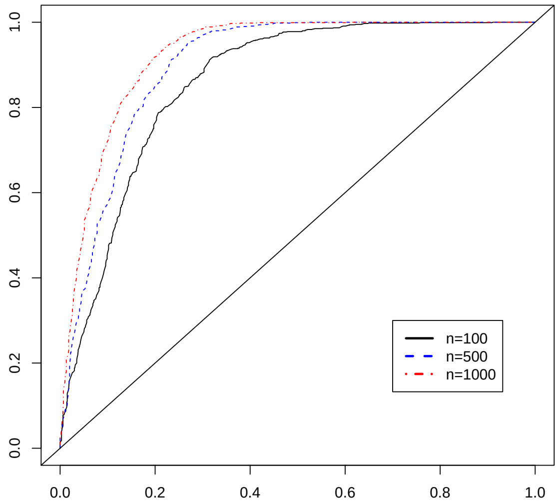

Figure 18: Size-power curve when $\phi_n = 1 - \frac{log(n)}{n}$

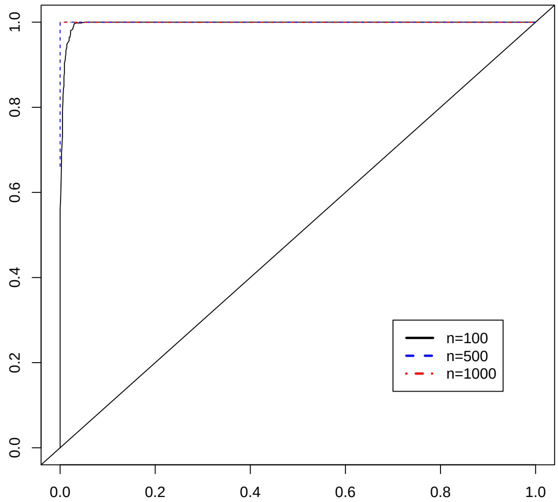

Figure 19: Size-power curve when $\phi_n = 1 - \frac{n/log(n)}{n}$

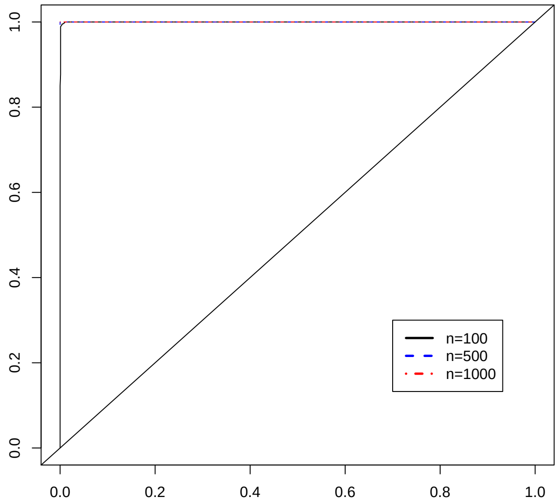

Figure 20: Size-power curve when $\phi_n = 1 - \frac{n^{3/4}}{n}$

5 CONCLUSIONS

In this master thesis nearly nonstationary AR(1) processes and nonstationarity testing with local alternatives were examined. The test statistic under local alternatives was investigated in this paper. The results obtained in Chapter 2 were graphically showed in Chapter 3 with empirical cumulative distribution functions. From size-adjusted test power analysis we can conclude that when coefficient ϕ_n in AR(1) model is replaced by $e^{\gamma/n}$ with $\gamma = -2, -5, -10$ there is a big possibility to make Type II error accepting that the process is nonstationary. When coefficient $\gamma = -20, -50$ from size-adjusted test power we can conclude that the possibility to make Type II error is very small. When coefficient ϕ_n in AR(1) model is replaced by $1 - \frac{n^{1/4}}{n}$ and $1 - \frac{log(n)}{n}$ from size-adjusted test power analysis we can say that there is a big possibility to make Type II error. When coefficient of AR(1) model is $\phi_n = 1 - \frac{n/log(n)}{n}$ and $\phi_n = 1 - \frac{n^{3/4}}{n}$, the possibility to accept null hypothesis when it is false is very small.

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