VILNIUS UNIVERSITY

FACULTY OF MATHEMATICS AND INFORMATICS

Master Thesis

Estimation of a New Keynesian Currency Union Model under Behavioral Expectations Using Kalman Filter

Naujojo Keinsizmo Valiutų Sąjungos Modelio su Elgsenos Lūkesčiais Vertinimas Taikant Kalmano Filtrą

Akvilė Bertašiūtė

VILNIUS 2018

Darbo vadovas dr. Dmitrij Celov

Darbo recenzentas dok. Saulius Jokubaitis

Darbas apgintas ___2018 m. sausio 15 d.___

Darbas įvertintas _______________

Registravimo NR. 111000-9.1-5/ _________________

2018-01-15 __________________________

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Abstract

The postulate of unbounded rationality – the mainstream way of economic thinking for several decades – is receiving an increasing amount of criticism. We relax the assumption of a perfectly informed agent by introducing a heuristics switching model which is developed based on cognitive limitations. We also turn to a more realistic setting of modeling a currency union not as a homogeneous economy but rather as an open economy among other open to trade currency union members. Inflation predictions obtained using a behavioral macroeconomic model significantly outperforms predictions delivered by the same macro-model under full rationality. The difference between the results of modeling currency union with and without our extension on spillovers is much more subtle.

Key words : Behavioral Macroeconomics; Currency Union; Expectation Formation; Rational Expectations; Kalman Filter; Inflation

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Santrauka

Neriboto racionalumo prielaida jau kelis dešimtmečius yra dominuojanti makroekonomikos kryptis, tačiau pastaruoju metu pradedama vis labiau kritikuoti šios prielaidos pagrįstumą. Šiame darbe nukrypstame nuo visiškai racionalaus agento ir įvedame euristikas keičiantį mechanizmą, kuris remiasi agento sprendimo priėmimui esant ribotai informacijai. Taip pat įvedame labiau realistišką valiutų sąjungos modelio apibrėžimą, pagal kurį valiutų sąjunga būtų modeliuojama ne kaip homogeniška ekonomika, bet kaip viena su kita per pinigų politiką susijusių atvirų ekonomikų sąjunga. Infliacijos prognozės rezultatai, gauti remiantis elgsenos makroekonominiu modeliu, yra reikšmingai tikslesni nei makroekonominio modelio su racionaliais lūkesčiais. Rezultatų skirtumai tarp valiutų sąjungos, modeliuojamos su ir be mūsų priklausomybės tarp šalių apibrėžimo plėtinio, yra nevienareikšmiai.

Raktiniai žodžiai : Elgsenos Makroekonomika; Valiutų Sąjunga; Lūkesčių Formavimas; Racionalūs Lūkesčiai; Kalmano filtras; Infliacija

Contents

1 Introduction

The time of 'Great Moderation' has led many to believe that new economic insights were able to achieve great stabilizing success. This went hand in hand with the mainstream macroeconomic theory that supports the world of rationality. In this world, there is no heterogeneity in the behavior of economic agents because everyone use the same complete information about the underlying model and shock distributions, as well as continuously perform the same utility-optimizing procedures. The eruption of recent crisis has shed light on many unresolved issues that involve these simplifying assumptions. There is now an overwhelming support of evidence that show departures from the assumption of rational expectations (see Kahneman et al., 1982; Mullainathan and Thaler, 2001; Della Vigna, 2009; Malmendier and Nagel, 2016; among many others). This has set the stage for many academic researchers to further explore the behavioral theory that allows agents to deal with imperfect information and enter the zone of irrationality (see Brock and Hommes, 1997; Woodford, 2003; De Grauwe, 2012b; Hommes et al., 2015; among many others). The appeal of this approach is that rationality is not disregarded per se. It only makes assumptions that agents are exposed to cognitive limitations in understanding the complexity of all information and optimization methods. More specifically, it makes the agents use simple rule-of-thumbs to deal with this overwhelming task of problem solving (see Gabaix et al., 2006).

In this thesis we set the stage for two main deviations from the mainstream macroeconomic theory that is embodied in the New Keynesian model framework. The first extension is changing the fundamental way of modeling expectations in a monetary union. In the academic literature it is customary to model currency unions as one homogeneous economy where expectations are formed only as currency-union aggregates. This means that, for example, price changes at the currency-union level affect a country just as much as price changes in the according country. We lay out a different framework that models a country as an open economy that is only a part of currency union and is linked to other currency union countries through open trade and common monetary policy channels (see Galí and Monacelli, 2005). We believe that this extension serves as a more realistic setting to analyze dynamic changes. The second extension is to depart from rational expectation formation and incorporate cognitive limitations. We do this by introducing the 'heuristics switching mechanism' which allows economic agents to rely on different set of simple heuristics while constantly adjusting their choice based on prediction performance (Brock and Hommes, 1997; see Hommes et al., 2015; Massaro et al., 2017 for a more recent overview). More specifically, we allow heterogeneous agents to select one of the adaptive, trend following and learning, anchoring and adjustment expectation formation rules at each time point. We believe that even if the assumption of rational expectations has its practical and logical appeal, ultimately it needs to be judged based on its ability to produce accurate predictions when confronted with real-data. Therefore, in this thesis we aim to tackle the question of whether it is worth to admit agents irrationality and that certain rule-of-thumbs can better describe their behavior, or rather to stick with perfectly rational agents without any major loss of empirical predictive power if that is proven to be the case.

The most important finding in our thesis is the fact that our suggested behavioral model pass the test of being confronted with real-time data extremely well. Once agents are allowed to deviate from rational expectation formation this leads to much richer dynamic movements and much more accurate inflation predictions in terms of MSE/other accuracy criterion (set of accuracy diagnostic checks, etc.). These superior predictions under behavioral expectations in comparison to mainstream macroeconomic model in the world of rationality persist for every alternative way of expectation formation and for every country under consideration. This persistent robustness of our key result promotes bounded rationality and provides a potential to further move away from the bottlenecks of this narrow rationality concept that is dominating the modern economic literature. Another important result regarding our extension of modeling expectations with and without spillovers is the fact that according to our empirical findings the prediction accuracy has not changed very significantly. However, considering that even a very small improvement of prediction power is very important for policy decisions involving macroeconomic data this result should not be easily discarded. Overall, we believe that our findings have a contribution in exploring new ways of dealing with mainstream assumptions. On the other hand, they are not refuted and require much more empirical evidence as well as potential extensions.

This thesis is organized as follows. In Section 2 we introduce a behavioral New Keynesian model and describe the underlying modeling choices. We also describe how we model expectation formation according to a heuristics switching model. In Section 3 we explain the methodology of using Kalman filter for our empirical testings. In Section 4 we present the results and interpretations of our findings. We also discuss an additional structural break extension. Section 5 concludes.

2 The Behavioral New Keynesian Model

For our analysis we consider the log-linear version of aggregate demand-supply relation often referred to as New Keynesian (hereinafter – NK) model. It is defined in terms of output gap and inflation, both of which are an important objective for economic policy regulation. Furthermore, we specifically use behavioral version of NK model widely applied in the strand of behavioral economics literature including authors like Woodford (2003), De Grauwe (2012a), Hommes et al. (2015) among others.

2.1 Closed Economy

We describe our closed economy model by the following equations:

$$y_t = \tilde{E}_t y_{t+1} - \frac{1}{\sigma}(i_t - \tilde{E}_t \pi_{t+1}) + \varepsilon_t, \tag{1}$$

$$\pi_t = \beta \tilde{E}_t \pi_{t+1} + \kappa y_t + \eta_t, \tag{2}$$

$$i_t = \bar{\pi} + \Phi_\pi(\pi_t - \bar{\pi}) + \Phi_y(y_t - \bar{y}), \tag{3}$$

where y_t is the output gap which is the deviation of long-run actual real GDP from equilibrium level of output in the economy.[1] Inflation term is denoted as π_t, and i_t is the short-term nominal interest rate. $\tilde{E}_t y_{t+1}$ and $\tilde{E}_t \pi_{t+1}$ are output gap and inflation expectations for time moment $t+1$, respectively. ε_t and η_t are each exogenous white noise disturbances. We also note that this macroeconomic system of equations is micro-founded under both rational and behavioral expectations, i.e. is derived from optimizing behavior of both consumers and producers [2].

Equation (1) is the aggregate demand equation. Its derivation is based on linearized Euler equation for representative households' consumption and is sometimes called the "intertemporal IS relation". Here $r_t = i_t - \tilde{E}_t \pi_{t+1}$ is the 'natural' real interest rate. σ is a positive parameter that denotes the intertemporal interest rate elasticity of output. The shock process ε_t is interpreted as a shock to demand (or preferences). According to a relatively simple interpretation of this aggregate equation, if the future income (output gap, in this case) is expected to decrease and/or real interest rate increases then utility maximizing agents are willing to spend less of their income at the present (De Grauwe, 2012b).

Equation (2) is the aggregate supply equation (or the so-called "New Keynesian Phillips curve"). According to its micro-foundations, it summarizes price setting behavior by profit-maximizing firms. Here $0 < \beta < 1$ is the discount factor of the representative household (or a country in our case), and κ is a positive parameter that depends on the average frequency of price adjustment as well as other features of our underlying structure. The shock process η_t is interpreted as a shock to marginal costs or, more specifically, a "cost-push shock" which consists of all the exogenous shifts between inflation and output in the equilibrium that do not correspond to potential output.[3]

[1] Here we follow the description provided by Woodford (2003) where he describes equilibrium level of output as a state when all wages and prices are completely flexible.

[2] Micro-foundations under rational expectations can be found in Woodford (2003) and under behavioral expectations – in Massaro (2013).

[3] All of the presented definitions and interpretations of NK Philips curve variables are based on Giannoni and Woodford (2003).

Equation (3) is the Taylor rule that describes how nominal interest rate responds to current deviations from the inflation target $\bar{\pi}$ and deviations from 'natural' output gap $\bar{y}$ in the equilibrium level. Taylor rule treats i_t as a central bank's instrument, objective of which is price and output stability in the economy: if actual inflation and/or output gap are above their potential levels, central bank should increase the interest rate to bring them back to their stable levels (and vice versa). This implies that Φ_π, and Φ_y should be non-negative parameters.

One can notice that in both aggregate demand and supply equations we do not include any of y_{t-1} or π_{t-1} lags that would satisfy the realistic need for inertia and put more emphasis on actual realizations. This is important because adjustment of consumption is not instantaneous (De Grauwe, 2012b). Inclusion of lags is also consistent with some specific types of NK model which are mixed with behavioral expectations (signals a la Calvo (1983)). However, in this paper we intentionally choose a more extreme case to focus more on persistence mechanism carried out by learning and expectation formation. Nevertheless, introduction of lags in the model would be a meaningful exercise in order to check the persistence of the results and potentially improve the predictive power of the model.

Next, we calculate the equilibrium state of the system in order to obtain the unobserved 'natural' output gap. We do this by replacing all of the time-varying inflation and output gap terms, including their expectations, by their long-run equilibrium states. We denote these steady states by $\tilde{y}$ and $\tilde{\pi}$ for output gap and inflation respectively. Our NK model obtains the form:

$$\tilde{y} = \tilde{y} - \frac{1}{\sigma}(i_t - \tilde{\pi}), \tag{4}$$

$$\tilde{\pi} = \beta \tilde{\pi} + \kappa \tilde{y}, \tag{5}$$

$$i_t = \bar{\pi} + \Phi_\pi(\tilde{\pi} - \bar{\pi}) + \Phi_y(\tilde{y} - \bar{y}). \tag{6}$$

Then, solving the system of equations, we get the expressions $\tilde{y} = \bar{y} = \frac{(1-\beta)\bar{\pi}}{\kappa}$ and $\tilde{\pi} = \bar{\pi}$. We see that both inflation and output gap in their equilibrium states are exactly equal to inflation target and 'natural' output gap respectively.

The model consisting of equations (1)–(3) can equivalently be rewritten in matrix form as:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \Theta^{-1} \begin{bmatrix} \bar{\pi}(\Phi_\pi - 1) + \Phi_y \bar{y} \\ \kappa\bar{\pi}(\Phi_\pi - 1) + \kappa\Phi_y\bar{y} \end{bmatrix} + \Theta^{-1} \begin{bmatrix} \sigma & 1 - \Phi_\pi\beta \\ \kappa\sigma & \kappa + \beta\sigma + \beta\Phi_y \end{bmatrix} \begin{bmatrix} \tilde{E}_t y_{t+1} \\ \tilde{E}_t \pi_{t+1} \end{bmatrix} + \Theta^{-1} \begin{bmatrix} \sigma & -\Phi_\pi \\ \kappa\sigma & \sigma + \phi_y \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix},$$

where $\Theta = \sigma + \kappa\Phi_\pi + \Phi_y$. We will use this expression for our further derivations in Section 3.

2.2 Open Economy

Next, we expand our (1)–(3) equation system into an open economy model following Galí and Monacelli (2005) [4]. Instead of treating an economy as completely closed, now we treat it as one of the parts that makes up a world economy – which in our case is a currency union that is

[4] Galí and Monacelli (2005) also provide micro-foundations for this system under rational expectations and assumption of continuum of countries. We do not have any references to micro-foundations for an open economy under behavioral expectations. However, this does not make our model less reliable or useful for our empirical testings (Massaro et al., 2017).

closed to the rest of the world. The rationale behind this decision is that by allowing countries to influence one another through intra-trade spillover effects we would obtain a richer and more realistic dynamics.

We describe our open economy model by the following equations:

$$y_t^i = \tilde{E}_t y_{t+1}^i - \frac{1}{\sigma^i}(i_t - \tilde{E}_t \pi_{t+1}^i) + \alpha^i \tilde{E}_t \Delta s_{t+1}^i + \varepsilon_t^i, \tag{7}$$

$$\pi_t^i = \beta^i \tilde{E}_t \pi_{t+1}^i + \kappa^i y_t^i + \eta_t^i, \tag{8}$$

$$i_t = \bar{\pi} + \Phi_\pi(\pi_t^{cu} - \bar{\pi}) + \Phi_y(y_t^{cu} - \bar{y}^{cu}). \tag{9}$$

Let us denote the number of countries that makes up a currency union as N, then index $i = 1,...,N$ corresponds accordingly to a member country of this currency union. Just as before, equation (7) represents IS aggregate demand relation, equation (8) – New Keynesian Phillips curve relation and equation (9) – central bank's monetary policy expressed as Taylor rule. Also, the interpretation of all the previously defined parameters is kept exactly the same. However, there are some differences compared to the closed model.

First, in equation (7) we introduce a new term $\tilde{E}_t \Delta s_{t+1}^i = \tilde{E}_t \pi_{t+1}^{*i} - \tilde{E}_t \pi_{t+1}^i$ that is the expected (log) effective terms of trade change.[5] In other words, it means that if price change in a foreign currency union country is expected to rise by more than in home country i, then the output gap in country i would be higher due to increased export – a result of country's i increased competitiveness in the trading market. Parameter α_i shows by how much a country i is elastic to expected price changes in other currency union countries. $\tilde{E}_t \pi_{t+1}^{*i} = (\sum_{k=1}^{N,k\neq i} w(k) \tilde{E}_t \pi_{t+1}^k) / \sum_{k=1}^{N,k\neq i} w(k)$ is the expected (log) inflation rate in the rest of the currency union excluding country i. $w(i)$ is the weight for country i which is set according to a country's economic importance in the currency union.

Second, every country that is now a part of a currency union has the same nominal interest rate set by a central bank that only reacts at the whole monetary union level. More specifically, individual country's inflation and output gap are now replaced by a weighted average of inflation and output gap for the whole currency union, calculated as $\pi_t^{cu} = (\sum_{k=1}^N w(k) \pi_t^k) / \sum_{k=1}^N w(k)$ and $y_t^{cu} = (\sum_{k=1}^N w(k) y_t^k) / \sum_{k=1}^N w(k)$ respectively. Output gap in the equilibrium $\bar{y}$ is now replaced by $\bar{y}^{cu} = (\sum_{k=1}^N w(k) \bar{y}^k) / \sum_{k=1}^N w(k)$. This means that a country that has joined the monetary union loses the ability to mitigate its own inside shocks by using monetary policy. However, the monetary union's price stability maintenance objective can, under normal circumstances, stabilize the common monetary union shocks (Carnot et al., 2017).

Third, if we model a currency union according to equations (1)–(3) where we replace π_t and y_t terms with π_t^{cu} and y_t^{cu}, then expectations $\tilde{E}_t \pi_{t+1}$ and $\tilde{E}_t y_{t+1}$ are calculated for currency union aggregates. A very important difference is provided in equations (7)–(9) where expectations are now calculated at a country's i level. This means that economic agents in country i form their inflation and output gap expectations only according to changes in their home country, rather then a weighted average of the whole monetary union. The effect of expectation changes abroad are passing through the real exchange rate term.

[5] Galí and Monacelli (2005) provide a full form: $\tilde{E}_t \Delta s_{t+1}^i = \tilde{E}_t \Delta e_{t+1}^i + \tilde{E}_t \pi_{t+1}^{*i} - \tilde{E}_t \pi_{t+1}^i$. We exclude the change in expected nominal exchange rate term $\tilde{E}_t \Delta e_{t+1}^i$ because for a currency union, nominal exchange rate is fixed. This means that $\tilde{E}_t \Delta e_{t+1}^i$ becomes 0.

Next, we calculate the equilibrium state of the open economy system. Similarly as before, we denote the steady states of country's i inflation and output gap as $\tilde{y}^i$ and $\tilde{\pi}^i$ respectively.

$$\tilde{y}^i = \tilde{y}^i - \frac{1}{\sigma^i}(r_t - \tilde{\pi}^i) + \alpha^i(\tilde{\pi}^{*i} - \tilde{\pi}^i), \tag{10}$$

$$\tilde{\pi}^i = \beta^i \tilde{\pi}^i + \kappa^i \tilde{y}^i, \tag{11}$$

$$r_t = \bar{\pi} + \Phi_\pi(\tilde{\pi}^{cu} - \bar{\pi}) + \Phi_y(\tilde{y}^{cu} - \bar{y}^{cu}), \tag{12}$$

where $\tilde{\pi}^{*i} = (\sum_{k=1}^{N,k\neq i} w(k)\tilde{\pi}^k)/\sum_{k=1}^{N,k\neq i} w(k)$, $\tilde{\pi}^{cu} = (\sum_{k=1}^{N} w(k)\tilde{\pi}^k)/\sum_{k=1}^{N} w(k)$ and $\tilde{y}^{cu} = (\sum_{k=1}^{N} w(k)\tilde{y}^k)/\sum_{k=1}^{N} w(k)$.

Then, solving the system of equations, we get:

$$\tilde{y}^i = \bar{y}^i = \frac{(1-\beta^i)\bar{\pi}^i}{\kappa^i} \tag{13}$$

$$\tilde{\pi}^i = \frac{(\Phi_\pi - 1)\bar{\pi}^i - \alpha^i \tilde{\pi}^{*i}}{\Phi_\pi - 1 - \alpha^i} \tag{14}$$

Note that for $\tilde{\pi}^{*i} = \bar{\pi}^i$ we have the same values as for the closed economy: $\tilde{\pi}^i = \bar{\pi}^i$, $\tilde{y}^i = \frac{(1-\beta^i)\bar{\pi}^i}{\kappa^i}$.

The matrix form for equations (7)–(9) becomes quite inelegant and, therefore, is provided in Appendix A.2.

2.3 A Heuristic Switching Model of Expectation Formation

Expectation modeling in a stylized neoclassical model assumes that agents form their expectations rationally. This means that expectations are model-consistent and all of the agents know both the model and all the shocks hitting the economy, as well as able to calculate the equilibrium of the underlying model. There is no heterogeneity in the behavior of these economic agents because it is a common knowledge that they all use the same information and expect everyone else to do the same (Holden, 2012). However, this assumption of unbounded rationality ultimately is an empirical issue due to a great lack of supporting evidence (Mullainathan and Thaler, 2001; Kahneman et al., 1982; among many others). Conlisk (1996) was one of the early critics that supported the adoption of rule of thumb behavior as a way for people to deal with their limited brainpower and time while searching for an optimal solution to a difficult problem. In other words, people are "rational" but bounded with their information processing and accessing capabilities.

For this reason we introduce a heuristic switching model into our analysis which provides a way for heterogeneous agents to form their expectations under bounded rationality. The heuristics switching model (HSM) was first developed by Brock and Hommes (1997) and later comprehensively described by Hommes et al. (2015). The HSM can also be interpreted as "trial and error" selection mechanism because agents select the forecasting rule that has performed best in the past (De Grauwe, 2012b).

The model provides a framework where agents choose between three simple heuristics [6]: adaptive, weak trend following and learning, anchoring and adjustment expectation formation rules. An adaptive expectation rule (ADA) is given by:

$$x^e_{1,t+1} = 0.65x_{t-1} + 0.35x^e_{1,t}. \tag{15}$$

A weak trend-following rule (WTF) is given by:

$$x^e_{2,t+1} = x_{t-1} + 0.4(x_{t-1} - x_{t-2}). \tag{16}$$

A learning, anchoring and adjustment rule (LAA) is given by:

$$x^e_{3,t+1} = 0.5(x^{average}_{t-1} + x_{t-1}) + (x_{t-1} - x_{t-2}). \tag{17}$$

Here $x^{average}_{t-1}$ denotes the average of all observations up to time $t-1$. This specific set of heuristics is based on rich empirical research (see Hommes et al., 2005; Assenza et al., 2014, among others). The described heuristics can be thought of as rules of thumb because they do not require agents to know full information and are a parsimonious simplification of the real world forecasts. However, agents that choose between these heuristics are not fools. On the contrary, they show their rationality by demonstrating willingness to learn from their mistakes – agents update their forecasts according to past performances of these rules (De Grauwe, 2012b). Forecast performance (or fitness) of a particular rule f can be described as follows [7]:

$$U_{f,t-1} = -(x^e_{f,t-1} - x_{t-1})^2 + \eta U_{f,t-2}, \tag{18}$$

where x denotes either output gap or inflation. The first term in equation (18) computes the squared prediction error of the forecast rule f. $0 \leqslant \eta \leqslant 1$ is the so-called memory parameter and captures the rate at which agents tend to forget the past. If η is equal to 1 then all past forecast performances are weighted equally. If η is equal to 0 – agents forget all but the most recent past.

The next step consists in evaluating the probability of choosing heuristics f. If agents were fully rational, they would compare the fitness measures and simultaneously choose the single best option. However, it is observed that the choice between alternative rules is not fully deterministic because it depends on the agents' preferences and their willingness to learn (De Grauwe, 2012b). It is customary to assume that this random component that captures the heterogeneity in preferences is logistically distributed. Then one can obtain a multinomial logit expression [8] that represents the fraction of agents in the whole population using heuristics f in period t as follows:

$$n^f_{x,t} = \frac{\exp\left(\mu U_{f,t-1}\right)}{\sum_{f=1}^{F} \exp\left(\mu U_{f,t-1}\right)}\ , \tag{19}$$

where F is the number of forecasting rules. $\mu \geq 0$ is the so-called intensity of choice parameter. It measures the willingness to learn from past forecast performance. More specifically, if μ is

[6] Hommes et al. (2015) and Massaro et al. (2017) fit the model using an additional strong trend-following rule while Anufriev et al. (2013) use only adaptive and trend-following rules. We choose to analyze only three heuristics to keep the model relatively simple.

[7] The description and interpretation of the reinforcement learning model described in equations (18)–(20) is based on Massaro et al. (2017).

[8] More details on derivation can be found in Brock and Hommes (1997).

0, then agents do not show any willingness and they choose between the heuristics completely at random. If μ approaches ∞, then agents show a complete willingness to adapt their forecast rule to the most successful heuristics with probability 1. By further following Hommes et al. (2015) and Massaro et al. (2017) we make an additional assumption that not all agents choose to update their forecasting rule at each period. That is, we assume that:

$$n_{x,t}^{f} = \delta n_{x,t-1}^{f} + (1-\delta)\frac{\exp\left(\mu U_{f,t-1}\right)}{\sum_{f=1}^{F}\exp\left(\mu U_{f,t-1}\right)} \, . \tag{20}$$

To put differently, equation (20) allows for asynchronous updating. The parameter $0 \leqslant \delta \leqslant 1$ is the average fraction of agents that do not update their previous forecasting rule. If δ is equal to 1, the agents do not update their forecasting strategy.

Summing up the presented reinforcement learning model and our specific set of heuristics, the linear forecast in period $t+1$ is given by:

$$\begin{aligned} \tilde{E}_t x_{t+1} =& (0.65n_{x,t}^1 + 1.4n_{x,t}^2 + 1.5n_{x,t}^3)x_{t-1} + 0.35n_{x,t}^1 x_{1,t}^e + 0.5n_{x,t}^3 x_{t-1}^{average} \\ & - (0.4n_{x,t}^2 + n_{x,t}^3)x_{t-2} \, . \end{aligned} \tag{21}$$

3 Kalman Filter

The Kalman filter is an optimized method that uses recursive algorithm. It was formulated in the early 1960's and had a first publicly known application to navigation and guidance system for the Apollo space program. Kalman filter is proven to be successful and reliable for numerous engineering and aerodynamics applications and is widely used to this day (Grewal and Andrews, 2010).

Because Kalman filter is easily formulated, it also has an important use in solving various economic problems, namely those involving econometric models for the purpose of forecasting or estimating unobserved components. The motivation for using a filtering technique is the fact that the focus of our analysis is not retrospective reflection of historical data but rather the analysis of prediction accuracy obtained under different assumptions. The object of filtering is to take into account all of the observations up until the time point under consideration and update the knowledge of the system with every newly introduced observation. Therefore, it is specifically suitable for forecasting systems. On top of that, we have a fixed structure of the model as well as fixed priors of shock processes in the underlying model. This makes the Kalman filter appropriate for a straightforward implementation (Athans, 1974; Durbin and Koopman, 2012).

3.1 State Space Model

In the paper we use the general multivariate Gaussian state space model form based on Durbin and Koopman (2012):

$$\begin{aligned} \dot{y}_t =& Z_t\alpha_t + \xi_t, & \xi_t &\sim N(0,H_t), & & \\ \alpha_{t+1} =& B_t + T_t\alpha_t + R_t\delta_t, & \delta_t &\sim N(0,Q_t), & t &= 1,\dots,n, \\ & & \alpha_1 &\sim N(a_1,P_1), & & \end{aligned} \tag{22}$$

where n is the number of observations. $\dot{y}_t$ – $p \times 1$ vector of p endogenous time series and is called the *observation* vector. α_t – $m \times 1$ vector of m unobserved series and is called the *state* vector. Error terms ξ_t and δ_t are assumed to be normally distributed white noise processes that are serially independent at all time points. α_1 is the initial *state* vector value which depends on previously fixed distribution mean a_1 and variance P_1 and is assumed to be serially independent of ξ_t and δ_t for every t. The matrices $Z_t, B_t, T_t, R_t, H_t, Q_t$ are assumed to be known although they are still allowed to be time-varying. The later feature provides a convenient way to distinguish structural breaks. In practice, these matrices depend on some unknown parameters. The estimation of these parameters is discussed below.

In our specific case for the closed economy model described in Section 2.1, *observation* vector is defined as $\dot{y}_t = \pi_t$. *State* vector is defined as $\alpha_t = (y_t, \pi_t)^T$. One can notice that time series π_t is already treated as endogenous in the model therefore at time t the estimate is exactly equal to the provided observation and we are only interested in one-step ahead prediction provided by the filter. Output gap is treated as an unobserved component although we do use previously estimated output gap time series to calculate output gap expectations up to time point t which we treat as predetermined. Matrix $H_t = 0$ and matrix $Q_t = diag(\sigma_\varepsilon^2, \sigma_\eta^2)$ where *diag* denotes a diagonal matrix form. Considering the heuristics switching model case, we can rewrite equations (1)–(3) to the form (22) using the following matrices:

$$Z_t = \begin{bmatrix} 0 & 1 \end{bmatrix}, R_t = \Theta^{-1} \begin{bmatrix} \sigma & -\Phi_\pi \\ \kappa\sigma & \sigma + \phi_y \end{bmatrix}, T_t = \begin{bmatrix} n_{yt} & n_{\pi t} \\ n_{yt} & n_{\pi t} \end{bmatrix} \odot \Theta^{-1} \begin{bmatrix} \sigma & 1 - \Phi_\pi \beta \\ \kappa\sigma & \kappa + \beta\sigma + \beta\Phi_y \end{bmatrix},$$

$$B_t = \Theta^{-1} \begin{bmatrix} \bar{\pi}(\Phi_\pi - 1) + \Phi_y \bar{y} \\ \kappa\bar{\pi}(\Phi_\pi - 1) + \kappa\Phi_y \bar{y} \end{bmatrix} + \Theta^{-1} \begin{bmatrix} \sigma & 1 - \Phi_\pi \beta \\ \kappa\sigma & \kappa + \beta\sigma + \beta\Phi_y \end{bmatrix} (0.5 \begin{bmatrix} n_{yt}^4 y_{t-1}^{average} \\ n_{\pi t}^4 \pi_{t-1}^{average} \end{bmatrix} + \begin{bmatrix} n_{yt}^* y_{t-2} \\ n_{\pi t}^* \pi_{t-2} \end{bmatrix}$$
$$+0.35 \begin{bmatrix} n_{yt}^1 \tilde{E}_t y_t \\ n_{\pi t}^1 \tilde{E}_t \pi_t \end{bmatrix}), \Theta = \sigma + \kappa\Phi_\pi + \Phi_y.$$

Here $n_{yt} = 0.65 n_{yt}^1 + 1.4 n_{yt}^2 + 1.5 n_{yt}^3$, $n_{\pi t} = 0.65 n_{\pi t}^1 + 1.4 n_{\pi t}^2 + 1.5 n_{\pi t}^3$, $n_{yt}^* = -0.4 n_{yt}^2 - n_{yt}^3$, $n_{\pi t}^* = -0.4 n_{\pi t}^2 - n_{\pi t}^3$. Notation $\odot$ represents element-wise product of matrices, known as the Hadamard product. $\tilde{E}_t y_t$, $\tilde{E}_t \pi_t$, y_{t-2}, π_{t-2}, $y_{t-1}^{average}$, $\pi_{t-1}^{average}$ are taken as predetermined since they only depend on observations $\dot{y}_1, \ldots, \dot{y}_{t-1}$. Also, parameters $\phi_y, \phi_\pi, \bar{\pi}, \bar{y}$ are calibrated while parameters $\sigma, \kappa, \beta, \sigma_\varepsilon^2, \sigma_\eta^2$ are treated as unknown.

In the open economy case scenario described in Section 2.2, *observation* vector is defined as $\dot{y}_t = (\pi_t^1, \ldots, \pi_t^N)^T$ which is inflation time series of all N countries, therefore it is a N-dimensional vector. *State* vector is defined as $\alpha_t = (y_t^1, \ldots, y_t^N, \pi_t^1, \ldots, \pi_t^N)^T$. Again, matrix $H_t = 0$ and matrix $Q_t = diag(\sigma_{\varepsilon_1}^2, \ldots, \sigma_{\varepsilon_N}^2, \sigma_{\eta_1}^2, \ldots, \sigma_{\eta_N}^2)$. State space model matrices forms are shown in the Appendix A for brevity. As before, parameters $\phi_y, \phi_\pi, \bar{\pi}, \bar{y}$ are calibrated while parameters $\sigma^i, \kappa^i, \beta^i, \sigma_{\varepsilon,i}^2, \sigma_{\eta,i}^2$ for all $i = 1, \ldots, N$ are treated as unknown.

All of the unknown parameters that yet need to be estimated for each model case are contained in the parameter vector θ. A computationally efficient method in terms of numerical stability is an iterative optimization method Broyden-Fletcher-Goldfarb-Shanno (BFGS). BFGS is a numerical quasi-Newton method based on information from the gradient and is used to maximize the log-likelihood of our state space model with respect to θ numerically. Among other things, the number of iterations that are necessary to satisfy the conditions for optimality is sensitive to the provided initial parameters.[9] For our analysis we use a modified limited-memory BFGS method which allows each parameter to have a lower/upper bound in which it is optimized.[10]

3.2 Filtering Algorithm

In order to apply the filter we need to define the recursive filtering equations [11]. Let us denote Y_{t-1} as a set of past observations $\dot{y}_1,\dots,\dot{y}_{t-1}$. Then $v_t = \dot{y}_t - E(\dot{y}_t|Y_{t-1})$ is one step-ahead forecast error of $\dot{y}_t$ given Y_{t-1} and is sometimes referred to as *innovations*. The object of filtering is to calculate conditional mean and variance for both α_t and α_{t+1} given Y_t which we denote as $a_{t|t} = E(\alpha_t|Y_t)$ and $P_{t|t} = Var(\alpha_t|Y_t)$ for α_t, and $a_{t+1} = E(\alpha_{t+1}|Y_t)$ and $P_{t+1} = Var(\alpha_{t+1}|Y_t)$ for α_{t+1} respectively. Then the filtering algorithm takes the form:

$$\begin{aligned} a_{t|t} &= a_t + P_t Z_t' F_t^{-1} v_t, & P_{t|t} &= P_t - P_t Z_t' F_t^{-1} Z_t P_t, \\ a_{t+1} &= T_t a_{t|t} + B_t, & P_{t+1} &= T_t P_{t|t} T_t' + R_t Q_t R_t', \end{aligned} \tag{23}$$

for $t = 1,\dots,n$. Here $F_t = Z_t P_t Z_t' + H_t$. Assuming that $\alpha_1 \sim N(a_1, P_1)$, equation system (23) is the celebrated Kalman filter recursive algorithm. It can be shown that since our system is linear and matrices Z_t and T_t do not depend on y_t and other past observations, the filter provides minimum variance linear unbiased estimates together with minimum error variance linear estimates even if the variables under consideration are not normally distributed. Also, based on the derivation of the filter estimates, *innovations* v_t for every $t = 1,\dots,n$ should be independent of each other.

As in most practical cases, we do not know the true values of a_1 and P_1. However, as Durbin and Koopman (2012) points out, there is no easy solution to this initialization problem for our general multivariate Gaussian state space model. For this reason we accordingly set our initial value a_1 to the first π_1^i observation. Also, we set every P_1 diagonal element equal to 2 in both closed and open economy cases which appears somewhat reasonable considering that we work with macro-variables.

In order use limited-memory BFGS method we need to be able to obtain log-likelihood of our estimated model. The following Durbin and Koopman (2012) representation of log-likelihood given by:

$$logL(Y_n) = -\frac{np}{2} log 2\pi - \frac{1}{2}\sum_{t=1}^{n}(log|F_t| + v_t' F_t^{-1} v_t), \tag{24}$$

which is easily calculated using the Kalman filter output.

[9]The description of BFGS is based on Hoogerheide and Koopman (2015) and more details can be found in Durbin and Koopman (2012).

[10]For more details check Byrd et al. (1995).

[11]The derivation of the filter can be found in Durbin and Koopman (2012).

4 Results

4.1 Data

We use four member states of the Eurozone which have adopted the euro as their currency in 1999-01-01: Germany, France, Spain and Italy. These four countries have the largest economies according to their GDP share weights[12] and make up a total of 79.78% of total euro Area GDP size. Since the impact of foreign countries directly depends on the weight of their economic importance, countries with smaller weights can be neglected without any major loss of generality. Keeping a small number of countries is crucial, since every additional country also significantly increases the dimension of unknown parameters and computation time increases drastically. Taking these considerations into account, we treat these four countries as a fairly good approximation of a currency union. France on average takes up 27.37% with 36.30% for Germany, 14.07% for Spain and 22.26% for Italy of the new recalculated weights for our currency union approximation containing only four selected countries. Our sample of four macroeconomic variables (output gap, inflation, interest rate and nominal GDP) begins at 1999:Q1 and extends through 2014:Q4. This is the official start of European monetary union existence and when single interest rates for the euro became available. We end our observations at 2014 fourth quarter due to output gap data limitations.

We analyze quarterly inflation data, taken as HICP overall index and calculated as average of the monthly data of the period. This data (neither seasonally nor working day adjusted) is given as annual rate of change (% of change YoY)[13]. We also use quarterly output gap as the business cycle based on real GDP index 2010=100 (IMF IFS)[14]. Since these estimates are calculated as deviations from zero, for this analysis we add the calculated steady state of the output gap to the data.

4.2 Estimation of Heuristics Switching Mechanism

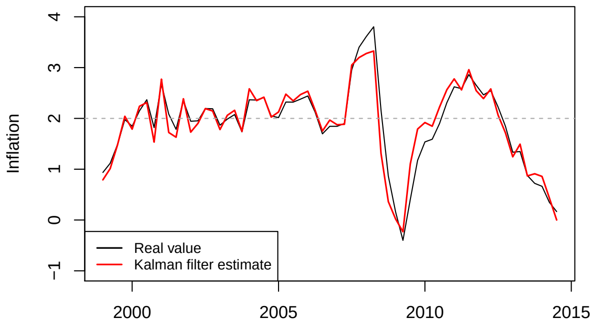
The calibration that we use for the Taylor rule coefficients is $\phi_\pi = 1.5$, $\phi_y = 0.5$. These coefficients were first proposed by Taylor (1993) and are often used as a standard calibration choice in the literature (Blattner and Margaritov, 2010; Curdia et al., 2012). Inflation target $\bar{\pi}$ is set to 2% which is a public objective of the ECB in the medium term. Many researchers believe that after the crisis Taylor rule coefficient became more uncertain than before and possibly encountered a structural break. We acknowledge this problem in the Section 4.5 where we stop relying on the Taylor rule since the start of quantitative easing program in the euro area.

In both closed economy and open economy model cases we use a set of calibration that was used by Clarida et al. (2000), Hommes et al. (2015) and Massaro et al. (2017). The initial

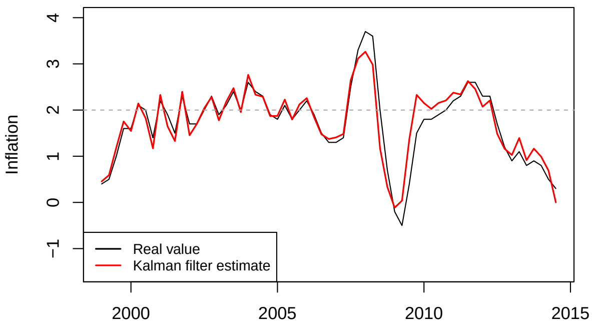
[12] Weights of individual countries: France - 21.83%, Germany - 28.96%, Spain - 11.22%, Italy - 17.76%. GDP data is taken as total economy's gross domestic product at market prices. Millions of euros at current prices. Data source: ECB (http://sdw.ecb.europa.eu/). The data (millions of euro) is not transformed: neither seasonally adjusted nor calendar adjusted.

[13] European Commission (Eurostat) and European Central Bank calculations based on Eurostat data. Data source: ECB (http://sdw.ecb.europa.eu/).

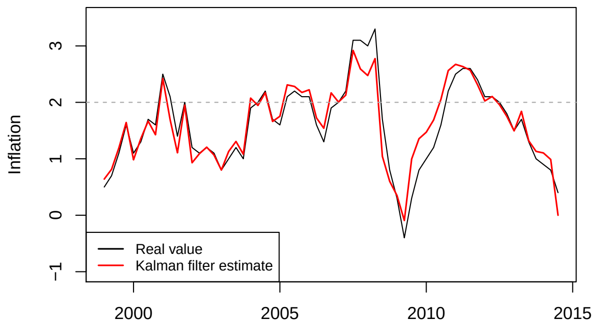
[14] Seasonally adjusted by X11 in Rats. Data provided by Comunale (2015).



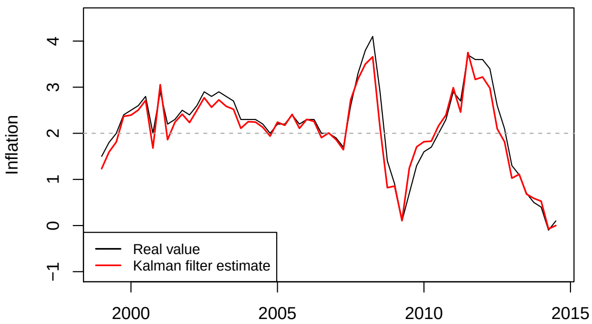
(a) Currency Union



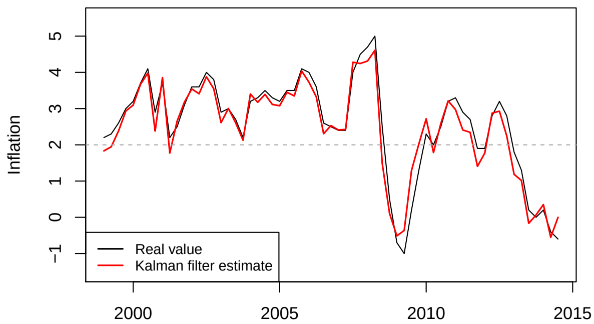
(b) France



(c) Germany



(d) Italy



(e) Spain

Figure 1: Inflation one-quarter ahead Predictions in a Closed Economy

parameters (with their respective lower/upper bounds in parentheses) for the discount factor are $\beta = 0.9$ $[0.7, 0.99]$[15], for the coefficient of relative risk aversion – $\sigma = 1$ $[0.1, 3]$, for the output elasticity of inflation – $\kappa = 0.3$ $[0.01, 0.5]$[16], for the error standard deviations – $sd_{\varepsilon} = 0.1$ $[0.01, 1]$, $sd_{\eta} = 0.1$ $[0.01, 1]$. In open economy case an additional economic integration coefficient is added which we set to $\gamma = 0.7$ $[0.1, 0.9]$[17]. We want to emphasize the fact that the discount factor β is restricted to be strictly lower than unity. Proposition that inflation expectations are not fully incorporated is the key feature of Tobin's neo-Keynesian Phillips curve (Palley, 2011). The bound choice was made according to the robustness check results found in Massaro et al. (2017). In the corresponding paper these specific set of bounds were found to capture most of the instability dynamics while still remaining in line with economic theory. Choosing slightly different parameters would not alter the basic logic, because we are more interested in comparison between certain models rather than finding the best possible prediction fit.

All of the estimates throughout the paper are one-step ahead (one quarter) predictions which are plotted against the real-time data at the same time point for which the prediction was made. We estimate each country case separately as well as a currency union case, calculated as a weighted average of our four considered countries as one economic unit. Figure 1 shows the results obtained by the heuristic switching mechanism as described in Section 2.3 for the closed economy model. It is visibly clear that the inflation one-step ahead prediction provided by the Kalman filter fit the data reasonably well, especially during the period before the 2007 crisis when the economy was relatively stable. The result do not depend on the country considered.

Table 1: MSE Prediction Error Accuracy of HSM

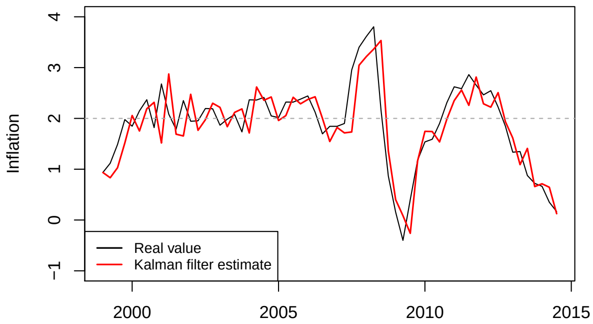
	France	Germany	Spain	Italy	Currency Union
Closed Economy	0.074	0.065	0.120	0.056	0.054
Open Economy	0.088	0.049	0.178	0.084	0.052

The results obtained for the open economy model is presented in Figure 2. The estimated one-step ahead predictions appear hardly distinguishable in the sense that no highly significant changes occurred to the deviations from the actual inflation in each country case. However, according to mean square estimates presented in Table 1 open economy case prediction is slightly more accurate for Germany and all countries modeled as a currency union. However, the same prediction accuracy of introducing an economic integration factor is slightly lower for France, Spain and Italy. A possible theoretical explanation, according to Massaro et al. (2017), is that having an unstable level of reaction to the price changes in other countries, instead of pulling toward the steady state, can do the opposite. This might even lead to diverging deviations or high oscillations around the steady state with an increasing amplitude when the integration level is too low or too high respectively.

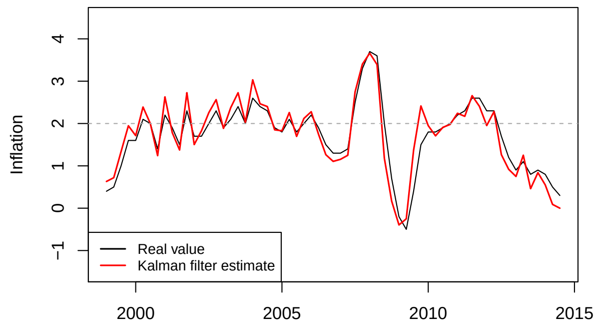
[15] Standard calibration in the referred papers sets discount factor equal to 0.99. However, it can be shown that our specific choice of β initialization does not alter the results significantly due to our optimization algorithm convergence results according to which $\beta = 0.99$ in majority of cases.

[16] There is no consensus on the proper calibration of κ in the literature. For example, De Grauwe and Ji (2016) set this coefficient equal to 0.05 and Galí and Monacelli (2005) – to 0.355.

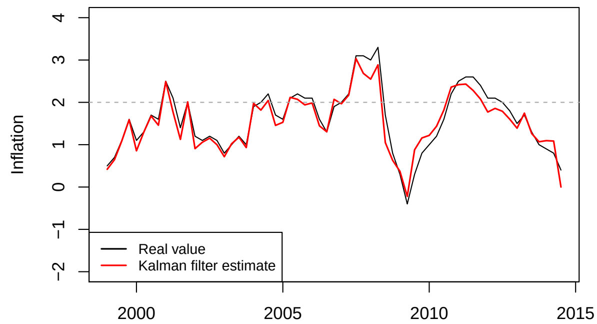
[17] Initialization for γ is solely based on Massaro et al. (2017).



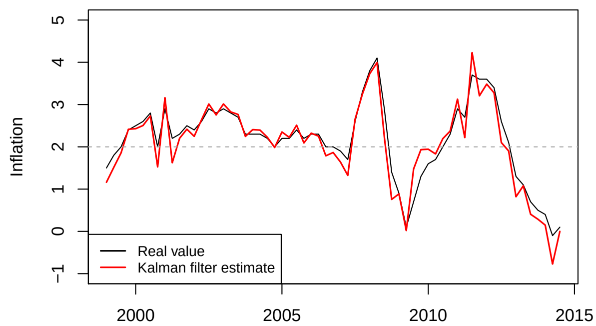
(a) Currency Union



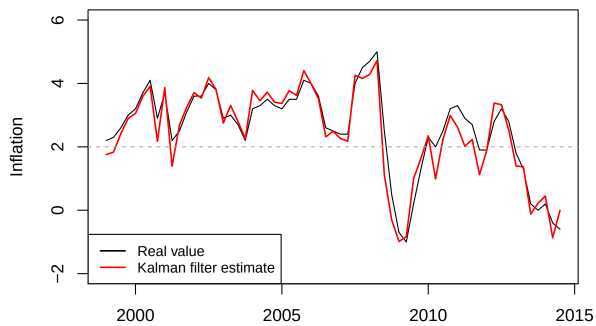
(b) France



(c) Germany



(d) Italy

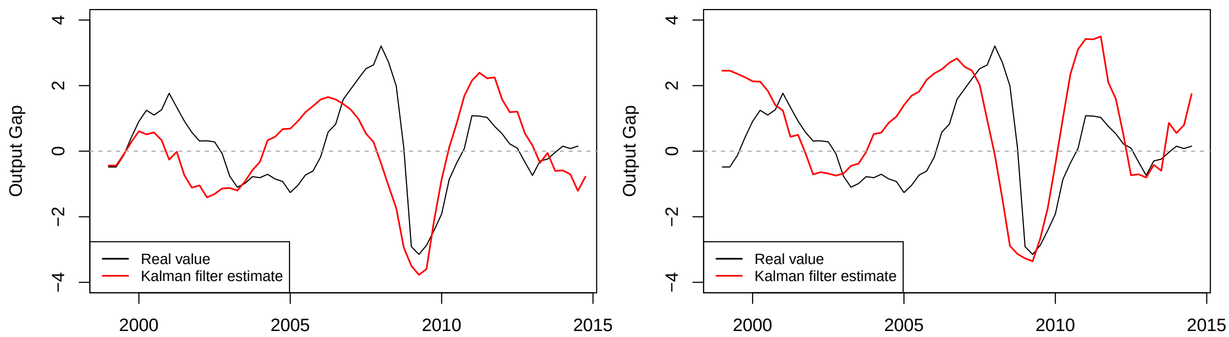


(e) Spain

Figure 2: Inflation one-quarter ahead Predictions in an Open Economy

Diagnostic analysis is presented in Appendix C where we check the assumptions underlying our models. Since we assume that our error terms η_t and ε_t are normally distributed and serially independent, our one-step ahead prediction errors should have the same properties. Looking at diagnostic plots for standardized prediction errors and performing normality tests we show that these specific assumptions are satisfied with a reasonable level.

Another matter of interest is checking the estimated unobserved state of output gap. Figure 3 presents only aggregated currency union state estimates for both open and closed economy model cases. It appears that the Kalman filter handled the unobserved output gap component reasonably well since it was able to replicate the over-all dynamics of Hodrick–Prescott filter based output gap estimates. However, there is a large real-time uncertainty that causes a partial loss of effectiveness. Also, since output gap is unobserved, it inevitably relies on the estimation method. There is a great amount of political disputes regarding the precise method of evaluation (Carnot et al., 2017). Thus, we do not focus our analysis on neither comparing these methods nor trying to find a better output gap prediction. The result of nothing completely "crazy" or counter-intuitive is satisfactory enough in this paper, although a further expanded research is needed [18].



(a) Closed economy

(b) Open economy

Figure 3: Output Gap modeled as a Currency Union

Another possible impact on the results is not optimal intensity of choice, memory and asynchronous updating parameters. So far we calibrated them as $\eta = 0.7$, $\mu = 0.4$ and $\delta = 0.9$ following Massaro et al. (2017) and Hommes et al. (2015). However, these parameters might be different for each country as well as depend on country integration level. Therefore we calculate the accuracy of prediction mean square errors as we change these three parameters one at a time. Other parameters are fixed to previously defined individually optimized and generally calibrated values. Appendix B Figure 6 shows the result for parameter μ. We observe a clear heterogeneity across countries. Interestingly, we do not observe possible cases when intensity of choice parameter on its own would change the outcome of a better closed economy prediction fit. The same can be said about the memory parameter δ and asynchronous updating

[18] De Grauwe (2012b) points out that it is quite usual for empirically tested output gap to have a lot of dynamics that is not described by the New Keynesian model and is found in the error term. This implies that there is a rather systematic problem with the model regarding the view of the business cycles.

parameter η for which the results are presented in Appendix B Figure 7 and Appendix B Figure 8 respectively. This experiment only goes to show that our results are quite robust in respect to this specific set of fixed parameters. However, Germany stands out as a slight exception. In particular, closed economy case under different assumptions, for example, very significantly lowering the fraction of agents that do not synchronously update their expectations, increasing agents memory, or increasing the intensity of choice, can potentially yield more accurate one-quarter inflation predictions compared to open economy. Summing up, our calibration choice of these highly uncertain parameters does not alter previously obtained results very significantly. However, it remains a potentially insightful track for future research.

4.3 Alternative Expectation Formation Results

In order to check the robustness of our conclusions drawn from model under behavior expectations we also discuss the results using alternative expectation formation. More specifically, we simplify our heuristics switching mechanism and treat all tree heuristics, described in Section 2.3, as separate cases. Changes in the state space model matrices forms are shown in the Appendix A.

Estimation results together with their according confidence intervals (CI) with 95% confidence level are illustrated in Appendix B. The conclusion that predictions produced by introducing an economic integration factor provide a (more or less) significant change to the prediction accuracy compared to closed economies are persistent for each ADA, WTF and LAA heuristics case scenario. Table 2 presents the comparison of one-step ahead prediction error and the actual inflation at each sample time point as according to their mean square errors for both closed and open economy. The results show that, according to the measure of fit, open economy case under adaptive expectations does outperform closed economy case for Spain. However, closed economy model prediction errors under WTF expectations produce a better fit for both Spain and Italy. Considering LAA expectations, open economy model predictions unanimously provide a better fit for each considered country. This leads us to believe that our four countries are not homogeneous in our analyzed framework.

Table 2: Prediction Error Accuracy

	France	Germany	Spain	Italy	Currency Union
Rational expectations	0.727	0.711	2.145	0.847	0.727
MSE for Adaptive Expectations					
Closed Economy	0.074	0.051	0.161	0.064	0.060
Open Economy	0.080	0.063	0.152	0.073	0.064
MSE for WTF Expectations					
Closed Economy	0.040	0.060	0.084	0.050	0.050
Open Economy	0.036	0.047	0.182	0.082	0.031
MSE for LAA Expectations					
Closed Economy	0.207	0.191	0.541	0.273	0.156
Open Economy	0.067	0.089	0.219	0.097	0.047

One can also notice that weak-trend following expectations in general seem to provide the most accurate predictions out of all considered heuristics taken separately, and, consequently, the heuristics switching mechanism as a whole. HSM provides a much more sophisticated description of expectation formation by economic agents where weights given to different heuristics depend on various factors, such as agents memory, intensity of choice and other. Because of these factors it takes a certain amount of time to pass for the accustomed weights to change significantly based on heuristics performance. This explains why HSM provides larger deviations than WTF or ADA expectations for certain countries, at least in the short run or during highly destabilizing periods such as the 2008 crisis. We conclude that having economic agents with different expectation formation heuristics can significantly influence the inflation prediction. For example, having more agents with LAA expectations would lead to less accurate one-quarter inflation forecasts. This result only adds more depth into the complexity that is brought in by the open economy model extension.

4.4 Rational Expectations

One might wonder, how important are behavioral expectations to our conclusions and would they be any different when we revert to rational expectations. After all, they are dominating the related mainstream literature. Also, which of the two (rational or behavioral) would present better forecasts.

Let us assume that errors $\varepsilon_t^i, \eta_t^i$ follow an AR(1) process with $\rho \in (0,1)$. Following the notation provided in Appendix A.2[19] the minimal state variable rational expectation solution for the model

$$x_t = T^* \tilde{E}_t x_{t+1} + B^* + R^* \varepsilon_t^*$$

is found to be

$$\tilde{E}_t x_{t+1} = (I - T^*)^{-1} B^* + \rho^2 (I - \rho T^*)^{-1} R^* \varepsilon_{t-1}^*$$

as according to to Massaro et al. (2017). The proof is based on the law of motion and the method of undetermined coefficients. Inserting the solution to the model we get:

$$x_t = T^*(I - T^*)^{-1} B^* + B^* + (R^* + \rho^2 T^*(I - \rho T^*)^{-1} R^*) \varepsilon_t^*.$$

Taking mathematical expectations of this equation leads to:

$$Ex_t = T^*(I - T^*)^{-1} B^* + B^*$$

which eventually solves as $Ex_t = (\bar{y}, \bar{\pi})^T$ and is exactly the steady state of the system. Since the solution does not depend on observations, Kalman filter one-step ahead prediction is equal to the systems steady state at each step. One can notice that in the end the rational expectations solution for the Kalman filter does not depend on ρ as well as AR(1) process assumption all together.

We see that with rational expectations inflation is exactly the same in both long-run and short-run which means that the Phillips curve becomes vertical. Palley (2011) explains that in this

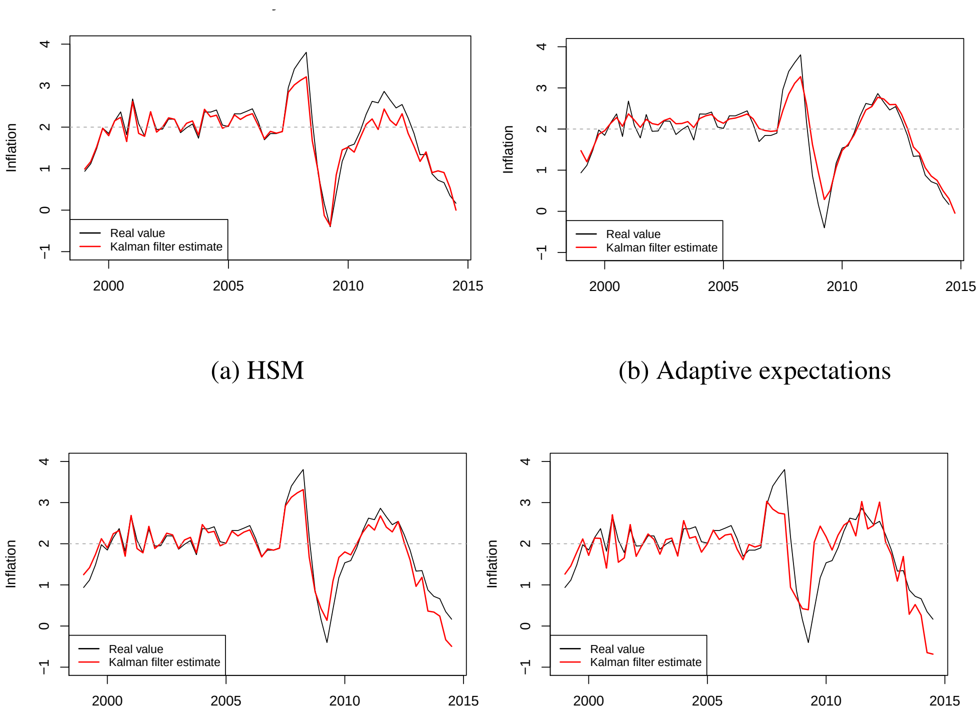
[19]The model form is derived for an open economy consisting of N countries. The closed economy case is derived analogously taking $N = 1$.

situation monetary policy authority can not exploit inflation to systematically move output or unemployment outcomes and its' best policy is to offset fluctuations around the natural rates. Also, he points out that optimal policy rate becomes crucial because in order for the rate to be effective, it needs to be understood and believed by the rational expectations model agents. This causes other important problems such as time inconsistency of policy, central bank independence, credibility and so on.

Looking at Table 1 and Table 2 we see that forecasts made under rational expectations are much less precise. Therefore, we conclude that rational expectations can be misleading and artificially simplistic. All of the complexity from behavioral models with all of their nuances and subtleties are completely lost.

4.5 Structural Break

Since 2009 May, the European Central Bank announced the start of asset purchasing program or the so-called quantitative easing. The relevance of Taylor rule since the crisis emerged in 2008 is broadly discussed in the academic literature. Many researchers express their views that the European Central Bank's interest rates in the presence of unconventional monetary policy deviates from the estimates of Taylor rule (Folkerts-Landau, 2016; Alcidi et al., 2017; Joyce et al., 2012; Gorter et al., 2009; Hofmann and Bogdanova, 2012). For this reason in this section



(a) HSM

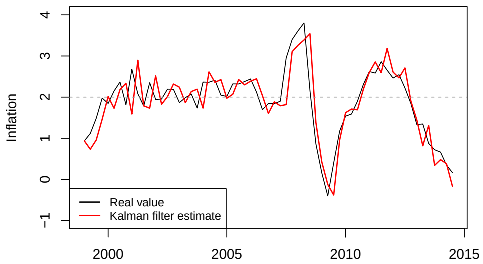
(b) Adaptive expectations

(c) WTF expectations

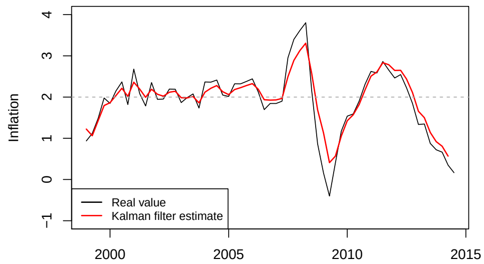
(d) LAA expectations

Figure 4: Inflation one-quarter ahead Predictions for Currency Union in Closed Economy with Structural Break

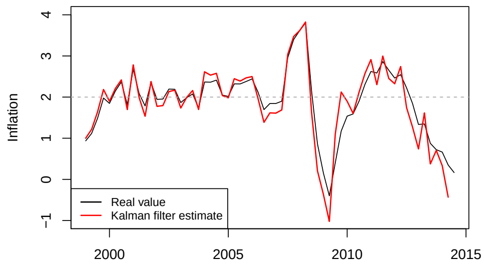
we abstain from using Taylor rule all-together and starting 2009 second quarter we replace the rule with real-time interest rate data [20] and use it as an exogenous variable. Changes in the state space model matrices form regarding this structural change are shown in the Appendix A. Same set of initial parameters and their bounds are used for every country and for both pre- and post-crisis periods as before (if not told otherwise).



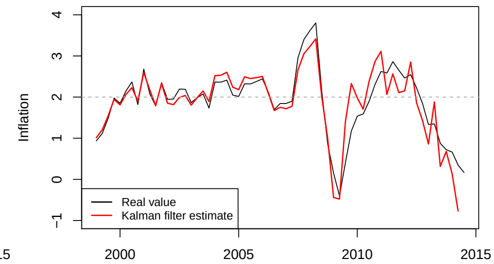
(a) HSM



(b) Adaptive expectations



(c) WTF expectations



(d) LAA expectations

Figure 5: Inflation one-quarter ahead Predictions for Currency Union in Open Economy with Structural Break

Figure 4 and Figure 5 show the results for economic union representation of currency union in both open and closed economies. Individual country case under heuristic switching mechanism is presented in Appendix B Figure 15 and Figure 16 while alternative expectation formation heuristics are excluded for brevity. One-quarter ahead inflation predictions appear to fit the actual data relatively well showing larger deviations during heating economy period in 2007 and 2008-09 crisis, which goes in line with our previous results. Also, filtered estimates appear to have a quite smooth transition between the separated periods since 2009 Q2 and do not present any visible structural shift. Saying that, for both WTF and LAA expectations our filtered prediction estimates show a significant negative deviation from the actual data at the end of the sample. In Appendix C we present normality tests results for our estimated standard one-step ahead prediction errors including a structural break. We conclude that our normality assumptions are once again specified with enough precision.

[20] Interest rate for main refinancing operation: fixed rate by ECB. Adjusted according to ECB rate manipulation. Data provided by Comunale (2015).

Table 3: Prediction Error Accuracy with Structural Break

	France	Germany	Spain	Italy	Currency Union
Rational expectation	0.727	0.711	2.145	0.847	0.727
MSE for Heuristics Switching Mechanism					
Closed Economy	0.083	0.068	0.197	0.064	0.054
Open Economy	0.070	0.060	0.205	0.087	0.059
MSE for Adaptive Expectations					
Closed Economy	0.089	0.057	0.165	0.067	0.069
Open Economy	0.093	0.073	0.153	0.082	0.075
MSE for WTF Expectations					
Closed Economy	0.073	0.072	0.202	0.093	0.068
Open Economy	0.134	0.131	0.178	0.107	0.096
MSE for LAA Expectations					
Closed Economy	0.229	0.191	0.578	0.226	0.218
Open Economy	0.149	0.174	0.397	0.251	0.132

Table 3 presents mean square errors between one-step ahead predictions and actual inflation observations for both closed and open economy. Our estimated prediction error accuracy with an introduced structural break yields almost identical conclusions to those previously obtained. However, the overall size of MSE has slightly increased in all cases. Important to note, that this structural break extension doubles the unknown parameters that we estimate in each model case. That imposes a significantly larger computational burden and the size of estimation error (in terms of unknown parameter optimization in a given range), especially in open economy case. Therefore, we conclude that our proposed extension did not provide other than theoretical improvement.

5 Conclusions

In this thesis we have relaxed the assumption of rational expectations and took a behavioral approach in modeling a currency union approximation in real-time using a linear dynamic system. On top of that, we extended the mainstream New Keynesian model by allowing an interdependence between the monetary union members. The results can be summarized as follows:

1. In most cases modeling a currency union as a set of open economies with intra union trade would provide less accurate predictions than those obtained by modeling a currency union as a closed and homogeneous economy. Even if the difference is not highly significant, it could matter for policy making purposes.

2. Assuming rational expectations while modeling macroeconomic New Keynesian models might not be as harmless as it might seem. Our results reveal a very remarkable improvement by omitting the assumption of perfectly rational economic agents and instead allowing them to rationally deal with imperfect information.

3. The result seems to persist for different expectation formation approaches including both fundamentalist and extrapolative rules.

4. From a practical point of view, Taylor rule managed to decently provide a rule of thumb in times when unconventional monetary policy took action.

Overall, we believe that our behavioral expectations approach could complement the conventional way of dealing with expectation formation assumptions while estimating real-time behavioral New Keynesian models.

Many improvements can still be made. First of all, we do not deal with a possibility of correlated errors. This is especially important considering how very likely it is that currency union members receive common shocks. For example, changes in common currency's exchange rate would lead to highly correlated output shocks. Second, one could account for the additional parameter estimation uncertainty by calculating prediction mean square errors using bootstrap methods (Roodriguez and Ruiz, 2010). Also, it could be valuable to check the robustness of our results by putting more relevance to past realizations, which is often observed in practical policy decision making tendencies (De Grauwe, 2010).

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A Appendix: Matrix Forms

A.1 Closed Economy State space model matrices

Adaptive Expectations:

Before the crisis: $Z_t = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $T_t = 0.65\Theta^{-1} \begin{bmatrix} \sigma & 1-\Phi_\pi\beta \\ \kappa\sigma & \kappa+\beta\sigma+\beta\Phi_y \end{bmatrix}$, $R_t = \Theta^{-1} \begin{bmatrix} \sigma & -\Phi_\pi \\ \kappa\sigma & \sigma+\phi_y \end{bmatrix}$,
$B_t = \Theta^{-1} \begin{bmatrix} \bar{\pi}(\Phi_\pi - 1) + \Phi_y\bar{y} \\ \kappa\bar{\pi}(\Phi_\pi - 1) + \kappa\Phi_y\bar{y} \end{bmatrix} + 0.35\Theta^{-1} \begin{bmatrix} \sigma & 1-\Phi_\pi\beta \\ \kappa\sigma & \kappa+\beta\sigma+\beta\Phi_y \end{bmatrix} \begin{bmatrix} \tilde{E}_t y_t \\ \tilde{E}_t \pi_t \end{bmatrix}$.

After the crisis: $Z_t = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $T_t = 0.65 \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \frac{1}{\kappa} & \frac{\kappa}{\sigma}+\beta \end{bmatrix}$, $R_t = \begin{bmatrix} 1 & 0 \\ \kappa & 1 \end{bmatrix}$,
$B_t = \begin{bmatrix} 0.35 & 0.35\frac{1}{\sigma} & -\frac{1}{\sigma} \\ 0.35\kappa & 0.35(\frac{\kappa}{\sigma}+\beta) & -\frac{\kappa}{\sigma} \end{bmatrix} \begin{bmatrix} \tilde{E}_t y_t \\ \tilde{E}_t \pi_t \\ r_t \end{bmatrix}$.

Here $\tilde{E}_t y_t$ and $\tilde{E}_t \pi_t$ are taken as predetermined.

Weak Trend Following Expectations:

Before the crisis: $Z_t = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $T_t = 1.4\Theta^{-1} \begin{bmatrix} \sigma & 1-\Phi_\pi\beta \\ \kappa\sigma & \kappa+\beta\sigma+\beta\Phi_y \end{bmatrix}$, $R_t = \Theta^{-1} \begin{bmatrix} \sigma & -\Phi_\pi \\ \kappa\sigma & \sigma+\phi_y \end{bmatrix}$,
$B_t = \Theta^{-1} \begin{bmatrix} \bar{\pi}(\Phi_\pi - 1) + \Phi_y\bar{y} \\ \kappa\bar{\pi}(\Phi_\pi - 1) + \kappa\Phi_y\bar{y} \end{bmatrix} - 0.4\Theta^{-1} \begin{bmatrix} \sigma & 1-\Phi_\pi\beta \\ \kappa\sigma & \kappa+\beta\sigma+\beta\Phi_y \end{bmatrix} \begin{bmatrix} y_{t-2} \\ \pi_{t-2} \end{bmatrix}$.

After the crisis: $Z_t = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $T_t = 1.4 \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \frac{1}{\kappa} & \frac{\kappa}{\sigma}+\beta \end{bmatrix}$, $R_t = \begin{bmatrix} 1 & 0 \\ \kappa & 1 \end{bmatrix}$,
$B_t = -0.4 \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma}+\beta \end{bmatrix} \begin{bmatrix} y_{t-2} \\ \pi_{t-2} \end{bmatrix} - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} r_t$.

Here y_{t-2}, π_{t-2} and r_t are taken as predetermined.

Learning, Anchoring and Adjustment Expectations:

Before the crisis: $Z_t = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $T_t = 1.5\Theta^{-1} \begin{bmatrix} \sigma & 1-\Phi_\pi\beta \\ \kappa\sigma & \kappa+\beta\sigma+\beta\Phi_y \end{bmatrix}$, $R_t = \Theta^{-1} \begin{bmatrix} \sigma & -\Phi_\pi \\ \kappa\sigma & \sigma+\phi_y \end{bmatrix}$,
$B_t = \Theta^{-1} \begin{bmatrix} \bar{\pi}(\Phi_\pi - 1) + \Phi_y\bar{y} \\ \kappa\bar{\pi}(\Phi_\pi - 1) + \kappa\Phi_y\bar{y} \end{bmatrix} + \Theta^{-1} \begin{bmatrix} \sigma & 1-\Phi_\pi\beta \\ \kappa\sigma & \kappa+\beta\sigma+\beta\Phi_y \end{bmatrix} (0.5 \begin{bmatrix} y_{t-1}^{average} \\ \pi_{t-1}^{average} \end{bmatrix} - \begin{bmatrix} y_{t-2} \\ \pi_{t-2} \end{bmatrix})$.

After the crisis: $Z_t = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $T_t = 1.5 \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \frac{1}{\kappa} & \frac{\kappa}{\sigma}+\beta \end{bmatrix}$, $R_t = \begin{bmatrix} 1 & 0 \\ \kappa & 1 \end{bmatrix}$,

$B_t = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma}+\beta \end{bmatrix} (0.5 \begin{bmatrix} y_{t-1}^{average} \\ \pi_{t-1}^{average} \end{bmatrix} - \begin{bmatrix} y_{t-2} \\ \pi_{t-2} \end{bmatrix}) - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} r_t.$

Here $y_t^{average} = \frac{1}{t}\sum_{k=1}^{t} y_k$, $\pi_t^{average} = \frac{1}{t}\sum_{k=1}^{t} \pi_k$ are taken as predetermined.

Heuristics Switching:

After the crisis: $Z_t = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $T_t = \begin{bmatrix} n_{yt} & n_{\pi t} \\ n_{yt} & n_{\pi t} \end{bmatrix} \odot \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \frac{1}{\kappa} & \frac{\kappa}{\sigma}+\beta \end{bmatrix}$, $R_t = \begin{bmatrix} 1 & 0 \\ \kappa & 1 \end{bmatrix}$,

$B_t = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \frac{\kappa}{\sigma}+\beta \end{bmatrix} (0.5 \begin{bmatrix} n_{yt}^4 y_{t-1}^{average} \\ n_{\pi t}^4 \pi_{t-1}^{average} \end{bmatrix} + \begin{bmatrix} n_{yt}^* y_{t-2} \\ n_{\pi t}^* \pi_{t-2} \end{bmatrix} + 0.35 \begin{bmatrix} n_{yt}^1 \tilde{E}_t y_t \\ n_{\pi t}^1 \tilde{E}_t \pi_t \end{bmatrix}) - \begin{bmatrix} \frac{1}{\sigma} \\ \frac{\kappa}{\sigma} \end{bmatrix} r_t.$

Here $n_{yt} = 0.65n_{yt}^1 + 1.4n_{yt}^2 + 1.5n_{yt}^3$, $n_{\pi t} = 0.65n_{\pi t}^1 + 1.4n_{\pi t}^2 + 1.5n_{\pi t}^3$, $n_{yt}^* = -0.4n_{yt}^2 - n_{yt}^3$, $n_{\pi t}^* = -0.4n_{\pi t}^2 - n_{\pi t}^3$. Notation $\odot$ represents element-wise product of matrices, known as the Hadamard product.

A.2 Open Economy Model Matrices

The model for an open economy of N countries consisting of equations (7)–(9) can be rewritten in matrix form as:

$$x_t = T^* \tilde{E}_t x_{t+1} + B^* + R^* \varepsilon_t^*, \tag{25}$$

with the following notation:

$$x_t := \begin{bmatrix} y_t^1 \\ \vdots \\ y_t^N \\ \pi_t^1 \\ \vdots \\ \pi_t^N \end{bmatrix}, \ \tilde{E}_t x_{t+1} := \begin{bmatrix} \tilde{E}_t y_{t+1}^1 \\ \vdots \\ \tilde{E}_t y_{t+1}^N \\ \tilde{E}_t \pi_{t+1}^1 \\ \vdots \\ \tilde{E}_t \pi_{t+1}^N \end{bmatrix}, \ \varepsilon_t^* := \begin{bmatrix} \varepsilon_t^1 \\ \vdots \\ \varepsilon_t^N \\ \eta_t^1 \\ \vdots \\ \eta_t^N \end{bmatrix},$$

$$\Theta := \begin{bmatrix} 1+\frac{\phi_y w(1)}{\sigma^1 \sum_{k=1}^N w(k)} & \cdots & \frac{\phi_y w(N)}{\sigma^1 \sum_{k=1}^N w(k)} & \frac{\phi_\pi w(1)}{\sigma^1 \sum_{k=1}^N w(k)} & \cdots & \frac{\phi_\pi w(N)}{\sigma^1 \sum_{k=1}^N w(k)} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ \frac{\phi_y w(1)}{\sigma^N \sum_{k=1}^N w(k)} & \cdots & 1+\frac{\phi_y w(N)}{\sigma^N \sum_{k=1}^N w(k)} & \frac{\phi_\pi w(1)}{\sigma^N \sum_{k=1}^N w(k)} & \cdots & \frac{\phi_\pi w(N)}{\sigma^N \sum_{k=1}^N w(k)} \\ -\kappa^1 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -\kappa^N & 0 & \cdots & 1 \end{bmatrix},$$

$$T^* := \Theta^{-1}\begin{bmatrix} 1 & \cdots & 0 & \frac{1}{\sigma^1}-\alpha^1 & \cdots & \frac{\alpha^1 w(N)}{\sum_{k=1}^{N,k\neq 1} w(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & \frac{\alpha^N w(1)}{\sum_{k=1}^{N,k\neq N} w(k)} & \cdots & \frac{1}{\sigma^N}-\alpha^N \\ 0 & \cdots & 0 & \beta^1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & \beta^N \end{bmatrix},$$

$$B^* := \Theta^{-1}\begin{bmatrix} \frac{\Phi_y w(1)}{\sigma^1 \sum_{k=1}^{N} w(k)} & \cdots & \frac{\Phi_y w(N)}{\sigma^1 \sum_{k=1}^{N} w(k)} & -\frac{1-\Phi_\pi}{\sigma^1} & \cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ \frac{\Phi_y w(1)}{\sigma^N \sum_{k=1}^{N} w(k)} & \cdots & \frac{\Phi_y w(N)}{\sigma^N \sum_{k=1}^{N} w(k)} & 0 & \cdots & -\frac{1-\Phi_\pi}{\sigma^N} \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix}\begin{bmatrix} \bar{y}^1 \\ \vdots \\ \bar{y}^N \\ \bar{\pi} \\ \vdots \\ \bar{\pi} \end{bmatrix},$$

$$R^* := \Theta^{-1}\begin{bmatrix} 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1 \end{bmatrix}.$$

After the crisis:

$$T^*_{ac} := \begin{bmatrix} 1 & \cdots & 0 & \frac{1}{\sigma^1}-\alpha^1 & \cdots & \frac{\alpha^1 w(N)}{\sum_{k=1}^{N,k\neq 1} w(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & \frac{\alpha^N w(1)}{\sum_{k=1}^{N,k\neq N} w(k)} & \cdots & \frac{1}{\sigma^N}-\alpha^N \\ \kappa^1 & \cdots & 0 & \beta^1+\kappa^1(\frac{1}{\sigma^1}-\alpha^1) & \cdots & \frac{\kappa^1\alpha^1 w(N)}{\sum_{k=1}^{N,k\neq 1} w(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \kappa^N & \frac{\kappa^N\alpha^N w(1)}{\sum_{k=1}^{N,k\neq N} w(k)} & \cdots & \beta^N+\kappa^N(\frac{1}{\sigma^N}-\alpha^N) \end{bmatrix},$$

$$B^*_{ac} := \begin{bmatrix} -\frac{1}{\sigma^1} \\ \vdots \\ -\frac{1}{\sigma^N} \\ -\frac{\kappa^1}{\sigma^1} \\ \vdots \\ -\frac{\kappa^N}{\sigma^N} \end{bmatrix} r_t,\ R^*_{ac} := \begin{bmatrix} 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 0 \\ \kappa^1 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \kappa^N & 0 & \cdots & 1 \end{bmatrix}.$$

A.3 Open Economy State space model matrices

Adaptive Expectations:

$Z_t = \begin{bmatrix} \mathbb{0}_{N\times N} & \mathbb{1}_{N\times N} \end{bmatrix}.$

Before the crisis: $T_t = 0.65T^*$, $R_t = R^*$, $B_t = B^* + 0.35T^*\tilde{E}_t x_t$.

After the crisis: $T_t = 0.65T^*_{ac}$, $R_t = R^*_{ac}$, $B_t = B^*_{ac} + 0.35T^*_{ac}\tilde{E}_t x_t$.

Weak Trend Following Expectations:

$Z_t = \begin{bmatrix} \mathbb{0}_{N\times N} & \mathbb{1}_{N\times N} \end{bmatrix}.$

Before the crisis: $T_t = 1.4T^*$, $R_t = R^*$, $B_t = B^* - 0.4T^* x_{t-2}$.

After the crisis: $T_t = 1.4T^*_{ac}$, $R_t = R^*_{ac}$, $B_t = B^*_{ac} - 0.4T^*_{ac} x_{t-2}$.

Learning, Anchoring and Adjustment Expectations:

$Z_t = \begin{bmatrix} \mathbb{0}_{N\times N} & \mathbb{1}_{N\times N} \end{bmatrix}.$

Before the crisis: $T_t = 1.5T^*$, $R_t = R^*$, $B_t = B^* - T^* x_{t-2} + 0.5T^* x^{average}_{t-1}$.

After the crisis: $T_t = 1.5T^*_{ac}$, $R_t = R^*_{ac}$, $B_t = B^*_{ac} - T^*_{ac} x_{t-2} + 0.5T^*_{ac} x^{average}_{t-1}$.

Heuristics Switching:

$Z_t = \begin{bmatrix} \mathbb{0}_{N\times N} & \mathbb{1}_{N\times N} \end{bmatrix}.$

Before the crisis: $T_t = \begin{bmatrix} n_{y_1 t} & \cdots & n_{y_4 t} & n_{\pi_1 t} & \cdots & n_{\pi_4 t} \\ \vdots & & & & & \vdots \\ n_{y_1 t} & \cdots & n_{y_4 t} & n_{\pi_1 t} & \cdots & n_{\pi_4 t} \end{bmatrix}_{2N\times 2N} \odot T^*$, $R_t = R^*$,

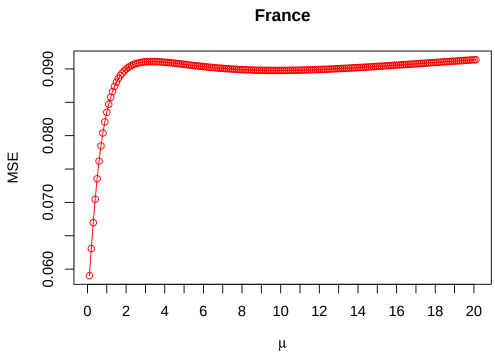
$$B_t = B^* + T^*(0.5 \begin{bmatrix} n^4_{y_1 t} y^{average,1}_{t-1} \\ \vdots \\ n^4_{y_N t} y^{average,N}_{t-1} \\ n^4_{\pi_1 t} \pi^{average,1}_{t-1} \\ \vdots \\ n^4_{\pi_N t} \pi^{average,N}_{t-1} \end{bmatrix} + \begin{bmatrix} n^*_{y_1 t} y^1_{t-2} \\ \vdots \\ n^*_{y_N t} y^N_{t-2} \\ n^*_{\pi_1 t} \pi^1_{t-2} \\ \vdots \\ n^*_{\pi_N t} \pi^N_{t-2} \end{bmatrix} + 0.35 \begin{bmatrix} n^1_{y_1 t} \tilde{E}_t y^1_t \\ \vdots \\ n^1_{y_N t} \tilde{E}_t y^N_t \\ n^1_{\pi_1 t} \tilde{E}_t \pi^1_t \\ \vdots \\ n^1_{\pi_N t} \tilde{E}_t \pi^N_t \end{bmatrix}).$$

After the crisis: $T_t = \begin{bmatrix} n_{y_1t} & \cdots & n_{y_4t} & n_{\pi_1t} & \cdots & n_{\pi_4t} \\ \vdots & & & & & \vdots \\ n_{y_1t} & \cdots & n_{y_4t} & n_{\pi_1t} & \cdots & n_{\pi_4t} \end{bmatrix}_{2N\times 2N} \odot T^*_{ac}$, $R_t = R^*_{ac}$,

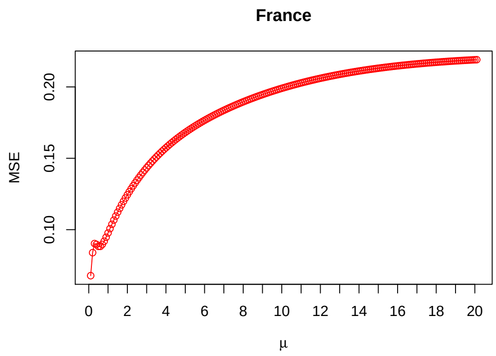
$$B_t = B^*_{ac} + T^*_{ac}(0.5 \begin{bmatrix} n^4_{y_1t}y^{average,1}_{t-1} \\ \vdots \\ n^4_{y_Nt}y^{average,N}_{t-1} \\ n^4_{\pi_1t}\pi^{average,1}_{t-1} \\ \vdots \\ n^4_{\pi_Nt}\pi^{average,N}_{t-1} \end{bmatrix} + \begin{bmatrix} n^*_{y_1t}y^1_{t-2} \\ \vdots \\ n^*_{y_Nt}y^N_{t-2} \\ n^*_{\pi_1t}\pi^1_{t-2} \\ \vdots \\ n^*_{\pi_Nt}\pi^N_{t-2} \end{bmatrix} + 0.35 \begin{bmatrix} n^1_{y_1t}\tilde{E}_t y^1_t \\ \vdots \\ n^1_{y_Nt}\tilde{E}_t y^N_t \\ n^1_{\pi_1t}\tilde{E}_t \pi^1_t \\ \vdots \\ n^1_{\pi_Nt}\tilde{E}_t \pi^N_t \end{bmatrix}).$$

Here $n_{y_it} = 0.65n^1_{y_it} + 1.4n^2_{y_it} + 1.5n^3_{y_it}$, $n_{\pi_it} = 0.65n^1_{\pi_it} + 1.4n^2_{\pi_it} + 1.5n^3_{\pi_it}$, $n^*_{y_it} = -0.4n^2_{y_it} - n^3_{y_it}$, $n^*_{\pi_it} = -0.4n^2_{\pi_it} - n^3_{\pi_it}$ where $i = 1, ..., N$.

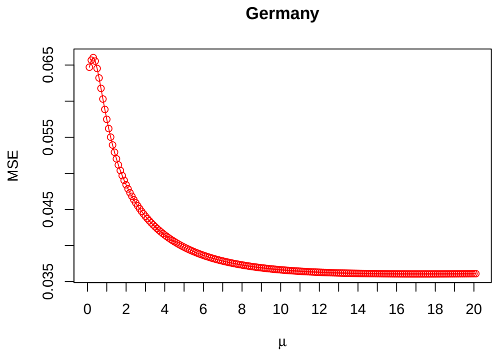
B Appendix: Additional Graphs



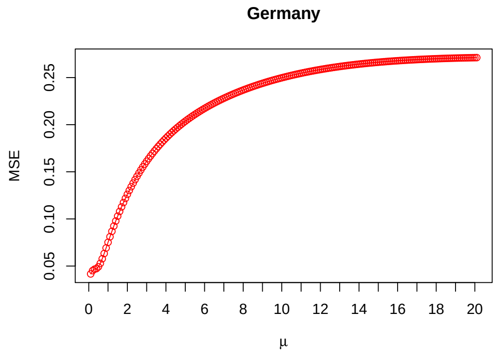
(a) Closed economy



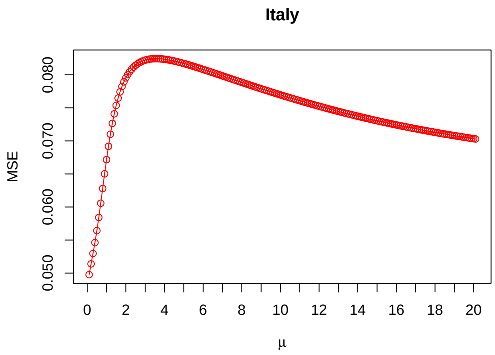
(b) Open economy



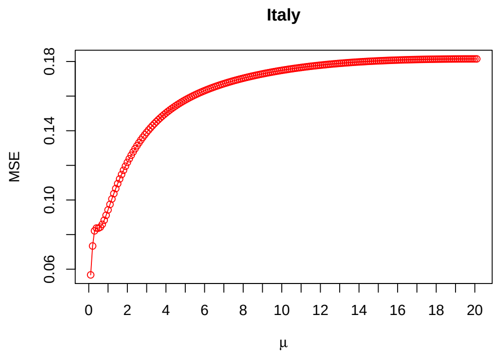
(c) Closed economy



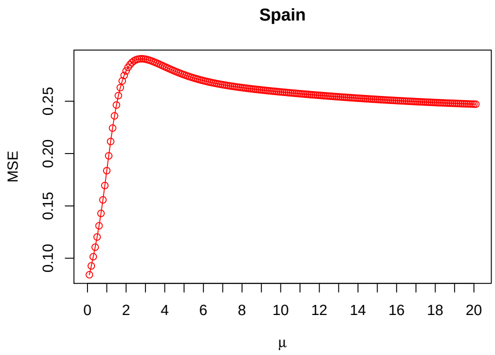
(d) Open economy



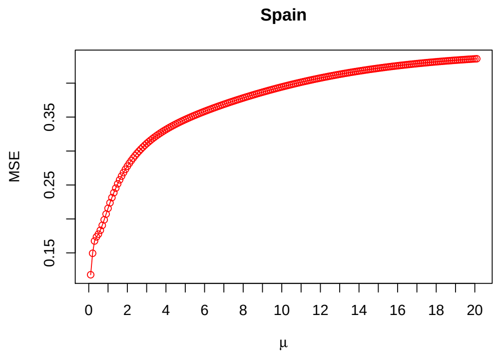
(e) Closed economy



(f) Open economy

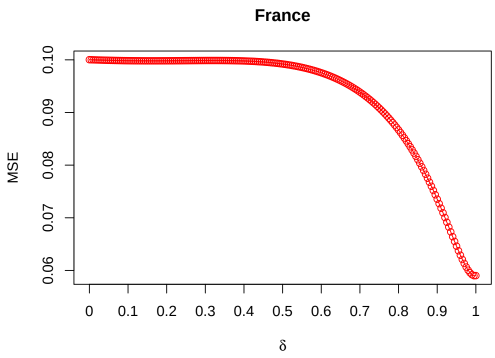


(g) Closed economy

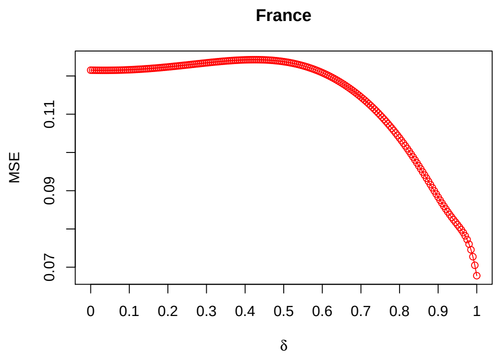


(h) Open economy

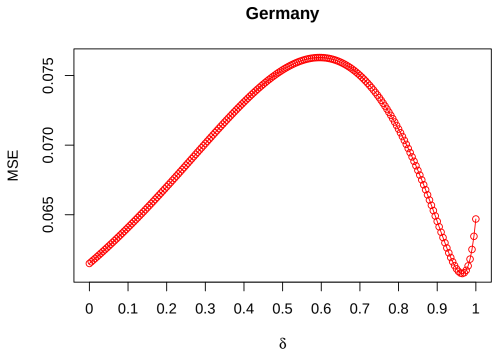
Figure 6: Prediction MSE accuracy for different values of μ



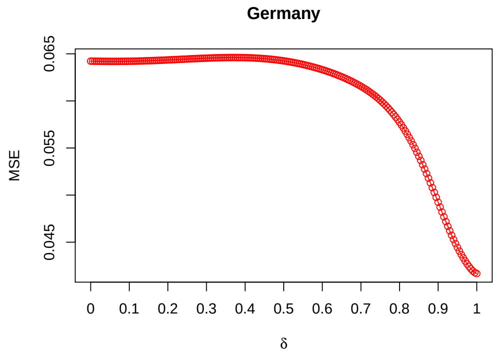
(a) Closed economy



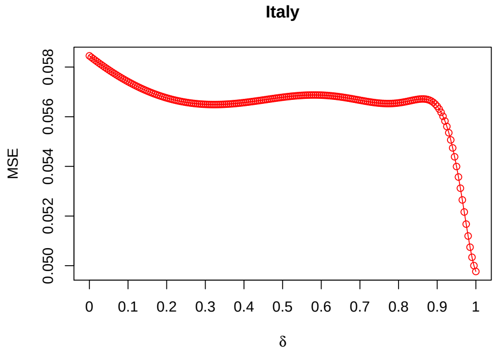
(b) Open economy



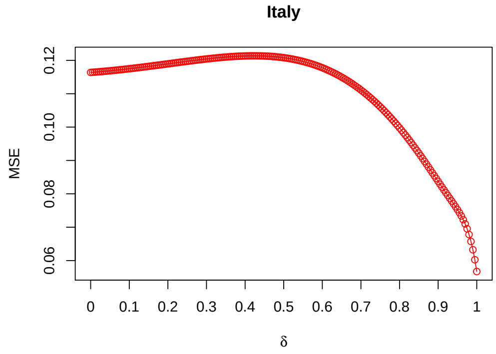
(c) Closed economy



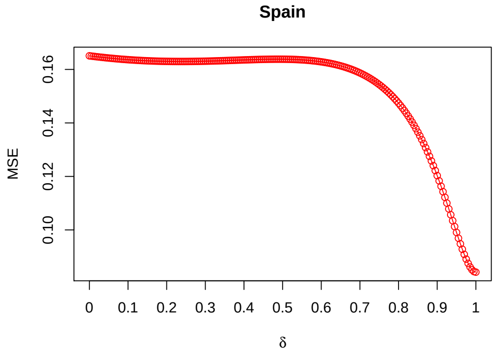
(d) Open economy



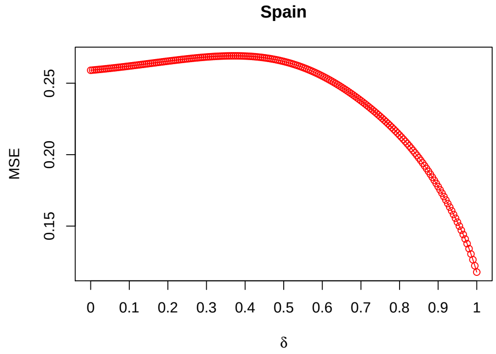
(e) Closed economy



(f) Open economy

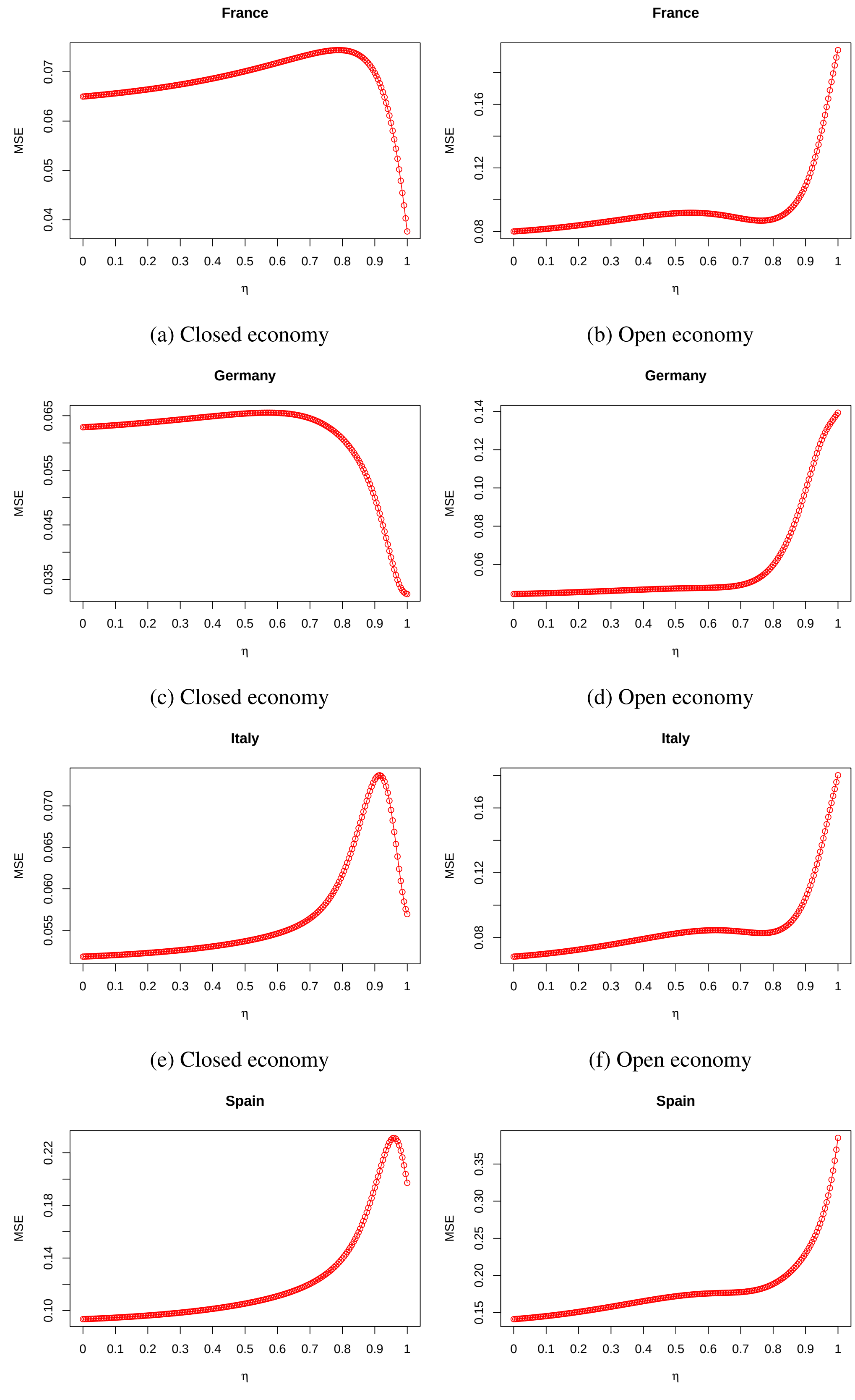


(g) Closed economy



(h) Open economy

Figure 7: Prediction MSE accuracy for different values of δ



(a) Closed economy (b) Open economy

(c) Closed economy (d) Open economy

(e) Closed economy (f) Open economy

(g) Closed economy (h) Open economy

Figure 8: Prediction MSE accuracy for different values of η

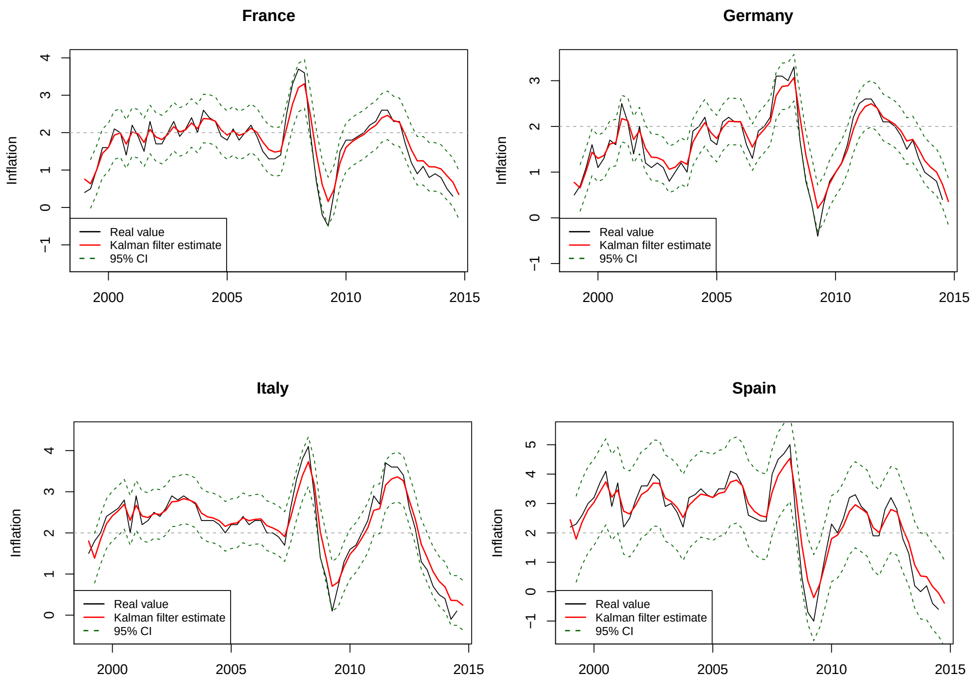


Figure 9: Inflation one-quarter ahead Predictions in Closed Economy under Adaptive Expectations

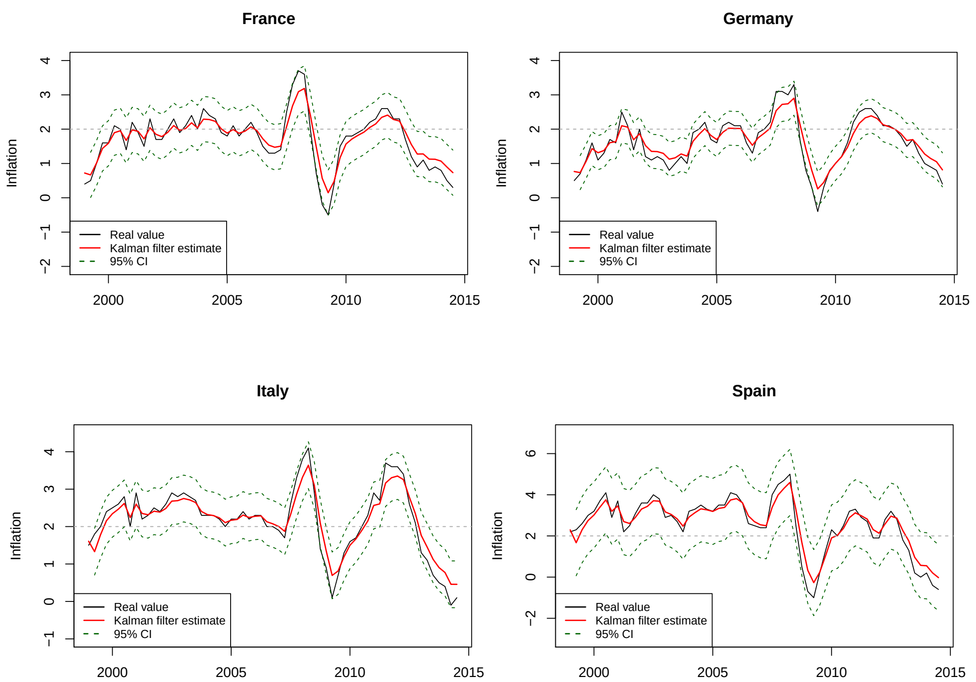


Figure 10: Inflation one-quarter ahead Predictions in Open Economy under Adaptive Expectations

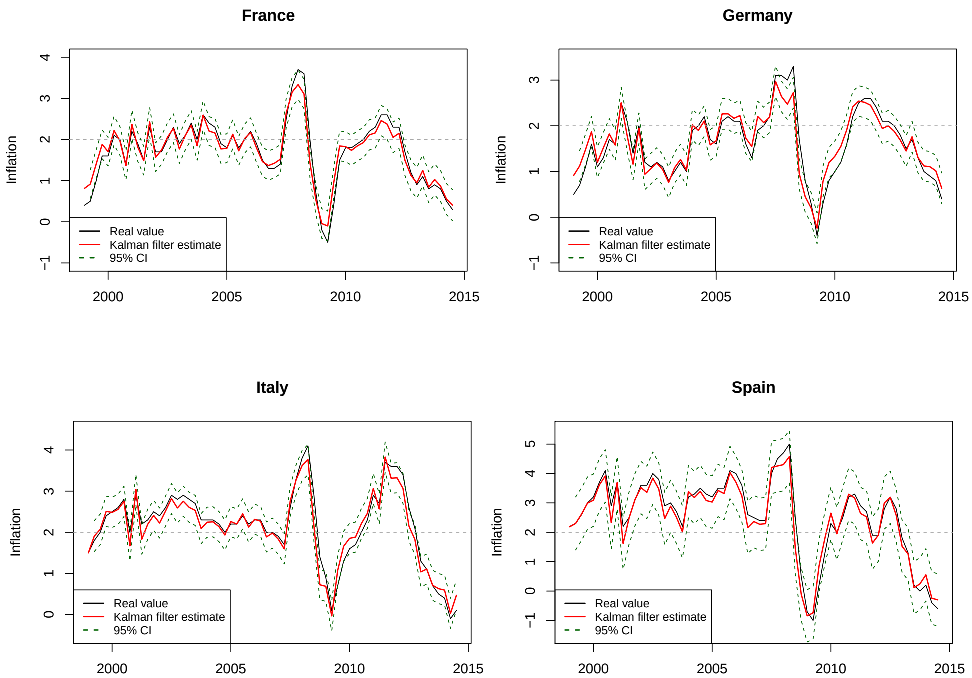


Figure 11: Inflation one-quarter ahead Predictions in Closed Economy under WTF Expectations

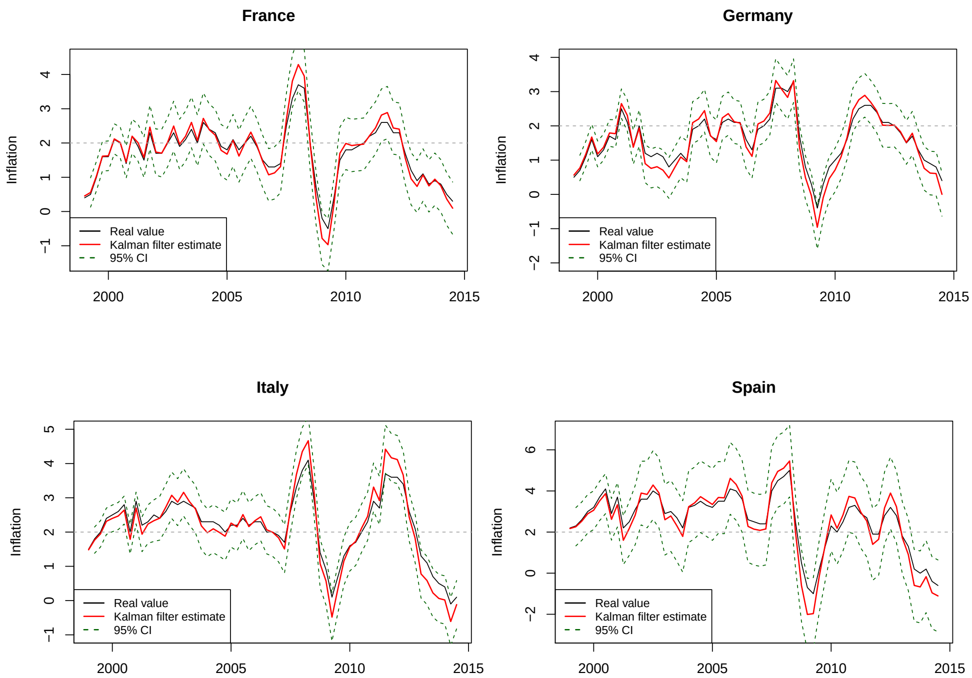


Figure 12: Inflation one-quarter ahead Predictions in Open Economy under WTF Expectations

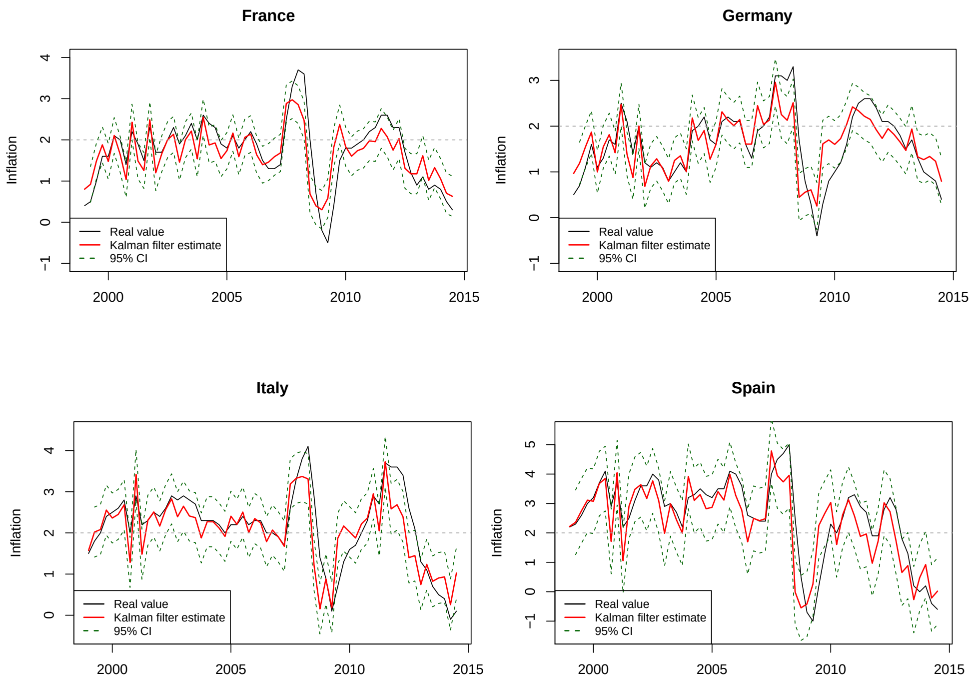


Figure 13: Inflation one-quarter ahead Predictions in Closed Economy under LAA Expectations

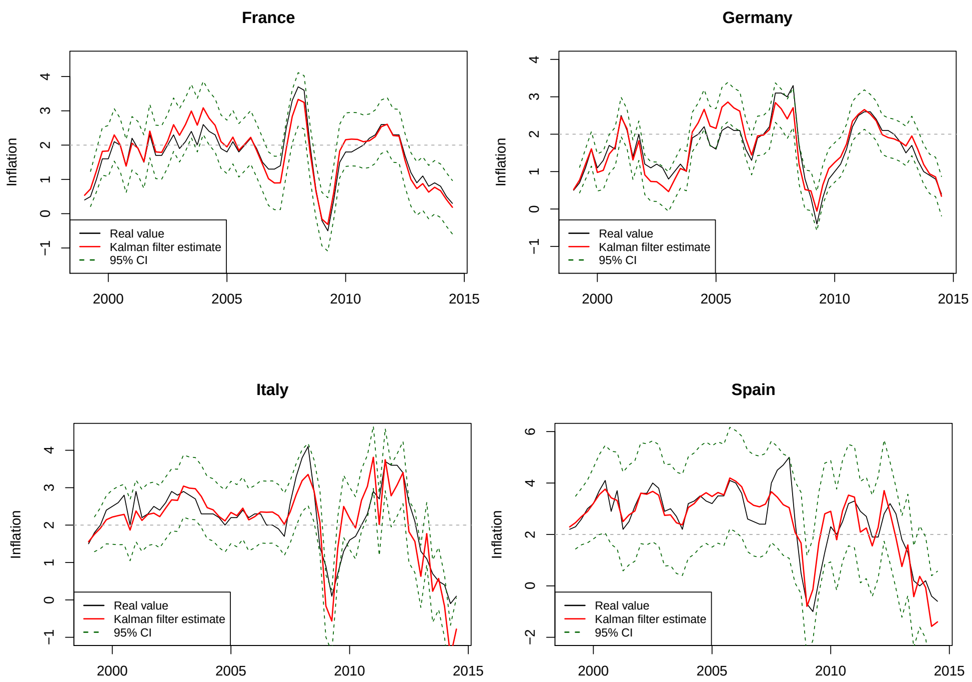


Figure 14: Inflation one-quarter ahead Predictions in Open Economy under LAA Expectations

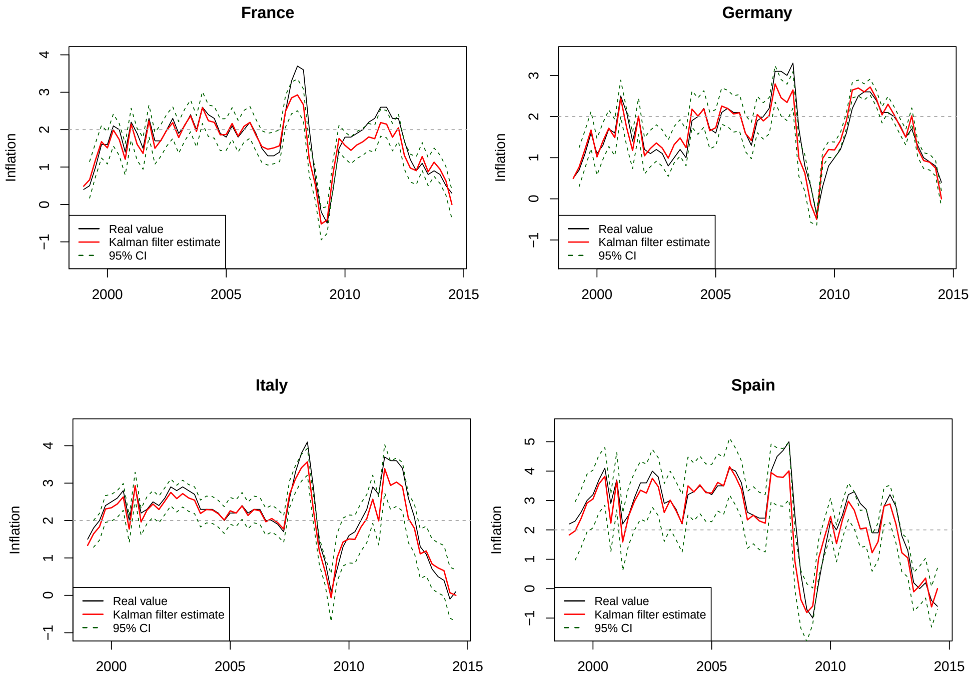


Figure 15: Inflation one-quarter ahead Predictions in Closed Economy under HSM with Structural Break

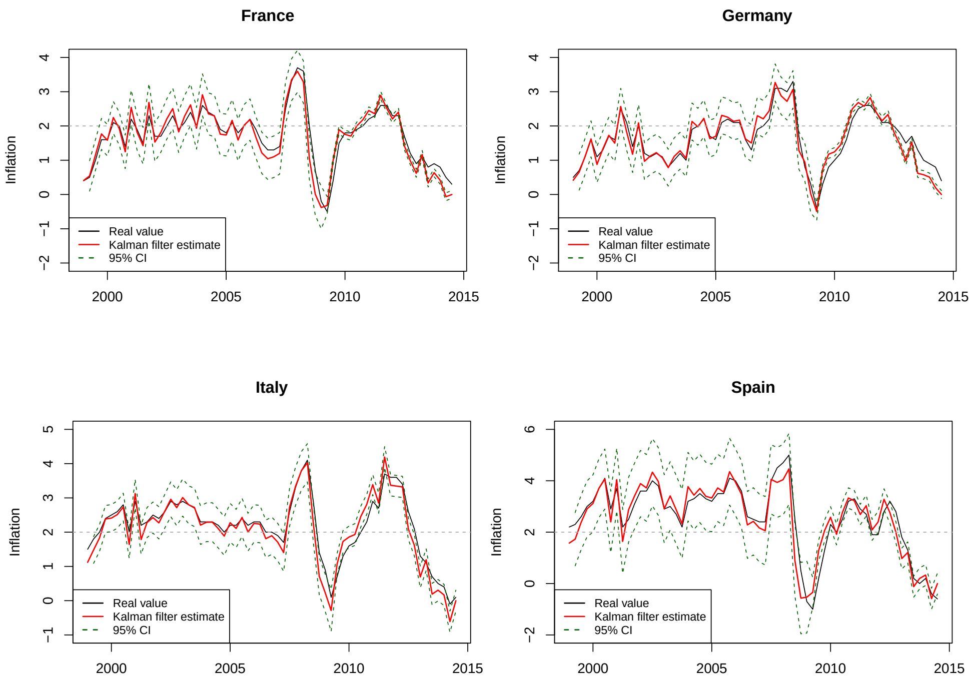
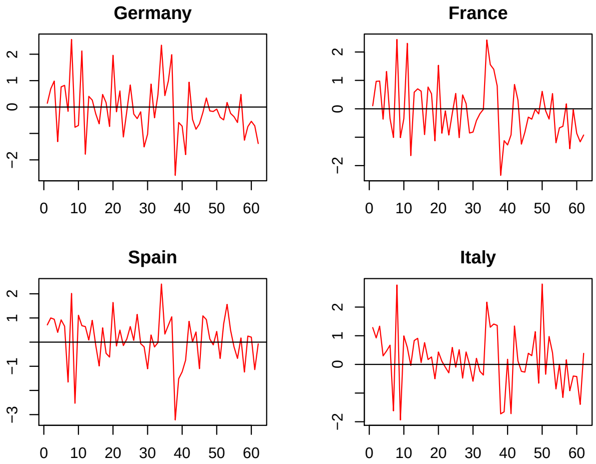


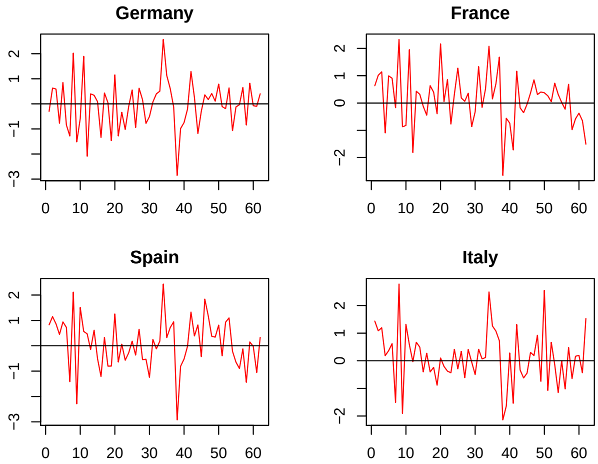
Figure 16: Inflation one-quarter ahead Predictions in Open Economy under HSM with Structural Break

C Appendix: Diagnostic Checking

Heuristics Switching Mechanism



(a) Std. Errors for Closed Economy



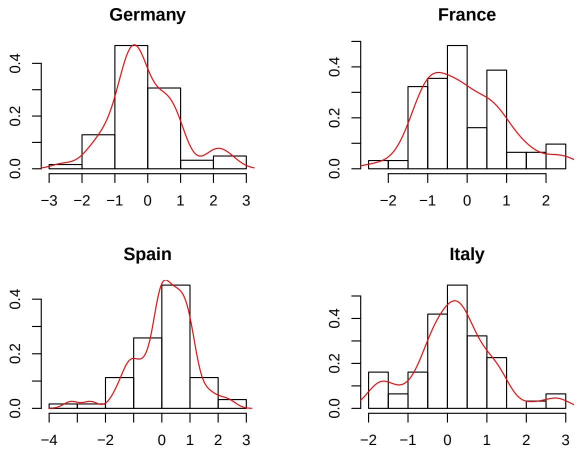
(b) Std. Errors for Open Economy

Figure 17: Standardized one-step ahead Forecast Errors under HSM

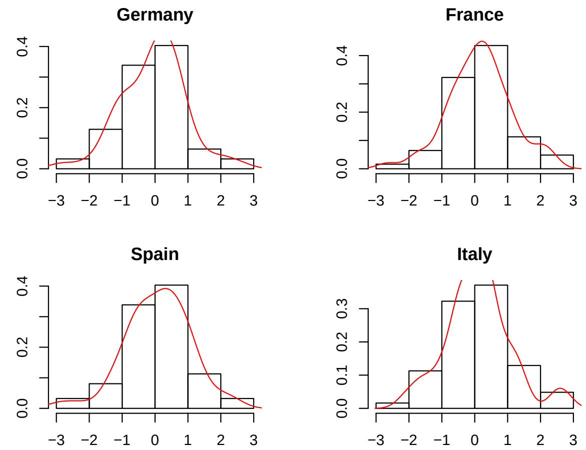
Table 4: Normality Test Statistics

	Closed Economy		**Open Economy**	
	JB p-value	SW p-value	JB p-value	SW p-value
Without Structural Break				
France	0.218	0.067	0.649	0.684
Germany	0.210	0.083	0.450	0.453
Spain	0.006	0.040	0.280	0.655
Italy	0.636	0.188	0.432	0.311
With Structural Break				
France	0.263	0.099	0.209	0.049
Germany	0.219	0.105	0.864	0.881
Spain	0.800	0.943	0.288	0.154
Italy	0.616	0.746	0.857	0.109

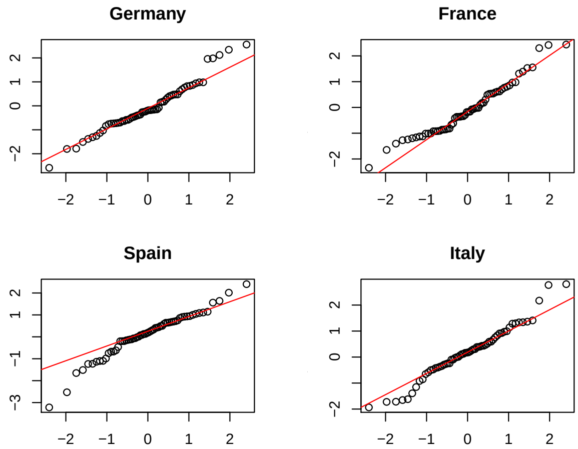
Notes: JB denotes Jarque-Bera normality test and SW – Shapiro-Wilk normality test. Using 96% confidence interval we accept the normality assumption in the error terms in all of the countries according to at least one of the tests.



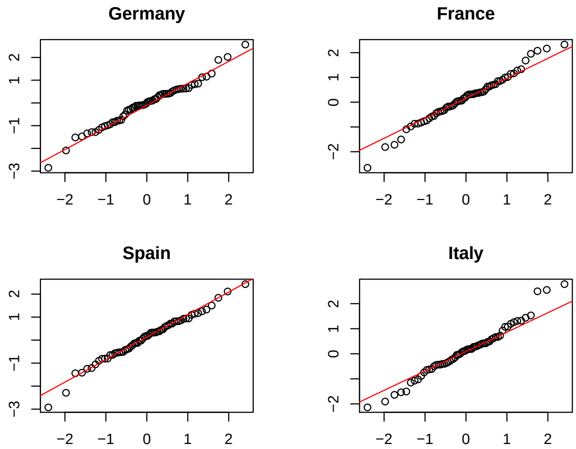
(a) Histogram and Density for Closed Economy



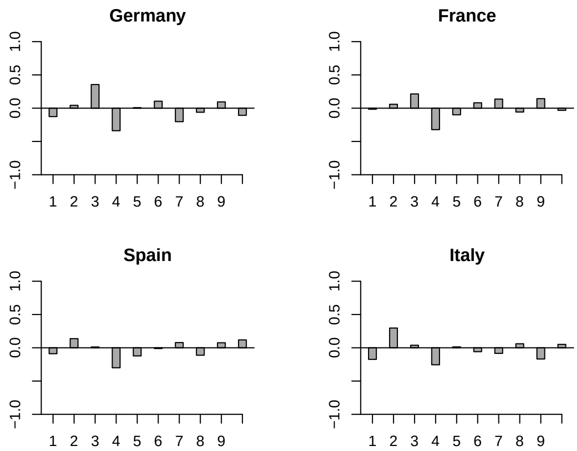
(b) Histogram and Density for Open Economy



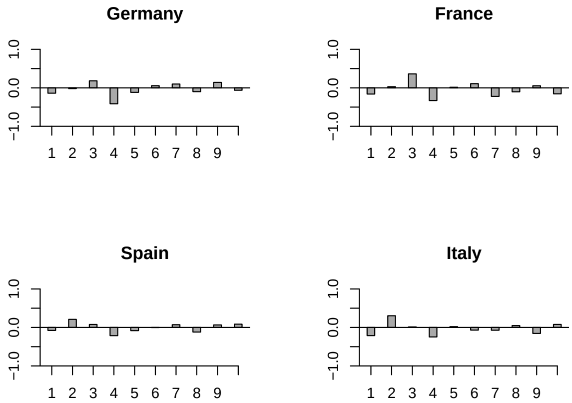
(c) QQ Plot for Closed Economy



(d) QQ Plot for Open Economy



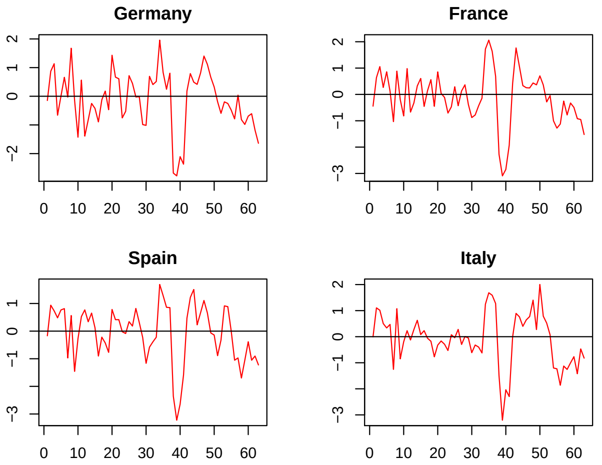
(e) Correlogram for Closed Economy



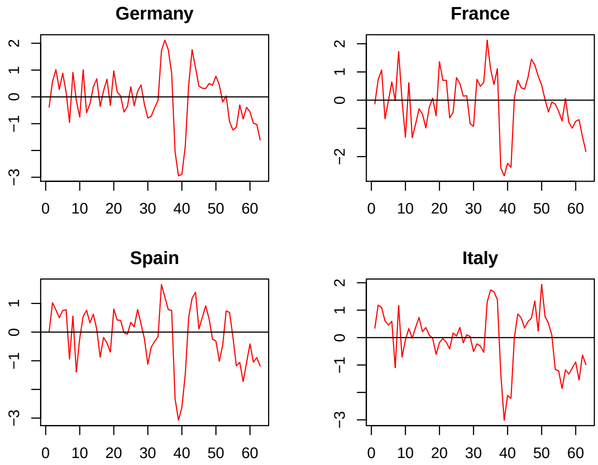
(f) Correlogram for Open Economy

Figure 18: Diagnostic Plots for Standardized Prediction Errors under HSM

Adaptive Expectations



(a) Std. Errors for Closed Economy



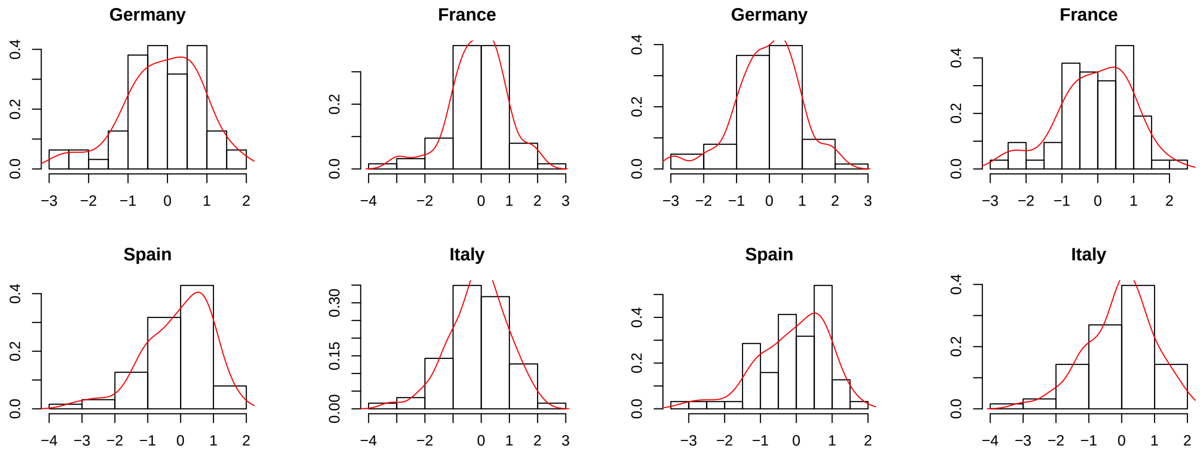
(b) Std. Errors for Open Economy

Figure 19: Standardized one-step ahead Forecast Errors under Adaptive Expectations

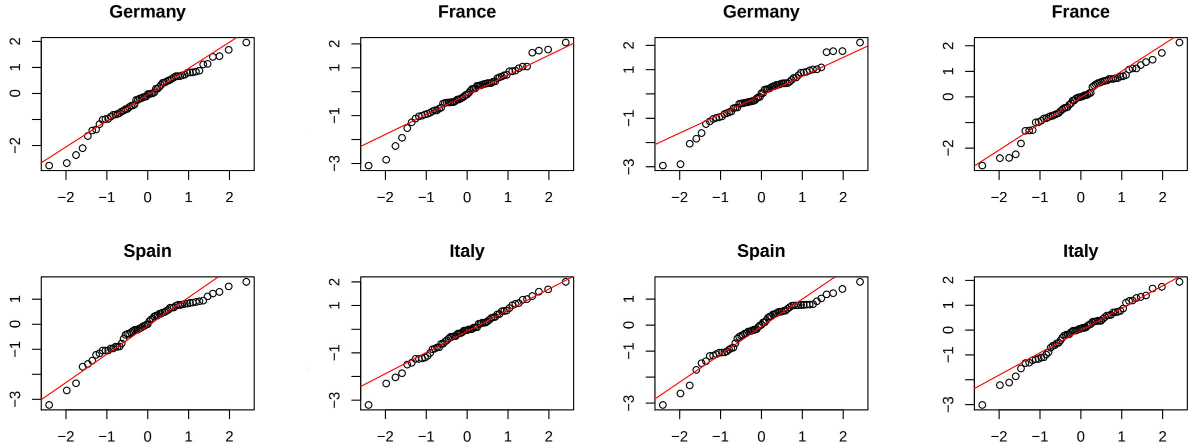
Table 5: Normality Test Statistics

	Closed Economy		**Open Economy**	
	JB p-value	SW p-value	JB p-value	SW p-value
Without Structural Break				
France	0.071	0.107	0.295	0.200
Germany	0.197	0.185	0.092	0.113
Spain	0.016	0.013	0.020	0.011
Italy	0.234	0.600	0.247	0.332
With Structural Break				
France	0.075	0.037	0.162	0.188
Germany	0.398	0.226	0.075	0.039
Spain	0.020	0.019	0.026	0.015
Italy	0.113	0.296	0.018	0.052

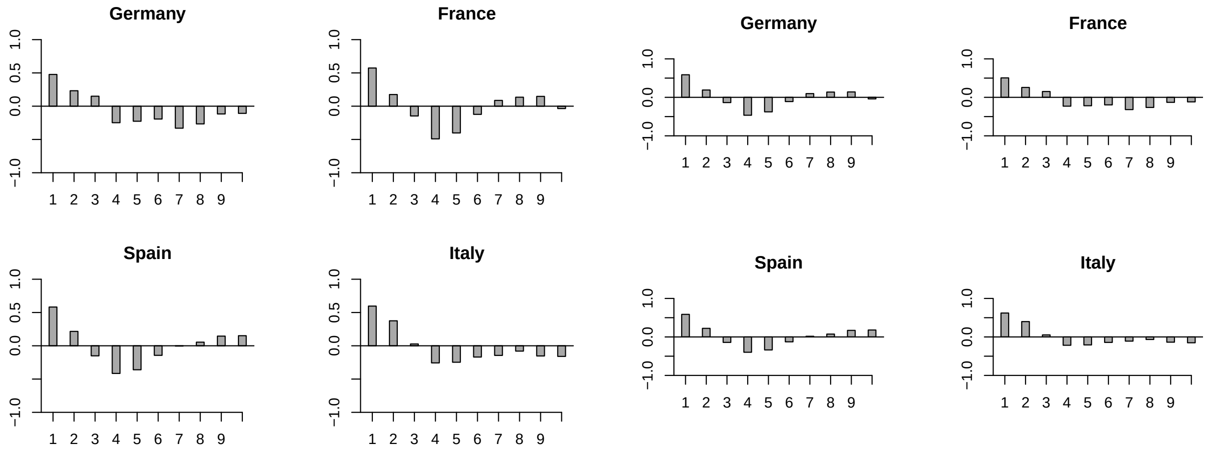
Notes: JB denotes Jarque-Bera normality test and SW – Shapiro-Wilk normality test. Using 96% confidence interval we accept the normality assumption in the error terms in all of the countries according to at least one of the tests.



(a) Histogram and Density for Closed Economy (b) Histogram and Density for Open Economy



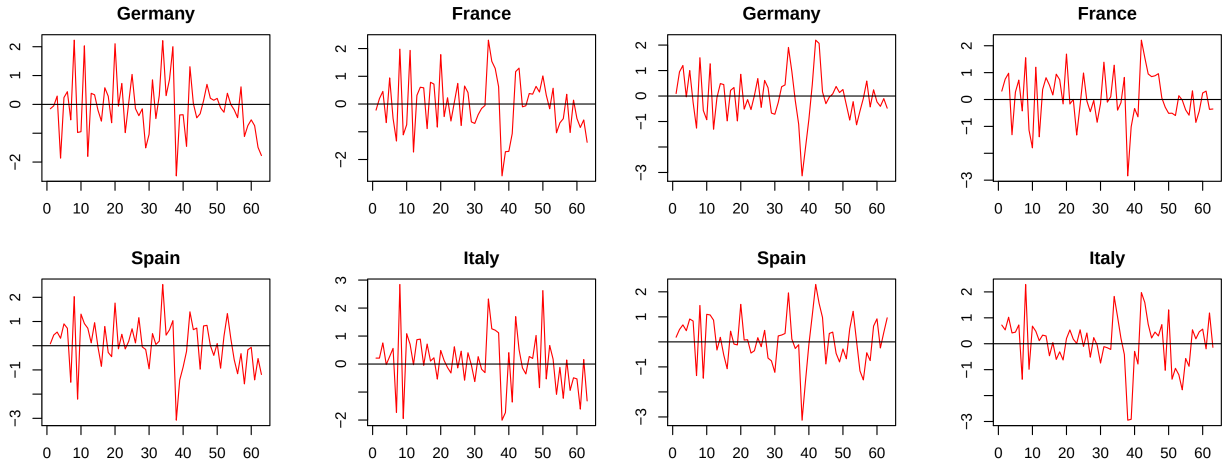
(c) QQ Plot for Closed Economy (d) QQ Plot for Open Economy



(e) Correlogram for Closed Economy (f) Correlogram for Open Economy

Figure 20: Diagnostic Plots for Standardized Prediction Errors under Adaptive Expectations

Weak Trend-following Expectations



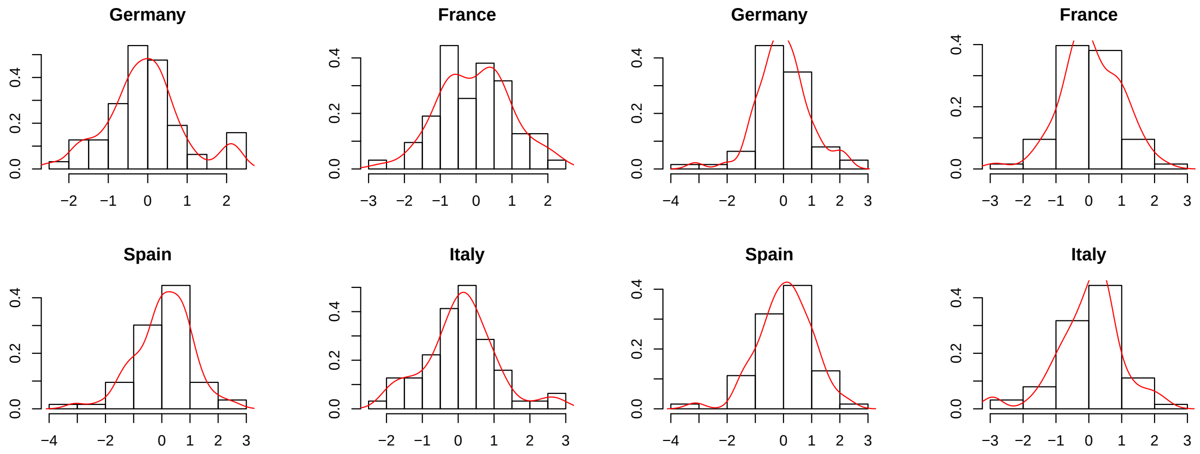
(a) Std. Errors for Closed Economy (b) Std. Errors for Open Economy

Figure 21: Standardized one-step ahead Forecast Errors under WTF Expectations

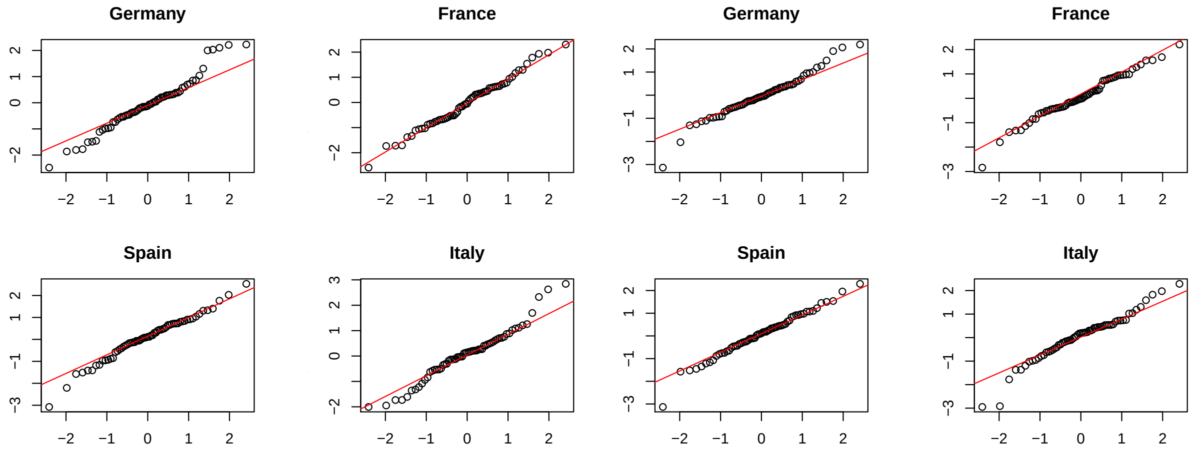
Table 6: Normality Test Statistics

	Closed Economy		Open Economy	
	JB p-value	SW p-value	JB p-value	SW p-value
Without Structural Break				
France	0.968	0.901	0.337	0.496
Germany	0.531	0.092	0.039	0.128
Spain	0.140	0.399	0.118	0.534
Italy	0.393	0.101	0.007	0.028
With Structural Break				
France	0.681	0.611	0.044	0.166
Germany	0.213	0.179	0.159	0.503
Spain	0.695	0.646	0.050	0.116
Italy	0.876	0.348	0.286	0.100

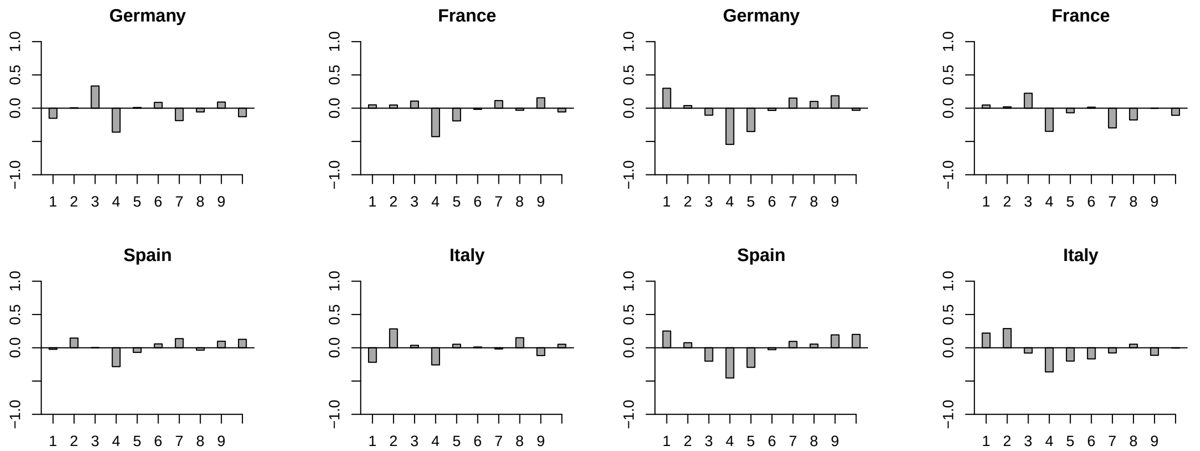
Notes: JB denotes Jarque-Bera normality test and SW – Shapiro-Wilk normality test. Using 99% confidence interval we accept the normality assumption in the error terms in all of the countries according to at least one of the tests.



(a) Histogram and Density for Closed Economy (b) Histogram and Density for Open Economy



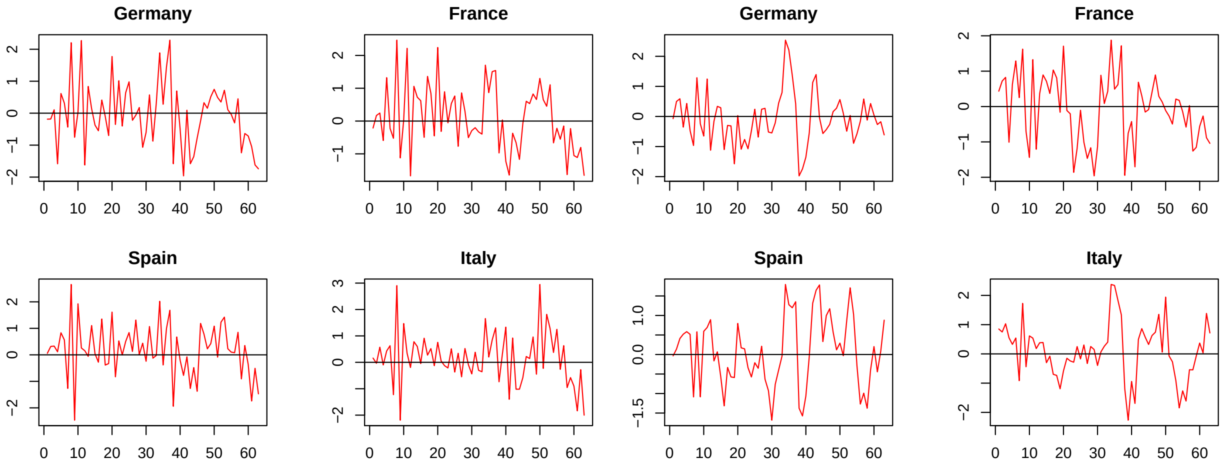
(c) QQ Plot for Closed Economy (d) QQ Plot for Open Economy



(e) Correlogram for Closed Economy (f) Correlogram for Open Economy

Figure 22: Diagnostic Plots for Standardized Prediction Errors under WTF Expectations

Learning, Anchoring and Adjustment Expectations



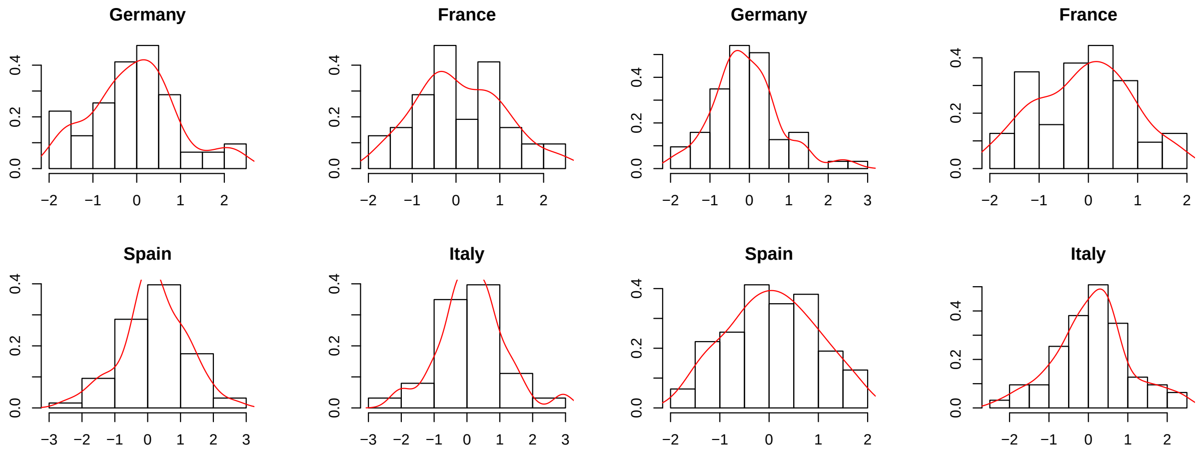
(a) Std. Errors for Closed Economy (b) Std. Errors for Open Economy

Figure 23: Standardized one-step ahead Forecast Errors under LAA Expectations

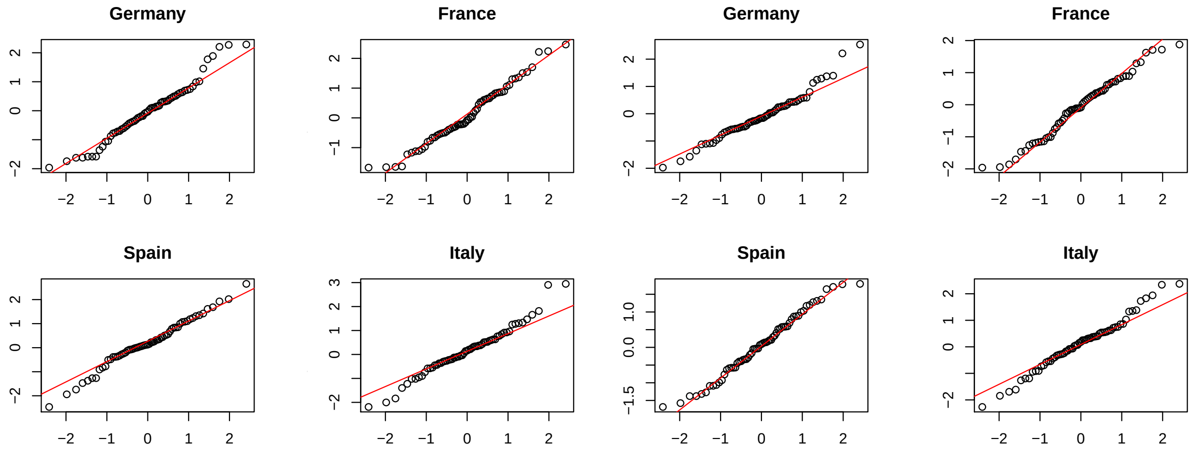
Table 7: Normality Test Statistics

	Closed Economy		Open Economy	
	JB p-value	SW p-value	JB p-value	SW p-value
Without Structural Break				
France	0.512	0.318	0.565	0.417
Germany	0.609	0.153	0.056	0.132
Spain	0.693	0.732	0.517	0.509
Italy	0.216	0.249	0.933	0.651
With Structural Break				
France	0.902	0.839	0.901	0.579
Germany	0.657	0.576	0.358	0.171
Spain	0.571	0.747	0.491	0.172
Italy	0.564	0.918	0.430	0.325

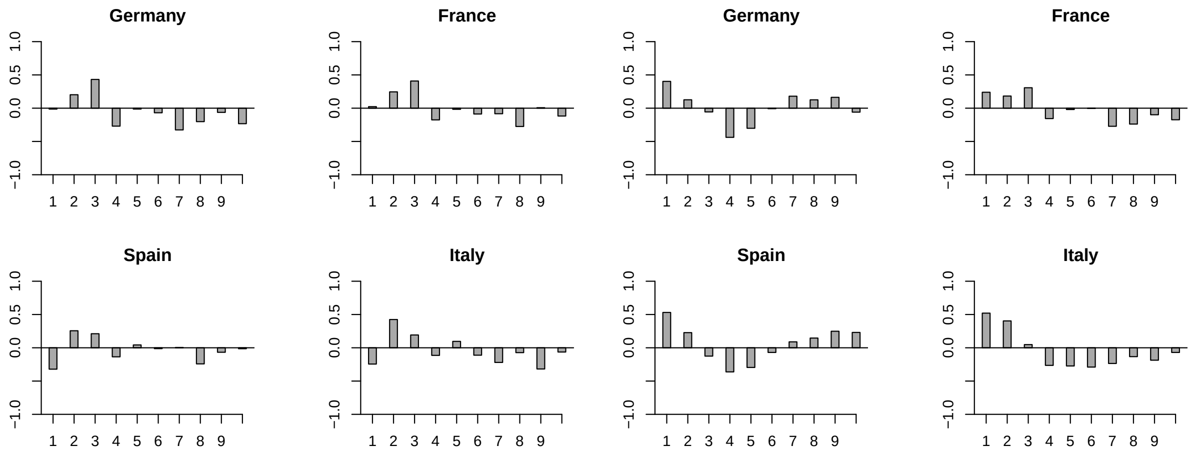
Notes: JB denotes Jarque-Bera normality test and SW – Shapiro-Wilk normality test. Using 95% confidence interval we accept the normality assumption in the error terms in all of the countries according to at least one of the tests.



(a) Histogram and Density for Closed Economy (b) Histogram and Density for Open Economy



(c) QQ Plot for Closed Economy (d) QQ Plot for Open Economy



(e) Correlogram for Closed Economy (f) Correlogram for Open Economy

Figure 24: Diagnostic Plots for Standardized Prediction Errors under LAA Expectations