

Coulomb Force Effects in Few-Nucleon Systems

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Abstract Theoretical predictions for sample observables of three-nucleon and four-nucleon reactions are reviewed. The focus is on Coulomb force effects. The calculations are based on the Alt-Grassberger-Sandhas version of the Faddeev equations. The calculations are done in momentum space. The calculational technique used to include the Coulomb repulsion between protons screens the infinite Coulomb tail, renormalizes the results and thereby corrects them for screening. The competition between three-nucleon force and Coulomb force effects as well as the Coulomb domination in special kinematic situations of reactions are discussed. Reactions connected by charge symmetry are reviewed. Special reaction observables are studied, in search for the hadronic violation of charge symmetry in the nuclear interaction and for its competition with the charge-asymmetric Coulomb force.

Keywords Three-nucleon scattering · four-nucleon reactions

1 Is an Accurate Treatment of the Coulomb Interaction between Protons Physically Necessary for a Reliable Description of Few-Nucleon Systems?

We study few-nucleon systems, in order to learn about the nuclear force. Its two-nucleon part is unknown in some details, whereas its three-nucleon part is unknown to a larger extent. Protons (p) also interact via the Coulomb force, but it is much weaker than the nuclear force. Thus, one could conjecture

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that Coulomb effects in few-nucleon reactions would be buried in the error bars of the experimental data, except near the forward direction in elastic scattering. However, this is not true. The Coulomb force becomes competitive with the nuclear one at small relative momentum transfers, given the long-range nature of the Coulomb force; this is the case at low scattering energies for most observables and for breakup cross sections in particular kinematic final-state situations at all reaction energies.

When the Coulomb force becomes competitive with the nuclear one, it may create special cross-section features, not accountable for by the nuclear force alone. It is then necessary to describe the Coulomb effects technically well, in order not to be misled to the false conclusion that an interesting feature of novel physics is showing up. Another particular Coulomb situation arises in charge-symmetric reactions, e.g., in the comparison of pp versus $n(\text{neutron})n$, $pd(\text{deuteron})$ versus nd , $p^3\text{He}$ versus $n^3\text{H}$, and $p^3\text{H}$ versus $n^3\text{He}$ reactions. In such a comparison, even small Coulomb effects are important for a reliable determination of the charge symmetry or its violation in the nuclear force; the masking of the nuclear force by the Coulomb force has to be "subtracted" theoretically from the data.

Thus, an accurate treatment of the Coulomb force in few-nucleon scattering is a prerequisite for the full study of the nuclear physics offered by few-nucleon systems, especially three- and four-nucleon reactions. This is the particular focus on few-nucleon scattering in this contribution.

2 Theoretical Treatment of the Coulomb Interaction between Protons by Screening and Renormalization

Given the long-range nature of the Coulomb force, a description of nuclear reactions with the inclusion of Coulomb appears most naturally done in the configuration space, where the differential Schrödinger equation for the scattering wave function with appropriate Coulomb boundary conditions is solved [1]. In contrast, the integral-equation approach, put on a firm mathematical basis by Faddeev [2], is usually employed in momentum space. That choice is most convenient for modern realistic short-range potentials with strong non-localities, but it is not tailored to the inclusion of a local $1/r$ potential such as Coulomb. For this reason the correct inclusion of Coulomb in momentum-space calculations only emerged several decades later [3]. In view of the Coulomb treatment, a configuration-space form of the Faddeev equations [4] was also developed and has come into successful use [5].

In the description of nuclear reactions without Coulomb we were used to work with the Alt-Grassberger-Sandhas (AGS) version of the integral Faddeev equations [6]. Therefore, we also opted for the AGS equations when Coulomb was added. Alt and Sandhas [7] had pioneered that approach, whose practicability and reliability we were able to improve significantly. This employed Coulomb treatment proceeds by screening the infinite tail of the Coulomb force first, thereby reducing the Coulomb potential to a finite-ranged poten-

tial that is tractable by standard scattering theory. Compared to Ref.[7] we changed the form of the screening by keeping the Coulomb potential almost untouched over the range of the nuclear force and making the screening rapid, but smooth outside that range; thereby screening becomes technically more effective, leading to a fast convergence with the screening radius. Another very important feature of our method is the ability to include high partial waves, necessary due to the longer range of the screened Coulomb potential. The screened results are then corrected by renormalization. Our procedure is described in detail in Ref.[3] together with our checks on the reliability of the chosen method. The procedure turns out to work well for elastic scattering and for breakup reactions with no more than two charged clusters. However, the procedure converges with the screening slowly when approaching thresholds, i.e., zero kinetic energies, becoming impractical in those cases.

Nature also screens the Coulomb force, and that screening takes place at atomic distances. In contrast, the screening of the Coulomb tail, required for technical reasons in our Coulomb treatment, has to set in at much smaller distances. Our screening therefore yields an unorthodox asymptotic wave function behavior, which has to be corrected for the unscreened Coulomb tail. However, one has to keep in mind that the configuration-space procedure [1; 5] calculates the scattering wave function in a finite domain only, i.e., in a screened cut-off configuration-space regime, much like the screening of our approach; the physically proper asymptotic behaviour has to be imposed on the wave function calculated in configuration space, i.e., it has to be added to the solution in the next step, comparable to the renormalization step in the momentum-space approach.

3 Examples for Coulomb Effects in Few-Nucleon Reactions

Few-nucleons systems are particularly suited to test nuclear force models, the need for a three-nucleon force and its detailed form. Although the relative strength of two- and three-nucleon forces are intertwined and dependent of the chosen theoretical formulation, the best examples for the need of a three-nucleon contribution to the nuclear force are the binding energies of the three- and four-nucleon bound states where the known realistic, standard two-nucleon forces underbind ${}^3\text{H}$ by about 0.5 to 1 MeV, ${}^4\text{He}$ by about 2 to 4 MeV.

In few-nucleon reactions in contrast, the evidence for an important contribution of the three-nucleon force is scarce. A good example for a substantial three-nucleon force contribution is elastic pd scattering at large proton energies and scattering angles larger than 40° . That part of the cross section is reliably described by the nuclear force alone without the inclusion of Coulomb into the calculation, and the three-nucleon force contribution is sizable. In Fig.1 we show sample results for 250 MeV proton energy: The Coulomb effect is tiny, compared to the three-nucleon force effect, and it is completely buried in the error bars of the measurement, except for a narrow part in the forward direction, i.e., at small relative momentum transfers. The theoretical

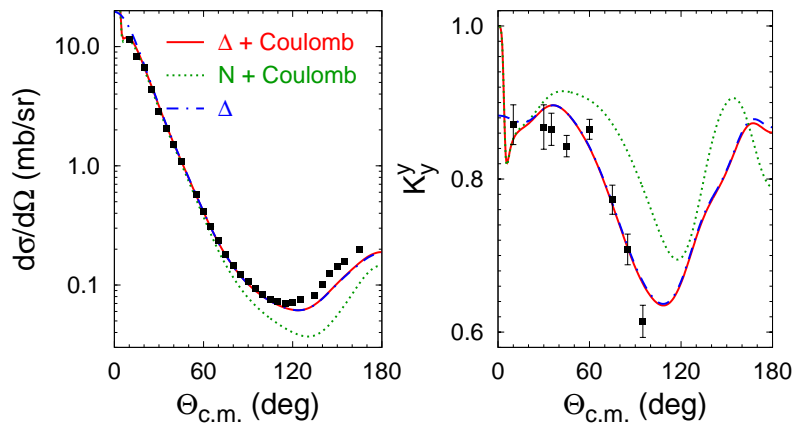


Fig. 1 (Color online) Differential cross section and nucleon-to-nucleon spin-transfer coefficient K_y^y of elastic pd scattering at 250 MeV proton lab energy as function of the c.m. scattering angle $\Theta_{\text{c.m.}}$. The theoretical predictions are based on the CD Bonn + Δ potential: The curve coded Δ + Coulomb uses the two- and effective three-nucleon parts of the nuclear potential and Coulomb, the one coded Δ leaves out Coulomb, and the one coded N + Coulomb uses the purely nucleonic CD Bonn potential. The experimental data are from Ref. [10].

description of Fig.1 is derived from the two-nucleon charge-dependent Bonn (CD Bonn) potential [8]; a consistent effective three-nucleon force contribution is added to the potential by the coupling of two-nucleon channels to channels in which one nucleon is excited to a Δ -isobar [9]; this potential will be referred to in the following as CD Bonn + Δ .

In contrast, the *Coulomb effect* can be *substantial at low scattering energies*. An example is shown in Fig. 2 for four-nucleon scattering, i.e., for elastic $p^3\text{He}$ scattering. The nuclear force, employed for the theoretical description, is CD Bonn + Δ . The three-nucleon force effect is considerably smaller than the Coulomb effect.

Furthermore, *in particular kinematic final-state situations* of breakup, the Coulomb effect can be quite massive even at higher energies. Fig.3 shows such a breakup situation in pd scattering. The nuclear force employed for the theoretical description is the CD Bonn + Δ potential; the nuclear force alone fails to describe the data. The Coulomb effect is substantial, and its inclusion is able to make the full theoretical description account well for the experimental data. Again, the three-nucleon force effect is considerably smaller than the Coulomb effect.

We conclude from the examples of Figs.1-3 that both generally small forces, i.e., the three-nucleon force and the Coulomb force have quite different realms in reactions in which they become separately important; that fact is due to their difference in range, the three-nucleon force being of especially short range, the Coulomb force being of extreme long range.

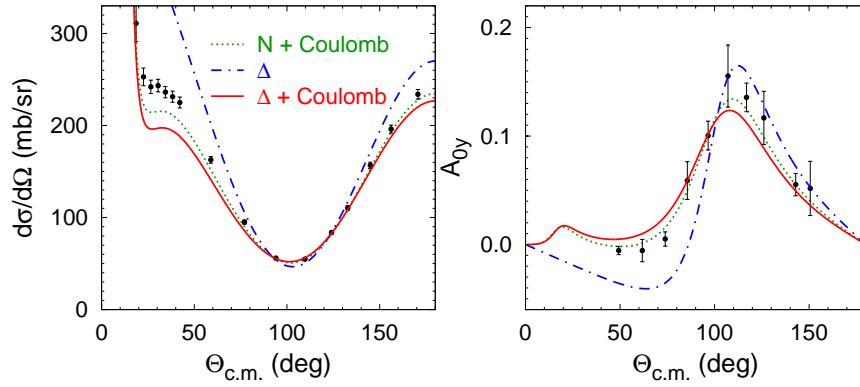


Fig. 2 (Color online) Differential cross section and target analyzing power A_{0y} of elastic $p^3\text{He}$ scattering at 5.54 MeV proton lab energy as functions of the c.m. scattering angle $\Theta_{\text{c.m.}}$. The theoretical predictions are done as for Fig.1 and coded as there. The data are from Ref. [11] for the differential cross section and from Ref. [12] for A_{0y} .

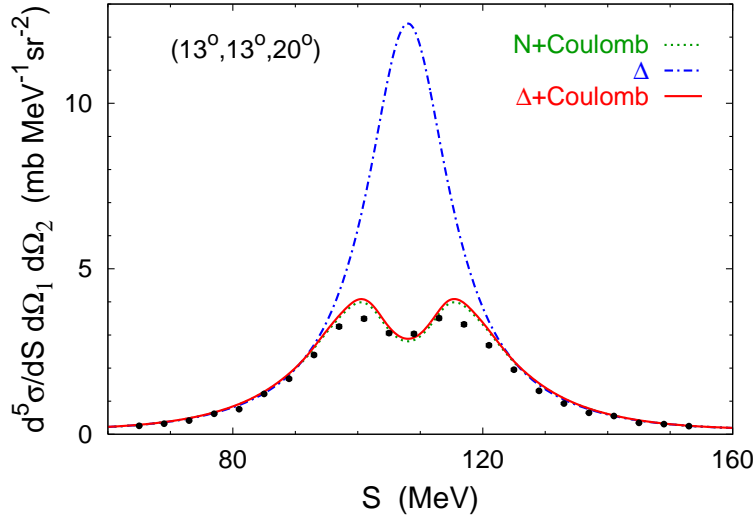


Fig. 3 (Color online) Differential cross section for pd breakup at 130 MeV deuteron lab energy as function of the arclength S along the kinematical curve. The theoretical predictions are done as for Fig.1 and coded as there. The experimental data are from Ref. [13].

Few-nucleon systems offer a wealth of *reactions connected by charge symmetry*. However, in reactions involving two or more protons the Coulomb interaction is also present and has to be theoretically taken into account, leading to a non-hadronic charge-symmetry violation. In order to reveal the hadronic charge symmetry underlying the reactions, the Coulomb effects have to be "subtracted" theoretically from the experimental data. Two well-known observables, in which both Coulomb and hadronic charge asymmetry are simul-

taneously important, are the two-nucleon 1S_0 scattering length [14] and the ^3H and ^3He binding energy difference [15].

In the following, we give four examples in few-nucleon reactions. In the display of experimental data and theoretical predictions we shall always show both reactions, connected by charge symmetry, in the same figure, in order to make their agreement or disagreement most transparent. The shown theoretical predictions are always for the full two- and three-nucleon forces, together with Coulomb when two protons are involved; the detailed contributions, arising from the three-nucleon force and from the Coulomb force, are given in the written description of the results.

First, the inability of nuclear theory with the standard and accepted nuclear potentials to account for the nucleon asymmetry around 10 MeV nucleon energy in elastic pd and nd scattering, is a long-standing puzzle. In Fig.4 we show that the theoretical predictions for pd and nd miss the peak considerably. The theoretical description of Fig.4 is derived from the CD Bonn + Δ potential, as used for Figs.1-3. In the comparison of pd and nd calculations the hadronic charge asymmetry built into the CD Bonn + Δ potential is taken into account, together with the hadronic charge asymmetry created in the three-nucleon force. The three-nucleon force effect is not shown in the figure, but it is quite small; in contrast the Coulomb effect in the pd prediction is sizable. Looking at the theoretical failures mildly, the miss of the experimental peaks in pd and nd are comparable. If that miss is due to an overall charge-symmetric miss by the nuclear force, then the puzzle of Fig.4 does not indicate a significant hadronic charge asymmetry, in addition to the one, already built into the employed nuclear potential.

Second, there are corresponding data for pd and nd breakup in the space-star configuration, as shown in Fig.5 for 13 MeV nucleon energy. The data are significantly different between pd and nd, indicating a strong violation of charge symmetry. The theoretical description of Fig.5 is derived from CD Bonn + Δ potential, as used for Figs.1 - 3; in the comparison of pd and nd calculations, the hadronic charge asymmetry, built into the CD Bonn + Δ potential, is taken into account. The three-nucleon force and the Coulomb-force contributions to the shown observables are both small, compared to the discrepancy between experimental data and theoretical prediction. In fact, this observable is dominated by two-nucleon S waves and is very insensitive to the nuclear Hamiltonian [18]. The experimental data suggest, together with the calculated small Coulomb effect, that the violation of charge symmetry in the data is big and has its origin in the nuclear force. That conclusion appears unescapable, assuming that the existing experimental data are correct. Nevertheless, such a large hadronic charge asymmetry is conceptually hard to understand and also experimentally hard to accept, since it does not show up with a corresponding strength in other observables of few-nucleon systems.

Third, there is also a multitude of four-nucleon reactions connected by charge symmetry as reviewed in Ref. [22]. Fig. 6 shows differential cross sections for the two sample reactions $D(d, p)^3\text{H}$ and $D(d, n)^3\text{He}$. In contrast to the use of the CD Bonn + Δ potential till now, the theoretical predic-

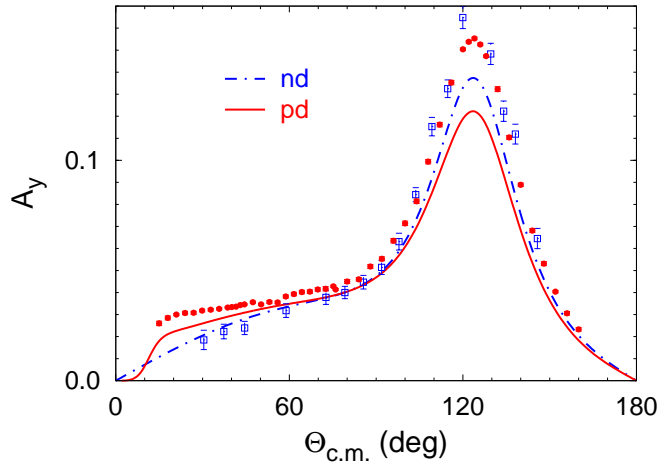


Fig. 4 (Color online) Nucleon analyzing power A_y for elastic nucleon-deuteron scattering at 10 MeV nucleon lab energy as function of the c.m. scattering angle $\Theta_{\text{c.m.}}$. Results for pd (solid curve) and nd (dashed-dotted curve) scattering using the CD Bonn + Δ potential are compared with the experimental pd data (bullets) from Ref. [16] and the experimental nd data (squares) from Ref. [17].

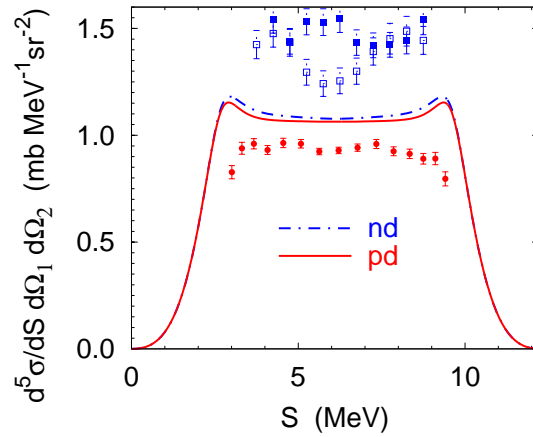


Fig. 5 (Color online) Differential cross section for nucleon-deuteron breakup at 13 MeV nucleon lab energy in the space-star configuration as function of the arclength S along the kinematical curve. The theoretical predictions for pd (solid curve) and nd (dashed-dotted curve) are derived from the CD Bonn + Δ potential. The experimental nd data (open and full squares) are from Refs. [19; 20], respectively; the pd data (bullets) are from Ref. [21].

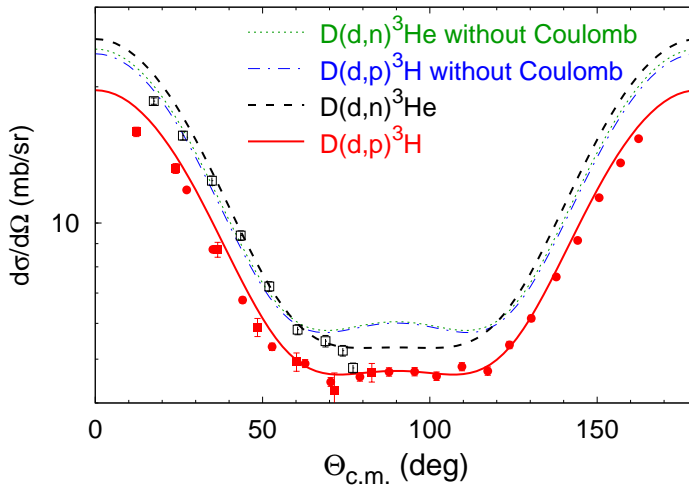


Fig. 6 (Color online) Differential cross section of $D(d,p)^3\text{H}$ (solid curve) and $D(d,n)^3\text{He}$ (dashed curve) reactions at 1.5 MeV deuteron lab energy as function of the nucleon c.m. scattering angle $\Theta_{\text{c.m.}}$. The theoretical predictions are derived from the INOY04 potential. The respective results without Coulomb are given by thin dotted and dashed-dotted curves. The $D(d,p)^3\text{H}$ data are from Refs. [25] (full squares) and [24] (bullets), the $D(d,n)^3\text{He}$ data are from Ref. [25] (open squares).

tions are based on the two-nucleon potential INOY04 [23] where a hadronic charge asymmetry is also built in. The advantage of the employed potential INOY04 is the fact that its phenomenological nonlocal part is adjusted to the binding energies of ^3H and ^3He , without an additional three-nucleon force. For the reactions of Fig.6, the effect of the Coulomb interaction is always included in the full calculations and is significant, whereas the effect of the built-in hadronic charge asymmetry, seen as the difference between the $D(d,p)^3\text{H}$ and $D(d,n)^3\text{He}$ results without Coulomb, is small. The asymmetry between $D(d,p)^3\text{H}$ and $D(d,n)^3\text{He}$ is present also in spin observables and at higher energies. Figure 7 shows the deuteron tensor analyzing power T_{20} at 10 MeV that is well reproduced by our calculations including the Coulomb force. In conclusion, in the considered four-nucleon reaction, the assumption of a small hadronic charge asymmetry in the nuclear interaction works convincingly, what is a dramatic difference relative to the charge asymmetry suspected for the three-nucleon breakup data of Fig.5.

4 Conclusions

The seminal work by Faddeev [2] gave impetus to the theoretical description of few-nucleon reactions by many researchers [28; 29]; his achievement made the field a playground for the microscopic theory of nuclear phenomena, made it conceptually attractive and created a continuous flow of calculations for the

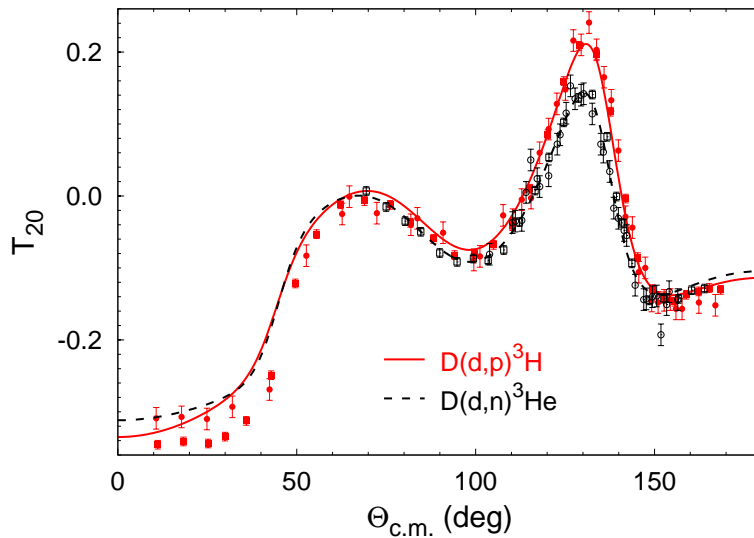


Fig. 7 (Color online) Deuteron tensor analyzing power T_{20} in $D(d,p)^3\text{H}$ (solid curve) and $D(d,n)^3\text{He}$ (dashed curve) reactions at 10 MeV deuteron lab energy as function of the nucleon c.m. scattering angle $\Theta_{\text{c.m.}}$. The theoretical predictions are derived from the INOY04 potential. The $D(d,p)^3\text{H}$ data are from Refs. [26] (bullets) and [24] (full squares), the $D(d,n)^3\text{He}$ data are from Refs. [26] (open circles) Ref. [27] (open squares), respectively.

few-nucleon systems. Nevertheless, theoretical achievements trailed behind experimental measurements of few-nucleon reactions for many decades, making reliable comparisons between theory and experiment difficult. An early exception were the predictions for nd elastic scattering and breakup, for which momentum-space calculations by the Bochum-Cracow collaboration [18] have existed since the late 1980's. Subsequent work by the Pisa group made a substantial contribution to the theoretical understanding of elastic pd reactions [1] and of low-energy $p^3\text{He}$ and $n^3\text{H}$ four-nucleon reactions [30]. Finally, it was the Coulomb treatment by us, using the AGS version of the Faddeev equations and following the foot steps of Alt and Sandhas. Our Coulomb treatment led to a decisive progress towards a complete theoretical description of three- [3] and four-nucleon [22] reactions, both below and above the breakup thresholds. Since the Coulomb force masks the underlying nuclear force, its reliable treatment is important for understanding a number of interesting physics situations. As we believe, the work, recalled in the present note, made a valuable contribution to the ongoing study of Coulomb effects in few-nucleon systems. With respect to computations: Whereas benchmark comparisons with other Coulomb techniques were done with colleagues for elastic scattering, the theoretical description of breakup channels and transfer reactions is, till now, unique of the present approach; benchmark comparisons for those experimental situations are clearly needed. Unfortunately, the present Coulomb treatment becomes impracticable at very low kinetic energies (< 1 MeV), namely

at threshold. This fact constitutes a drawback that hopefully will be removed, especially, since threshold physics may offer further interesting information on the nuclear force.

The main focus of few-nucleon systems has been the study of the three-nucleon force, its need and its detailed form. The original goal of a conclusive study of the three-nucleon force has not been reached yet, since three-nucleon force effects are rather scarce in the wide field of few-nucleon reactions scanned till now. Thus, instead of a quasi experimental study of the three-nucleon force, the attention has to be directed more towards its conceptual derivation and the subsequent test of the full nuclear force with all of its two-nucleon and many-nucleon components. We based most of our calculations recalled in this note - the only exception being those for the four-nucleon reactions of Figs. 6 and 7 - on the meson-theoretic CD Bonn + Δ potential; it has a two-nucleon force part and yields consistent effective three-nucleon and many-nucleon force contributions due to the coupling of the two-nucleon channels to channels in which one nucleon is turned into a Δ -isobar. This consistency between two-nucleon and many-nucleon forces is guaranteed and was the early advantage of this nuclear force model. Furthermore, the Δ -mechanism is considered, quantitatively, the important one for the three-nucleon force; the coupled-channel approach offers the additional possibility of going beyond pion production by further coupling the Δ -isobar to active one-pion channels [31]. However, a meson-theoretic approach is not considered any more conceptually satisfactory; modern nuclear interaction models are derived from effective field theories as two-nucleon potentials, usually together with consistent three- and many-nucleon potentials [32; 33]. This is the modern conceptual standard, and our successful inclusion of Coulomb into the theoretical description should be extended to those modern potentials. We are quite confident that the results presented in this note will remain qualitatively valid in such a recalculation, which is highly welcome and needed.

There are two important challenges ahead in few-nucleon physics:

On the experimental side: Most accelerators with p and n beams below 50 MeV got discontinued. Thus, it will be hardly possible to remeasure existing questionable data or study still missing observables, especially spin observables of three-nucleon and four-nucleon reactions. This is the *Achilles Heal* of few-nucleon physics, a major disadvantage for the future of the field.

On the conceptual side: We hope that effective field theories will eventually also provide the building blocks for an advanced coupled-channel potential, yielding effective Δ -mediated and simultaneously irreducible non- Δ -mediated many-nucleon potentials. Such a potential would merge the advantages of the major present-day approaches to the nuclear force. We do not see any reason why also such a conceptually improved approach to the nuclear force could not be combined with our Coulomb treatment for a further study of Coulomb force effects in few-nucleon systems.

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