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Testing AR(1) model*

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Abstract. In this paper we investigate a simple AR(1) model by testing a presence of changed segment in a data. We suggest test statistics based on a behavior of partial sums of residuals.

Keywords: AR(1) model, changed segment, partial sums of residuals.

1. Introduction

Structural stability of a time series is very important in applied econometrics. Estimates derived from unstable processes can be biased and forecasts lose accuracy. A considerable attention of testing the parameter constancy of time series have given Pickard [4], Lee and Park [3] and many others.

The CUSUM method has been utilized for testing a change of a mean, a variance and other parameters of regression type models, see, e.g., Kulperger [2], Bai [1]and references therein. Shin [7] established the weak limit of partial sums of residuals of AR models and investigated various tests for one change alternatives. We investigate in this paper a simple AR(1) model under changed segment type alternatives. The paper is organized as follows. Section 2 presents a model under consideration and test statistics. In Section 3 we study a behavior of test statistics under some alternatives. In section 4 some simulation results are presented.

2. Model and test statistic

In this paper we consider a simple AR(1) model:

$$y_k = \rho y_{k-1} + a_k + e_k, \quad k = 1,\ 2,\ \ldots,\ n,\ y_0 = 0, \tag{1}$$

where $e_1, \ldots, e_n$ are i.i.d. with mean zero and finite variance $\sigma^2 < \infty$, a sequence (a_k) will be specified later. We want to test the null hypothesis

$$H_0\colon\ a_k = 0 \quad \text{for all} \quad k = 1, .., n$$

against various type alternatives.

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Let $(\hat{e}_k, k = 1, \dots, n)$ denote the residuals of model (1) under the null hypothesis. Set $\hat{S}_0 = 0$, $\hat{S}_k = \hat{e}_1 + \cdots + \hat{e}_k$, $k = 1, \dots, n$. Define the test statistics

$$T(n;\alpha) = \max_{1<l<n} \frac{1}{l^\alpha} \max_{0\leqslant k\leqslant n-l} \left|\hat{S}(k+l) - \hat{S}(k) - \frac{l}{n}\hat{S}(n)\right|,$$

where $0 \leqslant \alpha < 1/2$. In Račkauskas and Rastenė [5] limiting distributions for normalized statistics $n^{-1/2+\alpha}\sigma^{-1}T(n;\alpha)$ are established under the null hypothesis.

3. Behavior of test statistics under alternatives

Consider model (1) where $|\rho| \neq 1$. Then $T(n;\alpha)$ can be estimated $T(n;\alpha) \geqslant T_1(n;\alpha) - T_2(n;\alpha)$, where

$$T_1(n;\alpha) = \max_{1<l<n} \frac{1}{l^\alpha} \max_{0\leqslant k\leqslant n-l} \left|\frac{1-\hat{\rho}}{1-\rho} \sum_{i=k}^{k+l} (a_i - \bar{a})\right|,$$

$$T_2(n;\alpha) = \max_{1<l<n} \frac{1}{l^\alpha} \max_{0\leqslant k\leqslant n-l} \left|\frac{1-\hat{\rho}}{1-\rho} \sum_{i=k}^{k+l} (e_i - \bar{e}) - \frac{\rho-\hat{\rho}}{1-\rho}\left(y_{k+l} - y_k - \frac{l}{n} y_n\right)\right|,$$

$\hat{\rho}$ denotes an estimate of ρ under null, $\bar{a} = n^{-1}\sum_{k=1}^n a_k$, $\hat{e} = n^{-1}\sum_{k=1}^n \hat{e}_k$.

By Račkauskas and Rastenė [5], Račkauskas and Suquet [6] assuming the conditions

$$\frac{1}{n}\sum_{i=1}^n a_i = O_p(1), \tag{2}$$

$$\lim_{t\to\infty} tP\left(|e_1| \geqslant t^{1/2-\alpha}\right) = 0 \tag{3}$$

it follows that $n^{-1/2+\alpha}\sigma^{-1}T_2(n;\alpha) = O_p(1)$. Hence, if under an alternative hypothesis we have $n^{-1/2+\alpha}\sigma^{-1}T_1(n;\alpha) \xrightarrow[n\to\infty]{\mathrm{P}} \infty$, then statistics $T(n;\alpha)$ are proper for testing. Next we consider two examples of changed segment alternatives.

Example 1. There exist l^*, k^*, $1 < l^*, k^* < n$, such that

$$a_k = a\mathbb{I}_{k^*<k\leqslant k^*+l^*}, \quad k = 1, \dots, n.$$

where $a \in R, a \neq 0$. Moreover, we assume that $l^* \to \infty$ and $l^*/n \to 0$ as $n \to \infty$. In this case

$$T_1(n;\alpha) \geqslant \left|\frac{1-\hat{\rho}}{1-\rho} a \left(1 - \frac{l^*}{n}\right) l^{*(1-\alpha)}\right|.$$

Hence, under conditions (2) and (3), we have $n^{-1/2+\alpha}\sigma^{-1}T_1(n;\alpha) \xrightarrow[n\to\infty]{\mathrm{P}} \infty$ provided $\sigma^{-1}|a| l^{*(1-\alpha)} n^{-1/2+\alpha} \to \infty$ as $n \to \infty$.

Example 2. There exists l^*, k^*, $1 < l^*, k^* < n$, such that

$$a_k = (1-\rho) y_{k-1}\mathbb{I}_{k^*<k\leqslant k^*+l^*}.$$

Under this alternative, model (1) takes the form

$$y_k = \begin{cases} 0, & if\ k = 0 \\ \rho y_{k-1} + e_k, & if\ 1 \leqslant k \leqslant k^*,\ k^* + l^* < k \leqslant n, \\ y_{k-1} + e_k, & if\ k^* < k \leqslant k^* + l^*, \end{cases}$$

i.e., there exists a segment where AR(1) process is non-stationary. This example is investigated by simulations.

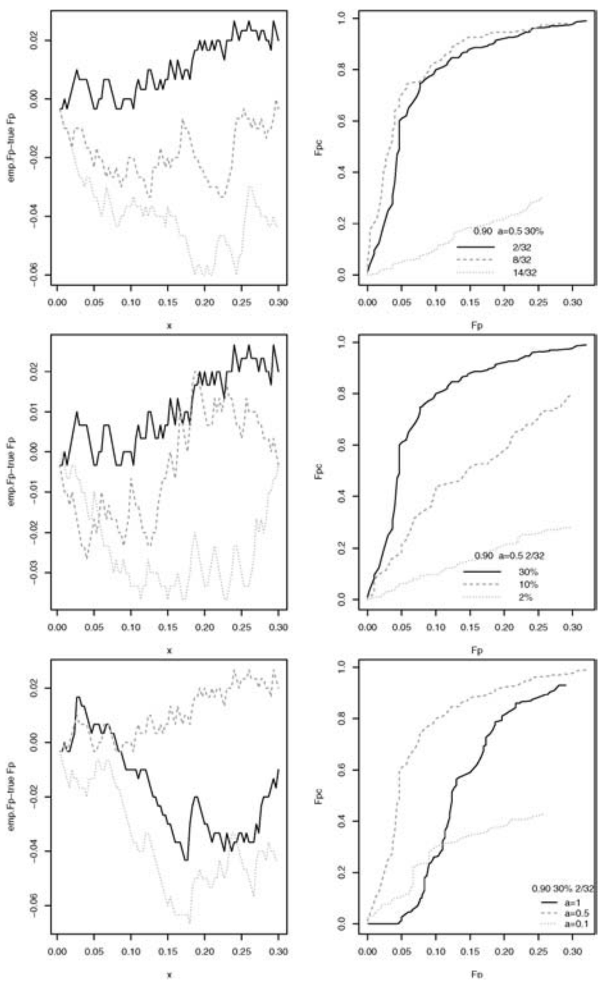

Fig. 1. Example 1.

4. Simulations

For different values of l^*, a, α, ρ we have computed 300 realizations of the test statistics, where n is equal to 1000. Residuals were generated from the standard normal distribution. For the p-values analysis we use p-values discrepancy plots. We compare the empirical distribution function for p-values with the distribution function of the true p-values. A difference between the empirical and the true distribution functions is set on y-axis and an argument of the distribution function on x-axis. For a power analysis we have presented size-power curves. On the x-axis we have set values of empirical p-values distribution function under the null hypothesis whereas on the y-

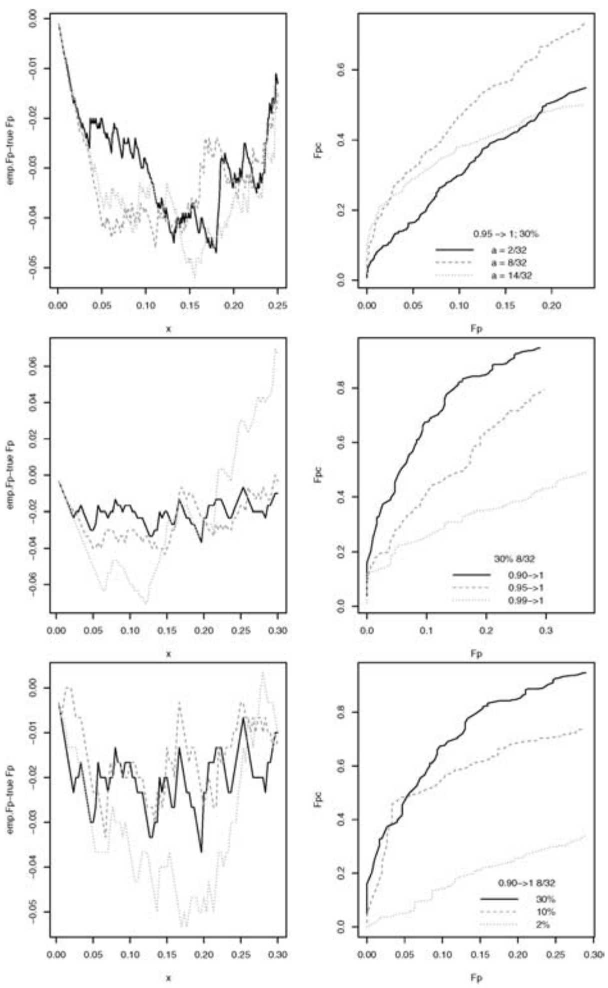

Fig. 2. Example 2.

axis values of empirical p-value distribution function under the alternative (empirical power function).

Example 1. From the Fig. 3 we see that almost in all cases the test is a bit conservative (in average accept the null hypothesis too often) except when $\alpha = 2/32$, $a = 0.5$ and a change segment is equal 30%. The right column of plots shows that the power increases when a changed segment and constant a increases and α is closer to 0 rather than to $1/2$.

Example 2. From the Fig. 3 we see that the test accept the null hypothesis too often. The size-power curves show that the test power decreases when ρ tends to 1 and change segment decreases. Best results gives $\alpha = 1/4$.

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REZIUMĖ

I. Rastenė. AR(1) modelio testavimas

Darbe nagrinėjamas AR(1) modelio galimas stebėjimų segmento pasikeitimas. Pasiūlyta testinė statistika, paremta modelio liekanų dalinių sumų elgesiu.